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Bivariate Modeling of US and European Short Rates

Charlotte Christiansen

School of Economics and Management
University of Aarhus
Building 1322, DK-8000 Aarhus C
Denmark

Aarhus School of Business
University of Aarhus

Handelshøjskolen
Aarhus Universitet

UNIVERSITY OF
COPENHAGEN

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Charlotte Christiansen[†]

CREATES, University of Aarhus

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[†]Charlotte Christiansen, School of Economics and Management, University of Aarhus, Building 1322, 8000 Aarhus C, Denmark. +45 8942 5477 (phone), CChristiansen@econ.au.dk (mail).

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Abstract: This paper introduces regime switching into level-ARCH models for the short rates of the US, the UK, and Germany. Once regime switching and level effects are included there are no gains from including ARCH effects. It is of secondary importance how the regime switching is specified. The estimated level parameters differ across countries. The corresponding new bivariate models show that the states of the US and UK short rate volatilities are not independent nor identical. There is Granger causality from the US to the UK short rate volatility state but not vice versa. There is no contagion between the US and UK volatility states. Equivalent results apply to the relation between the US and German volatility states.

1 Introduction

This paper models short rates which are known to be important for asset pricing and political decision making. The starting point is the short rate model of Chan, Karolyi, Longstaff & Sanders (1992) where the conditional volatility of the short rate changes is proportional to the γth power of the level of the short rate itself; the CKLS *level* model. By imposing different restrictions on γ various well-known short rate models are obtainable, e.g. when $\gamma \equiv \frac{1}{2}$ the square root model of Cox, Ingersoll & Ross (1985) emerges. In Chan et al. (1992) γ is a free parameter and it is estimated to be approximately $1\frac{1}{2}$ which is considered unreasonably large. Previous literature has extended the CKLS level model to also include ARCH volatility. Introducing heteroskedasticity into the conditional volatility reduces the estimate of γ. Moreover, short rate models including both level and heteroskedasticity effects are superior to models comprising only one or the other effect. On the other hand, previous literature has shown that regime switching models are generally preferable to single regime models. In this paper we further introduce regime switching conditional volatility into the level-ARCH model. In one model specification, the γ parameter is state dependent whereby we are able to investigate which short rate models apply in the two states.

Another contribution of this paper is to extend the regime-switching level-ARCH model to a bivariate setting to describe the short rates of the US and the UK simultaneously. No restrictions are imposed on the transition probabilities. Subsequently, we provide tests for identical volatility states, independent volatility states, no Granger causality between the US and UK volatility states, and contagion between the US and UK volatility states. This enables us to analyze the interdependence of the volatility states of the world's largest economy and the most important European economy outside the euro area which is a major concern for European policy makers. Similar analysis is conducted for the US and German short rates, whereby we analyze the relation between the volatility states of the US and the most important euro country.

Various specifications of level-ARCH short rate models have been analyzed in univariate frameworks, e.g. by Longstaff & Schwartz (1992),

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Brenner, Harjes & Kroner (1996), and Koedjik, Nisssen, Schotman & Wolff (1997). Using dummy variables, the previous literature has documented a structural shift in the parameters of the CKLS level model around the Monetary Experiment of the Federal Reserve, cf. Bliss & Smith (1998) and Dell'Aquila, Ronchetti & Trojani (2003). Furthermore, Dewachter (1996) and Dahlquist & Gray (2000) estimate the CKLS level model with truly regime switching parameters.

The previous literature has investigated short rate models with both regime switching volatility, level effects, and ARCH volatility. But (as far as the authors are aware) only with stronger assumptions than those applied here. In particular, Gray (1996) estimates the square root model ($\gamma \equiv \frac{1}{2}$) where the ARCH process is regime switching. In contrast, in our model γ is an estimated parameter. And regime switching either occurs in the γ parameter or in the ARCH process. Smith (2002) uses a short rate model with regime switching stochastic volatility (in place of ARCH volatility). Although γ is an estimated parameter, it is not allowed to be state dependent. Sun (2005) extends the analysis by Smith (2002) by amongst other things using time varying transition probabilities that depend on the lagged short rate. However, Sun (2005) finds that the transition probabilities are not significantly different from being constant.

The literature continues to find the univariate CKLS model interesting, e.g. Czellar, Karolyi & Ronchetti (forthcoming) study different estimation techniques for the CKLS short rate model and Bali & Wu (forthcoming) investigate extensions of the mean specification of the CKLS model.

The previous literature has offered bivariate short rate models, but (to our knowledge) not with both regime switching, level, and ARCH effects as we suggested here. Christiansen (2005) uses a bivariate level-ARCH model without regime switching to describe the long rate and the spread between the long and short rate. Ferreira (2005) applies a bivariate non-switching level-ARCH model to describe German and French short rate changes. Ang & Bekaert (2002) apply various trivariate models for the US, UK, and German short rates. In the level specification $\gamma \equiv \frac{1}{2}$ and ARCH effects are not included.

The main findings of the paper can be summarized as follows. Once the

univariate short rate model includes level effects and regime switching, there are no advantages of the conditional volatility following an ARCH process. This applies to all three countries. Moreover, it is undecided whether it is preferable that the regime switching occurs by letting the level parameter (γ) or the volatility (σ) be state dependent. For the US when γ is state dependent the following applies: In the low variance state the Cox et al. (1985) square root model applies ($\gamma = \frac{1}{2}$) and in the high variance state the Cox, Ingersoll & Ross (1980) model applies ($\gamma = 1\frac{1}{2}$). The applicable short rate models are very different for the two European countries. When the level parameter is not allowed to be state dependent, no well-known short rate model applies, $\gamma \in (1; 1\frac{1}{2})$, and it amounts to averaging across the two states. Again, the estimated level parameters are very different for the UK and Germany. The variance state is shown to be strongly correlate with the business cycle.

The bivariate short rate modeling for the US and UK short rates provide unaltered conclusions regarding the short rate volatilities. We find that the states of the US and UK short rate volatilities are not independent. Furthermore, the volatility states of the US and UK short rates are not identical, i.e. they are not in the same volatility state at the same time. The US volatility state appears to Granger cause the UK volatility state, whereas the UK volatility state does not Granger cause the US volatility state. There is no contagion between the US and UK volatility states. Equivalently, the states of the US and German short rates are neither independent nor identical. There is mixed evidence that the US volatility state Granger causes the German volatility state. The German volatility state Granger causes the US volatility state. There is no contagion between the US and German volatility states. Thus, European policy makers can hope to exert at least some influence upon the short rate volatility.

The remaining part of this paper is structured as follows. In Section 2 the regime-switching short rate models are introduced; first we provide the one-country models which are subsequently extended to a two-country setting. Section 3 introduces the short rate data and presents the empirical results: First the country-by-country results are discussed followed by the US-UK results and finally the US-German results. Section 4 concludes.

2 Regime-Switching Short Rate Model

In this section we introduce the econometric framework we use to describe the short rates. Firstly, we introduce a univariate regime-switching model with level and heteroskedasticity effects. Secondly, we present a bivariate extension hereof that describes the evolution of two short rate series simultaneously.

2.1 Univariate Short-Rate Model

2.1.1 CKLS Level Model

Here we describe the evolution of the short rate country by country. The model takes its offspring in the CKLS level model for the short rate (r_t), cf. Chan et al. (1992), namely the continuous-time diffusion:

$$dr_t = (\alpha_0 + \alpha_1 r_t)dt + \sigma r_t^{\gamma} dZ_t \tag{1}$$

where Z_t is a standard Brownian motion. The conditional mean of the short rate changes depends on the short rate, and the short rate process is mean reverting for $\alpha_1 < 0$. The conditional volatility of the short rate changes depends on the level of the short rate, in fact it is proportional to the γth power of the level of the short rate. The level parameter γ is non-negative. The larger γ is, the more important the level of the short rate is for the conditional volatility. Various well-known term structure models are nested in the CKLS level model and are obtainable by imposing different restrictions on γ. For $\gamma \equiv 0$ the Vasicek (1977) model is obtained, for $\gamma \equiv \frac{1}{2}$ the square-root model of Cox et al. (1985), for $\gamma \equiv 1$ the Brennan & Schwartz (1980) model, and for $\gamma \equiv 1\frac{1}{2}$ the Cox et al. (1980) model.

In the empirical analysis Chan et al. (1992) use a discrete version of the diffusion process in equation (1):

$$r_t - r_{t-1} = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \tag{2}$$

where the residual ϵ_t has mean zero and conditional variance

$$E[\epsilon_t^2] = h_t = \sigma^2 r_{t-1}^{2\gamma} \qquad (3)$$

As in the continuous-time framework, the short rate is mean reverting for $\alpha_1 < 0$, and the volatility of the short rate is proportional to the γth power of the short rate last period. Although the volatility of the short rate changes is time-varying it is not persistent. Chan et al. (1992) find an estimate of γ equal to around $1\frac{1}{2}$ which is in accordance with the Cox et al. (1980) model, but is at odds with the theoretically preferred square-root model of Cox et al. (1985) in which γ equals $\frac{1}{2}$. The estimated γ is therefore considered too high.

2.1.2 Level-ARCH Models

The CKLS level model has been extended to accommodate heteroskedasticity effects as well by assuming that the volatility of the short rate changes follow a GARCH-type model, cf. Engle (1982) and Bollerslev (1986). These level-GARCH models are empirically preferred to both level models and GARCH models. A popular level-GARCH specification is contained in Brenner et al. (1996).[1] The mean of the short rate evolves unchanged according to the CKLS level model in equation (2). The residual ϵ_t still has zero mean, but its conditional variance is slightly different:

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma} \qquad (4)$$

The only difference is that the σ_t^2 is now time varying. The heteroskedasticity is introduced by assuming that σ_t^2 evolves according to a GARCH-type specification, e.g. the GARCH(1,1) model. In this manner the short rate model of Brenner et al. (1996) includes both level effects and heteroskedasticity effects. Level-GARCH models for the short rate changes has also been applied by e.g. Longstaff & Schwartz (1992), Andersen & Lund (1997), Koedjik et al. (1997), and Bali (2000a, 2000b).

The previous literature finds that there are structural breaks in the short rate volatility. Bliss & Smith (1998) and Dell'Aquila et al. (2003) use dummy

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[1] Here we apply their "time-varying parameter" specification.

variables to find evidence that the parameters in the CKLS level model incur a structural break around the Monetary Experiment in 1979-1982. Dewachter (1996) and Dahlquist & Gray (2000) apply a CKLS level model with regime switching parameters.

2.1.3 Regime-Switching Level-ARCH Models

We suggest to use a regime-switching ARCH specification to describe the evolution of σ_t^2. In this way we combine previous findings of regime-switching volatility, level effects, and heteroskedasticity effects. The state of the economy is defined by the level of the volatility of the short rate changes, this is explained in more detail shortly. A first-order two-state Markov chain is assumed.[2] The state of the economy at time t is denoted s_t. At each point in time the economy is either in state 1 or state 2; $s_t \in \{1, 2\}$. The probability that s_t equals, say, 1 depends only on the value of the state variable last period, s_{t-1}. We assume constant transition probabilities:

$$
P = \begin{pmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{pmatrix}
\tag{5}
$$

where p_{ij} is the probability that the economy was in state i last period and is in state j this period; $p_{ij} = prob[s_t = j | s_{t-1} = i]$. Hereby the unconditional probability of being in e.g. state 1 is constant:

$$
prob[s_t = 1] = \frac{1 - p_{22}}{2 - p_{11} - p_{22}}
\tag{6}
$$

To access which state the economy is in at a given point in time, the smoothed probabilities are used. The smoothed probability of being in state i at time t is based on all observations in the sample; $prob[s_t = i | \Phi_T]$ where T is the last observation in the sample. When the smoothed probability of being in state i at time t is larger than 0.5 the economy is said to be in state i at time t.

Previous literature provides evidence that the most important part of the model specification stems from the variance part, cf. e.g. Durham

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[2]More information on regime switching models is available in Hamilton (1994, Chapter 22).

(forthcoming). Moreover, informal investigations provide no evidence of regime switching in the mean equation. Therefore, the mean equation for the short rate changes is unaltered as in the level CKLS model in equation (2).

The regime switching enters into the variance specification. State 1 is the low variance state and state 2 is the high variance state. The regime dependence of the conditional variance as specified in equation (4) can occur either from the specification of σ_t^2 or from the specification of γ. It is not sensible to make both σ_t^2 and γ regime dependent at the same time because then it is not possible to distinguish conclusively between the low and the high variance regimes.[3]

First, we let the regime switching enter via σ_t^2, and h_t is still given by equation (4). Moreover, σ_t^2 is still assumed to evolve according to a heteroskedastic model, but not the GARCH(1,1) as in Brenner et al. (1996). By introducing regime switching into the volatility process we accommodate for the finding in Laumoreux & Lastrapes (1990) that unaccounted structural changes in the volatility process might exaggerate the persistence of the volatility process. The simplest way to include both regime switching and heteroskedasticity in the volatility process, is by using the switching ARCH(1) model suggested independently by Hamilton & Susmel (1994) and Cai (1994). Here we apply the latter specification.

$$\sigma_t^2 = \beta_0(s_t) + \beta_1 \varepsilon_{t-1}^2 \tag{7}$$

where $\beta_0(s_t) = \left\{ \begin{array}{l} \beta_{01} \\ \beta_{02} \end{array} \right.$. The following restrictions are imposed to ensure that the conditional variance is strictly positive and non-exploding: $\beta_{01}, \beta_{02} > 0$ and $\beta_1 \leq 1$. We distinguish between the two states by the size of the $\beta_0(s_t)$ parameters, thus, we impose the following restriction in the estimation:

$$\beta_{01} \leq \beta_{02} \tag{8}$$

whereby state 1 is the low variance state, and state 2 the high variance state.

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[3]If σ_t^2 is in the low regime, then γ must also be in the low regime in order to conclude unambiguously that the economy is in the low variance regime.

A second option is to let the γ parameter in equation (4) be state dependent; $\gamma(s_t) = \left\{ \begin{array}{c} \gamma_1 \\ \gamma_2 \end{array} \right.$. Thus, the conditional variance for the short rate changes is

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma(s_t)} \tag{9}$$

In order to make state 1 the low variance state again the following restriction is imposed during estimation:

$$\gamma_1 \leq \gamma_2 \tag{10}$$

Note that the restriction makes state 1 the low variance state because the interest rates are measured in percentage terms (e.g. 5% compared to 0.05). The σ_t^2 process is now assumed to be a simple ARCH(1) process without regime switching:

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 \tag{11}$$

The two specifications are nested in the following general volatility equation:

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma(s_t)} \text{ where } \sigma_t^2 = \beta_0(s_t) + \beta_1 \varepsilon_{t-1}^2 \tag{12}$$

when appropriate parameter restrictions are imposed.

In the empirical analysis we estimate six models that all have the same mean equation given in equation (2) and variance specifications that are special cases of equation (12).

- I: Level model

$h_t = \sigma_t^2 r_{t-1}^{2\gamma}$ where $\sigma_t^2 = \beta_0$

- II: Level ARCH model

$h_t = \sigma_t^2 r_{t-1}^{2\gamma}$ where $\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2$

- III: Switching-level ARCH model

$h_t = \sigma_t^2 r_{t-1}^{2\gamma(s_t)}$ where $\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2$ and $\gamma(s_t) = \left\{ \begin{array}{c} \gamma_1 \\ \gamma_2 \end{array} \right.$

- IV: Level SWARCH model

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma} \text{ where } \sigma_t^2 = \beta_0(s_t) + \beta_1 \varepsilon_{t-1}^2 \text{ and } \beta_0(s_t) = \left\{ \begin{array}{l} \beta_{01} \\ \beta_{02} \end{array} \right.$$

- V: Switching-level model

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma(s_t)} \text{ where } \sigma_t^2 = \beta_0 \text{ and } \gamma(s_t) = \left\{ \begin{array}{l} \gamma_1 \\ \gamma_2 \end{array} \right.$$

- VI: Level switching-volatility model

$$h_t = \sigma_t^2 r_{t-1}^{2\gamma} \text{ where } \sigma_t^2 = \beta_0(s_t) \text{ and } \beta_0(s_t) = \left\{ \begin{array}{l} \beta_{01} \\ \beta_{02} \end{array} \right.$$

Two models are not regime switching; model I is the CKLS level model and model II is the level-ARCH model. Model III has a switching level parameter and ARCH volatility. Model IV has a constant level parameter and SWARCH volatility. Furthermore, we estimate regime switching models without heteroskedasticity; firstly in model V the level parameter is state dependent and the volatility constant, and secondly in model VI the level parameter is constant and the volatility parameter is constant in each regime.

Previous papers have also applied regime switching models for the short rate changes that include both level and heteroskedasticity effects, but with stronger assumptions on the γ parameter. The well-known paper by Gray (1996) is such an example. In Gray (1996) the level parameter is assumed fixed at $\frac{1}{2}$ ($\gamma \equiv \frac{1}{2}$), thus restricted to the square-root model whereas he uses a new SWARCH model to describe σ_t^2.

Dahlquist & Gray (2000) estimate regime switching CKLS level models with $\gamma \equiv \frac{1}{2}$ imposed. Another example is contained in Smith (2002) who uses a model with non-switching level effects, but regime switching stochastic volatility.

2.2 Bivariate Short-Rate Model

Here we introduce a model that describes the evolution of two countries' short rate changes simultaneously as well as their interdependence. The bivariate model is an extension of the regime switching level-ARCH models introduced above. We are not aware of any bivariate models for short rate

changes that include regime switching, level effects, and heteroskedasticity effects. Related papers include Christiansen (2005) who finds that a bivariate level-GARCH model for the long rate changes and (long rate-short rate) spread changes is preferred above a bivariate level model and a bivariate GARCH model. Ferreira (2005) uses a bivariate level-GARCH model to describe the German and French short rate changes. Ang & Bekaert (2002) apply various trivariate models for the US, UK, and German short rates (not short rate *changes* as is usual). The specification that includes level effects assumes the Cox et al. (1985) specification (with $\gamma \equiv \frac{1}{2}$) and does not include ARCH effects. Unlike our model, the transition probabilities for each country are assumed independent, more hereon follows.

We model the short rate changes for country $i = 1, 2$ (throughout, the i in superscript indicates country i). The mean equations are as in the univariate model:

$$r_t^i - r_{t-1}^i = \alpha_0^i + \alpha_1^i r_{t-1}^i + \epsilon_t^i \tag{13}$$

So are the conditional variance equations of the residuals, ε_t^i.

$$h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma(s_t^i)} \text{ where } (\sigma_t^i)^2 = \beta_0^i(s_t^i) + \beta_1^i(\varepsilon_{t-1}^i)^2 \tag{14}$$

where $\beta_0^i(s_t) = \begin{cases} \beta_{01}^i \\ \beta_{02}^i \end{cases}$ and $\gamma^i(s_t) = \begin{cases} \gamma_1^i \\ \gamma_2^i \end{cases}$. The conditional correlation between ε_t^1 and ε_t^2 is assumed to be constant (ρ) as in the Constant Conditional Correlation model in Bollerslev (1990). Hereby, the conditional covariance is proportional to the conditional volatilities:

$$h_t^{12} = \rho \sqrt{h_t^1 h_t^2} \tag{15}$$

Notice that there is a state variable for each country, $s_t^1 = 1, 2$ and $s_t^2 = 1, 2$. This implies that country 1 and 2 are not necessarily in the same volatility state at the same time. As in the univariate case, either the ARCH parameter ($\beta_0^i(s_t)$) is regime dependent or the level parameter ($\gamma(s_t)$) is regime dependent. With SWARCH, the following restrictions are

imposed to make states 1 the low volatility regimes:

$$\beta_{01}^1 \leq \beta_{02}^1 \text{ and } \beta_{01}^2 \leq \beta_{02}^2 \tag{16}$$

In the case of switching in the level parameter, the following restrictions are imposed:

$$\gamma_1^1 \leq \gamma_2^1 \text{ and } \gamma_1^2 \leq \gamma_2^2 \tag{17}$$

Combining two countries each with two volatility states, there are in total four joint states:

$s_t^* = 1$ for $s_t^1 = 1$ and $s_t^2 = 1$

$s_t^* = 2$ for $s_t^1 = 1$ and $s_t^2 = 2$

$s_t^* = 3$ for $s_t^1 = 2$ and $s_t^2 = 1$

$s_t^* = 4$ for $s_t^1 = 2$ and $s_t^2 = 2$

In state 1 both countries are in their low variance states, whereas in state 4 they are both in the high variance states. In states 2 and 3 there is one country in the low variance state and one in the high variance state. The transition probabilities are assumed to be constant. The matrix of transition probabilities P^* is a 4×4 matrix with typical element p_{ij}^* for $i, j = 1, 2, 3, 4$ where $p_{ij}^* = prob[s_t^* = j | s_{t-1}^* = i]$ and where each row sums to one, $\sum_{j=1}^4 p_{ij}^* = 1$ for $i = 1, 2, 3, 4$. The unconditional probability of being in state i can be calculated from the estimated transition matrix. Similarly, the smoothed probabilities are defined as in the univariate case: At a given point in time, the economy is said to be in the state which has connected the largest concurrent smoothed probability.

To sum up, we use four different volatility specification in the bivariate model. Below we list in bullet form the volatility equations for model A-D. Model A is the bivariate equivalent to model III, model B to model IV, model C to model V, and model D to model VI.[4] Model A has switching level parameters and ARCH volatilities whereas model B has constant level parameters and SWARCH volatilities. Model C has switching level parameters and constant volatility, whereas model D has constant level parameters

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[4]We do not estimate bivariate single regime models. Firstly, in the univariate case they are inferior to two state models. Secondly, they provide no information as to the relation between the countries' volatility states.

and the volatilities are constant in each regime.

- A: Switching-level ARCH model

  $h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma(s_t^i)}$ where $(\sigma_t^i)^2 = \beta_0^i + \beta_1^i (\varepsilon_{t-1}^i)^2$ and $\gamma(s_t^i) = \begin{cases} \gamma_1^i \\ \gamma_2^i \end{cases}$

- B: Level SWARCH model

  $h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma}$ where $(\sigma_t^i)^2 = \beta_0^i(s_t^i) + \beta_1^i (\varepsilon_{t-1}^i)^2$ and $\beta_0^i(s_t) = \begin{cases} \beta_{01}^i \\ \beta_{02}^i \end{cases}$

- C: Switching-level model

  $h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma(s_t^i)}$ where $(\sigma_t^i)^2 = \beta_0^i$ and $\gamma(s_t^i) = \begin{cases} \gamma_1^i \\ \gamma_2^i \end{cases}$

- D: Level switching-volatility model

  $h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma}$ where $(\sigma_t^i)^2 = \beta_0^i(s_t^i)$ and $\beta_0^i(s_t) = \begin{cases} \beta_{01}^i \\ \beta_{02}^i \end{cases}$

We estimate the bivariate model first for the US and the UK short rates and subsequently for the US and German short rates. Firstly, our interest lies in the relation between the short rate volatility state of US and the short rate volatility state of European countries, not the relation between European countries' volatility states. Secondly, an equivalent trivariate model for the US, UK, and German short rates that imposes no restrictions on the transition probabilities have in total $2^3 = 8$ states, so the transition matrix is an 8×8 matrix which is likely to cause severe estimation problems.

2.2.1 Regime Dependencies

The transition probabilities p_{ij}^* can be restricted to reflect various assumptions regarding the relation between the states of country 1 and 2, e.g. that they are independent or identical, cf. Ramchand & Susmel (1998). These are the two extreme cases that we consider first. As an example we consider the transition probability from state 2 to state 4, p_{24}^*, i.e. going from country 1 in the low variance state and country 2 in the high variance state to both

countries being in the high variance state. In the empirical analysis, the estimation poses no restrictions on the transition probabilities. Subsequently, the hypotheses of e.g. independent states are tested using Likelihood Ratio (LR) tests.

If the volatility states are **independent**, then $p_{24}^* = p_{12}^1 p_{22}^2$, i.e. the multiple of country 1 changing from state 1 to state 2, and country 2 staying in state 2. All elements in P^* are given by corresponding multiples of individual transitions probabilities. The assumption of independent volatility states is applied in the interest model of Ang & Bekaert (2002).

When the variance state of country 1 and 2 are **identical**, then all elements of P^* are zero except for $p_{11}^*, p_{41}^*, p_{14}^*$ and p_{44}^*. The term structure modeling of Christiansen (2004) assumes identical volatility states.

Following Bialkowski & Serwa (2005) we test for no Granger (1969) **causality** from s_t^1 to s_t^2. When s_t^1 does not Granger cause s_t^2 then the value of s_t^1 last period does not influence the outcome of s_t^2 today. More formally, s_t^1 does not Granger cause s_t^2 if

$$prob[s_t^2 = i | s_{t-1}^1 = 1, s_{t-1}^2 = 1] = prob[s_t^2 = i | s_{t-1}^1 = 2, s_{t-1}^2 = 1] \qquad (18)$$

and

$$prob[s_t^2 = i | s_{t-1}^1 = 1, s_{t-1}^2 = 2] = prob[s_t^2 = i | s_{t-1}^1 = 2, s_{t-1}^2 = 2]. \qquad (19)$$

No Granger causality translates into a number of restrictions on the transition matrix:

$$
\begin{aligned}
p_{11}^* + p_{13}^* &= p_{31}^* + p_{33}^* \\
p_{21}^* = p_{23}^* &= p_{41}^* + p_{43}^* \\
p_{22}^* + p_{24}^* &= p_{42}^* + p_{44}^* \\
p_{12}^* + p_{14}^* &= p_{32}^* + p_{34}^*
\end{aligned}
\qquad (20)
$$

Equivalently, we can test for no Granger causality from s_t^2 to s_t^1

Finally, we can impose restrictions on the transition matrix that implies that the state of country 1 causes **contagion** to the state of country 2 as

defined by Sola, Spagnolo & Spagnola (2002) and Bialkowski & Serwa (2005). There is contagion from s_t^1 to s_t^2 if country 2 is always in the state country 1 was in last period; $prob[s_t^2 = j | s_{t-1}^1 = j] = 1$. This means that half of the transition probabilities are equal to zero: $p_{31}^* = p_{41}^* = p_{12}^* = p_{22}^* = p_{33}^* = p_{43}^* = p_{14}^* = p_{24}^* = 0$. Equivalently, we can restrict the transition matrix to incorporate contagion from s_t^2 to s_t^1.

3   Empirical Results

In this section we present our empirical results. Firstly, the short-rate data are introduced. Subsequently, in Section 3.2 we show the results from estimating the set of univariate short-rate models country-by-country for the US, the UK, and Germany. In Section 3.3 the results from the bivariate models are introduced; first the US-UK results and then the US-German results.

3.1   Short-Rate Data

In the empirical analysis we make use of time series of short rates for the US, the UK, and Germany. We measure the US short rate by the 1-month Eurodollar rate, the UK short rate by the 1-month LIBOR rate in Great Britain Pound, and the German short rate by the 1-month Euromark rate.[5] We use the longest sample period for which data are available for all three countries, namely from January 1975 through December 2004. We use weekly observations and obtain a total of 1565 observations. By using weekly data instead of data of a higher frequency, we overcome at least two problems; on the one hand we diminish the problems arising from differences in trading hours and on the other hand we avoid potential day-of-the-week effects. The short rates are from Wednesdays when available, otherwise they are collected on Thursdays or Tuesdays (in that order). The Eurodollar bid rates are collected from the H.15 release of the Federal Reserve. The LIBOR (offered) rates are obtained from DataStream. The Euromark bid rates are also obtained from DataStream.

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[5] After January 1999, i.e. after the launch of the euro, the German Euromark rate is replaced by the euro Eurocurrency rate.

Table 1 provides summary statistics for the short rates and the short rate changes. The short rates are measured in percent per annum.[6] The average short rate is largest for the UK (9.23%), medium for the US (6.75%), and smallest for Germany (5.25%). The ordering of the standard deviations also follows this pattern and they fall between 3.70% (UK) and 2.44% (Germany). All the short rate series are somewhat right skewed. The US short rates are leptokurtic, the UK short-rates are platykurtic, and the German short rates are close to being mesokurtic. Both the short rates and the squared short rates are highly autocorrelated. The short rates are strongly correlated; the US-UK short rate correlation coefficient equals 0.74, US-Germany 0.59, and UK-Germany 0.66.

The short rates changes are the explained variables in the subsequent models, and therefore their properties are of major interest. As required, the series of the short-rate changes are stationary according to the augmented Dickey-Fuller unit root test where the stationary alternative includes a constant and four lags, cf. Dickey & Fuller (1979). The average short rate changes are close to zero, whereas the standard deviations amount to between 0.24% (Germany) and 0.38% (US). The US short rate changes are left skewed, whereas the UK and German series are right skewed. The short rate changes are all highly leptokurtic. The short rate changes show weak negative first order autocorrelation.[7] As is usual, the squared series of the short rate changes are significantly positively autocorrelated; the first order autocorrelation coefficients are between 0.04 and 0.27. This suggests using an econometric framework to describe the short rate changes that allows for heteroskedasticity effects. In contrast to the short rates themselves, the short rate changes show only weak correlations; the US-UK short rate changes correlation coefficient equals -0.03, US-Germany -0.06, and UK-Germany 0.04.

The short rate series show signs of mean reversion, in that the correlation coefficients between the level of the short rate last week and the current short rate change are negative.

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[6] In the empirical analysis below we also apply the short rates in percent per annum.

[7] The first order autocorrelation is only significantly negative for the German short rate changes, not for the US and UK short rate changes.

3.2 Country-by-Country Analysis

We use the Quasi Maximum Likelihood method described in Hamilton (1994, Chapter 22). The estimation is conducted in GAUSS using the constrained maximum likelihood module. The numerical optimization applies a combination of the Berndt-Hall-Hall-Hausman and the Newton-Raphson algorithm. Table 2 contains the results from estimating the univariate models for the short rate, model I-VI outlined in Section 2.1 above. First we present in detail the US results. Subsequently, we compare the UK results to the US results finally we consider the German results.

3.2.1 US Short Rate Models

Panel A of Table 2 concerns the US. The findings regarding the mean equation are qualitatively identical across the six volatility specifications. The short rate is mean reverting, i.e. $\widehat{\alpha}_1 < 0$. This corresponds to our a priori expectations: Firstly, the current short rate change is negatively correlated with the level of the short rate last period indicating mean reversion, cf. Section 3.1. Secondly, the existence of mean reversion is in accordance with previous findings, cf. e.g. Chan et al. (1992). Based on an ordinary t-test the mean reversion parameter is not significantly negative.[8] Note, however, that for the null hypothesis of α_1 equal to zero the process for the short rate changes is non-stationary, so the usual t-test is actually not valid.

Model I is the original CKLS level model without regime switching. The level parameter is significantly larger than 1 and significantly smaller than $1\frac{1}{2}$; $\widehat{\gamma} = 1.23$. The level parameter is lower than the $1\frac{1}{2}$ of Chan et al. (1992) most likely because of differences in sample period. Furthermore, the estimated level parameter does not point to any of the well-known short rate models.

Model II is not regime switching, but has both level and heteroskedasticity effects. The level parameter is around unity and insignificantly different here from ($\widehat{\gamma} = 1.09$). A unity level parameter is specified in the short rate model of Brennan & Schwartz (1980). The level parameter is smaller than for model I, which is typical when introducing heteroskedasticity into the

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[8]In the following, the default level of significance is 5%.

level model, cf. Brenner et al. (1996).

Model III and IV include regime switching, level effects, and ARCH effects. In both models, it is seen that the ARCH parameter, β_1, is statistically insignificant and small. Thus, it appears that once we introduce state dependent volatility into the short rate model, ARCH effects are superfluous. The remaining parameter estimates are hardly changed by excluding the ARCH effects ($\beta_1 \equiv 0$). Below we provide further evidence of leaving ARCH effects out of the regime switching models.

Model V has a state dependent level parameter. In state 1, the level parameter is insignificantly different from $\frac{1}{2}$. State 2 is the high variance state where the level parameter significantly exceeds unity ($\widehat{\gamma}_2 = 1.35$), yet is not significantly different from $1\frac{1}{2}$. This implies that state 1 is compatible with the Cox et al. (1985) square root model, whereas state 2 is compatible with the Cox et al. (1980) volatility specification. Table 3 summarizes the level parameter estimates for model V and VI.

There are no previous findings to compare the estimated level parameters to. The closest we get is Bliss & Smith (1998) and Dell'Aquila et al. (2003) who apply the CKLS level model with dummy variables to distinguish between the Monetary Experiment and the rest of the sample period. Our high volatility state covers a much longer fraction of the sample period than the Monetary Experiment, more below, and our high variance state is not deterministic. In Bliss & Smith (1998) and Dell'Aquila et al. (2003) both γ and σ^2 are "state dependent" at the same time, whereas we assume that only one of them is state dependent at a time, to make sure that states are unambiguously low and high volatility states.

The ordinary Wald test is invalid for testing the hypothesis that the level parameters are identical in state 1 and 2, because under the null hypothesis the model is not regime switching and the transition probabilities are not identified. Still, the Wald test can be used as a rough indication of the validity of the hypothesis, cf. e.g. Ramchand & Susmel (1998). The p-value of the Wald test is far below 1%, thereby pointing towards using two regimes instead of just one. In the level model without regime switching (model I), the γ estimate falls between the γ_i estimates for state 1 and 2. So neglecting regime switching amounts to averaging the parameter estimates across the

states.

In model VI σ_t^2 is equal to a different constant in each state. The ordinary Wald test provides informal evidence of significantly different states, the p-value is far below 1%. The variance is around 20 times larger in the high variance state than in the low variance state. The level parameter (γ) equals 1.26 which is significantly larger than unity and significantly smaller than $1\frac{1}{2}$, so it is not in accordance with any of the know short rate models. However, it is similar to the parameter estimate in the single regime level model (model I).

In both model V and VI the low variance state is most common. The unconditional probability of being in state 1 is 58% and 62% for model V and VI, respectively. From the vectors of smoothed probabilities, we conclude that the economy is in state 1 60% (model V) and 64% (model VI) of the time, respectively. This also implies that the low volatility state is not pronouncedly dominant. The correlation between the smoothed state 1 probabilities for model V and VI is 0.91. The model V and VI indicator functions for being in state 1 have a correlation coefficient of 0.81.[9] Thus, the two models roughly classify the same periods as being low and high variance states.

Figure 1 shows the smoothed state 1 probabilities for model V. An ideal model classify regimes sharply and has smoothed probabilities that are either close to zero or one. Around 28% of the smoothed state 1 probabilities are below 0.2 and 43% are above 0.8, leaving 29% blurred observations. The regime classification measure ($RMC1$) of Ang & Bekaert (2002) is close to zero for a perfect model and close to 100 for a model that cannot distinguish between the two states

$$RMC1 = \frac{400}{T} \sum_{t=1}^{T} prob[s_t = 1|\Phi_T]prob[s_t = 2|\Phi_T] \qquad (21)$$

where $prob[s_t = i|\Phi_T]$ is the time t smoothed state i probability. Table 4 shows the $RCM1$s for each of the four regime switching models. The $RCM1$ is smallest for model VI, for which it equals 29.6. The $RCM1$s are smallest

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[9]The state 1 indicator function equals 1 at time t if the contemporaneous smoothed state 1 probability exceeds 0.5.

for models with regime switching in β_0 compared to models with regime switching in γ. Overall, the regime classification is fairly good.

The beginnings and endings of the regimes is not as clear-cut as one could wish. The economy is mainly in the high variance regime from the beginning of the sample period in 1975 to the beginning of 1983. This period includes the NBER business cycle recession that ends in March 1975 and the Monetary Experiment of the Federal Reserve from 1979-1983. The economy is also in the high variance regime around the 1987 stock market crash, in the beginning of the 1990s, which is concurrent with a NBER business cycle recession (July 1990 to March 1991), at the beginning of the 2000s (NBER business cycle recession from March 2001 to November 2001), and at the very end of the sample period.

Another way to highlight that the variance state is highly correlated with the general state of the economy is to run a logit regression where the explained variable is the indicator function that equals one when the economy is in the high variance state ($prob[s_t = 2|\Phi_T] > .5$). The explanatory variables are the GDP growth rate and a constant.[10] For both model V and VI the GDP growth rate is strongly significant in explaining the volatility state, and the relation is positive as expected. Moreover, the simple logit model classifies as many as 65% (66%) correct for model V (VI).

The periods classified as low and high variance states correspond fairly well with our expectations based on previous findings.

Table 5 shows various statistics for the standardized residuals.[11] Panel A concerns the US. According to the Jarque & Bera (1980) test for non-normality (not reported), the standardized residuals for all models are not normally distributed. The main departure from normality occurs via highly leptokurtic residuals. The non normality does not invalidate the models as the estimation is conducted using the *Quasi* ML method. The Lagrange Multiplier test for no ARCH(1) effects in the standardized residuals show no

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[10] The nominal GDP series are obtained from DataStream. The quarterly observations are transformed into logarithmic growth rates that are constant for all weeks in a given quarter.

[11] The standardized residuals are calculated using a weighting scheme based on the ex ante probabilities. This is identical to Gray (1996) as it appears from his shareware GAUSS program used to estimate his Table 2.

evidence of remaining ARCH effects in any of the models apart from model I. Also, the squared standardized residuals of model II-VI are not autocorrelated. This further corroborates the conclusions from above that once regime switching is included, it is not necessary to incorporate ARCH effects as well. For model II-VI the first order autocorrelations of the standardized residuals are insignificant. In contrast, the fourth order autocorrelation coefficients are small, negative, and significant based on a 5% level of significance (although they are insignificant at a 1% level of significance). Overall, model II-VI appear to be well specified. In contrast, model I standardized residuals show problems of heteroskedasticity and serially correlation both at lag 1 and 4.

The standardized residuals indicate that the CKLS level model is inappropriate and that there is no gain from including ARCH effects into the regime switching models, i.e. model V and VI are preferred. Yet, we do not have preference of one of those against the other.

Like Bali (2000b) we access the predictive ability of the models by regressing the realized volatility on model forecasted volatility. We compare the explanatory power across models by comparing their R^2s. In line with the previous literature, the R^2s are not corrected for model complexity. The realized volatility is measured by the absolute short rate changes: $|r_t - r_{t-1}|$:

$$|r_t - r_{t-1}| = c_0 + c_1\sqrt{\widehat{h}_t} + e_t \tag{22}$$

It is noted that the regression does not consider true out-of-sample predictability, which is similar to Bali (2000b). Table 6 shows the R^2s from these predictive regressions. The R^2s are fairly large, above 0.25. The R^2s from models without regime switching are smaller than for the models with regime switching. The R^2s are largest for the models with level effects and regime switching (0.33 for model V and VI). So, we find evidence that including ARCH effects does not increase the predictive power of the model. Thus, also in this respects there is no value added from including ARCH effects.

In summary, the US short rate shows mean reversion. A model which includes level effects and regime switching is preferred, whereas there is no

need for it to include ARCH effects. There is no evidence in favor of using either the model in which the level parameter is regime dependent (model V) or the model in which the volatility parameter is regime dependent (model VI). Both specifications capture the essential properties of the US short rate data. This is not unlike the findings of Smith (2002) who uses a short rate model with regime switching stochastic volatility and a state independent level parameter. He finds that it suffices to include regime switching such that (in our notation) $\sigma_t^2 = \begin{cases} \beta_{01} \\ \beta_{02} \end{cases}$ or to include stochastic volatility, it is not necessary to include both regime switching and stochastic volatility.

3.2.2 UK Short Rate Models

The parameter estimates from the six short rate models for the UK are provided in Panel B of Table 2.

Like the US, the UK short rate shows mean reversion in all model specifications. In the regime switching models the mean reversion parameter α_1 is significantly negative according to the ordinary t-test, cf. the discussion of its deficiencies above.

In neither of the models is the ARCH parameter significant. So, as for the US, including ARCH in the UK short rate model is superfluous once regime switching is included. Therefore, we again focus on model V and VI.

In model V the level parameter provides regime switching. In the low volatility regime, the level parameter equals 0.91 which is insignificantly different form unity. In the high variance state the level parameter equals 1.68 which is insignificantly different from $1\frac{1}{2}$. Thus, for the UK short rate the low volatility state is described by the Brennan & Schwartz (1980) model, cf. Table 3. In contrast, for the US the Cox et al. (1985) square root process is appropriate. In the high variance state, the UK short rate is described by the Cox et al. (1980) model, just like the US short rate. The ordinary Wald test gives a rough indication that the level parameters are significantly different in the two states (the p-value is far below 1%). As for the US, the UK level parameter in the level model without regime switching (model I) is insignificantly different from unity. Thus, again it falls between the state 1 and state 2 level parameters, i.e. indicating the single regime amounts to

averaging across regimes.

In model VI the regime switching occurs via $\sigma^2(s_t)$. The variance is about 27 times larger in the high variance state than in the low variance state. So the point estimates of the difference between the low and high variance states is more pronounced than for the US. Yet, the ordinary Wald test indicates that the β_{0i} parameters are not significantly different in the two states (the p-value equals 26%). We take this as evidence against applying model VI to the UK short rate. The level parameter is just slightly larger than unity and insignificantly so, pointing towards the Brennan & Schwartz (1980) model. This is equivalent to the γ estimate in the single regime level model (model I). For the US the level parameters are larger than unity and smaller than $1\frac{1}{2}$ in model I and VI.

The economy spends most of its time in the low variance state. The unconditional probability of being in state 1 is 79% for model V and 77% for model VI. According to the smoothed probabilities, the economy spends 82% (model V) and 81% (model VI) in the low variance state. So, the dominance of the low variance state is more pronounced than for the US. The two models specify the same periods as low and high variance and the smoothed state 1 probabilities are almost perfectly correlated: The correlation coefficient exceeds 0.99. Figure 2 shows the smoothed state 1 probability of model V. The economy is mainly in the high variance state in the period 1976-1978. Unlike for the US, there are no official UK business cycle data available to compare to. From logit regressions we learn that the GDP growth rate is strongly significant in explaining the state 2 indicator, and the positive relationship indicates that the high variance state is mainly occurring during expansions. The logit model classifies 67% and 81% of the observations correct in model V and VI, respectively,

Table 4 shows the regime classification measures for model III-VI. The $RCM1$ is smallest for model V where it equals 23.8. This is unlike for the US where the $RCM1$ is smallest for model VI. The regime classification is sharper for the UK than the US.

The properties of the standardized residuals for the UK are identical to those of the US, cf. Table 5, Panel B.

The predictability of the six models is assessed by running the regression

in equation (22), cf. Table 6. The explanatory power is much lower than for the US, the R^2 ranges from 0.08 to 0.11. As for the US, the R^2 is lowest for the models without regime switching. There is no predictability gain from including ARCH effects into the regime switching models. The R^2 is almost identical for model V and VI, namely 0.10 and 0.11, respectively.

Overall, the findings are in favour of short rate models with level effects and regime switching but no ARCH effect. There is a slight tendency to prefer the specification with state dependent level parameter (model V).

3.2.3 German Short Rate Models

The results from estimating the short rate models for Germany are provided in Table 2, Panel C.

Like the US and UK, all models include mean reversion. The ARCH parameter, β_1, is only significant in the model without regime switching. So, we confirm for Germany that once regime switching is introduced there is no need for ARCH effects. This points towards models V and VI.

In model V the level parameter is regime dependent. In the low variance regime, the level parameter is in fact insignificant, whereby the conditional volatility is constant and equals σ^2, just like in Vasicek (1977). In the high variance state, the level parameter is not significantly different from unity, like the Brennan & Schwartz (1980) model. Thus, the $\hat{\gamma}_i$s are totally different from the US and UK, cf. Table 3. The ordinary Wald test provides evidence that the level parameters are not identical in the two states, the p-value is far below 1%. As for the US and the UK, $\hat{\gamma}$ of the single regime level model falls in between $\hat{\gamma}_1$ and $\hat{\gamma}_2$. It equals 0.87 which is significantly larger than zero and smaller than unity.

In model VI the ordinary Wald test informally indicates no regime switching, the p-value equals 37%. This is similar to the UK but unlike the US. The point estimate of the variance is about 17 times larger in the high variance state than in the low variance state. The difference between the variance across regimes is smaller than for the US and UK. The level parameter is of about the same size as in the single regime level model as for the US and UK.

State 1 is the most common state for model V and VI. The unconditional state 1 probability equals 61% (model V) and 62% (model VI). According to the smoothed probabilities, the economy spends 62% (model V) and 63% (model VI) of its time in the low variance state. These figures are close to the US but lower than the UK. The appointed low variance periods are close to being identical for models V and VI: The correlation between the smoothed state 1 probabilities is 0.95. The smoothed state 1 probabilities for model V are shown in Figure 3. The economy is mainly in the high variance regime during the first part of the sample period until the beginning of the 1980s. The German reunification in 1990 does not appear to cause the regime switching. There are no official business cycle data available for Germany. Instead, the logit regressions explaining the state 2 indicator by the GDP growth rate show that the volatility state is strongly and positively correlated with the business cycle. 61% (62%) of the observations are classified correct in model V (VI).

Although the division of the sample period into high and low volatility periods is not as clear-cut as desirable, the regime classification measures are low, cf. Table 4. The $RCM1$ is smallest for model V where it equals 26.0, about the same size as for the UK and much smaller than for the US.

Table 5, Panel C provides information about the standardized residuals. For models I to IV there are no differences to the US, whereas for model V there is evidence of unaccounted serial correlation and heteroskedasticity. For model VI there is evidence of unaccounted serial correlation.

The predictability of the models measured by the R^2s obtained from regression (22) are in between the US and UK for models IV and V. The R^2s are largest for model V and VI and amount to 0.13 and 0,14, respectively.

Overall, we take our results as providing evidence for the short rate model with level effects and regime switching but no ARCH effects. The results provide no preference for regime switching in the level parameter or in σ_t^2. This confirms the US and UK results.

3.3 Bivariate Analysis

The estimation of the bivariate models is conducted similar to the univariate models. Firstly, the US-UK results are examined at some length, and subsequently the US-German results are discussed briefly.

3.3.1 US-UK Results

Panel A of Table 7 shows the parameter estimates for the bivariate models for the US-UK short rate changes. The estimated correlation coefficients (ρ) are small, negative, and insignificant, between -0.015 and -0.038. This is perhaps not surprising because the raw correlation between the US and UK short rate changes amounts to -0.03, cf. Section 3.1.

As in the univariate models, the ARCH parameter, β_1^i, is insignificant in model A and B. So, we prefer model C and D with regime switching and level effects, but no ARCH effects. This is similar to the univariate results. The parameter estimates in both model C and D correspond to the univariate equivalents.

For model C, the ordinary Wald test for the null hypothesis of $\gamma_1^1 = \gamma_2^1 \wedge \gamma_1^2 = \gamma_2^2$ provides an informal indication of the validity of a one-regime model. For model D the null hypothesis is $\beta_{01}^1 = \beta_{02}^1 \wedge \beta_{01}^2 = \beta_{02}^2$. Notice, that under both null hypotheses, the transition probability matrix P^*, is not defined, cf. the discussion in Section 3.2.1 above. The resulting p-values are far below 1% suggesting that one-regime models are inadequate.

For clarity, Table 7 reports the unconditional state probabilities not the estimated 4×4 transition probability matrix P^*. The ordering of the unconditional probabilities is identical for model C and D. The unconditional probabilities of the low-low volatility state ($s_t^* = 1$) exceed 50%. The second most likely situation is the US high-UK low volatility state ($s^* = 3$), followed by both countries being in the high variance state ($s_t^* = 4$). The least likely is the US low-UK high volatility state ($s^* = 2$). For model C, the unconditional probability of the US being in the low variance state (sum of $s^* = 1$ and $s^* = 2$ probabilities) equals 58% (same as in univariate model). The probability of the UK being in the low variance state ($s^* = 1$ and $s^* = 3$) equals 79% again equivalent to the univariate case. In model

D, the unconditional probability of the US being in state 1 is slightly larger than in the univariate model and unaltered for the UK.

Based on the smoothed probabilities for model C (model D), the economy spends 54% (56%) of the time in the low-low volatility state ($s^* = 1$), 29% (25%) in the US low-UK high volatility state ($s^* = 3$), 11% (8%) in the high-high volatility state ($s^* = 4$), and 6% (10%) in the US low-UK high volatility state ($s^* = 2$). Model by model, the bivariate smoothed probabilities for the US being in the low volatility state are almost identical to the US univariate smoothed probabilities of the low volatility state. The same applies to the UK. The correlations are between 96% and 98%.

The regime classification measure for two states of Ang & Bekaert (2002) in equation (21) is extended to multiple states in Baele (2005). For four states it reads:

$$ RCM2 = 100 \left(1 - \frac{4}{3} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{4} \left(prob[s_t^* = j | \Phi_t] - \frac{1}{4} \right)^2 \right) \qquad (23) $$

The $RCM2$ lies between 0 (perfect) and 100 (worst). The regime classification measures are shown in Table 9. The $RCM2$ is smallest for the non-ARCH models, and for those it is slightly smaller for model D than for model C; 33.8 compared to 34.5. This is equivalent to the univariate US results, but opposite the univariate UK result.

In order to test if the volatility states of the US and UK are independent, we estimate model C including appropriate restrictions on the transition matrix, cf. the discussion in Section 2.2.1. We use a Likelihood Ratio test to test the null hypothesis of independent states, cf. Susmel (2000) and Baele (2005). The test statistic is χ^2 distributed where the degrees of freedom equal the difference in the number of parameters in the restricted and unrestricted model.[12] The LR test is also used to test for identical states. Here, the restricted model with identical states is actually not nested in the unrestricted model, still the ordinary χ^2 distribution is applied, cf. Baele (2005). The same set of LR tests are applied to model D. For both model C and D we strongly reject that the US and UK volatility states

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[12] Elements of P^* with zero point estimates do not count to the number of parameters.

are independent, implying that the volatility state of one country influences the volatility state of the other country and vice versa. The US and UK volatility states are significantly not identical either, so the economies are not always in the same volatility state. The LR p-values are all far below 1%. Thus, the dependence of the US and UK volatility states is somewhere between the two extreme cases and it makes sense to take account of the interdependence in empirical work, unlike e.g. Ang & Bekaert (2002) and Christiansen (2004).

In a similar fashion we use the LR test statistics to test for Granger causality and for contagion between the US and UK volatility states. To do so, we estimate the models imposing the relevant restrictions for no Granger causality and contagion to the transition matrix, cf. the discussion in Section 2.2.1. First, the null hypothesis is no Granger causality from the US volatility state to the UK volatility state. Both model C and D implies that there is Granger causality from the US volatility state to the UK volatility state (the p-values from the LR tests are below 1%). In contrast, we find no evidence of the UK volatility state Granger causing the US volatility state (the LR tests imply p-values of 91% and 39% for models C and D, respectively). Contagion is a stronger relationship between the two volatility states than Granger causality. Thus we might find causality from the US to the UK but not vice versa. However, we strongly reject the null hypothesis of contagion from the US volatility state to the UK volatility state (the p-values for the LR tests are well below 1%). As expected, there is no contagion from the UK volatility state to the US volatility state (the p-values for the LR tests are much lower than 1%).

The standardized residuals imply that the models are fairly well specified, cf. Table 9. The cross multiplied standardized residuals are not serially correlated, which indicates that there is no unaccounted heteroskedasticity in the covariances.

To compare the predictive power of the models we conduct similar predictive regressions as in equation (22); a regression for the US short rate and another regression for the UK short rate. In a third covariance regression the explained variable is $(\left| r_t^1 - r_{t-1}^1 \right| \left| r_t^2 - r_{t-1}^2 \right|)$ and the explanatory variable is the estimated covariance $(\widehat{h_t^{12}})$. The predictive power is much larger for the

non-ARCH models (model C and D) than for the ARCH models (model A and B). The R^2s are slightly larger for model D than C. The R^2s for the volatility regressions are of about the same size as in the univariate case. The R^2s from the covariance regressions fall between the US and UK R^2s.

To sum up, the bivariate models provide similar information as the univariate models as to the preferred short rate volatility specification. From the bivariate setting we learn about the coincidence and dependence of the US and UK volatility states, namely that the US and UK volatility states are neither independent nor identical, that the US volatility state Granger causes the UK volatility state and not vice versa, and that there is no contagion between the US and UK volatility states.

3.3.2 US-German Results

The parameter estimates for the US-German estimation are shown in Panel B of Table 7. As above, the parameter estimates are almost identical to those from the univariate regressions and the correlation coefficient (ρ) is small, negative, and insignificant. Again, the non-ARCH models are superior.

According to the $RCM2$ the preferred specification is model C, cf. Table 8 which is similar to German univariate results and opposite US univariate results. In contrast, the standardized residuals point towards model D, cf. Table 9, similar to the univariate German results. The predictive regressions do not select model C above model D, cf. Table 10.

According to the smoothed probabilities of model C (model D), the economy spends 57% (52%) in the low-low volatility state, 18% (17%) in the high-high volatility state, 13% (16%) in the US high-German low volatility state, and 12% (14%) in the US low-German high volatility state. The smoothed probabilities for the US low volatility (states $s^* = 1$ and $s^* = 2$) and the German low volatility (states $s^* = 1$ and $s^* = 3$) are strongly correlated with the univariate equivalents, the correlation coefficients are between 91% and 96%. So, the bivariate models classify the same periods as low volatility periods and high volatility periods as the univariate models. The ordinary Wald tests provide evidence against non-switching models.

The LR test for identical US and German volatility states is clearly

rejected for both model C and D. Similarly, the LR test rejects that the US and German volatility states are independent. This implies, that the dependence between the US volatility state and the German volatility state falls somewhere between these two extreme cases.

There is mixed evidence as to whether the US volatility state Granger causes the German volatility state, the LR test statistic for no such Granger causality results in a p-value less than 1% for model C, but equal to 20% for model D. The German volatility state Granger causes the US volatility state (both p-values are below 1%). This is in contrast to the UK not Granger causing the US volatility state. We strongly reject that there is contagion from the US volatility state to the German volatility state and vice versa (all p-values below 1%).

4   Conclusion

This paper investigates models for the short rates of the US, the UK, and Germany. Firstly, we analyze one country at a time. We extend the CKLS short rate model (where the conditional volatility is proportional to the γth power of the level of the short rate) to include both heteroskedasticity effects and regime switching volatility. The regime switching either occur in the level parameter or in the constant in the ARCH specification. For each country we estimate six different versions of the short rate model. We show that once the short rate model includes regime switching and level effects there are no further gains from including ARCH effects as well. This conclusion is based on the parameter estimates, the regime classification measure, the properties of the residuals, and predictive regressions. Moreover, the inclusion of regime switching is essential. Furthermore, it is of secondary importance whether the regime switching occurs in the level parameter or in the ARCH parameter. When the level parameter is state independent it is an average of the level parameters in the low and high variance states. When the regime switching occur via the ARCH parameter the volatility is several times higher in the high variance state than in the low variance state. The estimated level parameters are very different across countries. The estimated level parameters are compared to various popular short rate

models.

We also estimate equivalent bivariate models for the US-UK short rates as well as for the US-German short rates. The findings regarding the short rate volatilities are unaltered compared to the univariate models. Most importantly, we find that the volatility states of the US and UK are not independent. Moreover, the US and UK economies are not in the same volatility state at the same time. The US volatility state Granger causes the UK volatility state, whereas there is no contagion from the US volatility state to the UK volatility state. The UK volatility state neither Granger causes nor provides contagion to the US volatility state. Equivalent, the short rate volatility states of the US and Germany are not independent and not identical. There is mixed evidence of Granger causality from the US volatility state to the German volatility state and no contagion in that direction. The German volatility state Granger causes but provides no contagion to the US volatility state.

The papers highlights that it is important to account for the fact that the volatility states of short rates of different countries are neither independent nor identical. In future research it would be interesting to investigate in more detail the severity of applying these assumptions which are applied because they simplify the estimation. Moreover, whether or not the short rate volatility states Granger causes each other across the Atlantic and the fact that there is no contagion across the Atlantic have policy implications.

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Table 1: Summary Statistics

	US		UK		Germany	
	r	Δr	r	Δr	r	Δr
Mean	6.75	-0.005	9.23	-0.004	5.25	-0.003
Std dev	3.59	0.38	3.7	0.34	2.44	0.24
Skewness	0.99	-0.22	0.34	1.44	0.94	2.71
Kurtosis	4.47	24.38	2.09	20.76	3.07	66.43
AC, 1	0.994	-0.018	0.995	-0.039	0.995	-0.106
AC, 4	0.973	-0.016	0.981	0.069	0.982	-0.005
AC, sq, 1	0.971	0.265	0.983	0.148	0.975	0.038
AC, sq, 4	0.869	0.205	0.942	0.058	0.934	0.022
$\mathrm{cor}(r_{t-1}\Delta r_t)$	-0.051		-0.042		-0.043	

Notes: The table reports the summary statistics for the US 1-month Eurodollar rate, the UK 1-month LIBOR, and the German 1-month Euromark rate. The interest rates are measured in percent per annum. The weekly (Wednesdays) data cover the period 1975-2004. For each country the first column concerns the short rate (r), and the second column the short rate changes (Δr). The following statistics are reported: Mean, standard deviation, skewness, kurtosis, autocorrelation (order 1 and 4), autocorrelation of the squared series (order 1 and 4), and the correlation between the short rate last week and the current short rate changes.

Table 2: Univariate Results

Panel A: US

	I		II		III		IV		V		VI	
$100\alpha_0$	0.198		0.385		0.195		-0.071		0.209		-0.063	
	(0.408)		(0.473)		(0.370)		(0.147)		(0.371)		(0.151)	
$100\alpha_1$	-0.038		-0.103		-0.057		-0.010		-0.063		-0.015	
	(0.141)		(0.172)		(0.096)		(0.064)		(0.098)		(0.067)	
$\gamma(s_t=1)$	1.228	***	1.086	***	0.529	***	1.219	***	0.585	***	1.260	***
	(0.066)		(0.084)		(0.169)		(0.075)		(0.148)		(0.066)	
$\gamma(s_t=2)$					1.294	***			1.345	***		
					(0.099)				(0.091)			
$100\beta_0(s_t=1)$	0.065	***	0.083	***	0.089	**	0.007	***	0.083	***	0.007	***
	(0.018)		(0.025)		(0.027)		(0.002)		(0.026)		(0.002)	
$100\beta_0(s_t=2)$							0.150	***			0.141	***
							(0.044)				(0.040)	
$10\beta_1$			0.003		0.002		0.004					
			(0.002)		(0.002)		(0.004)					
p_{11}					0.901	***	0.901	***	0.908	***	0.908	***
					(0.039)		(0.031)		(0.032)		(0.022)	
p_{22}					0.881	***	0.842	***	0.875	***	0.849	***
					(0.046)		(0.075)		(0.046)		(0.056)	

Panel B: UK

	I		II		III		IV		V		VI	
$100\alpha_0$	1.813		-0.158		2.690	***	2.639	***	2.740	***	2.808	***
	(1.530)		(2.830)		(0.745)		(0.732)		(0.740)		(0.703)	
$100\alpha_1$	-0.234		-0.078		-0.484	***	-0.467	***	-0.493	***	-0.496	***
	(0.219)		(0.470)		(0.116)		(0.111)		(0.114)		(0.109)	
$\gamma(s_t=1)$	1.019	***	0.985	***	0.905	***	1.010	***	0.909	***	1.075	***
	(0.283)		(0.043)		(0.183)		(0.175)		(0.186)		(0.159)	
$\gamma(s_t=2)$					1.674	***			1.680	***		
					(0.162)				(0.164)			
$100\beta_0(s_t=1)$	0.112		0.067	***	0.029		0.015		0.028		0.012	
	(0.150)		(0.019)		(0.025)		(0.010)		(0.025)		(0.009)	
$100\beta_0(s_t=2)$							0.478				0.346	
							(0.483)				(0.303)	
β_1			0.018		0.002		0.001					
			(0.019)		(0.003)		(0.002)					
p_{11}					0.903	***	0.891	***	0.905	***	0.903	***
					(0.017)		(0.020)		(0.016)		(0.015)	
p_{22}					0.634	***	0.567	***	0.641	***	0.667	***
					(0.078)		(0.171)		(0.074)		(0.061)	

	I	II	III	IV	V	VI
$100\alpha_0$	2.012 **	1.636	0.158	0.153	0.055	0.338
	(1.001)	(1.055)	(0.758)	(0.383)	(0.768)	(0.705)
$100\alpha_1$	-0.449 *	-0.233	-0.068	-0.079	-0.051	-0.149
	(0.247)	(0.293)	(0.170)	(0.096)	(0.168)	(0.179)
$\gamma(s_t=1)$	0.871 ***	0.902 ***	0.203	0.815 ***	0.140	0.793 *
	(0.053)	(0.016)	(0.270)	(0.240)	(0.244)	(0.461)
$\gamma(s_t=2)$			1.018		1.004 ***	
			(0.292)		(0.255)	
$100\beta_0(s_t=1)$	0.283 ***	0.159 ***	0.299	0.032 *	0.400	0.052
	(0.053)	(0.043)	(0.229)	(0.017)	(0.274)	(0.046)
$100\beta_0(s_t=2)$				0.645 *		0.863
				(0.349)		(0.952)
β_1		0.028 ***	0.008	0.016		
		(0.010)	(0.007)	(0.015)		
p_{11}			0.930	0.913 ***	0.935 ***	0.926 ***
			(0.023)	(0.032)	(0.016)	(0.035)
p_{22}			0.903	0.865 ***	0.899 ***	0.877 ***
			(0.045)	(0.043)	(0.039)	(0.042)

Notes: The table shows the parameter estimates from the univariate model for the short rate changes for the US, UK, and Germany: $r_t - r_{t-1} = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t$ where the conditional variance of the residual is given by: $h_t = \sigma_t^2 r_{t-1}^{2\gamma(s_t)}$ where $\sigma_t^2 = \beta_0(s_t) + \beta_1 \varepsilon_{t-1}^2$. Bollerslev & Wooldridge (1992) robust standard errors in parentheses. Model I: Level. Model II: Level ARCH. Model III: Switching-Level ARCH with $\beta_{01} = \beta_{02}$. Model IV: Level SWARCH with $\gamma_1 = \gamma_2$. Model V: Switching-Level with $\beta_{01} = \beta_{02}$. Model VI: Level switching-volatility with $\gamma_1 = \gamma_2$. *, **, *** indicates that the parameter is significant at 10%, 5%, and 1% level of significance, respectively.

Table 3: Corresponding Short-Rate Models

	Model V		Model VI
	State 1	State 2	
US	$\gamma = \frac{1}{2}$	$\gamma = 1\frac{1}{2}$	$\gamma \in (1; 1\frac{1}{2})$
	Cox et al. (1985)	Cox et al. (1980)	NA
UK	$\gamma = 1$	$\gamma = 1\frac{1}{2}$	$\gamma = 1$
	Brennan & Schwartz (1980)	Cox et al. (1980)	Brennan & Schwartz (1980)
Germany	$\gamma = 0$	$\gamma = 1$	$\gamma = \frac{1}{2}$
	Vasicek (1977)	Brennan & Schwartz (1980)	Cox et al. (1985)

Notes: The table summarizes the estimated level parameters and the corresponding short rate models. Model V: Switching-Level with $\beta_{01} = \beta_{02}$. Model VI: Level switching-volatility with $\gamma_1 = \gamma_2$.

Table 4: Regime Classification Measures - Univariate Models

	III	IV	V	VI
US	36.6	32.9	36.1	29.6
UK	24.0	27.5	23.8	24.5
Germany	28.5	34.3	26.0	28.1

Notes: The table reports the regime classification measure (RCM1) of Ang & Bekaert (2002) for the univariate model for the short rate changes for the US, UK, and Germany: Model III: Switching-Level ARCH with $\beta_{01} = \beta_{02}$. Model IV: Level SWARCH with $\gamma_1 = \gamma_2$. Model V: Switching-Level with $\beta_{01} = \beta_{02}$. Model VI: Level switching-volatility with $\gamma_1 = \gamma_2$.

Table 5: Standardized Residuals - Univariate Models

Panel A: US

	I	II	III	IV	V	VI
Skewness	0.42	0.97	0.60	-0.20	0.57	-0.24
Kurtosis	17.13	18.23	25.81	25.13	26.06	26.24
ARCH(1)	48.40	0.56	0.51	0.02	1.34	0.41
AC,1	-0.057 **	-0.038	-0.031	-0.026	-0.032	-0.028
AC, 4	-0.013 ***	-0.013 **	-0.016 **	-0.021 **	-0.018 **	-0.021 **
AC, sq, 1	0.176 ***	0.019	0.018	0.004	0.029	0.016
AC, sq, 4	0.049 ***	0.049	0.019	0.015	0.019	0.014

Panel B: UK

	I	II	III	IV	V	VI
Skewness	3.36	3.31	3.42	2.89	3.39	2.80
Kurtosis	45.59	32.30	36.62	24.56	36.33	24.77
ARCH(1)	7.45 ***	0.03	0.45	0.00	0.75	0.28
AC,1	-0.036	0.008	-0.013	-0.01	-0.017	-0.014
AC, 4	0.076 ***	0.0074 **	0.085 ***	0.093 ***	0.086 ***	0.094 ***
AC, sq, 1	0.069 ***	-0.004	0.017	-0.001	0.022	0.013
AC, sq, 4	0.023 **	0.048	0.034	0.06	0.034	0.055

Panel C: Germany

	I	II	III	IV	V	VI
Skewness	0.92	1.24	0.52	2.20	0.58	1.69
Kurtosis	23.15	30.7	14.86	43.52	14.56	30.01
ARCH(1)	42.57 ***	0.03	0.07	0.05	7.86 ***	0.64
AC,1	-0.092 ***	-0.051 **	-0.052 **	-0.053 **	-0.067 ***	-0.070 ***
AC, 4	0.006 ***	0.014	0.014	0.011	0.013 **	0.011 **
AC, sq, 1	0.165 ***	-0.005	0.007	-0.005	0.071 ***	0.020
AC, sq, 4	0.048 ***	0.012	0.010	-0.000	0.018 ***	0.004

Notes: The table reports various statistics for the standardized residuals from the univariate model for the short rate changes for the US, UK, and Germany: Model I: Level. Model II: Level ARCH. Model III: Switching-Level ARCH with $\beta_{01} = \beta_{02}$. Model IV: Level SWARCH with $\gamma_1 = \gamma_2$. Model V: Switching-Level with $\beta_{01} = \beta_{02}$. Model VI: Level switching-volatility with $\gamma_1 = \gamma_2$. The following statistics are reported: Skewness, Kurtosis, LM test statistic for ARCH(1), autocorrelation (order 1 and 4), and autocorrelation for squared standardized residuals (order 1 and 4). (Except for the skewness and kurtosis) *, **, *** indicates significance at 10%, 5%, and 1% level of significance.

Table 6: Predictive Power - Univariate Models

	I	II	III	IV	V	VI
US	0.29	0.25	0.33	0.31	0.33	0.33
UK	0.08	0.08	0.10	0.11	0.10	0.11
Germany	0.05	0.05	0.08	0.08	0.13	0.14

Notes: For the univariate models for the US, UK, and Germany, the table reports the R^2s from regressing the absolute short rate change ($|r_t - r_{t-1}|$) on its estimated volatility ($\sqrt{\widehat{h_t}}$). Model I: Level. Model II: Level ARCH. Model III: Switching-Level ARCH with $\beta_{01} = \beta_{02}$. Model IV: Level SWARCH with $\gamma_1 = \gamma_2$. Model V: Switching-Level with $\beta_{01} = \beta_{02}$. Model VI: Level switching-volatility with $\gamma_1 = \gamma_2$.

Table 7: Bivariate Results

Panel A: US-UK

	A		B		C		D	
	US	UK	US	UK	US	UK	US	UK
$100\alpha_0$	0.297	2.648 ***	-0.036	2.119 ***	0.304	2.283 ***	-0.045	2.564 ***
	(0.312)	(0.753)	(0.138)	(0.782)	(0.367)	(0.859)	(0.158)	(0.760)
$100\alpha_1$	-0.052	-0.477 ***	-0.031	-0.379 ***	-0.095	-0.419 ***	-0.036	-0.456 ***
	(0.077)	(0.116)	(0.056)	(0.121)	(0.098)	(0.135)	(0.070)	(0.117)
$\gamma(s_t=1)$	0.355 ***	0.878 ***	1.072 ***	0.793 ***	0.552 ***	0.756 ***	1.255 ***	0.957 ***
	(0.102)	(0.167)	(0.090)	(0.165)	(0.115)	(0.230)	(0.074)	(0.184)
$\gamma(s_t=2)$	1.199 ***	1.638 ***			1.322 ***	1.513 ***		
	(0.079)	(0.147)			(0.077)	(0.225)		
$100\beta_0(s_t=1)$	0.085 ***	0.033	0.005 **	0.034	0.088 ***	0.052	0.007 ***	0.021
	(0.020)	(0.026)	(0.002)	(0.021)	(0.027)	(0.050)	(0.002)	(0.016)
$100\beta_0(s_t=2)$			0.198 ***	1.218			0.149 ***	0.560
			(0.071)	(1.054)			(0.044)	(0.500)
$100\beta_1$	9.858 *	0.002	0.173	0.468				
	(5.397)	(0.003)	(0.138)	(0.554)				
ρ	-0.018		-0.038		-0.015		-0.028	
	(0.030)		(0.030)		(0.031)		(0.030)	
$prob[s_t=1]$	0.620		0.447		0.502		0.523	
$prob[s_t=2]$	0.131		0.077		0.074		0.112	
$prob[s_t=3]$	0.170		0.360		0.285		0.245	
$prob[s_t=4]$	0.079		0.117		0.145		0.119	

Panel B: US-Germany

	A		B		C		D	
	US	Germany	US	Germany	US	Germany	US	Germany
$100\alpha_0$	0.140	0.424	-0.069	0.285	0.171	0.339	-0.068	0.464
	(0.372)	(0.661)	(0.155)	(0.397)	(0.366)	(0.649)	(0.166)	(0.671)
$100\alpha_1$	-0.036	-0.105	-0.033	-0.096	-0.048	-0.094	-0.032	-0.149
	(0.105)	(0.142)	(0.066)	(0.098)	(0.100)	(0.139)	(0.070)	(0.148)
$\gamma(s_t=1)$	0.562 ***	0.135	1.166 ***	0.693 ***	0.582 ***	0.094	1.233 ***	0.681 ***
	(0.100)	(0.207)	(0.099)	(0.194)	(0.086)	(0.189)	(0.073)	(0.239)
$\gamma(s_t=2)$	1.313 ***	0.946 ***			1.346 ***	0.956 ***		
	(0.075)	(0.227)			(0.069)	(0.197)		
$100\beta_0(s_t=1)$	0.097 ***	0.441 *	0.007 ***	0.048	0.093 ***	0.533 **	0.007 ***	0.083
	(0.027)	(0.229)	(0.002)	(0.031)	(0.025)	(0.253)	(0.002)	(0.068)
$100\beta_0(s_t=2)$			0.188 ***	1.019			0.162 ***	1.394
			(0.065)	(0.634)			(0.053)	(1.046)
$100\beta_1$	0.020	1.220	0.085	2.072				
	(0.024)	(1.195)	(0.091)	(1.619)				
ρ	-0.058*		-0.048		-0.058*		-0.043	
	(0.030)		(0.031)		(0.030)		(0.036)	
$prob[s_t=1]$	0.511		0.462		0.525		0.488	
$prob[s_t=2]$	0.139		0.179		0.132		0.150	
$prob[s_t=3]$	0.157		0.164		0.150		0.181	
$prob[s_t=4]$	0.192		0.194		0.193		0.181	

Notes: The table shows the parameter estimates from the bivariate model for the short rate changes for the US-UK and US-Germany: $r_t^i - r_{t-1}^i = \alpha_0^i + \alpha_1^i r_{t-1} + \epsilon_t$ for $i = 1,2$ where the conditional variance of the residual is given by: $h_t^i = (\sigma_t^i)^2 (r_{t-1}^i)^{2\gamma^i(s_t)}$ where $(\sigma_t^i)^2 = \beta_0^i(s_t^i) + \beta_1^i(\varepsilon_{t-1}^i)^i$ and the conditional covariance is given by: $h_t^{12} = \rho\sqrt{h_t^1 h_t^2}$. Bollerslev & Wooldridge (1992) robust standard errors in parentheses. Model A: Switching-Level ARCH with $\beta_{01}^i = \beta_{02}^i$. Model B: Level SWARCH with $\gamma_1^i = \gamma_2^i$. Model C: Switching-Level with $\beta_{01}^i = \beta_{02}^i$. Model D: Level switching-volatility with $\gamma_1^i = \gamma_2^i$. *, **, *** indicates that the parameter is significant at 10%, 5%, and 1% level of significance, respectively.

Table 8: Regime Classification Measure - Bivariate Models

	A	B	C	D
US-UK	35.5	40.2	34.5	33.8
US-Germany	37.5	43.1	34.9	36.3

Notes: The table reports the regime classification measure (RCM2) of Baele (2005) for the bivariate model for the short rate changes for the US-UK and US-Germany: Model A: Switching-Level ARCH with $\beta_{01}^i = \beta_{02}^i$. Model B: Level SWARCH with $\gamma_1^i = \gamma_2^i$. Model C: Switching-Level with $\beta_{01}^i = \beta_{02}^i$. Model D: Level switching-volatility with $\gamma_1^i = \gamma_2^i$.

Table 9: Standardized Residuals - Bivariate Models

Panel A: US-UK

	A US	A UK	B US	B UK	C US	C UK	D US	D UK
Skewness	0.61	3.39	0.03	3.33	0.18	1.00	-0.69	3.01
Kurtosis	29.75	35.48	19.35	32.05	24.56	3.32	23.33	27.91
ARCH(1)	0.11	0.37	0.05	0.02	1.66	0.64	0.57	0.20
AC,1	-0.022	-0.015	-0.040	-0.019	-0.034	-0.036	-0.029	-0.026
AC, 4	-0.017 **	0.083 ***	-0.020 ***	0.072 **	-0.015	0.068 **	-0.023 **	0.073 **
AC, sq, 1	-0.008	0.015	0.006	-0.003	0.033	0.020	0.019	0.011
AC, sq, 4	0.021	0.031	0.035	0.027	0.016	0.021	0.011	0.035
AC, cr, 1	-0.007		-0.015		-0.013		-0.005	
AC, cr, 4	-0.008		-0.022		-0.011		-0.016	

Panel B: US-Germany

	A US	A Germany	B US	B Germany	C US	C Germny	D US	D Germany
Skewness	-0.37	0.60	-0.46	2.15	-0.43	0.65	-1.00	1.49
Kurtosis	22.35	14.29	22.24	41.70	22.50	13.08	27.31	26.13
ARCH(1)	0.52	0.003	0.009	0.05	1.42	5.85 **	0.42	0.44
AC,1	-0.020	-0.062 **	-0.029	-0.055 **	-0.021	-0.078 ***	-0.027	-0.073 ***
AC, 4	0.018 ***	0.013	0.006 **	0.009	0.018	0.010 **	0.010 ***	0.006 **
AC, sq, 1	0.018	-0.001	0.002	-0.006	0.030	0.061 **	0.016	0.017
AC, sq, 4	0.007	0.010	0.016	0.002	0.006	0.018 **	0.009	0.007
AC, cr, 1	-0.005		-0.009		-0.007		-0.007	
AC, cr, 4	0.001		0.002		0.001		0.005	

Notes: The table reports various statistics for the standardized residuals from the bivariate model for the short rate changes for the US-UK and US-Germany: Model A: Switching-Level ARCH with $\beta_{01}^{i} = \beta_{02}^{i}$. Model B: Level SWARCH with $\gamma_{1}^{i} = \gamma_{2}^{i}$. Model C: Switching-Level with $\beta_{01}^{i} = \beta_{02}^{i}$. Model D: Level switching-volatility with $\gamma_{1}^{i} = \gamma_{2}^{i}$. The following statistics are reported: Skewness, Kurtosis, LM test statistic for ARCH(1), autocorrelation (order 1 and 4), autocorrelation for squared standardized residuals (order 1 and 4), and autocorrelation for cross multiplied standardized residuals (order 1 and 4). (Except for the skewness and kurtosis) *, **, *** indicates significance at 10%, 5%, and 1% level of significance.

Table 10: Predictive Power - Bivariate Models

Panel A: US-UK

	A	B	C	D
US volatility	0.23	0.28	0.33	0.34
UK volatility	0.11	0.11	0.12	0.12
Covariance	0.13	0.16	0.21	0.22

Panel B: US-Germany

	A	B	C	D
US volatility	0.35	0.31	0.36	0.35
German vol.	0.09	0.09	0.13	0.14
Covariance	0.15	0.16	0.24	0.24

Notes: For the bivariate models for US-UK and US-Germany, the table reports the R^2s from regressing the absolute short rate change ($\left|r_t^i - r_{t-1}^i\right|$) on its estimated volatility ($\sqrt{\widehat{h_t^i}}$). for $i = 1, 2$, and from regressing ($\left|r_t^1 - r_{t-1}^1\right|\left|r_t^2 - r_{t-1}^2\right|$) on the estimated covariance ($\widehat{h_t^{12}}$). A: Switching-Level ARCH with $\beta_{01}^i = \beta_{02}^i$. Model B: Level SWARCH with $\gamma_1^i = \gamma_2^i$. Model C: Switching-Level with $\beta_{01}^i = \beta_{02}^i$. Model D: Level switching-volatility with $\gamma_1^i = \gamma_2^i$.
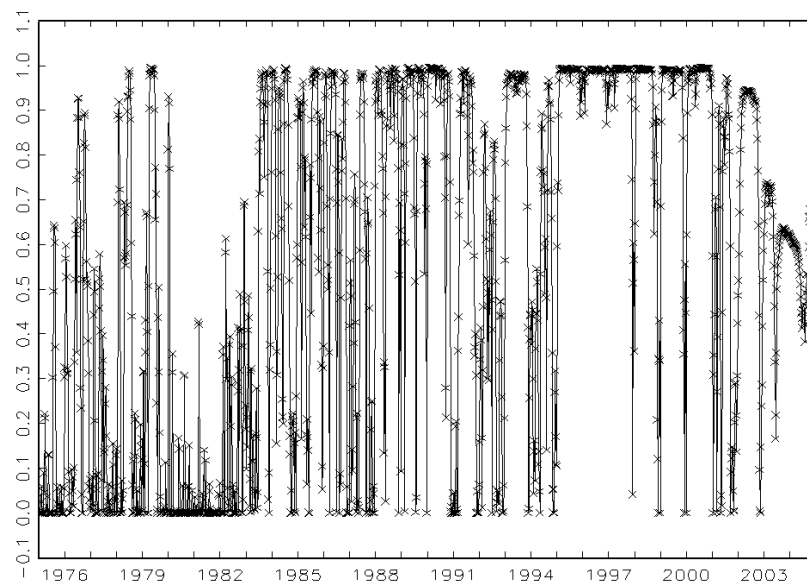
Figure 1:

Note: The figure shows the smoothed state 1 probabilities for the US based on the univariate Model V.
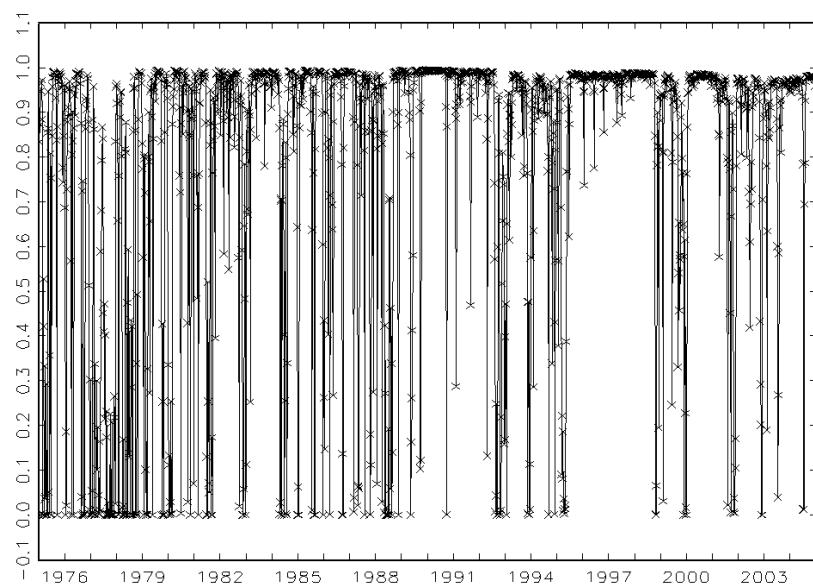
Figure 2:

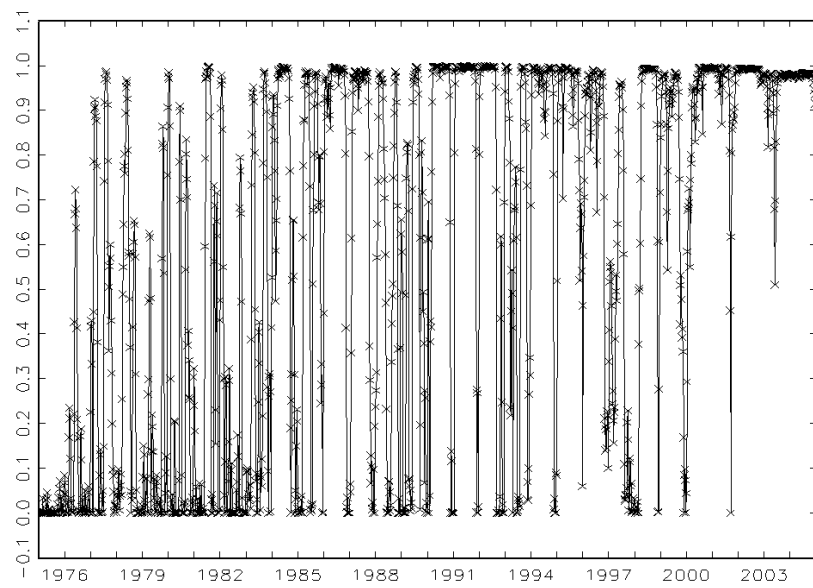Note: The figure shows the smoothed state 1 probabilities for the UK based on the univariate Model V.

Figure 3:

Note: The figure shows the smoothed state 1 probabilities for Germany based on the univariate Model V.

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