# Market Forces and Dynamic Asset Pricing

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## Abstract

We study a dynamic model of asset pricing which is driven by two characteristic market features: the law of investor demand (e.g. 'buy low, sell high') and the law of the market institution (which codifies the trading rules under which the market operates). We demonstrate in a simple investor-specialist trading market that these features are sufficient to guarantee an equilibrium where investors' trading strategies and the specialist's rule of price adjustments are best responses to each other. The drift term appearing in the resulting equation of the asset price process may be interpreted using Newtonian mechanics as the acceleration of a 'market force'. If either of the market participants is risk-neutral, the result leads to risk-neutral asset pricing (e.g. the Black and Scholes option pricing formula).



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