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Abstract

A regime dependent VAR model is suggested that allows long memory (fractional integration) in each of the regime states as well as the possibility of fractional cointegration. The model is relevant in describing the price dynamics of electricity prices where the transmission of power is subject to occasional congestion periods. For a system of bilateral prices non-congestion means that electricity prices are identical whereas congestion makes prices depart. Hence, the joint price dynamics implies switching between essentially a univariate price process under non-congestion and a bivariate price process under congestion. At the same time it is an empirical regularity that electricity prices tend to show a high degree of fractional integration, and thus that prices may be fractionally cointegrated. An empirical analysis using Nord Pool data shows that even though the prices strongly co-move under non-congestion, the prices are not, in general, fractional cointegrated in the congestion state.

Keywords: Cointegration, electricity prices, fractional integration, long memory, Markov switching.

JEL Classification: C32.

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1 Introduction

Over the past decade or so electricity markets have been strongly liberalized throughout the world. In particular, the Nordic power market consisting of Norway, Sweden, Finland, and Denmark has developed remarkably towards liberalization and the establishment of competitive market conditions, and today this market serves as a model for the restructuring of other power markets. The Nordic power market is characterized by a grid of physical exchanges of power across geographical regions where the actual exchange is constrained by the flow capacity. Naturally, this has implications for the way prices are formed: When there are no bilateral capacity restrictions then there is a free flow of power, and prices will be identical. On the other hand, when there is congestion prices tend to depart to meet the supply and demand conditions subject to restricted access to power from other regions. In order to model electricity prices it is thus natural to consider regime dependent price processes reflecting the presence or absence of flow congestion. This particular feature of the market has been addressed in recent work by Haldrup and Nielsen (2006a,b). Another important property of electricity prices modeled in these works is the presence of long memory. Statistical tests strongly reject price series to be I(0) and I(1), whereas I(d) processes with d being fractional (see Granger (1980), Granger and Joyeux (1980) and Hosking (1981)) provide a nice characterization of the data.

The combination of fractional integration and regime switching gives rise to some challenges. Ding and Granger (1996), Diebold and Inoue (2001), and Granger and Hyung (2004) argue that under certain conditions time series variables can spuriously have long memory when measured in terms of their fractional order of integration, when in fact the series exhibit non-linear features such as regime switching. In the model framework of Haldrup and Nielsen (2006a,b) separate long memory price dynamics is allowed in adjacent power regions depending upon whether the power exchange is subject to congestion or non-congestion. The model is of the Markov switching type originally defined by Hamilton (1989). However, because the defining property of e.g. a non-congestion state is that prices are identical, the state variable is observable as opposed to being a latent variable. An important feature of the model is that the price processes in the different regimes can have different degrees of long memory, which gives rise to a number of interesting possibilities. For instance, consider the state with non-congestion and assume that the associated bivariate prices are fractionally integrated of a given order. It follows that prices are fractionally cointegrated in this case, i.e. extending the notion of Granger (1981, 1986) and Engle and Granger (1987), in the sense that individual prices are fractionally integrated but price differences are identically zero. Thus, an extreme form of cointegration occurs in this situation because the prices are identical and hence are governed by exactly the same price shocks. The price behavior in the congestion state can (and typically will) be very different. That is, the bivariate prices can be fractionally cointegrated in a more conventional way or the prices can appear not to cointegrate. Hence the model can potentially exhibit state dependent fractional cointegration. By not appropriately conditioning on the congestion state, i.e. when having a model with no regime switching, the full sample estimates are likely to be a convex combination of the behavior in the individual states and hence misleading inference is likely to result. In fact, this is one of the major empirical findings in Haldrup and Nielsen (2006a).

The modeling approach used in Haldrup and Nielsen (2006a) is limited in the sense that the individual price series and the relative price series are analyzed separately as univariate models. When the focus of analysis is the potential (fractional) cointegration amongst multiple series a system approach is more natural, but clearly also more complex in the present context given the particular features the model should allow. In principle, the full set of price series should be modeled jointly, and, depending upon the market conditions, should shrink to a limited number of price series reflecting periods with non-congestion at some grid points.

We distinguish between price areas and geographical regions. Each geographical region corresponds to a physical exchange (e.g., West Denmark, South Norway, etc.) and is therefore constant over time. On the other hand, a price area is defined simply as an area with the same price and may therefore clearly change over time. Thus, West Denmark and South Norway always constitute two geographical regions, but in the case of non-congestion the same price prevails in both geographical regions and they hence constitute just one price area.

In this paper we model multiple price series jointly in a vector autoregression (VAR), which allows for fractionally integrated time series that potentially cointegrate in the congestion state. In the non-congestion state, prices are identical by definition and hence a univariate model for the price process is applied in this particular regime. Thus, our VAR model for fractionally cointegrated processes allows for the possibility of regime switching, and in particular differs from other specifications offered in the literature in the sense that our VAR model collapses to a pseudo-univariate model when a specific state arises.

There are different reasons why the identification of separate price dynamics is important. The operation of electricity markets is similar to the operation of financial markets with electricity power derivatives being priced and traded in highly competitive markets and hence appropriate modeling of both means and variances is crucial. Furthermore, the price dynamics is of interest with respect to competition analysis of electricity markets where market delineation is a central issue, see e.g. Sherman (1989) and Motta (2004). Even though most power markets are highly liberalized there is still scope for regulating authorities to closely follow the market behavior, see also Fabra and Toro (2005). Under non-congestion there is obviously a single price existing in the market and the relevant geographical market consists of the regions with identical prices. However, when there is congestion it is of interest to follow the price dynamics closely because suppliers can have a dominating position. The geographical market delineation thus becomes less straightforward in this case. If the price dynamics appears to be very different there is scope for further examination of the market conditions by regulatory authorities.

In our empirical analysis we find that generally the behavior of electricity prices in geographical price regions are different across states. The analysis shows that it is important to condition on congestion/non-congestion as non-switching models can generate misleading conclusions with regard to the fractional integration orders and potential fractional cointegration. Three leading types of misclassification of the model dynamics may arise. First, non-switching models may indicate that the price series are fractionally cointegrated, whereas when conditioning on states this is only the case in the non-congestion state (which is cointegrated by definition). Secondly, the non-switching model could indicate that there is no fractional cointegration when in fact there is cointegration in the non-congestion state. Finally there is the possibility of fractional cointegration in both regimes, but not in the non-switching model. Conditioning on states is also important when looking at the adjustment coefficients, as the non-switching models can lead to wrong conclusions about the convergence of geographical price regions towards equilibrium.

The remainder of the paper is structured as follows: We next offer a brief description of the structure of the Nordic electricity market. Section 3 introduces the data and argues for the importance of allowing for long memory, regime switching and seasonality when building a model to describe the geograhical region price processes. In section 4 the VAR modeling framework with long memory and regime switching is presented. In section 5 the empirical results are discussed and section 6 concludes.

2 The operation of the Nordic power market

Within the Nordic countries (Denmark, Finland, Norway, and Sweden), major electricity reforms were implemented during the 1990s. The deregulation process started in Norway in 1991, continued in Sweden 1996, in Finland 1998, and was finally completed in Denmark in 2000. As part of the liberalization the national electricity markets were opened up for cross-border trade by establishment of a common power exchange, Nord Pool. Today all member countries of the Nordic power market have adapted to the new competitive environment and the Nordic exchange serves as a model for the restructuring of other power markets throughout the world.[1]

The per capita consumption of electricity is very high in Norway and Sweden, slightly lower in Finland and at EU average in Denmark. The relatively high consumption level in the Nordic countries is caused by a relatively electricity intensive industrial production, a cold climate, and extensive use of electric heating in homes and offices, especially in Norway and Sweden. The sources of electricity power production are rather mixed in the Nordic area as a whole. The major energy source is hydropower supplying approximately 65% of total electricity in years with normal precipitation. On the national level the power generation systems differ significantly and are generally dominated by one or two technologies. In Norway the share of hydropower is close to 100%, in Sweden it is close to 50%, in Finland around 15% and in Denmark 0%. With respect to nuclear power the share is 50% in Sweden 30% in Finland, and 0% in Denmark and Norway. Power generation from fossil fuels is of major significance in Denmark and Finland, minor in Sweden, and close to non-existent in Norway. In Denmark 15-20% of the power supply originates from wind power turbines.[2]

Because hydropower production is mainly found in the northern parts of the Nordic power web and thermal power plants are located in the south, the relatively cheap hydropower generation is transmitted to the heavily populated southern region which of course requires a well established power grid transmission capacity to facilitate the flow. When the reservoir levels are adequate, the less costly hydropower production causes low spot prices. In these cases national and cross-border transmission systems will be used to their capacity in order to level out price discrepancies across regions. On the other hand, when reservoir levels are low there will be a net flow from south to north, and the market will see relatively high prices for thermally generated electricity.

>From an institutional point of view there is a common Nordic market for electricity; however, even though key market institutions are common this does not mean that the Nordic electricity market is an integrated market in the sense that "the law of one price" applies. The reason is that the transmission of power is subject to possible capacity constraints. The Nordic electricity market constitutes a number of distinct geographical regions different

[1] For a detailed description of the Nordic power market, see Nord Pool (2003a) or Amundsen and Bergman (2007).

[2] Increasing the relative production of electricity by renewable energy sources has considerable political focus in Denmark. According to official energy plans 50% of the Danish electricity production will come from wind power in 2030.

from the countries themselves and several price areas may coexist. Whenever the relevant interconnector capacity is insufficient, the Nord Pool area is divided into two or more price areas. The separate power regions consist of Sweden (SWE), Finland (FIN), West Denmark (WDK), East Denmark (EDK), North Norway (NNO), Mid Norway (MNO), and South Norway (SNO). Thus Denmark and Norway are each divided into multiple geographical regions in Nord Pool.[3] This division reflects the grid of physical exchanges of power and the bidding areas with respect to the pricing of electricity as we shall explain shortly. Figure 1 displays the actual electricity exchange points.

Figure 1 about here

The power spot market[4] operated by Nord Pool Spot A/S is an exchange where market participants trade power contracts for physical delivery the next day. This is referred to as a day-ahead market. The spot market is based on an auction with bids for purchase and sale of power contracts of one hour duration covering the 24 hours of the following day. At the deadline for the collection of all buy and sell orders the information is gathered into aggregate supply and demand curves for each power delivery hour. From these supply and demand curves the equilibrium spot prices - referred to as the system prices - are calculated.[5] Therefore, the system price is determined under the assumption that no transmission constraint is binding, and thus in a situation where no grid congestions exist across neighboring interconnectors there will be a single identical price across the areas with no congestions.

The actual trade is not necessarily carried out at the system price. When there is insufficient transmission capacity in a sector of the grid, a grid congestion will arise and the market system will establish different price areas across the geographical division of the Nord Pool area. The Nordic market is then partitioned into separate bidding areas which therefore become separate price areas when the contractual flow between bidding areas exceeds the capacity allocated by the transmission system operators for spot contracts. Within each price area the buyers pay, and the generators are paid, the corresponding area price. The difference between the area prices in two adjacent price areas determines the congestion charge. Because separate prices may coexist depending upon regional supply and demand conditions, the relevant market definition will vary with time. In practice, several price area combinations will occur. Some hours there will only be a single price area (given by the system price), other hours there will be two or more price areas.

3 Data

The data used in this paper are (log transformed) hourly electricity spot prices for the Nord Pool area; West Denmark (WDK), East Denmark (EDK), South Norway (SNO), Sweden (SWE) and Finland (FIN).[6] The data set is the same as that analyzed in Haldrup and Nielsen (2006a,b) and covers the period 3 January 2000 to 25 October 2003, including weekends and

[3] For the purpose of analysis of the Norwegian regions, only the SNO link is considered in the present paper.

[4] Since only the spot market will be relevant for the present study, only this market will be described here, see also Nord Pool (2003b). Nord Pool (2003c) describes the futures and forward markets of the Nordic power exchange which are used for price hedging and risk management.

[5] The system price is the reference price in the financial power contracts like futures, forwards, and options traded at Nord Pool.

[6] Mid and North Norway are also member areas of Nord Pool, but are left out from the present analysis because these areas coincide with South Norway for most of the year.

holidays. This yields a total of 33,404 observations. For EDK the sample period starts 1 October 2000 and thus covers 26,880 sample points. The data series are displayed in Figure 2. Some stylized facts about the data are reported in Haldrup and Nielsen (2006a).

Figure 2 about here

A pronounced characteristic of electricity markets is the abrupt and generally unanticipated extreme changes in spot electricity prices. These jumps or spikes generally occur within a very short period of time, implying that the general level of the different series tend to be highly persistent possible with mean reversion, see Escribano et al. (2002), Haldrup and Nielsen (2006a,b) and Koopmann et al. (2007). In Haldrup and Nielsen (2006a) a range of tests document that prices are neither I(0) nor I(1). Estimating the memory parameter for fractionally integrated, FI(d), processes shows that the series generally exhibit long memory with d in the range 0.31-0.52 with the SNO area being most persistent and in fact being nonstationary. The remaining areas have estimates of d in the stationary region. It should be noted, however, that these estimates do not allow for regime dependence.

Another important aspect of electricity prices is the very strong seasonal behaviour characterizing the series. Seasonality is mainly driven from the demand side and appears as seasonal variation within the day, within the week, and over the year. However, the supply side also contributes to seasonal variation as electricity production is highly dependent upon weather conditions. In particular, the seasonal variation in precipitation affects water reservoir levels in the generation of hydropower, and seasonal variation in wind conditions also plays an increasing role due to the growing number of wind turbines, especially in West Denmark.

Figure 3 about here

In Figure 3 scatter plots of log prices for adjacent Nord Pool areas are shown. When there are no capacity contraints across neigboring regions the prices will be identical, whereas congestion makes prices differ. Observations on the 45° line therefore represent non-congestion hours, whereas observations off the 45° line represent congestion hours. It is especially this marked difference in observations that motivates the present analysis.

4 Modeling of regime dependent long memory

4.1 A univariate model

We here briefly discuss the univariate model setup used in Haldrup and Nielsen (2006a). The main features that the estimation model should allow include seasonality, long memory, and regime switching of the type described above. Assume that individual electricity prices across adjacent regions are fractionally integrated in the non-congestion state. This means that an extreme form of fractional cointegration will exist in this state because the prices are *identical* across the two areas and thus price differences will be identically zero. On the other hand, the behavior of the two individual price series in the congestion state can be very different. If prices are compared without considering the different regime possibilities it is unclear what to expect from the data. However, the mixing of the two processes is likely to produce price series with a behavior that is a convex combination of the two state processes.

Consider the following model specification, which we denote a regime switching multiplicative RS-SARFIMA[7] model:

$$A_{s_t}(L)\left(1-a_{s_t}L^{24}\right)(1-L)^{d_{s_t}}\left(y_t-\mu_{s_t}\right)=\varepsilon_{s_t,t}, \qquad \varepsilon_{s_t,t}\sim nid\left(0,\sigma^2_{s_t}\right). \tag{1}$$

Here $A_{s_t}(L)$ is a lag polynomial and $s_t \in \{c, nc\}$ denotes the regime (c :congestion, nc: non-congestion), determined by a Markov chain with transition probabilities

$$P=\begin{bmatrix} p_{11} & 1-p_{11} \\ 1-p_{22} & p_{22}\end{bmatrix}. \tag{2}$$

Thus, for example, p_{11} denotes the probability that a congestion state will follow a congestion state. Note that because identical prices mean that we are in a non-congestion state, all regimes are observable, which contrasts the standard regime switching model of Hamilton (1989) where the regimes follow a latent Markov process.

The (univariate) series y_t may denote one of two individual log price series or the associated log relative price. The series y_t has been corrected for deterministic seasonality prior to the estimation whilst allowing interaction with the two observable regimes, that is, the coefficients on the dummy variables are allowed to differ across states. When y_t denotes a log relative price, all parameters are put to zero when $s_t = nc$, including σ^2_{nc}. Estimation of the above model is by conditional maximum likelihood and is discussed in detail in Haldrup and Nielsen (2006a).

4.2 A bivariate model

A disadvantage of the model described above is that parameters are estimated separately when in fact the price series to a large extent are governed by the same price shocks. We therefore consider the following fractional error correction model specification for a bivariate regime switching vector stochastic process subject to being in the congestion state:

$$\begin{aligned}\begin{pmatrix}\Delta^{d_1} & 0\\ 0 & \Delta^{d_2}\end{pmatrix}\begin{pmatrix}p_{1t}\\ p_{2t}\end{pmatrix} &= \begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}\Delta^{\gamma}(p_{1,t-1}-p_{2,t-1}) \\ &\quad +\sum_{i=1}^{k}\Gamma_{c,i}\Delta^{\xi_{s_{t-i}}}\begin{pmatrix}p_{1,t-i}\\ p_{2,t-i}\end{pmatrix}+\boldsymbol{\varepsilon}_{c,t},\end{aligned} \tag{3}$$

where $\boldsymbol{\varepsilon}_{c,t}\sim N(0,\Omega)$ and

$$\begin{aligned}\Gamma_{c,i} &= \begin{bmatrix}\Gamma^c_{11,i} & \Gamma^c_{12,i}\\ \Gamma^c_{21,i} & \Gamma^c_{22,i}\end{bmatrix},\\ \Delta^{\xi_{s_{t-i}}} &= \begin{cases} diag\left(\Delta^{d_1},\Delta^{d_2}\right) \text{ if } s_{t-i}=c,\\ \Delta^{d_{nc}} \text{ if } s_{t-i}=nc,\end{cases}\end{aligned}$$

such that the lagged fractional differences reflect whether a particular observation is associated with a congestion or non-congestion state. Thus, d_{nc} is the common fractional integration order in the non-congestion state, whereas d_1 and d_2 are the integration orders of the two price areas in the congestion state.

[7]RS-SARFIMA: Regime Switching Seasonal Autoregressive Fractionally Integrated Moving Average.

In the non-congestion state bilateral prices are identical, $p_{1t} = p_{2t} = p_t$, and hence the bivariate setup collapses to a pseudo-univariate model, i.e.

$$\Delta^{d_{nc}} p_t = \sum_{i=1}^{k} \Gamma_{nc,i} \Delta^{\xi_{s_{t-i}}} \begin{pmatrix} p_{1,t-i} \\ p_{2,t-i} \end{pmatrix} + \varepsilon_{nc,t} \tag{4}$$

where $\varepsilon_{nc,t} \sim N(0, \sigma^2)$ and

$$\Gamma_{nc,i} = \left(\Gamma_{11,i}^{nc}, \Gamma_{12,i}^{nc} \right).$$

Essentially, the price process switches between being generated from (3) or (4) where switching takes place in accordance with the transition probabilities (2).

We limit our study to the bivariate setup and disregard potential spill-overs from the other areas. From a theoretical point of view, it is conceptually easy to extend the present bivariate model to the multivariate case, and thereby model spill-overs using more advanced dynamics. However, from a computational point of view this appears infeasible as the number of regimes, and thereby the number of parameters, grows very fast. Indeed, in a multivariate setup with M geographical regions, there are 2^{M-1} different regimes.

A number of remarks are in order. Consider first the non-congestion state. In this regime the two price series are forced to be governed by the same process (4) and hence any conditional forecast for this regime will remain identical for both price series. This feature is not captured in the univariate model of Haldrup and Nielsen (2006a) and indeed requires our multivariate setup. Thus, in particular, forecasts of each price series in the non-congestion state may appear different when based on (1), whereas forecasts based on (4) will be identical for the two price series in the non-congestion state. Note that in the non-congestion state the prices are fractionally integrated of order d_{nc} and fractionally cointegrated in the sense that the series perfectly co-move. This notion of (fractional) cointegration is somewhat different than originally suggested by Granger (1986) and Engle and Granger (1987).

Next, consider the congestion regime. We will discriminate between two situations, i.e. when p_{1t} and p_{2t} cointegrate or do not cointegrate. (i) Assume first the situation with fractional cointegration. In this case it must hold that $d_1 = d_2 = d$, i.e. the price series have to be of the same order of fractional integration. Notice that whilst the single price series are $FI(d)$, the log relative prices are $FI(\gamma)$ where $\gamma < d$. At the same time we require that $(\alpha_1, \alpha_2)' \neq (0, 0)'$ with either $\alpha_1 < 0$ and/or $\alpha_2 > 0$ such that the model is truly error correcting. (ii) When prices do not cointegrate in the congestion regime nothing guarantees that $d_1 = d_2 = d$. Most importantly, there is no error correction towards equilibrium in this case and the usual interpretation of the parameters $(\alpha_1, \alpha_2)'$ and γ is invalid.

The adjustment coefficients, $(\alpha_1, \alpha_2)'$, may give an indication of whether the specific price areas adjust towards equilibrium, which we expect them to do under cointegration. Specifically, if $\alpha_1 > 0$ then p_{1t} is moving away from equilibrium (non-congestion), whereas if $\alpha_2 > 0$ then p_{2t} is moving towards equilibrium. Note that the full stability of the model requires that the entire system dynamics is included in the calculation, but in any case the values of α_1 and α_2 give a rough idea of the system dynamics under a ceteris paribus assumption. An alternative interpretation of the adjustment coefficients follows from the market setup and varying costs of electricity production in different geographical regions. For example, if there is no congestion between SNO and WDK prices are identical and electricity flows from the cheaper area (usually SNO because of the hydropower) to the more expensive area (WDK). However, if there is congestion, prices in WDK will be higher reflecting the higher costs of electricity production. This increase in price in WDK corresponds to $\alpha_1 > 0$

in the WDK-SNO bivariate model, i.e. a move away from equilibrium. Importantly, this is not due to system instability but rather to electricity being more expensive to produce in WDK compared to SNO.

The model analyzed in this paper is unique in the literature on regime switching and/or (fractionally) cointegrated models since it collapses to a pseudo-univariate model in one of the regimes. The error correction model specification (3)-(4) reflects the particular structure and features of the market design. For discussions of representation theory in the context of (non-switching) fractional cointegration, see Granger (1986), Davidson (2002), Robinson and Yajima (2002), and Johansen (2007).

4.3 Estimation

In our case congestion/non-congestion is an observed state such that regimes are observable, and the maximum likelihood estimates of the transition probabilities are

$$\hat{p}_{11} = \frac{n_{c,c}}{n_{c,c} + n_{c,nc}}, \tag{5}$$

$$\hat{p}_{22} = \frac{n_{nc,nc}}{n_{nc,c} + n_{nc,nc}}, \tag{6}$$

where n_{ij} is the number of times we observe regime i followed by regime j for $i, j \in \{c, nc\}$.

Estimation of the remaining parameters of the two states is done by conditional maximum likelihood. The regime-specific log-likelihood functions, omitting the constant, is

$$\begin{aligned} l_c(d_c, \theta_c) &= -\frac{\sum_t \mathbf{1}\{s_t = c\}}{2} \log|\Omega| - \frac{1}{2} \sum_t \operatorname{trace}\left(\Omega^{-1} \varepsilon_{s_t,t} \mathbf{1}\{s_t = c\} \varepsilon'_{s_t,t} \mathbf{1}\{s_t = c\}\right), \\ l_{nc}(d_{nc}, \theta_{nc}) &= -\frac{\sum_t \mathbf{1}\{s_t = nc\}}{2} \log \sigma^2 - \frac{1}{2} \sum_t \left(\sigma^{-2} \varepsilon_{s_t,t} \mathbf{1}\{s_t = nc\} \varepsilon'_{s_t,t} \mathbf{1}\{s_t = nc\}\right), \end{aligned}$$

where $\mathbf{1}\{A\}$ is the indicator function of the event A. The full-sample log-likelihood function is given by

$$l(d_c, d_{nc}, \theta) = -\frac{T}{2} \log(2\pi) + l_c(d_c, \theta_c) + l_{nc}(d_{nc}, \theta_{nc}). \tag{7}$$

When using a numerical optimization algorithm to maximize the log-likelihood function, concern must be given to the selection of starting values. The reason for concern is that the log-likelihood function is not globally concave and hence the results of the selected numerical optimization algorithm may depend on the choice of starting values. In our case we have used the fractional integration estimates from Haldrup and Nielsen (2006a) as our starting values. For the remaining parameters, i.e. autoregressive and variance-covariance terms etc., we find starting values by letting the fractional integration parameters be fixed at their initial values and maximizing the log-likehood with respect to the remaining parameters.

Finally, we remark that our model framework assumes that states are observable and that the cointegrating vector in the congestion state, $\beta = (1, -1)$, is given. Therefore, asymptotic distribution theory for the remaining parameters will be standard under suitable regularity conditions on the errors $\varepsilon_{s_t,t}$, such as serial independence and moment conditions. In particular, Gaussianity of the errors is not a necessary condition for the asymptotic distribution theory, but is used only to derive the likelihood function.

Table 1: Estimated transition probabilities (mean duration of states)

EDK-SWE		
	congestion	non-congestion
congestion	0.7848 (4.65)	0.2152
non-congestion	0.0131	0.9869 (76.57)

WDK-SWE		
	congestion	non-congestion
congestion	0.8216 (5.60)	0.1784
non-congestion	0.1259	0.8740 (7.94)

WDK-SNO		
	congestion	non-congestion
congestion	0.9247 (13.28)	0.0753
non-congestion	0.1221	0.8779 (8.19)

SNO-SWE		
	congestion	non-congestion
congestion	0.9478 (19.16)	0.0523
non-congestion	0.0462	0.9538 (21.64)

SWE-FIN		
	congestion	non-congestion
congestion	0.8505(6.51)	0.1495
non-congestion	0.0210	0.9790(48.78)

5 Empirical Results

Prior to estimation, each log price series had deterministic seasonality removed by regression on a constant, a time trend, dummy variables for hour-of-day, day-of-week, month-of-year, and a holiday dummy. The parameter estimates for the constant, trend, and dummy variables are allowed to differ across states. For computational reasons we have selected to set $k = 4$ to capture the within-the-day effects and also include a 24th lag, to capture the daily stochastic seasonality. The gain from introducing more lags and/or e.g. a weekly lag instead of a daily, was not significant enough in terms of whiteness of the residuals to compensate for the considerable estimation time.

5.1 Estimation of transition dynamics

Since the states are observable, as discussed earlier, estimates of the transition probabilities for each state are easily calculated and are reported in Table 1. It is clear that some grid points are more subject to congestion than others. This fact may be explained by demand and supply fluctuations, but there is also the possibility that congestion may be caused by exploitation of market power.

The estimated transition probabilities indicate a high degree of persistence in the states. The probability of staying in the congestion regime, $\hat{p}_{11}$, is highest for the grid point SNO-SWE, 0.9478, whereas it is lowest for EDK-SWE link, 0.7848. This corresponds to a mean duration of 19.16 and 4.65 hours, respectively. In general, the probability of staying in the non-congestion regime, $\hat{p}_{22}$, is higher, estimated at $0.8740 - 0.9870$, corresponding to mean duration of $7.94 - 76.57$ hours.

5.2 Estimation of fractional integration and cointegration parameters

In Tables 2-6 we present the estimates of the fractional integration d for a number of different cases. The models estimated under the heading "No switching" use pooled data, i.e. there is no separation of data connected with congestion and non-congestion periods. The estimates

Table 2: Estimates for the EDK-SWE link

				Switching					
	No switching			Non-congestion			Congestion		
Model	$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{d}_1^{nc}$	$\hat{d}_2^{nc}$	$\hat{\gamma}^{nc}$	$\hat{d}_1^{c}$	$\hat{d}_2^{c}$	$\hat{\gamma}^{c}$
Univariate	0.43	0.43	0.05	0.46	0.46	0	0.03	0.03	−0.26
	(0.012)	(0.012)	(0.018)	(0.012)	(0.011)		(0.013)	(0.012)	(0.077)
VAR estimates	0.45	0.49	0.21	0.32		0	0.09	0.10	0.04
	(0.011)	(0.018)	(0.019)	(0.011)			(0.021)	(0.038)	(0.04)
VAR estimates Restricted $d_1 = d_2$	0.49		0.21	0.32		0	0.09		0.00
	(0.009)		(0.047)	(0.013)			(0.040)		(0.049)

Notes: Subscripts denote the geographical region and superscripts denote the state. Standard errors are given in parentheses.

Table 3: Estimates for the WDK-SWE link

				Switching					
	No switching			Non-congestion			Congestion		
Model	$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{d}_1^{nc}$	$\hat{d}_2^{nc}$	$\hat{\gamma}^{nc}$	$\hat{d}_1^{c}$	$\hat{d}_2^{c}$	$\hat{\gamma}^{c}$
Univariate	0.31	0.42	0.27	0.38	0.33	0	0.28	0.46	0.37
	(0.015)	(0.011)	(0.017)	(0.024)	(0.013)		(0.021)	(0.014)	(0.015)
VAR estimates	0.31	0.54	0.51	0.19		0	0.12	0.39	0.23
	(0.010)	(0.020)	(0.025)	(0.021)			(0.020)	(0.012)	(0.012)
VAR estimates Restricted $d_1 = d_2$	0.56		0.53	0.25		0	0.33		0.11
	(0.011)		(0.072)	(0.018)			(0.033)		(0.029)

Notes: Subscripts denote the geographical region and superscripts denote the state. Standard errors are given in parentheses.

of d_1, d_2 refer to the fractional orders estimated for the first and second region, respectively, whereas the estimate γ is the fractional integration order of the log relative price. The results presented under the heading "Switching" refer to similar estimates when data is partitioned into congestion and non-congestion periods, where we use superscripts c or nc to denote estimates under the congestion and non-congestion regimes, respectively. Note that by definition $\gamma^{nc} = 0$ in the non-congestion state because the single price series are identical and hence the series are fractionally cointegrated in an extreme form. Results are reported using three different models. For comparison, "Univariate" reproduces the estimates reported in Haldrup and Nielsen (2006a), i.e. this corresponds to estimates using the model (1) for both the regime switching and non-regime switching cases. The row named "VAR estimates" displays estimates based on the model (3)-(4). Note that, as opposed to the univariate estimates, $d_1^{nc} = d_2^{nc}$ by construction since the price series follow the same process in these cases. Finally, VAR estimates are reported where we restrict $d_1 = d_2$ in the non-switching case and $d_1^c = d_2^c$ in the congestion state under the regime switching case.

Consider first the East Denmark-Sweden connection exhibited in Table 2, and consider initially the pooled data set without regime switching. The estimates of d for the two regions are rather similar regardless of the underlying model being estimated, i.e. estimates are in the range $0.43 - 0.49$ and hence on the borderline of the stationary region. The estimates of γ are somewhat lower: 0.05 when the univariate model is used for estimation and 0.21 when the VAR model is used. These results indicate that when data is not classified according to regimes, then there is evidence of fractional cointegration amongst the series. Now, the question is whether this result is caused by the non-congestion state dominating the sample or whether both regimes contribute to the cointegration finding. In the regime switching case, the non-congestion estimates clearly indicate cointegration (as expected) with estimates of d

Table 4: Estimates for the WDK-SNO link

	No switching			Switching					
				Non-congestion			Congestion		
Model	$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{d}_1^{nc}$	$\hat{d}_2^{nc}$	$\hat{\gamma}^{nc}$	$\hat{d}_1^{c}$	$\hat{d}_2^{c}$	$\hat{\gamma}^{c}$
Univariate	0.30	0.44	0.28	0.30	0.16	0	0.31	0.63	0.37
	(0.015)	(0.011)	(0.016)	(0.026)	(0.008)		(0.017)	(0.017)	(0.015)
VAR estimates	0.30	0.57	0.91	0.43		0	0.20	0.22	- 0.07
	(0.009)	(0.018)	(0.019)	(0.016)			(0.017)	(0.013)	(0.015)
VAR estimates Restricted $d_1 = d_2$	0.92		1.13	0.37		0	0.34		0.10
	(0.012)		(0.018)	(0.021)			(0.047)		(0.032)

Notes: Subscripts denote the geographical region and superscripts denote the state. Standard errors are given in parentheses.

Table 5: Estimates for the SNO-SWE link

	No switching			Switching					
				Non-congestion			Congestion		
Model	$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{d}_1^{nc}$	$\hat{d}_2^{nc}$	$\hat{\gamma}^{nc}$	$\hat{d}_1^{c}$	$\hat{d}_2^{c}$	$\hat{\gamma}^{c}$
Univariate	0.45	0.41	0.31	0.38	0.41	0	0.32	0.21	0.39
	(0.011)	(0.012)	(0.016)	(0.008)	(0.012)		(0.013)	(0.013)	(0.018)
VAR estimates	0.60	0.59	0.06	0.49		0	0.32	0.18	0.31
	(0.016)	(0.017)	(0.010)	(0.018)			(0.007)	(0.007)	(0.013)
VAR estimates Restricted $d_1 = d_2$	0.46		0.21	0.39		0	0.28		0.25
	(0.007)		(0.010)	(0.012)			(0.007)		(0.015)

Notes: Subscripts denote the geographical region and superscripts denote the state. Standard errors are given in parentheses.

in the range $0.32 - 0.46$. In the congestion case, the memory parameter for each of the price series are similar but somewhat lower, i.e. $0.09 - 0.10$. Also, there is indication of a weak form of fractional cointegration in the congestion state since the relative price is FI(0.04). When we restrict $d_1 = d_2$ over the different scenarios, we see the same story as not restricting the parameters.

Next, we turn to the West Denmark-Sweden link in Table 3. For the model without regime switching both restricted and unrestriced parameter estimates using the VAR model indicates no presence of fractional cointegration which is similar to what is found in the univariate case. Under regime switching there is clearly cointegration in the non-congestion state, however, for the model with unrestricted integration orders there is no cointegration in the congestion state. The results from the no switching models are thus some combination of their regime switching counterparts, and it is clear that by not taking regime switching into account we falsely conclude that there is no sign of fractional cointegration, whereas it is evident that fractional cointegration is present in the non-congestion state. When we restrict $d_1 = d_2$ the VAR model in fact shows cointegration also in the congestion state.

The West Denmark-South Norway link with estimates in Table 4 is an interesting case where there seems to be no fractional cointegration in the non-switching models. However, looking at the VAR models where we condition on congestion/non-congestion we see that there is in fact fractional cointegration in both states. That is, an extreme form in the non-congestion state by definition and in the congestion state because $\hat{d}_1^c \approx \hat{d}_2^c$ (or $\hat{d}_1^c = \hat{d}_2^c$) and we have a reduction of fractional order for the relative price series ($\hat{\gamma}^c$). In the univariate model there is no sign of fractional cointegration.

As seen from Table 5 the link between South Norway and Sweden indicates fractional cointegration in the model without regime switching. However, when conditioning on states,

Table 6: Estimates for the SWE-FIN link

				Switching					
	No switching			Non-congestion			Congestion		
Model	$\hat{d}_1$	$\hat{d}_2$	$\hat{\gamma}$	$\hat{d}_1^{nc}$	$\hat{d}_2^{nc}$	$\hat{\gamma}^{nc}$	$\hat{d}_1^{c}$	$\hat{d}_2^{c}$	$\hat{\gamma}^{c}$
Univariate	0.39 (0.012)	0.38 (0.012)	0.24 (0.017)	0.42 (0.011)	0.43 (0.012)	0	−0.02 (0.012)	−0.02 (0.005)	0.48 (0.022)
VAR estimates	0.52 (0.009)	0.60 (0.015)	0.34 (0.017)	0.31 (0.014)		0	0.02 (0.010)	0.02 (0.009)	0.01 (0.012)
VAR estimates Restricted $d_1 = d_2$	0.49 (0.009)		−0.07 (0.037)	0.31 (0.013)		0	0.02 (0.011)		0.01 (0.013)

Notes: Subscripts denote the geographical region and superscripts denote the state. Standard errors are given in parentheses.

it is seen that it is only in the non-congestion state that cointegration takes place. In the congestion state the fractional orders of the single price series and the relative price series are almost identical for all three models.

Finally, for the Sweden-Finland link in Table 6 there is some evidence of fractional cointegration in the non-switching models. For the univariate model, the regime switching results do not make much sense because $\hat{\gamma}^c > \max\{\hat{d}_1^c, \hat{d}_2^c\}$. The two regime switching VAR models (with and without the restriction $d_1 = d_2$) give identical results in the regime switching case. There is cointegration the non-congestion state whereas all series seem to be I(0) in the congestion state. Hence, the non-congestion state seems to dominate the data when there is no conditioning on state.

5.3 Estimation of adjustment coefficients

By modeling the data using the multivariate switching VAR model (3)-(4) we obtain estimates of the adjustment coefficients in the congestion state which is not possible when estimating univariate models. The adjustment coefficients indicate (ceteris paribus) whether a specific geographical price region is moving towards or away from equilibrium in response to a particular price gap. An alternative interpretation of the adjustment coefficients follows from the market setup and varying costs of electricity production in different geographical regions, i.e. if an inexpensive electricity supply from another geographical region is suddenly stopped due to a congestion, prices are expected to be higher until non-congestion is restored which may result in adjustment parameters indicating a move away from equilibrium. Parameter interpretation is of course an issue here, because we force the cointegrating vector to be $(1, -1)$ and the parameter estimates α_1, α_2, and γ do not have the usual interpretation in the congestion state if in fact there is no cointegration present in that state (due to lack of identification). Therefore, if cointegration is not present the interpretation of $(\hat{\alpha}_1, \hat{\alpha}_2)'$ should be made with caution.

In Table 7 the adjustment coefficients (α_1, α_2) associated with the VAR models are reported, both with restricted and unrestricted d parameters and for the switching and non-switching cases. Numbers in boldface font indicate situations where, based upon the d_1, d_2, and γ estimates, some degree of fractional cointegration is likely to take place. In the regime switching models, boldface indicates situations where there appears to be cointegration in the congestion state.

Consider first the East Denmark-Sweden connection. When we do not condition on regime switching and $d_1 \neq d_2$, neither East Denmark nor Sweden appear to correct towards equilibrium. On the other hand, when $d_1 = d_2$ is enforced, East Denmark moves towards

Table 7: Estimated adjustment coefficients

	No switching				Switching			
	$d_1 \neq d_2$		$d_1 = d_2$		$d_1 \neq d_2$		$d_1 = d_2$	
Series	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
EDK-SWE	**0.1775****	**−0.3526****	**−0.3495****	**−0.0263**	**0.1592****	**0.3130****	**0.1987****	**0.2587****
	(0.0169)	(0.0172)	(0.1054)	(0.0218)	(0.0331)	(0.0443)	(0.0397)	(0.0538)
WDK-SWE	0.0328	0.0576	−0.1239	−0.0647**	0.4798**	0.0284	**0.0033**	**0.0059**
	(0.0943)	(0.0678)	(0.1244)	(0.0321)	(0.0374)	(0.025)	(0.0891)	(0.0503)
WDK-SNO	0.9253**	−0.0220	0.0173	−0.0200	**0.0429****	**−0.0048**	**0.1247****	**−0.0784****
	(0.0219)	(0.0243)	(0.0271)	(0.0212)	(0.0161)	(0.0143)	(0.0246)	(0.0171)
SNO-SWE	**−0.0433**	**−0.0017**	**0.8682****	**0.0376**	−1.039**	0.3130**	−0.9871**	0.5623**
	(0.0263)	(0.0448)	(0.0522)	(0.0224)	(0.1231)	(0.1043)	(0.1126)	(0.1526)
SWE-FIN	**0.3106****	**0.9949****	**−0.0154***	**0.0862****	0.7152**	−0.2694**	0.6991**	−0.3479**
	(0.0527)	(0.0842)	(0.0090)	(0.0177)	(0.0210)	(0.0313)	(0.0304)	(0.0154)

Notes: Subscripts denote the geographical region. Numbers in bold face refer to situations with indication of fractional cointegration based on the d_1, d_2, and γ estimates reported in Tables 2-6. Standard errors are given in parentheses. One and two asterisks denote significance at the 10% and 5% levels, respectively.

equilibrium whereas Sweden's adjustment coefficient is insignificant. When we condition on regime switching, East Denmark moves away from equilibrium, whereas Sweden now moves towards equilibrium.

Next, we look at the West-Denmark-Sweden link. Only the case with $d_1 = d_2$ for the switching model makes sense in this case, i.e. this is the only situation where some degree of cointegration was found. However, since both adjustment parameters are small and insignificant the power of the error correction mechanism should be questioned in this case.

Looking at the West Denmark-South Norway connection we found no immidiate sign of cointegration in the non-switching model, see Table 4. When we condition on regimes there is cointegration, and we see that both areas appear to move away from equilibrium. This would appear to contradict error correction adjustment. However, there may be other reasons for these seemingly contradictory results. For example, if there is no congestion between SNO and WDK prices are identical and electricity flows from the cheaper area (usually SNO because of the hydropower) to the more expensive area (WDK). However, when congestion occurs prices in WDK will be higher reflecting the higher costs of electricity production. If demand continues to increase in WDK during the congestion more expensive generators will be taken into use thus increasing marginal cost of production even further. This increase in price in WDK corresponds to $\alpha_1 > 0$ in the WDK-SNO bivariate model, i.e. a move away from equilibrium. Importantly, this is not due to system instability but rather due to electricity being more expensive to produce in WDK compared to SNO.

The South Norway-Sweden and Sweden-Finland cases are similar in the sense that no cointegration was found in the congestion state. However, in the non-switching model fractional cointegration was suggested by the data. Enforcing $d_1 = d_2$ seems to affect the adjustment mechanisms rather radically, which we attribute to the lack of conditioning on states.

To sum up, appropriate modeling of the regime switching feature is seen to have a major impact on the dynamic price adjustment mechanism. In addition to giving estimates of the adjustment process specific to the particular state, conditioning on congestion/non-congestion allows interpretation of the adjustment coefficients in terms of the prices in each geographic region under the congestion regime and not necessarily in terms of the stability of the system.

6 Conclusion

In this paper we have proposed a multivariate extension of the univariate framework of Haldrup and Nielsen (2006a). This extension enables us to describe the dynamic structure of congestion and non-congestion of electricity prices within the Nord Pool area. The notions of congestion and non-congestion are motivated by the organization of the Nord Pool market, which is characterized by physical exchanges of power across geographical regions. When the actual transmission of electricity is constrained by the flow capacity, congestion occurs. Therefore, the presence or absence of transmission bottlenecks may have implications for the way prices are formed. Our multivariate modeling framework allows us to explicitly take into account the fact that, in non-congestion periods, prices are the same across geographical regions are therefore also subject to the same price shocks. This, in particular, is not possible in the univariate frameworks in previous studies.

>From our empirical analysis it is clear that conditioning on states, i.e. congestion vs. non-congestion has a major impact on the implications for the dynamics of the electricity prices. That is, when not conditioning on the specific states, misleading conclusions in regards to potential fractional cointegration and the adjustment to equilibrium may be drawn.

There are three possible types of misclassification of the model dynamics in the empirical analysis. That is, (1) non-switching models may indicate that the price series are fractionally cointegrated, whereas when conditioning on states this is only the case in the non-congestion state (which is cointegrated by definition); (2) the non-switching model could indicate that there is no fractional cointegration when in fact there is cointegration in the non-congestion state; and (3) there is the possibility of fractional cointegration in both regimes, but not in the non-switching model. A feature of our model that is particular to its multivariate nature is that we are able to estimate adjustment coefficients in the error correction representation. Again, it is important to condition on congestion/non-congestion, since we may otherwise draw false conclusion about the adjustment to equilibrium (non-congestion).

Some geographical regions are indirectly connected, e.g. West Denmark and East Denmark are indirectly connected through Sweden, so there are regimes where West Denmark and East Denmark constitute the same price area. The effects of these indirect links between geographical regions and how they potentially affect fractional cointegration and the adjustment in the system are therefore of major interest. A detailed analysis which includes indirect links is conceptually straightforward using a higher-dimensional model, but computionally infeasible and therefore left for future research.

References

[1] Amundsen, E.S. and L. Bergman, 2007, Integration of multiple national markets for electricity: The case of Norway and Sweden, *Energy Policy* **35**, 3383-3394.

[2] Davidson, J., 2002, A model of fractional cointegration, and tests for cointegration using the bootstrap, *Journal of Econometrics* **110**, 187-212.

[3] Diebold, F. X., and A. Inoue, 2001, Long memory and regime switching, *Journal of Econometrics* **105**, 131-159.

[4] Ding, Z. and C.W.J. Granger, 1996, Modeling volatility persistence of speculative returns: A new approach, *Journal of Econometrics* **73**, 1, 185-215.

[5] Engle, R. F., and C. W. J. Granger, 1987, Co-integration and error correction: Representation, estimation and testing, *Econometrica* **55**, 251-276.

[6] Escribano, A., J. I. Peña, and P. Villaplana, 2002, Modelling electricity prices: International evidence, working paper 02-27, Universidad Carlos III De Madrid.

[7] Fabra, N., and J. Toro, 2005, Price wars and collusion in the Spanish electricity market, *International Journal of Industrial Organization* **23**, 155-181.

[8] Granger, C. W. J., 1980, Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics* **14**, 227-238.

[9] Granger, C. W. J., 1981, Some properties of time series data and their use in econometric model specification, *Journal of Econometrics* **16**, 121-130.

[10] Granger, C. W. J., 1986, Developments in the study of cointegrated economic variables, *Oxford Bulletin of Economics and Statistics* **48**, 213-228.

[11] Granger, C. W. J., and R. Joyeux, 1980, An introduction to long memory time series models and fractional differencing, *Journal of Time Series Analysis* **1**, 15-29.

[12] Granger, C. W. J., and Z. Ding, 1996, Varieties of long memory models, *Journal of Econometrics* **73**, 61-77.

[13] Granger, C. W. J. and N. Hyung, 2004, Occasional structural breaks and long memory with an application to the S&P 500 absolute stock returns, *Journal of Empirical Finance* **11**, 399-421.

[14] Haldrup, N. and M.Ø. Nielsen, 2006a, A regime switching long memory model for electricity prices, *Journal of Econometrics* **135**, 349-376.

[15] Haldrup, N. and M.Ø. Nielsen, 2006b, Directional congestion in a regime switching long memory model for electricy prices, *Studies in Nonlinear Dynamics and Econometrics* **10**, 1-24.

[16] Hamilton, J. D., 1989, A new approach to the economic analysis of nonstationary time series and the business cycle, *Econometrica* **57**, 357-384.

[17] Hosking, J. R. M, 1981, Fractional differencing, *Biometrika* **68**, 165-176.

[18] Johansen, S., 2007, A representation theory for a class of vector autoregressive models for fractional processes, forthcoming in *Econometric Theory*.

[19] Koopman, S. J., M. Ooms, and M. A. Carnero, 2007, Periodic Seasonal Reg-ARFIMA-GARCH Models for Daily Electricity Spot Prices, *Journal of the American Statistical Association* **102**, 16-27.

[20] Motta, M., 2004, *Competition Policy, Theory and Practice*, Cambridge University Press.

[21] Nord Pool, 2003a, The Nordic power market, Electricity power exchange across national borders, www.nordpool.no.

[22] Nord Pool, 2003b, The Nordic spot market, The world's first international spot power exchange, www.nordpool.no.

[23] Nord Pool, 2003c, Derivatives trade at Nord Pool's financial market, www.nordpool.no.

[24] Robinson, P.M. and Y. Yajima, 2002, Determination of cointegrating rank in fractional systems, *Journal of Econometrics* **106**, 217-241.

[25] Sherman, P., 1989, *The Regulation of Monopoly*, Cambridge University Press.

7 Appendix: Figures

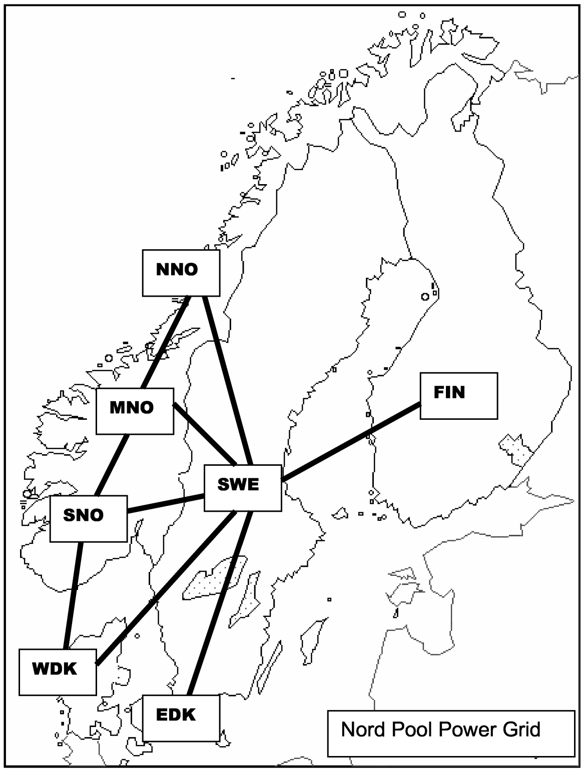

Figure 1: Map of the Nord Pool area.

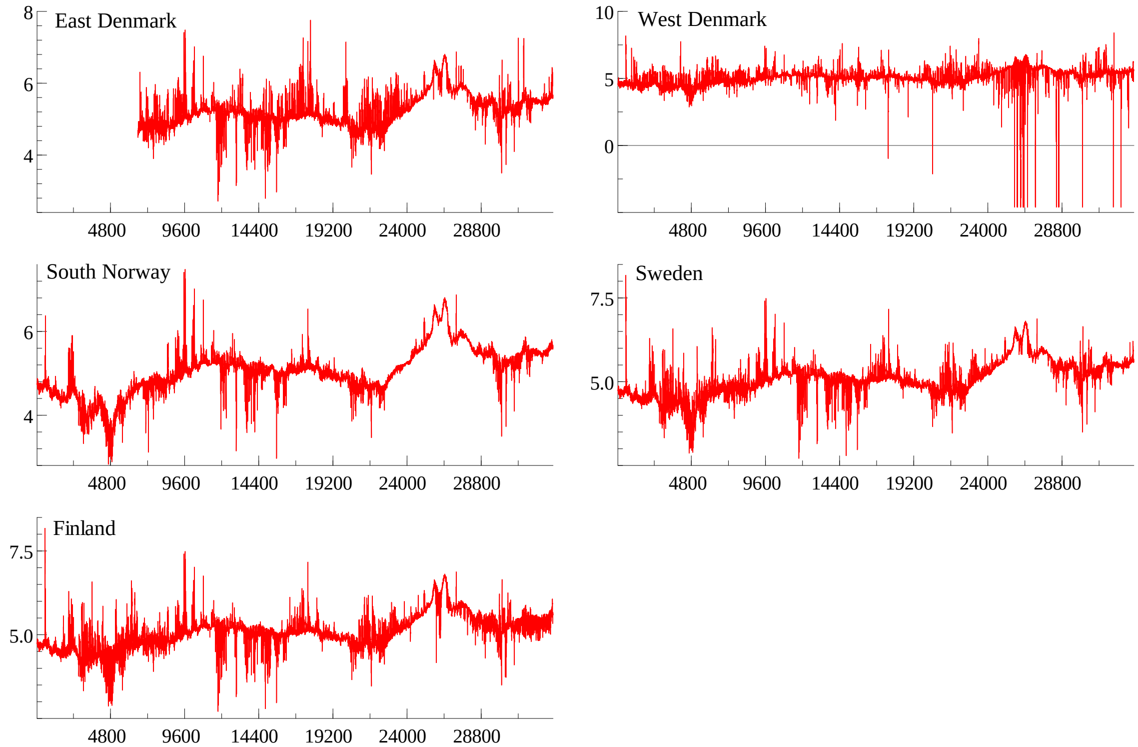

Figure 2: Hourly log spot electricity prices for the Nord Pool area covering the period 3 January 2000 to 25 October 2003.

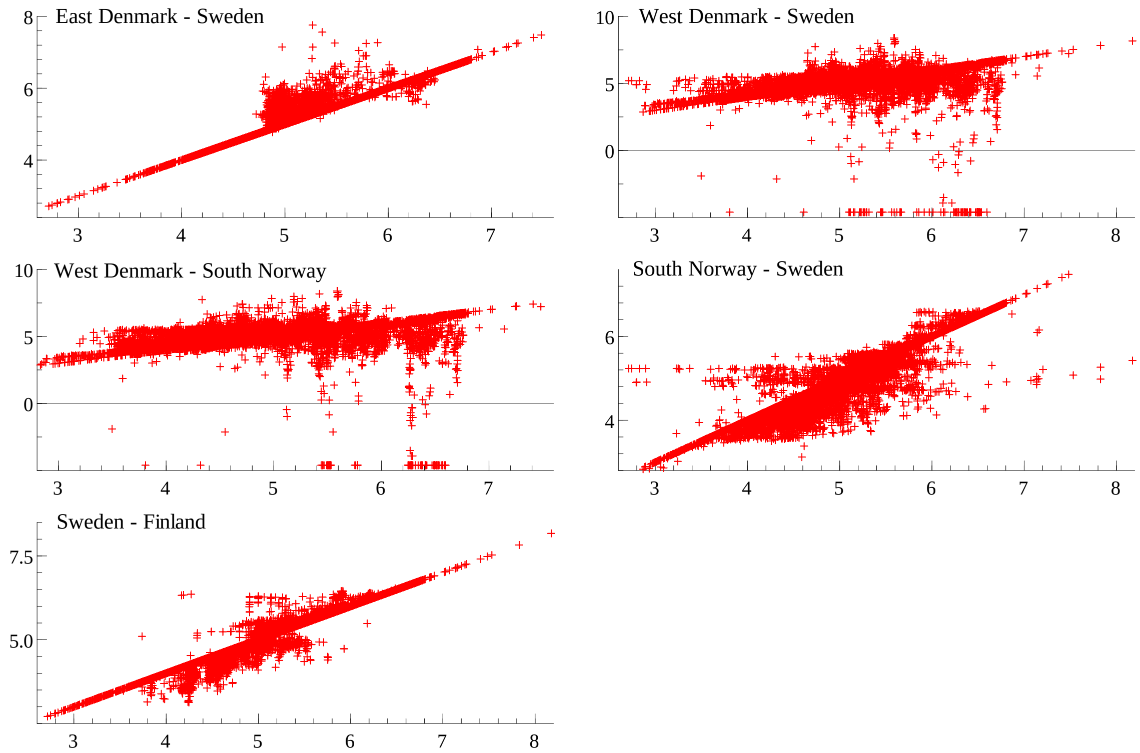

Figure 3: Scatter plots of hourly log prices across Nord Pool regions.

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