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# Imperfectively Competitive Labour Markets and the Productivity Puzzle

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**Abstract**

Standard models of imperfectly competitive labour market predict that real wages are unaffected by productivity. This is in conflict with empirical evidence. We integrate imperfectly competitive labour markets in a fully specified dynamic macromodel. While temporary shocks are consistent with the business cycle fact of little real wage and large employment responsiveness, we find in accordance with empirical evidence that permanent productivity changes affect real wages and not employment. Hence, the model solves the productivity puzzle and is capable of matching both business cycle and long-run facts concerning movements of real wages and employment.

*Keywords*: Productivity, unions, real wages and unemployment.

*JEL*: E32, J31, J51.

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# 1 Introduction

Imperfectly competitive labour markets or so-called bargaining models can account for an inefficiently low employment level. Moreover, under plausible assumptions the real wage is rigid to fluctuations in labour demand driven by e.g. relative price or productivity changes (see McDonald and Solow (1981)). From a business cycle perspective this implies that these models explain why the adjustment burden over the cycle falls on quantities (employment) rather than prices (real wages)[1]. This feature is often highlighted as showing that these models pass the first test of matching an important stylized fact concerning the movements of real wages and employment over the business cycle (see e.g. Blanchard and Fischer (1989)).

However, this property of the model also implies that productivity changes in the long-run are not affecting real wages but only employment. This is counterfactual for a number of reasons. First, it implies in a longer time perspective with underlying productivity improvements a continuous increase in employment at an unchanged real wage contrary to what has been observed. Second, all estimated wage equations based on bargaining models (so-called real wage equations) show that productivity is an important explanatory factor, and the elasticity of real wages with respect to productivity is usually found to be close to or equal to one. (see eg Tyrväinen (1995)). Finally, the model implies that the adjustment path is the same no matter whether the change is transitory or permanent.

The invariance of real wages to shifts in labour demand turns out to be robust with respect to the specification of the bargaining set- up (Nash-bargaining under right to manage, monopoly union, efficient bargaining etc) as long as the labour demand elasticity with respects to real wages is constant. While it is easy to change the technology assumption such that the labour demand elasticity is non-constant, this does not solve the problem since the effect of productivity changes under plausible assumptions will remain small. Accordingly, there is a productivity puzzle in models with imperfectly competitive labour markets, or in the words of Nickell (1998):

[1]For a recent survey of the empirical evidence on the cyclical behaviour of real wages see Abraham and Haltiwanger (1995).

> "In this model, a favourable shift in $\theta$ [productivity] leaves profits or quasi-rents *per employee* unchanged. So wages are unchanged. Since the whole purpose of these models is to capture the basic intuition that when the firm does well, it pays higher wages, the fact that the models do not accord with this intuition in a framework which is one of the bedrock of macroeconomics [constant elasticity production function] is unfortunate, to say the least."

The aim of this paper is to suggest a solution to this productivity puzzle. The key to this is to introduce imperfectly competitive labour markets in an explicit intertemporal model. This turns out to change the incentives underlying wage setting in a qualitative way even under standard assumptions like a utilitarian union and a constant elasticity of labour demand. Moreover, this framework allows an explicit analysis of how real wages and employment respond to temporary and permanent changes in productivity. This distinction turns out to be important since the persistency in productivity shocks is critical for the adjustment of real wages and employment.

The model framework used in the present paper is very close to that known from the real business cycle literature with the important exception that we allow for market power in the labour market[2]. Specifically we assume a monopoly union under a right to manage structure. We choose this specification since it is well-known to be a useful work-horse version of more general bargaining set-ups with the attraction that is allows considerable simplification without loss of qualitative insights.

We part company with most of the real business cycle literature also by aiming at finding an analytical solution rather than taking direct resort to numerical simulations. This has the advantage of producing a more transparent understanding of the mechanisms at play. In solving analytically for the equilibrium to the stochastic intertemporal model, we make use of the solution method proposed in Campbell (1994). The analytical solution shows that the dynamic adjustment process is unaffected by imperfect competition in the labour market. Moreover,

[2] See also Gali (1995). Danthine and Donaldson (1990) consider a real business cycle model with efficiency wage considerations while Andolfatto (1996) introduces search. None of these studies deal with the productivity puzzle.

it brings out that the problem encountered by real-business cycle models in accounting satisfactorily for the movements of real wages (much too pro-cyclical, cf eg Stadler (1994)) over the business cycles is due to a focus on very persistent shocks[3]. Temporary shocks can account for small real wage changes over the business cycle.

The paper is organized as follows. Section 2 presents the structure of the model, while section 3 describes the dynamic stochastic equilibrium. Section 4 considers real wage and employment adjustment, while section 5 gives some numerical illustrations. Section 6 concludes the paper.

# 2 A Dynamic Model with Imperfectly Competitive Labour Markets

We develop a dynamic model for a closed economy in which the only imperfection is imperfect competition in the labour market.

**Firms**

The economy consists of a large number of identical firms. They use labour $(N_t)$and capital $(K_t)$ to produce output $Y_t$. The production function is Cobb-Douglas:

$$Y_t = (A_t N_t)^{\alpha} K_t^{1-\alpha}, \qquad 0 < \alpha < 1. \tag{1}$$

The parameter $A_t$ is an exogenous technology parameter (see below).

A fraction $\delta$ of capital depreciates each period. Therefore, the capital accumulation process is:

$$K_{t+1} = (1-\delta) K_t + Y_t - C_{t,} \tag{2}$$

where $C_t$ is consumption. Thus, $Y_t - C_t$ is the gross investment at time $t$ $(I_t)$. Maximizing profit with respect to $K_t$ and $N_t$, we find the demand for the two

[3]This is done to ensure enough persistency in the real variables like output and employment, see eg Cogley and Nason (1995).

factors of production:

$$R_t = (1-\alpha)\left(\frac{A_t N_t}{K_t}\right)^{\alpha} + (1-\delta), \tag{3}$$

$$W_t = \alpha A_t^{\alpha}\left(\frac{K_t}{N_t}\right)^{1-\alpha}, \tag{4}$$

where $R_t$ is the gross rate of return on a one period investment in capital and $W_t$ is the real wage.

**Households**

The representative household possesses the following utility function for period $t$:

$$U(C_t, 1-N_t) = \log(C_t) + \theta\frac{(1-N_t)^{1-\gamma}}{1-\gamma}, \qquad \theta > 0, \tag{5}$$

where $1/\gamma$ represents the elasticity of intertemporal substitution for leisure. In models with indivisible labour, $\gamma = 0$ (see Hansen (1985) and Appendix A). The subjective discount factor equals to $\beta$ $(0 < \beta < 1)$, and the household maximizes

$$E\sum_{i=0}^{\infty}\beta^i\left[\log(C_{t+i}) + \theta\frac{(1-N_{t+i})^{1-\gamma}}{1-\gamma}\right] \tag{6}$$

subject to the budget constraints

$$C_t + I_t \leq W_t N_t + R_t K_{t.} \tag{7}$$

This determines consumption $(C_t)$ and labour supply $(N_t)$ when the real wage $(W_t)$ and the gross return to capital $(R_t)$ are taken as given.

The optimal consumption profile is determined by the well-known Euler-equation

$$\frac{1}{C_t} = \beta E_t\frac{R_{t+1}}{C_{t+1}}. \tag{8}$$

and labour supply is given by:

$$\theta(1-N_t)^{-\gamma} = \frac{W_t}{C_t}. \tag{9}$$

**Wage Setting**

Under perfect competition, the intersection between labour supply (9) and labour demand (4) determines the equilibrium real wage and employment level.

Consider alternatively the case of an imperfectly competitive labour market. Specifically, assume a monopoly union which determines the real wage ($W_t$) under a right to manage structure, that is, firms determine the employment level given the wage determined by the union. In this case the firm will be on its labour demand function, while the household in general will be off the individual labour supply curve (wanting to work more at the given wage).

We assume that the union chooses the wage that maximizes the representative household's utility, knowing that an increase of the wage will reduce employment (number of hours worked)[4]. In the terminology of the trade-union literature, we consider a utilitarian monopoly union. With divisible labour, we assume that every household works the same number of hours; there is unemployment in the sense that every worker would like to work more hours at the prevailing wage. Since all households are identical, it is natural to assume that the union maximizes the utility of the representative household. Thus, the union maximizes (6) subject to (7), (2) and labour demand (4). This gives:

$$\frac{1}{\alpha}\theta\left(1-N_t\right)^{-\gamma}=\frac{W_t}{C_t}. \tag{10}$$

Although this equation has the interpretation of a wage-setting scheme, it is very similar to the labour supply function (9) holding in the case of a competitive labour market; the only difference is the presence of a mark-up factor ($\frac{1}{\alpha}>1$) which raises the wage in the case of union wage determination. This suggests that the dynamics will be identical for competitive and imperfectly competitive labour markets; only the levels will differ.

Notice that intertemporal considerations in wage setting are captured by the fact that consumption enters the wage rule (10). An alternative derivation of

[4] Since the capital stock is predetermined, there is the potential that unions perceive that their wage policy affects the future capital stock and thus employment level. Under the assumption that labour is allocated randomly among firms between periods (see appendix A), it follows that no link is perceived between current wages and future labour demand if the number of firms is large.

(10) may bring the intuition out more clearly. The union provides its members with wage income yielding consumption possibilities at the cost of lost leisure. The benefit can be written as $\lambda_t W_t N_t$ where $\lambda_t$ is the shadow value of income to the household, the utility of leisure is $\theta\frac{(1-N_t)^{1-\gamma}}{1-\gamma}$, hence the objective of the union is to maximize

$$\lambda_t W_t N_t + \theta\frac{(1-N_t)^{1-\gamma}}{1-\gamma}$$

subject to (4). This yields

$$\frac{1}{\alpha}\theta(1-N_t)^{-\gamma} = \lambda_t W_t$$

Next, we have to determine the shadow value of income to the household. From the first-order condition for optimal consumption determination, we have

$$u_c(C_t, 1-N_t) = \frac{1}{C_t} = \lambda_t$$

where $\lambda_t$ is the Lagrange-multiplier associated with the intertemporal budget constraint. Combing these two expressions yields the wage equation (10). The point is that the current level of consumption measures the shadow value of income. Equation (10) implies that the higher the consumption level, the higher the wage has to be to induce a given labour supply. This suggests why productivity changes may feed into wages in a way not properly captured by static models. Productivity increases will induce higher consumption and thereby change incentives in wage formation.

The preceding argument was based on the assumption that labour is divisible. The standard models of imperfectly competitive labour markets assume that labour is indivisible, either you are employed or you are unemployed. This raises the question of the extent to which the results generalize to a situation with indivisible labour. A crucial aspect is how rationing in the labour market affects income distribution. Under the assumption that there exists an insurance market in which the income risk associated with unemployment can be diversified away[5], it follows that the model straightforwardly can be extended to the case of indivisible labour (see appendix A).

[5]This can also be achieved directly by the union via mandatory contributions to an unemployment benefit scheme, see eg Holmlund and Lundborg (1988).

To sum up: Under perfect competition, the model is composed of equations (1)-(4), (8) and (9); while the version with an imperfectly competitive labour market is composed of equations, (1)-(4), (8) and (10). The specific process for technological shocks will be specified later.

# 3 Stochastic Equilibrium

The next step is to find an analytical solution to the dynamic model. Although the model is specified such that a number of key relations are log-linear, we need to log-linearize a few relationships. This is done by replacing the main equations of the model with loglinear approximations, using Taylor approximations around the steady-state. Then we solve the equations by using the method of undetermined coefficients. The method is explained in Campbell (1994).

In steady-state, technology, capital, output and consumption grow at the constant rate $G = A_{t+1}/A_t$, and the gross rate of return on capital is a constant $R$. We use lower-case letters to denote logs and we suppress all constant terms so that the variables can be thought of as zero-mean deviations from the steady-state growth path. The resulting expressions are given below.

Production (1) becomes immediately and without approximation

$$y_t = \alpha a_t + \alpha n_t + (1-\alpha) k_t. \tag{11}$$

The capital accumulation process (2) becomes:

$$k_{t+1} = \lambda_1 k_t + \lambda_2 (a_t + n_t) + (1 - \lambda_1 - \lambda_2) c_t, \tag{12}$$

where

$$\lambda_1 \equiv \frac{1+r}{1+g}, \qquad \lambda_2 \equiv \frac{\alpha (r+\delta)}{(1-\alpha)(1+g)}.$$

The gross return to capital (3) becomes:

$$r_t = \lambda_3 (a_t + n_t - k_t), \qquad \lambda_3 \equiv \frac{\alpha (r+\delta)}{1+r}. \tag{13}$$

Consumption (8) becomes:

$$E_t \triangle c_{t+1} = \lambda_3 (a_{t+1} + n_{t+1} - k_{t+1}). \tag{14}$$

Finally, we assume that the technology parameter $a_t$ follows a first-order autoregressive process:

$$a_t = \phi a_{t-1} + \varepsilon_t, \qquad \varepsilon_t \ iid \ N\left(0, \sigma_\varepsilon^2\right) \tag{15}$$

where $\phi$ measures the persistence of technology shocks.

To complete the characterization of the model, we need the labour market relations and we state these explicitly for both a competitive and an imperfectly competitive labour market.

**Competitive Labour Market**

Using (9) and replacing $W_t$ by its value in (4) gives:

$$\theta \left(1 - N_t\right)^{-\gamma} = \alpha \frac{A_t^\alpha}{C_t} \left(\frac{K_t}{N_t}\right)^{1-\alpha}.$$

Loglinearizing this expression yields

$$n_t = v \left[\left(1-\alpha\right) k_t + \alpha a_t - c_t\right], \qquad v \equiv \frac{\left(1-N\right)\left(\frac{1}{\gamma}\right)}{N + \left(1-\alpha\right)\left(1-N\right)\left(\frac{1}{\gamma}\right)}, \tag{16}$$

where $N$ is the mean labour supply. This is the employment equation in the perfect competitive case. Taking logs of (4) gives the wage in the perfectly competitive case:

$$w_t = \alpha a_t + \left(1-\alpha\right)\left(k_t - n_t\right). \tag{17}$$

**Imperfect Competitive Labour Market**

Using (10) and replacing $W_t$ by its value in (4) gives:

$$\theta \left(1 - N_t\right)^{-\gamma} = \alpha^2 \frac{A_t^\alpha}{C_t} \left(\frac{K_t}{N_t}\right)^{1-\alpha}.$$

Loglinearizing this expression and dropping constants gives:

$$n_t = v \left[\left(1-\alpha\right) k_t + \alpha a_t - c_t\right].$$

This is the employment equation in the union case; up to constants it is exactly the same as in the perfectly competitive case (equation (16)). Moreover, the wage is also given by (4). Thus, we have the interesting result that the dynamics is exactly the same in the perfect competitive case and in the union case, only levels differ.

In the appendix we prove that there exists an equilibrium where

$$n_t = \eta_{nk} k_t + \eta_{na} a_t \tag{18}$$

$$k_{t+1} = \eta_{kk} k_t + \eta_{ka} a_t \tag{19}$$

$$y_t = \eta_{yk} k_t + \eta_{ya} a_t \tag{20}$$

$$w_t = \eta_{wk} k_t + \eta_{wa} a_t \tag{21}$$

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t \tag{22}$$

with parameters defined in the appendix. We can now characterize the dynamics more precisely. For the capital stock, equation (19) gives[6]:

$$k_{t+1} = \frac{\eta_{ka}}{(1 - \eta_{kk} L)} a_t.$$

Using that the technology process can be written

$$a_t = \frac{1}{(1 - \phi L)} \varepsilon_t. \tag{23}$$

it follows that the capital stock can be written as an AR(2) process:

$$k_{t+1} = \frac{\eta_{ka}}{(1 - \eta_{kk} L)(1 - \phi L)} \varepsilon_t. \tag{24}$$

[6] $L$ is the lag operator.

Given (23) and (24), it is possible to derive the dynamics of all other variables in the model. When these equations are combined with (18), (20) and (21), respectively we find that employment, real wages and output follow an ARMA(2,1) process:

$$n_t = \frac{\eta_{nk}\eta_{ka}L}{(1-\eta_{kk}L)(1-\phi L)}\varepsilon_t + \frac{\eta_{na}}{(1-\phi L)}\varepsilon_t,$$

$$w_t = \frac{\eta_{wk}\eta_{ka}L}{(1-\eta_{kk}L)(1-\phi L)}\varepsilon_t + \frac{\eta_{wa}}{(1-\phi L)}\varepsilon_t,$$

$$y_t = \frac{\eta_{yk}\eta_{ka}L}{(1-\eta_{kk}L)(1-\phi L)}\varepsilon_t + \frac{\eta_{ya}}{(1-\phi L)}\varepsilon_t.$$

# 4 Real Wages and Productivity

We can now turn to an analysis of how real wages and employment respond to productivity changes.

The short-run response of wages to a productivity change is found from (21) to be

$$\frac{\partial w_t}{\partial \varepsilon_t} = \eta_{wa}$$

and we have that (see appendix) the impact effects of a productivity shock is larger, the more persistent is the productivity shock, ie

$$\frac{\partial \eta_{wa}}{\partial \phi} > 0$$

provided that the interest rate is not too sensitive to the shock $\left(\lambda_3 \leq \overline{\lambda}_3\right)$.

The intuition for this condition is that the intertemporal substitution effect should not be too large. A change in the interest rate affects the consumption profile, and the condition essentially bounds the incentive to substitute consumption forward in time.

Considering the employment effect, we find from (18) that

$$\frac{\partial n_t}{\partial \varepsilon_t} = \eta_{na}$$

and for $\lambda_3 \leq \overline{\lambda}_3$ we have

$$\frac{\partial \eta_{na}}{\partial \phi} < 0.$$

Real wages respond more and employment less, the more persistent the productivity shock is.

The intuition for this result is that the shadow value of real wages is measured by the level of consumption. The higher the level of consumption, the lower the shadow price of labour income and hence the higher the wage demands. Since a temporary shock affects wealth and thus consumption by less than a permanent shock, it follows straightforwardly that the wage response will differ.

The difference to the standard bargaining model is seen clearly by noting that the shadow value of labour income is not constant. Hence, it follows that the mechanism present in this analysis does not play any role in static models implicitly assuming the shadow value of labour income to be constant.

Finally, let us consider how a fully persistent change affects real wages and employment. To this end the period $t$ real wage can be written

$$\begin{aligned} w_t &= \left( \frac{\eta_{wk}\eta_{ka}L}{(1-\eta_{kk}L)} + \eta_{wa} \right) a_t \\ &= \left( \frac{\eta_{wk}\eta_{ka}L}{(1-\eta_{kk}L)(1-\phi L)} + \frac{\eta_{wa}}{1-\phi L} \right) \varepsilon_t. \end{aligned}$$

It is an implication that productivity shocks do have a persistent effect on the real wage only if the shock is persistent ($\phi = 1$). For $\phi = 1$ we have (see appendix)

$$\frac{\partial w_{t+T}}{\partial \varepsilon_t} \rightarrow \frac{\eta_{wk}\eta_{ka}}{1-\eta_{kk}} + \eta_{wa} \equiv 1 \quad for \quad T \rightarrow \infty.$$

That is, fully persistent productivity shocks are in the long-run proportionally reflected in real wages leaving employment unaffected.

These results solve the productivity puzzle. The long-run comovement of real wages and productivity is consistent with the finding that fully persistent productivity changes should be matched proportionately by real wages. The business cycle property of little cyclical movements of real wages is consistent with temporary shocks driving the cyclical movements.

# 5 Numerical Examples

To shed additional light on the dynamic adjustment process, we calibrate the model; we take the same benchmark values for parameters as Campbell (1994); for a quarterly model: $g = 0.005$ (steady-state growth rate), $r = 0.015$ (steady-state interest rate), $\alpha = 0.667$, $\delta = 0.025$ and $N = 1/3$.

Figures 1-4 show the results. We see (figure 1) that the long-run response to a permanent technology shock ($\phi = 1$) is a proportional increase in the wage. In the short and medium-run, the wage reacts progressively towards its new steady-state value. This conclusion holds for every value of $\gamma$. Therefore in models with an imperfectly competitive labour market, wages respond in the long-run proportionally to permanent technology shocks. This solves the productivity puzzle.

Other results are worth mentioning. First, when shocks are transitory, the wage increases in a first step but decreases then progressively to its former steady-state equilibrium (figure 1). Second (figure 2), the employment increases in the short-run when there is a productivity shock; it then progressively decreases towards its previous level even when the shock is permanent; when the shock is transitory, there is a period in which the employment is lower than its previous steady-state level; this is caused by the higher wage. Third, (figure 3) output rapidly approaches its new steady-state level in case of a permanent shock. Finally, the behavior of capital is shown in figure 4; in the long-run, capital adjusts to permanent shocks only. When the shock is transitory, capital first increases and then decreases towards its previous steady-state level.

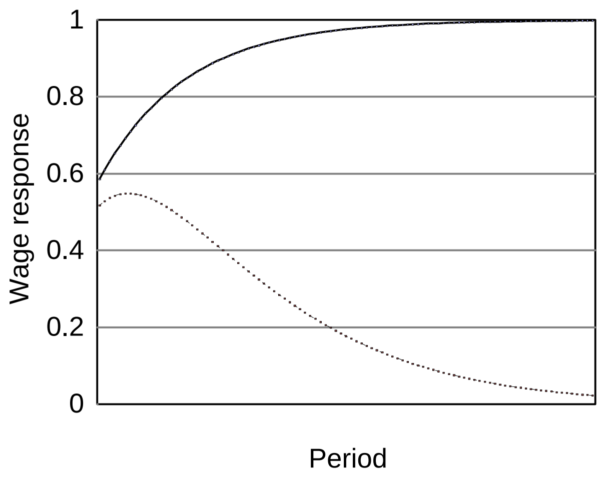


Figure 1: Wage response to a technology shock when $\gamma = 1$, $\phi = 0.95$ (dotted line) and $\phi = 1$ (solid line).

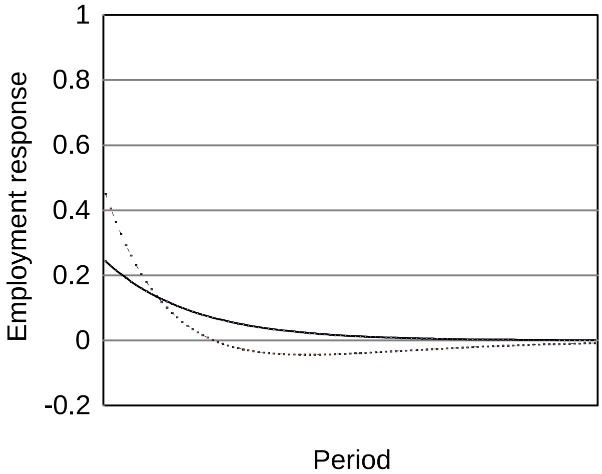


Figure 2: Employment response to a tecnology shock when $\gamma = 1$, $\phi = 0.95$ (dotted line) and $\phi = 1$ (solid line).

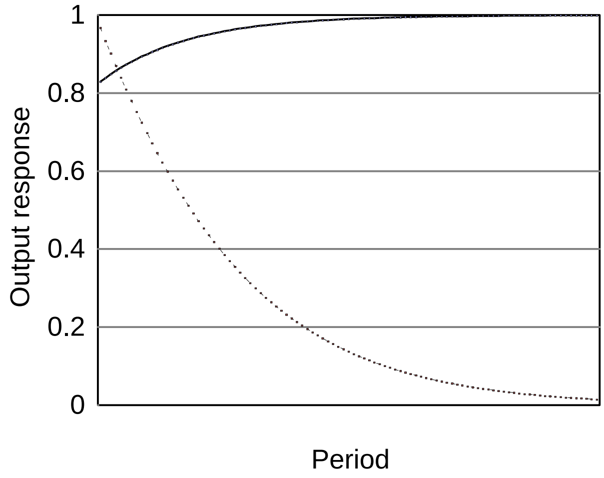


Figure 3: Output response to a technology shock when $\gamma = 1$, $\phi = 0.95$ (dotted line) and $\phi = 1$ (solid line).

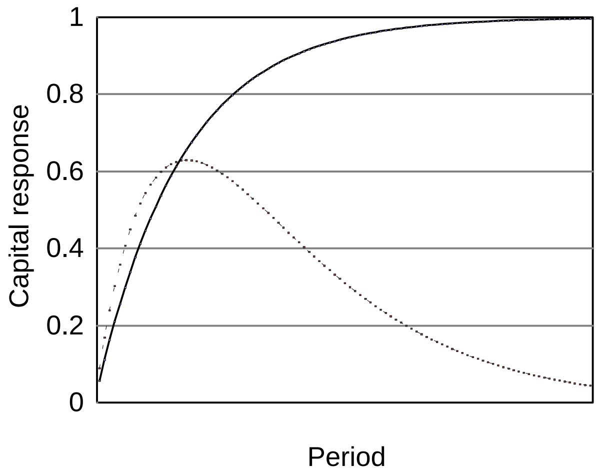


Figure 4: Capital response to a technology shock when $\gamma = 1$, $\phi = 0.95$ (dotted line) and $\phi = 1$ (solid line).

# 6 Conclusion

Popular models of unemployment suffer from the basic problem that they cannot explain how productivity changes are transmitted into wages. Since empirical evidence points to a strong relation between wages and productivity this is a severe shortcoming of imperfectly competitive labour market models.

The present analysis has shown that the paradox disappears once imperfectly competitive labour markets are introduced in a fully specified dynamic macro-model. The traditional standard models disregard that the marginal value of income received by employed workers depends on the shadow value of income, which in turn is affected by general productivity increases since they affect consumption. The basic intuition is thus that the higher the consumption level and thus the lower the shadow value of income the higher has the wage to be to induce a given labour supply. It follows straightforwardly that permanent productivity changes lead to wage increases.

The explicit dynamic structure of the model allows a comparison of transitory and permanent productivity shifts, and the model supports the intuitive assertion that permanent productivity shocks have a larger effect on wages than temporary shocks. A blend of temporary and permanent shocks can thus account both for the acyclicality of wages at business cycle frequencies and the long-run trend in wages following productivity.

It should be stressed that the findings in this paper relates to general or economy wide productivity changes, and the analysis does not offer an explanation of the channels through which firm specific productivity changes come to affect firm specific wages.

# Appendices

## A Indivisible Labour

In the main text we have assumed that all households are rationed in the same way on the labour market, that is, each household would like to work more hours at the prevailing wage but is unable to do it. It is probably more natural in a union context to assume that some households are working and other are (involuntary) unemployed. To account for this, we may assume that labour is indivisible: either the household is working an exogenous number of hours, or it is not working at all. To do this, we follow the steps of Hansen (1985) who introduces a market for unemployment insurance.

Households have access to an insurance market which allows them to insure their income wrt the risk of being unemployed. As Hansen shows, individuals will choose to be fully insured in equilibrium. Each period, the household chooses a consumption level contingent on labour market status and a level of unemployment compensation. The household's expected utility is:

$$U\left(C_t, 1-N_t\right) = \varrho\left[\log\left(C_t^w\right) + \theta\left(1-H\right)\right] + \left(1-\varrho\right)\left[\log\left(C_t^{nw}\right) + \theta\right] \tag{25}$$

where $\varrho$ is the probability of being employed, $C_t^w$ is the consumption when employed, $C_t^{nw}$ is the consumption when unemployed and $H$ is the exogenous number of hours that an employed worker must work. The household must choose $C_t^w$, $C_t^{nw}$ and a level of unemployment compensation $(Y)$ subject to the following budget constraints and depreciations constraints:

$$\begin{aligned}
C_t^w + I_t^w &\leq W_t H + R_t K_t - PY, \\
C_t^{nw} + I_t^{nw} &\leq Y + R_t K_t - PY, \\
K_{t+1}^w &= \left(1-\delta\right) K_t + I_{t,}^w \\
K_{t+1}^{nw} &= \left(1-\delta\right) K_t + I_{t,}^{nw}
\end{aligned}$$

where a subscript $w$ means that the household is working and a subscript $nw$ means that it not working; $P$ is the price of insurance. The revenue of the insurance company is $PY$ and its costs are $Y$ times the probability that the household be unemployed. Therefore, the profit is equal to $PY - (1-\rho)Y$; it is equal to zero only if $P = 1-\rho$ which will be the price of the insurance. As shown by Hansen (1985), given the price of the insurance, the household maximization implies that $C_t^w = C_t^{nw}$, $K_{t+1}^w = K_{t+1}^{nw}$, $I_t^w = I_t^{nw}$ and $Y = WH$, there is full insurance. This implies that all households have the same capital stock. Substituting in (25), we have the following maximization:

$$U\left(C_t, 1-N_t\right) = \log\left(C_t\right) + \varrho\theta\left(1-H\right) + \left(1-\varrho\right)\theta \quad \text{s.t.}$$

$$\begin{aligned} C_t + I_t &\leq \varrho W_t H + R_t K_t, \\ K_{t+1} &= \left(1-\delta\right)K_t + I_t. \end{aligned}$$

In equilibrium, the number of worked hours $N_t$ is equal to $\varrho H$. Therefore, $\varrho = N_t/H$. Substituting this expression in the above maximization, we find that the households have the following maximization:

$$U\left(C_t, 1-N_t\right) = \log\left(C_t\right) + \theta\left(1-N_t\right) \quad \text{s.t.}$$

$$\begin{aligned} C_t + I_t &\leq N_t W_t + R_t K_t, \\ K_{t+1} &= \left(1-\delta\right)K_t + I_t. \end{aligned}$$

which is equivalent to (5) and (7) when $\gamma = 0$. The analysis of the main text is unchanged when labour is indivisible.

# B The solution

In this Appendix we present the solution to the model formed by equations (11)-(15). Since the equations of the model are the same as in Campbell (1994), we have exactly the same solution to the model. Namely,

$$c_t = \eta_{ck} k_t + \eta_{ca} a_t, \tag{26}$$

where

$$\eta_{ck} = \frac{1}{2Q_2}\left(-Q_1 - \sqrt{Q_1^2 - 4Q_0Q_2}\right),$$

$$\eta_{ca} = \frac{(1+\alpha v)\{\lambda_3\phi - \lambda_2[\eta_{ck}(1+\lambda_3 v) - \lambda_3((1-\alpha)v-1)]\}}{[\eta_{ck}(1+\lambda_3 v) - \lambda_3((1-\alpha)v-1)](1-\lambda_1-\lambda_2-\lambda_2 v) - [1-\phi(1+\lambda_3 v)]},$$

$Q_0 = -\lambda_3 \left[ v \left(1-\alpha\right) - 1 \right] \left[ \lambda_1 + \lambda_2 v \left(1-\alpha\right) \right],$
$Q_1 = \left[ \lambda_1 + \lambda_2 v \left(1-\alpha\right) \right] \left(1+\lambda_3 v\right) - \left(1-\lambda_1-\lambda_2-\lambda_2 v\right) \lambda_3 \left[ v \left(1-\alpha\right) - 1 \right] - 1,$
$Q_2 = \left(1+\lambda_3 v\right) \left(1-\lambda_1-\lambda_2-\lambda_2 v\right).$
Using (16) and (26), we find:

$$\begin{aligned} n_t &= v(1-\alpha-\eta_{ck})k_t + v(\alpha-\eta_{ca})a_t, \\ &= \eta_{nk}k_t + \eta_{na}a_t, \end{aligned} \tag{27}$$

Using (12), (26) and (27), we find:

$$\begin{aligned} k_{t+1} &= \left[ \lambda_1 + \left(1-\lambda_1-\lambda_2\right) \eta_{ck} + \lambda_2 v(1-\alpha-\eta_{ck}) \right] k_t + \\ &\quad \left[ \lambda_2 + \left(1-\lambda_1-\lambda_2\right) \eta_{ca} + \lambda_2 v(\alpha-\eta_{ca}) \right] a_t, \\ &= \eta_{kk}k_t + \eta_{ka}a_t \end{aligned} \tag{28}$$

Using (17) and (27), we find:

$$\begin{aligned} w_t &= \left(1-\alpha\right) \left[ 1 - v(1-\alpha-\eta_{ck}) \right] k_t + \left[ \alpha - \left(1-\alpha\right) v \left(\alpha-\eta_{ca}\right) \right] a_t, \\ &= \eta_{wk}k_t + \eta_{wa}a_t \end{aligned} \tag{29}$$

Using (11) and (27), we find:

$$\begin{aligned} y_t &= \left[ \left(1-\alpha\right) + \alpha v \left(1-\alpha-\eta_{ck}\right) \right] k_t + \left[ \alpha + \alpha v \left(\alpha-\eta_{ca}\right) \right] a_t, \\ &= \eta_{yk}k_t + \eta_{ya}a_t. \end{aligned} \tag{30}$$

# C Proof: $\frac{\partial \eta_{ca}}{\partial \phi} > 0$ for $\lambda_3 \leq \overline{\lambda}_3$

From the definition of $\eta_{ca}$ we find that

$$\frac{\partial \eta_{ca}}{\partial \phi} = \frac{1}{N} \left[ \left(1+\alpha v\right) \lambda_3 - \left(1+\lambda_3 v\right) \eta_{ca} \right]$$

where

$$N \equiv \psi \left(1-\lambda_1-\lambda_2-\lambda_2 v\right) - \left(1 - \phi \left(1+\lambda_3 v\right)\right) < 0$$

$$\psi \equiv \eta_{ck} \left(1+\lambda_3 v\right) - \lambda_3 \left(\left(1-\alpha\right) v - 1\right) > 0$$

First we prove that $\eta_{ca}$ is monotone in $\phi$. To see this, notice that if $(1+\alpha v)\lambda_3 - (1+\lambda_3 v)\eta_{ca} < 0$ for $\phi = 0$, it follows that $\partial\eta_{ca}/\partial\phi > 0$ and hence no sign reversal is possible. A similar argument applies for $(1+\alpha v)\lambda_3 - (1+\lambda_3 v)\eta_{ca} > 0$.

Second using this monotonicity we search for a condition ensuring that $\frac{\partial\eta_{ca}}{\partial\phi} > 0$ for $\phi = 0$ which implies that $\frac{\partial\eta_{ca}}{\partial\phi} > 0 \quad \forall\phi \geq 0$.

From the condition given above we find that

$$\frac{\partial\eta_{ca}}{\partial\phi} > 0 \quad if \quad \eta_{ca} > \frac{(1+\alpha v)\,\lambda_3}{1+\lambda_3 v}$$

For $\phi = 0$ we have that

$$\eta_{ca} = \frac{(1+\alpha v)\,\psi\,(-\lambda_2)}{\psi\,(1-\lambda_1-\lambda_2-\lambda_2 v)-1}$$

Hence

$$\frac{-\lambda_2\,(1+\alpha v)\,\psi}{\psi\,(1-\lambda_1-\lambda_2-\lambda_2 v)-1} \geq \frac{(1+\alpha v)\,\lambda_3}{1+\lambda_3 v}$$

if

$$\lambda_3 \leq \overline{\lambda}_3 \equiv \frac{\lambda_2\psi}{\psi\,(\lambda_1+\lambda_2-1)+1}$$

# D Proof that $\frac{\eta_{wk}\eta_{ka}}{1-\eta_{kk}} + \eta_{wa} = 1$ for $\phi = 1$.

We have that $\frac{\eta_{wk}\eta_{ka}}{1-\eta_{kk}} + \eta_{wa} = 1$ requires

$$\eta_{wk}\eta_{ka} = (1-\eta_{wa})\,(1-\eta_{kk})\,.$$

We shall prove that this condition is fulfilled since

$$\eta_{wk} = 1-\eta_{wa}$$

and

$$\eta_{ka} = 1-\eta_{kk}.$$

To this end we note that

$$\eta_{wa} + \eta_{wk} = 1 + (1 - \alpha)\, v\, (\eta_{ca} + \eta_{ck} - 1)$$

$$\eta_{ka} + \eta_{kk} = 1 + (1 - \lambda_1 - \lambda_2 - \lambda_2 v)\, (\eta_{ca} + \eta_{ck} - 1)\,.$$

We shall prove that $\eta_{ca} + \eta_{ck} = 1$ for $\phi = 1$. Inserting the definition of $\eta_{ca}$ we find after some tedious steps that this condition is equivalent to

$$\begin{aligned}
&(1 + \lambda_3 v)\,(1 - \lambda_1 - \lambda_2 - \lambda_2 v)\,\eta_{ck}^2 \\
&+ [-\lambda_2\,(1 + \alpha v)\,(1 + \lambda_3 v) - (1 + \lambda_3 v)\,(1 - \lambda_1 - \lambda_2 - \lambda_2 v) \\
&-\lambda_3\,((1 - \alpha)\,v - 1)\,(1 - \lambda_1 - \lambda_2 - \lambda_2 v) + \lambda_3 v]\,\eta_{ck} \\
&+ [(1 + \alpha v)\,\lambda_3 + (1 + \alpha v)\,\lambda_2\lambda_3\,((1 - \alpha)\,v - 1) \\
&+ \lambda_3\,((1 - \alpha)\,v - 1)\,(1 - \lambda_1 - \lambda_2 - \lambda_2 v) - \lambda_3 v] = 0
\end{aligned}$$

This condition is precisely the same as the one defining $\eta_{ck}$. For $\eta_{ca} + \eta_{ck} = 1$ it follows that $\eta_{wa} + \eta_{wk} = 1$ and $\eta_{ka} + \eta_{kk} = 1$ implying that

$$\frac{\eta_{wk}\eta_{ka}}{1 - \eta_{kk}} + \eta_{wa} = 1$$

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