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Some Reflections on Time and Equilibrium

by

Ebbe Yndgaard

Abstract: The paper commences by a discussion of the philosophical foundation of the concept of time. Since the concept of equilibrium is inextricably connected with that of time, it is argued that economists should be (more) careful when applying the equilibrium method; it is not legitimate to apply the concept of equilibrium without an explicit definition of the time concept connected with it. Much confusion arises because of mixing state of affairs with that of equilibrium.

Key words: Equilibrium, Time, Methodology

JEL classification: A12, B41

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Preface

This essay is concerned with some fundamental aspects of the concept of time as it is used in economic theory in connection with the concept of equilibrium. The reader should not expect to meet a clarified exposition of the issue; if, however, I have contributed to 'confuse him on a higher level', my goal has been achieved.

Also, the reader should be informed that we disregard other essential aspects of the time concept, eg in connection with time consuming processes, economic growth theory, time series analysis etc.

1. Introduction

Time as such is void; yet, it is an indispensable organizing instrument for not only ordinary communication and interaction between human beings, but also to all sciences concerned with understanding (*Erklärung*) and causality in general. The basic and useful characteristic of time is its sequential nature, as it is utilized in the economics discipline, also.

The ultimate ambition of—at least part of—the discipline of economics is to become a science that is as rigorous as those of the natural sciences; that, however, can never be the case until economics abides fully with the precision requirements, characterizing eg mathematics and formal logics. In spite of its predominantly mathematical form, even contemporary economics suffers from ambiguity and imprecision, often stemming from unclear mixes of concepts of equilibrium and time.

The core concept of economic theory is that of *equilibrium*. Since it is inextricably connected with that of *time*, it is surprising—to say the least—that contributions to approach this relationship of inseparability explicitly are, if not totally absent, then very scarce in the professional literature of not only economics, but also that of philosophy—and mathematics for that matter.

Obviously, confusion and lack of precision can easily turn out when the three fundamental concepts of time, viz those of the meta-, calendar and historical types, in their turn split into two varieties: a discrete and continuous form, are connected with real economic phenomena. In practice, the ambiguity is often concealed by christening a specific equilibrium by a name of its own; let us give just some examples of the broadness of the vocabulary of ‘equilibrium’: partial, market, general, restricted, Walrasian, tatonnement, atemporal, temporary, intertemporal, stationary, evolutionary, replicative, static, dynamic, stable, short-run, long-run, rational expectations, model consistent, Cournot, Bertrand and disequilibrium. This list is far from exhaustive and it should be noticed that each of its items is related to a specific—not often explicitly identified—time concept.

A fundamental question is therefore, if it is possible to establish a concept of equilibrium that is void of time-anchoring or conversely—what is the more likely outcome—if precision of the economics profession must accept to be explicit with regard to the time foundation whenever an equilibrium concept is referred to.

The first formal concept of equilibrium¹ was quite simple and related to a single market (place), trading perishable goods. If the supply of goods happened to be equal to the demand for the same good, *at some specific price called the equilibrium price* then the market was considered to be in equilibrium. It is worth noticing that the concept was and often implicitly still is associated with that of a market.

The implication of the above definition of equilibrium is that *if ceteris paribus* supply and demand were functions of the price and that remained constant at its equilibrium level, then the market would be *repetitive*, ie both the quantity turned over and its price would be unchanged over time or in a different phraseology, the rate of change with respect to time would be zero for the variables involved. This example gives the flavour of one of the equilibrium concepts most widely applied in economics.

However, as is to be demonstrated in the sequel the concept of equilibrium as applied in economics often gives rise to, not only misunderstandings and misinterpretations, but worse to misleading, let alone abuse.

Therefore, it is natural to begin our Odyssey by asking the question if it is possible to find a consensus and universal definition of ‘equilibrium’ that could be imposed onto economics to avoid confusion and imprecision. Let us give two standard examples of a *definition of equilibrium, viz*

state of equal balance; equipose; equality of weight or force; a state of rest or balance due to the action of forces which counteract each other [Cassell, Concise English dictionary, 1989] or

A stable state, characterized by the cancellation of all forces by equal opposing forces [Roget’s II, The New Thesaurus, 1988].

Such definitions are not very helpful in economics, ia because they do not explicitly recognize the inseparability between equilibrium and time already pointed to above.

Of course, many economists have been speculating about the concept per se and its usefulness. One notable contribution is that of Sir John Hicks in his *Methods of Dynamic Economics* [1985], Ch. 2: *The Concept of Equilibrium*; he refuses the inheritance of the concept from mathematical mechanics by concluding,

¹The first economist to set up a demand curve was Gregory King (1696), according to Schumpeter (1954) p. 212-213, and the first economist to attempt to define a formal equilibrium was Isnard (1781), *ibid*, p. 217 & 307.

“It is safer, in my view, to define the static equilibrium of economics as an independent concept in its own right”.

We shall follow the scheme of Hicks quite closely—not least because he recognizes that equilibrium is an analytical tool, a method, not a state of the economy— and conclude that *equilibrium is a time-relativistic method*.

After having argued in favour of the necessity and expediency of the concept of equilibrium, in the sequel we shall take a tour d’horizon, firstly to scrutinize the time concept per se and secondly to present a few, widely differing applications of equilibria in economics.

2. The necessity, expediency and attractiveness of equilibrium

If equilibrium as an analytical tool is an imprecise instrument, why use it at all? Strong explicit arguments can be forwarded in favour of its playing a central role in economics, viz as an organizing tool. Let us give just two convincing examples.

The first one is by G. L. S. Shackle in his *The Nature of Economic Thought* [1966], p. 226 which states:

“One dichotomy is between equilibrium and development theories. Equilibrium is a test that selects for the economist one particular situation out of an infinity of situations and justifies his calling attention to it as something special. Judged by the smallness of the ratio of what it accepts to what it rejects, no other test seems able to rival its selective power. No other test, it may be claimed can state so sharply in what the accepted differs from the rejected situations. By contrast, no test of comparable power and conviction can be found for selecting among paths of development. On the most general grounds, equilibrium has great claims as an economizer of thought”.

The second one, in fact reasoning along the same lines, is Peter Skott, *Conflict and effective demand in economic growth*, Ch. 2, *Methodological Issues* [1989], which says,

“In general, any non-vacuous and internally consistent theory will describe a number of regularities and define a non-empty set of outcomes satisfying the regularities. This set of consistent outcomes constitutes the equilibria of the theory. A proof of the existence of equilibrium therefore is simply a check on the logical consistency of the theory. A theory without equilibrium (..) is logically false”

and also

“If the equilibrium of (*a special theory, my insertion*) T_1 is stable under the assumptions of (*a more general theory, my insertion*) T_0 then the predictions of T_1 should be almost as accurate as those of T_0 ”.

The second quotation from Skott is close to the Shackle-point of view, according to which equilibrium is useful as an organizing principle in that it dismisses the extensive if not infinite set of irrelevant alternatives; the first Skott-quotation, on the other hand, cuts the concept down to size in that it stresses its ‘neutral’ and methodological character in relation to realia; disequilibrium becomes a precise concept also, cf below.

Apart from the above arguments of the necessity and usefulness of applying the concept of equilibrium in economics, I would add some further comments which are of a more controversial nature, viz the apparent fascination by equilibrium when mixed up with real phenomena, either when interpreted as a state of affairs or even more misleading when implicitly used as a normative guideline for policy makers.

The rock bottom reason for its popularity is indisputably that economic theory has been developed in a market economic context; *en passant* it is important to distinguish between the concepts of equality and equilibrium.

To state that two magnitudes are the same, eg $A=B$, does not in itself signal an equilibrium; on the other hand, an equilibrium implies that $A=B$. The basic difference between the two concepts is that in an economic context the variable A is the outcome of the decision making of one group of agents and B that of another—not necessarily mutually exclusive—group. In this sense *the economic equilibrium signals coincidence between two decision makers’ plans or wants*.

In contrast to this, the equality ‘=’ in general only states that two magnitudes are the same, reflecting an equilibrium or not. Hence, the equilibrium concept relies on a typically very complex set of (behavioural) assumptions, cf the section below on equality and the symbol: ‘=’.

To understand the popularity of the equilibrium concept, especially among Western or market oriented economists, we must go back to the origin of the construct ie the very nature of a market economic or capitalistic system. In such a system, the sovereign authority in the economic sphere is the market, ie the in principle non-interfered-with-by-authorities outcome of interactions between the agents of the economy. This mechanism is the famous ‘invisible hand’ as it was christened by Adam Smith.

To clarify the construct further, let us apply an instructive example, created by Joan Robinson, viz a so-called (peace-full) concentration camp economy. All prisoners

receive identical Red Cross packages, containing raisins, tobacco, chocolate, socks etc.

Since, the prisoners are not identical they do not have the same preferences for the various goods in the packages; therefore, allow them to exchange commodities and realize that the original identical distribution will be replaced by a different distribution that comes closer to the individual preferences—saturation being excluded of course. The resulting rates of exchange, eg raisins versus socks constitute the so-called relative prices, the basic instrument of most economic theory. When an equilibrium has been established in the camp, nobody is able to make any gains by indirect trading, ie arbitrage; he cannot buy raisins for socks in one end of the camp and sell them at a higher ‘price’ at the other end of the camp. The established prices reflect an equilibrium solution to this simple exchange economy.

Even though this example looks like a caricature, it contains all the essential features of the so-called general equilibrium model of economic theory. In technical terms, the economist would say that the individuals are furnished with personal preferences and an initial endowment (the Red Cross package); add to this that they strive for individual maximum utility, ie the highest possible level of preference. Given that people are selfish but honest, the outcome of the exchange will be as described above. *The anonymous working of the ‘market’ is the highest authority* in this system and does not call for any interference; in other words ‘we are living in a Panglossian best of all worlds’.

The reader will probably speculate about the possibility of modifying and improving the result above; the thought is both relevant and well justified. The question raises two fundamental problems.

Firstly, even in the non-interfered economy the individuals do not care about others; they are truly selfish and the mother does not care about her child. Obviously the above result of the exchange changes significantly if we introduce interdependent utility functions, allowing for interpersonal externalities².

Secondly, the system does not allow—nor need—interpersonal utility comparisons; even if the Robinson—packages are identical they do not necessarily produce the same level of satisfaction to all individuals—unless they are identical also³, an absurdity. However, we could think of some benevolent dictator who felt called for to redistribute

²Imagine, that the mother does not like milk; still she gets hold of some to be able to care her child, cf Laursen and Yndgaard (1994), *Nytte og moral* in Festskrift til Erik Harsaae, Universitetsforlaget

³In that case, obviously the system degenerates in the sense that no exchange will take place!

commodities between prisoners, ensuring better supplies for the weak and elderly and less for the strong and younger people; the point here is that we cannot do so without introducing interpersonal comparisons of utilities alias ‘incomes’ in this case. If we interfere in this way⁴ we violate the sovereign authority of the market.

In conclusion, the market is able to find an ‘optimal’ solution and should be left alone—*ie if we forget about interdependent utilities and interpersonal comparison of utilities*. Even though these two reservations are very restrictive, they are normally considered to be an irrelevant nuisance to economic theory and left out of consideration. If the economic modeller—also in a more general context—is able to find an equilibrium a la camp, above he typically feels that he has legitimized non-interference from political authorities; without such interference, the agents of the economy will be able to find a solution, typically by signalling messages indirectly between them via the markets, ie via prices in a broad sense. In short, if there exists an equilibrium, the market forces will coordinate ‘plans’ and make them coincide from both the supply and demand side in a general sense⁵.

Frank Hahn [1984], p. 4 attacks this attitude very fiercely in opposition to Robert Lucas,

“For instance, the competitive equilibrium gains its interest from the postulate that prices must change in all other states of the economy. But there are many other plausible dynamics and the competitive one suffers from having no theory of agents who change prices. But in any event if economic theory has anything to offer on the interaction of market signals and agents’ actions then it will need to formulate an equilibrium concept.

But there are also dangers. One of these is that one considers nothing but equilibrium. Professor Lucas (..) has in fact argued in favour of proceeding in just this manner but I have not been able to make any kind of sense of his argument.”

⁴In fact such interference is the foundation of redistribution of incomes etc via tax and social payment systems in the ‘interventionistic’ economies, the ‘Welfare State’.

⁵In the former—and present—CPE, ie centrally planned economies, the ‘two’ sides were represented by central planners and uncoordinated ‘demanders’; the fact that prices were fixed meant that they were deprived of the ability to equalize supply and demand at the single markets with the well-known effects of shortage at the ruling price, typically incurring black markets’ and redundant supply of unsalable goods. In market economic terms we would say that such an economy was in a state of disequilibrium.

Nonetheless, contemporary economic policy recommendations seem to rely on the Lucas point of view!

3. A first taste of the complexity of the concept of time⁶

The concept of time has two main dimensions, viz one of measurement and one of epistemology. The first one is relatively simple, since the measurement—always ex-post—is straightforwardly related to the astronomic system, potentially by means of a technical device, mimicking the astronomic system, ie a clock in a broad sense.

The second, more interesting and difficult dimension is concerned with that of perceiving time. Centuries of philosophic reflections have not been able to bring a clear answer to *what time is per se*; all attempts to define time have proven futile in that they seem always to end up in circularity.

Plato talked about time as ‘a moving image of eternity’—what is eternity? Aristotle tried with the phrase: ‘the number of movement in respect of the before and after’—what is before and after? And Augustine: ‘a present of things past, memory, a present of things present, sight, and a present of things future, expectations’—still we miss the ordering capability of time.

Naturally, many attempts have been made to infer some contents to time per se; one such example is the so-called A-B-series approach, claiming that ‘time consists in nothing but a fixed “B-series” of events running from earlier to later’. Conversely, the A-theory requires ‘that these events also form an “A-series” going from the future through the present into the past and, moreover, shift in respect to these determinations’. A central epistemological problem here is the concept of shift, referred to as “temporal becoming”. If ‘now’ is defined by the present state, then ‘now’ cannot be later—because then it is not ‘now’ anymore. In other words, we cannot identify the time now by the present state of affairs—*time as such is empty*. If we try to identify time by a sequence of events, the concepts of past, present and future escape us.

An illustrative approach to the perception of time has been presented by the French philosopher Henri Bergson [1934], who has compared time to a film; the pictures of a film are ordered, but consist of—if not discernible by the human eye during play—nonetheless separable pieces. The experience of continuity in this case is created by an illusion, due to the limited capacity of perception by the human eye or brain. The present becomes past, picture by picture in reality, but in a ‘continuous’ flow to the watching individual. The ordering of ‘events’ is captured by the film, but the continuity

⁶This section draws heavily on the article on time by John Earman and Richard M. Gale, in *The Cambridge Dictionary of Philosophy*, CUP, 1995.

between states escapes us; we are still stuck in the Zenoian problem of discerning state from movement.

The above considerations may seem far fetched to economic theorizing; as we shall argue below, however, that is not the case. On the contrary, the distinction between real continuous time and discrete perception of time is in the focus of all sincere economic modelling.

Let us begin with the introduction of the concept of continuous time; let B be an ordering relation of all pairs of instants $\{i_1, i_2\}$, meaning 'is before'. Then if $B i_1 i_2$ is true then there must exist some intermediate instant i_3 so that $B i_1 i_3$ and $B i_3 i_2$ are true also. Continuing this reasoning ad infinitum always introducing new division points leads to a dense set, which in its turn, however, does not exclude gaps; to ensure a continuous time concept we must also introduce the assumption that any convergent series of instants has a limit; in technical terms, the distance measure used must approach zero as the convergent series run through ever increasing instant numbers⁷.

With an eye on the purpose of this analysis, we must refer to the Anglo-American mathematician, logician and philosopher Alfred North Whitehead (1861-1947) who has challenged the applicability of a continuous time concept to capture real-life changes. (The existence of the work by the new-Zenoian philosopher Whitehead does not seem to have been realized by social scientists in general⁸, even though his contributions to methodology seem to have a profound bearing on especially economic modelling—his work deserves very close scrutiny, especially his *Process and Reality*, 1929). Whitehead's 'anthropomorphic demand that the operations of nature must be understood in terms of human agency' sounds convincing to this author.

As a preliminary result of the above discussion, we feel that we have found theoretical support to the view, that even though time per se moves on continuously, it does not follow that it is legitimate to model economic phenomena in a continuous time domain; in real life economic realm, decisions are made by individuals, who do not

⁷As an example, realize that the set of rational numbers meets the first of the requirements; it, however, does not constitute a continuum. The square root of 2 is not included in the set of numbers, but still belongs to the continuum. Therefore, to create a continuum of numbers we should add the set of irrational figures. The extended set is a continuum.

⁸One noticeable exception is N. Georgescu-Roegen, eg, [1970]

transmit continuous inputs to continuous outputs or decisions⁹.

4. A first connectivity of time and equilibrium: memory and expectations

As we shall see, the timing of events plays a crucial role in reality—that being so should be reflected in economic models. Economic agents as human beings have memory and form expectations when making their decisions, ie they take into account phenomena that have taken place in the past and are expected to happen in the future.

In economic modelling it is common practice—not as a matter of principle, but in reality—to consider *the economic sphere as a closed universe*, viz so that the agents' memory is only extended to cover economic phenomena, ie (a small subset of) past prices and economic quantities; the same restriction applies to expectations. To illustrate how restrictive this constraint is, think of an agent in the role as consumer; if he fell chronically ill last year, he changed his preferences accordingly, ie upgraded his preference for medical drugs and downgraded his expectations as to earnings opportunities and future need for medical treatment. In core economic models, the agents do not grow older; they do not remember their past 'careers' and their expectations are seldom related to their past experience.

5. A digression on the symbol: '='

Before proceeding with the main theme we subject the '='/equality symbol to further scrutiny. As we have already noted, an equality $A=B$ does not imply an equilibrium or stated differently, equality is only a necessary, but certainly not a sufficient condition for an equilibrium to prevail.

A fundamental problem here is the lax usage of this core mathematical symbol in economic analyses. We can distinguish between at least 4 different uses of the =-symbol:

- (1) - as a symbol that signals that the LHS variable is defined as indicated by the RHS; in this situation the LHS-variable can never deviate from the RHS-expression, ie a definition in ordinary terminology.
- (2) - as a symbol that indicates a typical causal relationship, viz from the RHS to the LHS; implicitly in this case it is assumed that there could exist—possibly time

⁹ As an example, we could accept that the driver of a car adjusts the wheel in a continuous way to the environment; but we would not find it fruitful to claim that he continuously adjusts his possession of car to the changing environment; in other words, he splits time into (varying) discrete time intervals.

dependent–mechanisms at the RHS that could lead to a deviation from the value of the LHS if the latter were ruled by (partly) independent forces. An obvious example of this situation could be the Keynesian so-called income equation

$$Y = C + I + G + X - M \quad (1)$$

The behavioural basic assumption of the so-called dual decision system is reflected here by claiming (implicitly) that sovereignty is with the demand (RHS) side; if Y were planned at least partly independently in the form of some planned Y^S , a priori that would not coincide with the sum of the RHS-(demand) constituents. In that case the RHS would have to include an unplanned positive change of stocks ΔJ , positive or negative or

$$Y^S = C^P + I^P + G^P + X^P - M^P + \Delta J^{UP} \quad (2)$$

to guarantee the validity of ‘=’

In (2) the LHS is the a priori planned supply and the RHS summarizes planned demand constituents and an unplanned residual. How is this brought in accordance with the first simple Keynesian ex-post equation above? Simply, by assuming that the economy does not realize any unplanned changes of stocks–or involuntary changes of any of the other demand constituents. A priori claiming that $\Delta J = 0$ and attaching sovereignty to the demand side leaves only Y^S to give in; suppliers adapt passively to demand.

In short, we jump from an a priori disharmony of uncoordinated–or only partly–coordinated plans to an ex-post identity by assuming that unplanned stocks’ changes are suppressed by some normally unspecified adaptation mechanism, consuming no real time, ie in comparison to the time dimension at an infinite speed.

- (3) - next the equality symbol is used in a hypothetical context, eg in an ordinary demand function; *if* RHS is so and so *then* the LHS would follow.
- (4) - finally we use the equality symbol in the much more information rich form, ie to signal an equilibrium, cf above.

Some exemplifications

To clarify the differences between the 4 types of equality symbols we introduce the following notational conventions:

- definitions and identities

$$A \stackrel{D}{=} B + C \quad (3)$$

- the second role of the equality symbol was to signal a causal ‘west-ward’ *ex-ante causality chain*, symbolized below by,

$$Y \stackrel{\Leftarrow}{=} C + I + G + X - M \quad (4)$$

If the above equation reads as an ex-post identity or definition the $\stackrel{D}{=}$ should replace the symbol $\stackrel{\Leftarrow}{=}$.

- the hypothetical equality symbol which has some similarity to the causal equality symbol,

$$x \stackrel{H}{=} f(p; \mathbf{a}) \quad (5)$$

Here the implied hypothetical causality/duality goes from the RHS to the LHS so that the arguments of the f-function are treated as if they were (autonomous) parameters, ie fixed values.

- equilibrium equality symbol

$$Y \stackrel{E}{=} C(Y) + I(Y) + G + X(Y) - M(Y) \quad (6)$$

Notice that in this case coincidence turns out between (1) and (2) because the RHS applied Y is equal to the LHS Y. In other words consistency exists between the two sides of the equation, but only because we have inferred a behavioural mechanism onto the system, viz so that ‘somehow’ the perception of the equilibrium level is the same for all agents of the economy, in casu suppliers and demanders.

Let us elaborate a bit further on this example by returning to the formula above including the unplanned stocks changes, ie

$$Y^S = C^P + I^P + G^P + X^P - M^P + \Delta J^{UP} \quad (7)$$

We could imagine that the consumers’ expectations as to income would reflect a level

corresponding to full employment, Y^* say which a priori would not coincide with that of the suppliers Y^S . Hence in more elaborate form (7) could read

$$Y^S = C(Y^*) + I(.)^P + G(.)^P + X(.)^P - M(.)^P + \Delta J^{UP} \quad (8)$$

The role of the ‘buffer’ ΔJ is to catch the ‘uncoordinated’ discrepancy or inconsistency between the two plans on each side of the equality symbol.

The implication of this small inspection into economic methodology has probably already left the reader in a bit confused state of mind and he should be on the alert whenever he meets an economist who works in the apparently stringent tradition of mathematics. His methodological instruments are most often imprecise and when he claims a result to be an equilibrium, without further ado he should be met by the questions: which one, how do you bring it about, and what is its time conceptual foundation?

6. The main constituents of an economic model

Next, in the graph below we apply standard terminology from economic modelling. The endogenous variables, \mathbf{Y} , collected in vectors for each period, are those variables that occur in an ordinary model, potentially implicitly, on the LHS of an equation in the model. Likewise the \mathbf{X} represents the so-called exogenous variables. The subscripts τ and θ refer to lags and leads respectively such that expected values are referred to the future time domain, and predetermined variables to the past. We assume that time (calendar) as such may be one of the exogenous variables and so the stochastic generators.

The symbol y symbolizes the ‘time’ series of a single variable, defined in calendar time.

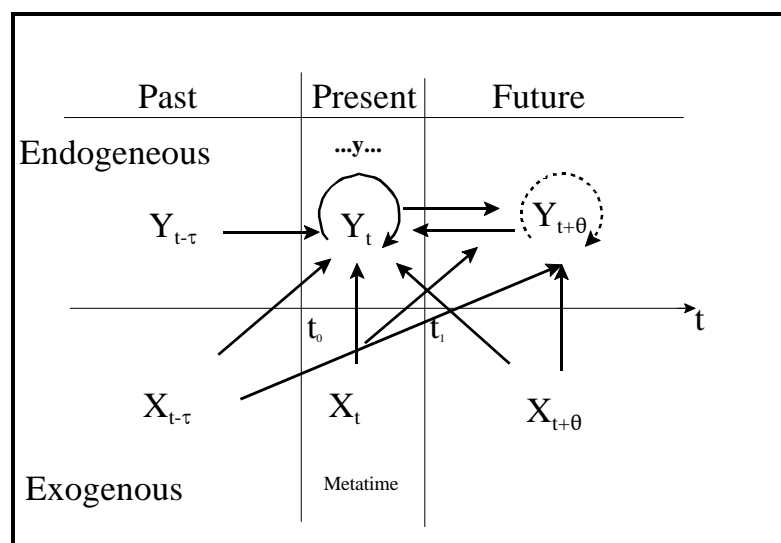
The arrows indicate channels of influence on the present value of \mathbf{Y} , ie past, simultaneous and expected inputs from lagged endogenous and exogenous variables,

simultaneous exogenous and endogenous¹⁰ variables and expected values of both types of variables.

The dotted curved arrow hints at the possibility that eg a learning process takes place to the effect that the transformation of inputs is revised over time; the parameters may depend on the realized values of both inputs and ‘outputs’; this aspect incorporates the so-called Lucas criticism, and could perhaps be reflected already in the current period.

It should be noticed that both the past and the future are indicated by open intervals, in principle extending from $-\infty$ to $+\infty$. The range of time extended by the present *period* is indicated by the two vertical lines at t_0 and t_1 - and its measure is not zero!

The time axis t refers to calendar time, either in a synthetical or historical sense;



A symbolic simplified representation of causal chains

within the present period we use an internal meta-time ‘runner’ v ; since by implication ie assumption we have decided that the *period* corresponds to our observation frequency; what goes on within the period is unobservable; that, however, does not mean that we cannot set up *fictive adaptation processes*, extended in meta-time only, ie with no real time extension. As an example, think of the usual tatonnement process which by auctionarius’s iterations converges to an equilibrium; here the single iteration is dated in meta-time by v ; since by assumption, the process terminates before we pass the line t_1 implicitly we have assumed that the *speed of adaptation* is infinite when related to the calendar time axis.

By construction it is noticeable that the temporary adaptation values cannot carry

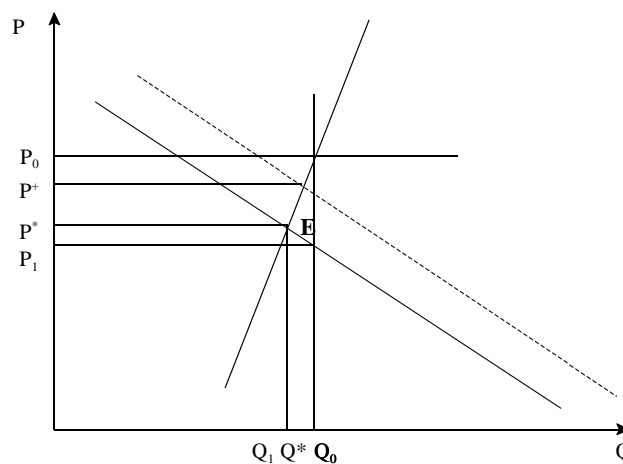
¹⁰The (solid) curved arrow symbolizes the simultaneous interaction between the endogenous variables.

any memory values to the ‘external’ calendar time dimension.

7. The ‘classical’, simple market and its partial¹¹ equilibrium

The conceptual framework introduced so far allows us to formulate a definition of the concept of an economic equilibrium:

An economic partial equilibrium, anchored in a specific time-concept, is a situation where the (behavioural) mechanisms that may produce changes in the time domain have been exhausted within the given environment.



The simple strawberry market

A closer interpretation is called for here. The anchoring in a specific time-concept refers to the phenomenon—as is to be clarified below—that the choice of a concept of time has an impact on the category of equilibrium brought about. The exhaustion aspect refers to the idea that no agents find any more incentives to revise ‘plans’ or behaviour in this specific period. Finally, the clause ‘given environment’ means that no stimuli are supposed to be inferred onto the system.

Also, a graphical illustration may be worthwhile. Let us look at the simplest imaginable case, *the market for strawberries in calendar time*. First we notice/assume that the commodity, exchanged at this market is perishable¹²; it cannot be carried over between periods.

¹¹By partial we mean that all potentially influential sources on the market are assumed to be constant; the restriction clause to reflect this assumption is the *ceteris paribus*.

¹²The ‘most’ perishable of all goods or services is that of labour!

In the figure, where we have drawn the usual negatively inclined demand and the positively inclined supply curves, we imagine that at day 1 the suppliers expect the price to be P_0 and therefore find it optimal to supply the quantity Q_0 . Since the commodity is perishable, the suppliers sell total supply Q_0 at the price available, ie P_1 ¹³. The following day under an *adaptive learning process* the suppliers expect the price to be P_1 ; hence, they cut their supply accordingly, cf the quantity Q_1 . Now their price expectation is too low; the price will increase etc. In short, we trace the well-known cobweb realisation of fluctuating prices and quantities, converging¹⁴ towards the intersection point.

If the learning process in this case could be replaced by a rational expectations mechanism under perfect information, the working of the market would be fully known to the suppliers; they could avoid the iterations and ‘jump to the conclusion’, ie the market intersection point directly.

Finally, accepting rational expectations, but imperfect information the suppliers may underestimate the demand, cf the dotted curve in the figure and meet a higher price P^+ than expected at the perceived equilibrium supply.

This simple example, which is much more complicated than it looks, has introduced two further aspects of the equilibrium formation mechanism, viz the ‘nature’ of the goods traded (perishable or not; producible or not) and the ‘learning mechanism’. As we shall clarify in the sequel, *the rational expectations mechanism is an artefact in the sense that it replaces calendar time by meta time*; no realizations take place outside equilibrium, cf the tatonnement process.

We can characterize a few more concepts here. Since, in our setup we allow realizations to take place outside the point E, we have a market that realizes *temporary equilibria*, ie situations where realizations in fact do take place, but where there exist underlying non-exhausted tendencies for behavioural plans to change over (calendar) time. We use the concept of *false trading* as a synonym for such an occurrence or *disequilibrium realization*.

If we replace the calendar time concept of the market with that of meta time, eo ipso we rule out the possibility of false trading or disequilibrium realizations. About such a market functioning we will also use the term *infinite adaptation speed*.

The information flow of the above construction was created by the feedback from the market to the adaptively learning suppliers. The cobweb-mechanism applied is,

¹³How this result is brought about is left unexplained!

¹⁴Stability is assumed, ie in this case the absolute value of the coefficient of inclination of the demand curve is smaller than that of the supply curve.

however, only one simple and very naive way of imagining, how a final outcome could be brought about¹⁵; in fact, if we postulate some, ie restricted, rationality, suppliers may pursue a less passive policy.

On the one hand, even if they adjust their supply passively in an ex post fashion, they might get the idea eg to interpolate between successive market realizations to shorten the necessary lapse of time outside equilibrium.

On the other hand, it may be more rational to restrict their supply to a level that is always less than—or equal to— Q^* ; depending on the circumstances, such a policy may very well mean a flow of incomes, the present value of which is higher than the one realized by the cobweb-system. On balance, this turns out to be a question of weighing the potential monopolistic rents against the risk of new entries to the group of producers. It is, however, not 'legitimate' to introduce such reflections into the above market situation, because it carries with it a (true) time perspective. In other words, our market is of an *atemporal* nature; so, if the future is disregarded - or repeats itself - we have a *repetitive market*, always *replicating* itself. It should be noticed that the information carrier of this market is the (lagged) price (and quantity).

In analogy to supply side considerations on the demand side, unsatisfied plans will give rise to revisions during an adaptation process in calendar time. In both cases the demand and supply curves are not valid as analytical instruments outside equilibrium; *the curves simply do not exist in a disequilibrium situation and in an equilibrium situation we do not need them!*¹⁶

To summarize, if we jump to the equilibrium point E without further ado, by implication we have accepted:

- a myopic behaviour, ie negligence of the potential gains from optimizing the adaptation path
- infinite speed of adjustment, ie meta time within the calendar period
- no false trading is taking place

¹⁵In this connection recall the principle of correspondence, introduced by P. A. Samuelson (1947).

¹⁶Of course, the Hicksian compensated demand curve represents an attempt to remedy this problem—but not fully successful.

- perishable goods or services¹⁷
- no (real time) memory in meta time
- only external forces, which could comprise a recognition of future stimuli, can create a new adjustment process and a new equilibrium
- there exists an–unspecified–convergent adjustment mechanism, cf the correspondence principle
- the time concepts used, meta or calendar, are of a discrete nature

Referring to the main issue of this paper, the equilibrium concept described above, pretended to be realized in calendar time, but relies on an adjustment mechanism that takes place in meta time. Also, notice that the equilibrium discussed in the professional literature is simply represented by $Q^S(\dots) = Q^D(\dots)$ where our extended ‘ $\overset{E}{=}$ ’ symbol is replaced by the ordinary ‘=’.

The relatively pedestrian way of describing the well-known market cross has served the purpose of illustrating how complex even the simplest case turns out to be under closer scrutiny. The lesson to be learned from this example is not that the construct is useless, but that it is important to realize what the underlying, typically implicit assumptions are.

8. The flagship of (neo-)classical economics: General Equilibrium (GE)

Next we turn to the extremely simple exchange economy case to be followed by the more general case of price equilibrium.

8.1 The simple exchange or barter economy

Let us analyse the general equilibrium model in a (two-person) barter economy. To produce an equilibrium, the fiction is that an auctionarius cries out excess-demand ruled price suggestions to the agents of the economy, one ‘vector’ per period in *meta-time*.

Under the usual assumptions the process will converge, *but not within a finite time horizon if it were to take place in calendar time*. If our aim is to legitimate a jump to the equilibrium conclusion, the latter aspect must be disregarded; we simply apply a meta-time construct with no extension in the calendar time domain. Then it does not matter that

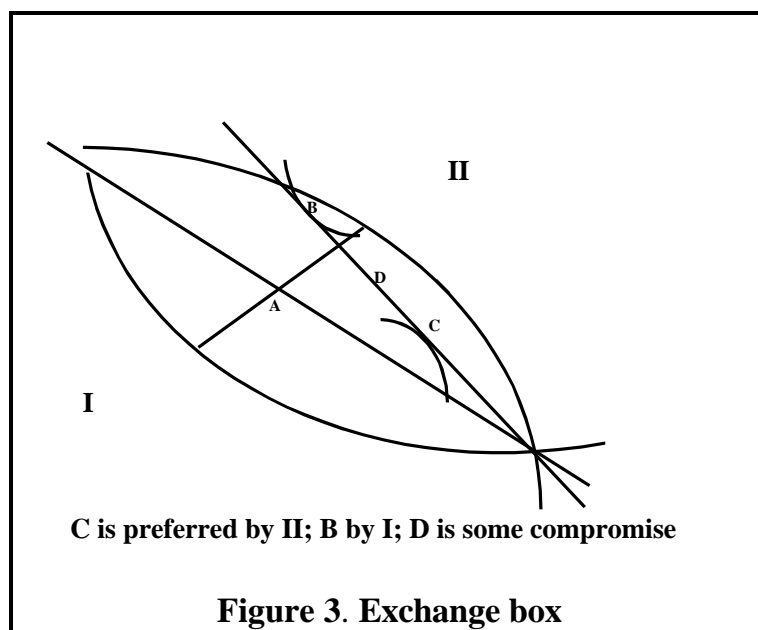
¹⁷If the goods exchanged were not perishable, they could be withdrawn from the market today and re-supplied tomorrow; such an optimization strategy, however, would reintroduce the true time perspective and conflict with the interpretation of the supply curve as derived from the marginal within-period cost curve of the supplier!

it takes meta time to converge; in calendar time, the process by assumption will be ruled by an infinite adaptation speed. It is part of the auctionarius setup that false trading does not take place, not even in meta time! This rules out the Hicksian concept of *temporary equilibria* of a *sequential economy*.

The matter becomes quite *different when we insist on letting the tatonnement process take place in calendar time*; since the auctionarius needs an infinite number of real time units to find the equilibrium, the agents cannot wait until the end of the process; they must trade at some intermediate point of time, ie realize a temporary equilibrium, a seminal theoretical contribution by Hicks to economics.

In the well-known Edgeworth-Bowley exchange box we could imagine that the fictive price equilibrium would be represented by the core point A.

In the first round on the other hand the (implicit) price line through the points CDB would lead to the responses B for agent I and C for agent II; obviously these reactions are



incompatible, since the suggested price ratio leads to excess supply of commodity 1 and excess demand for commodity 2.

Recalling that any interior point of the set between the two initial utility curves represents a gain to both agents in a regime with false trading, depending on bargaining strength we could expect any point on the line from C to B, D, say, to be the outcome of the first round.

If trade actually takes place at point D, in the second round the core of the game is reduced to a subset of the original core and A *will no longer be the 'equilibrium' result*.

Continuing a convergent auction pricing process will ultimately lead to a solution which is a point on the original core, but in fact it could be any point on the core. In other words we have translated the auctionarius process to a game problem, the outcome of which ultimately depends on the bargaining strength of the (two) parties¹⁸. Each realization outside the core represents false trading or a disequilibrium solution; after each trade, tendencies, ie incentives to change the temporary distribution/equilibrium still exist. The flow of temporary equilibria traces a *sequential economy*. It should be emphasized that both temporary equilibrium and the concept of a sequential economy are assigned to calendar time.

8.2 The general, neoclassical (deterministic) equilibrium

A first important aspect to notice in this case is that it is difficult to extend the future to infinity; the usual trick is to stick to a discounting approach by some positive rate of interest.¹⁹

Secondly, the single series of endogenous y's over time may easily vary, ie an equilibrium does not warrant stationarity. To illustrate, let us take an example: It is possible that some endogenous variable—which we could think of as wealth—is subject to variation over time; it could build up over some period and then gradually fade out again, eg the accumulation of a personal wealth variable over life time.

¹⁸It would be an interesting problem to analyse in greater depth what the outcome would be, if the auctionarius demands some compensation for doing his work. The final outcome will probably not meet the well-known characteristics of the classical model!

¹⁹In the limit ie for T going to infinity an often used normalization procedure breaks down. Let \mathbf{p}_t be a vector of (discrete) equilibrium prices for an internal period, ie a period equal to or later than the present. It is well-known that only relative prices matter in this kind of model; hence, we define a total set of prices by a simplex, viz so that

$$\hat{\mathbf{p}} = \frac{\{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \dots, \mathbf{p}_T\}}{\sum_1^T \sum_1^G p_{it}}$$

If the set of \mathbf{p}_t 's constitute a set of *intertemporal equilibrium* prices, then so does the vector $\hat{\mathbf{p}}$; but if the original p's are greater than zero—which should be the case for non-free goods—then letting T go to infinity would send the $\hat{\mathbf{p}}$'s to zero!

This observation means that we cannot rely on a definition of an intertemporal equilibrium as implying stationarity. In a looser sense what equilibrium means is that if the environment is unaltered, then the equilibrium would be the same in a repeated ‘tatonnement’ process.

Thirdly, all markets are predestined to clear, period by period; in that sense we have an equilibrium, also. Regarding all the markets over time, we can conclude that no market exhibits a positive excess demand feature—but possibly a negative one for periodically free (perishable) goods. In a different parlance, in plans all markets exhibit equality between supply and demand—for non free goods.

Fourthly, cf Bliss, there exists an implicit free capital market and eo ipso a set of consistent inter-temporal rates of interest; no arbitrage gains are collectable over time²⁰.

Fifthly, the agents of the economy, their preferences and endowments for all future periods are well-defined, including potential inheritance schemes.

Finally, a long series of other characteristics follow from the above set-up, cf eg Arrow & Hahn, Debreu and others for details.

The important matter on our excursion is to realize that the equilibrium concept can only be associated with the concept of time as such in the sense that the sequence of realizations/equilibria are mutually coordinated over time and so that all markets clear, period by period - and no room is left for surprises or revisions.

9. Keynesian macroeconomics

9.1 Different adaptation speeds

As a prominent example of this kind of model, let us look at the Keynesian model. The outstanding and original contribution made by Keynes was his explicit acceptance of so-called false trading. In the terminology of Hicks an economy realizes temporary equilibria. Referring to the figure on page 15, let us imagine that we squeeze the present period down to such a short interval, that even though forces are at work that try to realize a full coordinated solution they are not strong enough to reach the goal within the present period; ‘something must happen’ and the economy realizes the values found within the limited period, ie a temporary equilibrium. The adaptation within this period is ordered

²⁰A special problem is produced by perishable goods in this connection. If such a commodity disappears between periods, it gives very little meaning to calculate an own rate of interest. In its turn this fact produces an imperfection in the implied capital market(s) in that only own rates of non-perishable goods may be contrasted to the general rate of interest (structure).

by a meta-time construction; in the real time domain, however, the unsettled tensions are left over to the next real time period which is subjected to a new set of predetermined variables, also. Therefore, now we must operate with—at least—two adaptation speeds, viz the intra-period and the inter-period speeds.

If the intra-period adaptation speed is finite, then the realization becomes a temporary, disequilibrium solution. Next, if the inter-period speed of change is low enough the economy will gradually overcome the handed over disequilibria; but if the inter-period speed is relatively higher than that of the intra-period speed, then the economy will never settle in an equilibrium—ie unless the external sources are constant.

Part of the external impetus to the current period is not fully anticipated future values of not only exogenous variables, but also endogenous leaded variables. This is where the artefact of rational or model consistent expectations enters the picture. Future variables are replaced by their expected values, as predicted by the model²¹.

But, a strong reservation is called for here, cf the simple market example above, when the expected values under a stochastic representation of imperfect information are represented by their stochastic mean values. The problem is, that an *equilibrium in means* is not a proper equilibrium, unless we are dealing with a world of only perishable goods. One noticeable ‘commodity’ of this kind is labour; the excess supply of labour is bygone, ie of the strawberry type. All real commodities can at least partly be handed over between real time periods. If, for example, the supply exceeds that of demand in a future period by some (finite) stochastic quantity, this excess supply must be referred to an unplanned increase of stocks. The model cannot escape the problem of real time memory and a reformulation of behavioural assumptions accordingly. Seen from the current point of view, to plan the future actions according to expected values in means may be rational in isolation; but, the noise left over from the former period must be explicitly taken account of during this period. In other words, the economic model cannot remove real calendar time by such artefacts as REH, ie the neoclassical GE-fiction cannot be smuggled into the model by such a construction.

9.2 Realization and plans—the dual decision hypothesis

According to Walras’ law during a period the (nominal) values of all purchases and sales

²¹Even though such values are dated in the domain of future they can only be transformations of past (and present) values!

in an economy for simple accounting reasons must be the same²² and also according to Say's law the value of total supply must be equal to the value of total demand. These sentences apparently state the same message; that it is not the case, however.

Walras' law states that all exchanges must match by value, equilibrium or not while Say's law states that the value of supply must be counterbalanced by an equally valued income, again equilibrium or not.

Let us clarify this by setting up an utmost simple model,

$$Y = C + I + \Delta J \quad (9)$$

Let Y represent realized supply, C realized consumption and I realized investment for a given period. A priori the plans are not coordinated to the effect that when the plans of independent decision makers (cf the so-called dual decision hypothesis) are carried out, suppliers may be exposed to unplanned stocks' changes. If C+I are smaller than the planned and realized supply, 'firms' must realize an unplanned (positive) change of stocks, ΔJ. Formally they purchase their own excess production for stock piling; that is possible because they already possess it.

In a situation where C+I exceeds that of Y they can sell out from their capital stock, ie accumulated stocks piled up in the past. In other words, unplanned stocks' changes play the role of buffer which is true because of Say's law²³.

It is noticeable that *both Walras' and Say's law are always true whether we are in an equilibrium situation or not*. What is important then is to determine when a realization reflects an equilibrium. In our simple setup above that is the case when a coordination takes place between the dual decision makers, ie the left hand side (suppliers) and the right hand side (demanders) so that no unplanned realizations occur; in our case that is when ΔJ=0 or formally

$$\hat{Y} = C(\hat{Y}) + I(\hat{Y}) + 0! \quad (10)$$

In general that is only possible when firms refuse to carry out unplanned stocks' changes or stated differently, they leave the sovereign decision with the demand side. This is

²²A more commonly used version is formulated: If (n-1) markets clear, so does the n'th market.

²³We could also interpret Say's law in the following way: What demanders do not buy we buy ourselves.

precisely the basic idea of the Keynesian approach. The difference between (9) and (10) is that the first equation is not an equilibrium while the second is; both respect both Walras' and Say's law, though.

To perspectivize the point further let us write a more elaborate set of equations or model that does not a priori leave the buffer problem with the supply side alone. Here the 3 constituents' realizations are split into a planned and an unplanned part.

$$Y^R = Y^P + \overset{\vee}{Y} \quad (11)$$

$$I^R = I^P + \overset{\vee}{I} \quad (12)$$

$$C^R = C^P + \overset{\vee}{C} \quad (13)$$

According to Walras' law of course the (ex ante and ex post) realized income must be equal to the realized values of the two demand constituents, ie

$$Y^R = C^R + I^R \quad (14)$$

So far these equations are mere accounting truisms; to turn them into part of a model we must introduce (at least) one hypothesis and we could postulate that consumers passively plan to consume a share α of realized income, ie

$$C^P = \alpha Y^R \quad (15)$$

and that planned investment is equal to I_0 .

We insert the RHS specifications into (15) to get

$$Y^R = \left(\alpha Y^R + \overset{\vee}{C} \right) + I_0 + \overset{\vee}{I} \quad (16)$$

and replace the LHS Y^R by Y^P in (17), rearrange and get

$$Y^P = \alpha Y^R + I_0 + \left(\overset{\vee}{C} + \overset{\vee}{I} - \overset{\vee}{Y} \right) \quad (17)$$

Here it becomes clear that the Keynesian version is a very special case; if we insist that the demanders realize only $\left\{ \overset{\vee}{C}, \overset{\vee}{I} \right\} = 0$, then the only buffer left is with the suppliers who

can only avoid unplanned stocks changes by giving in to the demand side, ie by ‘planning’ $Y^P \Leftarrow C^R + I^R$.

Equation (14), which is necessarily always true by construction—assuming naturally that the consumption hypothesis is valid—has several interesting aspects. We shall point to a couple of them.

Firstly, only in the case where the RHS parenthesis adds to zero and all its constituents are zero, we have the ‘normal’ text book version of the Keynesian model. A less restrictive version turns out if we claim only that the sum should be equal to zero; that would open up to a situation where, say consumers reduced their consumption in comparison with their plans, because the goods they wanted to buy were not available. This reduction could be counterbalanced by the firms’ piling up the ‘wrong’ consumption good; in that case if investment were realized as planned, the unplanned realizations in the rest of the economy would add to zero. We would hardly call that situation an equilibrium.

Secondly, the basic structure of supply side models would correspond to a situation where planned supply is equal to full capacity output in combination with no unplanned stocks changes. This setup leaves only the consumers and investors to accommodate, ie convert their unplanned demands into planned demands, eg via the rate of interest.

Thirdly, we shall only point to the problem that arises when the model is clad in a stochastic dress; if a more general buffering than the simple Keynesian version is allowed, then it is no longer legitimate to add a stochastic element to for example the consumption function (15); the realized, ie observed C now includes not only a structural planned part, but also an *unplanned structural, spilled-over part* to the effect that the usual residual of the consumption function is no longer an ordinary exogenous stochastic variable. To be more precise, if some non zero $\overset{\vee}{C}$ is included in C , it has been spilled over from the endogenous, if not specified structural part of the model, ie the disturbance includes ‘endogenous’ behaviouristic or structural elements.

In conclusion to this section we can state that the Keynesian temporary equilibrium is a special case where the accommodation is left unilaterally with the supply side.

10. Expectations and time

If we adopt the institutional convention that decision makers decide on and implement their plans at discrete—for the sake of convenience by assumption equally spaced—points and intervals of time respectively and furthermore we take it for granted that the measure between decision points is a finite positive figure, then we are easily brought into—at least epistemological—troubles, let alone absurdities, eg in connection with the formation of

expectations.

As a prominent example of this, let us look more closely at the so-called rational expectations method, assuming that we are at the point of time 0.

Let the variables to be determined for this period be collected in the vector y_0 then a model with REH would claim that formally

$$y_0 = f(y_1; E_0, \hat{E}_1 | P_0) \quad (18)$$

where E_0 represents ‘environments’ or exogenous variables at time 0; E_1 expected ‘environments’ and P_0 past relevant values, ie lagged variables.

Since, by assumption y_1 in its turn depends on future expected values also, we must respect the following equation,

$$y_1 = f(y_2; E_1, \hat{E}_2 | P_0, y_0) \quad (19)$$

Here we have separated the \hat{E}_1 from period 0 into a part, E_1 assigned to period 1 and a ‘new’ future \hat{E}_2 ; likewise we have transferred the y_0 to period 2 as part of the history. By continued insertions for T forward looking periods we can replace the above setup by a general implicit equation,

$$F(y_0; y_1; y_2; \dots; y_T; E_{-t}, \dots, E_0, E_1, \dots, E_T; y_{-1}, y_{-2}, \dots, y_{-t}) = 0 \quad (20)$$

Here the set $\{y_0, y_1, y_2, \dots, y_T\}$ is the REH solution of the model for a finite horizon, represented by T periods. Lagged values of y are predetermined and all the E’s are exogenous, but so that lagged values are historic, while future and possibly simultaneous E’s are expected, but non-controllable variables. For simplicity we have assumed that the f-function is independent of time.

Now, let us ask the question: what happens when we let T grow—and τ ! As long as T and τ are finite, the mathematical setup is valid in principle, albeit troublesome in practice.

When T and τ are very large, but finite we still run into epistemological problems, however, if we adhere to the assumption that decisions take time, ie the duration between the separated decision points is finite in absolute time. Since time in our setup is additive, our model will stretch its period of coverage from the very far past to the very distant

future, a lapse of time that by far exceeds the life of the decision makers. Then we must include in our model that the transition between ‘generations’ needs an explicit modelling (heritage; how does the mother plan the future of her child and what does the child take over); this aspect ends in non-operational nonsense²⁴.

In conclusion, to make REH-models worthwhile, they must build on a finite calendar time horizon, past and future.

11. Concluding remarks

This Odyssey has tried to explicate some of the basic problems that are underlying economic modelling.

On the background above it seems clear that the philosophical reflections on the concept of time are not directly useful to the profession of theoretical economics. In ordinary economics, where the essential characteristic of time is solely its sequential nature, we normally consider such problems as unpleasant nuisances and do what we can to conceal or suppress them. For operational and pragmatic purposes such an approach is absolutely defensible—in fact we have no alternative.

When, however, it comes to scientific honesty, an overt recognition of the imperfection of our profession is called for.

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²⁴Obviously, if T and τ go to infinity, the model goes from past infinity to eternity; not a very operational setup.

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