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Negotiation-Proof Nash Equilibrium

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Abstract. This paper defines "negotiation-proof Nash equilibrium", a notion that applies to environments where players can negotiate openly and directly prior to the play of a noncooperative game. It recognizes the possibility that a group of self-interested players may choose, voluntarily and without binding agreement, to coordinate their choice of strategies and make joint objections; moreover, it takes the perfect foresight of rational players fully into account. The merit of the notion of negotiation-proof Nash equilibrium is twofold: (1) It offers a way to rectify the nestedness assumption and myopia embedded in the notion of *coalition-proof Nash equilibrium*. (2) The negotiation process is formalized by a "graph", which serves as a natural extension to the approach that models preplay communication by an extensive game.

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I. Introduction

The most fundamental solution concept for noncooperative games is that of Nash equilibrium. A Nash equilibrium is commonly interpreted as a self-enforcing agreement. That is, if players communicate and agree on a certain profile of strategies without a binding agreement, then these strategies must constitute a Nash equilibrium. But if preplay communication is possible, players should be able to negotiate their agreements. Nash equilibrium, as a noncooperative notion, does not exploit such a role for preplay communication. The objective of this paper is to explore the consequence of open negotiation prior to the play of a noncooperative game and ascertain the Nash equilibria that survive such open negotiation. The notion of "negotiation-proof Nash equilibrium"

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introduced here captures the idea that a group of self-interested players may choose to coordinate, voluntarily and without binding agreement, their choice of strategies; moreover, it takes the perfect foresight of rational players fully into account.

One approach to model preplay communication, following Nash (1951), is to explicitly model the procedure of communication as a dynamic game that specifies how messages are interchanged, the order of offers and counter-offers, etc. [see, e.g., Farrell (1987), Farrell and Saloner (1988), and Rabin (1994)]. The results, however, are sensitive to the exact procedure employed and strong restrictions often have to be made to isolate the desired outcome. Also, one may argue that modelling communication as a noncooperative game does not fully capture the coordinating role of communication, since such an approach precludes the possibility of the "joint choice of strategies". An alternative approach focuses on the possibility that players can coordinate their choice of strategies via agreements that are mutually beneficial, leaving the details of communication un-modelled [see, e.g., Aumann (1959), Bernheim et al. (1987), and Mariotti (1997)]. We shall first motivate our analysis by examining the literature of this second approach and then discuss the contribution of our analysis to the first approach.

One of the most important contributions to the second approach is Bernheim et al.'s (1987) notion of *coalition-proof Nash equilibrium* (CPNE), which "is designed to capture the notion of an efficient self-enforcing agreement for environments with unlimited but nonbinding, preplay communication" (*p*.3). One motivation of CPNE is that the notion of *strong Nash equilibrium* (SNE) fails to capture the fact that a coalitional deviation may be subject to further deviations in the absence of binding agreements. An agreement is coalition-proof if it is efficient within the class of "self-enforcing" agreements. In turn, an agreement is "self-enforcing" if and only if no *proper subset* of players, taking the strategies of its complement as fixed, can deviate in such a way that benefits all its members. Therefore, in the definition of CPNE, self-enforceability of agreements is restricted to an important aspect: only subsets of a deviating coalition can further deviate. While such a (*nestedness*) restriction enables CPNE to be defined recursively, it also implies that the definition of CPNE may involve agreements that are open to further deviations. Consider the 3-player game in Table 1, where player 1 chooses rows, player 2 chooses columns, and player 3 chooses matrices.

This game has two Nash equilibria (in pure strategies), (U, L, A) and (D, R, B), but only (D, R, B) is coalition-proof. (U, L, A) is not coalition-proof by the following argument: Players 1 and 2 can jointly deviate to (D, R, A) which renders both players 1 and 2 higher payoffs. Such a deviation is "self-enforcing" because, according to the

Table 1

	L	R
U	2,2,1	1,0,0
D	0,0,0	3,3,0

A

	L	R
U	0,0,0	0,2,0
D	0,0,0	1,4,1

B

nestedness restriction in the definition of CPNE, only subsets of $\{1, 2\}$ can further deviate. Without the nestedness restriction, however, the self-enforceability of players 1 and 2's deviation to (D, R, A) is no longer valid because player 3 has incentive to further deviate to (D, R, B) and such a deviation is obviously self-enforcing.

Aside from being criticised for its nestedness restriction, the definition of CPNE also fails to account for the foresight of rational players, as noticed by Chwe (1994). The myopia embedded in the definition of CPNE can be illustrated by the following example taken from Chwe (1994).

Table 2

	L	C	R
U	1,1,1	0,0,0	0,0,0
M	0,0,0	0,0,0	0,0,0
D	0,0,0	0,0,0	0,0,0

A

	L	C	R
U	0,5,0	0,0,0	4,4,0
M	0,0,0	2,2,2	0,0,0
D	0,0,0	0,0,0	3,3,0

B

For the game in Table 2, the unique CPNE is (M, C, B). Although (D, R, B) renders both players 1 and 2 higher payoffs than (M, C, B) does, players 1 and 2 will not jointly deviate to (D, R, B). According to the definition of CPNE, such a joint deviation is not self-enforcing, the reason being that player 1 can subsequently deviate to (U, R, B), a "self-enforcing agreement" under the nestedness assumption. But this implies, evidently, that players 1 and 2 are myopic: were they farsighted, their joint deviation to (D, R, B) should be encouraged, not deterred, by player 1's further deviation to (U, R, B).

This paper offers a model of preplay communication that overcomes the difficulties of CPNE as illustrated through the examples in Tables 1 and 2. The proposed notion, "negotiation-proof Nash equilibrium", exploits open nonbinding negotiation that takes place prior to the play of an one-shot noncooperative game. The preplay negotiation is conducted as follows: Suppose a strategy profile is considered by all the players. A coalition[1] of players *can* make a joint objection by announcing *openly*, "if the rest of you stick with your strategies, we shall adopt new strategies so-and-so". This objection is simply a declaration of joint intention or a joint "contingent threat" that comprises no binding power. Given the new, revised strategy profile, another coalition, not necessarily a subset of the original objecting coalition, *can* make a further objection by announcing *openly* the new strategies its members will adopt contingent on the strategies of non-members. The process continues in this manner, until no coalition has an incentive to make any further objection. Since players are rational and hence farsighted, at any stage of the negotiation, a coalition always consider the *ultimate* consequences of its action in deciding whether or not to make an objection or what objection to make. Moreover, given that binding agreements are not possible, if players reach an agreement, it has to be a Nash equilibrium.

The above preplay negotiation process takes after the "coalitional contingent threat situation" (Greenberg 1990) but for the following two distinct features:

(i) The preplay negotiation is followed by an *one-shot noncooperative game*, hence a meaningful agreement must be self-enforcing, i.e., it must be a Nash equilibrium;

(ii) In the preplay negotiation, players are farsighted and they do not simply look only at the next step. What matters to farsighted players is the final agreements that their actions will lead to; hence they may, for example, strategically "deviate" to an agreement, which is not necessarily a Nash equilibrium, in order to induce a final agreement (which is necessarily a Nash equilibrium) that benefits all its members. The negotiation process is modelled as a "graph", which fully captures the perfect foresight of the players.

Loosely speaking, a Nash equilibrium is negotiation-proof if and only if no coalition can make an objection to it in such a way that its objection will *ultimately* lead to another negotiation-proof Nash equilibrium that benefits all its members. Thus, a Nash equilibrium can be and can only be ruled out from the "solution" (the set of negotiation-proof Nash equilibria in this case) by another Nash equilibrium which itself is in the solition. Such a definition is intrinsically "circular"[2], and is achieved by employing von Neumann and Morgenstern's (1944) "abstract stable set", a framework that has been

[1] A coalition is a nonempty subset of players.

[2] Recall that the original recursive definition of CPNE was possible because of the nestedness restriction.

used in Greenberg (1989) to offer an alternative definition of CPNE (see also Section 5.1) and in Greenberg (1990) to derive several other solution concepts.

In the above preplay negotiation, it is *feasible* that any coalition can form and object to any strategy profile but a rational and self-interested player is not bounded to join any coalition. The formation of any coalition is voluntary and is driven by each member's pursuit of his own interest; for a group of *rational* players to form a coalition, it must be the case that no members have incentive to quit this coalition. Thus, our negotiation process captures the intrinsic "noncooperative behavior" of the players. In Table 1, for example, it is *feasible* for players 1 and 2 to form a coalition and jointly "deviate" from (U, L, A) to (D, R, A). But, player 1, being *rational* and hence *farsighted*, will not join player 2 to make such a deviation, knowing that player 3 (or players 2 and 3) will subsequently deviate to (D, R, B). Note that it is not essential who can make a proposal that players 1 and 2 form a coalition to jointly deviate from (U, L, A) to (D, R, A): player 1 will neither offer nor accept such a proposal.

While rationality can single out the coalitions that will not form to make objections to a given strategy profile, it may not pin down the exact coalition that will form. However, it is not necessary to do so in defining negotiation-proof Nash equilibrium (NPNE). Indeed, a strategy profile x is not negotiation-proof as long as *at least one* coalition of rational players will ultimately benefit by objecting to x. In defining NPNE, we take the view that "it is the possibilities for coalition forming, promising and threatening that are decisive, rather than whose turn it is to speak" (Aumann 1987). Our negotiation process allows the players to negotiate openly and directly, and to exercise their "bargaining power" embedded in the structure of the game, in particular, the intrinsic properties of payoffs. As discussed earlier, some models of preplay communication impose procedures that can be represented by extensive form games. For example, in Rabin's (1994) [see also Farrell (1987) and Farrell and Saloner (1988)] model of preplay communication (for two-player games), players make repeated simultaneous proposals of equilibria; if the players propose the same equilibrium, they have an agreement to play that equilibrium.[3] The preplay negotiation process postulated in this paper may be viewed as a natural extension of these models.

In the next section, the preplay negotiation process among rational (and farsighted) players is formalized as a "(directed) graph", which need not be acyclic and does not stipulate that each "node" belong to a single player. (This is in contrast to an extensive form game, which is an acyclic graph and requires that each node belong to

[3]Such a procedure is "at variance with common procedure" [see Rabin (1994, *p*.389)].

a single player.) Such a graph allows us to model the fact that when a strategy profile x is under consideration, any coalition can make an objection, and each coalition, in deciding whether to make an objection and what objection to make, considers the final outcome its actions will lead to. Therefore, we can study the consequence of coalition formation without imposing a rigid procedure under which coalitions form.

The rest of this paper is organized as follows: In the next section, following the formalization of the negotiation process, a formal definition of "negotiation-proof Nash equilibrium" is offered using von Neumann and Morgenstern abstract stable set. Section III provides a way to improve the notion of negotiation-proof Nash equilibrium. Section IV extends negotiation-proofness to dynamic games. Section V offers a brief discussion of several attempts in the literature to relax the nestedness restriction of CPNE and to capture the foresight of players in strategic settings. It also discusses the underlying assumptions of abstract stable set used in our definition of negotiation-proof Nash equilibrium. Finally, it briefly discusses the possibility of allowing for correlated strategies.

II. Negotiation-Proof Nash Equilibrium

Consider a strategic game $\mathcal{G} \equiv (N, \{Z_i\}_{i\in N}, \{u_i\}_{i\in N})$, where N is the set of players and for every $i \in N$, Z_i is the set of strategies of player i and u_i is the payoff function of player i with $u_i \colon Z \to \Re$, where $Z \equiv \prod_{i\in N} Z_i$. For $S \subset N$, let $Z_S \equiv \prod_{i\in S} Z_i$, and for all $x, y \in Z$, write $x \prec_S y$ if $u_i(x) < u_i(y)$ for all $i \in S$.

Assume that $x \in Z$ is under consideration. As discussed in the introduction, it is feasible for any coalition to form and to object to x. If a coalition $S \subset N$ objects to x by choosing $y_S \in Z_S$ contingent on $x_{N\setminus S}$, then the resulting new strategy profile is $y = (y_S, x_{N\setminus S})$; in this case, write $x \to_S y$ to denote "S objects to x via y". Thus, for all $S \subset N$, $\to_S$ is a (binary) relation on Z that specifies what S can do *if and when* it forms. Given that players are rational and hence farsighted, if a coalition S forms and objects to x via y, it must be the case that

(C) such an objection will lead to a final agreement z that benefits all members of S.

Recall the example in Table 1. Starting from (U, L, A), coalition $\{1, 2\}$ does not form for the exact reason that (C) is violated. The following example illustrates foresight when condition (C) holds.

Table 3

	L	R
U	2,2,2	0,0,1
D	0,0,1	2,2,0

A

	L	R
U	0,0,0	1,1,1
D	0,0,0	3,4,1

B

Both (U, L, A) and (D, R, B) are CPNE's. In fact, they are also SNE's. We shall argue, however, that players 1 and 2, being farsighted (as implied by rationality), will jointly deviate from (U, L, A) to (D, R, A), because player 3, for his own interest, will subsequently deviate to (D, R, B), which renders both players 1 and 2 higher payoffs than (U, L, A). Note that players 1 and 2 do not (strictly) prefer (D, R, A) to (U, L, A).[4] Thus, in contemplating a deviation, a coalition of farsighted players considers the *final* agreement its deviation leads to, as asserted by condition (C). Note that the joint deviation of players 1 and 2 from (U, L, A) to (D, R, A) is self-enforcing: neither player 1 nor player 2 has an incentive to object to such an agreement, knowing that the joint deviation leads to (D, R, B). Again, it is not essential who proposes this agreement; it is the existence of such an agreement that invalidates (U, L, A).

As condition (C) asserts, it is not sufficient for coalition $\{1, 2\}$ to form and make a joint objection that can *feasiblely* lead to a final agreement that makes both players 1 and 2 better off. Further deviations along the way to the final agreement that players 1 and 2 hope to reach, may result in an agreement that makes player 1 or 2 worse off. Consider the following example, which is a modification of Table 3.

Table 4

	L	R
U	2,2,2	0,0,1
M	2,0,1	4,0,1
D	0,0,1	2,2,0

A

	L	R
U	0,0,0	1,1,1
M	0,0,2	0,2,0
D	0,0,0	3,4,1

B

It is still the case that players 1 and 2's joint deviation from (U, L, A) to (D, R, A) can *feasiblely* lead to (D, R, B). But, given that players are rational, will player 2 join player 1 to deviate from (U, L, A) to (D, R, A) in the hope that player 3 will

[4]Note also that (D, R, A) is not a Nash equilibrium.

subsequently deviate to (D, R, B)? The answer is no. If (D, R, A) is reached, player 1 has incentive to engage a preemptive deviation to (M, R, A), which player 3 has no incentive to forestall; as a result, (M, R, A) will prevail. Thus, although it is feasible for players 1 and 2 to jointly deviate from (U, L, A) to (D, R, A), player 2, being self-interested and farsighted, will not form a coalition with player 1 to engage such a deviation.

The above two examples illustrate that a coalition of rational and farsighted players, in contemplating its objections, considers all further objections and recognizes the other players are also rational and farsighted; it will "strategically" object to an agreement *if and only if* such an objection *ultimately* leads to a final agreement that makes its members better off. A coalition also recognizes that for its objection to lead to a final agreement, it must be the case that no coalition has incentive to prevent this final agreement from being reached (by deviating along the way to this final agreement). Furthermore, a coalition does not compare such a final outcome only with the status quo. Recall the example in Table 4. If (D, R, A) is reached and player 1 contemplates a deviation to (M, R, A), he compares (M, R, A) with (D, R, B), which would arise were he not to act. These aspects of perfect foresight are captured by viewing the succession or "path" of objections that bring about a final agreement as a whole. Consider the following path in both Tables 3 and 4: (U, L, A) is objected to by the coalition of players 1 and 2 via (D, R, A), which is subsequently objected to by player 3 via (D, R, B). This path "prevails" in the example depicted by Table 3 because no coalition of rational and farsighted players will have incentive to deviate from such a path; that is, players 1 and 2's joint deviation from (U, L, A) to (D, R, A) will lead to (D, R, B). The same path, however, does not prevail in the example depicted by Table 4.

Thus, the preplay negotiation process can be represented by a (*directed*) *graph* that consists of the set of *vertices* (*nodes*) Z and a collection of *arcs* where for every $a, b \in Z$, ab is an arc if and only if there exists $S \subset N$ such that $a \rightarrow_S b$.[5] Assume that some $y \in Z$ can replace x through a succession of deviations and, at every "stage", the "deviating" coalition prefers y to the agreement it faces at this stage.[6] This succession of deviations that replace x with y is called a "path" (of deviations) from x to y. Formally,

Definition 1. A path from $x \in Z$ is a sequence of strategy profiles $(x^0, x^1, \ldots, x^m)$ in

[5]Obviously, $a \rightarrow_S a$ for every $a \in Z$ and $S \subset N$. Thus, "inaction" or "waiting" is accommodated.

[6]The latter condition implies that we restrict our attention to those "paths" that can possibly be followed by rational players.

Z, where $x^0 = x$, such that there exist coalitions $S^0, S^1, \ldots, S^{m-1}$ and $x^j \rightarrow_{S^j} x^{j+1}$ and $x^j \prec_{S^j} x^m$, for all $j = 0, 1, \ldots, m-1$.

For a strategic game $\mathcal{G}$, let Π denote the union of the set of all paths and Z, the set of all "degenerate paths". For $\alpha \in \Pi$, let $f(\alpha)$ denote the final "node" (strategy profile) that lies on path α, i.e., $f(\alpha) = x^m$, and if x is strategy profile that lies on α, write (with slight abuse of notation) $x \in \alpha$. For $\alpha, \beta \in \Pi$, if $f(\alpha) \prec_S f(\beta)$ for some $S \subset N$, write $\alpha \prec_S \beta$.

As discussed in the introduction, the open negotiation is followed by the play of a noncooperative game; hence a meaningful agreement must belong to the set of Nash equilibria (self-enforcing agreements). Therefore, only those paths that lead to Nash equilibria are of interest. Let NE denote the set of Nash equilibria of $\mathcal{G}$ and let $\Pi_{NE} \equiv \{\alpha \in \Pi \mid f(\alpha) \in NE\}$. In order to determine whether a path $\alpha \in \Pi_{NE}$ will prevail, deviations along α have to be considered. For any $x \in \alpha$, if a coalition can initiate another path β that makes its members better off than α does, then α is said to be "dominated" by β. That is,

Definition 2. For $\alpha \in \Pi_{NE}$ and $\beta \in \Pi_{NE}$, α is dominated by β, denoted $\alpha < \beta$, if there exist $S \subset N$, $x \in \alpha$ and $y \in \beta$ such that $x \rightarrow_S y$ and $\alpha \prec_S \beta$.[7]

β itself may be dominated by another path, say, γ. Thus whether α will prevail depends whether β will prevail; whether β will prevail depends, in turn, on whether γ will prevail; and so on. We wish to identify a set of paths $\Sigma \subset \Pi_{NE}$ such that it contains *those and only those paths* that are not ruled out by any coalition of rational players, whose members are aware of and believe in the specification of such Σ. That is, Σ is both consistent and self-policing; moreover, it "justifies" every path it excludes. This is precisely the intuition behind the von Neumann and Morgenstern abstract stable set. Recall,

Definition 3. An *abstract system* is a pair $(D, \angle)$ where D is an arbitrary nonempty set and $\angle$ is a binary relation, called the *dominance relation*, on D such that $a \angle b$ means that a is dominated by b. $K \subset D$ is *internally stable* if K is free of inner contradictions, i.e., there do not exist $a, b \in K$, such that $a \angle b$, and it is *externally stable* if K accounts for every element it excludes, i.e., if $a \in D \setminus K$, then there exists

[7] For the example in Table 3, let $\alpha \equiv ((U, L, A), (D, R, A), (D, R, B))$ (i.e., path α consists of players 1 and 2's deviation from (U, L, A) to (D, R, A) and player 3's further deviation to (D, R, B)) and $\beta \equiv ((D, R, A),\ (M, R, A))$; then, $\alpha < \beta$.

$b \in K$ such that $a \angle b$. K is an *abstract stable set* if K is both internally and externally stable.

Let Σ be an abstract stable set for $(\Pi_{NE}, <)$, then it contains those paths that are to prevail in the preplay negotiation, once Σ becomes common knowledge. Note that the definition of Σ takes the noncooperative behavior of self-interested players fully into consideration. If a path α in Σ involves any coalition, it implies that members of this coalition recognize the interdependence of their welfare and choose to coordinate their choice of strategies. Should some player find it not in his best interest to join such a coalition, α would have been ruled out from Σ. Consider, again, the path where players 1 and 2 deviate from (U, L, A) to (D, R, A) and player 3 subsequently deviates to (D, R, B) in the examples in both Tables 3 and 4. This path belongs to the unique abstract stable set for $(\Pi_{NE}, <)$ associated with the example in Table 3, implying that players 1 and 2 will form a coalition if (U, L, A) is under consideration. The same path, however, does not belong to the unique[8] abstract stable set for $(\Pi_{NE}, <)$ associated with Table 4, because player 2 knows that once (D, R, A) is reached, player 1 will deviate to (M, R, A) (which belongs to the abstract stable set); hence player 2 will not form a coalition with player 1 when (U, L, A) is under consideration. If Σ is an abstract stable set for $(\Pi_{NE}, <)$ then it is "dynamically consistent". That is, every "stable path" in Σ satisfies the "truncation property": the continuation of a "stable path" at any stage along the path is stable itself. Formally,

Lemma 1. *Assume that Σ is an abstract stable set for $(\Pi_{NE}, <)$ and that $\alpha \in \Sigma$. Then, $\alpha|_x \in \Sigma$ for all $x \in \alpha$, where $\alpha|_x$ is the continuation of α from x.*

Proof of Lemma 1. Assume in negation that $\alpha \in \Sigma$, but $\alpha|_x \notin \Sigma$ for some $x \in \alpha$. By external stability of Σ, there exists $\beta \in \Sigma$ such that $\alpha|_x < \beta$. By Definition 2, $\alpha < \beta$, contradicting the internal stability of Σ. ∎

If a Nash equilibrium x is not objected to by any coalition who believes in the specification of an abstract stable set Σ, then x is said to be "negotiation-proof".

Definition 4. Let Σ be an abstract stable set for the abstract system $(\Pi_{NE}, <)$ associated with a strategic game; then the set of Negotiation-Proof Nash Equilibria (NPNE's)

[8] To show the uniqueness of the abstract stable set, first note that both (M, R, A) and (D, R, B) belong to every stable set by external stability because they are not dominated. It is easy to see that from (D, R, A), the path to (M, R, A) is in a stable set while the path to (D, R, B) is not. Then, evidently, no joint deviation of players 1 and 2 form (U, L, A) can lead to (D, R, B). It follows that (U, L, A) also belongs to every stable set. The rest of the construction is straight-forward.

relative to Σ is given by

$$Q_\Sigma \equiv \{x \in NE \mid x \in \Sigma\} \equiv \{x \in Z \mid \exists \alpha \in \Sigma \text{ such that } x = f(\alpha)\}.$$

If Σ is an abstract stable set for $(\Pi_{NE}, <)$, then Q_Σ is nonempty (by the external stability of Σ). Q_Σ contains *those and only those* self-enforcing agreements from which no coalition can initiate such a deviation that will ultimately lead to some self-enforcing agreement in Q_Σ that benefits all its members. Consequently, if x belongs to Q_Σ, then *no* coalition (or player) *will* ultimately benefit by objecting to x; hence rational players will not form a coalition to object to x, although it is feasible for them to do so. Note that the procedure under which a coalition forms is not essential here, because no player will either offer or accept a proposal of forming a coalition to object to x. On the other hand, if x does not belong to Q_Σ, then it must be the case that *at least one* coalition *will* ultimately benefit by objecting to x. In this case, at least one coalition of rational players will form. If more than one coalitions of rational players have incentive to form to object to x, we cannot pin down exactly which coalition will actually form, particularly given that our negotiation process does not impose the order in which objections are made. However, as far as the notion of NPNE is concerned, such an order is inessential because whichever rational coalition forms, it will object to x and hence disqualify x as an NPNE.

For the game in Table 1, both (U, L, A) and (D, R, B) are negotiation-proof. For the game in Table 2, although the unique NPNE is (M, C, B), which coincides with the unique CPNE, the underlying logic is very different: In CPNE, players 1 and 2 will not deviate to (D, R, B) because of both the nestedness restriction and myopia embedded in the definition of CPNE as discussed in the introduction. According to the definition of NPNE, however, players 1 and 2 will not deviate to (D, R, B) because such a deviation cannot eventually benefit them.[9] For the example in Table 3, (D, R, B) is the unique NPNE, which "refines" CPNE and SNE. The set of NPNE's of the game in Table 4 comprises (U, L, A), (M, R, A), and (D, R, B).

Following von Neumann and Morgenstern (1944), the dominance relation $<$ on Π_{NE} is said to be *strictly acyclic* if there does not exist an infinite sequence of paths $\alpha^1, \alpha^2, \ldots$ in Π_{NE} such that $\alpha^j < \alpha^{j+1}$ for all $j = 1, 2 \ldots$.

Proposition 2. *If NE is nonempty and $<$ on Π_{NE} is strictly acyclic, then, the set of NPNE's of a strategic game is uniquely defined and nonempty.*

[9] Indeed, at (D, R, B), only paths leading to (M, C, B) are in the unique stable set.

Proof of Proposition 2. Since $<$ is strictly acyclic, by a theorem of von Neumann and Morgenstern (1944, 65:Y, *p.* 601), $(\Pi_{NE}, <)$ admits a unique abstract stable set Σ. By external stability, for every $\alpha \in \Pi_{NE} \setminus \Sigma$, there exists $\beta \in \Sigma$ such that $\alpha \angle \beta$; hence $\Sigma \neq \emptyset$. Therefore, the set of NPNE's is nonempty and uniquely defined. ∎

The examples in Tables 1 through 4 all satisfy the condition in Proposition 2.[10]

Corollary 3. *If NE is finite and all Nash equilibria can be weakly Pareto-ranked, then, the set of NPNE's of a strategic game is uniquely defined and nonempty. Moreover, if a strategic game has a unique Pareto efficient Nash equilibrium (within NE), then it is the unique NPNE.*

Proof of Corollary 3. Since NE is finite and the set of Nash equilibria can be weakly Pareto ranked, $<$ is strictly acyclic. Then, it follows from Proposition 2 that the set of NPNE's is uniquely defined and nonempty.

Let x be the Pareto efficient Nash equilibrium within NE and $\Sigma \equiv \{\alpha \in \Pi_{NE} \mid f(\alpha) = x\}$. Then, $\beta \in \Pi_{NE} \setminus \Sigma$ if and only if $f(\beta) \prec_N x$. Since $f(\beta) \rightarrow_N x$, $\beta < x$. Therefore, $\beta \in \Pi_{NE} \setminus \Sigma$ if and only if $\beta < x$. But $x \in \Sigma$. Hence Σ is stable for $(\Pi_{NE}, <)$. Uniqueness follows from the fact that Σ must be contained in any stable set, since for all $\alpha \in \Sigma$, there does not exists $\beta \in \Pi_{NE}$ such that $\alpha < \beta$. ∎

Thus, for games with common interests and coordination games, preplay negotiation achieves full efficiency; and if a game has a unique Nash equilibrium (for example, the Cournot oligopoly model), then it is also the unique NPNE. The property of NPNE in Corollary 3 is not shared by CPNE. It is easy to verify that the game in Table 5 does not admit a CPNE or an SNE. But the unique NPNE is (U, L, A).[11]

Table 5

	L	R
U	2,2,2	0,0,0
D	0,0,0	3,3,0

A

	L	R
U	2,2,0	0,0,0
D	0,0,0	1,1,1

B

[10] It is easy to verify the absence of cycles of dominance in these finite games.

[11] This game has two Nash equilibria, (U, L, A) and (D, R, B). (U, L, A) is not a strong Nash equilibrium because the coalition of players 1 and 2 has incentive to jointly deviate to (D, R, A). Moreover, such a joint deviation is self-enforcing under the nestedness assumption; hence (U, L, A) is also not coalition-proof. Similar argument goes for (D, R, B).

NPNE may differ from CPNE or SNE even for two-player games.

TABLE 6

	L	R
U	2,2	0,0
D	0,0	1,2

Both (U, L) and (D, R) are CPNE's and SNE's. However, the unique NPNE is (U, L): player 1, being farsighted, will object to (D, R) via (U, R), expecting (U, L) to prevail. For generic 2-player games, however, we have the following result.

Lemma 4. *For a generic strategic game with 2 players, every SNE is an NPNE.*

Proof of Lemma 4. Let $x = (x_1, x_2)$ be an SNE of a generic game with 2 players. Define $\Sigma \equiv \{\alpha \in \Pi_{NE} \mid f(\alpha) = x\}$. We shall show that Σ is a stable set. Internal stability of Σ is trivially satisfied by definition. To show external stability, let $\beta \in \Pi_{NE} \setminus \Sigma$ and assume that $y = (y_1, y_2) = f(\beta)$. Since x is an SNE and the game is generic, $u_i(y) < u_i(x)$ from some i. Assume, without loss of generality, that $i = 1$. Let $z = (x_1, y_2)$. Since the game is generic, $u_2(z) < u_2(x)$. Thus, (z, x) is a path and belongs to Σ. Note that $y \rightarrow_{\{1\}} z$ and $u_1(y) < u_1(x)$; then it follows that $\beta < (z, x)$ and hence Σ is externally stable. ∎

The converse of Lemma 4 is not true; an immediate example is a generic game resembling prisoner's dilemma where the unique Nash equilibrium is an NPNE but not an SNE. For generic strategic games with more than 2 players, an SNE need not be an NPNE and *vice versa*. Examples can be easily constructed based on Tables 3 and 1, respectively.

III. WEAKLY NEGOTIATION-PROOF NASH EQUILIBRIUM

The game in Table 6 illustrates that the foresight of rational players enables NPNE to provide "sharp prediction". However, the dominance relation $<$ may endow a deviating coalition (or player) with too much "power". To illustrate this, consider the familiar "battle of the sexes" game in Table 7.

TABLE 7

	L	R
U	2,1	0,0
D	0,0	1,2

In this case, paths $\alpha \equiv ((U,R),(D,R))$ and $\beta \equiv ((U,R),(U,L))$ dominate each other. Therefore, for $\Sigma \subset \Pi_{NE}$ to be (internally) stable, either α or β must be excluded from Σ. Indeed, $(\Pi_{NE}, <)$ admits two distinct stable sets: one stable set Σ^1 rules out α and the other Σ^2 rules out β. Consequently, (U,L) is an NPNE according to Σ^1 and (D,R) is an NPNE according to Σ^2. The exclusion of one path, say α from Σ^1, is attributed to that β belongs to Σ^1 and α is dominated by β. Note, however, that β itself is also dominated by α. Therefore, it does not seem sound to rule out one path based on another if these two paths dominate each other. For this reason, we define a "stronger" dominance relation $\ll$ based on $<$: $\alpha \ll \beta$ if $\alpha < \beta$ and $\beta \not< \alpha$. Since farsighted players look arbitrarily many steps ahead, the dominance relation $\ll$ relation can be generalized as follows.[12]

Definition 5. For $\alpha, \beta \in \Pi$, $\alpha \ll \beta$ if

(1) $\alpha < \beta$, and

(2) there do not exist $\beta^0, \beta^1, \ldots, \beta^m$ in Π_{NE}, where $\beta^0 = \beta$ and $\beta^m = \alpha$, such that for $j = 0, 1, \ldots, m-1$, $\beta^j < \beta^{j+1}$.

Using $\ll$ we can define the notion of "Weakly Negotiation-Proof Nash Equilibrium (WNPNE)" as follows.

Definition 6. Let Σ be an abstract stable set for the abstract system $(\Pi_{NE}, \ll)$ associated with a strategic game; then the set of WNPNE's relative to Σ is given by

$$W_\Sigma \equiv \{x \in NE \mid x \in \Sigma\} \equiv \{x \in Z \mid \exists \alpha \in \Sigma \text{ such that } x = f(\alpha)\}.$$

The abstract stable set of $(\Pi_{NE}, \ll)$ coincides with that of $(\Pi_{NE}, <)$ for each of the games in Tables 1 to 6. However, for the game in Table 7, $(\Pi_{NE}, \ll)$ admits a *unique* abstract stable set that includes both α and β; in fact it also includes both $\alpha' \equiv ((D,L),(D,R))$ and $\beta' \equiv ((D,L),(U,L))$. At (U,R) both players have incentive to wait, while at (D,L) both players have incentive to preempt. In each case, rationality alone cannot predict the exact outcome. In the presence of such a tension, WNPNE is permissive or inclusive because α and β (or α' and β') are equally "rational"; hence one path cannot be used to rule out the other. The set of WNPNE's $W_\Sigma = \{(U,L),(D,R)\}$ is uniquely defined for the example in Table 7. In contrast, NPNE requires a much

[12] Similar approach appears, for example, in Bernheim and Ray (1989)'s notion of "consistent set".

stronger internal consistency. Indeed, an abstract stable set of $(\Pi_{NE}, \angle)$ needs to rule out, for example, either α or β, thereby resulting in an asymmetric treatment of α and β.

WNPNE may exist when NPNE, CPNE, and SNE fail to exist. Consider the following example.

TABLE 8

	L	R
U	1,2,3	0,0,0
D	0,0,0	2,3,1

A

	L	R
U	0,0,0	3,1,2
D	0,0,0	0,0,0

B

The game does not admit a CPNE, an NPNE, or an SNE; however, there exists a unique stable set for $(\Pi_{NE}, \ll)$, giving rise to three WNPNE's: $(U, L, A,), (D, R, A)$, and (U, R, B). The implication of the examples in Tables 7 and 8 is that preplay negotiation cannot pin down the exact equilibrium to be played in these games.[13]

Proposition 5. *For a finite strategic game, the set of WNPNE's is nonempty and uniquely defined.*

Proof of Proposition 5. We need only to show that $(\Pi_{NE}, \ll)$ admits a unique abstract stable set. By a theorem of von Neumann and Morgenstern (1944, 65:Y, *p.* 601), it suffices to show that $\ll$ is strictly acyclic. That is, there does not exist an infinite sequence of paths $\alpha^1, \alpha^2, \ldots$ in Π_{NE} such that $\alpha^j \ll \alpha^{j+1}$ for all $j = 1, 2 \ldots$. Indeed, let $\alpha^1, \alpha^2, \ldots$ be a sequence of paths in Π_{NE} such that $\alpha^j \ll \alpha^{j+1}$ for all $j = 1, 2 \ldots$. We claim that $i < j$ implies that $\alpha^i \neq \alpha^j$. Otherwise, $\alpha^i \ll \alpha^{i+1} \ll \cdots \ll \alpha^j = \alpha^i$; hence $\alpha^i < \alpha^{i+1} < \cdots < \alpha^j = \alpha^i$. Then, by Definition 5, $\alpha^i \not\ll \alpha^{i+1}$, thereby yielding a contradiction. Π_{NE} is finite since NE is finite. Thus, $\alpha^1, \alpha^2, \ldots$ must be a finite sequence and hence $\ll$ is strictly acyclic. ∎

IV. EXTENSIVE GAMES

Although the primary concern of this paper is strategic games, the notions of NPNE and WNPNE can also be extended to dynamic games. We shall focus on the extension of WNPNE in this section. In doing so, we have to be explicit about whether there is on-going open negotiation as the game unfolds. In the absence of on-going negotiation, players negotiate openly only before they engage in an extensive game and will

[13]Such is the case whenever a game has multiple (weakly) negotiation-proof equilibria.

not have the opportunity to meet again once the game starts. In this case, we need only to consider negotiation-proof agreements. If on-going negotiation is exercised, then players negotiate prior to the start of every subgame; that is, players *renegotiate after every history of play.* In this case, agreements have to be "renegotiation-proof". Such a distinction is important particularly from the view point of a single player. Renegotiation-proofness entails that every player, in contemplating a deviation, is *certain* that all players *will* meet and renegotiate after his deviation; in fact, he believes negotiation will occur after any deviation by any player (or coalition) in any future period. If for whatever reason a player is uncertain whether renegotiation will take place after a unilateral deviation and is averse to such an uncertainty, then negotiation-proofness may well be relevant.[14]

Negotiation-proofness for extensive games can be defined in the same fashion as negotiation-proofness for strategic games except that for extensive games the only "meaningful" agreements are subgame perfect equilibria. Let SPE denote the set of subgame perfect equilibria of an extensive game and let $\Pi_{SPE} \equiv \{\alpha \in \Pi \mid f(\alpha) \in SPE\}$, where Π is the set of paths defined on the normal form representation of this game.

Definition 7. Let Σ be an abstract stable set for the abstract system $(\Pi_{SPE}, \ll)$ associated with an extensive game; then the set of Weakly Negotiation-Proof Nash Equilibria (WNPNE's) relative to Σ is given by

$$W_\Sigma \equiv \{x \in SPE \mid \exists \alpha \in \Sigma \text{ such that } x = f(\alpha)\}.$$

For a finite extensive game, the set of WNPNE's is uniquely defined, following from Proposition 5. Now consider the following game, taken from Bernheim et al. (1987), repeated twice without discounting.

TABLE 9

	L	C	R
U	5,5	0,6	0,0
M	6,0	4,4	0,0
D	0,0	0,0	2,2

[14] In the context of repeated games, the consideration that renegotiation might not take place after every history appears, for example, in Pearce (1987), Bergin and MacLeod (1993), and Xue (1999).

There exists a unique WNPNE: In the first period, players choose (U, L); the second period play is (D, R) if any player deviates in the first period and (M, C) otherwise. The equilibrium payoffs are $(9, 9)$.

Renegotiation-proofness entails that renegotiation precedes *every* subgame. For extensive games with finite number of stages, we can use a simple recursive definition as in, for example, Bernheim and Ray (1989) and Ferreira (1996).

Definition 8.

(1) For a single stage game, a strategy profile x is renegotiation-proof if and only if it is a WNPNE.

(2) Let $t > 1$. Assume that renegotiation-proof equilibrium has been defined for all games with less than t stages. Then for any game with t stages, a strategy profile x is renegotiation-proof if and only if x is a WNPNE for $\bar{\mathcal{G}} \equiv (N, \bar{Z}, \{u_i\}_{i \in N})$, where the set of strategy profiles is given by

$$\bar{Z} = \{x \mid \text{the restriction of } x \text{ to any proper subgame of } \mathcal{G} \\ \text{constitutes a WNPNE for that subgame}\}.$$

For the example in Table 9, the unique WNPNE is not renegotiation-proof: player 1, say, will deviate in the first period by playing M, being certain that in the second period player 2 will join him to renegotiate and abandon the punishment equilibrium (D, R) for (M, C).[15] The unique renegotiation-proof equilibrium is to repeat (M, C), which yields payoffs of $(8, 8)$.

Renegotiation-proof equilibrium exists for finite games.

Proposition 6. *For a finite game in extensive form, there exists a renegotiation-proof equilibrium. Moreover, every negotiation-proof equilibrium is subgame perfect.*

Proof of Proposition 6. For a finite single stage game, renegotiation-proof equilibrium exists because WNPNE exists. For a finte game with more than one stages, recursively applying Proposition 5 yields the existence of renegotiation-proof equilibrium, because in Definition 8, $\bar{\mathcal{G}}$ is a finte strategic game. The second assertion follows from Definition 8 and the "one-stage deviation principle" (note that the recursive definition resembles backward induction). ∎

[15] If the game is repeated more than twice, each player is certain that players 1 and 2 will renegotiate *every time* he or his opponent deviates.

V. Discussion

5.1 CPNE and the Nestedness Restriction

One of our motivations to define NPNE and WNPNE is to resolve the nestedness restriction and the myopia embedded in the definition of CPNE. We first discuss briefly several notions in the literature that attempt to relax the nestedness restriction.

Recall, first, the following definition of CPNE using von Neumann and Morgenstern abstract stable set (Greenberg 1989 and 1990). For a strategic game $\mathcal{G}$, let

$$D \equiv \{(S, x) \mid S \subset N \text{ and } x \in Z\},$$

and for (S, x) and (T, y) in D,

$$(S, x) \angle\ (T, y) \iff T \subset S, x_{N\setminus T} = y_{N\setminus T}, \text{ and } x \prec_S y.$$

Theorem 7 (Greenberg 1989). *Let K be an abstract stable set for $(D, \angle)$. Then the set of CPNE's is given by $\{x \mid (N, x) \in K\}$.*

The nestedness restriction is evident in the above definition. This (nestedness) restriction can be relaxed in several ways, depending on whether the agreements of a deviating coalition are common knowledge [see Greenberg (1994)]. In the "coalitional contingent threat situation" (Greenberg 1990), each deviation is made publicly (and is hence common knowledge) and further deviations are not restricted to subcoalitions. This negotiation process is delineated by a dominance relation on Z[16] defined as follows.

$$x \angle'\ y \iff \exists S \subset N, \text{ such that } x_{N\setminus S} = y_{N\setminus S}, \text{ and } x \prec_S y.$$

The abstract stable set for the abstract system $(Z, \angle')$ consists of those and only those agreements that players, who may be myopic, can reach in open preplay negotiation. Such an abstract stable set may contain strategy profiles that are not Nash equilibria [see Greenberg (1990)], in which case, it is necessary to enforce, via binding contracts, these agreements, or to assume that a strategic game is not played in such a way that there are publicly observable moves and counter-moves.

Arce M. (1994) argued that "coalition building" often occurs in political situations; that is, new members are added efficiently to an existing coalition so that the final outcome benefits all members of the new coalition. Therefore, the nestedness restriction

[16] Without the nestedness assumption it is sufficient to define the dominance relation on Z.

of CPNE is "inverted". This implies that cooperation becomes possible in prisoner's dilemma, since once a coalition forms, it will never break.

The nestedness assumption can also be relaxed under the assumption that the agreements of a deviating coalition are not common knowledge. Loosely speaking, the negotiation process underlying the definition of CPNE can be viewed as follows: A deviating coalition S, upon reaching an agreement among its members, leaves the scene of negotiation and members of S will never approach non-members. In Chakravorti and Kahn's (1993) definition of "universal coalition-proof equilibrium", a subset of S, say T, is allowed to approach and attract some members that are not in S, say some $Q \subset N \setminus S$, in contemplating further deviations. Since Q is not aware of the previous agreement of S, Chakravorti and Kahn postulated that Q joins T only if any actions of $T \cup Q$ that hurt some member of Q will also hurt some member of T. Moreover, in defining their notion, Chakravorti and Kahn employed semi-stable set (Roth 1976) rather than (abstract) stable set used in this paper.

5.2 Agreements among Farsighted Players

Recent study of agreements among farsighted players in strategic environments can be found, for example, in Chwe (1994), Mariotti (1997) and Xue (1998). In Chwe (1994) and Xue (1998), a strategic game is a special case of the model they analyzed. Chwe (1994) formalized one version of Harsanyi's (1974) "indirect dominance" in an attempt to capture foresight. For a strategic game, a strategy profile y is said to indirectly dominate another strategy profile x if y can be reached from x through a succession of deviations, and at each "stage", the deviating coalition prefers y to the agreement from which it deviates. Thus, this indirect dominance is defined on the set of strategy profiles. Based on such an indirect dominance, Chwe (1994) defined "the largest consistent set (LCS)" and applied it to the negotiation processes underlying the "coalitional contingent threat situation" and CPNE. In both cases, LCS may include agreements that are not Nash equilibria. Moreover, the implicit behavioral assumption [see Xue (1998)] underlying the LCS is different from the one embedded in the notion of abstract stable set that has been used to define NPNE and WNPNE. Chwe (1995) also applied LCS to open preplay negotiation but assumed that players only consider Nash equilibrium strategies in the negotiation; while in this paper a coalition may deviate to an agreement that is not necessarily a Nash equilibrium, as long as such a deviation will eventually lead to some final agreement (necessarily a Nash equilibrium) that benefits all its members. Furthermore, Xue (1998) showed that indirect dominance captures only partial foresight in that it ignores deviations on the way to the final agreements. Xue (1998) offered a formalization of *perfect foresight* by considering the "paths" of deviations. The notions of NPNE and WNPNE are built on this formalization of perfect foresight.

Mariotti's (1997) theory of agreements in strategic games is also based on the bargaining procedure of "coalitional contingent threat situation" (Greenberg, 1990). In analyzing this bargaining procedure, Mariotti treated coalitions just as players in that a coalition chooses a coalitional strategy (that specifies its members' actions in every contingency arising in the bargaining), which incorporates foresight of its members. Given the structure of strategic games, each coalitional strategy profile is associated with a set of payoff vectors; thus, a coalition, in deciding its strategics, needs to compare sets of payoff vectors. Mariotti introduced an explicit assumption (resembling yet different from optimism) to achieve such a comparison. Coalitional equilibrium arises if no coalition has incentive to deviate. A coalitional equilibrium also need not be a Nash equilibrium, necessitating binding agreements or a strategic game played in a way that deviations are public.

5.3 Abstract Stable Set and Behavioral Assumptions

The notion of abstract stable set used in the definition of NPNE builds on dominance relation $<$ that entails optimism on the part of a deviating coalition. Indeed, a path α (of objections) is dominated by another path β as long as some coalition has incentive to initiate an objection to a strategy profile on α via another strategy profile on β that makes its members better off. This implies that this coalition hopes for the best (e.g. β) when deviating from α, although different paths may arise. In addition to this implicit optimism on the part of a deviating coalition, the dominance relation $<$ also assumes that all players be fully aware of the dominated path α. One way to resolve this is to let a coalition compare different objections (deviations), which in turn involves comparing sets of paths induced by different objections. An assumption like that of Mariotti's (1997) need to be introduced in order for such a comparison to be made.

In a general framework, Greenberg (1990) established a formal link between abstract stable set and his optimistic stability. He also introduced conservative stability, assuming that when comparing a single outcome with a set of outcomes, a deviating coalition fears the worst outcome as opposed to hoping for the best one as stipulated by optimistic stability. Conservative stability has been used in both Chwe's (1994) and

Xue's (1998) study of agreements among farsighted players. Such a stability notion can also be applied, in place of abstract stable set, to define NPNE. For simplicity, however, this paper introduces an alternative dominance relation to resolve the optimism embedded in the dominance relation used in the definition of NPNE and offers the notion of WNPNE using this alternative dominance relation.

5.4 Correlated Strategies

CPNE has been extended to allow for correlated strategies. In Moreno and Wooders (1996), for example, a correlation device (or mediator) is available every time a coalition forms, and a coalitional deviation is carried out through such a correlation device. In their notion of "coalition-proof correlated equilibrium", self-enforceability of a deviation resembles that of CPNE. Correlated strategies can also be introduced to the preplay negotiation analyzed in this paper (with slight modifications), and then players bargain to determine which correlated equilibria are negotiation-proof. If a correlated equilibrium is negotiation-proof, then this equilibrium is implemented by the corresponding correlation device that makes a private recommendation to each player.

5.5 Concluding Remarks

To model preplay communication is no doubt a task of great difficulty; this difficulty is only magnified by the restrictive framework of dynamic games. Instead of modelling how messages are interchanged among the players, this paper offers a model of preplay communication in which players negotiate openly and directly. We assume that communication admits the possibility of *coalition formation* in that any group of players can coordinate their choice of strategies, thereby making joint objections in the negotiation. We set aside the details of communication that lead to the formation of a coalition; we assume instead that it is feasible for every coalition to form. The "noncooperative behavior" intrinsic to a noncooperative game is nevertheless captured in our analysis, because, by exploiting the rationality of self-interested players, we can single out the coalitions that will not form among rational players. Moreover, a strategy profile x is not negotiation-proof as long as there exists one coalition of rational and self-interested players who will ultimately benefit by making an objection to x. Thus, it is not necessary to stipulate that only a particular coalition or player can object to x; it is also not necessary to pin down the exact coalition that will form among rational players. As discussed earlier, alternative formulations of negotiation-proofness are of course possible.

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