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The formation of inflation expectations under changing inflation regimes*

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Abstract

The present paper offers a careful description on empirical identification of possible multiple changes in regime. We apply recently developed tools designed to select between regime switching models among a broad class of linear and nonlinear regression models and provide a discussion on the impact on the formation of inflation expectations in the presence of multiple and recurrent changes in inflation regimes. Our empirical findings give a plausible explanation why the rational expectation hypothesis based on direct measures of inflation expectations from survey series is typically rejected due to large systematic differences between actual and expected inflation rates. In particular, our results indicate that in the case of changing and not perfectly observed inflation regimes, inference about rationality and unbiasedness based on a comparison of *ex ante* forecasts from survey series and actual inflation rate based on *ex post* realizations, will be ambiguous due to the presence of an *ex post* bias. The empirical findings are based on Danish inflation rates covering the period from 1957 -1998. We show that it is not possible to reject the hypothesis of multiple inflationary regimes and that the actual inflation rate can be represented

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by a two state Markov regime switching model. It turns out that the real time forecasts produced from this model exhibit a large degree of similarity when compared to the direct measures of inflation expectations. The result illustrates the important impact of switching regimes on the formation of actual and expected inflation and hence of *ex post* bias as a main contributor to the difference between actual and expected inflation observed directly from survey series.

JEL classification: C14; C22; C42; C51; C52; C53

Keywords: Markov regime switching; Ex post bias; Inflation and inflation expectations

1 Introduction

It is by now well established that expectations about future inflation matter in many different macroeconomic contexts. Savings and investments decisions are well known examples. The same is wage formation according to e.g. the expectations augmented Phillips-curve, and nominal interest rates should depend one-to-one on expected inflation according to the Fisher equation. Contrary to the monetary-business-cycles theory of Lucas focusing on expected actual rate of inflation, several new-Keynesian models predict a relationship between economic activity and the difference between actual and expected future inflation, see e.g. Roberts (1995). In a large number of empirical studies, notably Phillips-curve estimations, expected inflation is often proxied by current and lagged levels of inflation as suggested by simple backward-looking expectation formation. However, in many contexts this approach turns out to be too crude and more proper measures of expected inflation are warranted, e.g. in accordance with rational, forward-looking expectations.

In many countries survey-based data on expected future inflation exist and hence constitute a natural approach to the problem. In this context the perhaps two most intensively studied survey series on inflation expectations are the Livingston price expectation data and the SRC consumer survey data at the University of Michigan, see Rich (1989,1990) and the references herein. In general, the results on rationality/unbiasedness of expectations from these studies based on comparisons with actual inflation have been mixed. Engsted (1991) and Christensen (1996) represent survey-based studies of Danish households' inflation expectations. They find that inflation expectations have been markedly biased over extended periods. In particular, they find that the inflation expectations tend to over-predict actual inflation in periods with low inflation, especially in the nineties, and under-predict in periods with high inflation, like e.g. in the seventies. These features also seem to be present in the Livingston and the SRC survey series according to Evans and Wachtel (1993). This of course casts doubt on the rational expectations hypothesis in the sense that agents persistently ignore relevant information. However, evidence of bias and serial correlation in expectation errors does not definitely refute the hypothesis of rationality, as pointed out by Jonung and Laidler (1988) and Evans and

Wachtel (1993), who refer to "peso problems" and "possible changes in regime". They argue that if the inflation rate evolves according to a single regime for a sustained period, but households expect a regime shift in the inflation process with a non-zero probability, then expectations might actually be persistently biased and serially correlated.

The observed persistence in the difference between ex ante forecasts and ex post realizations, will be denoted *ex post* bias. In order to determine whether the observed wedge between the household sectors perceptions and expectations could be due to *ex post* bias we offer a careful examination of the stochastic process underlying the actual inflation rate process. The basic premise is that if it is not possible to reject the hypothesis that the actual inflation rate follows a regime shifting model capable of producing the wedge between perceptions and expectation of the magnitude actually observed, then it is not possible to conclude - as it is often done in the literature - that the household sector's inflation rate expectations are formed in contradiction with the rationality hypothesis.

As Evans and Wachtel (1993) we use the Markov regime switching approach as a workhorse in order to quantify the *ex post* bias. However, we elaborate extensively on their approach. Firstly, we present numerous tests for identification of instability of the linear forecasting model. Secondly, we provide a battery of specification tests ranging from tests based on flexible regression models such as the Neural Network test of White (1992) and the test recently suggested by Hamilton (1999) to the more well known Reset test in order to identify nonlinear components in the inflation rate. Thirdly, having identified the need for a nonlinear component in the regression model possibly in the shape of a regime shifting model, we test the Markov regime switching model specification against the linear model using the test proposed by Hansen (1996). Fourthly, using the approach recently suggested by Dahl and Hylleberg (1999) based on the flexible parametric regression model of Hamilton (1999) and the non-parametric projection pursuit regression model of Aldrin, Boelviken and Schweder (1993) , we are able to evaluate the adequacy of the Markov regime switching model against a wide range of competing nonlinear models/specifications of the inflation rate. Fifthly, using the Bayesian approach suggested by Hamilton (1999) we provide evidence that the nonlinearity in the conditional mean function produced by the Markov regime switching model is in accordance with the data. Finally, we test the Markov regime switching model for parameter stability and for dynamic misspecification. Contrary to Evans and Wachtel (1993) who use *in sample* forecasts, we suggest using real time forecasts produced from the Markov regime switching model when evaluating the size of the *ex post* bias. By our approach we are able to produce a forecast based on the same information set as that of the household sector when they form expectations. It therefore constitutes a more natural way for comparison.

We find that the exact real time forecasts produced from our model exhibit a large degree of similarity when compared to the direct measures of inflation expectations. In particular, our results show that one-year ahead expectation errors generated from the well-specified regime switching models do exhibit severe bias and serial correlation as it is the case for the expectation errors based on survey data. We believe that the result illustrates the important impact from possible regime switches on the formation of actual and expected inflation and of *ex post* bias as a significant contributor to the difference between actual and expected inflation observed directly from survey series.

The paper is organized as follows. In the next section we briefly discuss how the existence of unobserved shifts in regimes can lead to a persistent gap between actual and expected inflation. Against the background of existing studies on survey based measures of Danish households' inflation expectations we discuss common pitfalls and in particular why these studies tend to lead to a rejection of the rational expectations hypothesis. In Section 3 we test the rational expectations hypothesis based on the linear forecasting/regression model and based on actual inflation rates in Denmark while we in section 4 present and carefully test the Markov switching forecasting/regression model. Section 5 contains economic interpretations of the results, and finally section 6 concludes.

2 Preliminaries

2.1 Inflation forecasting in the presence of changes in regime: The notion of ex post bias

The consequences for the inflation rate forecast of a switch in regime can most easily be illustrated by a simple example with two inflation regimes[1]. Let us assume that the inflation rate π_t evolves over time according to the following rule

$$\pi_t = (1 - S_t)\pi_{0t} + S_t\pi_{1t} \tag{1}$$

where S_t is a dichotomous unobserved stochastic variable that can take only the values zero and one. Let us for simplicity assume that π_{0t} and π_{1t} are stochastic variables, independent of the current realizations of S_t. The interpretation of the above equation is that if $S_t = 1$, the inflation rate is said to be in regime 1, where it is governed by the process of π_{1t}. Otherwise, if $S_t = 0$ we say that the inflation rate is in regime 0 and is determined solely by the development in π_{0t}, that is supposed to be different from that of π_{1t}. If we assume that expectations are formed rationally and we let Y_{t-1} denote all the available information at the beginning of period t, the unconditional expected inflation rate conditional on $S_t = i$ for $i = \{0, 1\}$ is given by

$$E(\pi_t|S_t = i, Y_{t-1}) = E(\pi_{it}|Y_{t-1}) \tag{2}$$

Letting $P(S_t = 0|Y_{t-1})$ and $P(S_t = 1|Y_{t-1})$ denote the probability measures of being in regime 0 and regime 1 respectively, the unconditional expected inflation rate can be found by summation over all the possible realizations of S_t[2]. This

[1] This example is lending on the work by Evans and Wachtel (1993).

[2] Unconditional with respect to the unobserved state variable S_t.

yields the following expression for the expected inflation rate

$$E(\pi_t|Y_{t-1}) = P(S_t = 0|Y_{t-1})E(\pi_{0t}|Y_{t-1}) + P(S_t = 1|Y_{t-1})E(\pi_{1t}|Y_{t-1}) \quad (3)$$

and the inflation forecast errors become

$$\begin{aligned} \pi_t - E(\pi_t|Y_{t-1}) &= (1 - S_t)\pi_{0t} + S_t\pi_{1t} - \\ &\quad P(S_t = 0|Y_{t-1})E(\pi_{0t}|Y_{t-1}) - P(S_t = 1|Y_{t-1})E(\pi_{1t}|Y_{t-1}) \end{aligned} \quad (4)$$

From the above equation it is fairly easy to verify that *ex ante* the forecast error has a mean value equal to zero, that is $E\{\pi_t - E(\pi_t|Y_{t-1})|Y_{t-1}\} = 0$. However, the major implication of the model is that the rationally formed expectations can appear to be biased when viewed *ex post* . To illustrate this, let us assume that the actual inflation rate currently follows the regime one process. Recall that the economic agents do not observe the current regime so the best guess they can come up with is given by the unconditional expectation $E(\pi_t|Y_{t-1})$. Subtracting this term from the inflation rate currently following a regime-one process equals

$$\begin{aligned} \pi_{t|s_t=1} - E(\pi_t|Y_{t-1}) &= \{\pi_{1t} - E(\pi_{1t}|Y_{t-1})\} + \\ &\quad P(S_t = 0|Y_{t-1})\{E(\pi_{1t}|Y_{t-1}) - E(\pi_{0t}|Y_{t-1})\} \end{aligned} \quad (5)$$

Here, the first term on the right-hand side equals on average zero because of the assumptions of rationally formed expectation and because the inflation rate actually develops according to the regime-one process in period t. However, if the economic agents place some weight on the probability of the inflation rate being in a regime-zero process the second term on the right hand side will not have zero mean as long as the expected values of π_{1t} and π_{0t} differ. This implies that the forecast $E(\pi_t|Y_{t-1})$ will appear biased when viewed *ex post* even though agents are using all available information efficiently in making their forecast. This is the phenomenon known as *ex post* bias. In general ex post bias occurs when inflation follows a process allowing for several regimes, and agents when forming their expectations incorporate the possibility of a regime switch. As long as the actual process continues in - say - regime one and the possibility of a switch to regime zero persists, rational forecasts will appear biased. In order to get an estimate of the size of the *ex post* bias one must specify the processes generating π_{1t} and π_{0t}. In addition, one also has to specify a probability distribution of the stochastic variable S_t. In this setup the h-steps ahead forecast will be given by

$$\begin{aligned} E(\pi_{t+h-1}|Y_{t-1}) &= P(S_{t+h-1} = 0|Y_{t-1})E(\pi_{0t+h-1}|Y_{t-1}) + \\ &\quad P(S_{t+h-1} = 1|Y_{t-1})E(\pi_{1t+h-1}|Y_{t-1}) \end{aligned} \quad (6)$$

Evans and Wachtel (1993) suggest using a two state Markov regime switching approach with constant transition probabilities in order to determine the size of the *ex post* bias. In that case

$$\begin{bmatrix} P(S_{t+h-1} = 0|Y_{t-1}) \\ P(S_{t+h-1} = 1|Y_{t-1}) \end{bmatrix} = \begin{bmatrix} p & 1-q \\ 1-p & q \end{bmatrix}^h \begin{bmatrix} P(S_{t-1} = 0|Y_{t-1}) \\ P(S_{t-1} = 1|Y_{t-1}) \end{bmatrix} \quad (7)$$

where $p = P(S_t = 0|S_{t-1} = 0)$ and $q = P(S_t = 1|S_{t-1} = 1)$ denote the constant transition probabilities. Furthermore, if it is assumed that we can represent the inflation rate as autoregressive processes in both regimes such that

$$\pi_{st} = \Gamma'_{t-1}\beta_s + \epsilon_{st} \text{ for } s = \{0, 1\} \tag{8}$$

where $\Gamma_t = \{1, \pi_t, \pi_{t-1,..,}\pi_{t-k+1}\}$ the rational forecasting rule is given according to the following recursive scheme for $j = 1$ to h

$$\begin{aligned} E(\pi_{t+j-1}|Y_{t-1}) &= \widehat{\Gamma}'_{t+j-2}[\beta_0 + P(S_{t+j-1} = 1|Y_{t-1})(\beta_1 - \beta_0)] \\ \widehat{\Gamma}'_{t+j-2} &= \{1, E(\pi_{t+j-2}|Y_{t-1}), E(\pi_{t+j-3}|Y_{t-1}), .., E(\pi_{t+j-k-1}|Y_{t-1})\} \end{aligned} \tag{9}$$

The analytical expression for the inflation expectations is very tedious to write down for high dimensional models due to the assumptions on the dynamics of the inflation process given by equation (8) which differs from the dynamics of the inflation rate assumed by Evans and Wachtel. To illustrate the implications of these "new dynamics" to the agents rational forecasting rule and in order to compare it with the forecasting rule used in Evans and Wachtel (1993), let us write down the case of $k = 1$. If we let the inflation rate in the two regimes be given by

$$\pi_t = \begin{cases} \alpha_0 + \alpha_1\pi_{t-1} + \epsilon_{0t} & \text{for } S_t = 0 \\ \vartheta_0 + \vartheta_1\pi_{t-1} + \epsilon_{1t} & \text{for } S_t = 1 \end{cases} \tag{10}$$

the expectations h periods ahead will be given as

$$\begin{aligned} E(\pi_{t+h-1}|Y_{t-1}) = & [\alpha_0 + P(S_{t+h-1} = 1|Y_{t-1})(\vartheta_0 - \alpha_0)] \\ & + \textstyle\sum_{j=1}^{h}\{[\alpha_0 + P(S_{t+j-1} = 1|Y_{t-1})(\vartheta_0 - \alpha_0)] \\ & \times \textstyle\prod_{v=j+1}^{h}[\alpha_1 + P(S_{t+v-1} = 1|Y_{t-1})(\vartheta_1 - \alpha_1)]\} \\ & + \textstyle\prod_{j=1}^{h}[\alpha_1 + P(S_{t+j-1} = 1|Y_{t-1})(\vartheta_1 - \alpha_1)]\pi_{t-1} \end{aligned} \tag{11}$$

This expression is by far more complicated than the analogous equation (6) in Evans and Wachtel (1993, p. 491). This hinges on the fact that Evans and Wachtel assume the two underlying processes to be completely independent, i.e. $\pi_{st} = f_s(\pi_{st-1})$, $s = \{0, 1\}$, hence allowing for discrete jumps in the inflation rate process despite the underlying process being continuous, whereas we only allow for jumps in the parameters by assuming $\pi_{st} = f_s(\pi_{t-1})$, $s = \{0, 1\}$. Our two underlying processes are hence interrelated since both depend on the lagged values of the observed inflation rate, and it means that the possibility of the inflation rate shifting back and forth between regimes within the forecast interval must be taken into account. Under the simpler approach by Evans and Wachtel this complicating feature can be disregarded. A full description of the methodology for estimating the parameters of the Markov regime switching model and on how to construct the filter probabilities - $P(S_t = 1|Y_{t-1})$ - is given in section 4. However, before turning to the more complex nonlinear modelling procedures we will verify that a linear representation cannot provide an adequate description of the inflation rate process. Since parsimonious models often outperform less parsimonious models in terms of real time forecast accuracy, linear models will typically be preferred to nonlinear models implying that we have to make sure that a nonlinear representation is actually needed.

2.2 Quantification of qualitative data on inflation expectations

For most European member-countries of the European Union qualitative data on households' expectations about inflation one year ahead as well as the households' perceptions of last year's price development exist[3]. The data are produced by interviewing a representative sample of people of age between 16 and 74. The persons are asked two questions: "How is the price level today compared to the price level one year ago?" and "How will the price level be one year ahead compared to the price level today?". The five possible answers are "much higher" (weight: +1), "somewhat higher" (weight: $+\frac{1}{2}$), "slightly higher" (weight: 0), "unchanged" (weight: -$\frac{1}{2}$), and "slightly lower" (weight: -1). A single measure of the household sector's inflation expectations is then produced by adding up the weights of all the households. Assuming the same relationship for both past and future inflation between qualitative and quantitative data on inflation, a quantitative measure of inflation expectations can be deduced by simple regression, cf. Pesaran (1987). Such a regression represents nothing but a simple conversion from qualitative into quantitative measures of inflation expectations, and gives no causal explanation of inflation expectations. A quantification of the categorical/ordinal responses is done by the regression method, assuming the relationship between actual price changes and survey responses be given by

$$\pi_t = \alpha_1 MH_t^p + \alpha_2 SH_t^p + \alpha_3 LH_t^p + \alpha_4 U_t^p + \alpha_5 LL_t^p + \epsilon_t \tag{12}$$

where ε_t is a measurement error, assumed to follow a Gaussian distribution. MH_t^p denote the fraction of households with a perception that prices today are much higher than one year ago, SH_t^p somewhat higher, LH_t^p a little higher, U_t^p unchanged, and LL_t^p a little lower. Let $MH_{t-1|t}^e$, $SH_{t-1|t}^e$, $LH_{t-1|t}^e$, $U_{t-1|t}^e$ and $LL_{t-1|t}^e$ represent the same fractions with respect to one year ahead expectations, respectively. Expected inflation can then be derived by the following conversion formula

$$\widehat{\pi}_{t-1|t}^e = \widehat{\alpha}_1 MH_{t-1|t}^e + \widehat{\alpha}_2 SH_{t-1|t}^e + \widehat{\alpha}_3 LH_{t-1|t}^e + \widehat{\alpha}_4 U_{t-1|t}^e + \widehat{\alpha}_5 LL_{t-1|t}^e \tag{13}$$

where $\widehat{\alpha}_i$, for $i = \{1, .., 5\}$ are OLS-estimates of α_i, in (12). One very crucial assumption upon which Pesaran's approach rests is that the perceptions are rational so that the perception errors - given by ϵ_t in equation (12) - are without

[3]The Danish data are collected by Statistics Denmark since 1974, until 1983 three times a year. Since then the data frequency has increased, and since 1987 figures are available on a monthly basis, though not in June before 1997. After 1988 gross figures on the number of respondents in the different categories are no longer reported.

any bias and serial correlation. Secondly, it requires that the regression coefficients in (12) are stable and that the actual inflation is normally distributed. The consequences of the actual inflation rate following a nonlinear regime shifting model can now easily be understood. It basically implies failure of all the critical assumptions of Pesaran's regression models approach to hold. In particular, if the actual inflation rate follows a two state Markov regime switching model the perception errors can actually be temporally bias and serially correlated due to the fact that the regimes are not perfectly observable ex post. Obviously, the inflation rate will not be normally distributed asymptotically in this situation but instead distributed as a mixture of two normals. Finally, this implies that it is very unlikely that the regression coefficients in (12) will be even approximately stable. Consequently, before undertaking Pesaran's regression model approach one has to check carefully whether the basic assumptions stated above hold. Conditional on Danish survey data Engsted (1991) and Christensen (1996) represents two studies of the expectation hypothesis based on Pesaran's regression model approach. The data on the household sector inflation perceptions and expectations which they use are depicted in figure 1.

$$[Figure\ 1]$$

Notice, that when the actual inflation rate is high as in the seventies and the beginning of the eighties inflation expectations are below inflation perceptions, whereas in the nineties where actual inflation rates are very low inflation expectations are well above inflation perceptions. For the sub-period 1986 to 1996, Christensen (1996) shows that Danish households' inflation expectations may have a serious upward bias in the latter part of the investigated period. According to his study, expected inflation has been relatively stable around 3 per cent P.A. throughout the nineties, whereas actual inflation has been persistently lower. Engsted (1991) studies gross figures for the period 1975 to 1990, and his analysis confirms that inflation expectations have a marked upward bias in periods with low inflation, and vice versa. Unfortunately, in neither of the two studies the crucial assumptions discussed above are checked carefully. Hence one of the main contributions of our paper will be a careful discussion on how to perform a proper model evaluation and selection which should enable us to evaluate the validity of the results of Engsted (1991) and Christensen (1996). Furthermore, the methods discussed will provide us with an extensive guidance on how to specify the proper Markov regime switching model as a superior alternative the linear regression representation of the actual inflation rate.

Finally, it is worth mentioning that similar systematic over-predictions of the Danish inflation rate found by Engsted (1991) and Christensen (1996) have been made by official forecasters, such as the government, OECD, etc. Despite the difficulties to track the actual inflation rate, it is striking that apparently systematic forecast errors of opposite signs, i.e. under-predictions of the inflation rate, are present in the period with high rates of inflation, and vice versa.

2.3 Flexible regression models

In order to determine the existence of nonlinear components in the inflation rate process we use two different flexible regression models. We use a parametric approach recently suggested by Hamilton (1999) denoted FNL and we use the more familiar nonparametric projection pursuit approach in a form suggested by Aldrin, Boelviken and Schweder (1993). The basic idea underlying both approaches is to estimate the conditional mean function of the time series y_t without imposing any restrictions on the functional form of the conditional mean function of y_t The flexible regression model can be written as

$$y_t = \mu(x_t, \delta) + \epsilon_t \tag{14}$$

where ϵ_t is a sequence of $NI(0, \sigma^2)$ error terms and $\mu(x_t, \delta)$ is the conditional mean function. x_t is a $k \times 1$ vector, which may include lagged dependent variables. In Hamilton's approach the conditional mean function , i.e. $\mu_{fnl}(x_t, \delta)$, is represented as having a linear part and a stochastic nonlinear and stochastic part according to[4]

$$\mu_{fnl}(x_t, \delta) = x_t^{'}\beta + \lambda m(g \odot x_t) \tag{15}$$

where only the linear part is perfectly observable up to an unknown parameter β and the nonlinear random function $m(.)$ depends on the parameter vector g determining the curvature of the function and on the regressors x_t. λ is a parameter that determines the weight to assign to the nonlinear component in the conditional mean function. Notice, that if the hypothesis $\lambda = 0$ cannot be rejected the model is purely linear. Notice further that inference about whether the regressors x_{it}, $i = 1, .., k$ should enter nonlinear in the conditional mean function can be based on the null that $g_i = 0$. Estimation of the parameters of the model given by $\delta = \{\beta, \lambda, g, \sigma\}$ as well as the estimation of the nonlinear random function $m()$ is carried out by the principle of maximum likelihood, see Hamilton (1999) for details. In the projection pursuit approach the conditional mean function is represented as

$$\begin{aligned} \mu_{ppr}(x_t, \varrho) &= x_t'\beta + \sum_{j=1}^{v} \omega_j \varphi_j(x_t'\Phi_j) \\ \varrho &= \{\beta, \omega_1, ...\omega_v, \Phi_1, .., \Phi_v\} \end{aligned} \tag{16}$$

The parameters Φ_j define the projection of the input vector x_t onto a set of planes labelled by $j = 1, .., v$. These projections are transformed by the nonlinear activation functions denoted $\varphi_j(.)$ which in our case are taken to be a cubic spline function and these in turn are linearly combined with weight ω_j to form the output variable y_t. The algorithm for estimating the parameters is described in details by Aldrin et al. (1993).

[4]Here g is a $k \times 1$ vector of parameters and $\odot$ denotes element-by-element multiplication i.e. $g \odot x_t$ is the Hadamard product. β is a $kx1$ vector of coefficients.

3 The linear representation of the inflation rate

As a first attempt to model the actual inflation rate we begin by considering linear representations. But before we turn to the actual modelling Let us have a short look at the development in the Danish quarterly inflation rate in the period 1958q3-1998q4 depicted in figure 2.

$[Figure\ 2]$

The most striking feature is the shift in level that seemed to occur very rapidly in the middle of the eighties. A transition from very high and volatile inflation rates in the seventies to low and much more stable inflation rates in the nineties. Over the whole period the mean inflation rate has been about 1.4 per cent a quarter with a standard deviation around 1.2. Inspection of the results reported in table 1 reveals that it is very unlikely that the inflation rate follows a normal distribution based on a Jacque-Bera test. This evidence is confirmed in figure 3 that is a plot of the estimated density of the inflation rate. The density seems to be too fat right-tailed to be approximated by a normal density. Notice further that a mean and variance shift of the inflation rate could produce of mixture density of a shape consistent with the estimated density. We will return in detail to that subject later.

$[Table\ 1]$
$[Figure\ 3]$

The Augmented Dickey-Fuller tests for stationarity on the inflation rate are reported in table 2. In general is seems that the hypothesis of nonstationarity is strongly rejected when the number of lags included in the auxiliary regression do not exceed 2. Furthermore, a closer look at the auxiliary regressions equations in the case of up to 3 and up to 4 lags included in table 3 and table 4 respectively reveals that the third and fourth lags do not enter significantly into the auxiliary regression, hence causing a potential loss in power of the augmented Dickey-Fuller statistics.

$[Table\ \ 2]$
$[Table\ \ 3]$
$[Table\ \ 4]$

We are now ready to turn to the linear modelling of a univariate dynamic representation of the inflation rate. The aim is to analyze whether it is consistent to model the inflation rate by a dynamic one-regime econometric model. The model is represented as

$$\begin{aligned} \pi_t &= \alpha + \sum_{j=1}^{k} \beta_j \pi_{t-j} + \epsilon_t \qquad (17) \\ \epsilon_t &\sim N(0, \sigma^2) \end{aligned}$$

The AR(4)-model is estimated using recursive least squares and the estimation results are presented in table 5.

[$Table$ 5]

A first question of interest is whether the one year ahead forecasts produced by the estimated AR(4) model share some of the same characteristics with the survey based inflation expectations. Inflation rate forecasts produced by the AR(4) reveal a similarity with inflation expectations based on gross survey data, in the sense that forecast based on an AR(4) model produces a negative wedge between actual inflation and expectation in the seventies and a clear-cut positive wedge in the nineties see figure 4 for the one quarter ahead forecasts and figure 5 for the one year ahead forecasts.

[$Figure$ 4]

[$Figure$ 5]

The distribution of the one period ahead prediction of the linear model depicted in figure 6 confirms that the estimated median of the inflation rate expectation is higher that the median of the actual inflation rate. The density plot also reveals that the linear model is not able to provide sufficient fatness in the distribution to the right.

[$Figure$ 6]

The next question of interest is whether the observed expectation bias could arise due to a mis-specified model. If this was to be the case and the similarity of the forecast is in fact because the household sector uses an AR(4) representation when forecasting inflation, the observed bias is indeed a result of non-rational behavior and a clear-cut violation of the rational expectation hypothesis. Looking at the dynamic specification tests reported in table 6 it is not possible to reject the hypothesis that the linear model is dynamically well specified due to the absence at a 5 percent significance level of neglected autocorrelation and of ARCH effects up to an order of 4.

[$Table$ 6]

However the stability of the model seems more critical. Based on the well known 1-step Chow test for structural breaks within the sample period depicted in figure 7 and the forecast (N-step up) Chow tests - depicted in figure 8 - the null of stability of the model is rejected at a 5 percent level and the test gives some indications of one or more structural breaks over the period. In contradiction to this evidence stands the Break-point (N steps down) Chow test where stability cannot be rejected at a five percent level, see figure 9.

[$Figure$ 7]

[$Figure$ 8]

[$Figure$ 9]

In order to add some more information we now turn to some more powerful tests for detecting multiple structural break suggested by Andrews, Lee and Ploberger (1996) . Based on an extensive Monte Carlo study they cast some doubt on the power of the traditional Chow test for various kinds of structural breaks and regime shifts. Instead they suggest three optimal change-point tests for detecting the presence of non-stable coefficients in a normal linear regression model with unknown breakpoints[5]. The three test-statistics denoted $SupF$, $ExpF$ and $AvgF$ are presented in table 7. Based on all three statistics it is possible to test the joint hypothesis of stability of all the parameters in the model as well as the hypothesis that a single parameter is stable. By inspection of table 7 it is evident that the hypothesis that all the parameters are stable is clearly rejected at a 5 percent level by all three tests. In fact the three tests all agree on the fact that only the second and fourth lag of the inflation rate enter stably in the linear regression model while the stability of the constant term and the coefficient related to the first and third lag of the inflation rate clearly is rejected. Finally we consider Nyblom's (1989) test for parameter stability against the hypothesis that the regression coefficients follow a martingale process. This test for stability of the parameters of the linear model is also rejected at a 5 percent level.

[$Table$ 7]

Finally, we conclude this section by testing the linear AR(4) model for neglected nonlinearity. If instability of the linear model of the inflation rate is due to the fact that the true inflation rate were in fact generated by a regime shifting process we would expect these specification test to reject the null hypothesis of linearity. The results from the specification test are reported in table 8. The battery of specifications tests include Hamilton's (1999) test, the neural network test due to White (1992), Tsay's (1986) polynomial test, White's (1987) information matrix test, Ramsey's (1969) Reset test, and finally the nonparametric BDS-test due to Broch, Deckert and Scheinkman (1987). The evidence of the presence of a neglected nonlinear component seems a little mixed. While Hamilton's test, Tsay's test, White's test and the Reset test give support to the null of linearity the Neural Network test and the BDS test clearly rejects the null of linearity.

[$Table$ 8]

Summing up there seems to be clear evidence of the presence of one or more structural breaks in the inflation rate process during the period from 1958 to 1998. On this account it is fairly safe to conclude that the persistent expectation

[5] Based on a wide range of Monte Carlo experiments Andrews, Lee and Ploberger (1996) show that their test statistics appear to be superior in power. The probability distributions of the test statistics are non-standard under the null which makes them a bit more tedious to use. Hansen (1997) recently constructed a response surface for calculating the p-values for the three optimal change-point tests suggested by Andrews, Lee and Ploberger (1996) making inference straightforward.

bias observed is due to shifts in the parameters of the AR(4) model. This implies that if the households use this model to forecast the inflation rate, their behavior is indeed non-rational. This raises the issue whether there exists an alternative empirical - possibly nonlinear - representation that takes into account the regime shifts that apparently occur in the inflation process and at the same time produces the same degree of similarity when compared to the survey based measures of expected inflation as the forecasts from the AR(4) model apparently do. In the next section such a new candidate - the Markov regime switching model - will be presented and discussed.

4 A two-state Markov regime switching representation of the inflation rate

In this section we investigate whether the Danish inflation rate can be represented by a two state Markov regime switching model. The section begins with a short description of the statistic representation of the model and it is discussed how the unknown parameters of the model are estimated. The model we consider in this section is a "non-centered" version of Hamilton's (1989) Markov regime switching model[6]. In addition to Hamilton's "centered" model we allow both the constant term and the autoregressive coefficients of the AR(4)-representation of the inflation process to switch between possible regimes, thereby taking into account the evidence presented in the last section on the instability of these parameters in the one-regime AR(4) model. We also allow the variance of the inflation rate to shift across the different regimes. The inflation regime at date t is indexed by an unobserved random variable S_t. In our setup S_t has two possible outcomes - $S_t = 0$ if the inflation process is in regime 0 and $S_t = 1$ if the inflation process is in regime 1. We assume that the transitions between the regimes are governed by a two-state Markov chain. The "non-centered" Markov switching regime model can be given a simple state space representation with one measurement equation representing the actual inflation rate and one transition equation representing the unobserved state variable S_t. This writes

$$\pi_t = (1 - S_t)\{\alpha_0 + \sum_{j=1}^{k_0} \beta_{0j}\pi_{t-j} + \epsilon_{0t}\} + S_t\{\alpha_1 + \sum_{j=1}^{k_1} \beta_{1j}\pi_{t-j} + \epsilon_{1t}\}$$

$$\begin{bmatrix} \epsilon_{0t} \\ \epsilon_{1t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_0^2 & 0 \\ 0 & \sigma_1^2 \end{bmatrix}\right) \tag{18}$$

[6]Hamilton typically represents the dependent variable in deviations from its mean (the variable is said to be centered around its mean), where the mean of the lagged dependent variable is a function of the lagged value of the state variable. In our setup we do not represent the inflation rate in deviations from its mean and only the current value of the state variable matters.

and with the constant transition probabilities of the Markov chain defined as

$$\begin{aligned} p(S_t &= 0|S_{t-1} = 0) = p \\ p(S_t &= 1|S_{t-1} = 0) = 1 - p \\ p(S_t &= 0|S_{t-1} = 1) = 1 - q \\ p(S_t &= 1|S_{t-1} = 1) = q \end{aligned} \tag{19}$$

There are $6 + k_0 + k_1$ unknown parameters in the model to be estimated. These are collected in the vector $\lambda = \{\alpha_0, \alpha_1, \beta_{01}, .., \beta_{0k_0}, \beta_{11}, .., \beta_{1k_1}, \sigma_0^2, \sigma_1^2, p, q\}$. As shown by Hamilton (1989) the conditional density function for π_t in this setup is a mixture of two normal densities weighted by the filter probabilities of being either in state $S_t = 0$ or $S_t = 1$. If we let Y_t denote the information set containing the observations $\pi_1 ... \pi_t$ and use successive conditioning, the conditional likelihood function - given an initial observation for π_0 - can be written as

$$l(\lambda) = \prod_{t=1}^{T} \sum_{i=1}^{2} \frac{1}{(2\pi\sigma_i^2)} \exp\left(\frac{-(\pi_t - \alpha_i - \sum_{j=1}^{k_i} \beta_{ij}\pi_{t-j})^2}{2\sigma_i^2} \right) P(S_t = i|Y_{t-1} : \lambda) \tag{20}$$

Provided that the conditional probabilities $P(S_t = i|Y_{t-1} : \lambda)$ exist and can be evaluated at $t = 1, .., T$, the maximum likelihood estimates $\widehat{\lambda}$ can be found solving $arg\ max\ l(\lambda)$ using suitable numerical constrained optimization algorithms[7]. Before turning to the actual estimation of parameters of the Markov regime switching model, we first consider a specification test - proposed by Hansen (1992,1996) - in order to draw some inference whether the null of the AR(4) representation can be rejected in favor of the Markov switching model. Secondly using an approach based on forecast accuracy suggested by Dahl and Hylleberg (1999) we test the null of the Markov regime switching model against a wide class of linear and nonlinear models. By this approach the models under the alternative is assumed to be represented by one of two different flexible regression model approaches consisting of Hamilton's parametric flexible regression model - denoted FNL - and the non-parametric projection pursuit regression model of Aldrin, Boelviken and Sweder (1993) - denoted PPR-L.

As pointed out by Hansen (1992) classical test statistics such as the LR, $Wald$ and LM statistics are not asymptotically χ^2 distributed in this case. These statistics are all based on regularity conditions ensuring that the likelihood surface is locally quadratic and that the score-vector has a non-zero variance. These conditions are all violated here. First of all because there are two nuisance parameters (the transition probabilities) not identified under the null making the likelihood surface flat at the optimum. Secondly because the null hypothesis yields a local optimum of the likelihood surface, implying that the score-vector is identically equal to zero, hence contradicting the non zero variance condition. By working directly with the likelihood surface, viewing the

[7]In a series of papers Hamilton carefully describes an algorithm for obtaining the sequences of $\{P(S_t = i|Y_{t-1} : \lambda)\}_{t=1}^{T}$. The reader is refered to Hamilton (1994) for further details.

likelihood function as an empirical process for the unknown parameters Hansen (1992,1996) derives a test statistics that does not require the likelihood function to be locally quadratic or requires the scores (or for that matter any other higher order derivative) to have positive variance. The cost of working with empirical process theory is that Hansen is only able to derive a boundary and not an asymptotic distribution for the standardized likelihood ratio test he suggests. However, based on a Monte Carlo experiment Hansen (1992,1996) shows that the size and power properties of his standardized likelihood ratio statistics are reasonably good. Hansen's standardized likelihood ratio test is known to be rather cumbersome to compute even in small parametric models. In our fairly parsimonious representation this was not a serious problem. The results from conducting Hansen's (1992) tests are reported in table 9, where LR_T^* denotes the standardized likelihood ratio test statistics and M denotes the chosen bandwidth, see Hansen (1996) for details. Independent of the choice of bandwidth the associated p-values are all very small suggesting that we can reject the null of a linear AR(4) model as an adequate description of the Danish inflation rate in favor of the two-state Markov switching model.

[$Table$ 9]

A crucial question is whether the Markov regime switching model is the preferred nonlinear specification among the large class of nonlinear models that can mimic multiple regime shift or smooth transitions between regimes. Recently Dahl and Hylleberg (1999) have suggested a general to specific approach as a device for selecting between nonlinear models. In particular they suggest using flexible nonlinear regression models as baseline models under the null and to test - in this case - the Markov switching models against this broad class of models in terms of recursive real time forecast accuracy. Due to the fact that the statistics suggested by Dahl and Hylleberg do not depend on any unidentified parameters the approach turns out to be particularly useful when discriminating between models containing nonlinear components and parameters not identified under the alternative. The first step in the Dahl-Hylleberg procedure consists of generating an h-steps ahead forecast in real time. In our case we set $h = 1$ and select and estimate the models based on the initialization period given by 1958q3-1979q4. Conditional on this information set a forecast of 1980q1 is produced. In the next step, the various models are selected and estimated on the period 1958q3-1980q1 and a real time forecast for 1980q2 is made. This procedure continues until we have generated a sequence of true out-of-sample forecast on the period 1980q1-1998q4 from the linear model the Markov regime switching regression specification, Hamilton's flexible regression model and the Projection pursuit regression model. Notice that model selection in terms of choosing the lags included in the regression model is done every time the sample period is altered. This is just a simple way to allow for time varying influence of the nonlinear components in the regression model. LeBaron (1992) states the importance of this point by showing that it is often the case that nonlinearities kick-in some periods and completely vanishes in other periods. Since it is a relatively difficult task to apply the cross validation model selection principle to the Markov regime switching model we base the model selection entirely on the BIC criterion as recommended in Dahl and Hylleberg (1999). Measures of absolute forecast performance are reported in the top part of table 10. It is apparent that all of the nonlinear models have lower mean squared forecast error loss (MSE) than the linear model as well as lower mean absolute forecast error loss (MAD). Notice further that the lowest point estimate of MSE is obtained by the Markov regime switching model. Theil's U statistics gives the ratio of the mean squared error from the model under consideration relative to the mean squared error from the pure random walk model. Since the statistics are far below one in all cases we can conclude that all the models forecast the inflation rate better than the simple random walk model. In table 10 the outcome from the Granger-Newbold (1977) version of the Mincer-Zarnowitz regression of the actual value on a constant and the real time forecasts is also reported. The coefficient of determination (R^2) from this regression can directly be applied as a measure of goodness of fit if the intercept in the Mincer-Zarnowitz regression is zero and the slope coefficient equals one. The reported values for the $t-stat(intc=0)$ and $t-stat(slope=0)$ are the p-values associated with the t-statistics of the null that the intercept equals zero and the t-statistics of the null that the slope equals one respectively. The reported value in the $F-stat$ entry is the p-value associated with the joint hypothesis that the intercept and slope equal zero and one respectively. Again, the Markov regime switching model performs rather well in the sense that it has the highest R^2 measure associated and that it - based on the simple t-statistics - is not possible to reject the null that the intercept equals zero and the slope equals one. This feature is only shared with the PPR-L. However, a note of caution is needed here since the F-statistics reject the joint hypothesis at a five percent level implying that the R^2 is not a meaningful measure. We have also reported measures of the directional forecast performance of the various models in the lower part of table 10. These consist of the Henriksson-Merton (1981) test (HM), the χ^2 test for independence (χ^2), the confusion rate (CR) indicating how frequent the forecast is in a wrong direction, and finally the measure of the degree of diagonal concentration (ϕ) which can be interpreted almost like the R^2 hence is a measure of goodness of directional fit. Based on the HM-statistics and the χ^2 statistics independence between the directional forecast and the actual change in the inflation rate is rejected at a five percent level for all the models presented in table 10. However, it is apparent that the Markov regime switching model does not perform as well as the two flexible regression models when it comes to predicting the change in the inflation rate one period ahead.

[*Table* 10]

By inspection of the point estimates on absolute forecast performance the Markov regime switching specification seems very promising, but in order to conclude whether this performance actually is significantly more accurate we turn to the Diebold-Mariano test and to a range of forecast encompassing tests. The Diebold-Mariano test and the Modified Diebold-Mariano for evaluating relative, predictive accuracy based on MSE and MAD loss function are reported in table 11 and table 12 respectively. For any details on the Diebold-Mariano test and the Modified version, see Diebold and Mariano (1995) and Harvey, Leybourne and Newbold (1997). Even though the MSE and MAD produced from the Markov regime switching model seem much lower that the MSE and MAD from the linear model we are not able to reject the hypothesis of equal forecast accuracy of the two models. However the predictive accuracy of the FNL model turns out to be significantly better than the forecast accuracy of the linear model based on the MSE loss function, whereas both the two flexible regression models significantly outperform the linear model in terms of accuracy when the MAD loss function is used. It is important to notice, however, that it is not possible to reject the hypothesis at any reasonable significance level that the Markov regime switching model possesses the same predictive accuracy as the two flexible regression models based on the Diebold-Mariano test and this result is independent of the choice of loss function.

[$Table$ 11]

[$Table$ 12]

As pointed out by Dahl and Hylleberg (1999) a probably more powerful statistics for discriminating between the forecast accuracy of nonlinear models is the forecasting encompassing principle, in particular when based on the battery of robust statistics suggested by Harvey, Leybourne and Newbold (1998). The results on forecast encompassing are reported in table 13. Based on this test principle it is not possible to reject the hypothesis that the Markov regime switching model forecast encompasses the linear model, whereas it is clearly rejected by all the statistics considered that the linear model can forecast encompass the Markov regime switching model. Also the FNL-model can forecast encompass the linear model whereas the opposite hypothesis can be rejected, again indicating and supporting the existence of a nonlinear component in the inflation rate process. In addition it is also worth mentioning that the Markov regime switching model can forecast encompass the FNL and PPR-L models whereas the opposite hypothesis is rejected by some of the statistics reported.

[$Table$ 13]

From the empirical evidence reported in table 11-13, based on the general-to-specific approach for selecting between nonlinear model suggested by Dahl and Hylleberg (1999), it is not possible to reject the hypothesis that the Markov regime switching model is an adequate nonlinear specification of the Danish inflation rate.

In order to specify the number of lags to include in the Markov Switching model we use Hamilton's flexible regression model approach. Without putting any assumption on the functional form of the regression model it is possible to determine what lags contribute mostly to the nonlinear component of the inflation rate process as described in section 2.3. The estimation results are reported in table 14. The value of λ - the weight placed on the nonlinear component - equals about one and based on the p-value from a t-statistics the presence of the nonlinear component in the inflation rate process seems highly significant. From the estimated FNL model we infer that the third and fourth lag enter significantly in the nonlinear component at a five percent level. Also the second lag seems to make some contribution to the nonlinear component whereas the influence of the first lags is limited. Hence the basic message from the FNL model is that we should include at least four lags when modelling the nonlinear component. As a consequence we chose to include four lags in the conditional mean function of the Markov regime switching model.

[*Table* 14]

The estimated parameters of the Markov switching model and the corresponding standard errors are reported in table 15. As expected from the inference based on the FNL model the dynamics of the inflation rate seem to differ substantially between the two regimes and we see that the largest observed numerical difference between the estimated autoregressive coefficient associated with the third and fourth lag which correspond almost perfectly to the results obtained from the FNL model. Not surprisingly there is a considerable difference between the mean and variance of the inflation rate across the two different regimes. In the high inflation rate regime - $S_t = 1$ - the unconditional mean of the quarterly inflation rate equals $\widehat{\mu}_1 = 1.74$ whereas the unconditional mean in the low inflation rate regime - $S_t = 0$ - equals $\widehat{\mu}_0 = 0.81$. The estimators also confirm the conjecture of a positive relationship between the level of inflation and its volatility, which is often referred to as a reason to pursue an economic policy aiming at low inflation, see Barro (1997). In the high inflation rate regime the degree of inflation rate uncertainty is clearly higher compared to the low inflation rate regime, which is seen from the fact that $\widehat{\sigma}_0^2 = 0.25$ while $\widehat{\sigma}_1^2 = 1.24$.

[*Table* 15]

A comparison of the distribution of the predicted inflation rate based on the Markov switching model and the actual inflation rate reveals that the Markov regime switching model provides a good estimate of the median of the distribution and that it seems to be able to produce the same fatness in the right hand tail, see figure 10.

[*Figure* 10]

Contrary to the AR(4) model the expectation bias from the Markov switching model is generated by a well specified model. Dynamic misspecification tests based on the prediction errors of the estimated Markov regime switching model, see table 16, do not reveal any violations of the underlying assumptions of the model. In neither inflation regime there seems to be autocorrelation in the forecasts, nor are there signs of any serious ARCH effects[8]. The same holds

[8] Two types of tests were actually performed, namely on one hand well known asymptotic tests for dynamic mis-specification, cf. White (1987), with a χ^2 distribution, and on the other hand Langrange multiplier test (LM-test), adopted recently by Hamilton (1996) for Markov-switching time series models.

across regimes. Furthermore it is not possible to reject the assumption that regime switching actually follows a stochastic homogeneous Markov process at the traditional 5 per cent significance level.

[*Table* 16]

As for the linear model we conduct a series of tests for the stability of the parameters of the model, see table 17. Hamilton (1996) suggests the use of the statistics of Andrews (1993) in order to test whether there is evidence in favor of one or more additional shifts in the mean of the inflation rate not already accounted for. Based on Andrews' test statistics the null hypothesis of a stable mean cannot be rejected. Hansen (1992) suggests using the parameter stability test of Nyblom (1989) to the Markov switching model. Also according to Nyblom's test, stability of the Markov switching model cannot be rejected. Notice however, that the stability of the transition parameters is rejected at a 5 per cent level but not at a 1 per cent level. Hence there appears to be no evidence of serious misspecification in the Markov regime switching representation of the Danish inflation rate.

[*Table* 17]

As part of checking the selected nonlinear specifications Hamilton (1999) suggests comparing the shape of the conditional mean function of the flexible regression model with the shape of conditional mean function of the Markov regime switching model by varying every variable included in the conditional mean in turn keeping the other variables fixed at some predetermined level. By using a Bayesian approach Hamilton provides a method to calculate 95 per cent confidence bands around the partial conditional mean function of the FNL model. The partial conditional mean functions of the estimated FNL model,the estimated Markov regime switching model and the linear model with respect to all four lags, in addition with the 95 per cent. confidence band produced from the FNL model are presented in the figures 11-14. Since the partial conditional mean functions produced by the Markov switching model all lie within the 95 per cent confidence bands it is not possible to reject that the inflation rate actually follows a Markov regime switching model. However, notice that the confidence bands actually are so wide that based on his approach it is not possible to reject linearity as well.

[*Figure* 11]
[*Figure* 12]
[*Figure* 13]
[*Figure* 14]

5 Economic Interpretations

Having discussed all the statistical properties of the Markov regime switching model let us turn to the economic interpretation. From inspection of the smoothed probabilities and the classification of inflation regimes depicted in figure 15 the inflation rate appears to be in the high mean/high variance regime in the periods from 1958 to the end of 1983, 1986-1987 and again in 1990.

[*Figure* 15]

In the beginning of the 1980's the outlook of the Danish economy was very poor, both in terms of high unemployment and high government as well as current account deficits. The credibility of economic policy was very low, and long-term interest rates were above 20 per cent. In 1982 a new government took office and immediately after announced radical measures to tackle the crisis situation. The program consisted of three main components, namely 1) an abolition of the wage-indexation schedule, 2) a significant tightening of fiscal policy, and 3) the announcement of a strong commitment to a fixed-exchange rate policy. Important credibility was gained shortly after the announcement by keeping the exchange rate unchanged when Sweden - Denmark's second largest trading partner - devaluated by 16 per cent. Evidence from financial data, see Christensen (1988), supports the hypothesis that this policy gained credibility relatively fast, leading to an almost instant reduction in the variability in exchange rates and a lowering of the inflation rate expectations. The fast reduction in both nominal interest rates and inflation after the policy shift confirms this. Our analysis confirms to some extent this claim in the sense that the agents' perceptions of the underlying inflation rate regime shifts from a high mean/high variance regime to at more stable low mean/low variance regime relatively quickly after the introduction of the new economic policy in 1982. Furthermore our analysis indicates that the parameters of the inflation rate model are policy variant. In the one-regime AR(4) model this is exactly what leads to a contradiction of the basic assumptions of the model implying non-rational forecasts. In the Markov regime switching model the shifting parameters are allowed for, but this may lead to *ex post* biased forecasts. In particular, the fixed-exchange rate policy, despite the marked demonstration of a strong commitment to the policy quickly after the announcement in 1982, does not seem to have been considered fully credible - in terms of low inflation expectations - before the very late eighties. This is confirmed by the gradual reduction in the long term interest-rate differential to Germany from above 10 percentage points in 1982 to less than 1 percentage points in 1991.

In contrast to previous results the credibility of the policy seems to disappear in 1986-1987 as implied by the smoothed probabilities of the model. The Danish domestic demand was extremely buoyant in 1986/87 driven by the fall of interest rates since 1982 and a sharp pick-up in house prices. The current account deficit peaked in 1986 at a level of 6 percent of GDP and annual wage increases doubled in 1987 to around 10 percent. The return to high inflation expectations is confirmed by the fact that the long term interest rate differential to Germany widened substantially in 1986 and 1987. Also the unification of Germany in 1990 and the implied massive growth in the Danish export to Germany appear to have affected the inflation rate process to such an extent that there are indications of a temporary return to the high mean/high variance regime. In fact, the smoothed probabilities of the model reveal, that every time there has been an upward pressure on the Danish inflation rate, agents have anticipated a return to the high mean/high variance regime.

By comparing the in-sample predictions from the Markov switching model with the actual inflation rate over the entire estimation period it is seen that the model is not able to generate ex post of the size observed in the Household-sector data, particularly not in the nineties. Not even when the forecast horizon is expanded to four quarters, see figure 4 and figure 5. This conclusion changes when the inflation rate forecasts one year ahead are based on true out of sample predictions. In figure 16 actual inflation rates and expected inflation from the linear model and the Markov regime switching model one year ahead are compared. Our estimations seem to confirm the survey-based measures of inflation expectations, see above, in the sense of a protracted over-prediction of the inflation rate in the low-inflation period of the nineties, whereas the higher inflation rates, which occurred in large parts of the seventiess and early 1980's, were clearly under-predicted. Furthermore, notice that the size of the *ex post* bias seems to diminish gradually indicating that the household sector is attaching a decreasing probability to the event that the Danish economy will return to a high-inflation-rate regime. This seems plausible after a long period - 7-8 years - of historically low inflation rates in Denmark.

$[Figure\ 16]$

This suggests that the systematic forecast errors owe to a low, but non-zero probability of a regime shift one year ahead from today.

6 Conclusion

We have presented empirical evidence showing that the Danish inflation rate cannot be represented adequately by means of a simple linear regression model due to the presence of multiple and recurrent shifts in the mean and variance of the underlying stochastic process driving the inflation rate. By using Hansen's (1992) test and the general-to-specific approach of Dahl and Hylleberg (1999) we are not able to reject the Markov regime switching representation as the preferred nonlinear specification of the conditional mean function against a wide range of nonlinear and linear alternatives. Furthermore, we are not able to reject that the Markov regime switching representation is dynamically well specified and that the assumption of constancy of the estimated parameters of this representation is satisfied.

Studies of survey data on households' inflation perceptions tend to support evidence of a persistent difference between actual and regression based inflation rate expectations. Similar differences between actual and expected inflation are found in our analysis. Based on evidence of possible regime shifts in the Danish inflation rate process we claim that part of this difference is due to an *ex post* bias. This implies that persistent deviations between actual and expected inflation cannot be taken as an argument against rational expectations, but rather be explained by perceived non-zero probabilities of a change in the inflation regime. If forecasters forming rational expectations about future inflation are confronted with an additional problem of identifying the inflation regime, systematic forecast errors over sustained periods may actually arise. Our result shows that in large parts of the seventies and early eighties the high rates of inflation were underpredicted. The steep decline in inflation after 1982, followed by a more gradual disinflation throughout the second half of the eighties and into the nineties, has generally not been fully anticipated, hence implying an overprediction of the actual inflation rate. Moreover, contrary to what is sometimes argued in the literature, see e.g. Christensen (1988) the estimated regime probabilities suggest that the hard currency policy introduced in 1982 was not fully credible before well into the second part of the eighties.This is also to some extent backed by the protracted decline in the interest rates. A short-lived shift to a perceived high-inflation regime in 1990 follows immediately after the German unification, and is the forerunner of a persistent overprediction of the inflation rate in the nineties owing to a permanent non-zero probability of a change in regime. It must be emphasized that similar forecast errors were made by government and other official forecasters, see Christensen (1996), and hence do not indicate that useful information is systematically ignored.

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7 Graphs and tables

Figure 1: The perceived and expected inflation rates of the Danish Household sector in the period 1976m1 - 1997m10. a) Full line: The perceived changes in the price level over the previous year, b) Dashed line: The expected change in the price level one year ahead

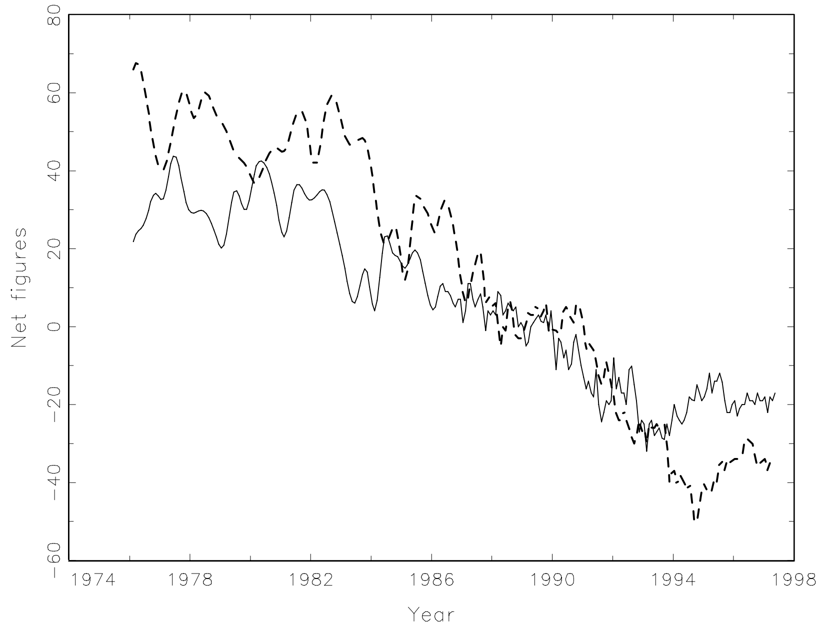

Figure 2: Quarterly growth rates in Danish consumer prices (seasonally adjusted), 1957q2 - 1998q4.

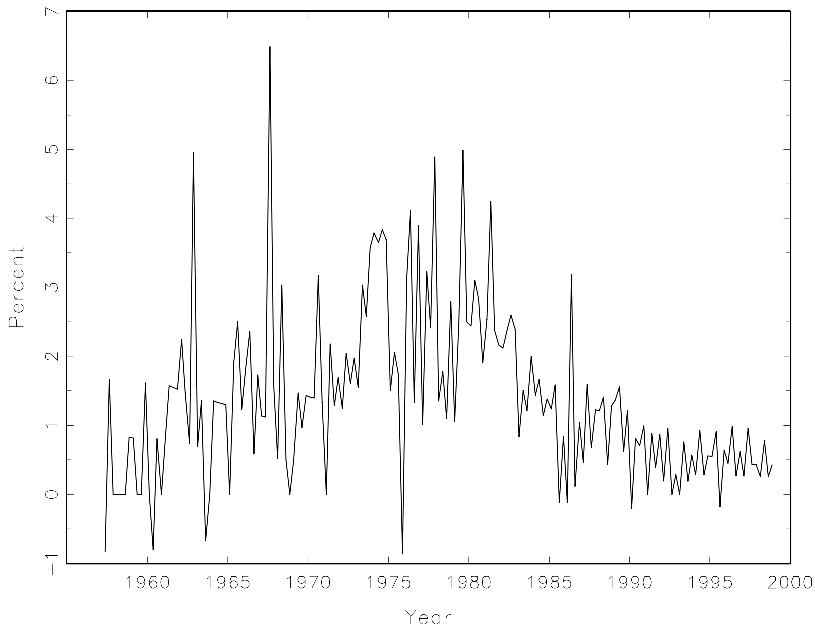

Figure 3: Density estimate of the quaterly growth rates in Danish consumer prices in the period 1957q1 - 1998q4. The density estimate is based on the Epanechnikov Kernel with data determined bandwidth equal to h=0.345. For details about the data dependent bandwidth selection procedure, see Silverman (1986) eq. (3.31).

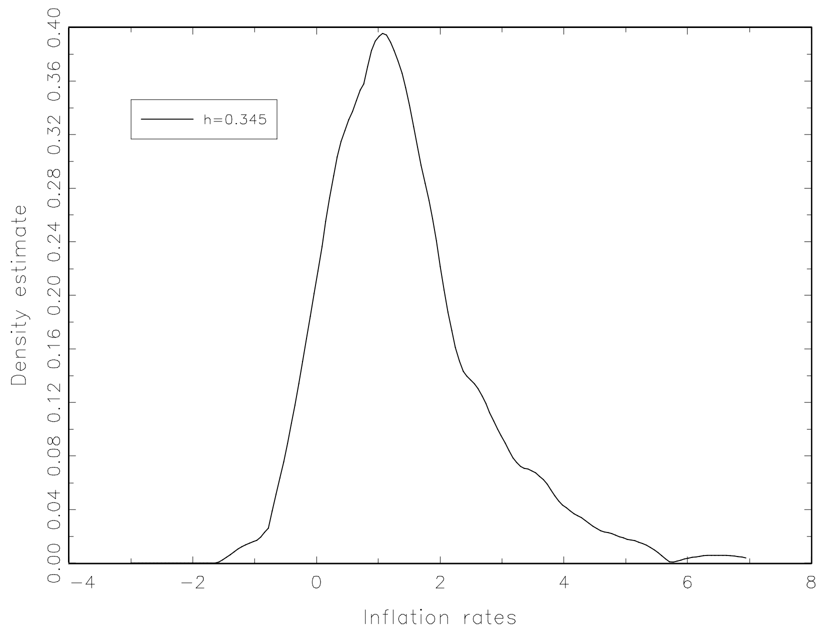

Figure 4: Actual and one period ahead predictions of the quaterly growth rates in Danish consumer prices. a) Full line: Actual inflation, b) Dashed line: In sample predictions from the Markov regime switching model, c) Dotted line: In sample predictions from the AR(4) model. Estimation period equals 1958q3 - 1998q4.

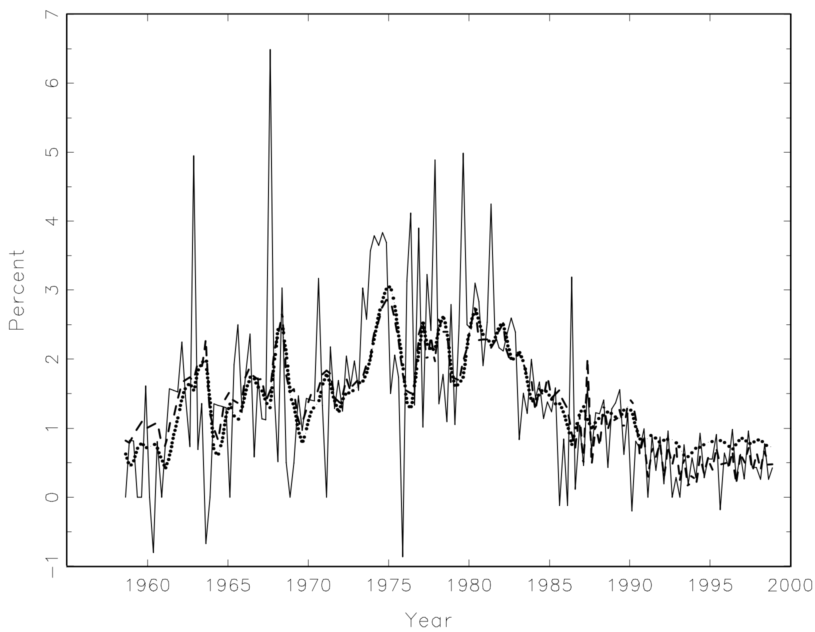

Figure 5: Actual and four periods ahead predictions of the quaterly growth rates in Danish consumer prices. a) Full line: Actual inflation, b) Dashed line: In sample predictions from the Markov regime switching model, c) Dotted line: In sample predictions from the AR(4) model. Estimation period equals 1958q3 - 1998q4.

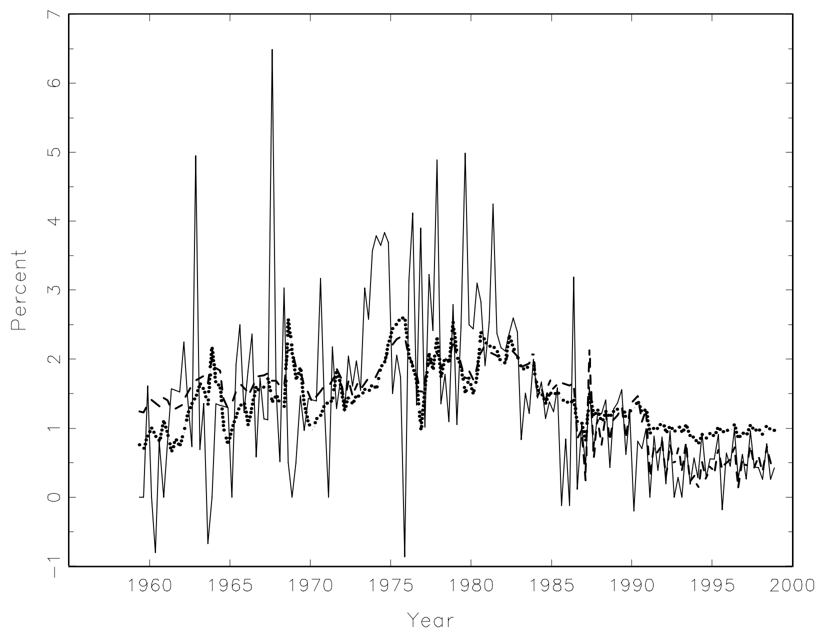

Figure 6: Density estimates. a) Full line: Actual quaterly growth rates in Danish consumer prices, b) Dashed line: One step ahead in sample predictions produced from the AR(4) model. The density estimates is both based on the Epanechnikov Kernel with data determined bandwidth equal to h=0.345. The data covers the period 1958q3 - 1998q4.

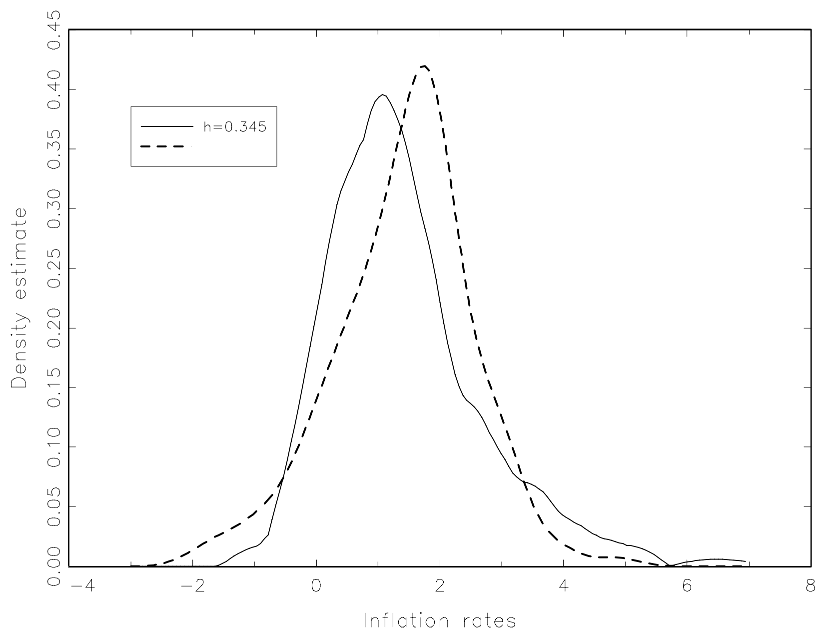

Figure 7: One step up Chow test for stability of the AR(4) model of the quaterly growth rates in Danish consumer prices. Estimation period 1958q3-1998q4. a) Dotted line: Chow statistic relative to the five percent critical value under the null of stability, b) Vertical full line: Values above indicates rejection of the null of stability at a five percent level.

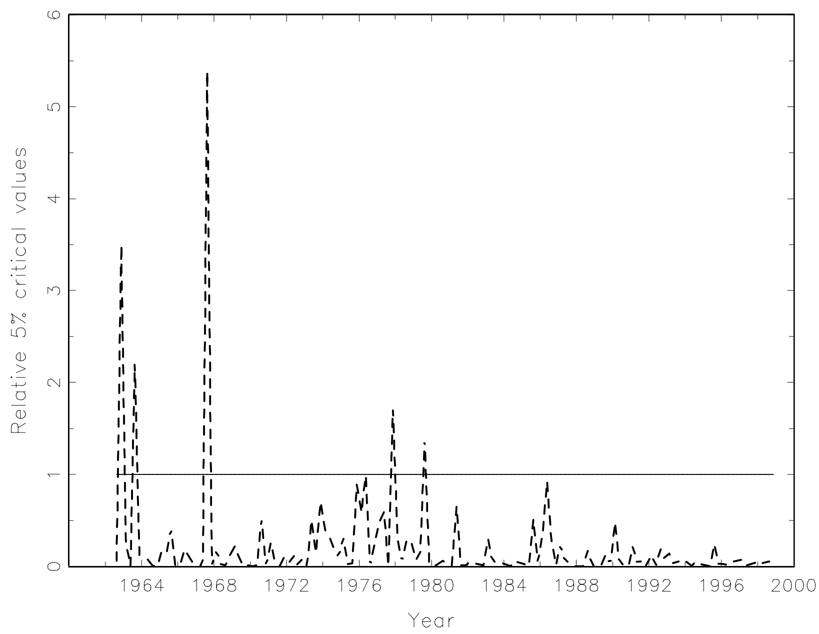

Figure 8: Forecast (N steps up) Chow test for stability of the AR(4) model of the quaterly growth rates in Danish consumer prices. Estimation period 1958q3-1998q4. a) Dotted line: Chow statistic relative to the five percent critical value under the null of stability, b) Vertical full line: Values above indicates rejection of the null of stability at a five percent level.

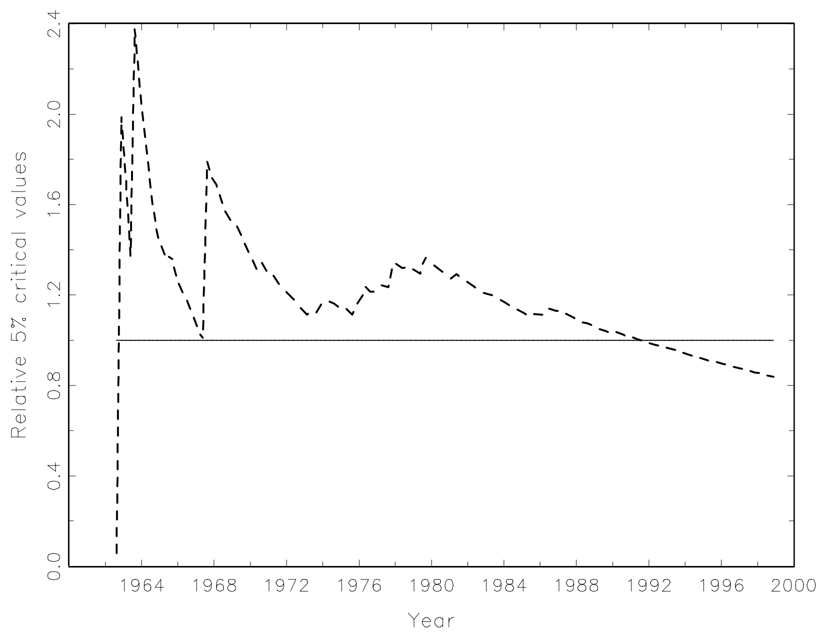

Figure 9: Breakpoint (N steps down) Chow test for stability of the AR(4) model of the quaterly growth rates in Danish consumer prices. Estimation period 1958q3-1998q4. a) Dotted line: Chow statistic relative to the five percent critical value under the null of stability, b) Vertical full line: Values above indicates rejection of the null of stability at a five percent level.

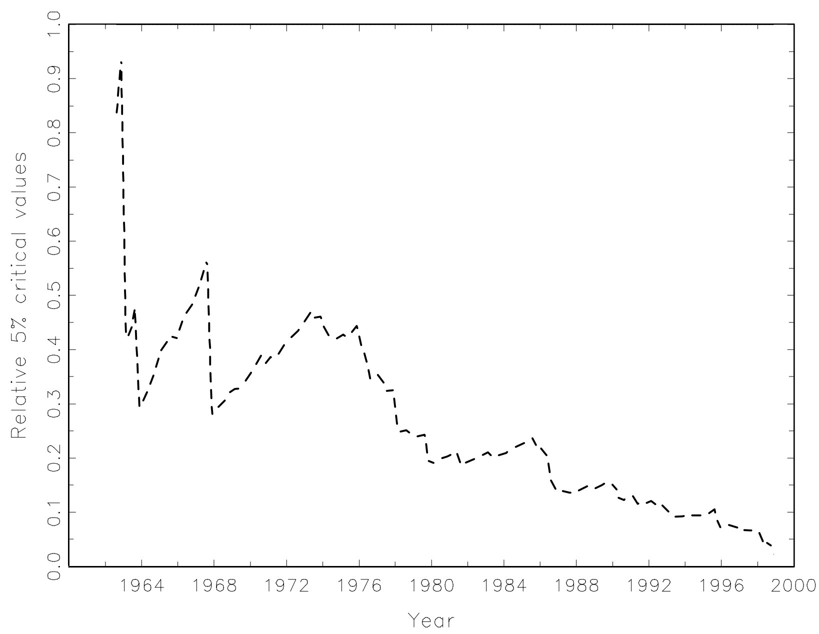

Figure 10: Density estimates. a) Full line: Actual quaterly growth rates in Danish consumer prices, b) Dashed line: One step ahead in sample predictions produced from the Markov regime switching model. The density estimates is both based on the Epanechnikov Kernel with data determined bandwidth equal to h=0.345. The data covers the period 1958q3 - 1998q4.

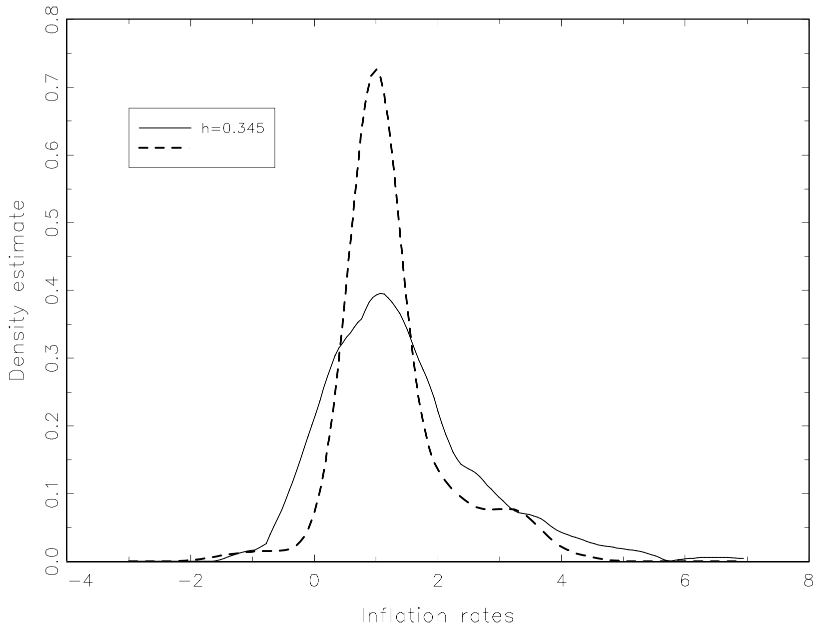

Figure 11: The figure plots posterior means $\widehat{E}(\mu(\pi_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4})|Y_T)$ as a function of π_{t-1} for $\overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T the given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$ and π_{t-4}. a) Dashed line. Posterior mean from the flexible regression model with 10000 Monte Carlo draws with a fixed sample size of 162 observation covering the period 1958q3 - 1998q4. b) Dotted-dashed line: 95% conficence intervals. c) Full line: Posterior mean from Markov regime switching model. d) Posterior mean from linear model.

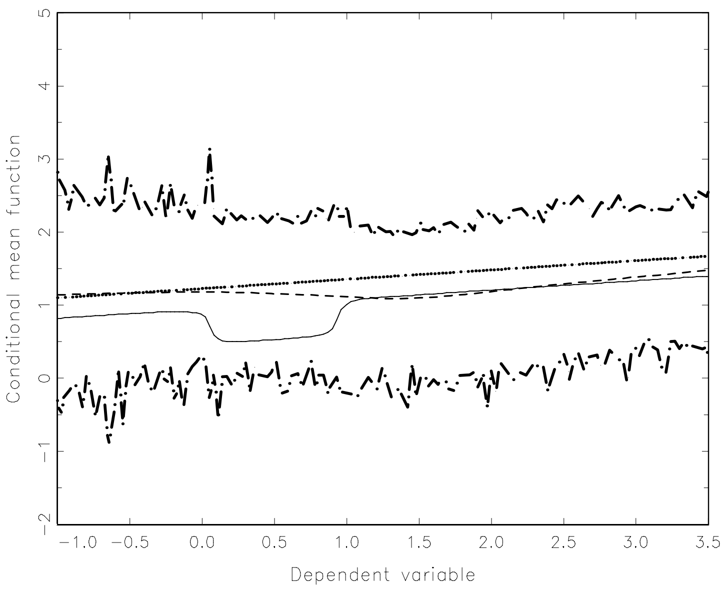

Figure 12: The figure plots posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \pi_{t-2}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4})|Y_T)$ as a function of π_{t-2} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-3}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T the given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$ and π_{t-4}. a) Dashed line. Posterior mean from the flexible regression model with 10000 Monte Carlo draws with a fixed sample size of 162 observation covering the period 1958q3 - 1998q4. b) Dotted-dashed line: 95% conficence intervals. c) Full line: Posterior mean from Markov regime switching model. d) Posterior mean from linear model.

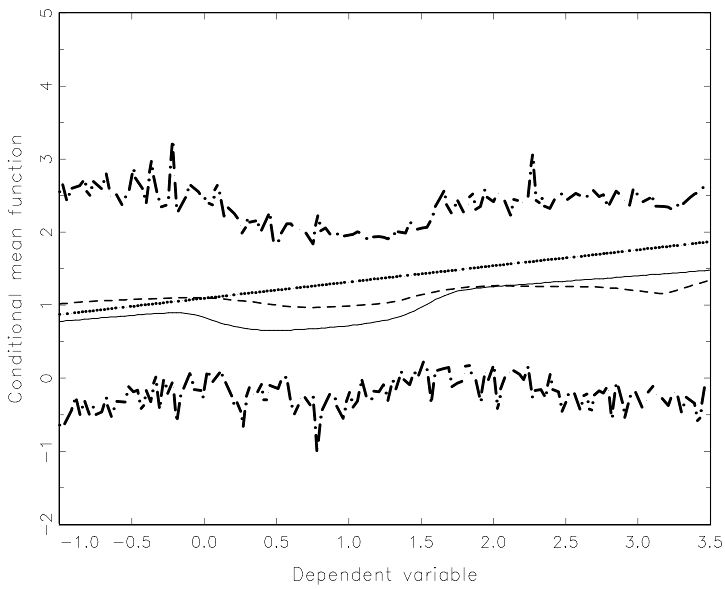

Figure 13: The figure plots posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \pi_{t-3}, \overline{\pi}_{t-4})|Y_T)$ as a function of π_{t-3} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-4}$ fixed at 1 and Y_T the given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$ and π_{t-4}. a) Dashed line. Posterior mean from the flexible regression model with 10000 Monte Carlo draws with a fixed sample size of 162 observation covering the period 1958q3 - 1998q4. b) Dotted-dashed line: 95% conficence intervals. c) Full line: Posterior mean from Markov regime switching model. d) Posterior mean from linear model.

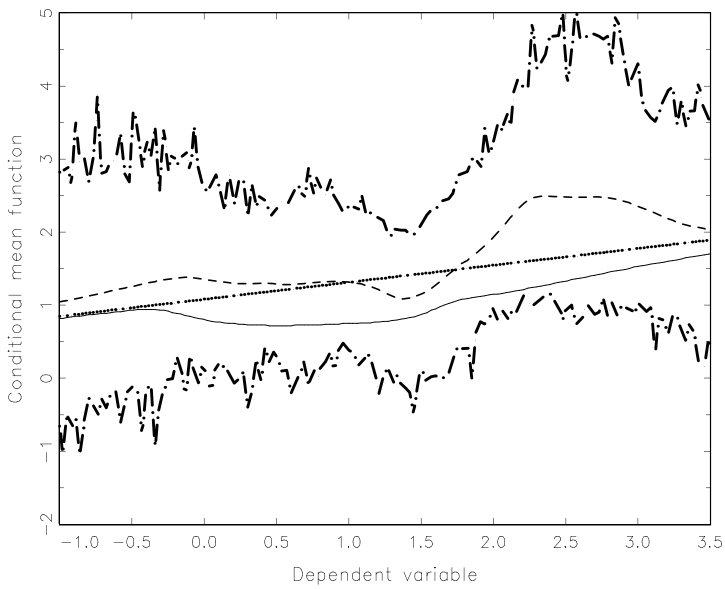

Figure 14: The figure plots posterior means $\widehat{E}(\mu(\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}, \pi_{t-4})|Y_T)$ as a function of π_{t-4} for $\overline{\pi}_{t-1}, \overline{\pi}_{t-2}, \overline{\pi}_{t-3}$ fixed at 1 and Y_T the given sample observations on π_t, $\pi_{t-1}, \pi_{t-2}, \pi_{t-3}$ and π_{t-4}. a) Dashed line. Posterior mean from the flexible regression model with 10000 Monte Carlo draws with a fixed sample size of 162 observation covering the period 1958q3 - 1998q4. b) Dotted-dashed line: 95% conficence intervals. c) Full line: Posterior mean from Markov regime switching model. d) Posterior mean from linear model.

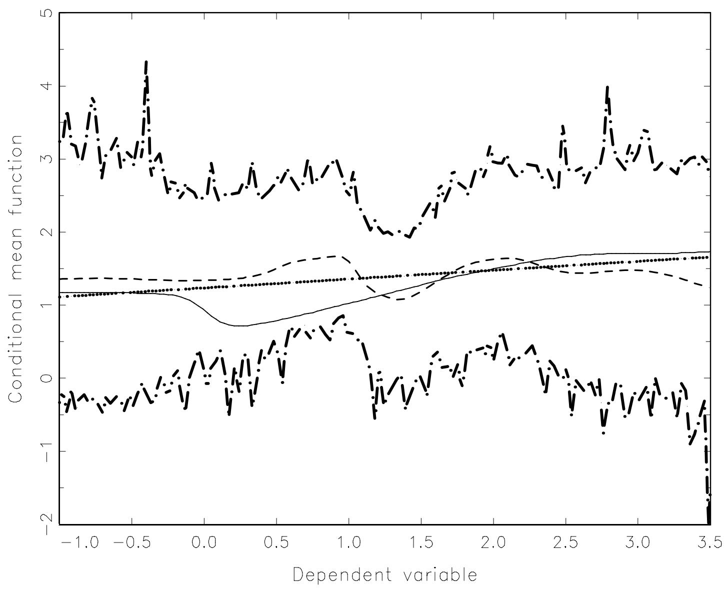

Figure 15: Classification of the inflation regimes in Denmark in the period 1958q3 - 1998q4. a) Full line: Actual inflation, b) Dashed line: Smoothed probabilities of being in an regime with high inflation rates and high inflation uncertainty $\{pr(S_t = 1|Y_T)\}$, c) Dotted line: Classification of inflation regimes. High inflation regime when $pr(S_t = 1|Y_T) > 0.5$.

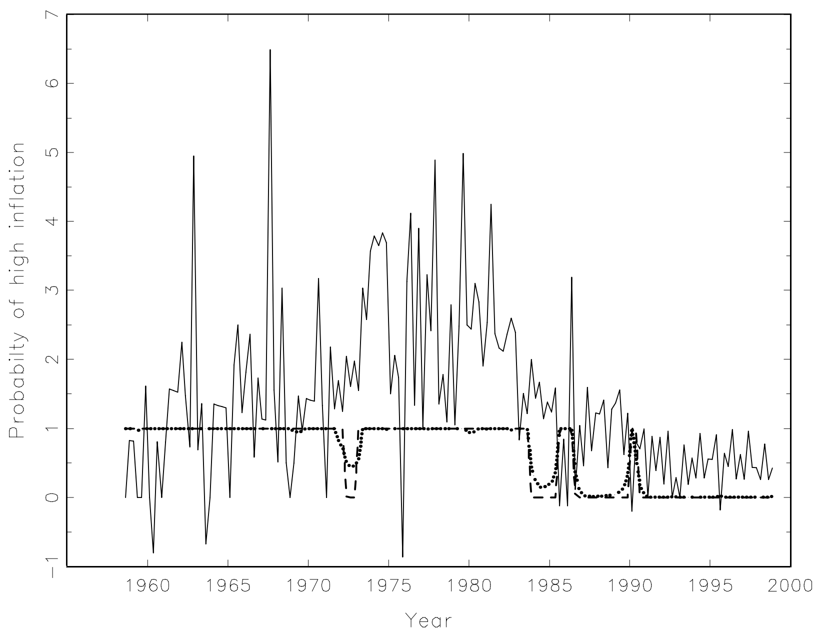

Figure 16: Actual and four periods ahead predictions in real time of the quaterly growth rates in Danish consumer prices. a) Full line: Actual inflation, b) Dashed line: Real time predictions from the Markov regime switching model, c) Dotted line: Real time predictions from the AR(4) model. Growing data window size with initial period covering 1958q3 - 1979q1 and with the final period covering 1958q3-1997q4.

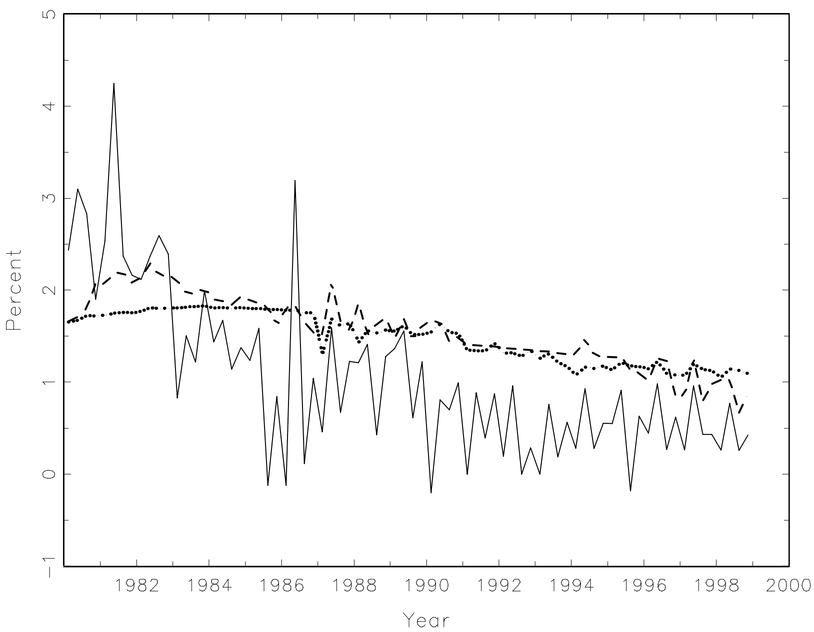

Table 1: Quarterly growth rates in the Danish consumer prices, 1957q1 - 1998q4. Simple descriptive statistics and the Jacque-Bera test for normality.

Mean	1.368
Standard Deviation	1.222
Skewness	1.109
Excess Kurtosis	1.785
Minimum	-0.863
Maximum	6.489
Normality test	
Statistic	34.804
p-value	0.0

Table 2: Augmented Dickey-Fuller test for nonstationarity of the Danish inflation rate. Sample period equals 19958q2 - 1998q4. Notice that ***, **, * denotes rejection of the null of nonstationarity at a 1, 5 and 10 percent level respectively

Lags	ADF coefficient on lagged level (t-ratio) For model including the following			Obs.
	No constant No trend	Constant No trend	Constant Trend	
0	-0.286	-0.654	-0.669	166
	(-5.26)***	(-9.01)***	(-9.18)***	
1	-0.152	-0.437	-0.453	165
	(-2.92)***	(-5.16)***	(-5.29)***	
2	-0.102	-0.329	-0.349	164
	(-2.00)**	(-3.70)***	(-3.86)**	
3	-0.083	-0.292	-0.316	163
	(-1.64)*	(-3.15)**	(-3.35)*	
4	-0.078	-0.296	-0.324	162
	(-1.53)	(-3.07)**	(-3.31)*	

Table 3: Augmented Dickey-Fuller test for nonstationarity of the Danish inflation rate. Estimation of the auxiliary regression under the inclusion of 3 lags of the left hand side variable. Sample period equals 19958q1 - 1998q4.

RHS var	ADF regression coefficients (t-ratio) For model including the following		
	No constant No trend	Constant No trend	Constant Trend
π_{t-1}	-0.083	-0.292	-0.316
	(-1.64)	(-3.15)	(-3.35)
$\Delta\pi_{t-1}$	-0.737	-0.578	-0.564
	(-8.55)	(-5.59)	(-5.44)
$\Delta\pi_{t-2}$	-0.459	-0.354	-0.347
	(-4.85)	(-3.52)	(-3.45)
$\Delta\pi_{t-3}$	-0.176	-0.127	-0.126
	(-2.27)	(-1.63)	(-1.62)
constant	.	0.410	0.442
	.	(2.67)	(2.85)
trend	.	.	-0.002
	.	.	(-1.30)
RSS	183	175	173

Table 4: Augmented Dickey-Fuller test for nonstationarity of the Danish inflation rate. Estimation of the auxiliary regression under the inclusion of 4 lags of the left hand side variable. Sample period equals 19958q1 - 1998q4.

RHS var	ADF regression coefficients (t-ratio) For model including the following		
	No constant No trend	Constant No trend	Constant Trend
π_{t-1}	-0.079	-0.296	-0.324
	(-1.53)	(-3.07)	(-3.31)
$\Delta\pi_{t-1}$	-0.750	-0.577	-0.561
	(-8.42)	(-5.29)	(-5.12)
$\Delta\pi_{t-2}$	-0.487	-0.358	-0.350
	(-4.66)	(-3.15)	(-3.08)
$\Delta\pi_{t-3}$	-0.215	-0.127	-0.125
	(-2.11)	(-1.21)	(-1.19)
$\Delta\pi_{t-4}$	-0.050	-0.007	-0.008
	(-0.63)	(-0.09)	(-0.10)
constant	.	0.420	0.458
	.	(2.65)	(2.86)
trend	.	.	-0.003
	.	.	(-1.42)
RSS	182	175	172

Table 5: The estimated AR(4) representation of the quarterly Danish inflation rate. Estimation period equals 1958q3-1998q4. π_t denotes the inflation rate.

RHS var	The linear model (LR)		
	Estimate	std. error	p-value
π_{t-1}	0.127	0.079	0.111
π_{t-2}	0.221	0.077	0.005
π_{t-3}	0.232	0.077	0.003
π_{t-4}	0.121	0.079	0.126
constant	0.423	0.155	0.007
Log likelihood		-235.910	
R^2		0.269	
RSS		174.552	

Table 6: Testing the adequacy of the AR(4) model. Lagrange multiplier tests for detecting dynamic misspecification in terms of neglected autocorrelation and ARCH effects.

Test	Statistic	p-value
LM tests for no autocorrelation		
AR(1;1)	0.009	92.4
AR(1;2)	5.743	5.6
AR(1;3)	5.688	12.8
AR(1;4)	6.828	14.52
LM-tests for no ARCH effects		
ARCH(1;1)	3.814	5.1
ARCH(1;2)	5.103	7.8
ARCH(1;3)	4.895	17.9
ARCH(1;4)	4.917	29.6

Table 7: Testing the adequacy of the AR(4) model. Nyblom's score test and the Andrews-Plobergers SupF, ExpF and AvgF tests for detecting parameter instability. The 1,5, and 10 percent critical values of Nyblom's statistics equals 1.870, 1.462 and 1.275 respectively. The p-values associated ved the Andrews-Ploberger test are calculated according to the method described by Hansen (1997)

Test	Statistic	p-value
Tests for structural stability		
Nyblom		
$\{\beta_1, .., \beta_5\}$	1.659	.
SupF		
$\{\beta_1, .., \beta_5\}$	19.336	3.3
β_1	10.980	1.7
β_2	11.993	1.0
β_3	6.479	13.0
β_4	13.121	0.6
β_5	3.183	51.7
ExpF		
$\{\beta_1, .., \beta_5\}$	7.618	1.3
β_1	3.517	0.6
β_2	3.923	0.3
β_3	1.355	12.6
β_4	4.534	0.1
β_5	0.704	34.2
AvgF		
$\{\beta_1, .., \beta_5\}$	10.930	0.8
β_1	3.963	1.2
β_2	4.056	1.1
β_3	1.969	10.4
β_4	4.515	0.6
β_5	1.206	26.7

Table 8: Testing the adequacy of the linear AR(4) representation of the Danish inflation rate. The linear models under the null are selected by Akaike's infomations criteria (AIC), the Schwart's infomation criterion (BIC), the Cross Validation selection criterion (CV). Critical values are reported with the associated p-values in parenthesis.

	AIC (lags=4)		BIC (lags=3)		CV (lags=3)	
Hamilton	1.57	(21.0)	1.65	(20.0)	1.65	(20.0)
Neural Network	7.90	(0.0)	4.46	(3.0)	4.46	(3.0)
Tsay	1.29	(24.0)	1.22	(30.0)	1.22	(30.0)
White	18.90	(46.0)	13.73	(39.0)	13.73	(39.0)
Reset	0.56	(45.0)	0.64	(42.0)	0.64	(42.0)
BDS	25.34	(0.0)	15.34	(0.0)	15.34	(0.0)

Table 9: Hansen's Standardized Likelihood Ratio test of the null of the linear AR(4) representation of the Danish inflation rate against the Markov Switching AR(4) model under the alternative. Sample period equals 1958q3-1998q4. Grid used: p,q from 0.1 to 0.925 in steps of 0.075. Regression parameters from 0.01 to 0.61 in step of 0.3. Total number of gridpoints equals 104976. M denotes the bandwidth. Internal Monte Carlo replications equals 1000. $p_0 = p_1 = 4$.

	Statistic	p-value				
		M=0	M=1	M=2	M=3	M=4
LR*	5.009	0.0	0.0	0.2	0.7	0.9

Table 10: One period ahead forecast performance in real time of the quarterly growth rates in Danish consumer prices, 1980.Q1 - 1998.Q4. All models selected by BIC.

	MS-AR	LR	FNL	PPR-L
Mean BIC	0.792	0.443	0.585	0.503
Absolute forecast performance.				
MSE	0.524	0.576	0.525	0.564
MAD	0.534	0.562	0.524	0.542
Theil's U	0.824	0.863	0.824	0.854
t-stat.(intc=0)[p-val.]	29.2	3.3	3.0	13.2
t-stat.(slope=1)[p-val.]	58.4	18.5	23.5	50.3
F-stat [p-val.]	0.5	1.5	1.0	4.7
R^2	44.3	36.4	42.8	35.3
Directional forecast performance				
HM [p-val.]	3.1	3.1	0.2	0.8
χ^2 [p-val.]	3.0	3.0	0.2	0.7
CR	39.5	39.5	34.2	36.8
ϕ	24.9	24.9	35.5	30.8

Table 11: Diebold-Mariano test for relative predictive ability in real time. Quarterly growth rates in Danish consumer prices, 1980.Q1 - 1998.Q4. The squared error loss function is used in the DM and Modified DM statistics. Let $\approx$ denote equal relative predictive ability. Associated p-values reported.

H_0	DM	MDM
MS-AR $\approx$ LR	47.5	48.0
FNL $\approx$ LR	0.9	1.2
PPR-L $\approx$ LR	48.2	48.7
MS-AR $\approx$ FNL	99.9	99.0
MS-AR $\approx$ PPR-L	58.9	80.1

Table 12: Diebold-Mariano test for relative predictive ability in real time. Quarterly growth rates in Danish consumer prices, 1980.Q1 - 1998.Q4. The absolute error loss function is used in the DM and Modified DM statistics. Let $\approx$ denote equal relative predictive ability. Associated p-values reported.

H_0	DM	MDM
MS-AR $\approx$ LR	38.7	39.3
FNL $\approx$ LR	0.0	0.1
PPR-L $\approx$ LR	2.7	3.1
MS-AR $\approx$ FNL	72.5	72.8
MS-AR $\approx$ PPR-L	80.1	80.3

Table 13: Forecast encompassing tests for relative predictive ability in real time. Quarterly growth rates in Danish consumer prices, 1980.Q1 - 1998.Q4. Let $\sqsubset$ denote encompassing. Associated p-values reported.

H_0			R	R_s	R_1	R_{dm}	R_{mdm}
MS-AR	$\sqsubset$	LR	22.1	47.6	34.8	39.1	39.4
LR	$\sqsubset$	MS-AR	0.3	0.3	2.2	1.3	1.3
FNL	$\sqsubset$	LR	6.0	12.1	21.4	9.1	9.3
LR	$\sqsubset$	FNL	0.1	0.3	2.7	0.1	0.1
PPR-L	$\sqsubset$	LR	85.2	15.6	85.3	85.2	85.3
LR	$\sqsubset$	PPR-L	21.6	0.2	21.9	22.3	22.3
MS-AR	$\sqsubset$	FNL	6.1	32.4	17.0	23.0	23.3
FNL	$\sqsubset$	MS-AR	6.0	2.6	16.9	13.8	14.1
MS-AR	$\sqsubset$	PPR-L	19.9	14.1	34.7	37.9	38.2
PPR-L	$\sqsubset$	MA-AR	0.6	41.5	4.6	4.0	4.1

Table 14: Maximum log likelihood estimates of the Hamilton's flexible regression representation of the Danish inflation rate. Sample period equals 1958q3 - 1998q4. Notice that the calculation of the likelihood ratio test is done under the null that the inflation rate follows an AR(4) model. Again it should be emphasized that the test statistics does not follow any standard distribution asymptotically, hence should only be considered suggestive.

	Linear part		
	Estimator	std. error	p-value.
π_{t-1}	0.117	0.084	16.5
π_{t-2}	0.197	0.085	2.3
π_{t-3}	0.221	0.086	1.1
π_{t-4}	0.087	0.089	33.0
constant	0.608	0.241	65.0
σ	0.194	0.427	
	Nonlinear part		
	Estimator	Std. error	p-value
$g_{\pi_{t-1}}$	0.368	0.234	13.2
$g_{\pi_{t-1}}$	1.413	0.784	7.4
$g_{\pi_{t-1}}$	3.230	0.924	0.1
$g_{\pi_{t-1}}$	2.132	0.988	3.3
λ	1.080	0.111	0.0
Log likelihood			-229.18
Likelihood ratio (LRA)			6.73
2*LRA			13.46

Table 15: Maximum log likelihood estimates of the Markov switching AR(4) representation of the Danish inflation rate. Sample period equals 1958q3 - 1998q4. Notice that the calculation of the likelihood ratio test is done under the null that the inflation rate follows an AR(4) model. Again it should be emphasized that the test statistics does not follow any standard distribution asymptotically, hence should only be considered suggestive.

RHS var			
	Regime $S_t = 0$		
	Estimate	std. error	p-value
π_{t-1}	0.123	0.082	6.8
π_{t-2}	0.244	0.076	0.1
π_{t-3}	0.109	0.078	8.2
π_{t-4}	0.537	0.069	0.0
constant	-0.013	0.085	55.9
σ_0	0.245	0.024	0.0
	Regime $S_t = 1$		
	Estimate	Std. error	p-value
π_{t-1}	0.128	0.101	10.3
π_{t-2}	0.162	0.099	5.2
π_{t-3}	0.241	0.099	0.8
π_{t-4}	0.018	0.105	43.1
constant	0.790	0.262	0.2
σ_1	1.239	0.090	0.0
p=pr(S_t=0|S_{t-1}=0)	0.963	0.023	0.0
q=pr(S_t=1|S_{t-1}=1)	0.937	0.037	0.0
Log likelihood			-191.941
Likelihood ratio (LRA)			43.972
2*LRA			87.944

Table 16: Testing the adequacy of the Markov switching AR(4) representation of the Danish inflation rate. White's dynamic misspecification tests are all $\chi^2(4)$ distributed while Hamiltons LM test are $\chi^2(1)$ distributed asymptotically. Sample period equals 1958q3 - 1998q4.

Test	Statistic	p-value
White's dynamic misspecification tests		
No autocorrelation	2.912	57.3
No ARCH effects	4.588	33.2
Validity of Markov assumption	5.031	28.4
Hamilton's LM-tests for no autocorrelation		
No autocorrelation in regime $S_t = 0$	0.598	43.9
No autocorrelation in regime $S_t = 1$	0.736	39.1
No autocorrelation across regimes	0.034	85.3
Hamilton's LM-tests for no ARCH		
No ARCH effects in regime $S_t = 0$	2.649	10.4
No ARCH effects in regime $S_t = 1$	2.078	14.9
No autocorrelation across regimes	4.581	3.2

Table 17: Testing the stability of the Markov switching AR(4) representation of the Danish inflation rate. The critical values of Nyblom's statisitc is only very sparce tabulated and only up to 10 parameters/degrees of freedom. Notice that a * indicates the critical value in the case of 10 degrees of freedom when 14 degrees of freedom is actually needed. ** denotes the approximate/interpolated critical value in the case of 5 degrees of freedom. Sample period equals 1958q3 - 1998q4.

Test	Statistic	Asymptotical critical values		
		10%	5%	1%
Nyblom's stability tests				
All parameters stable	2.462	*2.295	*2.533	*3.035
Autoregressive parameters stable	0.926	2.295	2.533	3.035
Autoregressive parameters in regime $S_t = 0$ stable	0.482	**1.275	**1.462	**1.870
Autoregressive parameters in regime $S_t = 1$ stable	0.448	**1.275	**1.462	**1.870
Standard error parameters stable	0.123	0.607	0.748	1.107
Transition parameters stable	1.012	0.607	0.748	1.107
Andrew's stability tests				
No additional shifts in mean	4.689	7.170	8.850	12.350

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