

**DEPARTMENT OF MANAGEMENT**

**AFDELING FOR VIRKSOMHEDSLEDELSE**

Working Paper 2006-3

The Role of Implied Volatility in Forecasting Future  
Realized Volatility and Jumps in Foreign Exchange,  
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**UNIVERSITY OF AARHUS • DENMARK**

ISSN 1398-6228

# The Role of Implied Volatility in Forecasting Future Realized Volatility and Jumps in Foreign Exchange, Stock, and Bond Markets

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June 26, 2006

## Abstract

We study the forecasting of future realized volatility in the stock, bond, and foreign exchange markets, as well as the continuous sample path and jump components of this, from variables in the information set, including implied volatility backed out from option prices. Recent nonparametric statistical techniques of Barndorff-Nielsen & Shephard (2004, 2006) are used to separate realized volatility into its continuous and jump components, which enhances forecasting performance, as shown by Andersen, Bollerslev & Diebold (2005). We generalize the heterogeneous autoregressive (HAR) model of Corsi (2004) to include implied volatility as an additional regressor, and to the separate forecasting of the realized components. We also introduce a new vector HAR (VecHAR) model for the resulting simultaneous system, controlling for possible endogeneity issues in the forecasting equations. We show that implied volatility contains incremental information about future volatility relative to both continuous and jump components of past realized volatility. Indeed, in the foreign exchange market, implied volatility completely subsumes the information content of daily, weekly, and monthly realized volatility measures, when forecasting future realized volatility or its continuous component. In addition, implied volatility is an unbiased forecast of future realized volatility in the foreign exchange and stock markets. Perhaps surprisingly, the jump component of realized return volatility is, to some extent, predictable, and options appear to be calibrated to incorporate information about future jumps in all three markets.

*Keywords:* Bipower variation, HAR, Heterogeneous Autoregressive Model, implied volatility, jumps, options, realized volatility, VecHAR, volatility forecasting.

*JEL classification:* C22, C32, F31, G1.

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# 1 Introduction

In both the theoretical and empirical finance literatures, volatility is generally viewed as one of the most important determinants of risky asset prices, such as exchange rates, stock and bond prices, and hence interest rates. Since valuation involves the level and riskiness of future payoffs, the forecasting of future volatility is particularly important for asset pricing as well as derivative pricing, hedging, and risk management. Indeed, for all these reasons, realized volatility (essentially, the summation of squared high-frequency returns over a specified time interval) is now a traded asset. In the recent literature, statistical techniques have been developed that allow separating the continuous sample path and jump components of the return volatility process and using them individually and in new combinations to build volatility forecasts. Andersen et al. (2005) present results from such an analysis for the foreign exchange market and the U.S. stock and Treasury bond markets. They show that for all markets, improved volatility forecasts may be obtained by splitting realized return volatility into its continuous and jump components and combining these optimally.

In the present paper, we consider the alternative route of including derivative prices and forecasting future volatility using implied volatility estimates. Specifically, we investigate whether implied volatility from options on foreign currency futures, S&P 500 index futures, or 30 year Treasury bond (T-bond) futures contains incremental information when assessed against volatility forecasts based on high-frequency (5-minute) current and past spot exchange rate returns, index futures returns, respectively T-bond futures returns, using the recently available statistical techniques to generate efficient measurements of realized volatility and its separate continuous and jump components. Furthermore, we investigate the predictability of the separate volatility components, including the role played by implied volatility in forecasting these.

The construction and analysis of realized volatility from high-frequency return data as a consistent estimator of integrated volatility or quadratic variation have received much attention in recent literature, see e.g. French, Schwert & Stambaugh (1987), Schwert (1989), Andersen & Bollerslev (1998*a*), Andersen, Bollerslev, Diebold & Ebens (2001), Andersen, Bollerslev, Diebold & Labys (2001), Barndorff-Nielsen & Shephard (2002*a*), Andersen, Bollerslev & Diebold (2004), and Barndorff-Nielsen & Shephard (2007). In particular, Andersen, Bollerslev, Diebold & Labys (2003) and Andersen, Bollerslev & Meddahi (2004) show that simple reduced form time series models for realized volatility constructed from historical returns outperform commonly used GARCH and related stochastic volatility models in forecasting future volatility. In recent theoretical contributions, Barndorff-Nielsen & Shephard (2004, 2006) derive a fully nonparametric separation of the continuous sample path and jump components of realized volatility. Applying this nonparametric separation technique, Andersen et al. (2005) extend results of Andersen, Bollerslev, Diebold & Labys (2003) and Andersen, Bollerslev & Meddahi (2004) by including both the continuous and jump components of past realized volatility as separate regressors in the forecasting

of future realized volatility. They show that the continuous sample path and jump components play very different roles in volatility forecasting. Significant gains in forecasting performance are achieved by splitting the explanatory variables into the separate continuous and jump components, compared to using only total past realized volatility. While the continuous component of past realized volatility is strongly serially correlated, the jump component is found to be distinctly less persistent, and almost not forecastable.

Many recent studies stress the importance of separate treatment of the jump and continuous sample path components, particularly in the stock market. This work considers both the estimation of parametric stochastic volatility models (e.g. Andersen, Benzoni & Lund (2002), Chernov, Gallant, Ghysels & Tauchen (2003), Eraker, Johannes & Polson (2003), Eraker (2004), Ait-Sahalia (2004), and Johannes (2004), who considers interest rates), non-parametric realized volatility modeling (e.g. Barndorff-Nielsen & Shephard (2004, 2006), Huang & Tauchen (2005), and Andersen et al. (2005), who also consider the stock, bond, and foreign exchange markets), empirical option pricing (e.g. Bates (1996*a*, 1996*b*) for the foreign exchange market, and Bates (1991) and Bakshi, Cao & Chen (1997) for the stock market), and information arrival processes (e.g. Andersen & Bollerslev (1998*b*) and Andersen, Bollerslev, Diebold & Vega (2003)). Indeed, in the stochastic volatility and realized volatility literatures, the jump component is found to be far less predictable than the continuous sample path component, clearly indicating separate roles for the two components in volatility forecasting.

We study high-frequency (5-minute) returns to the \$/DM exchange rate and monthly prices of \$/DM futures options, as well as 5-minute returns on S&P 500 index futures and 30 year Treasury bond futures and monthly prices of associated options. We compute alternative volatility measures from the two separate data segments, return-based resp. option implied measures. The return-based measures are realized volatility and its continuous and jump components from high-frequency returns, while the option-based measure is implied volatility. The latter is widely perceived as a natural forecast of integrated volatility over the remaining life of the option contract (under risk-neutral pricing). As discussed by Bollerslev & Zhou (2006), implied volatility is also a relevant forecast in a stochastic volatility setting even if volatility risk is priced, although in this case it would get a coefficient below unity in forecasting regressions. Since options expire at a monthly frequency, we consider the forecasting of one-month volatility measures. The issue is whether implied volatility retains incremental information about future integrated volatility when assessed against return-based measures from the previous month. Here, measures covering the entire previous month may not be the only relevant yardstick, since squared returns nearly one month past may not be as informative about future volatility as squared returns that are only one or a few days old. To accommodate this feature in our econometric framework, we apply the heterogeneous autoregressive (HAR) model proposed by Corsi (2004) for realized volatility analysis and extended by Andersen et al. (2005) to include the separate continuous ( $C$ ) and jump ( $J$ ) components of realized volatility as regressors. Specifically, we follow Andersen et al. (2005) and include daily, weekly, and monthly explanatory variables in the

HAR forecasting specifications. As a novel feature, we generalize the HAR framework to include implied volatility from option prices as an additional regressor. Furthermore, because of the different time series properties of the continuous and jump components, as documented in Andersen et al. (2005), separate forecasting of these is relevant for pricing and risk management purposes, and we extend the HAR methodology to the forecasting of each of the volatility components  $C$  and  $J$  individually, again using implied volatility as an additional explanatory variable.

We show that option implied volatility contains incremental information relative to both the continuous and jump components of realized volatility when forecasting subsequently realized return volatility in all three (\$/DM, stock, and bond) markets. In fact, implied volatility completely subsumes the information content of the daily, weekly, and monthly return-based measures in the foreign exchange market. However, implied volatility is not the only relevant forecaster in the stock and bond markets, where, in particular, it should be used in conjunction with the daily continuous and jump components in the stock market and the monthly jump component in the bond market when forecasting future realized volatility. Our results also show that although in all three markets there is clearly volatility information in option prices which is not contained in return data, implied volatility is only an unbiased forecast of future realized volatility in the foreign exchange and stock markets.

Using our extended HAR methodology for the separate forecasting of the continuous and jump components of future realized volatility, our results show that implied volatility has predictive power for both components, again largely subsuming the information content of the daily and weekly continuous and jump components of realized volatility. The forecasting of the continuous component is very much like the forecasting of realized volatility itself, whereas jumps are forecast quite differently. In particular, we show that even the jump component of realized volatility is, to some extent, predictable, with both option implied volatility and the past monthly components significant in the jump forecasting relation.

As an additional novel contribution, we also introduce a vector heterogeneous autoregressive (labelled VecHAR) model for the joint modeling of implied volatility and the separate components of realized volatility. This new system approach allows handling simultaneity issues which may arise from a number of sources. Since implied volatility is the new variable added in our study, compared to the realized volatility literature, and since it may potentially be measured with error stemming from non-synchronicity between sampled option prices and corresponding futures prices, bid-ask spreads, model error, etc., we take special care in handling this variable. The simultaneous VecHAR analysis controls for possible endogeneity issues in the forecasting equations including, e.g., measurement error in implied volatility and correlation among system errors. Furthermore, the simultaneous system approach allows testing interesting cross-equation restrictions.

The results from full information maximum likelihood (FIML) estimation of the VecHAR model reinforce our earlier conclusions. In particular, when forecasting the continuous component of future volatility, option implied volatility should be included in the information

set, and, indeed, it subsumes the information content of all return-based volatility measures in the foreign exchange market. On the other hand, implied volatility should be used together with monthly, but not weekly or daily, separate continuous and jump components of past realized volatility in case of the bond market, and together with daily continuous components in the stock market. Finally, the VecHAR results support the finding that implied volatility is a forecast of the sum of the two components, i.e., of total realized volatility, indeed an unbiased forecast in the foreign exchange and stock markets. In particular, even the jump component of realized volatility is, to some extent, predictable, and implied volatility has incremental information about future jumps in all three markets considered.

In the previous literature on the foreign exchange market, Jorion (1995) and Covrig & Low (2003) show that implied volatility outperforms past realized volatility as a forecast of future realized volatility, although it remains a biased forecast. Our work complements this literature by testing whether the conclusions hold up after allowing the two components of past realized volatility to act separately. In addition, the earlier literature on the relation between implied and realized volatility has considered realized volatility constructed from daily return observations, due to data limitations, and this could be one reason for imprecise measurement of realized volatility and might have biased the results on forecasting performance in favor of implied volatility from option prices, c.f. Poteshman (2000) and Blair, Poon & Taylor (2001), who consider the stock market, and Pong, Shackleton, Taylor & Xu (2004), who consider the foreign exchange market. In response, these authors include high-frequency returns, but they do not separate the continuous and jump components of realized volatility. In sum, what is needed is an assessment of the incremental forecasting power of implied volatility relative to the separate continuous and jump components of realized volatility based on high-frequency returns, and this is what we provide.

It seems plausible that complete reliance on return data possibly does not yield an efficient volatility forecast, given investors' information set. If option market participants are rational and markets are efficient, then implied volatility backed out from traded option prices should reflect available information about future volatility through expiration of the options, including that contained in past returns. Ignoring option price data in forecasting volatility therefore does not seem natural. In fact, in case of the stock market, Christensen & Prabhala (1998) consider more than a decade of return and option price data for the S&P 100 index and find that implied index option volatility is an unbiased and efficient forecast of future realized volatility, subsuming the information content of past realized volatility as a forecast. Other studies documenting the incremental information in implied volatility relative to past realized volatility in the stock market include Day & Lewis (1992), Canina & Figlewski (1993), Lamoureux & Lastrapes (1993), Fleming (1998), Poteshman (2000), and Blair et al. (2001), but none of these separate realized volatility into its continuous and jump components, and this may bias the results in favor of implied volatility. Our work contributes to this literature by examining the role of implied volatility from option prices in the context of the most recent realized volatility modeling and forecasting literature.

In case of the bond market, no previous study has compared the volatility forecasting performance obtained by using return-based volatility measures computed from high-frequency data with that obtained using implied volatility extracted from associated bond options. Amin & Morton (1994) use the Heath, Jarrow & Morton (1992) approach to calculate daily implied interest rate volatilities and compare observed option prices with model prices based on current futures prices and one-day lagged implied volatilities, but they do not consider realized volatility, jumps, or longer term volatility forecasting. In another study using the same data, along with daily spot interest rates, Amin & Ng (1997) examine the performance of implied interest rate volatility as a forecast of future interest rate volatility by including implied volatility in GARCH-type equations, but they do not consider bond returns, high-frequency data, or jumps. Bliss & Ronn (1998) use interest rate volatility implied from callable T-bonds to reveal empirical anomalies, but do not consider option data. In contrast to the other studies, we extract option implied bond return volatility and do not explicitly consider interest rate volatility, and our realized measures use high-frequency bond futures returns and the new nonparametric separation of the continuous and jump components of realized volatility. This allows us to investigate the incremental forecasting power of implied volatility in the bond market, too, relative to the improved realized volatility forecasting obtained by Andersen et al. (2005).

The remainder of the paper is laid out as follows. In the next section we briefly describe realized volatility and the nonparametric identification of its separate continuous sample path and jump components. In Section 3 we discuss the bond and exchange rate derivatives pricing models. Section 4 describes our data and provides summary statistics. In section 5 the empirical results are presented, and section 6 concludes.

## 2 The Econometrics of Jumps

Most contemporary continuous time asset pricing theory is cast in terms of a stochastic volatility model, possibly with an additive jump component. Thus, we assume that the logarithm of the asset price (exchange rate, stock price, or bond price),  $p(t)$ , follows the general stochastic volatility jump diffusion model

$$dp(t) = \mu(t) dt + \sigma(t) dw(t) + \kappa(t) dq(t), \quad t \geq 0. \quad (1)$$

The mean  $\mu(\cdot)$  is assumed continuous and locally bounded and the instantaneous volatility  $\sigma(\cdot) > 0$  càdlàg, both independent of the driving standard Brownian motion  $w(\cdot)$ . The counting process  $q(t)$  is normalized such that  $dq(t) = 1$  corresponds to a jump at time  $t$  and  $dq(t) = 0$  otherwise. Hence,  $\kappa(t)$  is the random jump size at time  $t$  if  $dq(t) = 1$ . We write  $\lambda(t)$  for the possibly time varying intensity of the arrival process for jumps.<sup>1</sup>

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<sup>1</sup>Formally,  $\Pr(q(t) - q(t-h) = 0) = 1 - \int_{t-h}^t \lambda(s) ds + o(h)$ ,  $\Pr(q(t) - q(t-h) = 1) = \int_{t-h}^t \lambda(s) ds + o(h)$ , and  $\Pr(q(t) - q(t-h) \geq 2) = o(h)$ . This rules out infinite activity Lévy processes, e.g. the normal inverse Gaussian process, with infinitely many jumps in finite time.

Stochastic volatility allows returns in the model (1) to have leptokurtic (unconditional) distributions and exhibit volatility clustering, which is empirically relevant.

An important quantity in this model is the integrated volatility (or integrated variance)

$$\sigma^{2*}(t) = \int_0^t \sigma^2(s) ds. \quad (2)$$

In option pricing, this is the relevant volatility measure, see Hull & White (1987). Estimation of integrated volatility is studied e.g. in Andersen & Bollerslev (1998*a*). Integrated volatility is closely related to quadratic variation  $[p](t)$ , defined for any semimartingale (see Protter (2004)) by

$$[p](t) = p \lim \sum_{j=1}^M (p(s_j) - p(s_{j-1}))^2, \quad (3)$$

where  $0 = s_0 < s_1 < \dots < s_M = t$  and the limit is taken for  $\max_j |s_j - s_{j-1}| \rightarrow 0$  as  $M \rightarrow \infty$ . In particular, the quadratic variation process for the model (1) is in wide generality given by

$$[p](t) = \sigma^{2*}(t) + \sum_{j=1}^{q(t)} \kappa^2(t_j), \quad (4)$$

where  $0 \leq t_1 < t_2 < \dots$  are the jump times,  $dq(t_j) = 1$ . In (4), quadratic variation is decomposed as integrated volatility plus the sum of squared jumps that have occurred through time  $t$  (see e.g. Andersen, Bollerslev, Diebold & Labys (2001, 2003)). Recent studies on the stock market (e.g., Andersen et al. (2002), Chernov et al. (2003), Eraker et al. (2003), Eraker (2004), and Ait-Sahalia (2004)), on interest rates (Johannes (2004)), and on exchange rates (Bates (1996*a*, 1996*b*)) all find that jumps are empirically important. To investigate the importance of jumps in foreign exchange, stock, and bond market volatility forecasting, we follow Andersen et al. (2005) and include the jump component explicitly in these markets, too. Rather than modeling (1) directly at the risk of adopting erroneous parametric assumptions, we use high-frequency return data and invoke a powerful nonparametric approach to identification of the two separate components of the quadratic variation process (4), integrated volatility respectively the sum of squared jumps, following Barndorff-Nielsen & Shephard (2004, 2006) and Andersen et al. (2005).

Let us assume that  $T$  months of intra-monthly asset price observations are available and denote the  $M+1$  evenly spaced intra-monthly observations for month  $t$  on the log-price by  $p_{t,j}$ . The one month time interval is used in order to match the sequence of consecutive non-overlapping one month option lives available given the monthly option expiration cycle. The  $M$  continuously compounded intra-monthly returns for month  $t$  are

$$r_{t,j} = p_{t,j} - p_{t,j-1}, \quad j = 1, \dots, M, \quad t = 1, \dots, T. \quad (5)$$

Realized volatility for month  $t$  is given by the sum of squared intra-monthly returns,

$$RV_t = \sum_{j=1}^M r_{t,j}^2, \quad t = 1, \dots, T. \quad (6)$$



Some authors refer to the quantity (6) as realized variance and reserve the term realized volatility for the square root of (6), e.g. Barndorff-Nielsen & Shephard (2001, 2002a, 2002b), but we shall use the more conventional term realized volatility. The nonparametric estimation of the separate continuous sample path and jump components of quadratic variation, following Barndorff-Nielsen & Shephard (2004, 2006), also requires the related bipower and tripower variation measures. The (first lag) realized bipower variation is defined as

$$BV_t = \frac{1}{\mu_1^2} \sum_{j=2}^M |r_{t,j}| |r_{t,j-1}|, \quad t = 1, \dots, T, \quad (7)$$

where  $\mu_1 = \sqrt{2/\pi}$ . In theory, a higher value of  $M$  improves the precision of the estimators, but in practice it also makes them more susceptible to market microstructure effects, such as bid-ask bounces, stale prices, measurement errors, etc., see e.g. Hansen & Lunde (2006) and Barndorff-Nielsen & Shephard (2007). These effects potentially introduce artificial (typically negative) serial correlation in returns. Huang & Tauchen (2005) show that the resulting bias in (7) is mitigated by considering the staggered (second lag, i.e. skip-one) realized bipower variation

$$\widetilde{BV}_t = \frac{1}{\mu_1^2(1-2M^{-1})} \sum_{j=3}^M |r_{t,j}| |r_{t,j-2}|, \quad t = 1, \dots, T. \quad (8)$$

The staggered version avoids the sharing of the price  $p_{t,j-1}$  which by (5) enters the definition of both  $r_{t,j}$  and  $r_{t,j-1}$  in the non-staggered version (7). A further statistic necessary for construction of the relevant tests is the realized tripower quarticity measure

$$TQ_t = \frac{M}{\mu_{4/3}^3} \sum_{j=3}^M |r_{t,j}|^{4/3} |r_{t,j-1}|^{4/3} |r_{t,j-2}|^{4/3}, \quad t = 1, \dots, T, \quad (9)$$

where  $\mu_{4/3} = 2^{2/3} \Gamma(7/6) / \Gamma(1/2)$ . The associated staggered realized tripower quarticity is

$$\widetilde{TQ}_t = \frac{M}{\mu_{4/3}^3(1-4M^{-1})} \sum_{j=5}^M |r_{t,j}|^{4/3} |r_{t,j-2}|^{4/3} |r_{t,j-4}|^{4/3}, \quad t = 1, \dots, T, \quad (10)$$

again avoiding common prices in adjacent returns. As the staggered quantities  $\widetilde{BV}_t$  and  $\widetilde{TQ}_t$  are asymptotically equivalent to their non-staggered counterparts  $BV_t$  and  $TQ_t$ , staggered versions of test statistics can be applied for robustness against market microstructure effects without sacrificing asymptotic results.

Using (3),  $RV_t$  in (6) is by definition a consistent estimator of the monthly increment to the quadratic variation process (4) as  $M \rightarrow \infty$  (Andersen & Bollerslev (1998a), Andersen, Bollerslev, Diebold & Labys (2001) and Barndorff-Nielsen & Shephard (2002a, 2002b)). At the same time,  $BV_t$  is consistent for month  $t$  integrated volatility, the component of the

increment to quadratic variation due to continuous sample path movements in the price process (1), i.e.,

$$BV_t \rightarrow_p \sigma_t^{2*} = \int_{t-1}^t \sigma^2(s) ds, \quad \text{as } M \rightarrow \infty, \quad (11)$$

as shown by Barndorff-Nielsen & Shephard (2004). It follows that the difference between realized volatility and realized bipower variation converges to the sum of squared jumps that have occurred during the course of the month. Of course, in finite samples,  $RV_t - BV_t$  may be non-zero due to sampling variation even if no jump occurred during month  $t$ , so a notion of a “significant jump component” is needed. Following Barndorff-Nielsen & Shephard (2004) and Huang & Tauchen (2005) we apply the (ratio) test statistic

$$Z_t = \sqrt{M} \frac{(RV_t - BV_t)RV_t^{-1}}{((\mu_1^{-4} + 2\mu_1^{-2} - 5) \max\{1, TQ_tBV_t^{-2}\})^{1/2}}, \quad (12)$$

which, in the absence of jumps, satisfies

$$Z_t \rightarrow_d N(0, 1), \quad \text{as } M \rightarrow \infty.$$

Thus,  $Z_t$  measures whether realized volatility exceeds realized bipower variation by more than what can be ascribed to chance, so large positive values of  $Z_t$  indicate the presence of jumps during month  $t$  in the underlying price process. This statistic was shown by Huang & Tauchen (2005) to have better small sample properties than the alternative asymptotically equivalent statistics in Barndorff-Nielsen & Shephard (2004, 2006). Using the staggered versions (8) and (10) of bipower variation and tripower quarticity yields a staggered version  $\tilde{Z}_t$  of the test, and this is recommended by Huang & Tauchen (2005) and Andersen et al. (2005).

With these definitions, the (significant) jump component of realized volatility is given by

$$J_t = I_{\{Z_t > \Phi_{1-\alpha}\}} (RV_t - BV_t), \quad t = 1, \dots, T, \quad (13)$$

where  $I_{\{A\}}$  is the indicator function of the event  $A$ ,  $\Phi_{1-\alpha}$  is the 100(1 -  $\alpha$ )% point of the standard normal distribution, and  $\alpha$  is the chosen significance level. Thus,  $J_t$  is exactly the portion of realized volatility not explained by realized bipower variation, and hence attributable to jumps in the sample path. Accordingly, the estimator of the continuous component of quadratic variation is the remaining portion of realized volatility,

$$C_t = RV_t - J_t, \quad t = 1, \dots, T. \quad (14)$$

This way, the month  $t$  continuous component equals realized volatility when there is no significant jump in month  $t$ , and it equals realized bipower variation when there is a jump, i.e.  $C_t = I_{\{Z_t \leq \Phi_{1-\alpha}\}}RV_t + I_{\{Z_t > \Phi_{1-\alpha}\}}BV_t$ . Note that for any standard significance level, both  $J_t$  and  $C_t$  from (13) and (14) are automatically positive, since  $\Phi_{1-\alpha} > 0$  for  $\alpha < 1/2$ . Since  $Z_t$  and  $BV_t$  enter the definition (13), there are staggered and non-staggered versions of both the continuous and the jump component. Consistency of the separate components

of realized volatility as estimators of the corresponding components of quadratic variation, i.e.

$$C_t \rightarrow_p \sigma_t^{*2} \text{ and } J_t \rightarrow_p \sum_{j=q(t-1)+1}^{q(t)} \kappa^2(t_j)$$

may be achieved by letting  $\alpha \rightarrow 0$  and  $M \rightarrow \infty$  simultaneously, whether staggered or non-staggered versions are used. Hence, this high-frequency data approach allows for month-by-month separate nonparametric consistent estimation of both components of quadratic variation, i.e. the jump component and the continuous sample-path or integrated volatility component, as well as the quadratic variation process for log-prices itself.

### 3 Derivative Pricing Models

We consider options written on \$/DM futures, S&P 500 index futures, and 30 year US Treasury bond (T-bond) futures contracts. Let  $c$  denote the (European call) option price,  $K$  the strike price,  $\tau$  the time to expiration of the option,  $F$  the price of the underlying futures contract expiring  $\Delta$  periods after the option, and  $r$  the riskless U.S. interest rate. Following Bates (1996a, 1996b), the futures option pricing formula is given by

$$\begin{aligned} c(F, K, \tau, \Delta, r, \sigma) &= e^{-r(\tau+\Delta)}[F\Phi(d) - K\Phi(d - \sigma\sqrt{\tau})], \\ d &= \frac{\ln(F/K) + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}, \end{aligned} \quad (15)$$

where  $\Phi(\cdot)$  is the standard normal c.d.f. and  $\sigma$  is the futures return volatility. This is a Black & Scholes (1973) and Merton (1973) (BSM) style option pricing formula, based on a geometric Brownian motion specification for the underlying asset, and for the case  $\Delta = 0$  (no delivery lag) it corresponds to the well-known Black (1976) and Garman & Kohlhagen (1983) futures option pricing formula, replacing the asset price in the BSM formula with the discounted futures price  $e^{-r(\tau+\Delta)}F$ . Jorion (1995) used the formula (15) with  $\Delta = 0$  for the currency option market, whereas Bates (1996a, 1996b) adjusted the formula for a delivery lag specific to the Philadelphia Exchange (PHLX) respectively the Chicago Mercantile Exchange (CME) by replacing  $\tau$  by  $\tau + \Delta$  in the discount factor where  $\Delta$  is the delivery lag from option expiration until the delivery date of the underlying futures contract.

Since we consider \$/DM and S&P 500 futures options traded at the CME, we follow Bates (1996a) and use the adjusted formula (15) with non-zero delivery lag,  $\Delta$  (typically,  $\Delta = 3/365$ , i.e., three day delivery lag). A similar argument applies to the American style T-bond futures options traded at the Chicago Board of Trade (CBOT), where  $\Delta = 3/365$ . Although CME options are American style, the Black (1976) formula produces implied volatilities that are very comparable to those from stochastic volatility option pricing formulas for short-term, at-the-money options, as noted by Jorion (1995). It now becomes an empirical question whether these implied volatilities also reflect information about future jumps, even though the derivation of (15) assumes no jumps.

Given observations on the option price  $c$  and the remaining variables  $F$ ,  $K$ ,  $\tau$ ,  $\Delta$ , and  $r$ , an implied volatility ( $IV$ ) estimate can be backed out from the option pricing formula in (15) by numerical inversion of the nonlinear equation

$$c = c(F, K, \tau, \Delta, r, IV^{1/2}) \quad (16)$$

with respect to  $IV^{1/2}$ . Newton’s method may be applied to compute  $IV$  estimates by iterating the scheme

$$IV_{n+1} = IV_n + \frac{c - c(F, K, \tau, \Delta, r, IV_n^{1/2})}{\mathcal{V}(F, K, \tau, \Delta, r, IV_n^{1/2})}$$

until convergence, where  $\mathcal{V}(F, K, \tau, \Delta, r, IV_n^{1/2}) = F\sqrt{\tau}\phi(d)e^{-r(\tau+\Delta)}$  is the vega of the option formula (see e.g. Hull (2002)) and  $\phi(\cdot)$  is the standard normal p.d.f. The last factor in vega,  $e^{-r(\tau+\Delta)}$ , does not enter, e.g., in the vega of the standard Black & Scholes (1973) formula, but enters here since the futures contract can be regarded as an asset paying a continuous dividend yield equal to the risk-free rate  $r$ . In our empirical work, the algorithm is stopped when  $|c - c(F, K, \tau, \Delta, r, IV_n^{1/2})| < 10^{-7}$ .

## 4 Data and Descriptive Statistics

For the currency options, we consider options on \$/DM futures traded at the CME over the period January 1987 (month where option price is sampled—expiration is following month) to May 1999.<sup>2</sup> The delivery dates of the underlying futures contract follow the quarterly cycle March, June, September, and December. In 1987 serial futures options with monthly expiration cycle were introduced. Thus, some of the options expire in the two months between the quarterly delivery dates of the futures contracts. The futures options are American with expiration dates two Fridays prior to the third Wednesday of each month. The delivery dates of the underlying futures contracts are on the third Wednesday of each of month in the quarterly (March) cycle. Upon exercise the holder of the option contract is provided a position at the strike price in the underlying futures contract on the following trading day, i.e., the following Monday, along with a cash amount equal to the intrinsic value of the option, which implies a delivery lag of  $\Delta = 3/365$  (from Friday to Monday).

Serial S&P 500 futures options (American style) with monthly expiration cycle similarly trade at the CME, and our sample covers the period January 1990 to November 2002. The underlying futures contract follows the same quarterly delivery cycle as the currency futures, with delivery date on the third Friday of the delivery month. To avoid “triple witching hour” problems associated with simultaneous maturing on the third Friday every

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<sup>2</sup>Trading in \$/DM options declined near the introduction of the Euro, and no January 1999 price of the relevant serial (monthly) option contract is available in the data from the Commodity Research Bureau, even though prices are available for the quarterly contracts expiring in March 1999. A monthly implied volatility estimate for January 1999 is constructed using linear interpolation between the December 1998 and February 1999 estimates.

third month of the S&P 500 futures contract, the associated futures options, and the options on the underlying stock index, the expiration date of the futures options was in the second quarter of 1986 (before the start of our sample period) shifted to the preceding Thursday, while keeping the delivery date for the underlying futures on Friday. Upon exercise the holder of the option contract is provided a position at the strike price in the underlying futures contract on the following trading day, plus the intrinsic value of the option in cash. This results in a delivery lag of  $\Delta = 1/365$  every third month, according to the March quarterly cycle, whereas  $\Delta = 3/365$  for the serial options expiring on the third Friday of each of the two intermediate months.

In October 1990 serial 30 year T-bond futures options with monthly expiration cycle were introduced at the CBOE, and our sample covers the period October 1990 to November 2002. The underlying futures contract follows the same quarterly delivery cycle of as the currency and stock futures. The particular T-bond serving as underlying asset for the futures is not uniquely identified by the contract specifications. It is simply required that the T-bond is not callable for at least 15 years from the first day of the contract month (the delivery month of the underlying futures contract), or, for a noncallable T-bond, has a maturity of at least 15 years from the first day of the contract month. For a detailed description of the 30 year T-bond futures, see e.g. Hull (2002). The options are American and expire on the last Friday followed by at least two business days in the month prior to the contract month. As for the currency options, upon exercise the holder of the option contract is provided a position at the strike price in the underlying futures contract on the following Monday, plus the intrinsic value in cash, again implying  $\Delta = 3/365$ .

The options data consist of daily open auction closing prices obtained from the Commodity Research Bureau. The US Eurodollar deposit one-month middle rate (downloaded from Datastream) is used for the risk-free rate. For the implied volatility ( $IV$ ) estimates we use at-the-money (ATM) calls with one month to expiration. The prices are recorded on the Tuesday after the last trading day of the preceding option contract. In total, a sample of 149 (currency market), 155 (stock market), and 146 (bond market) annualized monthly  $IV$  observations of ATM calls are available. Hence, although the underlying futures contracts expire at a quarterly frequency, the  $IV$  estimates are based on option contracts covering non-overlapping one-month time intervals. Furthermore, as suggested by French (1984), the option pricing formula in (15) is extended such that trading days are used for volatilities and calendar days for interest rates.

For  $RV$  and its separate components we use the same data as Andersen, Bollerslev, Diebold & Vega (2004) and Andersen et al. (2005), which are based on linearly interpolated five-minute observations (following Müller, Dacorogna, Olsen, Pictet, Schwarz & Morgeneegg (1990) and Dacorogna, Müller, Nagler, Olsen & Pictet (1993), among others) on  $\$/DM$  spot exchange rates, S&P 500 futures prices, and T-bond futures prices. There is round-the-clock trading in the  $\$/DM$  spot market, and the raw data are interbank quotes from Reuter's FXFX screen. There is open auction CME trading from 8:30 a.m. to 3:15 p.m. central time in S&P 500 futures, and the data are supplemented with GLOBEX

prices for the period from 7:10 a.m.. There is open auction T-bond futures trading at the CBOT from 7:20 a.m. to 2:00 p.m., with our price observations starting at 7:25 a.m. This provides us with a total of 288 high-frequency returns per day ( $r_{t,j}$  from (5)) for the \$/DM market, 97 per day for the stock market, and 79 per day for the bond market. The volatility measures are annualized and constructed on a monthly basis to cover exactly the same periods as the  $IV$  estimates. For the foreign exchange market we have  $T = 149$  and  $M$  about 6,336 (approx. 22 trading days with 288 returns per day – our data does not include weekends and holidays), for the stock market we have  $T = 155$  and  $M$  about 2,134 (approx. 22 trading days with 97 returns per day), and for the bond market we have  $T = 146$  and  $M$  about 1,738 (approx. 22 trading days with 79 returns per day). Our time index refers to the calendar month where implied volatility is sampled. Thus,  $IV_t$  can be regarded as a forecast of  $RV_{t+1}$ , since implied volatility is sampled at the beginning of the time interval covered by realized volatility for period  $t + 1$ . For example, in the foreign exchange market, if  $t$  and  $t + 1$  correspond to May and June, say,  $IV_t$  is sampled on the Tuesday after the May expiration date, which in this example is two Fridays prior to the third Wednesday of May, and  $RV_{t+1}$  is calculated from squared returns covering the period from the sampling of  $IV_t$  until two Fridays prior to the third Wednesday of June. As suggested by Andersen et al. (2005) a significance level of  $\alpha = 0.1\%$  is used to detect jumps, thus providing the series for the jump component  $J$  from (13) and continuous component  $C$  from (14) of realized volatility  $RV$ .

The \$/DM spot exchange rate differs from the futures rate, which is the price of the underlying asset for the option contract. However, through the interest rate parity  $\ln F = p + (r_{\$} - r_{DM})\tau$ , well known from international finance<sup>3</sup>, it is clear that the futures and spot \$/DM exchange rates primarily differ by the discounted interest rate differential. Using the spot rate instead of the futures price for realized quantities implies that our estimates of the forecasting power of  $IV$  (calculated from futures options) are on the conservative side in case of the currency market. For the bond market, rather than calculating  $RV$  from returns on T-bonds, which are not uniquely associated with the futures contracts, we consider  $RV$  from returns to the futures contract itself, i.e., precisely the underlying asset for the futures options. For the stock market, realized quantities are based on returns on the index futures contract, which is the underlying asset for the options considered, i.e., a perfect match.

### Table 1 about here

Summary statistics for the four different annualized volatility measures are presented in Table 1. Throughout the paper we use the staggered versions of the realized volatility measures as advocated by Huang & Tauchen (2005) and Andersen et al. (2005).<sup>4</sup> Panel A

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<sup>3</sup>The interest parity holds exactly with constant risk free interest rate, i.e. in this case the forward price equals the futures price (Cox, Ingersoll & Ross (1981)). It holds approximately when the interest rate is stochastic. Indeed, it is precisely under interest parity that the Garman & Kohlhagen (1983) spot exchange rate option pricing formula reduces to the Black (1976) futures option formula.

<sup>4</sup>The corresponding results for the non-staggered volatility measures, as well as results using the square-

presents statistics for the currency market, Panel B for the stock market, and Panel C for the bond market. If implied volatility were given by the conditional expectation of future realized volatility as traditional option pricing theory suggests, we would expect that  $RV$  and  $IV$  had equal unconditional means, and  $RV$  higher unconditional standard deviation in the time series than  $IV$ . This pattern is confirmed in the foreign exchange and stock markets (Panels A and B), where both  $RV$  and  $C$  have higher sample standard deviations than  $IV$ , and almost in the bond market (Panel C), where  $IV$  and  $RV$  have about the same standard deviation. The unconditional sample mean of  $IV$  is slightly higher than that of  $RV$  in the stock and bond markets, possibly reflecting a negative price of volatility risk (c.f. Bollerslev & Zhou (2006)) or an early exercise premium, whereas the opposite is the case for the foreign exchange market. None of these observations rule out that implied volatility has incremental information in forecasting future realized volatility. Finally, skewness and excess kurtosis are generally higher for  $J$  than for the other volatility measures, and they are lowest for  $IV$  in all three markets, i.e., implied volatility is closer to Gaussianity than realized volatility and both its continuous and jump components.

**Figure 1 about here**

Time series plots of the four monthly volatility measures are exhibited in Figure 1. The volatility measures from the foreign exchange market appear in Panel A, those for the stock market in Panel B, and those for the T-bond market in Panel C. In the foreign exchange and stock markets (Panels A and B), the continuous component of realized volatility is close to realized volatility itself. The new variable in our analysis, implied volatility, is also close to realized volatility, but not as close as the continuous component. In case of the T-bond market (Panel C), the continuous component of realized volatility is below realized volatility itself, and implied volatility hovers above both, consistent with the difference in means in Table 1. In all three markets, the jump component clearly behaves differently from the other volatility measures. There are 148 out of 149 months with significant jumps in the foreign exchange market, 120 out of 155 in stock market, and 138 out of 146 months with significant jumps in the T-bond market. Thus, jumps are non-negligible in all three markets, and the jump series clearly exhibit less serial dependence compared to the other series as found also by Andersen et al. (2005). Hence, Figure 1 provides clear indication of the importance of analyzing the continuous and jump components separately.

## 5 Econometric Models and Empirical Results

In this section we study the relation between realized volatility together with its disentangled components and implied volatility from the associated option contract. We apply both standard univariate regression models and heterogeneous autoregressive (HAR) models, and we introduce new multivariate extensions of the latter, which turn out to prove

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root and log-transformation of the volatility measures, are similar and available from the authors upon request.

useful in our context with implied volatility as well as separate continuous and jump measures of realized volatility. Each of the tables in the empirical analysis consists of three panels with results for the foreign exchange market in Panel A, the stock market in Panel B, and the T-bond market in Panel C.<sup>5</sup>

## 5.1 Forecasting Realized Volatility

We consider regression of realized return volatility,  $RV$ , on option implied volatility,  $IV$ , as well as lagged  $RV$  or its continuous and jump components. The general form of the one-month ahead forecasting equation is

$$RV_{t+1} = \alpha + \beta IV_t + \gamma x_t + \varepsilon_{t+1}, \quad t = 1, 2, 3, \dots, \quad (17)$$

where  $\alpha$  denotes the intercept,  $\beta$  is the slope parameter for  $IV$ , and  $\varepsilon_{t+1}$  is the forecasting error. Either  $RV$  or the vector  $(C, J)$  is contained in  $x_t$ , where  $\gamma$  is the associated coefficient (vector). For some of the regressions presented,  $\beta = 0$  or  $\gamma = 0$  is imposed.

### Table 2 about here

In Table 2 we report coefficient estimates (estimated standard errors in parentheses) together with adjusted  $R^2$ , and the Breusch (1978) and Godfrey (1978) (henceforth BG) test statistic for residual autocorrelation up to lag 12 (one year), which is used instead of the standard Durbin-Watson statistic due to the presence of lagged endogenous variables in several of the regressions. Under the null hypothesis of absence of serial dependence in the residuals, the BG statistic is asymptotically  $\chi^2$  with 12 degrees of freedom.

The first order autocorrelation coefficient in the regression of realized volatility on its own lag in the first row of each panel in Table 2 is .46, .63, and .55 in the foreign exchange, stock, respectively T-bond markets, with associated  $t$ -statistics of 6.3, 10.1, and 7.9. In the following, this serves as a useful benchmark for assessing the new nonparametric tools, as well as the incremental information in option prices. We first address the importance of separating realized volatility into its continuous and jump components when forecasting future volatility. It is immediately clear that the two components of realized volatility play very different roles in forecasting realized volatility. The coefficient on the continuous component is .54 (.62) [.63] for the foreign exchange (stock) [T-bond] market, clearly significant with a  $t$ -statistic of 4.8 (7.7) [8.9], and close to that of the realized volatility regression. In contrast, in the foreign exchange and bond markets, the coefficient on the jump component is negative and insignificant with  $t$ -statistics of -.2 and -1.2, suggesting very little impact from the jump component on future realized volatility. This accords well with the consensus in the empirical finance literature that jumps are very hard to predict (e.g. Andersen et al. (2005)). Our results complement this notion by showing that jumps

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<sup>5</sup>Again, results for non-staggered volatility measures and for other transformations, are similar and available from the authors upon request.



in the foreign exchange and bond markets are of little importance in predicting future realized volatility. On the other hand, the jump component is significant in the stock market with a  $t$ -statistic of 2.7 and a positive coefficient close to that of the continuous component. Thus,  $C$  and  $J$  generally play very different roles in a forecasting context, but there are important differences across markets. The results show that it is not generally appropriate to use total lagged realized volatility as in the regression in the first row, imposing equal coefficients on  $C$  and  $J$ .

As a novel contribution we introduce currency, stock, and bond option implied volatility into this forecasting regression framework with separate continuous and jump components. This allows assessing the incremental information from option prices relative to the non-parametric volatility measures extracted from high-frequency returns.

The third row of each panel in Table 2 examines the information content of implied volatility in forecasting future realized volatility. The regression coefficient on implied volatility is .89 (.106) [.56] in Panel A (Panel B) [Panel C] and strongly significant with a  $t$ -statistic of 10.0 (15.9) [8.7]. Based on the estimated standard errors, we fail to reject the unbiasedness hypothesis  $\beta = 1$  in (17) for the foreign exchange and stock markets, where also adjusted  $R^2$ , at 39% resp. 62%, is much higher than that in the regression on past realized volatility or its components in rows one and two, and the BG test does not indicate any misspecification. In the T-bond market, the unbiasedness hypothesis is rejected, adjusted  $R^2$  is in between those in the first two rows of the panel, and the BG test shows signs of residual serial correlation when  $IV$  is the sole regressor.

The incremental information in implied volatility vis-à-vis realized volatility and its separate components is next assessed by including these simultaneously as regressors. The results in the fourth line of each panel in Table 2 confirm the differences between the three markets. In the foreign exchange and stock markets, past realized volatility carries no incremental information relative to implied volatility. The coefficient on lagged  $RV$  is small in magnitude, negative, and insignificant in both markets. On the other hand, in the bond market, when both realized and implied volatility are included in the regression, each of them has incremental information and remains significant. The last line of each panel shows the results when including both implied volatility and the separate continuous and jump components of realized volatility. Implied volatility subsumes the information content of both the continuous and jump components of realized volatility in the foreign exchange market, but not in the stock market where the jump component remains significant with a negative coefficient, although adjusted  $R^2$  does not increase much relative to the regression where  $IV$  is the sole regressor (62.9% resp. 62.2%) and the BG test now shows signs of misspecification. Moreover, all variables are significant in the T-bond market, where this regression gets the highest adjusted  $R^2$  at 45%, although the BG tests show mild signs of misspecification.

Recall that our implied volatility measure is backed out from the modified Black (1976) futures option pricing formula (15), as is standard among practitioners and in the empirical literature. The formula is consistent with a time-varying volatility process for a continuous

sample path futures price process but does not incorporate jumps in prices, and hence the jump component may not be explained by implied volatility. Nevertheless, our results show that implied volatility can in fact predict return volatility, which does include a jump component. Further results below on the direct forecasting of the jump component of future volatility support this interpretation. This suggests that option prices, at least to some extent, are calibrated to incorporate the effect of jumps, thus reducing the empirical need to invoke a more general option pricing formula allowing explicitly for jumps in prices. Such an approach would entail estimating additional parameters, including prices of volatility and jump risk. This would be a considerable complication, but would potentially reveal that even more information is contained in option prices. While leaving the alternative, more complicated analysis for future research, we note that our approach yields a conservative estimate of the information content on future return volatility contained in option prices.

## 5.2 Heterogeneous Autoregressive (HAR) Model

In forecasting future realized volatility, the OLS regressions in Table 2 above use measures of past realized volatility and the components of this, where squared returns are assigned equal weight throughout the month. This way squared returns nearly one month ago are given the same weight as squared returns one or two days ago. This may not be relevant if squared returns observed close to one month ago are nearly obsolete, and recent squared returns much more informative about future volatility. Instead, it may be more relevant to place higher weight on recent squared returns than on squared returns that are more distant in the past, and one way to do so in a parsimonious fashion is to apply the heterogeneous autoregressive (HAR) model proposed by Corsi (2004). When applying this only to realized volatility itself, we follow Corsi (2004) and denote the model HAR-RV. We also follow Andersen et al. (2005) and separate the realized volatility regressors into their continuous ( $C$ ) and jump ( $J$ ) components in what they denote the HAR-RV-CJ model.

In our analysis we modify and generalize the HAR-RV-CJ model in four directions. First, we include implied volatility ( $IV$ ) as an additional regressor and abbreviate the model HAR-RV-CJIV. Secondly, in the following subsection HAR regressions are used to predict each of the separate continuous and jump components rather than total realized volatility, and we denote the corresponding models HAR-C-CJIV respectively HAR-J-CJIV. Thirdly, in HAR type estimations data are measured on different time scales, such as daily, weekly, and monthly. Both Corsi (2004) and Andersen et al. (2005) normalize the time series to the daily frequency. However, in line with our previous analysis and without loss of generality, we annualize the data instead of normalizing to the daily frequency. Fourth, Andersen et al. (2005) estimate HAR models with the regressand sampled at overlapping time intervals, e.g. monthly  $RV$  is sampled at the daily frequency, causing serial correlation in the error term. This does not necessarily invalidate the parameter estimates, although an adjustment must be made to obtain correct corresponding standard errors. However, options expire on a monthly basis and the analysis in Christensen & Prabhala (1998) suggests that use of overlapping data may lead to erroneous inferences in a setting with both implied volatility

from option prices and realized volatility. Hence, we sample monthly, weekly, and daily measures of realized volatility and its components at the monthly frequency, with the weekly (daily) measures covering the last five trading days (last day) of the corresponding monthly measures.

For the HAR-RV-CJIV model, which is the HAR equivalent of the least squares regression equation in (17), we denote the  $h$ -day realized variation normalized to the annual frequency by

$$RV_{t,t+h} = 252h^{-1} (RV_{t+1} + RV_{t+2} + \dots + RV_{t+h}),$$

where  $h = 1, 2, \dots$ . Here, and throughout the rest of the paper, we use  $t$  to indicate trading days rather than months for all volatility measures. For instance,  $RV_t$  now denotes the daily realized volatility for day  $t$ . Thus,  $RV_{t,t+h}$  may denote a daily ( $h = 1$ ), weekly ( $h = 5$ ), or monthly ( $h = 22$ ) realized volatility measure. Similar measures may be computed for the continuous ( $C_{t,t+h}$ ) and jump ( $J_{t,t+h}$ ) components of realized volatility. Note that  $RV_{t,t+1} = 252RV_{t+1}$  and under stationarity  $E[RV_{t,t+h}] = 252E[RV_{t+1}]$  for all  $h$ .

In our empirical work the monthly realized measures are constructed using a value of  $h$  exactly matching the number of trading days covered by the associated implied volatility. However, for notational convenience we continue to use  $h = 22$  to indicate monthly realized measures even when the exact number of trading days in a given option cycle is slightly different. Finally,  $IV_t$  now denotes the implied volatility backed out from the price of the relevant one-month option sampled on day  $t$ .

The monthly frequency HAR-RV-CJIV model is

$$RV_{t,t+22} = \alpha + \gamma_m x_{t-22,t} + \gamma_w x_{t-5,t} + \gamma_d x_t + \beta IV_t + \varepsilon_{t,t+22}, \quad t = 22, 44, 66, \dots, \quad (18)$$

where  $\varepsilon_{t,t+22}$  is the monthly forecasting error,  $x_{t-22,t}$  is either  $RV_{t-22,t}$  or the vector  $(C_{t-22,t}, J_{t-22,t})$ , and similarly for the weekly and daily variables  $x_{t-5,t}$  respectively  $x_t$ . When a variable is not included in the specific regression,  $\beta = 0$  or  $\gamma_m = \gamma_w = \gamma_d = 0$  is imposed. Note that  $x_{t-22,t}$  corresponds to the one-month lagged realized volatility measures included in the earlier regressions, whereas  $x_{t-5,t}$  and  $x_t$  allow extracting information about future volatility from the more recent history of past squared returns.

### Table 3 about here

Table 3 shows the results for the HAR-RV-CJIV model in (18). The format is the same as in Table 2, so Panel A is for foreign exchange data. In the first regression depicted in the panel, past realized volatility is not split into its separate continuous and jump components. The combined impact from the monthly, weekly, and daily realized volatility measures on future realized volatility is  $.22 + .10 + .17 = .49$ , strikingly close to  $.46$ , i.e. the corresponding estimate from the first regression in Table 2. The  $t$ -statistics for the monthly, weekly, and daily  $RV$  parameter estimates are 1.92, .68, and 2.06, respectively, indicating that the weekly variable contains only minor information concerning future monthly exchange rate volatility. In the stock market (Panel B), all three  $RV$  measures are significant and the

weekly measure gets a negative coefficient. In contrast to the foreign exchange market, the BG test is significant. Panel C is for Treasury bond data and the results in the first line are similar to those in the first line of Panel A, except that daily  $RV$  is insignificant. In all three markets, adjusted  $R^2$  increases considerably compared to the first line of each panel of Table 2 (from 19.3% to 26.0% in the foreign exchange market, from 39.7% to 53.0% in the stock market, and from 29.7% to 32.5% in the bond market). This shows the value of the HAR approach, i.e., of including volatility measures of different frequencies in predicting future volatility.

Turning to the results in row two of each panel of Table 3, where the continuous and jump components of realized volatility enter separately in the regression, the conclusions for the continuous component variable are similar to the those for  $RV$  in the first row of the table, except that the monthly and weekly continuous components become insignificant in the stock market. As in Table 2 the jump components are mostly insignificant, with the exception that the daily jump component is significant in the stock and bond markets. Adjusted  $R^2$  improves when moving from the second line of each panel of Table 2 to the second line of each panel of Table 3, showing that the HAR approach is useful also when considering the separate components of  $RV$ . Adjusted  $R^2$  also improves when moving from first to second line of each panel of Table 3, thus confirming the enhanced forecasting performance obtained by splitting  $RV$  into its separate components also found by Andersen et al. (2005).

Next, implied volatility is added to the information set at time  $t$  in the HAR regressions.<sup>6</sup> When  $RV$  is included together with  $IV$ , fourth row, all the realized volatility coefficients turn insignificant in the foreign exchange and bond markets, whereas daily  $RV$  remains significant in the stock market. However,  $IV$  gets  $t$ -statistics of 6.15, 6.84, and 4.46 in the three markets, providing clear evidence for the relevance of implied volatility in forecasting future volatility. The last row of each panel shows the results when including the separate continuous and jump components of realized volatility together with implied volatility, i.e. the new HAR-RV-CJIV model. In the foreign exchange market (Panel A) implied volatility subsumes the information content of both the continuous and jump components of realized volatilities at all frequencies. Adjusted  $R^2$  is about the same as in the third line of the panel where  $IV$  is the sole regressor. In the stock market (Panel B) both daily components of  $RV$  remain significant and the adjusted  $R^2$  increases from 62% to 68% relative to having  $IV$  as the sole regressor. In the bond market (Panel C), implied volatility gets the highest  $t$ -statistics, as in the other two markets. In this case, the monthly jump component  $J_{t-22,t}$  is also significant and adjusted  $R^2$  improves markedly, both between lines three and four (adding realized volatility) and between lines four and five (separating the  $RV$  components). The BG test shows mild signs of misspecification in all three markets.

The finding so far is that implied volatility as a forecast of future volatility contains incremental information relative to return-based measures in all three markets, even when

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<sup>6</sup>The regression in row three of Table 3 duplicates that in row three of Table 2, and is included for clarity.

allowing more weight to be placed on more recent squared returns and when separating the continuous and jump components of the realized volatility regressor. Indeed, in the foreign exchange market, our results show that implied volatility subsumes the information content of high-frequency realized volatility and its separate components. All relevant information about future exchange rate volatility is reflected in the option prices. This shows that the conclusions of Jorion (1995) hold up even when adding high-frequency return data, using the new nonparametric techniques to disentangle and optimally combine the separate continuous and jump components of the realized volatility forecast, and invoking the HAR methodology to exploit potential forecasting power of the  $RV$  components at different frequencies. We next use the HAR framework to assess whether the conclusions extend to separate forecasting of the continuous and jump components of future realized volatility.

### 5.3 Forecasting the Continuous and Jump Components

We now split realized return volatility,  $RV_{t,t+22}$ , into its separate continuous sample path and jump components,  $C_{t,t+22}$  and  $J_{t,t+22}$ , and examine how these are forecast by the variables in the information set at time  $t$ . This is particularly interesting since Andersen et al. (2005) have shown that the time series properties of the continuous and jump components are very different, consistent also with our findings in Section 4. This suggests that the two components should be forecast in different ways. In the following we extend the HAR methodology to the forecasting of the separate continuous and jump components of future volatility. Although Andersen et al. (2005) did not consider the forecasting of the separate components, our analysis below shows that the HAR methodology is well suited also for this purpose. In addition, since our generalized specification includes implied volatility as well, we are able to assess the incremental information in option prices on future continuous and jump components of volatility.

Our HAR-C-CJIV model for forecasting the continuous component of future volatility is given by

$$C_{t,t+22} = \alpha + \gamma_m x_{t-22,t} + \gamma_w x_{t-5,t} + \gamma_d x_t + \beta IV_t + \varepsilon_{t,t+22}, \quad t = 22, 44, 66, \dots, \quad (19)$$

where  $C_{t,t+22}$  replaces  $RV_{t,t+22}$  on the left-hand side of the regression compared to (18) and  $x$  now contains either  $C$  or the vector  $(C, J)$ . Table 4 shows the results from estimation of this model. The format is the same as in Table 3, except that  $C$  and  $J$  are always considered separately rather than combined in the form of  $RV$ .

#### Table 4 about here

The results in Table 4 are similar to the corresponding results in Table 3. The BG tests show only modest signs of misspecification, except in the bond market when  $IV_t$  is the sole regressor or in the stock and bond markets when only continuous components are included. Throughout, adjusted  $R^2$  is higher than in comparable specifications in Table 3, confirming that  $C$  is more amenable to forecasting than  $RV$ , and hence demonstrating the value of the

new approach of separate HAR modeling of the continuous and and jump components of realized volatility. In Table 4, when forecasting  $C$ , implied volatility generally gets higher coefficients and  $t$ -statistics than the lagged continuous and jump components of realized volatility, and adjusted  $R^2$  is highest when implied volatility is included along with these. In the foreign exchange market, implied volatility completely subsumes the information content of the realized volatility measures, just as in Panel A of Table 3. In the stock market, the daily continuous component remains significant, as in Table 3, but the daily jump component becomes insignificant. In the bond market, the monthly jump component remains significant as in Table 3.

We next consider the predictability of the jump component of realized volatility. The relevant HAR-J-CJIV model takes the form

$$J_{t,t+22} = \alpha + \gamma_m x_{t-22,t} + \gamma_w x_{t-5,t} + \gamma_d x_t + \beta IV_t + \varepsilon_{t,t+22}, \quad t = 22, 44, 66, \dots, \quad (20)$$

where  $x$  now contains either  $J$  or the vector  $(C, J)$ . Table 5 reports results from regression of the future jump component,  $J_{t,t+22}$ , on the relevant variables in the information set at time  $t$ .

#### **Table 5 about here**

In the first line of panels A and C (foreign exchange and bond markets), only the monthly jump component is significant while the daily and weekly jump components are insignificant. In line two of these two panels the continuous component is added to the regression but turns out insignificant. The monthly jump components remain significant, and in the foreign exchange market the weekly jump component now gets a significant (although negative) coefficient, too. In the stock market (Panel B), the weekly and daily jump components are significant in the first two lines with a negative coefficient on the daily component, and when adding the continuous components in line two, the daily component is significant. In line three of each panel, implied volatility is used to predict the jump component of future volatility. It is strongly significant in all three markets and gets higher  $t$ -statistics than all other variables considered. The highest adjusted  $R^2$ s in the table are obtained in the fourth line of each panel, where all variables are included. Here, the BG test shows no signs of misspecification in the foreign exchange and bond markets, although it does in the stock market. Implied volatility remains highly significant in all three markets and turns out to be the strongest predictor of future jumps (in terms of  $t$ -statistics) even when the continuous and jump components at all frequencies are included. The coefficient on implied volatility ranges between .10 and .23 across markets and specifications, consistent with the mean jump component being an order of magnitude smaller than implied volatility in Table 1. Indeed, in the bond market, implied volatility subsumes the information content of both components of realized volatility at all frequencies. In the foreign exchange market, the monthly continuous component gets a negative coefficient which is now significant, along with implied volatility. In the stock market, all three jump components remain significant, with positive coefficient on the weekly and negative on the monthly and daily measures.

Comparing across Tables 3, 4, and 5, it is clear that the results are most similar in Tables 3 and 4 and quite different in Table 5. Clearly, realized volatility and the continuous component of this behave similarly, also in this forecasting context, and our results show that implied volatility from option prices are important in forecasting both. The difference in results when moving to Table 5 again reinforces that the continuous and jump components should be treated separately. When doing so, we find that, firstly, jumps are predictable from variables in the information set, and, secondly, implied volatility retains incremental information, thus suggesting that option prices incorporate jump information. Both are interesting and novel results.

#### 5.4 The Vector Heterogeneous Autoregressive (VecHAR) Model

Our results so far show that realized volatility should be separated into its continuous and jump components for forecasting purposes, and that implied volatility from option prices has incremental information for the forecasting of both. We now introduce a simultaneous system approach for the joint analysis of implied volatility and the separate continuous and jump components of realized volatility. The reason a simultaneous system approach is needed is firstly that results up to this point have been obtained in different regression equations which are not independent, so the relevant joint hypotheses actually involve cross-equation restrictions. Secondly, our variables may be contaminated with measurement error. In particular, implied volatility may be measured with errors stemming from non-synchronous option and futures prices, misspecification of the option pricing formula, etc. Even a simple errors-in-variables problem in implied volatility of this kind generates correlation between the implied volatility regressor and the error terms in the forecasting equations for the continuous and jump components, and thus a particular case of an endogeneity problem. In addition, realized volatility and its separate components contain sampling error, as studied in detail by, e.g., Barndorff-Nielsen & Shephard (2002a, 2007). Our simultaneous system approach provides an efficient method for handling the resulting endogeneity issues.

Thus, we consider the vector heterogeneous autoregressive (VecHAR) system

$$\begin{aligned} \begin{pmatrix} 1 & 0 & \beta_1 \\ 0 & 1 & \beta_2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{t,t+22} \\ J_{t,t+22} \\ IV_t \end{pmatrix} &= \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} + \begin{pmatrix} A_{11m} & A_{12m} & 0 \\ A_{21m} & A_{22m} & 0 \\ A_{31m} & A_{32m} & A_{33m} \end{pmatrix} \begin{pmatrix} C_{t-22,t} \\ J_{t-22,t} \\ IV_{t-1} \end{pmatrix} \\ &+ \begin{pmatrix} A_{11w} & A_{12w} \\ A_{21w} & A_{22w} \\ A_{31w} & A_{32w} \end{pmatrix} \begin{pmatrix} C_{t-5,t} \\ J_{t-5,t} \end{pmatrix} \\ &+ \begin{pmatrix} A_{11d} & A_{12d} \\ A_{21d} & A_{22d} \\ A_{31d} & A_{32d} \end{pmatrix} \begin{pmatrix} C_t \\ J_t \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,t+22}^1 \\ \varepsilon_{t,t+22}^2 \\ \varepsilon_{t,t+22}^3 \end{pmatrix}. \end{aligned} \quad (21)$$

The first two equations in our simultaneous VecHAR system comprise the forecasting

equations (19) and (20) for the separate components of realized volatility and the third equation endogenizes implied volatility. In the representation (21) of our VecHAR model, the three coefficient matrices on the right hand side are associated with monthly, weekly, and daily volatility measures, respectively. There are two sources of simultaneity in the VecHAR system. Firstly, the off-diagonal terms  $\beta_1$  and  $\beta_2$  in the leading coefficient matrix accommodates dependence of  $C_{t,t+22}$  and  $J_{t,t+22}$  on the endogenous variable  $IV_t$ . Secondly, the system errors may be contemporaneously correlated. In the third equation, option prices at the end of the month may reflect return movement over the course of the month, although via the HAR type specification, more recent returns may receive higher weight. In addition, our specification allows dependence on one-day lagged implied volatility,  $IV_{t-1}$ , i.e., implied volatility sampled on Monday for the same option contract as in  $IV_t$ , which is sampled on Tuesday. The specification of the third equation is similar to using  $IV_{t-1}$  as an additional instrument for  $IV_t$  in an instrumental variables treatment of the endogeneity problem, but the full system approach in (21) is more general and efficient.

### Table 6 about here

In Table 6 we present the results of Gaussian full information maximum likelihood (FIML) estimation of the VecHAR system with robust standard errors (sandwich-formula,  $H^{-1}VH^{-1}$ , where  $H$  is the Hessian and  $V$  the outer-product-gradient matrix) in parentheses. Of course, the results are asymptotically valid even in the absence of Gaussianity. Similarly to the univariate HAR results in Tables 3-5, the BG tests show no signs of misspecification in the foreign exchange and bond markets, although the tests are significant in two of the equations for the stock market.

Implied volatility is strongly significant in the forecasting equations for both the continuous and jump components of future volatility in all three markets, showing that option prices contain incremental information beyond that in high-frequency return-based volatility measures. In the foreign exchange market, implied volatility subsumes the information content of all other variables in forecasting both the future continuous and jump components. The monthly continuous component that got a significantly negative coefficient in the HAR-J-CJIV model (Table 5, Panel A) turns insignificant upon VecHAR simultaneity correction ( $t$ -statistic of 1.80). In the stock market, we find that the daily continuous component is significant in the continuous component forecasting equation, as in the fourth row of Panel B in Table 4, and in the jump component equation the coefficient on the monthly jump component remains negative and significant, whereas the other two jump components now turn insignificant. In the bond market, the monthly and daily jump components are now significant in the continuous component forecasting equation and the monthly continuous component that was significant in Panel C of Table 4 now drops out, whereas implied volatility subsumes the information content of both components at all frequencies in the jump component forecasting equation of the VecHAR system, just as in the HAR-J-CJIV model.

Turning to the implied volatility forecasting equations in the last row of each panel of



Table 6, we find that lagged implied volatility is strongly significant and by far the most important regressor. In the foreign exchange market, the daily continuous component is also significant, which is natural and accords well with normal trading behavior, i.e. market participants incorporate recent return-based volatility information in setting option prices. In the bond market, the daily continuous component is the strongest return-based predictor, too, but here the  $t$ -statistic is lower, at 1.42, and insignificant. In the stock market, the monthly and daily jump components are both significant, with coefficients equal in magnitude and of opposite sign.

### Table 7 about here

Table 7 shows results of likelihood ratio (LR) tests of various hypotheses of interest in the VecHAR model. Most of these hypotheses are tested in the first equation, but simultaneity implies that testing in the system framework is most appropriate. Overall, earlier conclusions are confirmed. Firstly, implied volatility subsumes the information in the weekly measures of the continuous and jump components. Specifically, the hypothesis  $H_2 : A_{11w} = 0, A_{12w} = 0$  in (21) is the relevant forecasting efficiency hypotheses in the continuous component equation with respect to both weekly realized volatility components. From the VecHAR model we get  $p$ -values for the weekly measures ( $H_2$ ) of 98% in the foreign exchange market, 14% in the stock market, and 49% in the bond market. Implied volatility also subsumes the information content of the monthly measures ( $H_1 : A_{11m} = 0, A_{12m} = 0$ ) in the stock market, and the daily measures ( $H_3 : A_{11d} = 0, A_{12d} = 0$ ) in the bond market. In the foreign exchange market, implied volatility subsumes the information content of the continuous and jump measures at all frequencies, with  $p$ -values of  $H_1$  and  $H_3$  of 39% and 27% in this market. On the other hand, the daily measures retain incremental information in the stock market and the monthly measures in the bond market, where  $H_3$  respectively  $H_1$  are rejected at the 1% level. Implied volatility is also found to be a biased forecast of the continuous component of future volatility, but less so in the foreign exchange market where the  $p$ -value of the test of the unbiasedness hypothesis  $H_4 : \beta_1 = 1$  is two percent. Thus, unbiasedness of the implied volatility forecast is rejected at the 5% level in the foreign exchange market, but not at the 1% level, while it is rejected at the 1% level in the stock and bond markets. From the point estimates in the first row of each panel of Table 6 the bias in implied volatility is positive in all three markets in the sense that the estimated coefficient on implied volatility is below unity in all three panels, showing that implied volatility is upward biased as a forecast of the future continuous component. Possible reasons for this phenomenon are that volatility risk is priced (c.f. Bollerslev & Zhou (2006)) or that implied volatility may reflect information about future jump components as well, which we return to in  $H_{10}$  below.

In  $H_5$ - $H_7$ , the unbiasedness hypothesis  $H_4$  is tested jointly with the efficiency hypotheses  $H_1$ - $H_3$ . Consistent with previous results,  $H_5$ - $H_7$  are strongly rejected in the stock and bond markets, whereas  $H_6$ - $H_7$  (efficiency with respect to daily and weekly measures along with unbiasedness) are not rejected at the 5% level in the foreign exchange market ( $p$ -values

between six and eight percent). It is noted that although neither of the hypotheses  $H_1$  and  $H_4$  are rejected at the 1% level in the foreign exchange market, the joint hypothesis  $H_5$  is, suggesting that the monthly continuous and jump components are more informative about future volatility than the weekly and daily measures in the foreign exchange market, just as in the bond market, where  $H_1$  is rejected, but  $H_2$  and  $H_3$  are not.

Next, using the matrix notation

$$\bar{\mathbf{A}}_k = \begin{pmatrix} A_{11k} & A_{12k} \\ A_{21k} & A_{22k} \end{pmatrix}, \quad k = m, w, d,$$

we examine in  $H_8 : \bar{\mathbf{A}}_m = 0, \bar{\mathbf{A}}_w = 0, \bar{\mathbf{A}}_d = 0$  the possibility that all the coefficients on realized components in both the continuous and jump equations are jointly insignificant, which is a cross-equation restriction and hence requires the system approach. Here, the forecasting efficiency hypothesis is tested simultaneously for both the continuous and jump components, and our results from the VecHAR model reject this hypothesis in all three markets. Particularly in the foreign exchange market, where  $H_1$ - $H_3$  are not rejected, and the significance of the individual coefficients in the two first equations of the VecHAR system (Panel A of Table 6) suggests that implied volatility subsumes the information content of all return-based measures in forecasting both components of realized volatility, the joint test of the cross-equation restrictions in  $H_8$  is informative and shows that implied volatility is nevertheless not a sufficient statistic for all variables in the information set.

In  $H_9 : \beta_2 = 0$ , we examine the hypothesis that implied volatility carries no incremental information about the future jump component of realized volatility, relative to the return-based measures. This restriction leads to strong rejection in all three markets, thus providing evidence that option prices do contain incremental information about future jumps. Finally, in  $H_{10} : \beta_1 + \beta_2 = 1$ , again a cross-equation restriction, we test the hypothesis that implied volatility  $IV_t$  is an unbiased forecast of the sum of the continuous and jump components, i.e., of total realized volatility,  $RV_{t,t+22} = C_{t,t+22} + J_{t,t+22}$ . Although unbiasedness of implied volatility as a forecast of the future continuous component,  $H_4 : \beta_1 = 1$ , is rejected at the 5% level or better in all markets,  $H_{10} : \beta_1 + \beta_2 = 1$  is not rejected in the foreign exchange and stock markets. This reinforces earlier conclusions that implied volatility does forecast more than just the continuous component of realized volatility, that jumps are, to some extent, predictable, and, indeed, that option prices are calibrated to incorporate information about future jumps.

## 6 Concluding Remarks

This paper examines the role of implied volatility in forecasting future realized volatility and jumps in the foreign exchange, stock, and bond markets. We consider realized volatility constructed from high-frequency (5-minute) returns on \$/DM exchange rates, S&P 500 index futures, and 30 year Treasury bond futures, as well as implied volatility backed out from prices of associated currency, stock, and bond futures option contracts. Recent

nonparametric statistical techniques of Barndorff-Nielsen & Shephard (2004, 2006) are used to separate realized volatility into its continuous sample path and jump components, since Andersen et al. (2005) show that this leads to improved forecasting performance. We assess the incremental forecasting power of implied volatility relative to the improved realized volatility forecasting obtained by Andersen et al. (2005).

On the methodological side, we generalize the heterogeneous autoregressive (HAR) model proposed by Corsi (2004) and applied by Andersen et al. (2005) to include implied volatility from option prices as an additional regressor, and to the forecasting of the separate continuous and jump components of realized volatility. Furthermore, we introduce a new vector HAR (VecHAR) model for the simultaneous modeling of implied volatility and the separate components of realized volatility, controlling for possible endogeneity issues including, e.g., measurement error in implied volatility, the new variable added in the forecasting equations relative to Andersen et al. (2005).

On the substantive side, our empirical results show that in all three markets, option implied volatility contains incremental information about future return volatility relative to both the continuous and jump components of realized volatility. Indeed, implied volatility subsumes the information content of the weekly return-based measures in all three markets. The optimal forecasts combining information from both returns and option prices are based on implied volatility together with the most recent one-day realized volatility measures in case of the stock market, and implied volatility together with monthly realized measures in the bond market. In the foreign exchange market, implied volatility completely subsumes the information content of the return-based measures at all frequencies, when forecasting future realized volatility or its continuous component. In addition, implied volatility is an unbiased forecast of the sum of the continuous and jump components, i.e., of total realized volatility, in the foreign exchange and stock markets. Finally, our results show that even the jump component of realized return volatility is, to some extent, predictable, and that option implied volatility enters significantly in the relevant forecasting equation for all three markets. This suggests that option market participants in part base their trading strategies on information about future jumps in foreign exchange rates as well as in stock and bond prices, and hence interest rates.

## Acknowledgements

We are grateful to Per Frederiksen, Neil Shephard, participants at the Nordic Econometric Meeting 2005 in Helsinki, the Econometric Society World Congress 2005 in London, the American Finance Association Annual Meeting 2006 in Boston, the Duke Financial Econometrics Lunch Group Seminar, the CIREQ 2006 Conference on Realized Volatility in Montreal, and the 2006 Conference on Multivariate Modelling in Finance and Risk Management in Sandbjerg for comments, and especially to Tim Bollerslev for extensive comments and for providing the realized volatility, bipower variation, and tripower quarticity data used in this study. We are also grateful to Rolf Karlsen, Christian Kristensen,

and Christian S nderup for research assistance and the Center for Analytical Economics (CAE) at Cornell, the Center for Analytical Finance (CAF) at Aarhus, and the Danish Social Science Research Council (SSF/FSE) for research support. Part of this research was carried out during Morten Nielsen’s research visits at the School of Economics and Management, University of Aarhus, in connection with FSE grant no. 275-05-0220. This paper combines and further develops the material in our previous three papers “The implied-realized volatility relation with jumps in underlying asset prices,” “Forecasting exchange rate volatility in the presence of jumps,” and “The information content of Treasury bond options concerning future volatility and price jumps.”

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Table 1: Summary statistics

Panel A: Foreign exchange data				
Statistic	$RV_t$	$C_t$	$J_t$	$IV_t$
Mean	0.0128	0.0114	0.0015	0.0119
Std. dev.	0.0070	0.0062	0.0011	0.0050
Skewness	2.3511	2.2478	2.5018	1.0356
Kurtosis	12.070	11.275	11.415	4.4502
JB	643.66**	546.89**	591.02**	39.425**
Panel B: S&P 500 data				
Statistic	$RV_t$	$C_t$	$J_t$	$IV_t$
Mean	0.0290	0.0253	0.0037	0.0322
Std. dev.	0.0320	0.0274	0.0089	0.0239
Skewness	3.1905	3.2100	6.6904	2.2781
Kurtosis	15.598	16.155	56.179	11.886
JB	1,287.9**	1,383.9**	19,420**	644.06**
Panel C: Treasury bond data				
Statistic	$RV_t$	$C_t$	$J_t$	$IV_t$
Mean	0.0073	0.0062	0.0011	0.0089
Std. dev.	0.0027	0.0025	0.0007	0.0028
Skewness	1.3274	1.5243	1.7522	1.3448
Kurtosis	6.1635	6.7023	9.2871	5.0721
JB	103.75**	139.92**	315.17**	70.127**

Note: The annualized monthly realized volatility,  $RV_t$ , and its continuous and jump components,  $C_t$  and  $J_t$ , are constructed from about 6,336 5-minute  $\$/DM$  spot exchange rate returns (Panel A) with a total of 149 monthly observations, from about 2,134 5-minute S&P 500 index futures returns (Panel B) with a total of 155 monthly observations, and from about 1,738 5-minute 30 year US Treasury bond futures returns (Panel C) with a total of 146 monthly observations. The monthly implied volatility,  $IV_t$ , is backed out from the option pricing formula (15) applied to at-the-money call options on  $\$/DM$  futures, S&P 500 futures, and 30 year US Treasury bond futures. In all panels, each of the four volatility measures cover the same one-month interval between two consecutive expiration dates. One and two asterisks denote rejection of the null of normality for the Jarque & Bera (1980) test (JB) at the 5% and 1% significance levels, respectively.

Table 2: Realized volatility regression models

Panel A: Foreign exchange data						
Const.	$RV_t$	$C_t$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0069 (0.0011)	0.4586 (0.0723)	—	—	—	21.0%	9.35
0.0068 (0.0011)	—	0.5392 (0.1117)	-0.1560 (0.6530)	—	21.0%	9.28
0.0022 (0.0012)	—	—	—	0.8948 (0.0898)	38.7%	11.33
0.0022 (0.0011)	-0.0549 (0.0970)	—	—	0.9506 (0.1367)	40.4%	23.67*
0.0021 (0.0011)	—	0.0313 (0.1212)	-0.7217 (0.5721)	0.9527 (0.1365)	40.6%	25.77*
Panel B: S&P 500 data						
Const.	$RV_t$	$C_t$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0107 (0.0027)	0.6345 (0.0630)	—	—	—	39.7%	33.16***
0.0108 (0.0027)	—	0.6242 (0.0809)	0.6833 (0.2488)	—	39.3%	31.90**
-0.0050 (0.0027)	—	—	—	1.0557 (0.0665)	62.2%	17.25
-0.0057 (0.0027)	-0.1010 (0.0916)	—	—	1.1718 (0.1225)	62.2%	19.51
-0.0071 (0.0028)	—	-0.0557 (0.0936)	-0.5187 (0.2296)	1.2277 (0.1246)	62.9%	26.62**
Panel C: Treasury bond data						
Const.	$RV_t$	$C_t$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0033 (0.0005)	0.5507 (0.0700)	—	—	—	29.7%	18.40
0.0038 (0.0005)	—	0.6279 (0.0707)	-0.3085 (0.2487)	—	35.1%	19.34
0.0024 (0.0006)	—	—	—	0.5571 (0.0641)	34.3%	32.22**
0.0019 (0.0006)	0.2578 (0.0928)	—	—	0.3956 (0.0883)	38.0%	25.49*
0.0024 (0.0006)	—	0.3158 (0.0881)	-0.7623 (0.2442)	0.4374 (0.0835)	45.3%	23.63*

Note: The table shows ordinary least squares results for the regression specification (17) with asymptotic standard errors in parentheses. Adj  $R^2$  denotes the adjusted  $R^2$  for the regression and BG is the Breusch-Godfrey test statistic (with 12 lags) for the null of no serial correlation in the residuals. One and two asterisks denote rejection at the 5% and 1% significance levels, respectively.

Table 3: Realized volatility HAR models

Panel A: Foreign exchange data												
Const.	$RV_{t-22,t}$	$RV_{t-5,t}$	$RV_t$	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0061 (0.0011)	0.2186 (0.1138)	0.0981 (0.1438)	0.1706 (0.0828)	—	—	—	—	—	—	—	26.0%	10.89
0.0061 (0.0011)	—	—	—	0.2355 (0.1597)	0.0871 (0.1623)	0.2407 (0.0930)	0.1922 (0.6783)	-0.8014 (0.5640)	0.0897 (0.2192)	—	26.9%	13.55
0.0022 (0.0012)	—	—	—	—	—	—	—	—	—	0.8948 (0.0898)	38.7%	11.33
0.0021 (0.0011)	-0.1483 (0.1178)	0.0769 (0.1284)	0.0765 (0.0754)	—	—	—	—	—	—	0.8733 (0.1419)	41.1%	21.34*
0.0022 (0.0012)	—	—	—	-0.0517 (0.1526)	0.0097 (0.1471)	0.1114 (0.0869)	-0.7076 (0.6319)	0.0996 (0.5326)	-0.0474 (0.1993)	0.8715 (0.1515)	40.4%	22.72*
Panel B: S&P 500 data												
Const.	$RV_{t-22,t}$	$RV_{t-5,t}$	$RV_t$	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0053 (0.0025)	0.6240 (0.1132)	-0.3340 (0.1039)	0.6765 (0.1007)	—	—	—	—	—	—	—	53.0%	43.02**
0.0037 (0.0023)	—	—	—	0.1568 (0.1327)	0.0407 (0.1353)	0.9646 (0.1088)	0.2726 (0.2720)	-0.1427 (0.1812)	-0.7903 (0.3873)	—	61.9%	25.44*
-0.0050 (0.0027)	—	—	—	—	—	—	—	—	—	1.0557 (0.0665)	62.2%	17.25
-0.0052 (0.0027)	0.0378 (0.1311)	-0.1617 (0.0943)	0.3177 (0.1026)	—	—	—	—	—	—	0.9513 (0.1391)	64.0%	25.14*
-0.0051 (0.0027)	—	—	—	-0.1511 (0.1336)	0.0633 (0.1237)	0.6016 (0.1194)	-0.4486 (0.2809)	0.0454 (0.1690)	-0.7019 (0.3541)	0.7952 (0.1447)	68.2%	22.89*
Panel C: Treasury bond data												
Const.	$RV_{t-22,t}$	$RV_{t-5,t}$	$RV_t$	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0031 (0.0005)	0.3600 (0.1106)	0.1112 (0.1143)	0.1389 (0.0744)	—	—	—	—	—	—	—	32.5%	16.80
0.0037 (0.0005)	—	—	—	0.4203 (0.1347)	0.1436 (0.1363)	0.0826 (0.0776)	-0.1502 (0.2792)	-0.3378 (0.2567)	0.3660 (0.1764)	—	37.0%	16.19
0.0024 (0.0006)	—	—	—	—	—	—	—	—	—	0.5571 (0.0641)	34.3%	32.22**
0.0018 (0.0006)	0.0462 (0.1254)	0.1835 (0.1086)	0.0817 (0.0710)	—	—	—	—	—	—	0.3933 (0.0882)	40.4%	21.66*
0.0023 (0.0006)	—	—	—	0.1736 (0.1355)	0.1424 (0.1267)	0.0318 (0.0729)	-0.7018 (0.2842)	-0.0605 (0.2457)	0.2812 (0.1649)	0.4129 (0.0867)	45.5%	20.40

Note: The table shows HAR-RV-CJIV results for the specification (18) with asymptotic standard errors in parentheses. Adj R<sup>2</sup> denotes the adjusted R<sup>2</sup> for the regression and BG is the Breusch-Godfrey test statistic (with 12 lags) for the null of no serial correlation in the residuals. One and two asterisks denote rejection at the 5% and 1% significance levels, respectively.

Table 4: Continuous component HAR models

Panel A: Foreign exchange data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0053 (0.0009)	0.2273 (0.1128)	0.0634 (0.1370)	0.1892 (0.0779)	—	—	—	—	28.6%	10.18
0.0053 (0.0009)	0.2651 (0.1410)	0.0487 (0.1433)	0.2176 (0.0821)	-0.1025 (0.5989)	-0.5571 (0.4980)	0.0826 (0.1936)	—	27.9%	11.66
0.0020 (0.0010)	—	—	—	—	—	—	0.7885 (0.0804)	37.9%	13.85
0.0020 (0.0010)	0.0209 (0.1359)	-0.0171 (0.1310)	0.1077 (0.0774)	-0.8674 (0.5627)	0.2088 (0.4743)	-0.0340 (0.1775)	0.7409 (0.1349)	40.2%	23.86*
Panel B: S&P 500 data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0031 (0.0018)	0.1556 (0.0989)	0.0959 (0.0967)	0.7356 (0.0783)	—	—	—	—	67.2%	26.49**
0.0023 (0.0017)	0.1246 (0.0944)	0.0853 (0.0962)	0.8636 (0.0774)	0.3412 (0.1934)	-0.3477 (0.1288)	-0.4236 (0.2753)	—	73.7%	13.19
-0.0052 (0.0021)	—	—	—	—	—	—	0.9480 (0.0519)	68.6%	24.35*
-0.0040 (0.0019)	-0.0945 (0.0949)	0.1014 (0.0879)	0.6053 (0.0848)	-0.1721 (0.1997)	-0.2138 (0.1201)	-0.3606 (0.2517)	0.5659 (0.1028)	78.1%	17.07
Panel C: Treasury bond data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj R <sup>2</sup>	BG
0.0023 (0.0004)	0.4028 (0.1216)	0.1480 (0.1236)	0.0959 (0.0722)	—	—	—	—	39.4%	26.37**
0.0030 (0.0005)	0.4220 (0.1221)	0.1082 (0.1236)	0.1171 (0.0703)	-0.4624 (0.2531)	-0.3924 (0.2327)	0.2856 (0.1599)	—	43.1%	22.35*
0.0018 (0.0006)	—	—	—	—	—	—	0.4885 (0.0635)	29.1%	50.97**
0.0019 (0.0005)	0.2353 (0.1259)	0.1073 (0.1177)	0.0786 (0.0678)	-0.8798 (0.2641)	-0.1826 (0.2282)	0.2214 (0.1532)	0.3124 (0.0806)	48.3%	21.46*

Note: The table shows HAR-C-CJIV results for the specification (19) with asymptotic standard errors in parentheses. Adj R<sup>2</sup> denotes the adjusted R<sup>2</sup> for the regression and BG is the Breusch-Godfrey test statistic (with 12 lags) for the null of no serial correlation in the residuals. One and two asterisks denote rejection at the 5% and 1% significance levels, respectively.

Table 5: Jump component HAR models

Panel A: Foreign exchange data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0011 (0.0001)	—	—	—	0.3272 (0.0908)	-0.1192 (0.0910)	-0.0167 (0.0372)	—	7.3%	10.68
0.0009 (0.0002)	-0.0295 (0.0265)	0.0384 (0.0270)	0.0231 (0.0155)	0.2947 (0.1127)	-0.2443 (0.0937)	0.0071 (0.0364)	—	14.4%	14.27
0.0002 (0.0002)	—	—	—	—	—	—	0.1063 (0.0154)	23.8%	7.72
0.0003 (0.0002)	-0.0726 (0.0259)	0.0268 (0.0250)	0.0038 (0.0148)	0.1598 (0.1073)	-0.1092 (0.0904)	-0.0134 (0.0338)	0.1307 (0.0257)	27.2%	8.71
Panel B: S&P 500 data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0031 (0.0008)	—	—	—	0.0041 (0.0992)	0.2036 (0.0770)	-0.3052 (0.1571)	—	4.0%	63.60 <sup>***</sup>
0.0015 (0.0010)	0.0321 (0.0577)	-0.0446 (0.0588)	0.1010 (0.0473)	-0.0685 (0.1182)	0.2049 (0.0788)	-0.3668 (0.1684)	—	6.2%	57.71 <sup>**</sup>
0.0002 (0.0011)	—	—	—	—	—	—	0.1078 (0.0287)	8.3%	55.96 <sup>**</sup>
-0.0011 (0.0012)	-0.0567 (0.0613)	-0.0381 (0.0568)	-0.0037 (0.0548)	-0.2765 (0.1290)	0.2592 (0.0776)	-0.3413 (0.1626)	0.2293 (0.0664)	12.7%	48.37 <sup>**</sup>
Panel C: Treasury bond data									
Const.	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	Adj $R^2$	BG
0.0007 (0.0001)	—	—	—	0.3100 (0.0867)	0.0438 (0.0808)	0.0820 (0.0553)	—	12.2%	12.30
0.0007 (0.0002)	-0.0017 (0.0430)	0.0354 (0.0436)	-0.0345 (0.0248)	0.3122 (0.0892)	0.0546 (0.0821)	0.0804 (0.0564)	—	11.6%	12.13
0.0005 (0.0002)	—	—	—	—	—	—	0.0686 (0.0206)	6.9%	27.39 <sup>**</sup>
0.0004 (0.0002)	-0.0617 (0.0448)	0.0351 (0.0419)	-0.0469 (0.0241)	0.1780 (0.0940)	0.1221 (0.0812)	0.0598 (0.0545)	0.1005 (0.0287)	18.2%	15.71

Note: The table shows HAR-J-CJIV results for the specification (20) with asymptotic standard errors in parentheses. Adj  $R^2$  denotes the adjusted  $R^2$  for the regression and BG is the Breusch-Godfrey test statistic (with 12 lags) for the null of no serial correlation in the residuals. One and two asterisks denote rejection at the 5% and 1% significance levels, respectively.

Table 6: VecHAR models

Panel A: Foreign exchange data										
Dep. var.	Constant	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	$IV_{t-1}$	BG
$C_{t,t+22}$	0.0024 (0.0010)	0.0552 (0.1819)	-0.0078 (0.1457)	0.1231 (0.0741)	-0.7601 (0.6084)	0.1013 (0.6201)	-0.0177 (0.1481)	0.6369 (0.1948)	-	16.97
$J_{t,t+22}$	0.0004 (0.0002)	-0.0658 (0.0366)	0.0286 (0.0320)	0.0068 (0.0144)	0.1811 (0.1224)	-0.1305 (0.1067)	-0.0102 (0.0265)	0.1100 (0.0400)	-	8.23
$IV_t$	0.0013 (0.0006)	-0.0668 (0.0562)	-0.0521 (0.0417)	0.0788 (0.0376)	-0.0431 (0.1471)	-0.3046 (0.1927)	0.0242 (0.0448)	-	0.9028 (0.0685)	11.07
Panel B: S&P 500 data										
Dep. var.	Constant	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	$IV_{t-1}$	BG
$C_{t,t+22}$	-0.0054 (0.0022)	-0.1432 (0.1342)	0.1050 (0.0906)	0.5478 (0.1703)	-0.2863 (0.1953)	-0.1840 (0.1691)	-0.3466 (0.3923)	0.6918 (0.1643)	-	16.79
$J_{t,t+22}$	-0.0015 (0.0012)	-0.0732 (0.1007)	-0.0369 (0.0440)	-0.0232 (0.0441)	-0.3152 (0.1514)	0.2693 (0.1946)	-0.3365 (0.3564)	0.2720 (0.1045)	-	33.20**
$IV_t$	0.0009 (0.0008)	-0.0144 (0.0629)	0.0159 (0.0466)	0.0307 (0.0368)	0.3800 (0.0562)	-0.0158 (0.0462)	-0.3868 (0.0982)	-	0.9349 (0.0568)	29.10**
Panel C: Treasury bond data										
Dep. var.	Constant	$C_{t-22,t}$	$C_{t-5,t}$	$C_t$	$J_{t-22,t}$	$J_{t-5,t}$	$J_t$	$IV_t$	$IV_{t-1}$	BG
$C_{t,t+22}$	0.0019 (0.0005)	0.2253 (0.1244)	0.1073 (0.1120)	0.0766 (0.0664)	-0.9023 (0.2673)	-0.1713 (0.1866)	0.2180 (0.0627)	0.3292 (0.0831)	-	18.67
$J_{t,t+22}$	0.0003 (0.0003)	-0.0740 (0.0492)	0.0350 (0.0423)	-0.0494 (0.0297)	0.1506 (0.1174)	0.1358 (0.0790)	0.0556 (0.0605)	0.1210 (0.0550)	-	16.30
$IV_t$	0.0001 (0.0003)	0.0328 (0.0460)	-0.0264 (0.0386)	0.0517 (0.0365)	0.2547 (0.2350)	-0.0344 (0.1120)	0.0448 (0.0401)	-	0.9172 (0.0414)	8.62

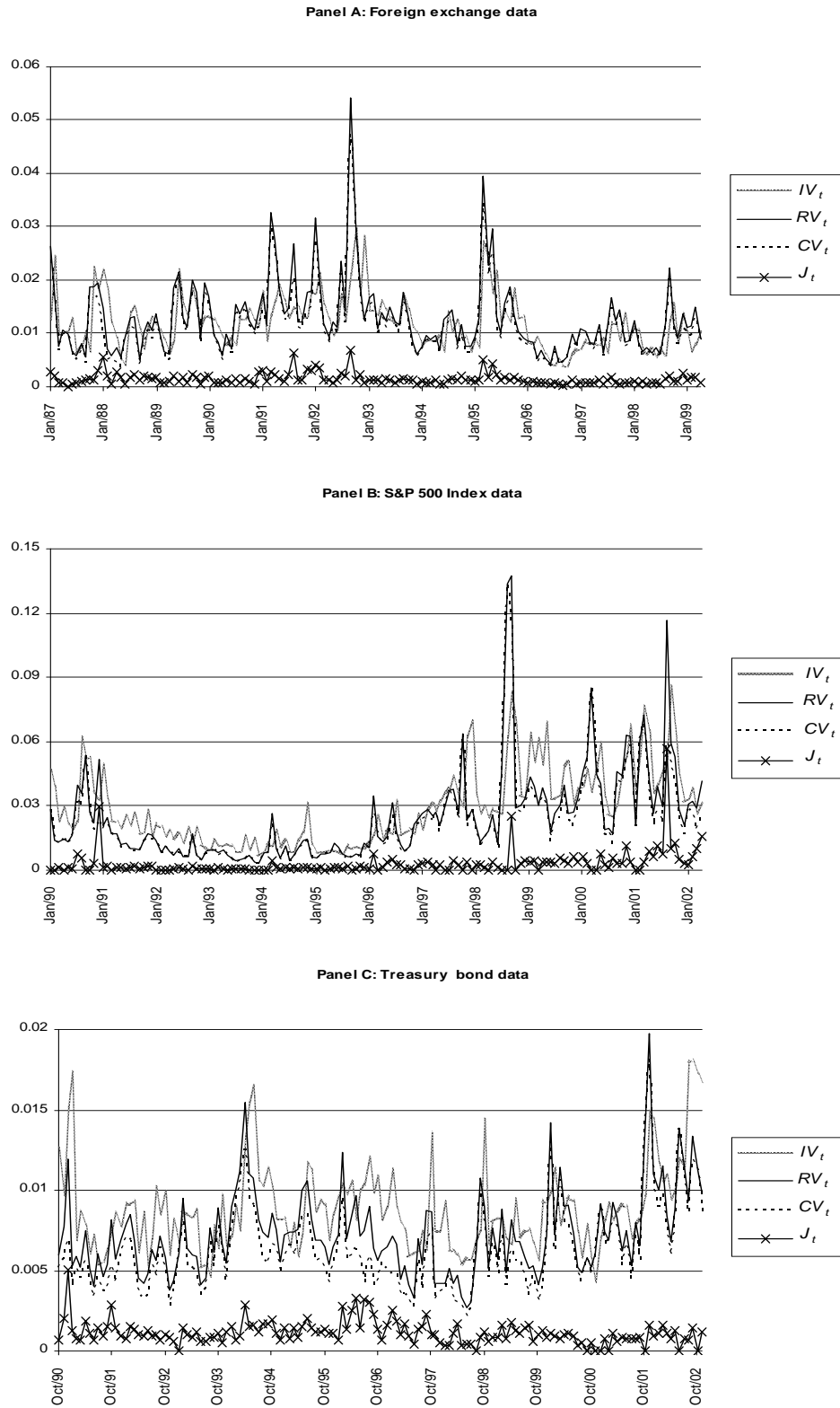
Note: The table shows FIML results for the simultaneous VecHAR system (21) with robust standard errors (sandwich-formula,  $H^{-1}VH^{-1}$ , where  $H$  is the Hessian and  $V$  the outer-product-gradient matrix) in parentheses. BG is the Breusch-Godfrey test statistic (with 12 lags) for the null of no serial correlation in the residuals. One and two asterisks denote rejection at the 5% and 1% significance levels, respectively.

Table 7: LR tests in VecHAR models

Hypothesis	Panel A: Foreign exchange data		Panel B: S&P 500 data		Panel C: Treasury bond data	
	Test statistics	d.f.	$p$ -values	Test statistics	d.f.	$p$ -values
$H_1 : A_{11m} = 0, A_{12m} = 0$	1.8922	2	0.3883	2.9715	2	0.2263
$H_2 : A_{11w} = 0, A_{12w} = 0$	0.0490	2	0.9758	3.8762	2	0.1440
$H_3 : A_{11d} = 0, A_{12d} = 0$	2.6273	2	0.2688	32.824	2	0.0000
$H_4 : \beta_1 = 1$	5.8950	1	0.0152	6.7789	1	0.0092
$H_5 : A_{11m} = 0, A_{12m} = 0, \beta_1 = 1$	14.560	3	0.0022	20.427	3	0.0001
$H_6 : A_{11w} = 0, A_{12w} = 0, \beta_1 = 1$	7.1047	3	0.0686	9.1775	3	0.0270
$H_7 : A_{11d} = 0, A_{12d} = 0, \beta_1 = 1$	6.9558	3	0.0733	35.498	3	0.0000
$H_8 : \mathbf{A}_m = 0, \mathbf{A}_w = 0, \mathbf{A}_d = 0$	29.377	12	0.0035	81.304	12	0.0000
$H_9 : \beta_2 = 0$	13.227	1	0.0003	13.403	1	0.0003
$H_{10} : \beta_1 + \beta_2 = 1$	2.2915	2	0.3180	0.0512	2	0.9747

Note: The table shows LR test results for the simultaneous VecHAR system (21) where the matrix notation  $\bar{\mathbf{A}}_k = \begin{pmatrix} A_{11k} & A_{12k} \\ A_{21k} & A_{22k} \end{pmatrix}$ ,  $k = m, w, d$ , is used.

Figure 1: Time series plots of monthly volatility measures





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