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Noisy Financial Signals and Persistent Effects of Nominal Shocks in Open Economies

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Abstract: Floating exchange rates display substantial short-run volatility implying that agents face a nontrivial information problem of disentangling temporary from permanent changes. Agents accumulate information and learn over time, and we analyze whether this basic information problem in the presence of nominal contracts can account for persistent effects of nominal shocks. Specifically we use a general equilibrium two-country model with specialized production and one-period nominal contracts and consider the propagation of nominal shocks over time. Informational problems are shown to have important qualitative and potentially strong quantitative importance for the propagation of nominal shocks.

JEL classification: E32; F41

Keywords: Exchange rates; Imperfect information; Learning; Nominal shocks; Persistence; Temporary and permanent shocks

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1 Introduction

Movements in exchange rates attract much attention and a great deal of resources are absorbed in interpreting the developments. Interpretation of any change in the exchange rate inevitably raises the question whether it is temporary or permanent. Information on this issue is clearly of value to traders in the foreign exchange markets, but also more widely as a number of decisions are dependent on exchange-rate movements in particular price and wage decisions. Time reveals whether given shocks were temporary or permanent, and this implies that information accumulation over time induces a learning process influencing the dynamic adjustment path.

That foreign exchange markets are characterized by substantial short-run volatility as well as substantial and lasting changes in exchange rates is well-known (Rogoff, 1996). It is also a well-established empirical fact that nominal exchange rates have persistent effects on relative prices between countries. It is an important puzzle to open macroeconomics to explain the empirical evidence indicating that the half-life of shocks on relative prices can be as long as 3-4 years (see, for example, McDonald, 1999, for a survey). This evidence shows that the problem of distinguishing between temporary and permanent changes in the exchange rate is by no means trivial. Since it moreover seems hard to explain the abovementioned facts without leaving a role for nominal shocks, it is natural to question what role nominal shocks have when agents are unable to distinguish temporary from permanent changes.[1]

The aim of this paper is to study how informational problems affect the transmission of nominal shocks into the real side of the economy within an explicit intertemporal general equilibrium model. The focus will in particular be on the adjustment process and the problem of persistency. To this end we need to model the sources of exchange-rate changes.

In the foreign exchange market agents trade one currency against another, and changes in supply and demand translate immediately into changes in exchange rates under a flexible exchange-rate regime. It follows that changes in exchange rates may originate from various forms of shocks arising on either the demand or the supply side. These shocks could be real or monetary in nature and leave a non-trivial problem of separating temporary from permanent changes in the exchange rate. Building this problem into a fully specified general equilibrium model is by no means trivial since it requires not only a specification of shocks which have temporary and permanent components

[1] The importance of distinguishing between temporary and permanent shocks goes back to Muth's (1960) discussion of the possible optimality of adaptive expectations, see also Sargent (1982). In standard macromodels the idea has been explored by Andersen (1985), Brunner et al (1983) and Gertler (1982).

but also an account of how these shocks affect the agents (preferences, endowments, technology, etc).

To simplify and to focus on the role of nominal shocks we analyze shocks arising in the financial sector. Thereby we also address the more difficult problem of explaining persistent effects of nominal shocks. An essential source of movements in exchange rates may be changes in the (relative) supply of liquidity which in turn may be driven by changes in the money base or in the liquidity created by the financial system. A variety of causes can thus give rise to a change in liquidity and leave a problem of disentangling temporary from permanent changes. Since the information problem of interpreting changes is essential to our story we exploit the model simplification which can be attained by assuming that changes in the available amount of liquidity are driven by changes in the money base which are either temporary or permanent. It is assumed that all current information is freely available, but agents face the problem of making inferences about its implications for the future. While the specific modeling approach taken here builds on the monetary (consumption) approach to exchange-rate determination, we think that the insights provided go beyond this specific way of modelling foreign exchange markets. The specific modeling approach adopted has the virtue of avoiding a specification of the financial system in great detail. The point that the determinants of exchange rates can have both temporary and permanent components is generic to any model of exchange-rate determination.

A substantial amount of effort has been put into analyzing the empirical role of monetary shocks for observed international business cycle fluctuations including exchange-rate movements. Our reading of the available evidence is that nominal shocks play a role, while it is less clear to what extent they are the dominant type of shock.[2] In the following we analyze the interplay between information problems and monetary shocks, not because monetary shocks necessarily are the empirically dominant type of shock, but because the theoretical literature so far has been unable to give a convincing explanation why nominal shocks have persistent real effects. In the same vein we assume one-period nominal contracts since it is well known from the literature that this type of contract does not generate any interesting dynamics absent information problems. The reference point is thus a setting where nominal shocks may have an impact effect due to one period nominal contracts but they do not by themselves produce any interesting dynamics, ie the impulse responses are trivial and implausible. We ask the simple question: to what

[2]Often cited papers are Clarida and Gali (1994) and Eichenbaum and Evans (1995). More recent contributions are Canova and de Nicoló (1998) and Rogers (1999). All find empirical support for the hypothesis that monetary shocks play a role (although to varying degrees).

extent is this changed when agents face the basic problem of disentangling temporary from permanent shocks?

The present analysis is related to the general discussion on propagation mechanisms in business cycle models. It is a well-known fact that the inherent propagation mechanism in closed-economy models is weak (see eg Cogley and Nason, 1995). This implies that even though short-run nominal rigidities can induce nominal shocks to have real effects, the effects are not persistent since the propagation mechanism is too weak. In an open economy context this usually shows up in the fact that the dynamics is essentially worked out over a time period equal to the contract length (see Obstfeld and Rogoff, 1995, 1996). For real shocks this problem can be circumvented by assuming persistency in the underlying shock. Obviously this procedure cannot readily be applied to nominal shocks since it is only the unanticipated part of the shock which has real effects. This has spurred an interest in strengthening the propagation mechanism via the introduction of staggered nominal price or wage contracts. While empirically relevant and strengthening propagation, it does not seem that this mechanism in itself can generate sufficient persistency in the adjustment process. The present paper takes an alternative route and shows that nominal shocks have persistent real effects when agents face the information problem of separating temporary from permanent shocks. This suggests a way of strengthening the endogenous propagation mechanism even if nominal contracts have short duration.

The specific structure of the model builds on the so-called New Open-Economy Macroeconomics launched by Obstfeld and Rogoff (1995) and which is a rapidly expanding field.[3] The literature has explored the consequences of both nominal price and wage contracts. We assume nominal wage contracts (see section 2.3 for further discussion) and a brief list of related New Open-Economy Macroeconomics papers all with sticky wages and perfect information include; Andersen and Beier (1999) who investigate a version of this paper's model with staggering as the propagation strengthening mechanism; Hau (1999) focuses on the role of nontradables on exchange-rate dynamics in a one-period deterministic model; Kollmann (1999) solves a stochastic model with both Calvo-style price and wage staggering numerically, and try to mimic cross country correlations of macro variables; and Obstfeld and Rogoff (1999) present a single-period stochastic model with exact closed form solutions, where they analyze the effects on welfare, expected output and expected terms of trade of the monetary regime. They rule out wealth reallocations by assumption.[4]

[3] For an excellent survey see Lane (1999). Several papers can be found on the New Open Economy Macroeconomics homepage http://www.princeton.edu/ ~bmdoyle/open.html.

[4] Apart form these sticky-wage papers there is a large literature on pricing-to-market

A basic lesson of models with one-period nominal wage or price contracts (see Lane, 1999) is that they generate real effects of nominal shocks but the dynamic implications are not very interesting since the steady-state effect is reached already after one period. Due to wealth reallocation induced by the impact effect and consumption smoothing, the shock has a permanent effect on relative prices (opposite in sign to the impact effect). The unit-root property of relative consumption thus carries over to the terms of trade.

A distinguishing feature of the present paper is that we want to be precise about the process followed by endogenous variables including their persistency properties. To this end we solve explicitly for the analytical solution to an intertemporal general equilibrium open economy model admitting wealth reallocations between countries and including nominal wage contracts as well as the information problem of separating temporary from permanent shocks. We show that this setting produces more plausible dynamics.

A closely related non-New Open-Economy Macroeconomics paper, which does introduce the notion of imperfect information and learning with respect to exchange-rate dynamics, is Gourinchas and Tornell (1996).[5] There are, however, some notable differences since; (i) they postulate an exogenous process for the interest rate and derive exchange rates from the uncovered interest rate parity; (ii) intertemporal aspects or interactions between the real and nominal side (including nominal rigidities) play no role; and (iii) they allow for agents misperceiving the parameters of the stochastic process. That said, the intuition underlying some of their results applies to our model as well.

The paper is organized as follows: The intertemporal two-country model with a flexible exchange rate is set up in section 2. The stochastic process for money and the information structure are defined in section 3. Section 4 describes the equilibrium. Section 5 considers the dynamics of nominal shocks. In section 6 we present numerical illustrations. Discussion and concluding remarks are offered in section 7.

and local-currency-pricing. From this literature Bergin and Feenstra (1999a,b) should be mentioned too. They consider pricing-to-market and find persistency in exchange rates in a model with price staggering, translog preferences and intermediate inputs. See Lane (1999) for other references and Obstfeld and Rogoff (1999) for a discussion on these pricing assumptions.

[5]For recent analyses of the relation between information and nonneutrality of nominal shocks, see Andersen (1997) and Kiley (1996).

2 A Stochastic Two-Country Model

Following Obstfeld and Rogoff (1995) we consider a two-country model with a flexible exchange rate and specialized production. There are two equally-sized countries and two goods, one produced by Home and one produced by Foreign firms. There are two assets in the economy: money and a real bond, where the former is motivated through money-in-the-utility (Feenstra, 1986) and the latter is traded in a perfect capital market. There is no capital in the model, no internationally mobile labor and to focus on interdependencies, the model is symmetric.

2.1 Consumers

The countries are inhabited by consumers who consume goods, supply labor, and hold money and bonds. The consumer's behavior is determined through maximizing expected lifetime utility. Let E_t be the expectations operator conditional on period t information (see next section for more on the information structure of the model), N labor supplied, M nominal balances, C a real consumption index and P the consumer price index, then the consumer's objective function is

$$U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[\frac{\sigma}{\sigma - 1} C_{t+j}^{\frac{\sigma-1}{\sigma}} + \frac{\xi}{1-\beta} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\beta} - \frac{\kappa}{1+\mu} N_{t+j}^{1+\mu} \right],$$

$$\sigma > 0, \quad \xi > 0, \quad \beta > 0, \quad \kappa < 0, \quad \mu > 0, \quad 0 < \delta \leq 1.$$

The real consumption index aggregates across consumption of the Home good and the Foreign good

$$C_t = \left[\left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^h \right)^{\frac{\rho-1}{\rho}} + \left(\frac{1}{2} \right)^{\frac{1}{\rho}} \left(C_t^f \right)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad \rho > 1,$$

where ρ is the elasticity of substitution between Home and Foreign goods.[6] The price index corresponding to composite consumption is

$$P_t = \left[\frac{1}{2} \left(P_t^h \right)^{1-\rho} + \frac{1}{2} \left(P_t^f \right)^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

[6] Assuming $\rho > 1$ ensures that the Marshall-Lerner condition is fulfilled, ie a nominal depreciation leads to an increase in the trade balance and, thus, a wealth reallocation in favor of Home. Andersen and Beier (1999) analyze the role of ρ for the adjustment process in more detail (see also Tille, 1999).

where $P_t^h(P_t^{*h})$ is the price of the Home good in Home (Foreign) currency and $P_t^f(P_t^{*f})$ is the price of the Foreign good in Home (Foreign) currency. As our focus will be on nominal wage rigidity we assume that law of one price holds for both goods, ie

$$P_t^h = S_t P_t^{*h}, \quad P_t^f = S_t P_t^{*f}.$$

An asterisk denotes Foreign variables and h (f) refers to variables originating in Home (Foreign). S is the nominal exchange rate defined as the Home price of Foreign currency. A direct implication of LOP is that purchasing power parity holds as well, that is, $P_t = S_t P_t^*$. As a consequence the subsequent analysis will focus on how nominal shocks affect the terms of trade. It can be shown in a setting including nontradables that the movements in the real exchange rate are qualitative equivalent to the movements in the terms of trade.

We assume that there is one internationally traded real bond denoted in the composite consumption good C. Let r_t be the consumption based real interest rate between dates t and t+1. The consumer's budget constraint for any period t is given by

$$P_t B_t + M_t + P_t C_t = (1 + r_{t-1}) P_t B_{t-1} + M_{t-1} + W_t N_t + \Pi_t + P_t \tau_t.$$

The right-hand side gives available resources as the sum of the gross return on bondholdings $(1+r_{t-1})P_t B_{t-1}$, initial money holdings M_{t-1}, labor income $W_t N_t$, nominal profit income Π_t and transfers from the government $P_t \tau_t$. Resources are allocated to consumption $P_t C_t$, nominal money holdings M_t and bondholdings $P_t B_t$.

Given the constant elasticity consumption index Home consumers' demands for the Home good and the Foreign good are

$$D_t^h = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t, \quad D_t^f = \frac{1}{2}\left(\frac{P_t^f}{P_t}\right)^{-\rho} C_t,$$

respectively, and mutatis mutandis for the demands by Foreign consumers. Aggregating demands, we find demands for the Home and Foreign goods to be

$$D_t \equiv D_t^h + D_t^{*h} = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t^W,$$

$$D_t^* \equiv D_t^f + D_t^{*f} = \frac{1}{2}\left(\frac{P_t^{*f}}{P_t^*}\right)^{-\rho} C_t^W,$$

where world consumption $C_t^W \equiv \frac{1}{2}C_t + \frac{1}{2}C_t^*$.

The first-order conditions arising from consumer maximization, written in log-linear form[7], are the Euler equation

$$E_t c_{t+1} = c_t + \sigma \log(1 + r_t), \tag{1}$$

and money demand

$$m_t - p_t = \eta_{mc} c_t - \eta_{mc}^1 E_t c_{t+1} + \eta_{mp}(p_t - E_t p_{t+1}), \tag{2}$$

where lower-case letters denote the natural logarithms of the corresponding upper-case variables.[8] All constants are neglected.[9] The labor-supply decision, or more generally, the wage setting issue is dealt with in section 2.3.

For later reference it is noted that (1) implies that relative consumption follows a martingale ie

$$E_t\left(c_{t+1} - c_{t+1}^*\right) = c_t - c_t^*.$$

The log-linearized versions of the price indices are

$$p_t = \frac{1}{2}\left(p_t^h + s_t + p_t^{*f}\right), \quad p_t^* = \frac{1}{2}\left(p_t^h - s_t + p_t^{*f}\right),$$

and the terms of trade is defined as

$$q_t \equiv p_t^h - p_t^f = p_t^h - p_t^{*f} - s_t.$$

2.2 Firms

Firms demand labor and produce the Home good. There is perfect competition in the product markets implying that firms are price takers. The wage is taken as given as well. The good is produced subject to a decreasing returns technology linking output Y^h and labor input N[10]

$$Y_t^h = N_t^\gamma, \quad 0 < \gamma < 1.$$

[7]The model is specified so as to yield a log-linear model. However, log-linearizations are needed for money demand and the budget constraint. See appendix A for details and definitions of constants.

[8]In the rest of the paper η_{xz} denotes the elasticity of the variable X with respect to the variable Z. Superscripts are included when the right hand side variable has more than one entry, eg lagged and leaded variables (cf η_{mc} and η_{mc}^1 in equation [2]).

[9]These constant terms include variance terms which are constant under the stochastic process considered.

[10]Y^h (Y^{*f}) is used as notation for Home (Foreign) output as we leave Y (Y^*) as notation for real incomes (see appendix A).

Maximizing profits subject to technology yields the following labor demand and output supply (in logs)

$$n_t = \eta_{nw}\left(p_t^h - w_t\right), \quad \eta_{nw} = (1-\gamma)^{-1}, \tag{3}$$

$$y_t^h = \eta_{yw}\left(p_t^h - w_t\right), \quad \eta_{yw} = \gamma(1-\gamma)^{-1}.$$

The elasticity of labor demand with respect to product real wage is $-\eta_{nw}$ and the elasticity of output supply with respect to product real wage is $-\eta_{yw}$. Profits are distributed to households.

2.3 Wage setting

To leave a role for nominal shocks we need to introduce nominal contracts. We do this by specifying one-period nominal wage contracts. While nominal rigidities may prevail in both product and labor markets, we focus on nominal wage rigidities as one possible source of nominal rigidity. Empirical evidence also supports that nominal wage rigidities might play a role (see eg Estevão and Wilson, 1998; Obstfeld and Rogoff, 1999; Spencer, 1998; Taylor, 1998).[11] With pre-setting of nominal wages it becomes of importance for wage setters how to separate temporary and permanent influences on the exchange rate. In this way informational problems get a non-trivial role for the interplay between the monetary and the real side of the economy.

We build on a rather extensive literature introducing imperfect competition into the labor market (see eg Moene and Wallerstein, 1993, for a survey and references). Workers are organized in (monopoly) unions, and each union represents a (small) subset of workers supplying labor to a given group of firms.[12] Each union is utilitarian choosing a wage for period t given all available information in period $t-1$ so as to maximize the expected utility of workers which in turn depends on the wage income received and the disutility of work. The level of employment is determined by firms given the wage set

[11] Taylor (1998) presents a more balanced view by concluding "... detailed studies do not provide evidence that one form of rigidity [wage] is more significant than the other [price] ..." (p 23).

[12] It is known from Hart (1982) that by assuming a sufficiently large number of unions, it is possible to maintain the property that they have market power in the labor market without introducing the possibility that they perceive that they can affect the whole economy (eg aggregate prices). By assuming symmetry we are able to attain a simple form for the optimal wage contract.

by the union (right-to-manage structure). Since all unions are identical, we can write the wage decision problem of a representative union as maximizing

$$E_{t-1}\left(\zeta_t \frac{W_t}{P_t} N_t - \frac{\kappa}{1+\mu} N_t^{1+\mu}\right),$$

where ζ_t measures the shadow value of wage income to the household ($\zeta_t = C_t^{-\frac{1}{\sigma}}$ cf the consumer's optimization problem). The union takes into account that employment is determined according to (3) and the optimal nominal wage to be quoted for period t can now be written

$$W_t = \kappa \frac{\eta_{nw}}{\eta_{nw}-1} \frac{E_{t-1}\left(N_t^{1+\mu}\right)}{E_{t-1}\left(C_t^{-\frac{1}{\sigma}} \frac{N_t}{P_t}\right)},$$

with the basic interpretation that the optimal wage is set so as to balance the marginal gain from a wage increase given as the (expected) marginal gain in consumption,

$$(1-\eta_{nw}) E_{t-1}\left(C_t^{-\frac{1}{\sigma}} \frac{W_t N_t}{P_t}\right),$$

with the marginal costs given by the (expected) negative effect of increased effort,

$$-\kappa \eta_{nw} E_{t-1}\left(N_t^{1+\mu}\right).$$

It is noted that the wage demands of the union are increasing in its market power measured by the wage elasticity of labor demand ($-\eta_{nw}$).

Using that all endogenous variables are log-normally distributed (cf below), we can by use of the labor demand (3) write the (log) nominal wage for period t as

$$w_t = E_{t-1}\left[\eta_{wp} p_t^h + (1-\eta_{wp})\left(s_t + p_t^{*f}\right) + \eta_{wc} c_t\right], \tag{4}$$

where

$$\eta_{wp} = (1+\mu\eta_{nw})^{-1}(0.5+\mu\eta_{nw}), \qquad \eta_{wc} = [\sigma(1+\mu\eta_{nw})]^{-1}. \tag{5}$$

Period-t employment is determined by the labor-demand equation given this wage.

It follows from the wage equation that nominal wages and thus prices depend on expected exchange rates. This captures the channel through which exchange rates affect the real side of the economy. Note also that (4) captures the basic homogeneity property which will be generic to any micro-founded wage-setting model.[13]

[13] It follows that adopting the framework of eg Obstfeld and Rogoff (1999) does not

2.4 Government

The only source of uncertainty in the model is changes in the money supplied by the government. We assume that the government balances its budget each period, ie

$$M_t - M_{t-1} = P_t \tau_t.$$

In other words, the only role of the government is to issue money. Money is transferred to consumers in a lump-sum fashion. Home money is only held by Home residents and vice versa for Foreign money. The stochastic process governing money supply along with the assumptions on the informational structure are described in detail in the next section.

We end the description of the model by noting that Foreign is completely symmetric and that a (symmetric) equilibrium exists (see appendix A) in which money is neutral.

3 Money Supply and Information Structure

The only source of uncertainty in the model is the (relative) money supply. A straightforward way by which to introduce the problem of separating between temporary or permanent influences is to assume that the relative money supply process is[14]

$$m_t - m_t^* = z_t + u_t, \tag{6}$$

and

$$z_t = \theta z_{t-1} + \varepsilon_t, \quad 0 \leq \theta \leq 1,$$

where u and ε are independent and normally distributed mean-zero shocks with variances σ_u^2 and σ_ε^2.

The money supply process captures that some changes are temporary (u) and some are permanent (z), and that agents cannot readily disentangle one type of shock form the other. The parameter θ determines the degree

change the results. In their framework there are differentiated labor inputs and CES production (linear in equilibrium). Workers have monopoly power in the labor market, set the wage a period in advance and firms are monopolistic in the product markets. In this setup we would get the exact same wage equation with $\frac{\eta_{nw}}{\eta_{nw}-1}$ replaced with $\frac{\phi}{\phi-1}$, where ϕ is the substitution elasticity between different kinds of labor inputs.

[14] Since information flows continuously in the foreign exchange market and the wage contracts are assumed to be fixed for a given period of time, it follows that some aggregation of information has already implicitly taken place in transforming financial data to match the length of wage contracts.

of persistency. Agents know current and past realizations of relative money supplies, but they do not know whether current changes are temporary or permanent. Accordingly, there is full current information, but agents learn over time as they accumulate information on the variables of interest. Notice that the present analysis does not build on the confusion between absolute and relative price changes underlying the New Classical macroeconomic models.

We distinguish between three types of shocks: a *transitory* shock ($u > 0, 0 < \theta \leq 1$), a *persistent* shock ($\varepsilon > 0, 0 < \theta < 1$) and a *fully permanent* shock ($\varepsilon > 0, \theta = 1$). The need to introduce the latter distinction arises because the dynamic properties following a shock to z change dramatically between the case where θ is strictly less than one and the case where it is exactly one (see below). When we present our results it is implicit that we condition on the type of shock which has hit the economy. Note, that to avoid confusion we use the terminology *temporary* and *permanent* shocks when we in general discuss the information disentangling problem (cf, for example, the introduction) and when we in general discuss the money supply process (6).[15] Hence, u is labelled as the temporary part, and z the permanent part. When specific shocks are considered, we use the three labels: *transitory*, *persistent* and *fully permanent*.

The specification (6) can be interpreted in several ways. The immediate interpretation is that money supply (monetary policy) changes may be either temporary or permanent. The temporary component may also represent short-run volatility or noise generated by the financial system (money multiplier), while the permanent part represents the underlying fundamental factors (money base). Finally, the temporary part may reflect measurement errors or errors included in preliminary data relative to the official data published with some lag.

Given the money supply process we can consider expectation formation, and it follows that

$$E\left(m_{t+1} - m^*_{t+1} \mid I_t\right) = E\left(z_{t+1} \mid I_t\right),$$

where we for the moment use the more compressed notation $E_t(\cdot)$ in favor of $E(\cdot \mid I_t)$ for the mathematical expectation conditioned on information available at time t (denoted by I_t). Observations of the relative money supply are in the present set-up the only source of information on its future movements and it follows that

$$E\left(z_{t+1} \mid I_t\right)$$

[15] This means that both persistent and fully permanent shocks in the general discussion are labelled permanent shocks.

$$
\begin{aligned}
&= E\left(z_{t+1} \mid I_{t-1,} m_t - m_t^*\right) \\
&= E\left(z_{t+1} \mid I_{t-1}\right) \\
&\quad + E\left[z_{t+1} - E\left(z_{t+1} \mid I_{t-1}\right) \mid m_t - m_t^* - E\left(m_t - m_t^* \mid I_{t-1}\right)\right],
\end{aligned}
$$

where the second term according to Graybill (1961) can be written

$$
\begin{aligned}
&E\left[z_{t+1} - E\left(z_{t+1} \mid I_{t-1}\right) \mid m_t - m_t^* - E\left(m_t - m_t^* \mid I_{t-1}\right)\right] \\
&= \theta h\left[m_t - m_t^* - E\left(m_t - m_t^* \mid I_{t-1}\right)\right],
\end{aligned}
$$

where

$$
0 \leq \theta h = \theta \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} \leq \theta.
$$

The h-coefficient is increasing in the variability of innovations to the permanent part of the (relative) money stock and decreasing in the variability of the temporary changes. That is, if all shocks are permanent we have $h = 1$, and if all shocks are temporary $h = 0$ reflecting that current signals contain all information of relevance for predicting future money supplies. Subsequently we label h as the noise coefficient.

Using that

$$
E\left(z_{t+1} \mid I_{t-1}\right) = \theta E\left(z_t \mid I_{t-1}\right) = \theta E\left(m_t - m_t^* \mid I_{t-1}\right),
$$

the conditional expectation can be written as

$$
E_t\left(m_{t+1} - m_{t+1}^*\right) = \theta\left(1 - h\right) E_{t-1}\left(m_t - m_t^*\right) + \theta h\left(m_t - m_t^*\right),
$$

or

$$
\begin{aligned}
&E_t\left(m_{t+1} - m_{t+1}^*\right) \qquad (7) \\
&= \theta E_{t-1}\left(m_t - m_t^*\right) + \theta h\left[m_t - m_t^* - E_{t-1}\left(m_t - m_t^*\right)\right],
\end{aligned}
$$

that is, expectations of tomorrow's relative money supply are determined by the part of past expectations which are believed to carry forward plus an update which depends on the new information provided by the most recent observation (the unanticipated change in the relative money supply). Notice, that the compressed notation for the conditional expectation has been re-adopted. If we interpret the temporary part of money evolution (u) as noise and the permanent part (z) as fundamentals, the updating formula has a very intuitive interpretation. The more noise in the model, the larger σ_u^2 and, hence, the smaller h. As h decreases the less weight is put on the current observation of money supply when updating expectations since agents know that current movements tend to reflect noise, ie current signals have a low information content.

One last convenient way of expressing the updating formula is

$$E_t\left(m_{t+1}-m_{t+1}^*\right)=\theta\left[(1-h)\,E_{t-1}\left(m_t-m_t^*\right)+h\left(m_t-m_t^*\right)\right],$$

where the noise coefficient measures the fraction of a given change in the money supply which is taken to reflect a permanent change. This highlights how the variance ratio (h) determines how much weight is put on current observations relative to last periods expectations in forming expectations. The term inside the brackets gives the best prediction of the permanent part of the relative money stock at time t, and θ times this gives the best prediction of the period t+1 relative money stock.

In general the information content of this periods relative money supply is measured by $\theta \cdot h$. The noise coefficient governs the weight put on current money as a function of noise (σ_u^2 relative to σ_ε^2) and given this weight θ determines how much that should carry over to next period.

Since the updating of expectations to shocks is crucial to the results of this paper, it is useful to consider how expectations evolve under our three types of shocks. Figures 1-3 describe the adjustment path for the actual and expected money stock to transitory, persistent and fully permanent shocks, respectively. In all cases we consider a 1 percent positive shock to the money stock.[16]

Figure 1 about here

For a transitory shock (figure drawn for $\theta = 1$, $h = 0.5$) we see that although the relative money supply is only affected in one period, it takes several periods for the agents to learn this. Accordingly a positive transitory shock will imply that the money stock is unanticipated low in subsequent periods until agents eventually learn that the shock was transitory. The learning period is prolonged and therefore persistency is increasing when θ is increasing and h decreasing. This follows since increasing θ leads to an increased weight on the first term in (7) and decreasing h leads to an decreased weight on the second term.

Figure 2 about here

Next we turn to a persistent shock (figure drawn for $\theta = 0.9$, $h = 0.5$) in which case it also takes several periods before expectations and de facto money converge, ie money is unexpectedly high. Furthermore, it is interesting

[16]We look at dynamics of expectations under the assumption that they have been zero up to date 1, where a one-time 1 percent positive monetary shock hits the economy. No shock occurs after that.

to see that there is delayed overshooting. Initially expectations rise, and first several periods later they begin to fall. Delayed overshooting occurs when

$$E_t\left(m_{t+1}-m_{t+1}^*\right) < E_{t+1}\left(m_{t+2}-m_{t+2}^*\right).$$

This implies that delayed overshooting in expectations is ensured if

$$\theta h\left(m_t-m_t^*\right) < \theta^2 h\left(m_t-m_t^*\right)+\theta h\left[\theta\left(m_t-m_t^*\right)-\theta h\left(m_t-m_t^*\right)\right],$$

or

$$1-2\theta+h<0.$$

This condition will turn up later when we reach exchange-rate dynamics. The persistency results are the same as for a transitory shock.

Figure 3 about here

Finally, for a fully permanent shock (figure drawn for $\theta = 1$, $h = 0.5$) we find that expectations slowly converges to the actual money stock from below, and the learning period is prolonged the smaller is h. This is due to the fact that agents perceive current observations to be highly infected by noise. Despite being surprised the period after the shock, agents still perceive this 'new' surprise to be due to noise in the money market, and so forth.

Evidently, the dynamics of relative money, and its expected value differ across the three types of shocks. These differences will turn out to be crucial when we turn to the dynamic adjustment of the other variables of the model.

4 Equilibrium

Characterizing the equilibrium analytically is not only complicated by the presence of nominal contracts and the information problem but also the intertemporal structure linking current and future decisions via changes in wealth and expectations. We demonstrate in appendix B how to find an analytical solution such that we can solve explicitly for the process for endogenous variables given the process for the exogenous variables (the money stock) and the information structure. Given this we are able to analyze how the adjustment pattern depends on various types of shocks under various assumptions concerning the information structure.

Specifically it is demonstrated in appendix B that the equilibrium process for the nominal exchange rate is given as[17]

$$\Phi_{ss}(L)s_t = \Phi_{sm}(L)\left(m_t-m_t^*\right), \tag{8}$$

[17] We omit the exact derivations and expressions as they do not add enough information relative to the space they consume. The reader is referred to the appendix for more technical details.

where

$$\Phi_{ss}(L) = 1 - \phi_{ss}^1 L - \phi_{ss}^2 L^2,$$

$$\Phi_{sm}(L) = \phi_{sm} + \phi_{sm}^1 L + \phi_{sm}^2 L^2,$$

and L is the lag-operator.[18] Thus the dynamics of the nominal exchange rate is determined by second-order lag polynomials for both the autoregressive and moving average parts.

Similarly, the terms of trade can be written

$$\Phi_{qq}(L) q_t = \Phi_{qm}(L) \left(m_t - m_t^*\right), \tag{9}$$

where

$$\Phi_{qq}(L) = \Phi_{ss}(L),$$

$$\Phi_{qm}(L) = \phi_{qm} + \phi_{qm}^1 L + \phi_{qm}^2 L^2.$$

Finally, relative consumption can be written

$$\Phi_{cc}(L) \left(c_t - c_t^*\right) = \Phi_{cm}(L) \left(m_t - m_t^*\right), \tag{10}$$

where

$$\Phi_{cc}(L) = \Phi_{qq}(L) = \Phi_{ss}(L),$$

$$\Phi_{cm}(L) = \phi_{cm} + \phi_{cm}^1 L.$$

Informational problems are seen to have the potential to generate a complicated dynamic adjustment path for both the nominal exchange rate, the terms of trade and relative consumption following nominal shocks. In particular we have non-trivial dynamics running beyond the length of nominal contracts (here one period).

[18] Along the same line as η_{xy} denotes the elasticity of a variable X with respect to another variable Y, $\Phi_{xx}(L)$ denotes the autoregressive lag polynomial determining the dynamics of the variable X. The entries are given by $\phi_{xx}^1 L$, $\phi_{xx}^2 L^2$, etc. Similarly, the 'moving-average' part has entries ϕ_{xm}, $\phi_{xm}^1 L$, etc.

5 Dynamics Under Imperfect Information

We now turn to a detailed analysis of the dynamic adjustment path under imperfect information. In particular we lay out the economy's response following an increase in relative money supply and we focus on the impact effects, the dynamics and the long-run effects. Given the amount of algebra involved, we, once again, refer to appendix B for technical details and proofs. Numerical illustrations are given in the next section.

For later comparison it is useful to note that under perfect information we can easily characterize the terms of trade following a one-time positive money shock as

$$q_t < 0,$$

$$q_{t+j} = q_{t+i} > 0, \ \ \forall j, i \quad j \geqslant 1, \ \ i \geqslant 1,$$

that is, the impact effect of the shock - due to one-period nominal wage contracts - is a terms-of-trade deterioration while the long-run effect is an improvement attained already after one period.[19] In the absence of information problems the adjustment to unanticipated shocks is ended after a period of time equal to the contract length, that is, the dynamics is trivial and the impulse-response functions are implausible. This also applies for relative consumption and the nominal exchange rate. Next we explore how this is changed under imperfect information.

5.1 Impact Effects

The impact effect of an expansion in (relative) Home money supply (regardless of the type of shock) is a depreciation of the nominal exchange rate and a terms-of-trade deterioration

$$\frac{\partial s_t}{\partial (m_t - m_t^*)} = \phi_{sm} > 0, \quad \frac{\partial q_t}{\partial (m_t - m_t^*)} = \phi_{qm} < 0,$$

as a result of the one-period nominal wage contracts. Consequently, demand is switched towards Home goods, Home output increases and Home runs a current-account surplus. This leads to a wealth increase which via consumption smoothing induces an increase in relative consumption

$$\frac{\partial (c_t - c_t^*)}{\partial (m_t - m_t^*)} = \phi_{cm} > 0.$$

[19] The perfect information relative money supply is $m_t - m_t^* = \theta \left(m_{t-1} - m_{t-1}^*\right) + \varepsilon_t$ where $\theta \in [0, 1]$.

Consumption only reacts to unanticipated changes in wealth and thus money supply. The impact effects differ across information regimes and the relative magnitude of this difference is determined by two things; the persistency parameter (or type of shock) θ and the noise coefficient h.

Under perfect information is might be conjectured that the impact effect is independent of the type of shock since the nominal rigidity is temporary. This is wrong due to intertemporal effects affecting the money market and therefore the nominal exchange rate (see appendix C). Following a positive monetary shock the real affects depend on the nominal depreciation, which in turn depends on the change in the money stock, consumption and expected future exchange rates. The depreciation can not be independent of the type of shock for the simple reason that it would imply the same real effects and thus the same change in (expected) future consumption, but that is inconsistent with the condition for money market equilibrium in the future which depends on the extent to which the money change was permanent. The nominal depreciation therefore has to be smaller for a temporary than for a permanent change in the money supply to ensure equilibrium in the money market. This brings out an important lesson from working with an explicit intertemporal model; the subsequent money supply has a crucial effect on what happens in the current period when the shock hits the economy. More persistence in the shock leads to larger impact effects.

Turning to the impact effects under imperfect information it is useful to note that it can be written as a weighted average of the impact effects of a temporary and a permanent shock under perfect information. More specifically, assume that $\theta = \widetilde{\theta} \in (0, 1]$, then with obvious notation[20]

$$\text{Impact Eff}|^{\text{imp info}} = (1-h)\ \text{Impact Eff}|^{\text{per info}}_{\theta=0} + h\ \text{Impact Eff}|^{\text{per info}}_{\theta=\widetilde{\theta}},$$

where

$$h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2},$$

and

$$\text{Impact Eff}|^{\text{per info}}_{\theta=0} < \text{Impact Eff}|^{\text{per info}}_{\theta=\widetilde{\theta}}.$$

It follows immediately that the impact effect of a persistent or a fully permanent shocks is smaller, and oppositely that the impact effect of a transitory shock is larger under imperfect information. The intuition is straightforward; part of a transitory shock is taken to be permanent and vice versa for a permanent shock. Moreover we have that more information confusion, ie the

[20] Consult appendix B for more on this expression.

lower h, lowers the impact effects of persistent and fully permanent shocks and increases the impact effect of transitory shocks. The impact effect under imperfect information depends positively on θ for all three types of shocks since the impact effect under perfect information is increasing in it.

5.2 Dynamics

The adjustment process is not ended after one period since agents over time acquire more information of relevance in deciding whether changes were temporary or permanent. There is thus a non-trivial dynamic adjustment process driven by accumulation of information over time.

Considering the interim dynamic process for the variables in more detail, it is interesting to note that the autoregressive part of the nominal exchange rate, the terms of trade and relative consumption are equivalent. This indicates that persistency in the adjustment process spreads out to all variables. Furthermore, the autoregressive parameters depend only on the parameters characterizing the stochastic process for money and therefore the information structure since

$$\phi_{ii}^1 = 1 + \theta\left(1 - h\right), \quad i = s, q, c,$$

$$\phi_{ii}^2 = 1 - \phi_{ii}^1 = \theta\left(h - 1\right), \quad i = s, q, c,$$

and there is a unit root in the autoregressive part, ie

$$\phi_{ii}^1 + \phi_{ii}^2 = 1, \quad i = s, q, c.$$

This brings out that the information structure has a potentially important role for the dynamic adjustment process, just as it did for the impact effects, and therefore also for persistency in the adjustment process even if nominal contracts only have a duration of one period. Specifically we find that more persistency in the permanent part of the shock (higher θ) increases the first-order autoregressive term and decreases the second order autoregressive term, and vice versa for a higher h.

Using that

$$\theta - \theta h = \theta\left(1 - \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2}\right) = \theta\frac{\sigma_u^2}{\sigma_\varepsilon^2 + \sigma_u^2} > 0,$$

we can infer that $\theta\left(1 - h\right) \in [0, 1]$ where the lower bound is reached when there is no confusion between temporary and permanent shocks (either all shocks are permanent $[\sigma_u^2 = 0]$ or all shocks are temporary $[\theta = 0]$) and the upper bound is reached when permanent shocks are fully permanent ($\theta = 1$) and short-run volatility dominates ($\sigma_u^2 \to \infty$). It follows that

$$\phi_{ii}^1 \in [1, 2], \quad i = s, q, c,$$

$$\phi_{ii}^2 \in [-1, 0], \quad i = s, q, c,$$

and it can be concluded that more persistency in the permanent part of the shocks (high θ) and more information confusion (low h) generate the strongest persistency in the response of the variables following nominal shocks. This supports that informational problems in interpreting financial signals may be an important propagation mechanism for nominal shocks. The reason for this can be found in the updating formula for expected money (7), where it is seen that a very low value of h decreases the information content of current money. The expected value of next periods relative money stock is determined solely by θ times last periods beliefs. A high θ yields persistency of these beliefs and, therefore, unexpected money.

To clarify the importance of the information structure for the dynamic adjustment process, we give in table 5.1 the characteristics of the ARIMA-processes followed for our three key variables under imperfect and perfect information. The top entry is the imperfect information process and the bottom entry is the process that the variable at hand would follow under perfect information

Table 5.1. Information Structure and Stochastic Processes for the Endogenous Variables.

Imperfect Information Perfect Information	s	q	$c - c^*$
Transitory	$ARIMA$(1,1,2) $ARIMA$(0,1,1)	$ARIMA$(1,1,2) $ARIMA$(0,1,1)	$ARIMA$(1,1,1) $ARIMA$(0,1,0)
Persistent	$ARIMA$(2,1,2) $ARIMA$(1,1,1)	$ARIMA$(2,1,2) $ARIMA$(0,1,1)	$ARIMA$(1,1,0) $ARIMA$(0,1,0)
Fully Permanent	$ARIMA$(1,1,1) $ARIMA$(0,1,0)	$ARIMA$(1,1,1) $ARIMA$(0,1,1)	$ARIMA$(1,1,0) $ARIMA$(0,1,0)

It is seen that going from perfect to imperfect information adds richer dynamics for all three variables. This will also be clear in the numerical illustrations below. It is noteworthy that, while the dynamics are the same for the terms of trade and relative consumption across the three types of shocks under perfect information, we find different processes under imperfect information. This simply reflects the importance of the learning process under imperfect information. To take an example, under perfect information relative consumption follows a random walk. Similarly, it can be shown that under imperfect information relative consumption follows a random walk in the surprise of relative money

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \phi_{cm} \left[m_t - m_t^* - E_{t-1} \left(m_t - m_t^* \right) \right],$$

but due to the learning problem in the latter term, we end up with a more complicated dynamic structure than under perfect information.

In the New Open-Economy Macroeconomics literature there is a growing consensus that the determinants of persistence in models with staggering are (i) marginal costs' sensitivity with respect to output; (ii) prices' sensitivity with respect to marginal costs (see Lane, 1999), and researchers try to find conditions under which the sensitivity is low in both cases. In this model the persistency is generated without these considerations as the persistency parameters only depend on the informational structure (θ,h).

5.3 Delayed Overshooting

Eichenbaum and Evans (1995) present impulse-response functions to monetary shocks which display delayed overshooting, that is, following an (unexpected) expansionary monetary policy the nominal exchange rate first follows a depreciating path and then appreciates towards its long-run value. This is different from the overshooting phenomena arising in the seminal work by Dornbusch (1976) in which the nominal exchange rate depreciates on impact and then appreciates. It turns out that a persistent shock may produce delayed overshooting since we have that

$$\frac{\partial s_t}{\partial \varepsilon_t} < \frac{\partial s_{t+1}}{\partial \varepsilon_t},$$

and since the shock fades out in the long run, for some period in time t+j

$$\frac{\partial s_{t+j+1}}{\partial \varepsilon_t} < \frac{\partial s_{t+j}}{\partial \varepsilon_t}, \quad j \geq 1,$$

is a possible dynamic adjustment pattern. A necessary condition for delayed overshooting is that

$$1 - 2\theta + \theta h < 0,$$

implying that a combination of a high degree of persistency in the shock (high θ) and much noise (low information content of signals, ie low h) tends to create this phenomenon. This condition on θ and h is the sufficient condition for generating delayed overshooting in money expectations (see section 3). Intuitively, this has to be fulfilled for delayed exchange-rate overshooting to occur at all, but is not enough. The reason is that h can become too small, or equivalently, learning can become too slow. There are two opposite effects relative to the impact effect; increased consumption pulls in the direction of an appreciation and the upward revision of expectations pulls in the direction of a depreciation. If learning is too slow the first effect outweighs the second. In the intermediate case it is the other way around and in the last case learning is too fast.[21]

To what extent does this capture the delayed overshooting found by Eichenbaum and Evans (1995)? There are two difficulties in interpreting their results relative to the present setting since their analysis does not allow for an explicit distinction between temporary and permanent shocks and since their results of course depends on the actual realization of temporary and permanent shocks over the sample period. With respect to the latter it might be that there are unusually many persistent shocks (compared to population moments) in the (small) sample they consider.

Gourinchas and Tornell (1996) find that unconditional delayed overshooting (irrespective of whether the shock is temporary or permanent) is not possible when agents know the parameters of the underlying stochastic processes. They show that unconditional delayed overshooting is possible if agents perceive both the importance of temporary shocks and the degree of persistency in the permanent part of the shock to be higher than what is actually the case. In this model, where agents know the parameters of the model, unconditional delayed overshooting can be ruled out as well (see appendix B). We do not pursue the issue of misperception.

Lastly, the Eichenbaum-Evans result might not be robust. New evidence (Faust and Rogers, 1999) points in that direction.

5.4 Long-Run Effects

Nominal shocks do in general have long-run real effects. That is, long-run neutrality does not prevail when nominal shocks induce a reallocation of wealth. Via consumption smoothing a change in wealth is transformed into a change in consumption. The unit root implied by consumption smoothing is thus recovered in the nominal exchange rate, the terms of trade and relative

[21] Actually, there are three effects. It can be seen from the guess for the nominal exchange rate (see appendix B): $s_t = \pi_{sc}(c_t - c_t^*) + \pi_{sm}(m_t - m_t^*) + \pi_{sm}^1 E_{t-1}(m_t - m_t^*)$ where $\pi_{sc} < 0$ and $\pi_{sm} > 0$, $\pi_{sm}^1 > 0$. The two effects discussed in the text correspond to the first and the last terms. The third effect is the middle term. In the period after the shock relative money has fallen relative to the impact effect. But delayed overshooting occurs only for θ 'large'. In other words, relative money supply is almost the same in the period after and for clarity of the argument we disregard this factor.

consumption. This is seen most clearly by writing the exchange-rate and the terms-of-trade equations as the following stochastic trend representation

$$s_t = \frac{1}{1+\phi_{ss}^2 L}[\phi_{sm}(m_t - m_t^*) + (\phi_{sm} + \phi_{sm}^1)(m_{t-1} - m_{t-1}^*) + (\phi_{sm} + \phi_{sm}^1 + \phi_{sm}^2)\sum_{j=0}^{\infty}(m_{t-2-j} - m_{t-2-j}^*)],$$

$$q_t = \frac{1}{1+\phi_{qq}^2 L}[\phi_{qm}(m_t - m_t^*) + (\phi_{qm} + \phi_{qm}^1)(m_{t-1} - m_{t-1}^*) + (\phi_{qm} + \phi_{qm}^1 + \phi_{qm}^2)\sum_{j=0}^{\infty}(m_{t-2-j} - m_{t-2-j}^*)],$$

and

$$c_t - c_t^* = \frac{1}{1+\phi_{cc}^2 L}[\phi_{cm}(m_t - m_t^*) + (\phi_{cm} + \phi_{cm}^1)\sum_{j=0}^{\infty}(m_{t-1-j} - m_{t-1-j}^*)].$$

The long run effects obviously depend on the type of shock as well as the information structure. The table given below summarizes the signs of the long-run effects

Table 5.2. Long-Run Effects.

	s	q	$c - c^*$	$\log(\frac{i}{i^*})$
Transitory	≤ 0	≥ 0	≥ 0	0
Persistent	< 0	> 0	> 0	0
Fully Permanent	$\gtreqless 0$	> 0	> 0	0

To see the intuition for these long run effects note that the shock has a real effect due to the nominal wage contracts. This creates a current account surplus which allows higher consumption in all future periods. This affects labor supply and therefore in turn all real variables. The unanticipated wealth changes induced by the nominal shock have long-run real effects. The interest rate spread is unaffected in the long run irrespective of the type of shock since the nominal exchange rate is constant in the long-run and the interest rate spread is determined by the UIP (see next subsection).

Under imperfect information the long-run effects differ between the three types of shocks, reflecting that the learning processes are different in the three cases. The differences in the learning processes are highlighted when we in almost the same manner as for the impact effects can write the long-run effect under imperfect information as (with obvious notation)[22]

$$\text{LR Effect}|^{\text{imp info}} = LE \cdot \left[(1-h) \text{ LR Effect}|^{\text{per info}}_{\theta=0} + h \text{ LR Effect}|^{\text{per info}}_{\theta=\tilde{\theta}} \right],$$

where

$$LE = \text{Learning Effect}.$$

That is, the long-run effect is the weighted average of the perfect information long-run effects multiplied by some term which we label "Learning Effect". This learning effect is dependent on the persistence of the shock and the informational setting ie the degree of noise. More specifically it can be shown that for a;

$$\begin{aligned} \text{transitory shock} &: \quad LE = \frac{1-\theta}{1-\theta+\theta h}; \\ \text{persistent shock} &: \quad LE = \frac{1}{1-\theta+\theta h}; \\ \text{fully permanent shock} &: \quad LE = \frac{1}{\theta h} = \frac{1}{h}; \end{aligned}$$

with the direct implication that the long-run effects of a temporary shock are smaller than the long-run effects of a permanent shock. Another intuitive implication worth emphasizing is that the long-run effects are decreasing in h $(\frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2+\sigma_u^2})$. More noise makes learning slower and increases the long-run effects.

While it is commonly asserted that nominal shocks are neutral in the long run we find this to be invalidated for open economies due to the induced wealth reallocation except in the case of a transitory shocks when the underlying permanent shocks are fully permanent ($\theta = 1$). The reason for long-run neutrality in this case is that the initial positive impact effect on relative consumption is subsequently redressed as agents perceive part of the shock to be fully permanent ($\theta = 1$), when the shock is actually transitory. It turns out that the positive and negative surprises precisely balance in the long run for $\theta = 1$ while for $\theta < 1$ the positive surprises always dominate.

Another puzzling finding is that a fully permanent shock may have an ambiguous effect on the nominal exchange rate in the long run. The likely

[22] There is one slight exception: the nominal exchange rate following a fully permanent shock. In this case the long-run effect under perfect information consists of the flexible price long-run effect (1) and a negative component arising from labor-leisure substitution. Under imperfect information it is only the last component that can be written as the postulated weighted average.

exchange-rate long-run outcome is a depreciation and a gradual adjustment from below to the new long-run equilibrium, ie undershooting. For extreme parameter values, more specifically a low value of h, a long-run appreciation is possible.[23] If there is confusion, or rather, enough short-run noise, we might get overshooting and a long-run appreciation. The reason for this is simply that the real effects generated by the extremely slow learning process are very strong. This outcome-dependence on the structural parameters of the model is also known from Dornbusch's overshooting model and in this case might be interpreted as showing that perverse nominal exchange-rate movements are possible if there is enough information confusion.

Comparing the relative sizes of the long-run effects, it turns out that they are smaller for transitory shock but larger for persistent and fully permanent shocks when compared to the long-run effects under perfect information.

The long-run effects following any shock are decreasing in h. There are two opposite effects. First, the impact effect increases. Second, learning is faster (see unexpected money dynamics in figures 1-3). The second effect outweighs the first. The dependence of the long-run effects of θ is a bit more mixed. For the transitory case the long-run effect is decreasing in θ. Again there are two effects; first, a positive effect since the impact effect increases, and, secondly, a negative effect since the period-after switch is increasing in θ. The second effect is strongest. For a persistent shock the long-run effects are increasing in θ.

5.5 Nominal Interest Rate Spread

Up to now we have only looked at three variables, but actually we have one more interesting variable: the nominal interest rate spread. The implications of nominal rigidities and imperfect information for the adjustment of nominal interest rates can easily be worked out by noting that the real asset available to households implies that it is possible to construct a nominal asset with a return determined as (in logs, disregarding constants)[24]

$$\log(1+i_t) = \log(1+r_t) + E_t p_{t+1} - p_t.$$

It follows that the nominal interest rate spread can be written in accordance with the uncovered interest rate parity as

$$\log(1+i_t) - \log(1+i_t^*) = E_t s_{t+1} - s_t,$$

[23] Numerical analysis showed that only for h very, very close to zero did this perverse dynamic adjustment occur.

[24] For the nominal interest rate we deviate from letting lower-case letters denote natural logarithms of upper-case counterparts. It is denoted i_t.

where it has been used that $p_t - p_t^* = s_t$.

Using the equilibrium value for the exchange rate, it follows that the interest-rate spread can be written in the following semi-reduced form

$$\begin{aligned} \log(1+i_t) - \log(1+i_t^*) &= (\theta - \theta h)\left[\log(1+i_{t-1}) - \log\left(1+i_{t-1}^*\right)\right] \\ &\quad + \Phi_{is}(L) s_t + \Phi_{im}(L)(m_t - m_t^*), \end{aligned}$$

where the lagpolynomials are defined in appendix B.

We have that a (relative) monetary expansion leads to a fall in the interest rate spread, ie

$$\frac{\partial\left[\log(1+i_t) - \log(1+i_t^*)\right]}{\partial(m_t - m_t^*)} < 0,$$

yielding the plausible result that a domestic monetary expansion leads to a fall in the domestic nominal interest relative to the foreign nominal interest rate.

Persistency in the adjustment of the interest rate spread is captured by the presence of both the lagged interest rate spread and nominal exchange rates changes on top of money supply changes. Under the perfect information regime there is persistency in the interest rate spread following a persistent shock. This is simply the mirror image of (correct) appreciation expectations. In the transitory case (and perfect information still) there is one-period dynamics and in the fully permanent case the interest differential does not move at all since the exchange rate follows a random walk. The information problem of disentangling temporary from permanent shocks strengthens inertia across types of shocks. That is, systematic differences in nominal interest rates arise as a result of the failure of agents to separate temporary from permanent nominal shocks. This does not leave any risk free arbitrage possibilities, since the real return is the same in both countries and the difference in nominal interest rates reflects the changes in the nominal exchange rate expected by all market participants.

It is a well established empirical fact that an appreciating (depreciating) exchange rate tends to be accompanied by a positive (negative) interest rate spread (see eg Frankel and Rose, 1995). The present setting explains that excess returns are persistent due to the interplay between nominal rigidities in the real side of the economy and the information problem of disentangling temporary from permanent shocks. To put it differently we find that expectational errors - in a rational expectations setting - can account for systematic interest rate spreads. It is also worth pointing out that it is an implication that the spread is time-varying reflecting the type of shock hitting the economy and the learning problem.

It is interesting to note that under perfect information the impact effect on the nominal interest rate spread is decreasing (absolute value) in θ. The reason for this is simply that the nominal depreciation is increasing and, hence, the demand switching effect is increasing in θ. Given a small demand switching effect, the interest rate has to fall more to clear the money market. This effect is found too in the imperfect information case. The same applies for h. Comparing across regimes the impact effect is larger in the persistent and fully permanent cases and smaller in the transitory. The reasoning is the same as above, the larger the effect on demand, the smaller the interest-rate movement warranted to clear the money market. This also means that the interest rate spread moves oppositely the three other variables in sensitivity analysis.

6 Numerical Illustrations

To make the preceding theoretical analysis more transparent it is useful to consider numerical illustrations of the effects of different types of shocks - transitory, persistent or fully permanent - to the relative money supply. The impulse-response functions for the nominal exchange rate, the terms of trade, relative consumption and the nominal interest spread are given in figures 4, 5 and 6, respectively. In all cases we consider a 1 percent increase in Home (relative) money in period 1. The parameter values used for the numerical illustrations are given in table 6.1

Table 6.1. Benchmark Values.[25]

ρ	γ	μ	σ	β	δ	h
2	0.67	10	0.75	9	1/1.05	0.5

In all illustrations it is assumed that $\theta = 1$ in the case of transitory and fully permanent shocks, while we generate a persistent shock by assuming that $\theta = 0.9$. The numerical illustrations are made under given assumptions about the signal-to-noise ratio (h), that is, the parameter determining the extent to which agents take changes to be temporary or permanent are given. We then feed the model with shocks which by construction are either transitory, persistent or fully permanent. The dynamic responses thus reflect the learning process when agents over time acquire more information. The figures also illustrate how the impulse-response functions are affected by variations

[25] The elasticity parameter ρ is chosen arbitrarily to be 2, γ is chosen to match the wage share of about 2/3 while μ is chosen so as to imply a labor-supply elasticity of 0.1. The next three coefficients correspond to those adopted in eg Hairault and Portier (1993) and Sutherland (1996). The last coefficient value is arbitrarily chosen to be 0.5.

in the parameters underlying the information structure (θ, h), and make a comparison to the case of full information. In reading the subsequent figures it is useful to keep the adjustment of expectations (figures 1-3) in mind.

6.1 Transitory Shock

The impulse-response functions for a transitory shock show that the initial exchange-rate depreciation is gradually worked out of the system. Relative consumption follows the same pattern, that is, it increases on impact and slowly decays towards its long-run value. Unsurprisingly the terms of trade deteriorates on impact, but interestingly the impact effect is reversed to an improvement in subsequent periods. This reflects that the monetary change disappears but agents still expect the money shock to have increased. As a consequence demand switches towards Foreign goods and this explains why the long-run effects are smaller when information is imperfect. The subsequent terms-of-trade improvement is gradually worked out of the system. The interest rate spread dynamics is the same as for the terms of trade; it falls on impact and then rises (relative to the initial value). From then on the adjustment is gradual towards the long-run value. Note, that we have a positive interest rate spread and an appreciating currency.

Figure 4 about here

Comparing across information regimes we see that trivial dynamics have been replaced by more persistency and non-trivial dynamics for all variables, and the impact effects are larger for all variables except for the nominal interest rate spread, cf the theoretical analysis. The relative magnitudes of the long-run effects are not visible from the graphs since they are small in absolute value.

The sensitivity results with respect to the noise coefficient h show two things. First, the impact effects are increasing in h, since agents tend to take the shock as being fully permanent. Secondly, the persistency is decreasing in h. The reason for this is that although a higher h, lowers a priori belief that the shock is transitory noise, it implies that it takes less time for agents to learn that the shock in fact was transitory. Along the same lines we find, that the impact effects are increasing in θ which reflects that a larger weight is attributed to the part of the shock perceived to be permanent. Persistency of the variables is increasing in θ reflecting that there is more persistency in the permanent part of the shock.

6.2 Persistent Shock

For a persistent shock we get on impact a nominal depreciation, a terms-of-trade deterioration, an increase in consumption and an interest-rate fall. The striking feature is the dynamics of the exchange rate, which is seen to display delayed overshooting. Following the initial depreciation the exchange rate depreciates even further in the next period before settling on an appreciating path towards the long-run value.

Figure 5 about here

Compared to the perfect information case, we find that imperfect information generates delayed overshooting as well as much richer dynamics for relative consumption displaying a kind of undershooting path and the terms of trade gradually rising after the impact fall. The impact effects are as analytically predicted smaller under imperfect information for the first three variables and, hence, larger for the nominal interest rate spread. The relative magnitudes for long-run effects across regimes are not visible except for relative consumption, but we know them to be larger under imperfect information (and zero for the spread).

The sensitivity results show that the delayed overshooting behavior disappears for lower values of the θ parameter. It is also seen that both impact effects (except for the interest rate spread) the long-run effects are increasing in θ as indicated by the relative consumption graph.

The parameter h affects the dynamics, and for low values of h delayed overshooting disappears as learning becomes too slow (cf section 5.3). The sensitivity of impact effects and persistency of the variables are as shown analytically; smaller h implies smaller impact effects (vice versa for the interest rate spread) and more persistency as agents take longer to learn the exact nature of the shock.

6.3 Fully Permanent Shock

For a fully permanent shock the initial effect is slowly worked out of the system for all four variables. For the specific parameter constellation used we see that the long-run exchange-rate effect is a depreciation, attained via undershooting behavior. Compared to the perfect information regime it is striking how the dynamics are much richer under imperfect information. In the former case the exchange rate and relative consumption jumps immediately to their new long-run values, the nominal interest rate spread does not move at all and the terms of trade have one-period dynamics. All four variables display much more plausible dynamics under imperfect information.

Figure 6 about here

Again we see that increasing the degree of noise (lowering h) increases persistency, reflecting that the learning process is prolonged. Moreover, we have a depreciating currency and a negative interest rate spread.

The numerical illustrations displayed above show that informational problems imply that nominal shocks have effects running well beyond the length of contracts. This supports that informational problems can play an important role as a propagation mechanism, though, it is clearly not the whole story since the model does not in general produce terms-of-trade half-lives of 3-4 years following shocks.

7 Discussion and Conclusions

We set out to investigate how informational problems would affect the transmission of nominal shocks. The information problem was that of separating temporary from permanent changes. Adopting a general equilibrium framework, we endogenously related the adjustment of nominal exchange rates, wages, prices and the nominal interest rate spread as well as real variables to monetary shocks. Even with short-term nominal contracts information problems could produce persistent effects of nominal shocks in excess of the contract length. In general, adding information interpretation to one-period contracts lead to much more plausible dynamics of all the variables compared to the case of one-period contracts only. Other interesting results were that the model was able to generate delayed exchange-rate overshooting and persistency in nominal interest rate differentials.

Our model should be seen as a complement rather than a substitute for other analyses (Bergin and Feenstra, 1999a,b; Kollmann, 1999) of persistency. We do not introduce persistency via asynchronized nominal contracts and, in some sense, show how information problems can circumvent the problem of creating conditions under which prices are insensitive to marginal cost, and marginal cost is insensitive to output changes. Persistency in our model is generated only by learning without direct assumptions on the abovementioned connections and should be seen as adding to the other mechanisms in the literature.[26]

[26] This is basically a question of the type: Is the glass half empty or half full? Learning in our model can be seen as driving a wedge in between marginal costs and output, or in other words, making marginal cost less dependent on output. When learning is slow (much noise) there are large and persistent effects on output without (relative) wage adjustment. Imperfect information implicitly makes marginal cost less sensitive to output changes.

We model the information problem in a very stylized way, that is, the informational structure (θ,h) is constant over time. In reality the relative variances of temporary and permanent shocks might vary and agents might not learn the new structure right away. In this paper we, therefore, have explored a minimal change from perfect information and shown that this can have drastic affects for the dynamic adjustment path. An obvious next step would be to add misperception (Gourinchas and Tornell, 1996) as a way to explain unconditional delayed nominal exchange-rate overshooting and predictability of excess returns.

We have chosen to consider the analytics in great detail. Much of the current business-cycle literature adopt the approach of proceeding directly to numerical simulations (calibration) after having set up a model (often featuring many new aspects). The precise working of the mechanisms and the critical parameters tend to become blurred by this approach. Since it is possible to find an analytical solution under assumptions no more restrictive than those employed in empirical simulations, we find that this yields more insights. Obviously, it does not make sense to take a - after all - simple model as the present directly to the data, except if one believes in a mono-causal explanation of observed business cycles facts (only nominal shocks, only informational problems as propagation mechanism). Still, the qualitative results and the quantitative illustrations in our numerical examples make us conclude that the realistic problem of interpreting whether changes are temporary or permanent add important dynamics to the adjustment process.

A Steady State and Log-Linearization

Our analysis builds on a version of the model set up in section 2 in log-deviations from steady state.
The consumer maximizes expected utility subject to the budget constraint and, the first-order conditions determining the optimal choice of B_t and M_t are readily found to be

$$C_t^{-\frac{1}{\sigma}} = \delta\left(1+r_t\right)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right), \tag{11}$$

$$C_t^{-\frac{1}{\sigma}} = \xi\left(\frac{M_t}{P_t}\right)^{-\beta} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right), \tag{12}$$

where it is assumed that the usual transversality condition is satisfied.
As is apparent from the first-order conditions not all expressions are linear in logs and subsequently we have to approximate around the steady state. The steady-state version of the model is similar to that analyzed in eg Obstfeld and Rogoff (1995) and Tille (1999). We focus on a symmetric steady state where $B = B^* = 0$ and

$$C = C^* = Y^h = Y^{*f} = Y = Y^* = \alpha_y\left[\alpha_n\left(\kappa\alpha_y^{\sigma}\right)^{\frac{1}{\mu}}\right]^{\frac{-\mu\eta_{yw}}{\mu\eta_{nw}+\sigma\eta_{yw}+1}}, \tag{13}$$

$$r = \delta^{-1} - 1, \tag{14}$$

$$\frac{P^h}{P} = \frac{P^f}{P} = \frac{P^{*h}}{P^*} = \frac{P^{*f}}{P^*} = 1, \tag{15}$$

$$\frac{W}{P} = \frac{W^*}{P^*} = \left[\alpha_n\left(\kappa\alpha_y^{\sigma}\right)^{\frac{1}{\mu}}\right]^{\frac{\mu}{\mu\eta_{nw}+\sigma\eta_{yw}+1}}, \tag{16}$$

and where money is neutral and the price level is determined from (12). Real incomes are

$$Y = \frac{P^h Y^h}{P}, \qquad Y^* = \frac{P^{*f}Y^{*f}}{P^*}.$$

Steady-state values are indicated by omission of time subscripts. The α-parameters stem from the firms' labor demands and output supplies

$$\begin{aligned} N_t &= \alpha_n\left(\frac{P_t^h}{W_t}\right)^{\eta_{nw}}, \qquad \alpha_n = \gamma^{\frac{1}{1-\gamma}}, \\ Y_t^h &= \alpha_y\left(\frac{P_t^h}{W_t}\right)^{\eta_{yw}}, \qquad \alpha_y = \gamma^{\frac{\gamma}{1-\gamma}}. \end{aligned}$$

Next step is to log-linearize the first-order conditions arising from consumer optimization (11)-(12). The log-linearized Euler equation (1) is obtained by using the convenient formula for log-normally distributed variables

$$\log E\left(X^f\right) = fE\left[\log\left(X\right)\right] + \frac{f^2}{2}Var\left[\log\left(X\right)\right],$$

where f is a scalar and X is log-normally distributed. Taking logs on both sides of the money demand equation (12) yields the log of a sum and it is easy to show that around a steady state (disregarding constants)

$$\log(X_t + Z_t) = \frac{X}{X+Z}\log(X_t) + \frac{Z}{X+Z}\log(Z_t).$$

Using this we get that

$$\begin{aligned} &\log\left[\xi\left(\frac{M_t}{P_t}\right)^{-\beta} + E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right] \\ = \;& (1-\delta)\log\left[\xi\left(\frac{M_t}{P_t}\right)^{-\beta}\right] + \delta\log\left[E_t\left(\delta C_{t+1}^{-\frac{1}{\sigma}}\frac{P_t}{P_{t+1}}\right)\right]. \end{aligned}$$

Equation (2) follows immediately with

$$\eta_{mc} = \frac{1}{\sigma\left(1-\delta\right)\beta}, \qquad \eta_{mc}^1 = \frac{\delta}{\sigma\left(1-\delta\right)\beta}, \qquad \eta_{mp} = \frac{\delta}{\left(1-\delta\right)\beta}.$$

Note, that following a shock the economy moves away from the initial steady state and does not return. The log-linearized first-order condition for money demand (2) still holds, though. Log-linearizing (12) around any non-inflationary steady state will yield (2) (disregarding constants). The wage is determined from

$$W_t = \kappa\frac{\eta_{nw}}{\eta_{nw}-1}\frac{E_{t-1}\left(N_t^{1+\mu}\right)}{E_{t-1}\left(C_t^{-\frac{1}{\sigma}}\frac{N_t}{P_t}\right)}.$$

Taking logs on both sides and using joint log-normality we end up with (4). While the model is specified so as to yield a log-linear structure, we have that the budget constraint is linear in levels, ie

$$B_t = \left(1 + r_{t-1}\right)B_{t-1} + Y_t - C_t. \tag{17}$$

Subtracting the steady-state version of the budget constraint from (17) and dividing by Y(=C) we get

$$\begin{aligned} \frac{B_t - B}{Y} \;=\;& \left(1+r\right)\frac{B_{t-1}-B}{Y} + \frac{Y_t - Y}{Y} - \frac{C_t - C}{C} \\ &+ \left[\left(1+r_{t-1}\right) - \left(1+r\right)\right]\frac{B_{t-1}-B}{Y}. \end{aligned}$$

The last term on the right-hand side is negligible as we look at small deviations around steady state. We end up with

$$b_t = \delta^{-1} b_{t-1} + y_t - c_t, \tag{18}$$

as $1 + r = \delta^{-1}$ and

$$\begin{aligned} y_t &= \log\left(\frac{Y_t}{Y}\right) \approx \frac{Y_t - Y}{Y}, \\ b_t &= \frac{B_t}{Y}, \\ c_t &= \log\left(\frac{C_t}{C}\right) \approx \frac{C_t - C}{C}. \end{aligned}$$

B Equilibrium with One-Period Nominal Wage Contracts and Imperfect Information

We solve for four variables: the nominal exchange rate, the terms of trade, relative consumption and the nominal interest rate spread. We use the method of undetermined coefficients and take each variable in turn. Our guesses are

$$s_t = \pi_{sc}\left(c_t - c_t^*\right) + \pi_{sm}\left(m_t - m_t^*\right) + \pi_{sm}^1 E_{t-1}\left(m_t - m_t^*\right), \tag{19}$$

$$q_t = \pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{qm}\left(m_t - m_t^*\right) + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right), \tag{20}$$

and

$$\begin{aligned} c_t - c_t^* = {} & \pi_{cb}\left(b_{t-1} - b_{t-1}^*\right) + \pi_{cc}\left(c_{t-1} - c_{t-1}^*\right) \\ & + \pi_{cm}\left(m_t - m_t^*\right) + \pi_{cm}^1 E_{t-1}\left(m_t - m_t^*\right). \end{aligned} \tag{21}$$

Combining the Euler equation and (21) we have that

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \pi_{cm}\left[m_t - m_t^* - E_{t-1}\left(m_t - m_t^*\right)\right]. \tag{22}$$

Next we apply a five step procedure where we have a look at the four variables in turn and lastly characterize the properties of the solution.

B.1 Nominal Exchange Rate

Before solving for consumption and the terms of trade we solve for the nominal exchange rate, which is determined by money market equilibrium. Relative money demand can be found from (2) and equating it with relative supplies yields

$$s_t = \frac{1}{1 + \eta_{mp}}\left[\left(\eta_{mc}^1 - \eta_{mc}\right)\left(c_t - c_t^*\right) + \eta_{mp} E_t s_{t+1} + m_t - m_t^*\right].$$

Consistency with (19) requires

$$\pi_{sc} = \left(1+\eta_{mp}\right)^{-1}\left[\eta_{mp}\pi_{sc} + \eta^1_{mc} - \eta_{mc}\right],$$

$$\pi_{sm} = \left(1+\eta_{mp}\right)^{-1}\left[\eta_{mp}\left(\pi_{sm} + \pi^1_{sm}\right)h + 1\right],$$

$$\pi^1_{sm} = \left(1+\eta_{mp}\right)^{-1}\left[\eta_{mp}\left(\pi_{sm} + \pi^1_{sm}\right)(\theta - h)\right].$$

Now rewrite (19) as

$$\begin{aligned} s_t &= s_{t-1} + \left(\pi_{sm} + \pi_{sc}\pi_{cm}\right)\left(m_t - m^*_t\right) \\ &\quad + \left(\pi^1_{sm}\frac{\theta h}{\theta - \theta h} - \pi_{sm}\right)\left(m_{t-1} - m^*_{t-1}\right) \\ &\quad + \left(\pi^1_{sm}\frac{\theta - \theta h - 1}{\theta - \theta h} - \pi_{sc}\pi_{cm}\right)E_{t-1}\left(m_t - m^*_t\right), \end{aligned}$$

where we have used

$$\begin{aligned} E_{t-1}\left(m_t - m^*_t\right) &= (\theta - \theta h)\,E_{t-2}\left(m_{t-1} - m^*_{t-1}\right) \\ &\quad + \theta h\left(m_{t-1} - m^*_{t-1}\right). \end{aligned} \tag{23}$$

Lagging (19) once and using (23) again we can write the nominal exchange rate as

$$\begin{aligned} s_t &= \phi^1_{ss}s_{t-1} + \phi^2_{ss}s_{t-2} \\ &\quad + \phi_{sm}\left(m_t - m^*_t\right) + \phi^1_{sm}\left(m_{t-1} - m^*_{t-1}\right) + \phi^2_{sm}\left(m_{t-2} - m^*_{t-2}\right), \end{aligned}$$

where

$$\phi^1_{ss} = 1 + \theta - \theta h,$$

$$\phi^2_{ss} = \theta h - \theta,$$

$$\phi_{sm} = \pi_{sm} + \pi_{sc}\pi_{cm},$$

$$\begin{aligned} \phi^1_{sm} &= \pi^1_{sm}\frac{h}{1-h} - \pi_{sm} - \left(\pi_{sm} + \pi_{sc}\pi_{cm}\right)(\theta - \theta h) \\ &\quad + \left(\pi^1_{sm}\frac{\theta - \theta h - 1}{\theta - \theta h} - \pi_{sc}\pi_{cm}\right)\theta h, \end{aligned}$$

$$\phi^2_{sm} = -\left(\pi^1_{sm}\frac{h}{1-h} - \pi_{sm}\right)(\theta - \theta h).$$

Playing around with this expression we retrieve the stochastic trend representation (see section 4) or

$$\begin{aligned}(1-L)\,s_t &= (\pi_{sm}+\pi_{sc}\pi_{cm})\,(m_t-m_t^*) \\ &+\left(\pi^1_{sm}\frac{h}{1-h}-\pi_{sm}\right)\left(m_{t-1}-m^*_{t-1}\right) \\ &+\left(\pi^1_{sm}\frac{\theta-\theta h-1}{\theta-\theta h}-\pi_{sc}\pi_{cm}\right)\theta h\sum_{j=0}^{\infty}(\theta-\theta h)^j\left(m_{t-1-j}-m^*_{t-1-j}\right).\end{aligned} \tag{24}$$

B.1.1 Impact Effects

In all three cases the impact effect is given by

$$\frac{\partial s_t}{\partial\,(m_t-m_t^*)}=\pi_{sm}+\pi_{sc}\pi_{cm}>0. \tag{25}$$

B.1.2 Dynamics

For a transitory shock the dynamics are described by an ARIMA(1,1,2)

$$(1-L)\left[1-\theta\,(1-h)\,L\right]s_t=\phi_{sm}u_t+\phi^1_{sm}u_{t-1}+\phi^2_{sm}u_{t-2}, \tag{26}$$

for a persistent shock an ARIMA(2,1,2)

$$\begin{aligned}&(1-L)\left[1-\theta\,(2-h)\,L-\theta^2\,(h-1)\,L^2\right]s_t \\ =\;&\phi_{sm}\varepsilon_t+\phi^1_{sm}\varepsilon_{t-1}+\phi^2_{sm}\varepsilon_{t-2},\end{aligned} \tag{27}$$

and finally for a fully permanent shock an ARIMA(1,1,1)

$$(1-L)\left[1-(1-\theta h)\,L\right]s_t=\phi_{sm}\varepsilon_t+\left(\phi_{sm}+\phi^1_{sm}\right)\varepsilon_{t-1}. \tag{28}$$

B.1.3 Long-Run Effects

The long-run effects in the three cases are[27]

$$\frac{\partial s_{t+j}}{\partial u_t}=\frac{(1-\theta)\,\pi_{sc}\pi_{cm}}{1-\theta+\theta h}<0\quad for\ j\to\infty, \tag{29}$$

$$\left.\frac{\partial s_{t+j}}{\partial \varepsilon_t}\right|_{\theta\in(0,1)}=\frac{\pi_{sc}\pi_{cm}}{1-\theta+\theta h}<0\quad for\ j\to\infty, \tag{30}$$

$$\left.\frac{\partial s_{t+j}}{\partial \varepsilon_t}\right|_{\theta=1}=1+\frac{\pi_{sc}\pi_{cm}}{\theta h}\lesseqgtr 0\quad for\ j\to\infty. \tag{31}$$

[27] For a process like $(1-L)\,s_t=\Psi(L)\varepsilon_t$, like equation (24), where ε_t is white noise, the long-run effect of a one time shock is $\Psi(1)=\sum\psi_i$. If instead ε_t follows an AR(1) with autoregressive parameter θ the long-run effect is $\frac{1}{1-\theta}\Psi(1)$. See eg Hamilton (1994).

B.1.4 Delayed Overshooting

In the following we will show two things: (i) if we condition on a shock being persistent then we can generate delayed nominal exchange-rate overshooting; (ii) if we do not condition on a given shock being persistent then we can rule out delayed overshooting.

Conditional Delayed Overshooting In the case of a persistent shock $[\varepsilon_t > 0,\ \theta \in (0,1)]$ we cannot reject the possibility of delayed overshooting. The impact effect and the effect one period later are

$$\frac{\partial s_t}{\partial \left(m_t - m_t^*\right)} = \phi_{sm},$$

and

$$\frac{\partial s_{t+1}}{\partial \left(m_t - m_t^*\right)} = \phi_{ss}^1 \phi_{sm} + \theta \phi_{sm} + \phi_{sm}^1,$$

respectively, implying that we need

$$\phi_{sm} < \phi_{ss}^1 \phi_{sm} + \theta \phi_{sm} + \phi_{sm}^1 \Rightarrow$$

$$\left(\theta h - 2\theta\right) \phi_{sm} < \phi_{sm}^1 \Rightarrow$$

$$\left(1 - \theta\right) \pi_{sm} - h \pi_{sm}^1 < \left(\theta - \theta h\right) \pi_{sc} \pi_{cm} < 0 \Rightarrow$$

$$\frac{1}{1 + \eta_{mp}} \left[\left(\pi_{sm} + \pi_{sm}^1\right) \eta_{mp} \theta h \left(1 - 2\theta + \theta h\right) + 1 - \theta\right] < \left(\theta - \theta h\right) \pi_{sc} \pi_{cm} < 0.$$

Hence, a necessary condition for delayed overshooting is $1 - 2\theta + \theta h < 0$. Given that

$$\frac{\partial s_{t+1}}{\partial \left(m_t - m_t^*\right)} > \frac{\partial s_t}{\partial \left(m_t - m_t^*\right)} > 0,$$

we have delayed overshooting as we know the long-run effect is an appreciation (see below), ie the exchange rate has to come down eventually.

Unconditional Delayed Overshooting If we do not condition on the shock being persistent we cannot generate delayed overshooting. Assume there is a unit impulse to relative money, then the unconditional exchange-rate response, s_t^U is

$$s_{t+j}^U = E_t\left(s_{t+j} | \, m_t - m_t^* = 1\right) = hs_{t+j}^P + (1-h)\, s_{t+j}^T, \quad j = 0, 1, 2, ...$$

where s_t^T and s_t^P are the responses for a temporary and a permanent shock respectively. The result follows simply from the linearity of the exchange-rate equation (19). We have that

$$\frac{\partial s_t}{\partial (m_t - m_t^*)} = \phi_{sm},$$

$$\left.\frac{\partial s_{t+1}}{\partial (m_t - m_t^*)}\right|_{Temporary} = \phi_{ss}^1 \phi_{sm} + \phi_{sm}^1,$$

$$\left.\frac{\partial s_{t+1}}{\partial (m_t - m_t^*)}\right|_{Permanent} = \phi_{ss}^1 \phi_{sm} + \theta\phi_{sm} + \phi_{sm}^1,$$

so we have to look at

$$\begin{aligned} & s_{t+1}^U - s_t^U \\ = \; & (1-h) \left.\frac{\partial s_{t+1}}{\partial (m_t - m_t^*)}\right|_{Temporary} + h \left.\frac{\partial s_{t+1}}{\partial (m_t - m_t^*)}\right|_{Permanent} - \frac{\partial s_t}{\partial (m_t - m_t^*)}, \end{aligned}$$

when assessing the possibility of obtaining unconditional delayed overshooting. Inserting we get

$$\begin{aligned} & (1-h)\left(\phi_{ss}^1 \phi_{sm} + \phi_{sm}^1\right) + h\left(\phi_{ss}^1 \phi_{sm} + \theta\phi_{sm} + \phi_{sm}^1\right) - \phi_{sm} \\ = \; & \phi_{ss}^1 \phi_{sm} + \theta h \phi_{sm} + \phi_{sm}^1 - \phi_{sm} \\ = \; & \left(\phi_{ss}^1 + \theta h - 1\right) \phi_{sm} + \phi_{sm}^1 \\ = \; & (1 + \theta - \theta h + \theta h - 1)\, \phi_{sm} + \phi_{sm}^1 = \theta\phi_{sm} + \phi_{sm}^1. \end{aligned}$$

Utilizing that

$$\phi_{sm} = \pi_{sm} + \pi_{sc}\pi_{cm},$$

and

$$\phi_{sm}^1 = \theta h \pi_{sm}^1 - (1 + \theta - \theta h)\, \pi_{sm} - \theta\pi_{sc}\pi_{cm},$$

we get that

$$\begin{aligned}
\theta\phi_{sm}+\phi_{sm}^{1} &= \theta\left(\pi_{sm}+\pi_{sc}\pi_{cm}\right)+\theta h\pi_{sm}^{1}-\left(1+\theta-\theta h\right)\pi_{sm}-\theta\pi_{sc}\pi_{cm} \\
&= \theta h\left(\pi_{sm}+\pi_{sm}^{1}\right)-\pi_{sm} \\
&= \frac{\theta h}{1+\eta_{mp}\left(1-\theta\right)}-\left[\frac{1}{1+\eta_{mp}}+\frac{1}{1+\eta_{mp}}\frac{\theta h\eta_{mp}}{1+\eta_{mp}\left(1-\theta\right)}\right] \\
&= \frac{\theta h\left(1+\eta_{mp}\right)-1-\eta_{mp}\left(1-\theta\right)-\theta h\eta_{mp}}{\left[1+\eta_{mp}\left(1-\theta\right)\right]\left(1+\eta_{mp}\right)} \\
&= \frac{\theta h-1-\eta_{mp}\left(1-\theta\right)}{\left[1+\eta_{mp}\left(1-\theta\right)\right]\left(1+\eta_{mp}\right)}<0.
\end{aligned}$$

In conclusion, unconditional delayed nominal exchange-rate overshooting can be ruled out.[28]

B.2 Terms of Trade

Relative demand is given as

$$d_t-d_t^{*}=-\rho q_t,$$

and relative supply is

$$\begin{aligned}
y_t^{h}-y_t^{*f} &= \eta_{yw}[q_t-\left(2\eta_{wp}-1\right)E_{t-1}q_t \\
&\quad +s_t-E_{t-1}s_t-\eta_{wc}E_{t-1}\left(c_t-c_t^{*}\right)].
\end{aligned}$$

Product market equilibrium implies that

$$\begin{aligned}
q_t &= \frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}}\left(c_{t-1}-c_{t-1}^{*}\right)+\frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}}E_{t-1}q_t \\
&\quad -\frac{\eta_{yw}}{\rho+\eta_{yw}}\left(s_t-E_{t-1}s_t\right),
\end{aligned}$$

where we have invoked the Euler equation. Invoking the expressions for the nominal exchange rate and (22) we obtain

$$\begin{aligned}
q_t &= \frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}}\left(c_{t-1}-c_{t-1}^{*}\right)+\frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}}E_{t-1}q_t \\
&\quad -\frac{\eta_{yw}}{\rho+\eta_{yw}}\left(\pi_{sm}+\pi_{sc}\pi_{cm}\right)\left[m_t-m_t^{*}-E_{t-1}\left(m_t-m_t^{*}\right)\right].
\end{aligned}$$

[28]Note that we do not prove that the exchange rate two periods after the shock can rise relative to the exchange rate in the period after the shock. It is unlikely it can occur given our result and as we are interested in the Eichenbaum-Evans result we only want to rule out the 'smooth' delayed overshooting.

From our guess (20) we have

$$E_{t-1}q_t = \pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \left(\pi_{qm} + \pi_{qm}^1\right) E_{t-1}\left(m_t - m_t^*\right),$$

and inserting this and equating coefficients yield the following restrictions

$$\pi_{qc} = \frac{\eta_{yw}\eta_{wc}}{\rho + \eta_{yw}}\pi_{qc} + \frac{\eta_{yw}\left(2\eta_{wp} - 1\right)}{\rho + \eta_{yw}},$$

$$\pi_{qm} = -\frac{\eta_{yw}}{\rho + \eta_{yw}}\left(\pi_{sm} + \pi_{sc}\pi_{cm}\right),$$

$$\pi_{qm}^1 = \frac{\eta_{yw}\eta_{wc}}{\rho + \eta_{yw}}\left(\pi_{qm} + \pi_{qm}^1\right) + \frac{\eta_{yw}}{\rho + \eta_{yw}}\left(\pi_{sm} + \pi_{sc}\pi_{cm}\right) = -\pi_{qm}.$$

Using (20), (22) and (23) we can rewrite the terms of trade as

$$\begin{aligned} q_t &= q_{t-1} + \pi_{qm}\left(m_t - m_t^*\right) \\ &\quad + \left[\pi_{qc}\pi_{cm} - \pi_{qm} + \frac{h}{1-h}\left(\pi_{qm}^1 + \pi_{qc}\pi_{cm}\right)\right]\left(m_{t-1} - m_{t-1}^*\right) \\ &\quad + \left(\pi_{qm}^1 - \frac{\pi_{qm}^1}{\theta - \theta h} - \frac{\pi_{qc}\pi_{cm}}{\theta - \theta h}\right) E_{t-1}\left(m_t - m_t^*\right). \end{aligned}$$

Lagging this once and using (23) we end up with (9) where

$$\phi_{qm} = \pi_{qm},$$

$$\begin{aligned} \phi_{qm}^1 &= \pi_{qc}\pi_{cm} - \pi_{qm} + \frac{h}{1-h}\left(\pi_{qm}^1 + \pi_{qc}\pi_{cm}\right) \\ &\quad + h\left(\pi_{qm}^1 - \frac{\pi_{qm}^1}{\theta - \theta h} - \frac{\pi_{qc}\pi_{cm}}{\theta - \theta h}\right) - \pi_{qm}\left(\theta - \theta h\right), \end{aligned}$$

$$\phi_{qm}^2 = -\left[\pi_{qc}\pi_{cm} - \pi_{qm} + \frac{h}{1-h}\left(\pi_{qm}^1 + \pi_{qc}\pi_{cm}\right)\right]\left(\theta - \theta h\right).$$

This can be rewritten along the same lines as the nominal exchange rate as the stochastic trend representation presented in the text or

$$\begin{aligned} (1 - L)\, q_t &= \pi_{qm}\left(m_t - m_t^*\right) \\ &\quad + \left[\pi_{qc}\pi_{cm} - \pi_{qm} + \frac{h}{1-h}\left(\pi_{qm}^1 + \pi_{qc}\pi_{cm}\right)\right]\left(m_{t-1} - m_{t-1}^*\right) \\ &\quad + h\left(\pi_{qm}^1 - \frac{\pi_{qm}^1}{\theta - \theta h} - \frac{\pi_{qc}\pi_{cm}}{\theta - \theta h}\right) \\ &\quad \cdot \sum_{j=0}^{\infty}\left(\theta - \theta h\right)^j \left(m_{t-1-j} - m_{t-1-j}^*\right). \end{aligned}$$

B.2.1 Impact Effects

The impact effect in all three cases is

$$\frac{\partial q_t}{\partial (m_t - m_t^*)} = \pi_{qm} < 0. \tag{32}$$

B.2.2 Dynamics

The dynamics in the three cases are

$$(1-L)\left[1-\theta(1-h)L\right]q_t = \phi_{qm}u_t + \phi_{qm}^1 u_{t-1} + \phi_{qm}^2 u_{t-2}, \tag{33}$$

$$\begin{aligned} & (1-L)\left[1-\theta(2-h)L-\theta^2(h-1)L^2\right]q_t \\ = \; & \phi_{qm}\varepsilon_t + \phi_{qm}^1\varepsilon_{t-1} + \phi_{qm}^2\varepsilon_{t-2}, \end{aligned} \tag{34}$$

and[29]

$$(1-L)\left[1-(1-\theta h)L\right]q_t = \phi_{qm}\varepsilon_t + \left(\phi_{qm} + \phi_{qm}^1\right)\varepsilon_{t-1}, \tag{35}$$

respectively.

B.2.3 Long-Run Effects

The long-run effect of a transitory shock is

$$\frac{\partial q_{t+j}}{\partial u_t} = (1-\theta)\frac{\pi_{qc}\pi_{cm}}{(1-\theta+\theta h)} > 0 \quad for\ j \to \infty. \tag{36}$$

The long-run effect of a persistent shock is

$$\left.\frac{\partial q_{t+j}}{\partial \varepsilon_t}\right|_{\theta\in(0,1)} = \frac{\pi_{qc}\pi_{cm}}{(1-\theta+\theta h)} > 0 \quad for\ j \to \infty. \tag{37}$$

The long-run effect of a fully permanent shock is

$$\left.\frac{\partial q_{t+j}}{\partial \varepsilon_t}\right|_{\theta=1} = \frac{\pi_{qc}\pi_{cm}}{\theta h} > 0 \quad for\ j \to \infty. \tag{38}$$

[29] When $\theta = 1$ we obtain that $\pi_{qm}^1 = -\pi_{qm}$, $\phi_{qm} = \pi_{qm}$, $\phi_{qm}^1 = -2\pi_{qm} + \pi_{qc}\pi_{cm}$ and $\phi_{qm}^2 = \pi_{qm} - \pi_{qc}\pi_{cm}$.

B.3 Relative Consumption

From equation (18) we have

$$b_t - b_t^* = \delta^{-1}\left(b_{t-1} - b_{t-1}^*\right) + \left(y_t - y_t^*\right) - \left(c_t - c_t^*\right),$$

and since

$$\begin{aligned} y_t - y_t^* &= (1-\rho)\, q_t \\ &= (1-\rho)\left[\pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{qm}\left(m_t - m_t^*\right)\right. \\ &\quad \left. + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right)\right], \end{aligned}$$

we have that

$$\begin{aligned} b_t - b_t^* &= \delta^{-1}\left(b_{t-1} - b_{t-1}^*\right) \\ &\quad + (1-\rho)\left[\pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{qm}\left(m_t - m_t^*\right)\right. \\ &\quad \left. + \pi_{qm}^1 E_{t-1}\left(m_t - m_t^*\right)\right] \\ &\quad - \left(c_t - c_t^*\right). \end{aligned}$$

The next step is to find an expression for relative consumption consistent with the Euler equation. Leading our guess we find

$$\begin{aligned} E_t\left(c_{t+1} - c_{t+1}^*\right) &= \pi_{cb}\left(b_t - b_t^*\right) + \pi_{cc}\left(c_t - c_t^*\right) \\ &= \pi_{cb}\delta^{-1}\left(b_{t-1} - b_{t-1}^*\right) + \pi_{cb}(1-\rho)\,\pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) \\ &\quad + \left[\pi_{cb}(1-\rho)\,\pi_{qm} + \theta h\left(\pi_{cm} + \pi_{cm}^1\right)\right]\left(m_t - m_t^*\right) \\ &\quad + [\pi_{cb}(1-\rho)\,\pi_{qm}^1 \\ &\quad + (\theta - \theta h)\left(\pi_{cm} + \pi_{cm}^1\right)] E_{t-1}\left(m_t - m_t^*\right) \\ &\quad + \left(\pi_{cc} - \pi_{cb}\right)\left(c_t - c_t^*\right). \end{aligned}$$

Using the Euler equation yields

$$\begin{aligned} c_t - c_t^* &= \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1}\{\delta^{-1}\pi_{cb}\left(b_{t-1} - b_{t-1}^*\right) \\ &\quad + \pi_{cb}(1-\rho)\,\pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) \\ &\quad + \left[\pi_{cb}(1-\rho)\,\pi_{qm} + \theta h\left(\pi_{cm} + \pi_{cm}^1\right)\right]\left(m_t - m_t^*\right) \\ &\quad + [\pi_{cb}(1-\rho)\,\pi_{qm}^1 + (\theta - \theta h)\left(\pi_{cm} + \pi_{cm}^1\right)] E_{t-1}\left(m_t - m_t^*\right)\}. \end{aligned}$$

It follows that

$$\pi_{cb} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1}\delta^{-1}\pi_{cb},$$

$$\pi_{cc} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1}\pi_{cb}(1-\rho)\,\pi_{qc},$$

$$\pi_{cm} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1}\left[\pi_{cb}(1-\rho)\,\pi_{qm} + \theta h\left(\pi_{cm} + \pi_{cm}^1\right)\right],$$

and

$$\begin{aligned}\pi_{cm}^{1} &= (1+\pi_{cb}-\pi_{cc})^{-1}\left[\pi_{cb}(1-\rho)\pi_{qm}^{1}+(\theta-\theta h)\left(\pi_{cm}+\pi_{cm}^{1}\right)\right] \\ &= -\pi_{cm}.\end{aligned}$$

It is worth noticing that relative consumption does not follow a random walk (as it does under perfect information) but evolves according to richer dynamics. This is seen by subtracting the lagged version of

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \pi_{cm}\left[m_t - m_t^* - E_{t-1}(m_t - m_t^*)\right],$$

from itself. After some simple algebra we end up with (10) where

$$\phi_{cc}^{1} = 1-\theta+\theta h,$$

$$\phi_{cc}^{2} = \theta h - \theta,$$

$$\phi_{cm} = \pi_{cm},$$

$$\phi_{cm}^{1} = -\theta\pi_{cm},$$

or the stochastic trend representation in the text. Relative consumption can also be written

$$\begin{aligned}&(1-L)(c_t-c_t^*) = [1-\theta(1-h)L]^{-1}\left[\phi_{cm}(m_t-m_t^*)+\phi_{cm}^{1}\left(m_{t-1}-m_{t-1}^*\right)\right] \\ &= \pi_{cm}\sum_{j=0}^{\infty}(\theta-\theta h)^{j}\left(m_{t-j}-m_{t-j}^*\right)-\pi_{cm}\theta\sum_{j=0}^{\infty}(\theta-\theta h)^{j}\left(m_{t-1-j}-m_{t-1-j}^*\right).\end{aligned}$$

B.3.1 Impact Effects

The impact effect is

$$\frac{\partial(c_t-c_t^*)}{\partial(m_t-m_t^*)} = \pi_{cm} > 0. \qquad (39)$$

B.3.2 Dynamics

The dynamics for a transitory shock, persistent shock and fully permanent shock, respectively, are

$$(1-L)[1-\theta(1-h)L](c_t-c_t^*) = \phi_{cm}u_t + \phi_{qm}^{1}u_{t-1}, \qquad (40)$$

$$\begin{aligned}&(1-L)\left[1-\theta(2-h)L-\theta^2(h-1)L^2\right](c_t-c_t^*) \qquad (41)\\ &= \phi_{qm}\varepsilon_t + \phi_{qm}^{1}\varepsilon_{t-1},\end{aligned}$$

and

$$(1-L)[1-(1-h)L](c_t-c_t^*) = \phi_{cm}\varepsilon_t. \qquad (42)$$

B.3.3 Long-Run Effects

The long-run effects are given by

$$\frac{\partial\left(c_{t+j}-c_{t+j}^{*}\right)}{\partial u_t}=\frac{\pi_{cm}\left(1-\theta\right)}{\left(1-\theta+\theta h\right)}>0 \quad for\ j\rightarrow\infty, \tag{43}$$

in the transitory case and

$$\left.\frac{\partial\left(c_{t+j}-c_{t+j}^{*}\right)}{\partial\varepsilon_t}\right|_{\theta\in(0,1)}=\frac{\pi_{cm}}{\left(1-\theta+\theta h\right)}>0 \quad for\ j\rightarrow\infty, \tag{44}$$

in the persistent case. The long-run effect in the fully permanent case is

$$\left.\frac{\partial\left(c_{t+j}-c_{t+j}^{*}\right)}{\partial\varepsilon_t}\right|_{\theta=1}=\frac{\pi_{cm}}{\theta h}>0 \quad for\ j\rightarrow\infty. \tag{45}$$

B.4 Nominal Interest Rate Spread

The Euler equation reads

$$C_t^{-\frac{1}{\sigma}}=\delta\left(1+r_t\right)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right).$$

If we instead of a real bond assume a nominal bond with gross return $1+i_t$ the Euler equation reads

$$C_t^{-\frac{1}{\sigma}}=\delta E_t\left[\frac{\left(1+i_t\right)P_t}{P_{t+1}}C_{t+1}^{-\frac{1}{\sigma}}\right].$$

Equalizing right-hand sides yields

$$1+i_t=\frac{\left(1+r_t\right)E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right)}{E_t\left(\frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}}\right)P_t},$$

implying that

$$\log\left(1+i_t\right)=\log\left(1+r_t\right)+\log E_t\left(C_{t+1}^{-\frac{1}{\sigma}}\right)-\log E_t\left(\frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}}\right)-\log P_t.$$

From joint log-normality of C and P we get

$$\log\left(1+i_t\right)=\log\left(1+r_t\right)+E_tp_{t+1}-p_t,$$

where we disregard a constant term. The nominal interest rate spread is then

$$\log(1+i_t) - \log(1+i_t^*) = E_t s_{t+1} - s_t.$$

Inserting for the nominal exchange rate we get that

$$\begin{aligned}\log(1+i_t) - \log(1+i_t^*) &= (\theta - \theta h)\left[\log(1+i_{t-1}) - \log\left(1+i_{t-1}^*\right)\right] \\ &\quad + \Phi_{is}(L)\, s_t + \Phi_{im}(L)(m_t - m_t^*),\end{aligned}$$

where

$$\Phi_{is}(L) = \phi_{is} + \phi_{is}^1 L + \phi_{is}^2 L^2 + \phi_{is}^3 L^3,$$

$$\Phi_{im}(L) = \phi_{im} + \phi_{im}^1 L + \phi_{im}^2 L^2 + \phi_{im}^3 L^3,$$

with entries

$$\phi_{is} = \phi_{ss}^1 = 1 + \theta - \theta h,$$

$$\phi_{is}^1 = \phi_{ss}^2 - \left(\phi_{ss}^1\right)^2,$$

$$\phi_{is}^2 = -2\phi_{ss}^1 \phi_{ss}^2,$$

$$\phi_{is}^3 = -\left(\phi_{ss}^2\right)^2,$$

$$\phi_{im} = \phi_{sm}(\theta h - 1) + \phi_{sm}^1,$$

$$\phi_{im}^1 = \theta(1-h)\,\phi_{sm} - \phi_{ss}^1 \phi_{sm}^1 + \phi_{sm}^2,$$

$$\phi_{im}^2 = \theta(1-h)\,\phi_{sm}^1 - \phi_{ss}^1 \phi_{sm}^2,$$

$$\phi_{im}^3 = \theta(1-h)\,\phi_{sm}^2.$$

Compared to the perfect information case we have non-trivial dynamics for all three kinds of shocks. The impact effect is

$$\frac{\partial\left[\log(1+i_t) - \log(1+i_t^*)\right]}{\partial(m_t - m_t^*)} = \phi_{im} + \phi_{is}\frac{\partial s_t}{\partial(m_t - m_t^*)} < 0, \tag{46}$$

and the long-run effect is zero

$$\frac{\partial\left[\log(1+i_{t+j}) - \log\left(1+i_{t+j}^*\right)\right]}{\partial(m_t - m_t^*)} = 0 \quad for\ j \to \infty. \tag{47}$$

B.5 Analytical Characterization of the Solution

In this subsection we analytically characterize the solution found. Our final aim is to characterize the impact effect, the dynamics and the long-run effects of the variables following the three different kind of shocks we consider in the paper. This strategy has the disadvantage that we end up with far too many Lemmas and Propositions. But in-depth analysis warrants this and to keep a bit structure this subsection is split into three subsubsections. First we present Lemmas needed later on, then we concentrate on impact effects and finally on long-run effects. Dynamics are considered in the main text. Note, furthermore, that we introduce **new notation**. When introducing imperfect information it is obvious that we want to compare the results with the perfect information regime. Therefore we introduce superscripts imp (for imperfect information) and per (for perfect information) when the same parameters in the different regimes need to be compared. We consider **positive monetary shocks** (cf section 3). We introduce new terminology, that is **Case 1** corresponds to the situation of a 1 percent increase in u_t, **Case 2** corresponds to the case of a 1 percent increase in ε_t $[\theta \in (0,1)]$ and lastly **Case 3** is the case of a 1 percent increase in ε_t and $\theta = 1$. The perfect information regime is laid out in appendix C.

B.5.1 Useful Lemmas

Lemma 1 $0 < \pi_{sm} \leq 1$ *and* $\pi_{sc} < 0$.

Proof. It follows from the restrictions for π_{sm} and π^1_{sm} that $\pi_{sm} > 0$, $\pi^1_{sm} > 0$ and $\pi_{sm} + \pi^1_{sm} = \frac{1}{\left(1+\eta_{mp}-\theta\eta_{mp}\right)} < 1$ implying that $\pi_{sm} \in (0,1]$.

The second part of the lemma follows straightfowardly from the restriction for π_{sc} and the definitions of η_{mp}, η_{mc} and η^1_{mc} (appendix A). ∎

Lemma 2 $\pi^{imp}_{qc} = \pi^{per}_{qc} > 0$.

Proof. The equality follows straightforwardly from comparison of the definitions of the two parameters. The inequality follows from $\pi_{qc} = \frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}} + \frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}}\pi_{qc} \Rightarrow \pi_{qc} = \left[1 - \frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}}\right]^{-1}\left(\frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}}\right) > 0$ as the last two terms are positive. ∎

Lemma 3 $\pi^{imp}_{cc} = \pi^{per}_{cc} < 0$.

Proof. The equality follows straightforwardly from comparison of the definitions of the two parameters. The inequality follows from substituting π_{cb} in the restriction for π_{cc} implying that $\pi_{cc} = \frac{(1-\delta)(1-\rho)\pi_{qc}}{1-\delta(1-\rho)\pi_{qc}} < 0$ as $\delta \in (0,1]$, $\rho > 1$ and $\pi_{qc} > 0$ (by Lemma 2). ■

Lemma 4 $\pi_{cb}^{imp} = \pi_{cb}^{per} > 0.$

Proof. The equality follows straightforwardly from comparison of the definitions of the two parameters. The inequality follows from $\pi_{cc} = \delta\pi_{cb}(1-\rho)\pi_{qc} < 0$ (Lemma 3) and since $\delta > 0$, $(1-\rho) < 0$ and $\pi_{qc} > 0$ (Lemma 2), π_{cb} must be positive. ■

Lemma 5 $\pi_{sm}^{per} > \pi_{sm}^{imp}$ *if* $\theta^{per} = \theta^{imp}$ *and vice versa if* $\theta^{imp} > \theta^{per} = 0.$

Proof. Solving for π_{sm}^{imp} we find that $\pi_{sm}^{imp} = \left(1+\eta_{mp}\right)^{-1}\left(1 + \frac{\theta h \eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right) = \left(1+\eta_{mp}\right)^{-1}\left(1+\theta h \eta_{mp}\pi_{sm}^{per}\right) > 0$ which is increasing in h and for $h \to 0$ we clearly get that $\pi_{sm}^{imp} > \pi_{sm}^{per}$ and for $h \to 1$, $\pi_{sm}^{imp} \to \pi_{sm}^{per}$ implying the first part of the lemma.

The second part follows by observing that $\pi_{sm}^{imp} = \frac{1}{1+\eta_{mp}} + \frac{\theta h \eta_{mp}}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp}-\theta\eta_{mp}\right)} > \frac{1}{1+\eta_{mp}} = \pi_{sm}^{per}$. ■

Lemma 6 $|\pi_{q\varepsilon}| > |\pi_{qm}|$ *if* $\theta^{per} = \theta^{imp}$ *and vice versa if* $\theta^{imp} > \theta^{per} = 0.$

Proof. In both informational regimes $\pi_{qi} = -\frac{\eta_{yw}}{\rho+\eta_{yw}}\left(\pi_{sm} + \pi_{sc}\pi_{ci}\right),\ i = m, \varepsilon$

$\Rightarrow\ \pi_{qi} = -\left[1 - \frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{\delta\pi_{cb}(1-\rho)}{\sigma\beta}\right]^{-1}\frac{\eta_{yw}}{\rho+\eta_{yw}}\pi_{sm},\ i = m, \varepsilon$ as

$\pi_{ci} = \delta\pi_{cb}(1-\rho)\pi_{qi},\ i = m, \varepsilon$. The result follows from Lemma 5. ■

Lemma 7 $\pi_{c\varepsilon} > \pi_{cm}$ *if* $\theta^{per} = \theta^{imp}$ *and vice versa if* $\theta^{imp} > \theta^{per} = 0.$

Proof. Under both informational regimes $\pi_{ci} = \delta\pi_{cb}(1-\rho)\pi_{qi},\ i = m, \varepsilon$. The result follows from Lemma 6. ■

The need to augment the three last lemmas stems from the fact that there is an asymmetry when we want to compare across regimes. In the persistent and fully permanent cases the obvious perfect information benchmark is simply $\theta^{per} = \theta^{imp}$ for the process $m_t - m_t^* = \theta^{per}\left(m_{t-1} - m_{t-1}^*\right) + \varepsilon_t$. In the transitory case, though, we restrict θ^{per} to be zero, whereas θ^{imp} is a free parameter.

Lemma 8 $\pi_{sm}^{per} < \frac{\pi_{sm}^{imp}}{1-\theta+\theta h}$ $(h < 1)$.

Proof. $\pi_{sm}^{per} < \frac{\pi_{sm}^{imp}}{1-\theta+\theta h} \Rightarrow (1-\theta+\theta h)\,\pi_{sm}^{per} < \pi_{sm}^{imp} \Rightarrow (1-\theta+\theta h)\,\pi_{sm}^{per} < \frac{h\eta_{mp}}{1+\eta_{mp}}\pi_{sm}^{per} + \frac{1}{1+\eta_{mp}} \Rightarrow \left[\left(1+\eta_{mp}\right)(1-\theta+\theta h) - h\eta_{mp}\right]\pi_{sm}^{per} < 1 \Rightarrow \frac{\left(1-\theta+\theta h+\eta_{mp}-\theta\eta_{mp}\right)}{1+\eta_{mp}-\theta\eta_{mp}} < 1$, which always holds true. ■

Lemma 9 $\frac{1-\theta}{1-\theta+\theta h}\pi_{sm}^{imp} < \pi_{sm}^{per}$ $(h > 0)$ *in case 1.*

Proof. In case 1 under perfect information $\pi_{sm}^{per}|_{\theta=0} = \frac{1}{1+\eta_{mp}}$ and under imperfect information $\pi_{sm}^{imp}|_{\theta\in(0,1]} = \left(1+\eta_{mp}\right)^{-1}\left(1+\frac{\theta h\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right)$. Substituting in yields the result $\frac{1-\theta}{1-\theta+\theta h}\pi_{sm}^{imp} < \pi_{sm}^{per} \Rightarrow \frac{1-\theta}{1-\theta+\theta h}\frac{1}{1+\eta_{mp}}\left(1+\frac{\theta h\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right) < \frac{1}{1+\eta_{mp}} \Rightarrow \theta h > 0$, which always holds. ■

B.5.2 Impact Effects

Proposition 10 *Relative consumption rises on impact.*

Proof. The proof runs by contradiction. We have $\pi_{cm} = \delta\pi_{cb}(1-\rho)\,\pi_{qm}$ and $\pi_{qm} = -\frac{\eta_{yw}}{\rho+\eta_{yw}}\left[\pi_{sm} - (\sigma\beta)^{-1}\pi_{cm}\right]$. Assume $\pi_{cm} < 0$. By the first expression we get that $\pi_{qm} < 0$ but by the first equation this must imply that $\pi_{cm} > 0$, ie a contradiction. ■

Proposition 11 *The terms of trade fall on impact.*

Proof. The result follows immediately from $\pi_{cm} = \delta\pi_{cb}(1-\rho)\,\pi_{qm} > 0$ (Proposition 10) and $\pi_{cm} > 0$, $\delta > 0$, $\pi_{cb} > 0$ and $(1-\rho) < 0$. ■

Proposition 12 *The nominal exchange rate depreciates on impact.*

Proof. The result follows straightforwardly from Proposition 10 as $\pi_{qm} = -\frac{\eta_{yw}}{\rho+\eta_{yw}}\left[\pi_{sm} + \pi_{sc}\pi_{cm}\right] < 0$ implying the term inside the brackets, which is $\frac{\partial s_t}{\partial(m_t - m_t^*)}$, is positive. ■

Proposition 13 *The nominal interest rate spread falls on impact.*

Proof. $\frac{\partial[\log(1+i_t)-\log(1+i_t^*)]}{\partial(m_t-m_t^*)} = \phi_{im} + \phi_{is}\frac{\partial s_t}{\partial(m_t-m_t^*)} = \phi_{sm}(h-1) + \phi_{sm}^1 + \phi_{is}\phi_{sm}$

$= -\left(\pi_{sm} + \pi_{sc}\pi_{cm}\right) + \left(1+\eta_{mp}-\theta\eta_{mp}\right)^{-1}\left(\frac{\theta h-1-\eta_{mp}+\theta\eta_{mp}}{1+\eta_{mp}}\right) < 0$ as $0 < \theta h < 1,\ 0 < \eta_{mp}$ and $0 < \pi_{sm} + \pi_{sc}\pi_{cm}$ (Proposition 12). ■

Proposition 14 *The impact effect on the nominal exchange rate is; Case 1: larger; Case 2: smaller; Case 3: smaller; under imperfect information than under perfect information.*

Proof. We need to compare $\pi_{sm}^{imp} + \pi_{sc}\pi_{cm}$ with $\pi_{sm}^{per} + \pi_{sc}\pi_{c\varepsilon}$.

Using $\pi_{ci} = \delta\pi_{cb}(1-\rho)\pi_{qi} = \delta\pi_{cb}(1-\rho)\left\{-\left[1-\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{\delta\pi_{cb}(1-\rho)}{\sigma\beta}\right]^{-1}\frac{\eta_{yw}}{\rho+\eta_{yw}}\right\}\pi_{sm}$ $(i = m, \varepsilon)$ we can write $\pi_{sm}^{per}+\pi_{sc}\pi_{c\varepsilon}-\pi_{sm}^{imp}-\pi_{sc}\pi_{cm} = \left[(\pi_{sm}^{per}-\pi_{sm}^{imp}) - \frac{(\pi_{c\varepsilon}-\pi_{cm})}{\sigma\beta}\right] = \frac{1}{1+k_0}(\pi_{sm}^{per}-\pi_{sm}^{imp})$, $\left[k_0 = \frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{\delta\pi_{cb}(\rho-1)}{\sigma\beta} > 0\right]$, where the sign depends on the case cf Lemma 5. ■

Proposition 15 *The impact effect on the terms of trade is (numerically); Case 1: larger; Case 2: smaller; Case 3: smaller; under imperfect information than under perfect information.*

Proof. This follows directly from Lemma 6 as the impact effect under imperfect information is π_{qm} and under perfect information $\pi_{q\varepsilon}$. ■

Proposition 16 *The impact effect on relative consumption is; Case 1: larger; Case 2: smaller; Case 3: smaller; under imperfect information than under perfect information.*

Proof. This follows directly from Lemma 7 as the impact effect under imperfect information is π_{cm} and under perfect information $\pi_{c\varepsilon}$. ■

Proposition 17 *The (absolute value of the) impact effect on the nominal interest rate spread is; Case 1: smaller; Case 2: larger; Case 3: larger; under imperfect perfect information than under perfect information.*

Proof. Case 1: Perfect information: $\frac{-1}{1+\eta_{mp}}$. Imperfect information: $\left(\frac{1+\eta_{mp}-\theta\eta_{mp}-\theta h}{1+\eta_{mp}-\theta\eta_{mp}}\right)\left(\frac{-1}{1+\eta_{mp}}\right)$. The result follows since $\frac{1+\eta_{mp}-\theta\eta_{mp}-\theta h}{1+\eta_{mp}-\theta\eta_{mp}} \in (-1, 0)$.

Cases 2 and 3: The results follow directly by noting that the impact effect under perfect information is $(\theta-1)\pi_{sm}^{per} = \frac{\theta-1+\theta\eta_{mp}-\eta_{mp}}{(1+\eta_{mp})(1+\eta_{mp}-\theta\eta_{mp})}$. And under imperfect information it is $\phi_{im} + \phi_{is}\frac{\partial s_t}{\partial(m_t-m_t^*)} = \frac{\theta h-1-\eta_{mp}+\theta\eta_{mp}}{(1+\eta_{mp})(1+\eta_{mp}-\theta\eta_{mp})}$, which is numerically larger than the perfect information impact effect. ■

Proposition 18 *The (absolute values of the) impact effects on the nominal exchange rate, the terms of trade and relative consumption are increasing in θ and h.*

Proof. By substitution we find that $\pi_{cm} = \frac{\delta\pi_{cb}(\rho-1)\frac{\eta_{yw}}{\rho+\eta_{yw}}}{1+\delta\pi_{cb}(\rho-1)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}\pi_{sm}$, $\pi_{qm} = \frac{\pi_{cm}}{\delta(1-\rho)\pi_{cb}}$, $\pi_{sm} + \pi_{sc}\pi_{cm} = \left(1 + \frac{\delta\pi_{cb}(1-\rho)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}{1-\delta\pi_{cb}(1-\rho)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}\right)\pi_{sm} = (1+k_1)\,\pi_{sm}$ and $\pi_{sm} = \frac{1}{1+\eta_{mp}}\left[1 + \frac{\theta h\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right]$. The impact effects are $\pi_{sm} + \pi_{sc}\pi_{cm}$, π_{qm} and π_{cm}, respectively.

The results follow from observing that;

(i) $\frac{\partial\pi_{sm}}{\partial i} > 0$ $\left(\Rightarrow \frac{\partial\pi_{c\varepsilon}}{\partial i} > 0 \text{ and } \frac{\partial\pi_{q\varepsilon}}{\partial i} < 0\right)$, $i = \theta, h$;

(ii) $k_1 \in (-1, 0)$;

(iii) π_{cb}, π_{sc} and the rest of the parameters above are independent of θ and h. ■

Proposition 19 *The (absolute value of the) impact effect on the nominal interest rate spread is decreasing in θ and h.*

Proof. The proof follows straightforwardly from the writing out the expression for the impact effect and evaluating the partial derivatives. We have that $\frac{\partial[\log(1+i_t)-\log(1+i_t^*)]}{\partial(m_t-m_t^*)} = \phi_{im} + \phi_{is}\frac{\partial s_t}{\partial(m_t-m_t^*)} < 0$ and inserting for ϕ_{im}, ϕ_{is} and $\frac{\partial s_t}{\partial(m_t-m_t^*)}$ we find that $\frac{\partial[\log(1+i_t)-\log(1+i_t^*)]}{\partial(m_t-m_t^*)} = \phi_{sm}(\theta h - 1) + \phi_{sm}^1 + (1+\theta-\theta h)\,\phi_{sm} = \theta\phi_{sm} + \phi_{sm}^1 = \theta h\,(\pi_{sm} + \pi_{sm}^1) = \frac{\theta h - 1 - \eta_{mp} + \theta\eta_{mp}}{(1+\eta_{mp})(1+\eta_{mp}-\theta\eta_{mp})}$ which is increasing in both θ and h. Since the impact effect is negative, we have the desired result. ■

B.5.3 Long-Run Effects

Proposition 20 *The long-run effect of a transitory or a persistent shock on the nominal exchange rate is an appreciation for $\theta \in (0,1)$. In the case of a transitory shock and $\theta = 1$ the long-run effect is zero.*

Proof. This follows directly from equations (29) and (30) as well as Lemma 1 ($\pi_{sc} < 0$) and Proposition 10 ($\pi_{cm} > 0$). ■

Proposition 21 *The long-run effect of a fully permanent shock on the nominal exchange rate is either an appreciation or a depreciation.*

Proof. The long-run effect is $1 + \pi_{sc}\frac{\pi_{cm}}{\theta h}$ and the easiest way to show it can be either positive or negative is to give examples. The baseline case yields a depreciation while $\sigma = 0.5$, $\beta = 0.5$ and $h = 0.005$ produces an appreciation.[30] ■

As this last proposition might seem to indicate, the case of an appreciation only occurs in the case of h very small, that is, for reasonable parameter values the likely outcome is a depreciation.

Proposition 22 *The long-run effect on the terms of trade is an improvement, except in the special case of a transitory shock and $\theta = 1$, where the long-run effect is zero.*

Proof. This follows directly from (36), (37), (38) and Lemma 2 ($\pi_{qc} > 0$), Proposition 10 ($\pi_{cm} > 0$) and that $\theta(1 - \theta + \theta h) - \theta h > 0$, since $\theta h < \theta$. ■

Proposition 23 *The long-run effect on relative consumption is an improvement, except in the special case of a transitory shock and $\theta = 1$, where the long-run effect is zero.*

Proof. The proof follows directly from equations (43), (44), (45) and the fact that $\pi_{cm} > 0$ (Proposition 10). ■

Proposition 24 *The long-run effects on the nominal exchange rate is; Case 1: numerically smaller; Case 2: numerically larger; Case 3: numerically smaller (if the long-run effect is a depreciation); under imperfect information than under perfect information.*

Proof. The proof is split into three cases:

Case 1: The result follows from Lemma 9: The long-run effect under imperfect information is $\frac{1-\theta}{1-\theta+\theta h}\pi_{sc}\pi_{cm}$ and under perfect information it is $\pi_{sc}\pi_{c\varepsilon}$. By Lemma 9 $\frac{1-\theta}{1-\theta+\theta h}\pi_{cm} < \pi_{c\varepsilon}$ implying that the long-run effect must be numerically smaller under the imperfect information regime.

Case 2: Lemma 8 states that $\pi_{sm}^{per} < \frac{\pi_{sm}^{imp}}{1-\theta+\theta h}$ from which it follows that $\pi_{c\varepsilon} < \frac{\pi_{cm}}{1-\theta+\theta h} \Rightarrow |\pi_{sc}\pi_{c\varepsilon}| < \left|\frac{\pi_{sc}\pi_{cm}}{1-\theta+\theta h}\right|$.

Case 3: Given the long-run effect is a depreciation this result follows from Lemma 8 as $\pi_{c\varepsilon} < \frac{\pi_{cm}}{\theta h} \Rightarrow 1 + \pi_{sc}\frac{\pi_{cm}}{\theta h} < 1 + \pi_{sc}\pi_{c\varepsilon}$, where the right-hand side is both the impact and long-run effect under perfect information and the left-hand side is the long-run effect under imperfect information. ■

[30] Program to check this available on request from the authors.

Proposition 25 *The long-run effects on the terms of trade are; Case 1: smaller; Case 2: larger; Case 3: larger; under imperfect information than under perfect information.*

Proof. The proof is split into the three cases:

Case 1: It follows from Lemma 9 that $\frac{1-\theta}{1-\theta+\theta h}\pi_{cm} < \pi_{c\varepsilon}$ implying that the long-run effect under imperfect information $\left(\frac{1-\theta}{1-\theta+\theta h}\pi_{qc}\pi_{cm}\right)$ is smaller than under perfect information $(\pi_{qc}\pi_{c\varepsilon})$ as $\pi_{qc}^{imp} = \pi_{qc}^{per}$ (Lemma 2).

Case 2: The imperfect information long-run effect is $\frac{\pi_{qc}\pi_{cm}}{1-\theta+\theta h}$ and it is a direct implication of Lemma 8 that $\pi_{c\varepsilon} < \frac{\pi_{cm}}{1-\theta+\theta h}$ implying that $\pi_{qc}\pi_{c\varepsilon} < \frac{\pi_{qc}\pi_{cm}}{1-\theta+h}$ which was what we wanted to show.

Case 3: By Lemma 8: $\pi_{sm}^{per} < \frac{\pi_{sm}^{imp}}{h} \Rightarrow \frac{\pi_{qm}}{h} < \pi_{q\varepsilon} \Rightarrow \pi_{c\varepsilon} < \frac{\pi_{cm}}{h}$ and the long-run effects are $\frac{\pi_{qc}\pi_{cm}}{h}$ and $\pi_{qc}\pi_{c\varepsilon}$ implying the result for Case 3. ■

Proposition 26 *The long-run effects on relative consumption are; Case 1: smaller; Case 2: larger; Case 3: larger; under imperfect information than under perfect information.*

Proof. The proof is split into the three cases:

Case 1: A direct implication of Lemma 9 is that $\frac{1-\theta}{1-\theta+\theta h}\pi_{cm} < \pi_{c\varepsilon}$.

Case 2: The long-run effect is $\frac{\pi_{cm}}{1-\theta+\theta h}$ which is larger than the long-run effect under perfect information $(\pi_{c\varepsilon})$ by Lemma 8.

Case 3: We need to compare $\frac{\pi_{cm}}{h}$ with $\pi_{c\varepsilon}$. A direct implication of Lemma 8 is that $\pi_{c\varepsilon} < \frac{\pi_{cm}}{h}$. ■

Proposition 27 *The long-run effect on the nominal interest rate spread is zero.*

Proof. This is most easily seen by writing the nominal interest rate spread as

$$\log(1+i_t) - \log(1+i_t^*) - (\theta - \theta h)\left[\log(1+i_{t-1}) - \log\left(1+i_{t-1}^*\right)\right] =$$
$$\phi_{ss}^1(s_t - s_{t-1}) - \phi_{ss}^1(\theta - \theta h)(s_{t-1} - s_{t-2})$$
$$+\phi_{ss}^2(s_{t-1} - s_{t-2}) + \phi_{ss}^2(\theta - \theta h)(s_{t-2} - s_{t-3}) + \phi_{sm}^1(1-L)(m_t - m_t^*)$$
$$+\phi_{sm}^1(\theta - \theta h)(1-L)\left(m_{t-1} - m_{t-1}^*\right) + \phi_{sm}^2(1-L)\left(m_{t-1} - m_{t-1}^*\right)$$
$$+\phi_{sm}^2(\theta - \theta h)(1-L)\left(m_{t-2} - m_{t-2}^*\right) + \phi_{sm}(\theta h - 1)(m_t - m_t^*)$$
$$+\phi_{sm}(\theta - \theta h)\left(m_{t-1} - m_{t-1}^*\right).$$

In the long run, the nominal exchange rate has converged to its long run value rendering the first four terms on the right-hand side zero. For cases 1 and 2 the relative money supply eventually returns to zero rendering the rest of the right-hand side terms zero. In the fully permanent case ($\theta = 1$) the last two terms (as well as the other money terms) are zero as well. In conclusion, $\log(1 + i_{t+j}) - \log\left(1 + i^*_{t+j}\right)$ must be zero as $j \to \infty$. ■

Proposition 28 *Following a transitory shock the (absolute value of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption are decreasing in θ.*

Proof. The long-run effects are $\frac{(1-\theta)\pi_{sc}\pi_{cm}}{1-\theta+\theta h}$, $\frac{(1-\theta)\pi_{qc}\pi_{cm}}{1-\theta+\theta h}$ and $\frac{(1-\theta)\pi_{cm}}{1-\theta+\theta h}$. We have that $\frac{1-\theta}{1-\theta+\theta h}$ is decreasing and π_{cm} is increasing in θ (see proof of Proposition 18), so no immediate conclusion can be drawn. We can write π_{cm} as $k_1\pi_{sm}$ (see proof of Proposition 18), where k_1 is independent of θ. The procedure then boils down to finding $sign\left[\frac{\partial\left(\frac{1-\theta}{1-\theta+\theta h}\pi_{sm}\right)}{\partial\theta}\right]$. Utilizing that $\pi_{sm} = \left(1+\eta_{mp}\right)^{-1}\left(1 + \frac{\theta h\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right)$ this derivative is easily found to be $\frac{-h}{(1-\theta+\theta h)^2}\frac{1}{1+\eta_{mp}}\left(1 + \frac{\theta h\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right) - \frac{1-\theta}{1-\theta+\theta h}\left(\frac{h\eta_{mp}}{1+\eta_{mp}}\frac{1+\eta_{mp}}{1+\eta_{mp}-\theta\eta_{mp}}\right)$ which is clearly negative. ■

Proposition 29 *Following a persistent shock the (absolute values of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption are increasing in θ.*

Proof. The long-run effects are $\frac{\pi_{sc}\pi_{cm}}{1-\theta+\theta h}$, $\frac{\pi_{qc}\pi_{cm}}{1-\theta+\theta h}$ and $\frac{\pi_{cm}}{1-\theta+\theta h}$. The result follows from the fact that $1-\theta+\theta h$ is decreasing and π_{cm} is increasing in θ (see proof of Proposition 18). ■

Proposition 30 *The (absolute values of the) long-run effects on the nominal exchange rate, the terms of trade and relative consumption following any shock are decreasing in h.*

Proof. The result follows from the fact that
$\frac{\partial\left(\frac{\pi_{sm}}{1-\theta+\theta h}\right)}{\partial h} = (1-\theta+\theta h)^{-2}\frac{-\theta}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp}-\theta\eta_{mp}\right)} < 0 \Rightarrow \frac{\partial\left(\frac{\pi_{cm}}{1-\theta+\theta h}\right)}{\partial h} < 0.$ ■

B.6 Perfect versus Imperfect Information[31]

Having mechanically characterized the solution we can now go a bit more into depth with respect to the connection to the perfect information case and some interesting interpretations. Solving the π-system we get that

$$\pi_{sm} = \frac{1}{1+\eta_{mp}} + \theta h f(\theta),$$

[31] The derivation of this subsection's main result is not obvious without having read the whole appendix, in particular the perfect information part. The intuition underlying the results should be accessible.

where

$$f(\theta) = \frac{\eta_{mp}}{\left(1+\eta_{mp}\right)\left(1+\eta_{mp}-\theta\eta_{mp}\right)}, \quad f' > 0.$$

Under perfect information (appendix C) it turns out that we can write the same parameter as

$$\frac{1}{1+\eta_{mp}-\theta\eta_{mp}} = \frac{1}{1+\eta_{mp}} + \theta f(\theta).$$

The interesting part stems from the fact that π_{sm} for given $\widetilde{\theta}$ under imperfect information can be written as a weighted average of the impact effects under perfect information with $\theta = 0$ (temporary shock) and with $\theta = \widetilde{\theta}$ (permanent shock). This is seen from noting that

$$\pi_{sm}|_{\theta=0}^{\text{perfect information}} = \frac{1}{1+\eta_{mp}},$$

$$\pi_{sm}|_{\theta=\widetilde{\theta}}^{\text{perfect information}} = \frac{1}{1+\eta_{mp}} + \widetilde{\theta} f(\widetilde{\theta}),$$

and

$$\pi_{sm}|_{\theta=\widetilde{\theta}}^{\text{imperfect information}} = \frac{1}{1+\eta_{mp}} + \widetilde{\theta} h f(\widetilde{\theta}).$$

From this it is clear that

$$\pi_{sm}|_{\theta=\widetilde{\theta}}^{\text{imperfect information}} = (1-h)\, \pi_{sm}|_{\theta=0}^{\text{perfect information}} + h\, \pi_{sm}|_{\theta=\widetilde{\theta}}^{\text{perfect information}}$$

with

$$h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} \quad \text{and} \quad 1-h = \frac{\sigma_u^2}{\sigma_\varepsilon^2 + \sigma_u^2}.$$

That is, the variance ratio determines the weight. This reflects the information properties of the model. If h (σ_u^2 large) is small, there is much noise and no weight is put on the persistency term. Note that as the imperfect information case converges to the perfect information case, π_{sm} converges too.

There are two important implications of this intuitive result[32]

[32]The reason these results go through is that the cited effects depend on two sets of coefficients. The first set is independent of θ and h. The second set is dependent on θ and h, but they can be written as linear functions of π_{sm}, and, hence, the results follow from all the expressions for impact and long-run effects. See below.

1. The impact effects under imperfect information can in the same way be written as the weighted average of the impact effects under perfect information.

2. The long-run effects can be written as a corrected weighted average of the long-run effects under perfect information. The correction is a term depending on θ and h multiplied with the weighted average. This correction can be interpreted as capturing the learning process.

The (technical) upshot is, though, that the difference in π_{sm} under the two regimes is driving the differences of all the other important coefficients (see below).

In the perfect information case (see appendix C) it is shown that relative magnitudes of the impact and long-run effects are determined by the interplay between the persistence of the shock and the income elasticity of money demand. With perfect information this changes. Below are given the impact and long-run effects for the nominal exchange rate, the terms of trade and relative consumption.

Table B.6.1. Impact and Long-Run Effects Under Imperfect Information.

	Impact Effect	Long-Run Effect[33]
s	$\pi_{sm} + \pi_{sc}\pi_{cm}$	$k\pi_{sc}\pi_{cm}$ $(1 + k\pi_{sc}\pi_{cm})$
q	π_{qm}	$k\pi_{qc}\pi_{cm}$
c-c*	π_{cm}	$k\pi_{cm}$

where the constant k is shock-dependent

Table B.6.2.

Type of Shock	k
Transitory	$\frac{1-\theta}{1-\theta+\theta h}$
Persistent	$\frac{1}{1-\theta+\theta h}$
Fully Permanent	$\frac{1}{\theta h}$

Only three coefficients are dependent on θ and it is[34]

$$\pi_{sm}\left(\theta, \eta_{mp}, h\right) = \left(1 + \eta_{mp}\right)^{-1}\left[1 + \frac{\theta h \eta_{mp}}{1 + \eta_{mp}\left(1 - \theta\right)}\right],$$

[33]The long-run effect for the nominal exchange rate is $k\pi_{sc}\pi_{cm}$ if the shock is either transitory or persistent and $1 + k\pi_{sc}\pi_{cm}$ if the shock is fully permanent.

[34]The results that the impact and long-run effects can be written as weighted averages follow from these expressions. See also tables B.6.1 and C.6.1.

$$\pi_{qm} = \frac{-\frac{\eta_{yw}}{\rho+\eta_{yw}}}{1-\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}\delta\pi_{cb}\left(1-\rho\right)}\pi_{sm}\left(\theta,\eta_{mp},h\right),$$

$$\pi_{cm} = \frac{-\frac{\eta_{yw}}{\rho+\eta_{yw}}\delta\pi_{cb}\left(1-\rho\right)}{1-\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}\delta\pi_{cb}\left(1-\rho\right)}\pi_{sm}\left(\theta,\eta_{mp},h\right).$$

It is seen that the dependence for π_{qm} and π_{cm} arises because they can be written as functions of π_{sm}. Now, when assessing the relative magnitudes of the impact and long-run effects across the type of shock we find differences compared to the perfect information case.

Under perfect information the agents know exactly what has hit them when a (one-time) shock occurs. The nominal depreciation creates a wealth reallocation which is smoothed over time. The relative magnitude of this wealth increase is determined by the amount of liquidity in the economy the period after the shock has hit (represented by θ) and the income (consumption) elasticity of money demand (represented by η_{mp} [β]). The relative impact and long-run effects on relative consumption and the terms of trade are increasing in θ and β (decreasing in the income elasticity). The result for the long-run effects follow since they are tied to the impact effects as wages are only fixed for one period.

Under imperfect information the agents do not know what has hit them. Therefore, the impact effects of a temporary and a permanent shock are the same.[35] The absolute magnitude is dealt with above.

The more interesting part is the long-run effects. Under imperfect information it is seen that the relative size of the long run effects for a persistent shock with $\theta = \widetilde{\theta} < 1$ compared to a transitory shock (with $\theta = \widetilde{\theta}$ for the underlying permanent part) is

$$\frac{\frac{1}{1-\widetilde{\theta}+h}\pi_{sm}\left(\widetilde{\theta},\eta_{mp},h\right)}{\frac{1-\widetilde{\theta}}{1-\widetilde{\theta}+h}\pi_{sm}\left(\widetilde{\theta},\eta_{mp},h\right)} = \frac{1}{1-\widetilde{\theta}}.$$

In the perfect information case this ratio is (see appendix C.6)

$$\frac{1+\eta_{mp}}{1+\eta_{mp}\left(1-\widetilde{\theta}\right)} < \frac{1}{1-\widetilde{\theta}}.$$

[35] We implicitly assume that when comparing a permanent shock with $\theta = \widetilde{\theta}$ and a temporary shock with $\theta = 0$, then the underlying permanent part for the temporary case has $\theta = \widetilde{\theta}$.

Thus, the link to η_{mp} has been broken under imperfect information. In fact it can be shown that ceteris paribus for $\beta \to 0$ under perfect information the ratio is $\frac{1}{1-\theta}$ as well. The reason for this is that when β is small the money demand elasticity (with respect to income) is large and in the period after the shock the money demand is highly sensitive with respect to income (consumption) thereby negatively affecting the effect on impact (perfect information). So, for imperfect information long-run effects we get the limiting case of the perfect information case with infinite income elasticity of money demand. This follows from the fact that agents do not know which shock hit them and, hence, the impact effects do not depend on whether the shock in fact was either temporary or permanent. The impact effects are the same and from then on the relative long-run effects depend solely on how much money that is in the economy (θ).

One of the reasons why the ratio increases under imperfect information is the period-after terms-of-trade improvement. The main conclusion, though, is that the ratio is unaffected (for given θ) by variations in other parameters under imperfect information whereas it is not under perfect information.

C Equilibrium with One-Period Nominal Wage Contracts and Perfect Information

In this section we go through the reference case of perfect information for completeness. We consider a money supply process where there is no confusion in disentangling permanent and temporary shocks

$$m_t - m_t^* = \theta\left(m_{t-1} - m_{t-1}^*\right) + \varepsilon_t, \quad \varepsilon_t \sim nid\left(0, \sigma_\varepsilon^2\right), \quad \theta \in [0,1],$$

and where the case of fully permanent shocks ($\theta = 1$) is analyzed in Andersen and Beier (1999). Again we solve for four variables in this appendix: the nominal exchange rate, the terms of trade, relative consumption, and finally, the nominal interest rate spread. We apply the method of undetermined coefficients. Our guesses are

$$s_t = \pi_{sc}\left(c_t - c_t^*\right) + \pi_{sm}\left(m_t - m_t^*\right), \tag{48}$$

$$q_t = \pi_{qc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{q\varepsilon}\varepsilon_t, \tag{49}$$

and

$$c_t - c_t^* = \pi_{cb}\left(b_{t-1} - b_{t-1}^*\right) + \pi_{cc}\left(c_{t-1} - c_{t-1}^*\right) + \pi_{c\varepsilon}\varepsilon_t. \tag{50}$$

Note that the guess for relative consumption and the Euler equation implies that relative consumption follows a random walk

$$c_t - c_t^* = c_{t-1} - c_{t-1}^* + \pi_{c\varepsilon}\varepsilon_t.$$

C.1 The Nominal Exchange Rate

Again we have

$$s_t = \frac{1}{1+\eta_{mp}} \left[\left(\eta_{mc}^1 - \eta_{mc}\right)(c_t - c_t^*) + \eta_{mp} E_t s_{t+1} + m_t - m_t^* \right].$$

Going through the usual steps produces the following restrictions

$$\pi_{sc} = \eta_{mc}^1 - \eta_{mc} = -(\sigma\beta)^{-1},$$

$$\pi_{sm} = \left(1 + \eta_{mp} - \theta\eta_{mp}\right)^{-1}.$$

The dynamic adjustment is found by using (48)

$$\begin{aligned} & (1-L)(1-\theta L)s_t \\ = \; & (1-L)(\pi_{sm} + \pi_{sc}\pi_{c\varepsilon})\varepsilon_t + (1-\theta)\pi_{sc}\pi_{c\varepsilon}\varepsilon_{t-1}, \end{aligned}$$

ie an ARIMA(1,1,1). Restricting θ to be 0 or 1 implies the process boils down to an ARIMA(0,1,1). The impact effect equals

$$\frac{\partial s_t}{\partial \varepsilon_t} = \pi_{sm} + \pi_{sc}\pi_{c\varepsilon} > 0. \tag{51}$$

The long-run effects are

$$\left.\frac{\partial s_{t+j}}{\partial \varepsilon_t}\right|_{\theta\in[0,1)} = \pi_{sc}\pi_{c\varepsilon} < 0 \quad for\ j \to \infty, \tag{52}$$

and

$$\left.\frac{\partial s_{t+j}}{\partial \varepsilon_t}\right|_{\theta=1} = \pi_{sm} + \pi_{sc}\pi_{s\varepsilon} = 1 + \pi_{sc}\pi_{c\varepsilon} > 0 \quad for\ j \to \infty. \tag{53}$$

C.2 The Terms of Trade

Following the same steps as in the imperfect information case we retrieve

$$\begin{aligned} q_t \;=\; & \frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}}\left(c_{t-1} - c_{t-1}^*\right) + \frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}} E_{t-1} q_t \\ & - \frac{\eta_{yw}}{\rho+\eta_{yw}}(\pi_{sm} + \pi_{sc}\pi_{cm})\varepsilon_t. \end{aligned}$$

After some simple algebra we find that consistency with the guess requires that

$$\pi_{qc} = \frac{\eta_{yw}\left(2\eta_{wp}-1\right)}{\rho+\eta_{yw}}\pi_{qc} + \frac{\eta_{yw}\eta_{wc}}{\rho+\eta_{yw}},$$

$$\pi_{q\varepsilon} = -\frac{\eta_{yw}}{\rho + \eta_{yw}} \left(\pi_{sm} + \pi_{sc}\pi_{cm}\right).$$

Using our guess we can rewrite the terms of trade as

$$\left(1 - L\right) q_t = \left(1 - L\right) \pi_{q\varepsilon}\varepsilon_t + \pi_{qc}\pi_{c\varepsilon}\varepsilon_{t-1},$$

or

$$q_t = \pi_{q\varepsilon}\varepsilon_t + \pi_{qc}\pi_{c\varepsilon} \sum_{j=0}^{\infty} \varepsilon_{t-1-j}.$$

That is the terms of trade follow an ARIMA(0,1,1) with impact effect $\pi_{q\varepsilon}$ and long-run effect $\pi_{qc}\pi_{c\varepsilon}$.

C.3 Relative Consumption

Proceeding as for relative consumption in the imperfect information case one finds that consistency with the Euler equation requires

$$\pi_{cb} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1} \delta^{-1}\pi_{cb},$$

$$\pi_{cc} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1} \pi_{cb} \left(1 - \rho\right) \pi_{qc},$$

$$\pi_{c\varepsilon} = \left(1 + \pi_{cb} - \pi_{cc}\right)^{-1} \pi_{cb} \left(1 - \rho\right) \pi_{q\varepsilon}.$$

Note that relative consumption follows a random walk.

C.4 Nominal Interest Rate Spread

Again we have that the nominal interest rate spread is

$$\log(1 + i_t) - \log(1 + i_t^*) = E_t s_{t+1} - s_t.$$

Let us for completeness consider the dynamics in our three cases. For a transitory shock ($\varepsilon_t > 0,\ \theta = 0$) we have that

$$\begin{aligned} \log\left(1 + i_{t+i}\right) - \log\left(1 + i_{t+i}^*\right) &= E_{t+i}s_{t+1+i} - s_{t+i} \\ &= s_{t+i} - \pi_{sm}\varepsilon_{t+i} - s_{t+i} \\ &= -\pi_{sm}\varepsilon_{t+i}, \quad i = 0, 1, 2, ..., \end{aligned} \tag{54}$$

ie one-period dynamics. For a persistent shock [$\varepsilon_t > 0,\ \theta \in (0, 1)$] we have

$$\begin{aligned} &\log\left(1 + i_{t+i}\right) - \log\left(1 + i_{t+i}^*\right) \\ = \ &\theta\left(s_{t+i} - s_{t-1+i}\right) - \left(\pi_{sm} + \theta\pi_{sc}\pi_{c\varepsilon}\right)\varepsilon_{t+i}, \quad i = 0, 1, 2, ..., \end{aligned} \tag{55}$$

which is a slowly decaying pattern. In the last case of a fully permanent shock ($\varepsilon_t > 0,\ \theta = 1$) the nominal exchange rate is a random walk leading to

$$\log\left(1 + i_{t+i}\right) - \log\left(1 + i^*_{t+i}\right) = 0, \quad i = 0, 1, 2, ... \tag{56}$$

Note that in the three cases we have one-period dynamics, infinite dynamics with the initial effect being slowly worked out of the system and no dynamics, respectively.

C.5 Analytical Characterization of the Solution

Lemma 31 *$0 < \pi_{sm} \leq 1$ and $\pi_{sc} < 0$.*

Proof. It follows from the restriction for π_{sm} that $\pi_{sm} = \left[1 + \eta_{mp}\left(1 - \theta\right)\right]^{-1} \in (0, 1]$ as $\eta_{mp} > 0$ and $\theta \in [0, 1]$.

The second part of the lemma follows straightforwardly from the restriction for π_{sc} and noting that $\sigma > 0$ and $\beta > 0$. ■

Proposition 32 *Relative consumption rises on impact.*

Proof. The proof runs by contradiction. We have that $\frac{\partial(c_t - c^*_t)}{\partial \varepsilon_t} = \pi_{c\varepsilon} = \delta\pi_{cb}\left(1 - \rho\right)\pi_{q\varepsilon}$ and $\pi_{q\varepsilon} = -\frac{\eta_{yw}}{\rho + \eta_{yw}}\left[\pi_{sm} - \left(\sigma\beta\right)^{-1}\pi_{c\varepsilon}\right]$. Assume $\pi_{c\varepsilon} < 0$. By the second expression we get that $\pi_{q\varepsilon} < 0$ but by the first equation this must imply that $\pi_{c\varepsilon} > 0$, ie a contradiction. ■

Proposition 33 *The terms of trade fall on impact.*

Proof. The result ($\frac{\partial q_t}{\partial \varepsilon_t} = \pi_{qc} < 0$) follows immediately from $\pi_{c\varepsilon} = \delta\pi_{cb}\left(1 - \rho\right)\pi_{q\varepsilon}$ and $\pi_{c\varepsilon} > 0$ (Proposition 32), $\delta > 0$, $\pi_{cb} > 0$ (Lemma 4) and $(1 - \rho) < 0$. ■

Proposition 34 *The nominal exchange rate depreciates on impact.*

Proof. The result follows straightforwardly from Proposition 33 as $\pi_{q\varepsilon} = -\frac{\eta_{yw}}{\rho + \eta_{yw}}\left[\pi_{sm} - \left(\sigma\beta\right)^{-1}\pi_{c\varepsilon}\right] < 0$ implying the term inside the brackets, which is $\frac{\partial s_t}{\partial \varepsilon_t}$, is positive (see equation [51]). ■

Proposition 35 *The impact effect on the nominal interest rate spread is; Case 1: a fall; Case 2: a fall; Case 3: zero.*

Proof. This follows directly from equations (54), (55), (56) and that $\pi_{sm} > 0$ (Lemma 31), $\pi_{sm}+\pi_{sc}\pi_{c\varepsilon} > 0$ (Proposition 34)$\Rightarrow \pi_{sm}+\theta\pi_{sc}\pi_{c\varepsilon} > 0$. ■

Proposition 36 *The long-run effect of a transitory shock* $\{\theta \in (0,1]\}$ *or a persistent shock* $[\theta \in (0,1)]$ *on the nominal exchange rate is an appreciation.*

Proof. This follows directly from (52), Lemma 31 ($\pi_{sc} < 0$) and Proposition 31 ($\pi_{c\varepsilon} > 0$). ■

Proposition 37 *The long-run effect of a fully permanent shock* $(\theta = 1)$ *on the nominal exchange rate is a depreciation.*

Proof. The result follows from Proposition 34 and noting that the impact effect equals the long-run effect, cf equation (53). ■

Proposition 38 *The long-run effect on the terms of trade is an improvement.*

Proof. This follows directly from noting that the long-run effect is $\pi_{qc}\pi_{c\varepsilon}$ and that $\pi_{qc} > 0$ (Lemma 2) and $\pi_{c\varepsilon} > 0$ (Proposition 32). ■

Proposition 39 *The long-run effect on relative consumption is an improvement.*

Proof. The result follows from Proposition 32 and noting that the long-run effect equals the impact effect. ■

Proposition 40 *The long-run effect on the nominal interest rate spread is zero.*

Proof. Follows directly from equations (54), (55) and (56). ■

Proposition 41 *The (absolute values of the) impact and long-run effects on the nominal exchange rate, the terms of trade and relative consumption are increasing in* θ*.*

Proof. By substitution we find that $\pi_{c\varepsilon} = \frac{\delta\pi_{cb}(\rho-1)\frac{\eta_{yw}}{\rho+\eta_{yw}}}{1+\delta\pi_{cb}(\rho-1)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}\pi_{sm}$, $\pi_{q\varepsilon} = \frac{\pi_{c\varepsilon}}{\delta(1-\rho)\pi_{cb}}$, $\pi_{sm} + \pi_{sc}\pi_{c\varepsilon} = \left(1 + \frac{\delta\pi_{cb}(1-\rho)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}{1-\delta\pi_{cb}(1-\rho)\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}}\right)\pi_{sm} = (1+k_1)\,\pi_{sm}$ and $\pi_{sm} = \frac{1}{1+\eta_{mp}-\theta\eta_{mp}}$. The impact effects are $\pi_{sm} + \pi_{sc}\pi_{c\varepsilon}$, $\pi_{q\varepsilon}$ and $\pi_{c\varepsilon}$. The long-run effects for the terms or trade and relative consumption are $\pi_{qc}\pi_{c\varepsilon}$ and $\pi_{c\varepsilon}$ respectively. The long-run effect for the nominal exchange rate is $\pi_{sc}\pi_{c\varepsilon}$ in the first two cases and in case 3 it is $1+\pi_{sc}\pi_{c\varepsilon}$.

The results follow from observing that;
(i) $\frac{\partial \pi_{sm}}{\partial \theta} = \eta_{mp}\left(1+\eta_{mp}-\theta\eta_{mp}\right)^{-2} > 0 \left(\Rightarrow \frac{\partial \pi_{c\varepsilon}}{\partial \theta} > 0 \text{ and } \frac{\partial \pi_{q\varepsilon}}{\partial \theta} < 0\right)$;
(ii) $k_1 \in (-1, 0)$;
(iii) π_{cb}, π_{sc} and the rest of the parameters above are independent of θ.
■

Proposition 42 *The (absolute value of the) impact effect on the nominal interest rate spread is decreasing in θ.*

Proof. The impact effect in the general case can be seen from (55) to be $\frac{\partial[\log(1+i_t)-\log(1+i_t^*)]}{\partial \varepsilon_t} = \theta\frac{\partial s_t}{\partial \varepsilon_t} - \pi_{sm} - \theta\pi_{sc}\pi_{c\varepsilon} = (\theta-1)\pi_{sm}$ and using that $\pi_{sm} = \left(1+\eta_{mp}-\theta\eta_{mp}\right)^{-1}$ we easily find that $\frac{\partial\left\{\frac{\partial[\log(1+i_t)-\log(1+i_t^*)]}{\partial \varepsilon_t}\right\}}{\partial \theta} = \frac{\partial[(\theta-1)\pi_{sm}]}{\partial \theta}$ is equal to $\pi_{sm} + (\theta-1)\eta_{mp}\left(1+\eta_{mp}-\theta\eta_{mp}\right)^{-2} = \left(1+\eta_{mp}-\theta\eta_{mp}\right)^{-2} > 0$. The result follows from observing that the impact effect is $-\left(1+\eta_{mp}\right)^{-1} < 0$ for $\theta = 0$ and 0 for $\theta = 1$. ■

C.6 A Closer Look

Having characterized the solution it might be enlightening to have a closer look at the effects, or more precisely; the differences across types of shock. In the following table we present the impact and long-run effects for the nominal exchange rate, the terms of trade and relative consumption.

Table C.6.1. Impact and Long-Run Effects Under Perfect Information.

	Impact Effect	Long-Run Effect[36]
s	$\pi_{sm}+\pi_{sc}\pi_{c\varepsilon}$	$\pi_{sc}\pi_{c\varepsilon}$ $\left(\pi_{sm}+\pi_{sc}\pi_{c\varepsilon}\right)$
q	$\pi_{q\varepsilon}$	$\pi_{qc}\pi_{c\varepsilon}$
c-c*	$\pi_{c\varepsilon}$	$\pi_{c\varepsilon}$

[36] The long-run effect for the nominal exchange rate is $\pi_{sc}\pi_{c\varepsilon}$ if $\theta \in [0,1)$ and $1+\pi_{sc}\pi_{c\varepsilon}$ if $\theta = 1$.

The interesting part is to note that we have

$$\pi_{sm}\left(\theta,\eta_{mp}\right) = \left[1+(1-\theta)\,\eta_{mp}\right]^{-1},$$

$$\pi_{q\varepsilon} = \frac{-\frac{\eta_{yw}}{\rho+\eta_{yw}}}{1-\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}\delta\pi_{cb}\,(1-\rho)}\pi_{sm}\left(\theta,\eta_{mp}\right),$$

$$\pi_{c\varepsilon} = \frac{-\frac{\eta_{yw}}{\rho+\eta_{yw}}\delta\pi_{cb}\,(1-\rho)}{1-\frac{\eta_{yw}}{\rho+\eta_{yw}}\frac{1}{\sigma\beta}\delta\pi_{cb}\,(1-\rho)}\pi_{sm}\left(\theta,\eta_{mp}\right),$$

where we have stressed that π_{sm} is a function of the persistency of the shock θ and the parameter η_{mp}. This parameter arises from the log-linearization of the money demand and for convenience we restate it here

$$\eta_{mp} = \frac{\delta}{(1-\delta)\,\beta}.$$

In the following we will take δ for given and focus on the role of β. If we take the (log) steady-state version of the money demand we find that (disregarding constants)

$$m-p = \frac{1}{\sigma\beta}c,$$

which highlights that β is an important determinant of the income (consumption) elasticity of money demand.

Having completed the technicalities we can turn to the dependence of the relative effects on θ and η_{mp}. As is evident from the expressions for π_{sm}, $\pi_{q\varepsilon}$ and $\pi_{c\varepsilon}$ this dependence stems from the fact that all three coefficients can be written as functions of θ and η_{mp}. Notice that π_{sm} is the only parameter which is dependent on θ and as a result so are $\pi_{q\varepsilon}$ and $\pi_{c\varepsilon}$. Comparing the relative effects of a temporary ($\theta = 0$) and a permanent shock with $\theta = \widetilde{\theta} \in (0,1)$ we can see that they are determined by

$$\begin{aligned}\frac{\pi_{sm}\left(\widetilde{\theta},\eta_{mp}\right)}{\pi_{sm}\left(0,\eta_{mp}\right)} &= \frac{1+\eta_{mp}}{1+\eta_{mp}\left(1-\widetilde{\theta}\right)} = \frac{1+\frac{\delta}{(1-\delta)\beta}}{1+\frac{\delta}{(1-\delta)\beta}\left(1-\widetilde{\theta}\right)}\\ &= \frac{(1-\delta)\,\beta+\delta}{(1-\delta)\,\beta+\delta\left(1-\widetilde{\theta}\right)}.\end{aligned}$$

From this it is seen that for if $\beta \to 0$ the relative effects (both impact and long-run effects) are $\frac{1}{1-\widetilde{\theta}} > 1$ and for $\beta \to \infty$ the relative effects are 1. The last result follows since the income elasticity of money demand becomes zero. Following any shock the money demand is unchanged in the period after. On the other hand, if β becomes arbitrarily small, then money demand is extremely sensitive to income changes. As a result, the effects are highly dependent on how much money there is in the economy the period after the shock to accommodate the increased money demand. In the case of a transitory shock there wont be any rise in liquidity the period after and subsequently the effects are small relative to the situation of a large θ in which there is leeway for a more considerable wealth increase in the period of the shock. It is important to stress that we consider relative effects, since the absolute effects are determined by several coefficients. We isolate the reasons why the effects differ across type of shock. When $\beta \rightarrow 0$ it is worth noting that the absolute long-run effects are zero, which is seen by inserting in the expressions above. This is due to the extreme sensitiveness of money demand on income (consumption). Note, that the relative long-run effects might not be that sensitive to changes in β since δ is large (the benchmark value is approximately 0.95).

The dependence of the effects of θ is, of course, the mirror argument; for a given money demand income elasticity a large θ makes room for a larger wealth (consumption) increase when the shock occurs relative to a small θ simply because there is more money in the economy the period after.

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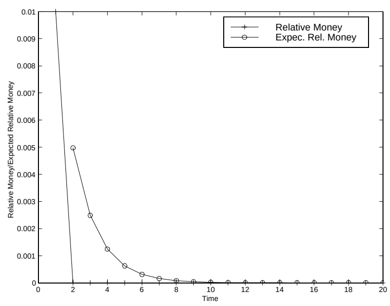
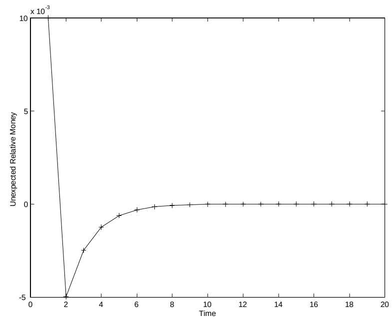
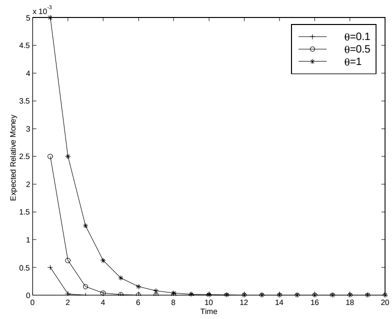
Figure 1.
1 Percent Transitory Home Monetary Expansion.
Dynamics of Expected and Unexpected Relative Money.[37]



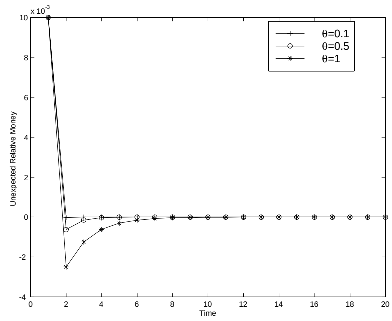
i) De Facto and Expected R. Money



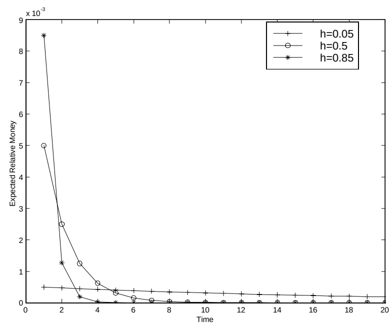
ii) Unexpected Rel. Money



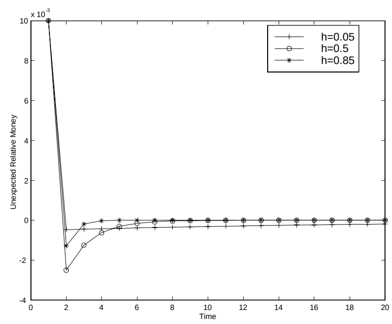
iii) Expected Rel. Money



iv) Unexpected Rel. Money



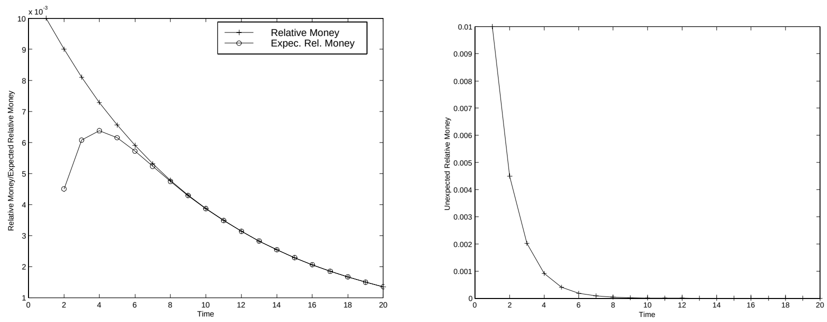
v) Expected Rel. Money



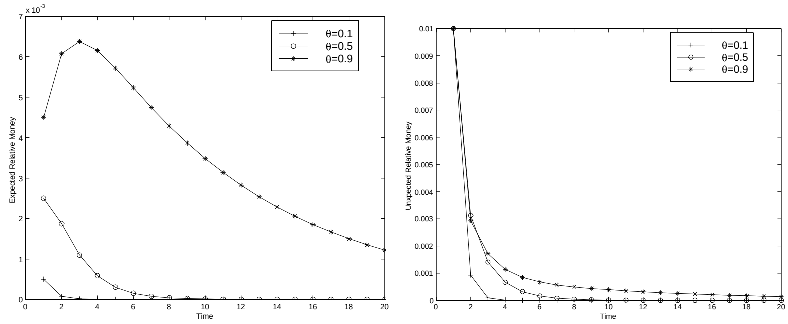
vi) Unexpected Rel. Money

[37] Figures i-ii and v-vi: $\theta = 1$. Figures i-iv: $h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} = 0.5$.

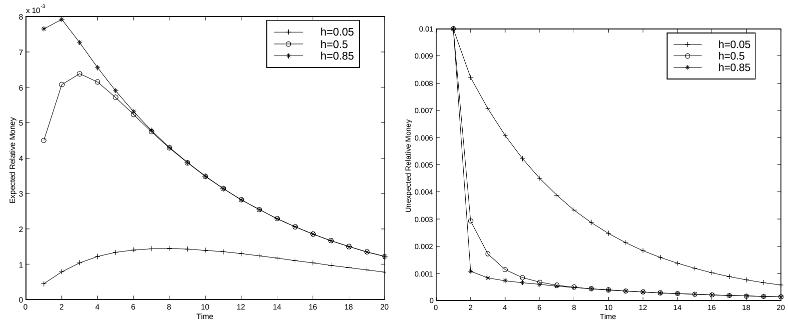
Figure 2.
1 Percent Persistent Home Monetary Expansion.
Dynamics of Expected and Unexpected Relative Money.[38]



i) De Facto and Expected R. Money ii) Unexpected Rel. Money



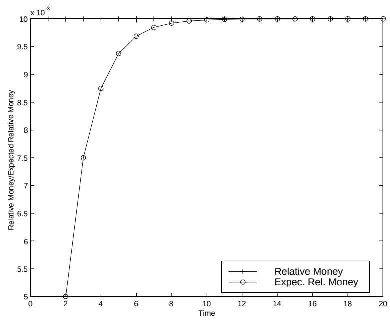
iii) Expected Rel. Money iv) Unexpected Rel. Money



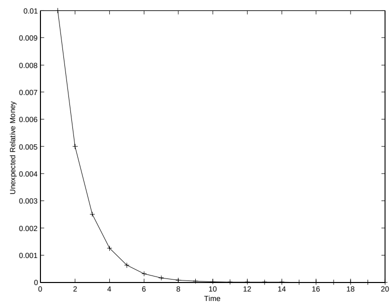
v) Expected Rel. Money vi) Unexpected Rel. Money

[38] Figures i-ii and v-vi: $\theta = 0.9$. Figures i-iv: $h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} = 0.5$.

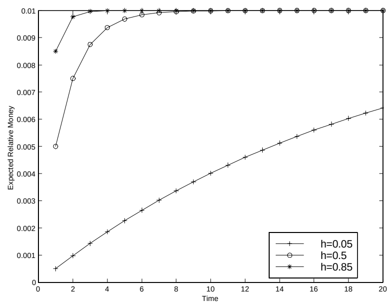
Figure 3.
1 Percent Fully Permanent Home Monetary Expansion.
Dynamics of Expected and Unexpected Relative Money.[39]



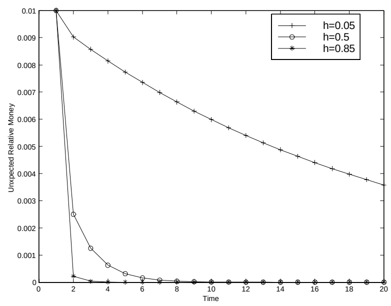
i) De Facto and Expected R. Money



ii) Unexpected Rel. Money



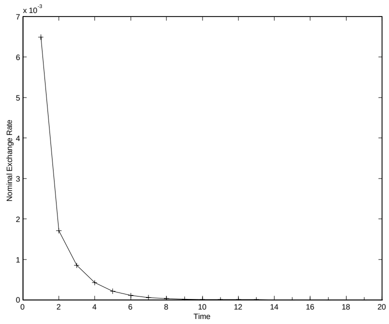
iii) Expected Rel. Money



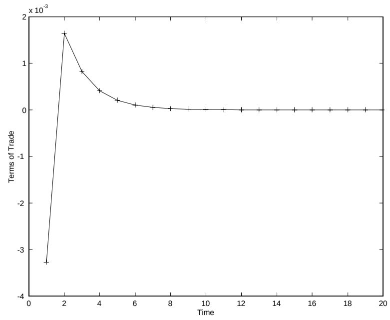
iv) Unexpected Rel. Money

[39]Figures i-ii: $h = \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_u^2} = 0.5$.

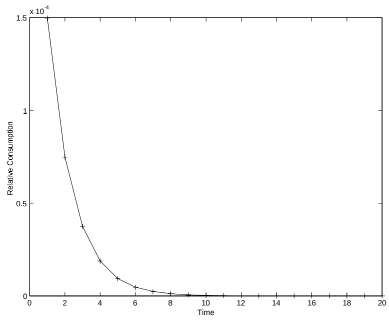
Figure 4a.
1 Percent Transitory Home Monetary Expansion.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[40]



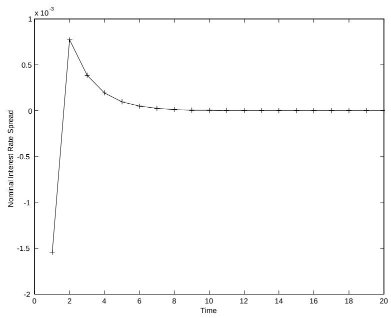
i) Nominal Exchange Rate



ii) Terms of Trade



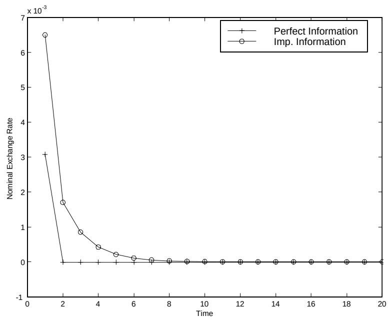
iii) Relative Consumption



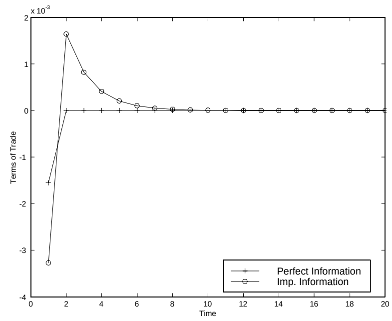
iv) Interest Rate Spread

[40] $\theta = 1$.

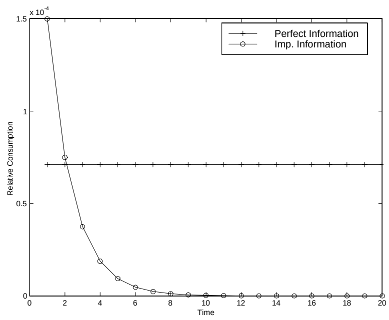
Figure 4b.
1 Percent Transitory Home Monetary Expansion.
Perfect Information versus Imperfect Information.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[41]



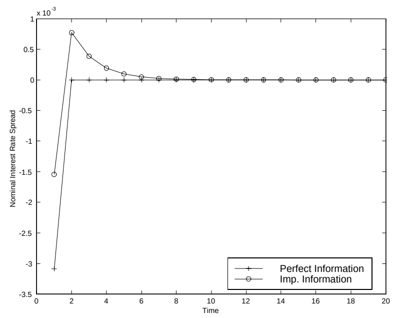
i) Nominal Exchange Rate



ii) Terms of Trade



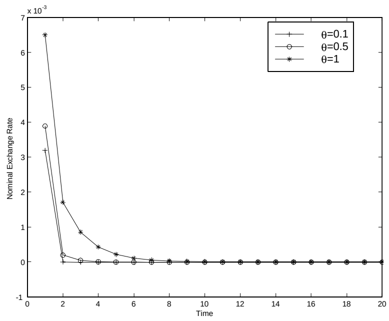
iii) Relative Consumption



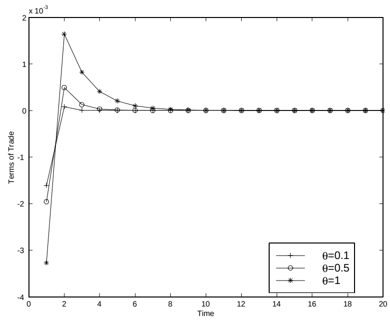
iv) Interest Rate Spread

[41] $\theta = 1$.

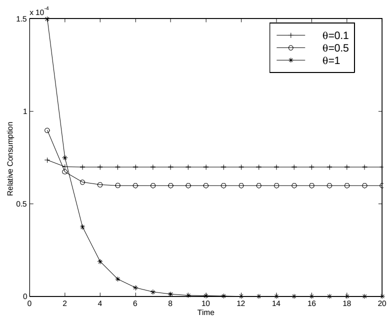
Figure 4c.
1 Percent Transitory Home Monetary Expansion.
Sensitivity wrt θ.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.



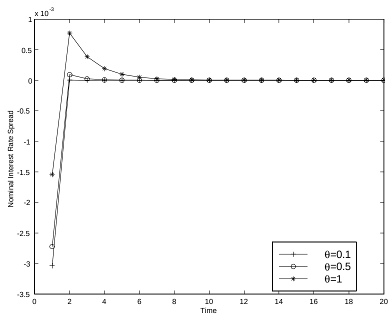
i) Nominal Exchange Rate



ii) Terms of Trade

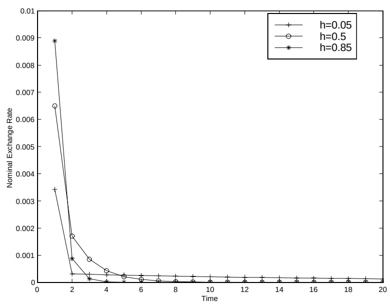


iii) Relative Consumption

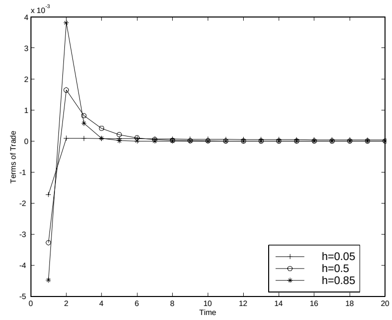


iv) Interest Rate Spread

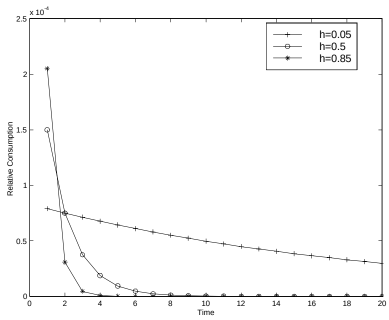
Figure 4d.
1 Percent Transitory Home Monetary Expansion.
Sensitivity wrt h.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[42]



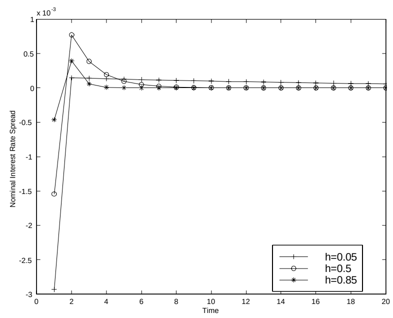
i) Nominal Exchange Rate



ii) Terms of Trade



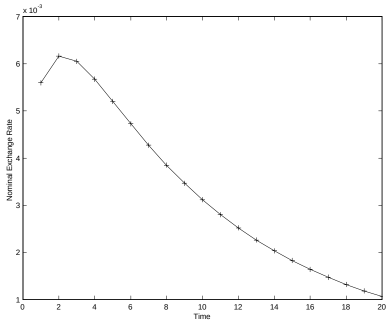
iii) Relative Consumption



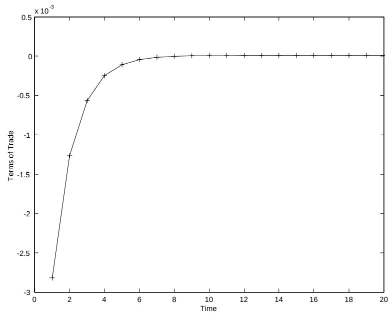
iv) Interest Rate Spread

[42] $\theta = 0.9$.

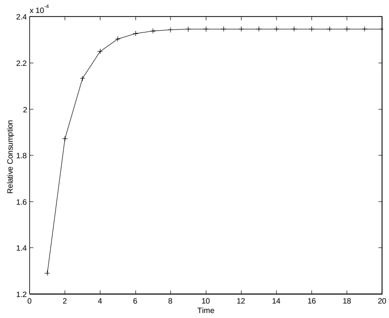
Figure 5a.
1 Percent Persistent Home Monetary Expansion.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[43]



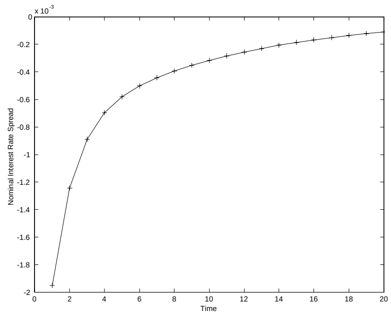
i) Nominal Exchange Rate



ii) Terms of Trade



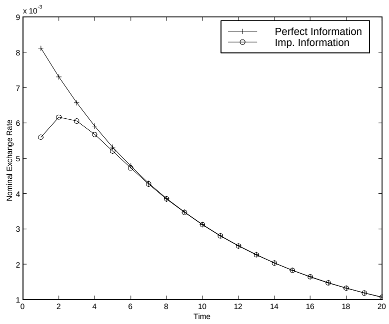
iii) Relative Consumption



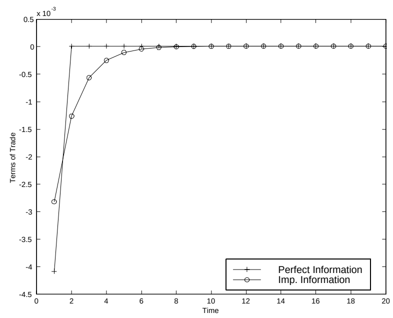
iv) Interest Rate Spread

[43] $\theta = 0.9$.

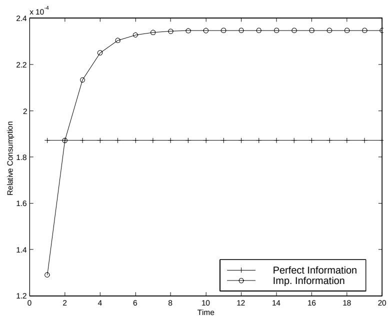
Figure 5b.
1 Percent Persistent Home Monetary Expansion.
Perfect versus Imperfect Information.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[44]



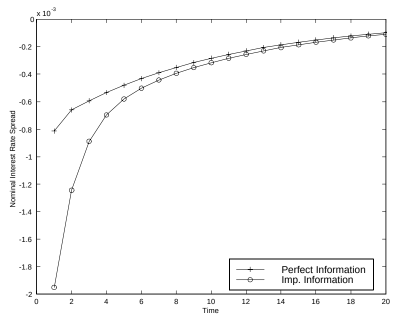
i) Nominal Exchange Rate



ii) Terms of Trade



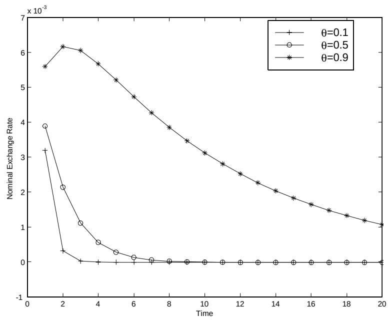
iii) Relative Consumption



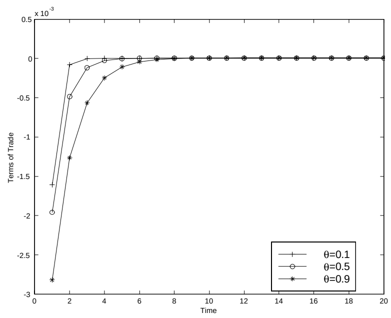
iv) Interest Rate Spread

[44] $\theta = 0.9$.

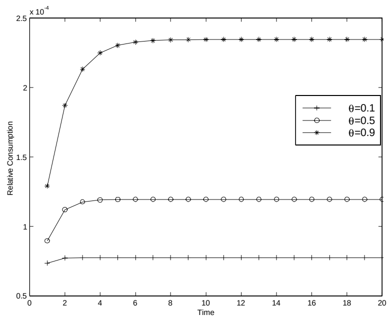
Figure 5c.
1 Percent Persistent Home Monetary Expansion.
Sensitivity wrt θ.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.



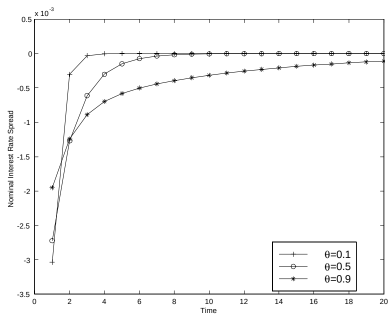
i) Nominal Exchange Rate



ii) Terms of Trade

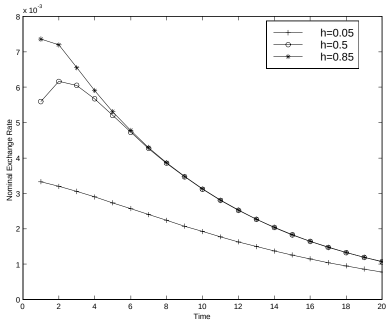


iii) Relative Consumption

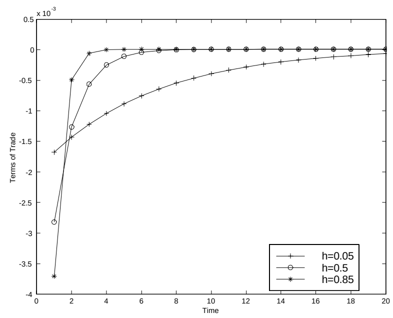


iv) Interest Rate Spread

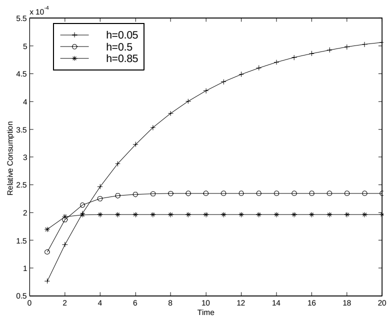
Figure 5d.
1 Percent Persistent Home Monetary Expansion.
Sensitivity wrt h.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.[45]



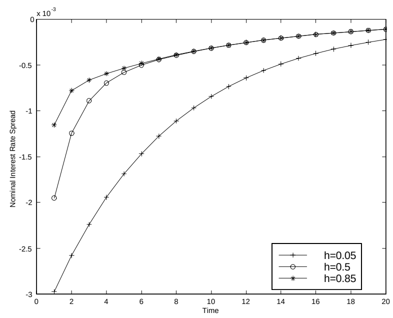
i) Nominal Exchange Rate



ii) Terms of Trade



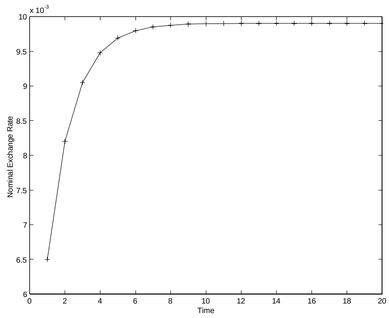
iii) Relative Consumption



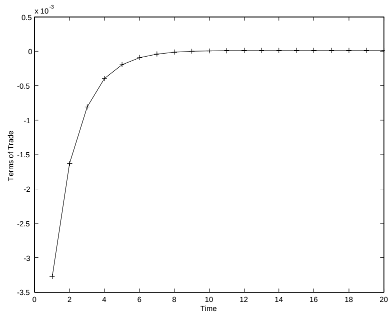
iv) Interest Rate Spread

[45] $\theta = 0.9$.

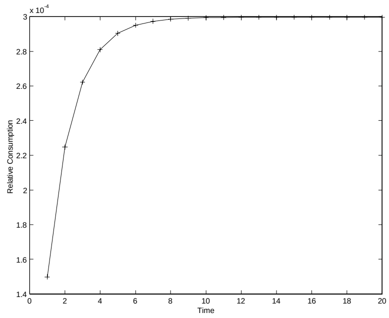
Figure 6a.
1 Percent Fully Permanent Home Monetary Expansion.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.



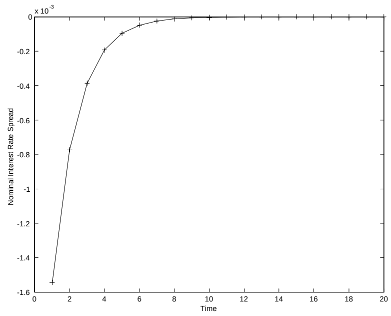
i) Nominal Exchange Rate



ii) Terms of Trade

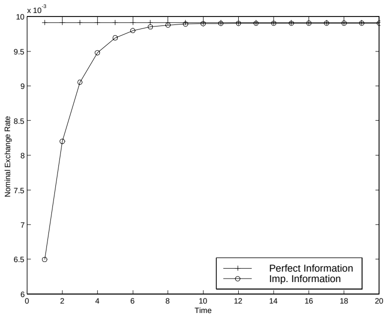


iii) Relative Consumption

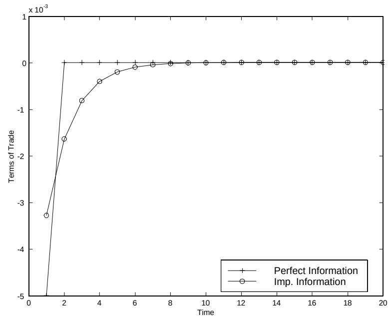


iv) Interest Rate Spread

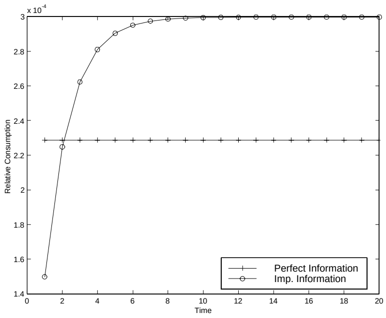
Figure 6b.
1 Percent Fully Permanent Home Monetary Expansion.
Perfect versus Imperfect Information.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.



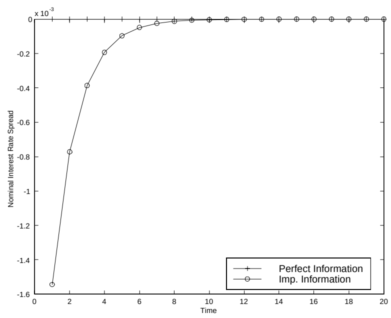
i) Nominal Exchange Rate



ii) Terms of Trade

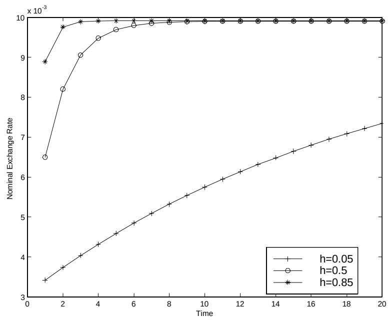


iii) Relative Consumption

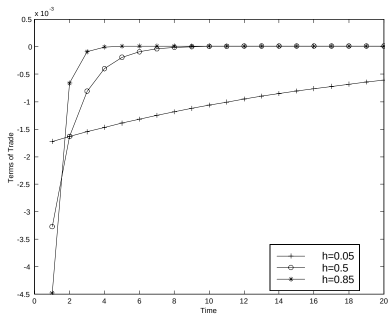


iv) Interest Rate Spread

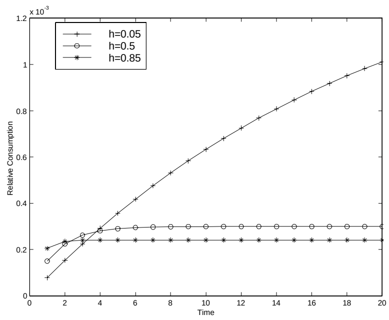
Figure 6c.
1 Percent Fully Permanent Home Monetary Expansion.
Sensitivity wrt h.
Dynamics of Nominal Exchange Rate, Terms of Trade, Relative Consumption and Nominal Interest Rate Spread.



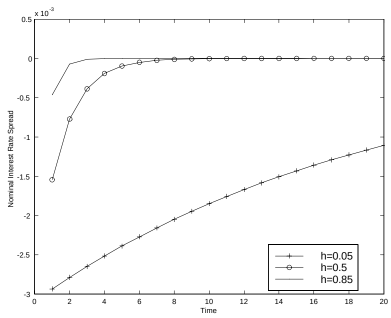
i) Nominal Exchange Rate



ii) Terms of Trade



iii) Relative Consumption



iv) Interest Rate Spread

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