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Svend Jespersen

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UNIVERSITY OF AARHUS C DENMARK

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 350
8000 AARHUS C - F 89421133 - TELEFAX 86136334

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SCHOOL OF ECONOMICS AND MANAGEMENT - UNIVERSITY OF AARHUS - BUILDING 350
8000 AARHUS C - DENMARK F +4589421133 - TELEFAX +4586136334

# Economic Development without Fisher Separation: "Trickle-up" or "Trickle-down"?*

Svend Jespersen†

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**Abstract**

This paper shows that if development is a process of sectorial change, and agents are credit constrained, then rising inequality is inherent to the initial phases of development. Moreover, wealth inequality is self-enforcing during the initial phases of development, where entrepreneurs are credit constrained. Later wealth inequality becomes self-defeating if entrepreneurs accumulate enough to become non-constrained. We find the conditions under which the long run distribution of wealth is equal, and characterize unequal long run distributions of wealth.

Unlike much related research, this paper does not assume Fisher separation, that is, that utility maximization implies profit maximization.

**JEL classification numbers**: D31, O11, O14, O16.

**Keywords**: Imperfect Credit Markets, Choice of Occupation, Endogenous Inequality, Trickle-Down, The Kuznets Curve, Long Run Distribution of Wealth.

## 1 Introduction

Is rising inequality inherent to early phases of development? Some empirical work finds that this is indeed the case, moreover, much research finds that

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†University of Aarhus, Department of Economics, Universitetsparken, Bygn. 350, DK-8000 Aarhus C. Tel.: +45 8942 1589. Fax: +45 8613 6334.

inequality shows no tendency to decline in later phases of development, as it should according to the Kuznets (1955) hypothesis.[1][2][3]

There are many economic arguments for, why inequality may be detrimental to long run growth,[4] a key issue is therefore: does the long run preserve inequality that arose in the early phases of development, or are "trickle-down" mechanisms, whereby wealth accumulation by the rich benefit the poor, strong enough to bring back equality? If one believes the empirical evidence against the Kuznets curve, the theoretical question to ask is: why does inequality not vanish in the long run?

Much research addressing these questions starts from Kuznets' original intuition that economic development is a process of sectorial change, where agents change occupation from a sector in which incomes have a low mean and a low variance to a sector with high mean, high variance income. When agents start changing occupation, inequality arises both as a result of the mean income differential, and because incomes in the new occupation have greater variance. As the number of agents working in the new occupation grows, total inequality falls, because the mean income differential becomes less important.

It is fairly general for research, which considers trickle-down in the above framework, that agents are assumed to choose the sector, which brings the highest present value of income, and one thereby discards the breakdown of Fisher separation when credit markets are imperfect. This criticism is especially relevant for models of development and personal income distribution, because one considers individual investments like education, migration, the purchase of farming machinery, etc.[5]

This model analyzes individuals' choice of occupation, trickle-down, and personal income distribution when Fisher separation fails.

A recent paper, Aghion and Bolton (1997), formalizes Kuznets' intuition above, but formally models the information problem, which prevents the optimal allocation of agents to occupations: moral hazard in lending prevents poor agents from choosing the modern occupation (investing), forcing them instead to lend their funds—for low rates of interest— to rich investors, who

[1]In this paper, wealth inequality and income inequality are directly and perfectly related, hence the term "inequality" is used in the remainder of this paper.

[2]Evidence to support this is given in e.g. Deininger and Squire (1998).

[3]An early paper which tests directly for—and rejects—the presence of a Kuznets curve is Anand and Kanbur (1993). For a more recent example, see Deininger and Squire (1998).

[4]See e.g. Benabou (1998) for a survey of these.

[5]A substantial literature considers the macroeconomic effects of credit constraints. In this literature, assuming Fisher separation appears to be less of a problem, because the focus is on firms' investment decisions. It seems likely that firms worry less about individual shareholders' short run consumption possibilities than an individual himself would.

thus grow richer at their expense. Later, when the rich have invested the efficient amount of capital, they start supplying credit, making it possible for poor agents to invest and thereby catch up with rich agents—a phenomenon referred to as "trickle-down".

A major contribution by Aghion and Bolton (1997) is isolating the circumstances under which the trickle-down mechanism eventually makes everyone invest. They do not, however, consider the distribution of wealth that arises when some agents are unable to invest and neither do they protocol the possible long run distributions of wealth. These analyses need to be made for at least two reasons: first, knowing the conditions under which a given long run outcome can arise, and what level of inequality is associated with this outcome improves the basis for countries' development strategies. Second, economists must be able to identify the efficiency problems that remain in the long run: do credit constraints hamper investment by incumbent entrepreneurs or do they prevent agents from setting up firms, or both? In the first case it may be relevant to support the entrepreneurial sector to let it act as a motor of growth for the rest of the economy, whereas in the second case, such a policy might just increase the rich agents' consumption.

This paper analyzes the evolution of the wealth distribution and addresses the efficiency of long run outcomes. The framework of analysis is a two-period overlapping generations model, where agents' status as lender or borrower is endogenously determined, and where some agents' investments are subject to a credit constraint a la Hart and Moore (1989). Wealth is transmitted through generations by means of "warm glow" bequests, and agents' only market interaction takes place in the credit market. The rate of interest provides powerful spill-over effects from lenders' wealth to borrowers' investment and conversely from borrowers' wealth to lenders' wealth accumulation.

The main results of this paper are first that sectorial change can lead to increasing inequality. Second, agents' interaction in credit markets can serve to increase wealth inequality in poor economies, but decrease wealth inequality in rich economies. Third, in the long run, credit constraints appear to more seriously affect the choice of occupation than entrepreneurs' investment.

In the following, section 2 sets up the model and finds agents' optimal choices of consumption, bequests and investment. Section 3 finds the static equilibrium for the unconstrained and constrained economies, section 4 analyzes the evolution of the economy and section 5 concludes. All proofs are gathered in Appendix B.

# 2 The Model

Consider a population of overlapping generations, each of which is a continuum of measure 1 of agents, and each agent lives for two periods. In the first period of life an agent is young, and in the second, old. At the beginning of each period old agents reproduce themselves, resulting in one offspring per old agent, and at the end of each period, old agents die and young agents ascend into old age. The population size is thus constant over time.

There is only one good in the economy. The good is perishable, so it cannot be stored, but it can be used as capital input and it can be consumed.

An agent born at date $t$ receives a transfer, $b_t$, from his parent, and has preferences described by the utility function

$$U(c_t, b_{t+1}) = c_t^{\gamma} b_{t+1}^{1-\gamma}, \tag{1}$$

where $c_t$ is his consumption when young, $b_{t+1}$ is a bequest which is left to his offspring in the second period of his life, and $0 < \gamma < 1$.[6] When we later refer to "wealth", we mean the endowment, $b_t$, whereas "bequest" refers to $b_{t+1}$.

In this model, the only transaction between members of different generations is the bequest. It is assumed that agents have a joy of giving to their children, rather than a genuine interest in the welfare of their children. This assumption is quite common in much recent research on the evolution of income and wealth distributions, e.g. Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), and Piketty (1997). The main reasons for working with this bequest motive are that it is more tractable and that empirical evidence tends to favor this "warm glow" bequest motive against the altruistic bequest motive.[7] In addition, McGarry (1997) finds conditions under which an altruistic bequest motive is well approximated by the warm glow bequest motive.

[6] This importance of timing in this formulation of utility distinguishes this paper from much related research, e.g. Matsuyama (1998).

[7] Specifically, two implications of the altruistic model are counterfactual: 1) in a growing economy altruistic bequests should be negative on average, 2) parents should leave greater bequests to poor children. Recent evidence against the latter one is presented by McGarry (1997) who found that in most cases parents give equal amounts to their offspring. An important empirical difficulty remains with our choice of bequest motive, though: if agents cared about the amount of bequest given to their offspring, then we should observe an extensive use of capital markets to insure the bequest against shocks. Moreover, if the preference is for giving, why don't we observe a wider use of gifts instead of bequests? These points have been made by Bernheim, Shleifer and Summers (1985) in a paper in which they also find evidence that the bequest motive is linked to the desire to influence the actions of the offspring.

A young agent chooses one of two possible occupations: she can either become a gatherer or an entrepreneur. Gatherers produce one unit of the consumption good in the first period of life, and therefore must save in order to leave a bequest in the second period. Entrepreneurs, on the other hand, invest an amount of the consumption good in a production process in order to produce output in the second period. Specifically, we assume that if an entrepreneur invests $k_t \in [0, 1]$ date $t$, she obtains $(1 + A)k_t$ in gross output at date $t + 1$. The upper bound, 1, introduces decreasing returns to scale for entrepreneurs. This is a crucial assumption for obtaining the "trickle-down" result of Section 4, because it implies that sufficiently rich agents lend to poor agents, allowing these to invest and thereby catch up with rich agents.[8][9]

It thus takes time before a newly started business generates positive returns, e.g. because setting up the business or gaining a position in the market takes time. This means that entrepreneurs may want to borrow for consumption and investment in order to smooth the intertemporal allocation of consumption and bequest.

Assume that capital is "more productive than gathering":

$$A > 1.$$

The timing of events in an agent's life is summarized in Figure 1.

A gatherer's choice must satisfy the intertemporal budget constraint:

$$c_t + \frac{b_{t+1}}{1 + r_t} = b_t + 1, \tag{2}$$

where $r_t$ is the rate of interest on her savings. Condition (2) requires the present discounted value of consumption and bequest to equal the sum of the bequest received and the income from gathering.

An entrepreneur's intertemporal budget constraint is given by

$$c_t + \frac{b_{t+1}}{1 + r_t} = b_t + \frac{A - r_t}{1 + r_t} k_t, \tag{3}$$

which has the same interpretation as (2) except that the second term on the right hand side of (3) is the net present value of investing $k_t$.

Given that gatherers want to save and that entrepreneurs may want to borrow, there is scope for credit market interaction. Such interaction is,

[8] Another purpose of introducing the upper bound on capital input is to avoid capital being used as a means of storage. The importance of this assumption will become clear later.

[9] Fixing gatherers' income and the upper bound on capital input to one involves a loss of generality only because it fixes the ratio of gatherers' income to the maximum capital input to one. The key mechanisms of the paper easily survive this restriction.

however, assumed to be hampered by lack of enforceability of credit contracts. Assume that borrowers are subject to a borrowing constraint of the form

$$(1 + r_t)d_t \leq \nu k_t, \tag{4}$$

where $d_t$ is the amount of borrowing, and $1 \leq \nu < 1 + A$. The parameter $\nu$ can be interpreted as a measure of creditors' bargaining power in a debt renegotiation procedure. A micro foundation for this particular constraint is given in Appendix A.

Substituting $d_t = c_t + k_t - b_t$ into (4) then gives

$$(1 + r_t - \nu)k_t \leq (1 + r_t)(b_t - c_t). \tag{5}$$

Now we are ready to state a newborn agent's maximization problem:

$$\max_{c_t, b_{t+1}} c_t^\gamma b_{t+1}^{1-\gamma}$$

subject to either (2), or (3) and (5).

In the following, we first look at the optimal choices of consumption and investment for a given choice of occupation and derive the resulting lifetime utilities. Then the choice of occupation is analyzed, assuming agents choose the occupation, which gives them greatest lifetime utility.

## 2.1 Gatherers' optimal decisions

Substitute (2) into (1) to get the unconstrained maximization problem:

$$\max_{c_t} c_t^\gamma \left((1 + r_t)(b_t + 1 - c_t)\right)^{1-\gamma},$$

the solution to the gatherer's problem can be found to be

$$\begin{aligned} c_t^G &= \gamma(b_t + 1), \\ b_{t+1}^G &= (1 - \gamma)(1 + r_t)(b_t + 1). \end{aligned} \tag{6}$$

These choices imply that a gatherer saves the amount

$$-d_t^G = (1 - \gamma)(b_t + 1), \tag{7}$$

and obtains the utility

$$V^G(b_t, r_t) = h(\gamma)(1 + r_t)^{1-\gamma}(b_t + 1), \tag{8}$$

where $h(\gamma) = \gamma^\gamma(1 - \gamma)^{1-\gamma}$.

Interesting findings are that gatherers increase credit supply as they accumulate wealth, and that gatherers' bequests increase as the rate of interest increases.

## 2.2 Entrepreneurs' optimal decisions

Substituting (3) into (1) reduces the entrepreneur's problem to

$$\max_{c_t, k_t} c_t^{\gamma} \left( (1 + r_t) \left( b_t + \frac{A - r_t}{1 + r_t} k_t - c_t \right) \right)^{1-\gamma} \tag{9}$$

subject to (5).

We classify entrepreneurs by their optimal choices of consumption, bequest and investment. In this setting, an agent is "unconstrained" if the solution to (9) has (5) non-binding. Conversely, if (5) is binding, the entrepreneur is said to be "credit constrained". For credit constrained entrepreneurs, a further distinction is made between those for whom the credit constraint hampers both investment and consumption, and those for whom only consumption is hampered. We refer to investment constrained entrepreneurs as "poor" and consumption constrained entrepreneurs as "rich", because $b_t$ determines whether they are investment or consumption constrained. This further distinction between "rich" and "poor" credit constrained entrepreneurs is necessary in order to understand the dynamics of wealth accumulation, because entrepreneurs who are investment constrained have stronger incentives to save than do consumption constrained entrepreneurs.

Our approach in what follows is to solve the entrepreneur's problem (9) assuming that (5) does not bind and then look for conditions under which the optimal choice has (5) non-binding. Having found these conditions, we proceed to look at credit constrained entrepreneurs' behavior.

### Unconstrained entrepreneurs

If the credit constraint does not bind, an entrepreneur essentially behaves like a gatherer, and treats the present value of profits as an additional endowment.[10] Assuming without loss of generality (see Lemma 1 below) that $A > r_t$, investment gives a higher rate of return than saving, so the unconstrained entrepreneur invests as much as she can, that is, $k_t = 1$. Substituting this into (9) reduces an unconstrained entrepreneur's problem to

$$\max_{c_t} c_t^{\gamma} \left( (1 + r_t) \left( b_t + \frac{A - r_t}{1 + r_t} - c_t \right) \right)^{1-\gamma}.$$

The solution to this problem is (the superscript, $U$, means "unconstrained")

$$c_t^U = \gamma \left( b_t + \frac{A - r_t}{1 + r_t} \right),$$

[10] More precisely, Fisher separation applies.

$$b_{t+1}^U = (1-\gamma)(1+r_t)\left(b_t + \frac{A-r_t}{1+r_t}\right). \quad (10)$$

The difference between an unconstrained entrepreneur's and a gatherer's problem is that the unconstrained entrepreneur spends one unit of the consumption good to purchase capital in the first period of life and does not earn income in the first period. Debt therefore equals

$$d_t^U = \gamma\left(b_t + \frac{A-r_t}{1+r_t}\right) + 1 - b_t, \quad (11)$$

which implies that when unconstrained entrepreneurs accumulate wealth, credit demand decreases.

The value function is

$$V^U(b_t, r_t) = h(\gamma)(1+r_t)^{1-\gamma}\left(b_t + \frac{A-r_t}{1+r_t}\right). \quad (12)$$

An entrepreneur is unconstrained if she can consume $c_t^U$ and invest 1 without violating the credit constraint. This is the case if

$$b_t \geq b_t^*(r_t) = \frac{\gamma}{1-\gamma}\frac{A-r_t}{1+r_t} + \frac{1+r_t-\nu}{(1-\gamma)(1+r_t)}. \quad (13)$$

The critical bequest can be split into two terms, one of which reflects the propensity to consume out of the present value of the investment, and one which reflects the value of investment as collateral. Call this second term a "down-payment"—the amount which the entrepreneur has to supply herself for a unit investment. If second period earnings is large, or the agent has a high preference for first period consumption, then the agent borrows heavily against future earnings. This means that $c_t^U$ is large, which makes a high $b_t$ necessary to not violate the credit constraint. If the ratio $\frac{\nu}{1+r_t}$ is high, then the down-payment is small, which makes it easier for an agent to consume $c_t^U$. Thus, the greater is $\frac{\nu}{1+r_t}$, the lower is $b_t^*$.

$b_t^*$ depends in a non-trivial fashion on $r_t$, because the rate of interest affects both the present value of investment and the down-payment. Increasing rates of interest drive down the present value of investment, and thereby also $b^*$, but increasing rates of interest decrease the ratio $\frac{\nu}{1+r_t}$, making investment less worth as collateral, which increases $b^*$. Differentiate $b^*$ with respect to $r_t$ to find the conditions under which a particular effect dominates:

$$\frac{db_t^*}{dr_t} = \frac{\nu - \gamma(1+A)}{(1-\gamma)(1+r_t)^2},$$

so a high unit return on investment tend to make the derivative negative, whereas a high creditors' bargaining power tends to make it positive.

### Poor constrained entrepreneurs

If the constraint, (5), binds, then by substituting $k_t$ from (5) into (3) and using the resulting expression to eliminate $b_{t+1}$ from (1), the entrepreneur's problem becomes

$$\max_{c_t} c_t^{\gamma}\left((1+r_t)\frac{1+A-\nu}{1+r_t-\nu}(b_t-c_t)\right)^{1-\gamma}.$$

The constrained entrepreneur's problem is different from the gatherer's problem because the slope of the intertemporal budget line is $-(1+r_t)\frac{1+A-\nu}{1+r_t-\nu}$, unlike the budget lines of gatherers and unconstrained entrepreneurs, which have slopes of $-(1+r_t)$. The propensity to save is thus higher for poor constrained entrepreneurs than for gatherers and unconstrained entrepreneurs. Compare this to the argument made by some development economists, that rich agents have higher propensity to save than poor agents, and therefore inequality is good, because it increases capital accumulation. We have found that when agents have identical preferences, some entrepreneurs do indeed have high savings rates, but, as we shall see later, these entrepreneurs are rather the middle class than the upper class.

If $1+r_t-\nu \leq 0$, then this problem is trivial—the entrepreneur can increase consumption and bequest at the same time by investing, because each unit of capital invested allows the agent to borrow more and thereby consume more. Also, because $A > r_t$, increased investment results in increased bequests too. In this case then the optimal choice of investment is 1 regardless of $b_t$, thus all agents are "rich".

In order to focus attention on poor constrained entrepreneurs, therefore, assume for now that $1+r_t-\nu > 0$.

The solution to the poor entrepreneur's problem is given by

$$\begin{aligned} c_t^P &= \gamma b_t, \\ b_{t+1}^P &= (1-\gamma)(1+r_t)\frac{1+A-\nu}{1+r_t-\nu}b_t. \end{aligned} \tag{14}$$

The investment associated with this solution is

$$k_t^P = \frac{(1-\gamma)(1+r_t)}{1+r_t-\nu}b_t,$$

and the credit constrained agent borrows to the maximum, $\frac{\nu}{1+r_t}k_t$:

$$d_t^P = \frac{1-\gamma}{1+r_t-\nu}\nu b_t.$$

$k_t^P$ is increasing with initial wealth, thus as poor credit constrained entrepreneurs accumulate wealth they increase investment, which again increases the wealth of future members of their dynasties. $d_t^P$ increases when initial wealth increases because higher initial wealth increases investment, and thereby also collateral and debt.

The relationship between poor constrained entrepreneurs' initial wealth and their debt implies that wealth accumulation by these agents tends to drive up the rate of interest, thereby increasing the income of gatherers. On the other hand, we saw earlier on, that wealth accumulation by gatherers and unconstrained entrepreneurs drive down the rate of interest.

The value of the poor constrained entrepreneur's problem is

$$V^P(b_t, r_t) = h(\gamma)\left((1+r_t)\frac{1+A-\nu}{1+r_t-\nu}\right)^{1-\gamma} b_t \tag{15}$$

and it can be seen to decrease when the rate of interest increases, implying that wealth accumulation by gatherers and unconstrained entrepreneurs makes this agent better off.

As long as $k_t^P < 1$, we call the entrepreneur "poor". This can be seen to be the case when

$$b_t < b_t'(r_t) = \frac{1+r_t-\nu}{(1-\gamma)(1+r_t)},$$

which shows that a poor constrained entrepreneur cannot pay the down-payment on investing 1. $b_t'$ is just the second term in (13), hence $b_t'$ is increasing in $r_t$, and given $A > r_t$ we know from the discussion in relation to $b_t^*$ that $b_t^* > b_t'$.

### Rich constrained entrepreneurs

For bequests in the range $[b_t', b_t^*]$ the optimal investment is 1, and (5) holds with equality. Substituting $k_t^R = 1$ into (5), solving for $c_t^R$, and then substituting $c_t^R$ into (3) gives

$$\begin{aligned} c_t^R &= b_t - \frac{1+r_t-\nu}{1+r_t}, \\ b_{t+1}^R &= (1+A-\nu)\,. \end{aligned} \tag{16}$$

Because the credit constraint is binding, we have

$$d_t^R = \frac{\nu}{1+r_t},$$

which shows that when entrepreneurs reach $b_t'$ they stop increasing investment, but they still want to increase consumption. However, as they are credit constrained, they cannot increase borrowing.

Substituting the solutions for $c_t^R$ and $b_{t+1}^R$ into the utility function, the value of the problem is:

$$V^R(b_t, r_t) = (1 + A - \nu)^{1-\gamma} \left( b_t - \frac{1 + r_t - \nu}{1 + r_t} \right)^\gamma . \qquad (17)$$

The most important implication of rich constrained entrepreneurs' behavior is that wealth accumulation stops feeding itself once entrepreneurs become rich, but still are credit constrained.[11] At this point, gatherers no longer benefit from entrepreneurs' accumulation, there is thus a chance that gatherers' wealth accumulation is stopped because the interest rate starts to fall.

## 2.3 Agents' optimal choice of occupation

If credit markets were perfect, then agents' choice of occupation would reduce to a comparison of present values of incomes in different occupations. This implies that an agent who is indifferent between unconstrained entrepreneurship and gathering must make the same consumption and bequest decisions in either occupation.

Credit constrained agents cannot use credit markets to transform future wealth into current consumption. This means that the timing of income matters because it affects the consumption possibilities at different points in time, whereas for unconstrained agents it matters only because it affects the present value of income.

For agents who consider choosing constrained entrepreneurship instead of gathering, the utility cost of consuming little in the first period may outweigh the utility gain from higher second period bequests, even though investing is in fact advantageous in terms of net present value. This is the additional effect of credit constraints in this model in relation to much of the literature on credit constraints and the choice of occupation, in which the credit constraint only affects the choice of occupation through its effect on the net present value of investment.

Using the value functions (8), (15) and (17) from above we find the critical bequest, $b_t^{**}$, which makes agents indifferent between choosing gathering and credit constrained entrepreneurship:

$$b^{**}(r_t) = \begin{cases} b_t : V^G(b_t, r_t) = V^P(b_t, r_t) \text{ for } b_t \leq b_t' \\ b_t : V^G(b_t, r_t) = V^R(b_t, r_t) \text{ for } b_t \geq b_t' \end{cases} . \qquad (18)$$

[11] This follows because credit constrained entrepreneurs stop investing wealth increases, a phenomenon which also arises in cases with non-linear technology. The specific form of the utility function and credit constraint are the direct causes of this phenomenon.

Given that both $\frac{\partial V_t^P}{\partial b_t}$ and $\frac{\partial V_t^R}{\partial b_t}$ exceed $\frac{\partial V_t^G}{\partial b_t}$, we find that $b_t^{**}$ is unique and increasing in $r_t$, meaning that as the rate of interest increases, more agents choose gathering.

An example of the composition of the population is given in Figure 2, where we assume that $b_t^{**} < b_t' < b_t^*$[12]. Knowing the optimal choice of occupation allows us to depict the value function and optimal bequest from sections 2.1 and 2.2, taking into account changes in the optimal occupation.

In Figure 3 the value function is shown, and the main feature is the kink at $b_t^{**}$.[13] This kink reflects the high return on investment relative to saving at the point where the agent chooses entrepreneurship. When later the agent becomes unconstrained, the marginal return to wealth accumulation is just the rate of interest, and hence the slope of the value function must decrease as wealth reaches $b_t^*$.

Figure 4 shows the dynamics of the optimal bequest. Notice the jump at $b_t^{**}$ and the kinks at $b_t'$ and $b_t^*$. The kinks arise because of changing returns to investment. At $b_t'$ the agent stops investing wealth increases in capital, choosing instead to consume wealth increases. At $b_t^*$ the agent is no longer credit constrained and starts increasing $b_{t+1}$ further.

The jump in $b_{t+1}$ at $b_t^{**}$ can be understood by looking at Figure 5. This graph depicts the choices of consumption and bequests for an agent who is just indifferent between being a gatherer and a poor constrained entrepreneur. If the agent chooses entrepreneurship, then she operates on the steep part of the budget line $((1+r_t)b_t + A - r)$—$b_t$ and makes optimal choices as indicated by the point $b_{t+1}^P$. If the agent chooses gathering, the optimal choice is $b_{t+1}^G$ on the budget line $(1+r_t)(1+b_t)$—$(1+b_t)$. The kink in the entrepreneur's budget line is at the level of consumption, which makes 1 just affordable. If this agent chooses entrepreneurship, then in order to compensate for lower consumption, she gives a greater bequest than she would if she had chosen gathering. This implies that for any level of bequest, constrained entrepreneurs leave bigger bequests than do gatherers, and thus an initially equitable distribution of wealth can become unequal. Moreover, wealth inequality tends to increase as wealth is accumulated because the offspring of poor agents chooses gathering; we return to this in a later section.[14]

[12] In Section 3 the equilibrium of this economy is defined, and it turns out that $b_t^{**} \leq b_t^*$ in equilibrium, because if this were not the case, then no agent would choose entrepreneurship, and there could be no credit market clearing. Unfortunately there is no general ordering of $b_t' \lesseqgtr b_t^{**}$.

[13] The value function is differentiable at all other points than $b^{**}$. The proof of differentiability at $b_t'$ and $b_t^*$ is not difficult and is omitted for the sake of brevity.

[14] For examples of this phenomenon, see e.g. Banerjee and Newman (1993) or Piketty

# 3 Static market equilibrium

This section defines our notion of static equilibrium and illustrates how the credit constraint makes the distribution of wealth relevant for equilibrium rates of interest and allocations of agents on occupations.

For the purpose of the definition, formalize agents' choice of occupation as follows:

$$O(b_t, r_t) = \begin{cases} G \text{ if the agent chooses gathering} \\ E \text{ if the agent chooses entrepreneurship} \end{cases} .$$

Assume that agents' initial wealth is distributed according to the distribution function $G_t(b)$ and collect all agents' choices in a set, $\{c_t, b_{t+1}, O_t\}_{b_t \in [0,\infty[}$. Finally denote by $\alpha_t$ the fraction of gatherers in the economy, $\alpha_t \in [0,1]$, and an equilibrium can be defined as follows:

**Definition 1** *An equilibrium at time t is a set of choices, $\{c_t, b_{t+1}, O_t\}_{b_t \in [0,\infty[}$ and a rate of interest, $r_t$ such that:*

1. *For each agent, the choices $c_t(b_t, r_t)$, $b_{t+1}(b_t, r_t)$, and $O(b_t, r_t)$ solve (1) subject to either (2), or (3) and (5).*
2. *The credit market clears at $r_t$.*

## 3.1 First best equilibrium

In this case all agents are unconstrained in their access to credit, which means that individuals' wealth does not affect the optimal level of investment nor the optimal choice of occupation. Thus, if there is to be both gatherers and entrepreneurs, the equilibrium rate of interest $r^{FB}$ must equate the lifetime utilities from gathering and entrepreneurship:

$$\begin{aligned} V^G &= V^U \Leftrightarrow \\ r^{FB} &= \frac{1}{2}(A-1) < A. \end{aligned}$$

(1997). In those papers the jump in bequests compensates for a discontinuous effort choice. In Galor and Zeira (1993), credit constrained agents face a steeper intertemporal budget line because of monitoring costs. This means that constrained entrepreneurs accumulate faster and may lead to a poverty trap in individual dynamics. Dynasties may therefore become concentrated in two groups: rich agents always become entrepreneurs and poor agents always become rentiers. The same effect is present in our model. In the Galor and Zeira (1993) model, however, there is no jump in the accumulation locus, but only a change of slope.

Given a rate of interest, which makes agents indifferent between the occupations, $\alpha_t$ is determined from credit market clearing. Substitute $r^{FB}$ into (11), multiply (11) by $1-\alpha_t$, multiply (7) by $\alpha_t$ and integrate over bequests to have the market clearing condition:[15]

$$\alpha_t^{FB} = \frac{1}{2}\left(1 + \gamma\left(\int_0^{\infty} b\ dG_t(b) + 1\right) - \int_0^{\infty} b\ dG_t(b)\right).$$

The allocation of agents on occupations does not depend on the distribution of wealth, but only on aggregate wealth, and the number of gatherers is decreasing in aggregate wealth. This is because when aggregate wealth is large, fewer gatherers are needed to finance entrepreneurs' investments.

$r^{FB}$ is independent of aggregate wealth and its distribution, except if aggregate wealth is sufficiently large that the equilibrium share of gatherers is zero. $r^{FB}$ starts varying with aggregate wealth at this point because in the absence of gatherers, only entrepreneurs supply and demand credit. Wealth accumulation by entrepreneurs must be absorbed by themselves in the credit market, which requires decreasing rates of interest.

## 3.2 Equilibrium with credit constrained agents

Unlike for the first best case, where a dichotomy in the determination of $r^{FB}$ and $\alpha^{FB}$ arose, the equilibrium rate of interest and population shares are jointly determined from the market clearing condition

$$\int_0^{b^{**}(r_t)} -d_t^G(b)\ dG_t(b) = \int_{b^{**}(r_t)}^{b^*(r_t)} d_t^{Constr.}(b, r_t)\ dG_t(b) + \int_{b^*(r_t)}^{\infty} d_t^U(b, r_t)\ dG_t(b), \tag{19}$$

where $d_t^{Constr.}$ denotes the relevant function, $d_t^P$ or $d_t^R$.

The number of gatherers is the integral

$$\alpha_t^{SB} = \int_0^{b_t^{**}(r_t)} dG_t(b),$$

which in general is different from $\alpha_t^{FB}$.[16] The dependence of $\alpha_t^{SB}$ on the distribution of wealth comes through two channels: firstly the value of the

[15] In finding this equilibrium it is implicitly assumed that agents at all level of wealth have the same probability, $1-\alpha_t$, of becoming entrepreneurs.

[16] It is difficult to say in general whether $\alpha_t^{FB}$ is greater or less than $\alpha_t^{SB}$. What is clear, though, is that the first best economy has all entrepreneurs investing 1, whereas the second best economy may have agents investing less than 1. For a given rate of interest the second best economy may therefore have many small entrepreneurs with small individual credit demand, thereby keeping the rate of interest low. From a point of view

integral below $b_t^{**}$ depends on the shape of the distribution, and secondly the rate of interest and hence the boundary, depends on the distribution of wealth.

Given that $d_t^R \to d_t^U$ as $b_t \to b_t^*$, entrepreneurs' debt decreases continuously as they change status from rich constrained to unconstrained, and using Leibnitz' formula shows that the right hand side is decreasing in $r_t$. Since the left hand side of (19) is increasing in $r_t$, there can at most be one equilibrium rate of interest. Existence of an equilibrium rate of interest is ensured by dividing—if necessary—groups of agents who are indifferent between entrepreneurship and gathering to obtain market clearing. This is also the way $\alpha^{FB}$ was obtained in the previous subsection.[17]

The equilibrium rate of interest can be bounded in the following way:

**Lemma 1** *At any date, the equilibrium has* $-1 < r_t \leq r^{FB} < A$.

Interest rates are thus higher in first best equilibria than in equilibria with constrained agents. This prediction is contrary to the effect of credit constraints found in models without endogenous choice of occupation. The reason is that the elasticity of credit demand is high at the external margin in first best equilibrium: the moment the rate of interest falls below $r^{FB}$, credit supply drops to zero. In the second best case, however, the elasticity at the external margin is smaller, because of the consumption loss and possibly inefficient scale of investment associated with being a constrained entrepreneur. This implies that the rate of interest can fall below $r^{FB}$ without credit supply going to zero. It cannot exceed $r^{FB}$, however because this would imply that unconstrained entrepreneurs prefer gathering, hence so too must constrained entrepreneurs, who derive less utility from entrepreneurship than do unconstrained entrepreneurs.

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of productive efficiency, this high number of entrepreneurs results in a loss of gathering income. On the other hand it may be that agents can make the efficient investment in spite of the credit constraint, but that the loss of consumption prevents them from choosing entrepreneurship. In this case we would expect the number of entrepreneurs in the credit constrained economy to be inefficiently low.

[17]This way of obtaining market clearing makes wealth equality a highly unstable distribution of wealth, when agents are potentially poor constrained. This is because if a group of identical agents is divided into poor constrained entrepreneurs and gatherers, then the credit constraint ensures that they leave different bequests. A similar way of obtaining market clearing when there are jumps in excess demand is used in Banerjee and Newman (1993). It follows from the analysis of the previous section that this way of splitting the population does not result in wealth inequality when credit markets are perfect, one may therefore interpret this result to mean that credit constraints result in wealth inequality.

# 4 The evolution of the economy

In section 2 we touched briefly on how wealth inequality arises endogenously and how wealth accumulation by one group of agents can benefit other groups. This section first formalizes that intuition in a dynamic example of how wealth accumulation affects the distribution of wealth. With the intuition from the example in mind, we then consider possible long run outcomes of the economy.

## 4.1 Some rudimentary dynamics

Consider the economy at date $t$, when entrepreneurship has just been invented, and assume that wealth is equally distributed, with everyone receiving $b_t$. At this date, all therefore have equal opportunities, and hence there are two requirements for equilibrium. First, agents must be indifferent between the two occupations, and second, the market for credit must clear.[18][19]

The first requirement states that $V^G(b_t) = V^P(b_t)$, which defines a rate of interest,

$$r_t = (1 + A - \nu)\left(\frac{b_t}{b_t + 1}\right)^{\frac{1}{1-\gamma}} + \nu - 1.$$

Insert this rate of interest in the market clearing condition to find the market clearing share of gatherers

$$\alpha_t = \frac{\frac{\nu}{1+A-\nu}\left(\frac{b_t+1}{b_t}\right)^{\frac{\gamma}{1-\gamma}}}{1 + \frac{\nu}{1+A-\nu}\left(\frac{b_t+1}{b_t}\right)^{\frac{\gamma}{1-\gamma}}}.$$

One can now find conditions under which $b_t < b_t'$, and we have found a case in which the invention of entrepreneurship leads to inequality.

[18] If aggregate wealth is sufficiently big, there is an alternative to the first requirement: all agents may prefer entpreneurship, and finance investment out of their own initial endowment. As this happens only in rich economies, this case is not interesting for the purpose of describing a process of development. We therefore consider the evolution of an economy in which the initial equilibrium has poor constrained entrepreneurs and gatherers.

[19] If the economy starts out with no wealth at all, the rate of interest has to be so low that $1+r-v < 0$ in equilibrium. In this case the economy starts out with all entrepreneurs being rich constrained. This case is more difficult to handle, because solving for $r$ such that $V^R = V^G$ cannot be done analytically. This equilibrium features rising inequality, like the case in the example below, but because rich constrained entrepreneurs do not use bequests to increase investment, accumulation within dynasties either stops or results in changes in the occupational structure quite quickly. This case is therefore less useful to illustrate the process of accumulation for a given occupational structure.

At date $t+1$ there are two groups of agents: in one group, $(P)$, members receive $b_{t+1}^{P}(b_t)$ and in another, $(G)$, members receive $b_{t+1}^{G}(b_t)$, and the number of group $P$-agents is $1-\alpha_t$. From this point on the dynamics of the economy entails both accumulation and changes in the occupational structure of the economy. In general this process is difficult to track, and to study the dynamics one generally looks at special cases in which either accumulation or changes in the occupational structure are unimportant.[20] In this setting there is a special case in which one can separate accumulation and changes in the occupational structure. Specifically, first there is a period of accumulation without changes in the occupational structure, and then, at some point, accumulation stops and there may be a change in the occupational structure. To understand the dynamics of this economy, we therefore look for a sequence of equilibria, $\{(r_j,\alpha_j)\}_{j=t}^{t_1}$ such that $\alpha_j=\alpha_t$ for all $j$ and $t_1>t$.

Unlike in period $t$, equilibrium at $t+1$ does not require that agents are indifferent between occupations, because they no longer have equal opportunities, the only requirement for equilibrium is thus that the credit market clears. To find an equilibrium with $\alpha_{t+1}=\alpha_t$, first calculate the equilibrium rate of interest assuming $\alpha_{t+1}=\alpha_t$, and then check whether this assumption is consistent with equilibrium, that is, whether $b_{t+1}^{G}<b^{**}(r_{t+1}(\alpha_t))<b_{t+1}^{P}<b'(r_{t+1}(\alpha_t))$. That is, the equilibrium rate of interest must be such that the offspring of gatherers prefers gathering and the offspring of poor constrained entrepreneurs prefers poor constrained entrepreneurship. Moreover, neither group of agents may afford the downpayment for buying $k=1$.

The equilibrium rate of interest is

$$r_{t+1}(\alpha_t)=\left(\frac{1-\alpha_t}{\alpha_t}\frac{b_{t+1}^{P}}{1+b_{t+1}^{G}}+1\right)\nu-1. \tag{20}$$

Maintaining the assumption that $b_{t+1}^{P}<b'(r_{t+1}(\alpha_t))$, $b^{**}(r_{t+1}(\alpha_t))$ can be found using (8) and (15).[21] Insert $r_{t+1}$ from (20) into $b^{**}(r_{t+1}(\alpha_t))$, and rewrite using (14), (16) and $r_t$ from (20) to find

$$\begin{aligned} b_{t+1}^{G} &< b_{t+1}^{**}(r_{t+1}(\alpha_t)) \Leftrightarrow \\ \left(\frac{1+b_{t+1}^{G}}{b_{t+1}^{G}}\right)^{\gamma} &> 1. \end{aligned} \tag{21}$$

(21) therefore always holds. This has a strong implication: if poor constrained entrepreneurs are accumulating wealth, then they put so much upward pressure on interest rates that it becomes too costly for the offspring of

[20] For more on this, see e.g. Banerjee and Newman (1993).

[21] Maintaining this assumption is done for expositional reasons. Below we check what happens when this assumption is violated and the structure of the system changes.

gatherers to choose entrepreneurship. Upward wealth mobility is therefore hampered endogenously in this model.

Performing similar steps one finds

$$\begin{aligned} b^{**}_{t+1}(r_{t+1}(\alpha_t)) &< b^P_{t+1} \Leftrightarrow \\ & \frac{1}{(1-\gamma)}\frac{1}{(1+A-\nu)}\left(\frac{\alpha_t}{1-\alpha_t}(b_t+1)+b_t\right)^{-1}+1 < \\ & \left(\frac{1}{(1-\gamma)}\frac{\alpha_t}{1-\alpha_t}\nu\left(\frac{\alpha_t}{1-\alpha_t}(b_t+1)+b_t\right)^{-1}+1\right)^{1-\gamma} \end{aligned} \tag{22}$$

(22) says that if the productivity of capital is high and/or the share of gatherers is big, then entrepreneurship is preferred by the offspring of entrepreneurs. Rewrite (22) as

$$z_1 x_t + 1 < (z_2 x_t + 1)^{1-\gamma}, \tag{23}$$

where $x_t = \left(\frac{\alpha_t}{1-\alpha_t}(b_t+1)+b_t\right)^{-1}$ and $z_1$ and $z_2$ are constants from (22). In Figure 6, the left hand side (LHS) and right hand side (RHS) of (23) are plotted against $x_t$ and it is seen that they are equal at $x = 0$ and at one $x = x' > 0$. Thus, if at date $t+1$ (23) is satisfied, then as wealth is accumulated, $x_{t+1} < x_t$ and (23) remains satisfied. So if young agents start choosing the occupation of their parents, then they do so until entrepreneurs are no longer poor credit constrained.

If (21) and (22) are satisfied, we can use (6), (14) and (20) to represent the equilibrium dynamics of the wealth distribution by a system of two linear difference equations:[22]

$$\begin{bmatrix} b^P_{t+1} \\ b^G_{t+1} \end{bmatrix} = \mathbf{\Gamma} + \mathbf{\Lambda}\begin{bmatrix} b^P_t \\ b^G_t \end{bmatrix} \tag{24}$$

where

$$\begin{aligned} \mathbf{\Lambda} &= \begin{bmatrix} (1-\gamma)(1+A-\nu) & \frac{\alpha_t}{1-\alpha_t}(1-\gamma)(1+A-\nu) \\ \frac{1-\alpha_t}{\alpha_t}(1-\gamma)\nu & (1-\gamma)\nu \end{bmatrix}, \\ \mathbf{\Gamma} &= \begin{bmatrix} \frac{\alpha_t}{1-\alpha_t}(1-\gamma)(1+A-\nu) \\ (1-\gamma)\nu \end{bmatrix}. \end{aligned}$$

The economic process captured by the system (24) is a process by which gatherers' accumulation leads to increased investment by entrepreneurs, again

[22]Given that $\alpha$ is constant while the system evolves according to these equations, we choose to omit the subscript on $\alpha$.

increasing entrepreneurial wealth. This again drives up credit demand and interest rates, thereby increasing the wealth of gatherers. As long as the economy evolves according to (24), wealth inequality is increasing and there is no mobility between occupations.

Entrepreneurs may at some point reach $k^P = 1$ in which case the structure of the dynamical system changes. Specifically, $k^P$ reaches one if the fixed point of (24) has $k^P(b^P) \geq 1$, or if the fixed point is unstable, so that bequests increase without bound.

The fixed point of (24) is

$$\begin{bmatrix} b^P \\ b^G \end{bmatrix} = (\gamma - A(1-\gamma))^{-1} \begin{bmatrix} \frac{\alpha_t}{1-\alpha_t}(1-\gamma)(1+A-\nu) \\ (1-\gamma)\nu \end{bmatrix} \tag{25}$$

and the eigenvalues of $\mathbf{\Lambda}$ are $\{0, (1-\gamma)(1+A)\}$, so if $(1-\gamma)(1+A) > 1$, bequests tend to infinity, and hence the system must change at some point.[23] Similarly, the condition for $b^P \geq b'$ is $(\gamma - A(1-\gamma)) \leq \frac{\alpha_t}{1-\alpha_t}(1-\gamma)^2$, hence if this condition is satisfied, the structure of the system changes.

If at some point $b_t^P > b_t'$, the rate of interest is determined by $(1-\alpha_t)d_t^R = -\alpha_t d_t^G$ :

$$r_t = \frac{1-\alpha_t}{\alpha_t} \frac{\nu}{(1-\gamma)(1+b_t^G)} - 1. \tag{26}$$

Rich entrepreneurs' credit demand is independent of wealth, and hence accumulation by gatherers leads to falling interest rates. It turns out that the decline in the rate of interest exactly chokes off gatherers' accumulation, meaning that once entrepreneurs stop increasing credit demand, gatherers stop accumulating too. To see this, consider the change in change in the rate of interest, which just chokes off an increase in a gatherer's endowment. Totally differentiate (6) and set the derivative equal to zero:

$$\frac{dr_t}{db_t^G} \mid_{db_{t+1}^G = \mathbf{0}} = -\frac{1+r_t}{1+b_t^G}.$$

Then use (26) to find the change in the rate of interest, which follows a change in $b^G$ :

$$\frac{dr_t}{db_t^G} \mid_{Marketclearing} = -\frac{1-\alpha_t}{\alpha_t} \frac{\nu}{(1-\gamma)(1+b_t^G)^2},$$

[23] $(1-\gamma)(1+A) > 1$ implies saddle-point stability, but by (25), $(1-\gamma)(1+A) > 1$ implies that the fixed point is one at which bequests are negative. This is precluded by agents always leaving non-negative bequests, hence saddle-point stability implies that bequests must increase without bound.

and substitute for $r_t$ from (26) to find that $\frac{dr_t}{db_t^G}\mid_{db_{t+1}^G=0}=\frac{dr_t}{db_t^G}\mid_{Marketclearing}$. Inserting for $r_t$ from (26) into (6) yields $b_{t+1}^G=\frac{1-\alpha_t}{\alpha_t}\nu$, which is constant over time.

Thus, when entrepreneurs become rich, aggregate accumulation continues only if the rate of interest drops to allow some gatherers to change to entrepreneurship.

It is difficult to say whether gatherers change occupation, because $b^G$ is unknown at the point of change, but one can check whether $b^R > b^*(r_t)$, so that rich constrained entrepreneurs become unconstrained. Using (16), (26), $b_{t+1}^G=\frac{1-\alpha_t}{\alpha_t}\nu$, and (13) some manipulations leads to the result that if

$$\frac{(1-\gamma)(1+A)}{\nu} < 1 < \gamma(1-\gamma)(1+A)$$

then rich constrained agents become unconstrained.[24] A necessary condition for the above inequality to hold is that $A > \frac{\gamma}{1-\gamma}$, which defines a region in $\gamma - A$ space where the long run occupational structure of the economy can have gatherers and unconstrained agents. This region is depicted in Figure (7).

Summing up, entrepreneurs becoming rich constrained is a hurdle for the continuing accumulation in this economy. Above we gave an example where the economy may overcome the hurdle, but it is still unclear, whether full economic efficiency will be achieved. This is analyzed in the following subsection.

## 4.2 Steady states

At this point we turn attention to steady states of this economy. Firstly, as shown above a sufficiently poor economy develops wealth inequality as it evolves, and this process may end at a point where wealth inequality is quite big, or it may be reversed at some point. One way to conclude whether a Kuznets curve arises is to find the conditions under which the long run wealth distribution is equal, in which case we know that wealth inequality must have described a Kuznets curve. Secondly, the system (24) does not capture the evolution of the economy starting from a more general distribution of wealth, hence our best chance to know how wealth inequality has evolved is to look for the steady state distribution.

Define a steady state as follows:

[24] Specifically, substitute $r_{t+1}$ and $b_{t+1}^G = \frac{1-\alpha}{\alpha}\nu$ into $b_{t+1}^*$ and compare the result to $1+A-\upsilon$.

**Definition 2** *A steady state is choices,* $\{c_t, b_{t+1}, O_t\}_{b_t \in [0,\infty[}$ *and outcomes,* $\{r_t, \alpha_t\}$*, such that if these are an equilibrium at date* $t$*, then they will be an equilibrium at date* $t+1$ *too.*

Given that the static equilibrium is unique for any given distribution of wealth, we can characterize the evolution of the economy completely by the sequence of distribution functions, $\{G_t(b)\}_{t=0}^{\infty}$. Starting from some initial distribution, $G_0(b)$, the distribution of wealth at a given date determines the rate of interest and composition of the population, and thereby the next period distribution of wealth. The dynamics of $G_t$ are captured using the function $b_{t+1}$, which describes the optimal bequest, taking into account agents' optimal choice of occupation:

$$G_{t+1}(b) = \begin{cases} \Pr(b_{t+1}(b_t, r_t) \leq b) = \Pr(b_t \leq b_{t+1}^{-1}(b, r_t)) \\ = G_t(b_{t+1}^{-1}(b, r_t)), \text{ where } b_{t+1}^{-1} \text{ is a function} \\ = G_t(b_t^{**}) \text{ if } b \in [b_{t+1}^G(b_t^{**}), b_{t+1}^P(b_t^{**})] \\ = G_t(b_t^*) \text{ if } b = (1 + A - \nu). \end{cases} \tag{27}$$

The approach in the following will be to see if the sequence (27) converges; we call such a limit an "invariant distribution of wealth". Lemma 2 shows that there are convergent subsequences, but we proceed on the assumption that the sequence itself does converge.

**Lemma 2** *The sequence* $\{G_t(b)\}_{t=0}^{\infty}$ *has a convergent subsequence.*

In an invariant distribution of wealth, uniqueness of equilibrium ensures that the invariant distribution is associated with a constant rate of interest. As a consequence of this, the accumulation locus, $b_{t+1}(b_t, r_t)$, is fixed over time and given as: (in the following, omission of time subscript refers to steady state values)

$$b_{t+1} = \begin{cases} (1-\gamma)(1+r)(b_t+1); \text{ for } b_t < b^{**}(r) \\ (1-\gamma)(1+r)\left(\frac{1+A-\nu}{1+r-\nu}\right) b_t; \text{ for } b^{**}(r) \leq b_t < b'(r) \\ (1+A-\nu), \text{ for } b'(r) \leq b_t < b^*(r) \\ (1-\gamma)(1+r)\left(b_t + \frac{A-r}{1+r}\right), \text{ for } b^*(r) \leq b_t \end{cases},$$

which is also depicted in Figure 4. Again we assume that $b^{**}(r) < b'$.

In this case, bequests must be constant for all agents, otherwise the distribution cannot be invariant. Therefore the invariant distribution must be associated with a steady state.

By simple inspection of Figure 4 one notices four possible steady state supports (*supp*) of the distribution of wealth:[25]

$$supp\,(G(b)) = \begin{cases} 1) : \{b^G, b^U\} \\ 2) : \{b^G, b^R\} \\ 3) : \{b^G, \{b^P\}\} \\ 4) : b^U \end{cases} . \qquad (28)$$

where $b^G = \frac{(1-\gamma)(1+r)}{1-(1-\gamma)(1+r)}$, $b^U = \frac{(1-\gamma)(1+r)}{1-(1-\gamma)(1+r)}\frac{A-r}{1+r}$ and $b^R = 1 + A - \nu$. In case 3. the bequest locus coincides with the $45^\circ$ line and the support of the invariant distribution can be an interval. In the following, we shall refer to 1) as a "type 1" steady state, 2) as "type 2", and 3) as "type 3" steady state. 4) will be called the efficient steady state

It is clear that the long run rate of interest is important for the steady state distribution of wealth. High long run rates of interest direct substantial funds to gatherers, whereas low rates of interest serve to concentrate wealth with rich entrepreneurs. The following lemma places restrictions on long run rates of interest:

**Lemma 3** *1. The equilibrium $r_t$ is such that $(1-\gamma)(1+r_t)\frac{1+A-\nu}{1+r_t-\nu} < 1$ at most a finite number of periods.*

*2. The equilibrium $r_t$ is such that $r_t > \frac{\gamma}{1-\gamma}$ at most a finite number of periods.*

Lemma 3 says that very high rates of interest are impossible, because they would make the economy grow without bound, an implication which is inconsistent with the finiteness of aggregate production. It also implies that there is at most one fixed point of $b_{t+1}$ that lies below $b^{**}(r)$ and at most one above $b'$. For the part of the bequest locus in the interval $[b^{**}(r), b'(r)]$, there can be many fixed points if the rate of interest is such that $(1-\gamma)(1+r)\frac{1+A-\nu}{1+r-\nu} = 1$, otherwise there is no fixed point on this part of the bequest locus.

If the bequest locus is such that the only fixed point is above $b^*$, then the steady state is first best. The next subsection looks for conditions under which this steady state obtains.

[25] A related paper, Ortalo-Magné (1995), simulates a land economy, but where individuals, not dynasties accumulate wealth. An important result is that in the long run, credit constraints are not important for land price fluctuations because the number of credit constrained agents is small.

### First best steady state

The following proposition uses Lemma 3 to check whether bequests are likely to exceed $b^*(r)$ in the long run. If this is the case, then as in the first best equilibrium, the accumulation loci are $b^U_{t+1}$ and (possibly) $b^G_{t+1}$. Inserting $r^{FB}$ into these loci results in $b^G_{t+1}(b_t, \cdot) = b^U_{t+1}(b_t, \cdot)$ and hence all agents are on the same linear accumulation locus. This linear difference equation has one fixed point, so all agents hold equal wealth in the steady state.

**Proposition 4** *1. If $1 + \gamma(1 + A) \leq \nu$ then as $t \to \infty$, the equilibrium is almost surely first best.*

*2. If $\gamma(1 + A) < \nu < 1 + \gamma(1 + A)$ and*

$$1 \leq \frac{(\nu - \gamma(1 + A))}{1 - (\nu - \gamma(1 + A))}$$

*then as $t \to \infty$, the equilibrium is almost surely first best.*

The idea of the first part is that if $\nu > \gamma(1 + A)$ then $b^*(r)$ is increasing in the rate of interest, but by Lemma 3 the "long run" rate of interest is bounded, and hence so is $b^*(r)$. If the upper bound on $b^*(r)$ is less than zero, then we have the result.[26] If, on the other hand, we require that $b^*(r) \geq 0$, then there is a rate of interest such that this is exactly satisfied. The extension in the second part is in noting that $b^G_{t+1}$ is increasing in $r_t$, and hence we can use the rate of interest, which ensures that $b^*(r) = 0$ to compare the smallest bequest in any equilibrium to the highest equilibrium $b^*(r)$.

The conclusion from Proposition 4 is that high rates of saving (low $\gamma$) and good access to credit (high $\nu$) make the first best equilibrium the likely long run outcome.

### Second best steady state

If the conditions underlying Proposition 4 are not satisfied, the economy may converge to a steady state which features inequality. The possible distributions of wealth in such steady states are given in (28), and this knowledge can be used to characterize steady states of this economy.

[26] A stronger result can be obtained by noting that there exists a minimum rate of interest that makes all agents choose entrepreneurship. This is seen directly by comparing $V^G(b = 0)$ to $V^R(b = 0)$. For this rate of interest a minimum bequest can be found and this can be compared to $b^*$. The problem is that a closed form expression for the minimum rate of interest generally does not exist. It is therefore difficult to compare this minimum bequest to $b^*$.

There is a non-zero measure of parameters, $\{\gamma, A, \nu\}$, which can result in long run inequality. These steady states are characterized by distributions of wealth, $G(b)$, and rates of interest, $r$, and can take the following forms:

1. $supp(G(b)) = \{b^G, b^U\}$, $r = \frac{\frac{\gamma}{1-\gamma}(1+A)}{\frac{\alpha}{1-\alpha}+A} - 1$, where $\alpha$ is such that the following conditions are satisfied:[27]

$$\begin{aligned} V^G(b^U) &\leq V^U(b^U), \\ V^{Constr.}(b^G) &< V^G(b^G), \\ b^G &< b^*(r) \leq b^U. \end{aligned}$$

2. $supp(G(b)) = \{b^G, b^R\}$, $r = \frac{\frac{1-\alpha}{\alpha}\nu}{(1-\gamma)(1+\frac{1-\alpha}{\alpha}\nu)} - 1$, where $\alpha$ is such that the following conditions are satisfied:

$$\begin{aligned} V^G(b^R) &< V^R(b^R) \\ V^{Constr.}(b^G) &< V^G(b^G) \\ b^G &< b^*(r),\, b' < b^R < b^*(r). \end{aligned}$$

3. $supp(G(b)) = \{b^G, \{b^P\}\}$, where $\max\{b^P\} \leq b'(r)$, $r = -\frac{\nu}{(1-\gamma)(1+A-\nu)-1} - 1$ and $\alpha$ is such that

$$\frac{\alpha(1-\gamma)}{1-(1-\gamma)(1+r)} = (1-\alpha)\frac{(1-\gamma)}{1+r-\nu}\nu \int_{b\in[b^{**}(r),b']} b\, dG(b).$$

For cases 1. and 2. the first two requirements state that agents must prefer the occupation of their parent, given the bequest received. The third condition states that gatherers' steady state bequest must not enable them to overcome the credit constraint, and that entrepreneurs leave bequests that allow their offspring to have the same status (constrained/unconstrained) as themselves.

Based on these requirements it is possible to preclude certain types of steady states for some parameter configurations:

**Proposition 5** *1. If $\frac{\nu}{\gamma} - 1 < A < \nu$ then type 1. steady states are not feasible.*

*2. If $\nu \leq A < \frac{\gamma}{1-\gamma}$ then type 1. steady states are not feasible.*

*3. If $\frac{\gamma}{1-\gamma} < A < \nu$ then type 2. steady states are not feasible.*

[27] Here $V^{Constr.}$ is the appropriate function, $V^P$ or $V^R$.

4. *If* $\nu \leq A < \frac{\nu}{\gamma} - 1$ *then type 2. steady states are not feasible.*

5. *If* $A \geq \frac{\gamma}{1-\gamma}$ *then type 3. steady states are not feasible.*

The results of propositions 4 and 5 are summarized in Figure 7. In this figure the solid lines bound regions of parameters, where certain types of steady state can arise. T1 refers to steady state of type 1, T2 to steady states of type 2 and T3 to steady states of type 3. It is seen that if $\nu$ is high, then efficiency is a likely long run outcome in the sense that the area of parameters in which efficiency arises is big. In steady states of type 3, entrepreneurs' investment is inefficient, and one notices that this can happen if the propensity to consume is high. In steady states of type 1 and 2, entrepreneurial investment is efficient, hence a tentative conclusion is that this is a likely long run outcome, as entrepreneurial investment is also efficient in the efficient regions. It is likely, though, that credit constraints will hamper establishment of entrepreneurial firms in the long run—a finding which is consistent with much empirical research on entrepreneurial choice and credit constraints.[28]

# 5 Conclusion

We set up a general equilibrium model featuring borrowing constraints and a choice of occupation when Fisher separation breaks down, and showed that this could be consistent with a Kuznets cycle pattern of inequality, but that some inequality is likely to remain even in the long run.

Trickle-down in the sense of Aghion and Bolton (1997) is powerful when the propensity to save is high, our results thus follow Aghion and Bolton (1997) and demonstrate the generality of their result.

It seems that credit constraints are more likely to hamper establishment of firms than the investment of incumbent firms in the long run, so trickle-down is not likely to bring about equal opportunities in the long run.

An important problem is that the theoretical underpinnings for the credit constraint are derived from a bilateral borrowing contract, but given the endogeneity of the numbers of borrowers and lenders in this model, a fully-fledged analysis would specify the workings of the credit market. Specifically, the optimal number of creditors for a given project should be derived and enter into the determination of market equilibrium.

Also, the absence of uncertainty makes the model of this paper a poor description of wealth mobility, which is altogether absent in the steady state.

[28] Some key references are Evans and Jovanovic (1989) and Blanchflower and Oswald (1998).

Probably the best way to model uncertainty in this model is to assume stochastic abilities, but deterministic project outcomes, because this leaves unaffected the modeling of the credit constraint. Furthermore, the one-sided bequest motive, where agents give bequests to their offspring is hardly descriptive of a developing country, where one often thinks of the offspring as providing for the agent's old age consumption.

An interesting extension of the paper would be a simulation exercise, in which the dynamics of the economy are more thoroughly explored. Similarly, an needed extension is to actually prove that the distribution of wealth in general converges to a limiting distribution, but this remains to be done for much related research. In effect this paper has given an example in which the distribution converges, and described what forms a steady state can take.

Similarly, Proposition 5 deals only with the steady states, which are consistent in the sense that agents who we hypothesize to be credit constrained are in fact credit constrained given the steady state interest rate. A complete analysis would take into account agents' preferences and work out whether the occupational structure of the economy is corresponds to the hypothesized optimal choice. Thus, some work remains to be done to find out more precisely which steady states can arise under certain parametric conditions.

# Appendix A

This is a standard incomplete contracts setting as specified in e.g. Hart and Moore (1989), where the lender can always confiscate the capital input, $k_t$, if the borrower defaults.

Imagine that even though agents have preferences defined over consumption and bequests at date $t$ and $t+1$ only, there is an intermediate phase in which project returns accrue and debt repayments can be made. A contract then consists of a loan at date $t$ and promised repayments at dates $t+\frac{1}{2}$ and $t+1$. Cash flows at date $t+\frac{1}{2}$ are transferred to date $t+1$ at a one-to-one rate and the project pays off only when capital is in place. Specifically, assume that the returns are $\Delta Ak_t$ at date $t+\frac{1}{2}$ and $(1-\Delta)Ak_t$ at date $t+1$, where $0 \leq \Delta \leq 1$. It is clear that the borrower cannot commit to repay more than $k_t$ at date $t+1$ because at date $t+1$ it can default costlessly on any repayment in excess of $k_t$.

Assume that when first a borrower defaults, it immediately offers the lender a new repayment scheme. The lender can accept or reject this offer, and if it rejects, then a bargaining process starts. In the bargaining process, the lender has an outside option of $k_t$ and the borrower an outside option of $\Delta A k_t$, because the lender can confiscate assets[29], but not returns. By continuing the project, an additional cash flow of $(1-\Delta)Ak_t$ arrives at date $t+1$, and this additional surplus is what agents bargain over. Assume that bargaining entails one agent making the opponent a take-it-or-leave-it offer, meaning that the opponent receives the outside option only. Assume also that agents are risk neutral and that the probability of the borrower making the take-it-or-leave-it offer is $\beta$. The lender then expects to receive a total repayment of

$$\begin{aligned} E(\text{lender's returns}) &= \beta k_t + (1-\beta)\left((1-\Delta)Ak_t + k_t\right) \\ &= k_t\left(1+(1-\beta)(1-\Delta)A\right) = \nu k_t \end{aligned}$$

in the renegotiation, whereas the borrower expects to make

$$E(\text{borrower's returns}) = \beta A k_t + (1-\beta)\Delta A k_t = A k_t\left(\beta + (1-\beta)\Delta\right).$$

Assume that

**Assumption A1**

$$\Delta \geq \frac{1-\beta}{2-\beta}.$$

> This assumption ensures that the borrower can make the payment $(1-\beta)(1-\Delta)Ak_t$ out of date $t+\frac{1}{2}$ returns and that the renegotiated contract is always feasible[30].

Risk neutrality implies that the borrower can offer the lender exactly $\nu k_t$ before the actual bargain begins, and the lender will accept this. Continuation is preferred by both parties, and even though the exact timing of repayments is moot, the borrower cannot commit to repay more than $\nu k_t$. The lender knows this and will not require greater repayment, because if he did so, he would certainly lose. The credit constraint is then

$$(1+r_t)d_t \leq \nu k_t;\ 0 \leq k_t \leq 1$$

and we note immediately that

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[29]For simplicity we do not consider partial liquidation in this model. For this generalisation, see Hart and Moore (1998).

[30]By making this assumption, we also avoid considering whether a borrower wants to put aside savings in order to make the $t+\frac{1}{2}$ payment.

The generalation to a case in which the borrower may not be able to make the above renegotiated repayment is beyond the scope of this paper. The assumption could be replaced by an assumption that partial liquidation is feasible, but this would complicate the analysis needlessly.

**Lemma 6**

$$1 \leq \nu \leq 1 + (1 - \Delta)A \leq 1 + \frac{1}{2}A.$$

*So, by assumption A1, borrowers cannot commit the entire future value of investment.*

# Appendix B

**Proof.** of Lemma 1:

Suppose $r^{FB} < r_t$ then by inspection of (8), (12), (15) and (17) it is clear that $V^P < V^R < V^U < V^G$ and therefore nobody choose entrepreneurship. But then credit demand equals zero, whereas credit supply is positive, meaning that $r_t$ cannot be an equilibrium. The second part of the proof follows because if $r_t \leq -1$ then all agents would demand infinite amounts of credit and this cannot be an equilibrium. ■

**Proof.** of Lemma 2:

This is shown by finding a compact set which contains the support of $G_t(b)$ at all $t$, and then applying Helly's theorem[31]. The lower bound comes from the fact that agents receiving a non-negative bequest always leave a non-negative bequest to their offspring. Establishing the upper bound can be done using the lack of a storage technology. Bequests are given out of second period income, which equals gross production plus the income from savings. But all the income from savings must come from the gross production of entrepreneurs, hence no second period income can exceed entrepreneurs' total gross production, and therefore the bequest of any agent is limited by $\bar{b} = \max\{(1-\gamma)(1+A), \bar{b}_0\}$. ■

**Proof.** of Lemma 3:

1. Suppose not, then $\frac{db_{t+1}^P}{db_t} < 1$ an infinite number of periods. The $b_{t+1}$-locus is continuous for $b_t \geq b_t^{**}$ and $\frac{db_{t+1}^R}{db_t} < \frac{db_{t+1}^U}{db_t} < \frac{db_{t+1}^P}{db_t}$, hence in terms of Figure 4 the bequest locus lies below the 45° line for $b_t \geq b_t^{**}$. This means that all entrepreneurs are decumulating wealth, and eventually everyone will choose gathering. This cannot result in credit market clearing. ■

2. Suppose $r_t > \frac{\gamma}{1-\gamma}$ an infinite number of periods.

[31] For a statement of the theorem and its proof, see Stokey and Lucas (1989) chapter 12.

i): $b_t^* < \infty$ for all $t$. To see this, note that $b_t^*$ is continuous in $r_t$ and defined on the interval $r \in \left[\frac{\gamma}{1-\gamma}, A\right]$, and hence finite.

ii): $b_{t+1}^G - \min\{b_{t+1}^*, b_{t+1}^{**}\} \to \infty$ as $t \to \infty$. It is clear that $b_{t+1}^*$ is finite, hence the minimum must always be finite. Now we need to show that $b_{t+1}^G \to \infty$ as $t \to \infty$, but this becomes apparent if we rewrite $b_{t+1}^G$ :

$$b_{t+1}^G = \sum_{i=0}^{t}(1-\gamma)^{t-i+1}\prod_{j=i}^{t}(1+r_j) + b_0(1-\gamma)^{t+1}\prod_{j=0}^{t}(1+r_j)$$

which tends to infinity as $t$ tends to infinity, if $r_t > \frac{\gamma}{1-\gamma}$ an infinite number of times.

iii): $b_{t+1}^P \to b'_{t+1}$. Our assumption that $r_t > \frac{\gamma}{1-\gamma}$ combined with part 1) of Lemma 3 implies that $(1-\gamma)(1+r_t)\frac{1+A-\nu}{1+r_t-\nu} > 1$, because if this did not hold, then 1) would imply that $A < r_t$, which cannot hold by lemma 1. Given this, we have that $\frac{db_{t+1}^P}{db_t} > 1$. Using 1) again we have that $b'_t$ is finite, and hence that $b_{t+1}^P \to b'_{t+1}$ as $t \to \infty$.

iv): $b_{t+1}^* < b_{t+1}^R$. Either $\nu \leq \gamma(1+A)$ in which case $b_t^*$ is non-increasing in $r_t$, and $b_{t+1}^* \leq (\gamma(1+A)+1-\nu)$, whereas $b_{t+1}^R = (1+A-\nu)$. $b_{t+1}^R \geq b_{t+1}^*$ if $A \geq \gamma(1+A)$, but this is always the case, because if $A < \gamma(1+A)$ then $A < \frac{\gamma}{1-\gamma} < r_t$, which cannot be the case by lemma 1. If $\nu > \gamma(1+A)$ then $b_t^*$ is increasing in $r_t$ and hence, by lemma 1, $b_{t+1}^* < \frac{1+A-\nu}{(1-\gamma)(1+A)} < (1+A-\nu) = b_{t+1}^R$ whenever $\frac{\gamma}{1-\gamma} < r_t < A$.

v): $d_t^U \to -\infty$ as $t \to \infty$. To see this, rewrite $b_{t+1}^U$ as

$$b_{t+1}^U = \sum_{i=0}^{t}(1-\gamma)^{t-i+1}(A-r_i)\prod_{j=i+1}^{t}(1+r_j) + b_0(1-\gamma)^{t+1}\prod_{j=0}^{t}(1+r_j)$$

and we see that if $r_t > \frac{\gamma}{1-\gamma}$ an infinite number of periods, then $b_{t+1}^U \to \infty$ as $t \to \infty$. Insert $b_{t+1}^U$ in (11) to see that $d_t^U \to -\infty$.

So, all agents eventually become entrepreneurs (parts i) and ii)) and get to invest the efficient amount of capital (part iii)). But investing the efficient amount always makes an agent unconstrained (parts i) and iv)). Now, given that $b_{t+1}^U \geq b_{t+1}^R \geq b_{t+1}^*$ we know that all agents eventually become unconstrained and remain so forever. Then, by part v) we know that credit supply tends to infinity. Combining these facts implies that aggregate credit supply tends to infinity and credit demand to zero. Thus $\frac{\gamma}{1-\gamma} < r_t$ cannot result in long-run market clearing. ■

**Proof.** of Proposition 4:

1. This is seen by inspection of (13) and using Lemma 3. ■

2. In order for $b_t^* \geq 0$, we must have $r_t \geq \frac{\nu-1-\gamma A}{1-\gamma}$, and we know from lemma 3 that $r_t < \frac{\gamma}{1-\gamma}$, hence we can compare $b^*\left(r_t = \frac{\gamma}{1-\gamma}\right)$ and $b_{t+1}^G\left(r_t = \frac{\nu-1-\gamma A}{1-\gamma}, b = 0\right)$ to have the result. ■

**Proof.** of Proposition 5:

1. Compare an entrepreneur's bequest to the critical one, $b^*$:

$$\begin{aligned} b^U &\geq b^* \Leftrightarrow \\ r\left((1-\gamma)(A-\nu)\right) &\geq -(1-\gamma)A + \gamma(1+A-\nu). \end{aligned} \tag{29}$$

If $A < \nu$, the condition becomes

$$r \leq \frac{\gamma(1+A)-\nu}{(1-\gamma)(A-\nu)} - 1,$$

which is possible ($r > -1$) only if $\frac{\nu}{\gamma} - 1 \geq A$.

2. On the other hand, if $A > \nu$, we must have

$$r \geq \frac{\gamma(1+A)-\nu}{(1-\gamma)(A-\nu)} - 1,$$

which is possible ($r < \frac{\gamma}{1-\gamma}$) only if $A \geq \frac{\gamma}{1-\gamma}$. If $A = \nu$ then we note from (29) that type 1 steady states are feasible only if $A \geq \frac{\gamma}{1-\gamma}$.

3. The condition here is

$$\begin{aligned} b' &> b^R \Leftrightarrow \\ -r\left((1-\gamma)(A-\nu)\right) &\geq (1-\gamma)A - \gamma(1+A-\nu). \end{aligned}$$

Using the same reasoning, if $A < \nu$ this is feasible only if $A < \frac{\gamma}{1-\gamma}$.

4. Similarly, if $A > \nu$ then this is feasible only if $A \geq \frac{\nu}{\gamma} - 1$.

5. We note immediately that $\frac{-\nu}{(1-\gamma)(1+A-\nu)-1} < 0$ is a requirement for $r > -1$, hence it must hold that $A < \frac{\gamma}{1-\gamma} + \nu$, and even if this holds, $r < \frac{\gamma}{1-\gamma}$ only if $A < \frac{\gamma}{1-\gamma}$.

Figure 1: The timing of events in the life of a generation $t$ agent


Figure 2: Agents' choice of occupation dependent on initial wealth

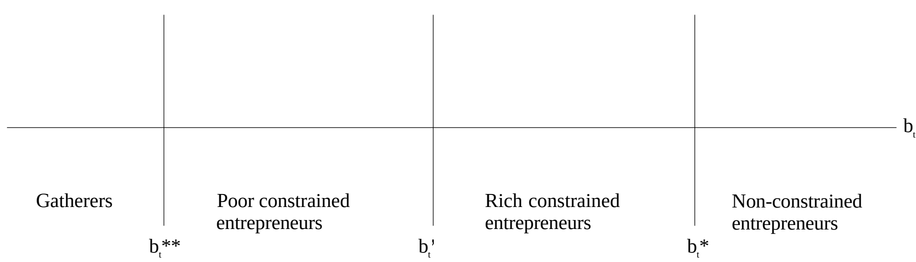


Figure 3: The value function

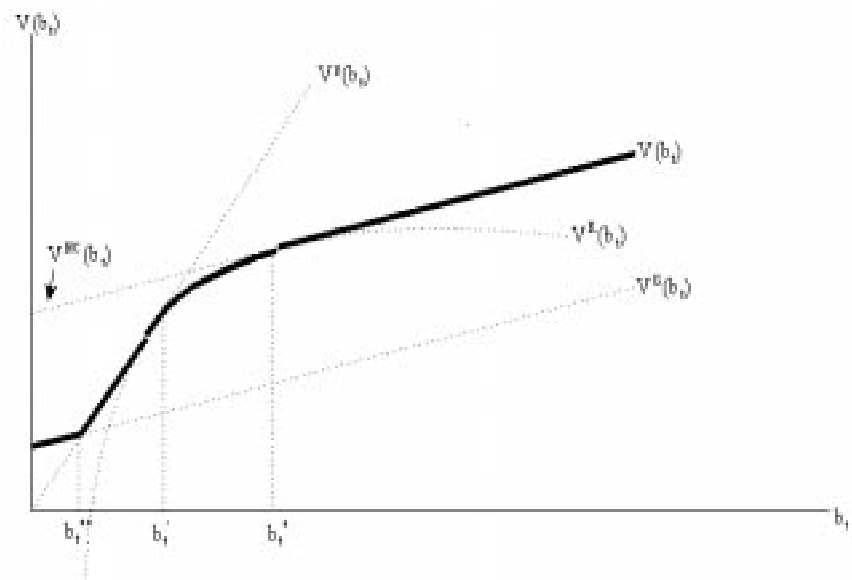


Figure 4: The optimal bequest, taking into account the choice of occupation

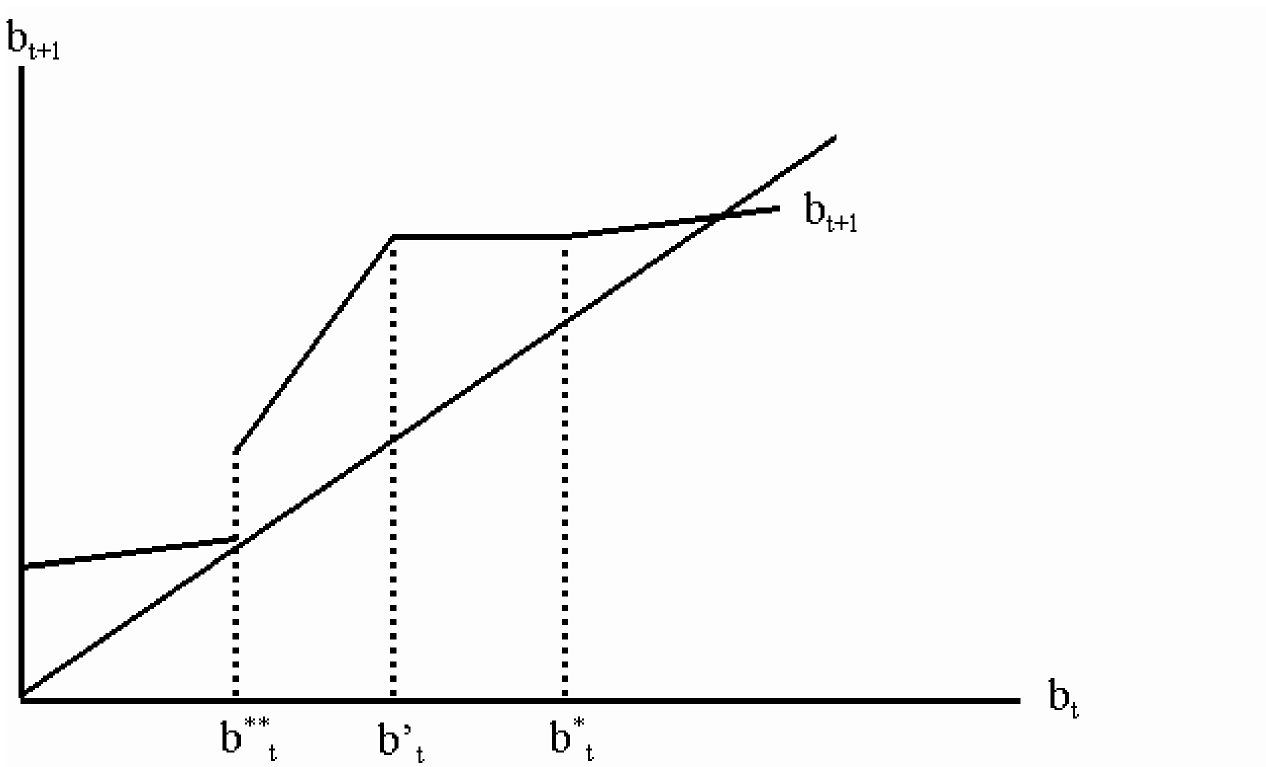


Figure 5: Optimal choices by gatherers and credit constrained entrepreneurs

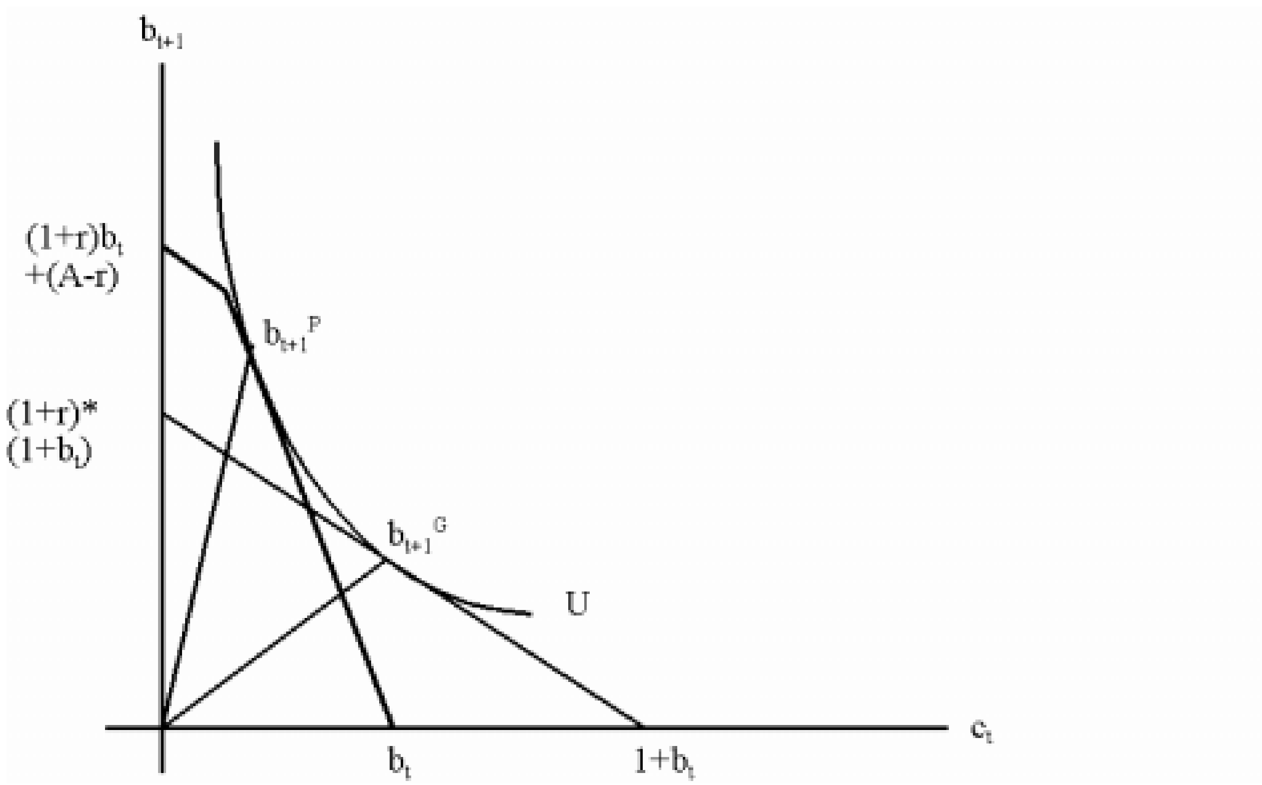


Figure 6: The behavior of (23) as wealth changes

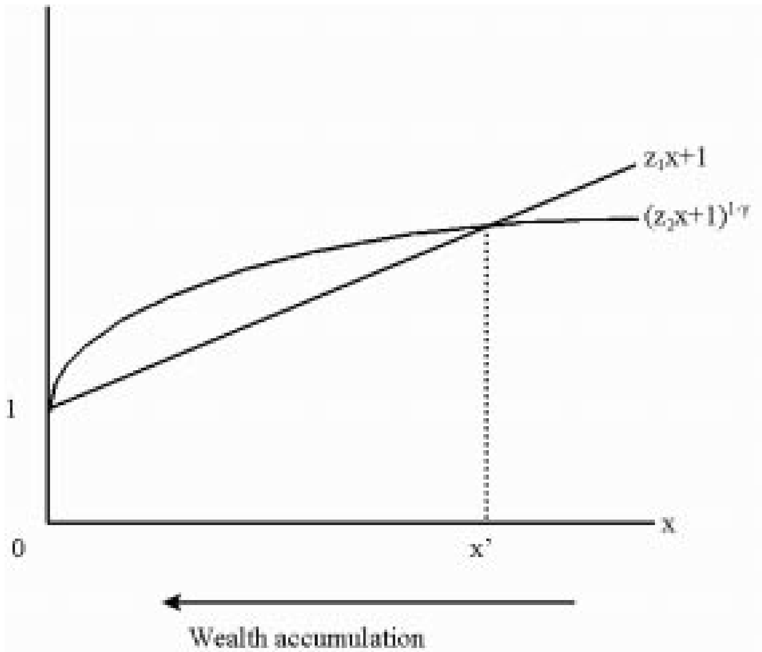
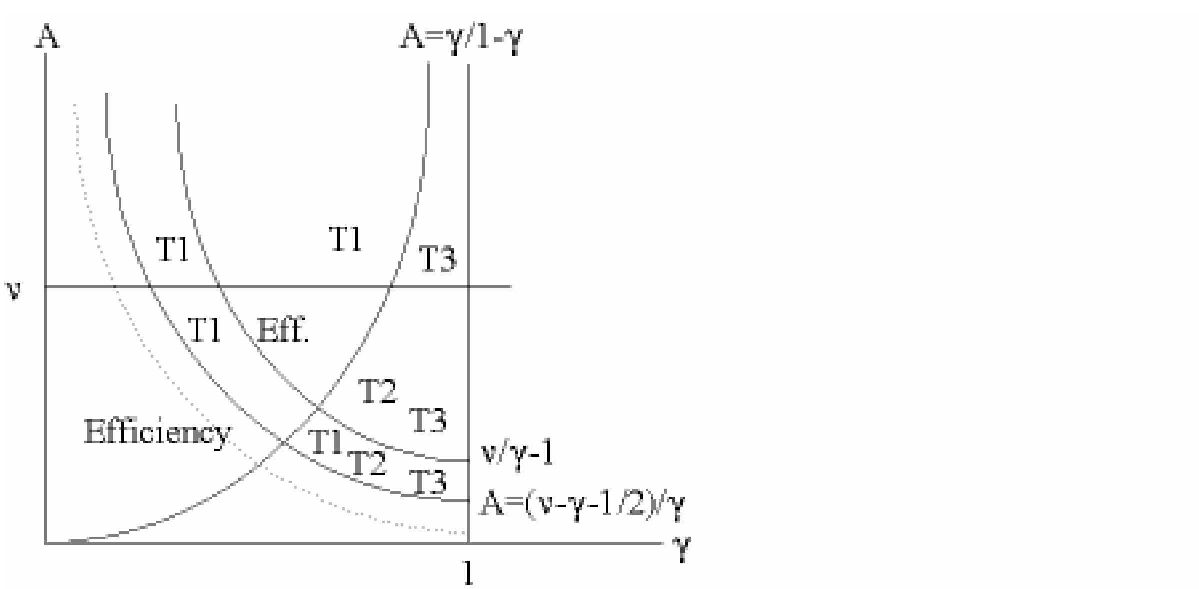



Figure 7: Possible steady states, depending on the parameter configuration



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