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SOCIAL INSURANCE AND THE PUBLIC BUDGET

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Abstract

Restraints on the public budget limit the ability of the public s
markets for intertemporal substitution. This interferes with the rol
as a buffer which provides insurance and possibly stabilizes income
consumption. We consider this insurance or stabilizing role of public
why a progressive taxation system may be optimal even when the disto
taxation are taken into account. Balanced budget restrictions interfere
effect, and they do not necessarily imply that a lower level of publ
optimal.

JEL: H20, H61, E62, D61, D80.
Keywords: Insurance, Optimal Taxation, Budget Regimes.

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1. Introduction

What is the role of budget deficits? It is a widespread view that l from a political bias in policy-making inducing politicians to pre increased public consumption to precede the costs in terms of higher[1] A large part of the literature has focused on the crowding out effects of public c run costs[2] Accordingly, it is often advocated that the public sector sh a balanced budget constraint or that the possibility for running restricted.

In the US many states have adopted a balanced budget rule[3], and for the federal budget there is a continuous debate on the Gramm-Rudmann-Hollings amendment r government to finance current expenditures from current (distortion Europe budget norms on the maximum size of budget deficits relative part of the convergence criteria for the Economic and Monetary Union adopted stability pact strengthens the interpretation of this norm and towards members states violating it.

Most countries have a public budget which is very sensitive to the b number of European countries it has been estimated (CEC(1997)) that borrowing requirement measured relative to GDP increases by between percentage points when GDP drops by 1 %. The sensitivity of budget rev fluctuations is significantly higher than that of government spending. budget sensitivity is positively related to the size of the public Empirical evidence also indicates that the automatic stabilizers i

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[1] This bias is discussed in a growing political economy literature, see eg Alesina Corsetti and Roubini (1997).

[2] For a discussion, see eg Chang (1990) and Ball and Mankiw (1995).

[3] See Poterba (1996) for an outline of how these are formulated and implemented.

sensitivity may stabilize economic activity. There is thus a negative co government size and macroeconomic volatility (Gali (1994)). Empirica single states of the USA shows that institutional restraints on the budget position the cyclical responsiveness of public finances and therefore poten automatic stabilizers among other things by forcing tax rates to mo (Bayoumi and Eichengreen (1995)).[4]

The primary budget position depends on the timing of taxation and insight of the "tax-smoothing" principle (Barro 1979) is that mini distortionary costs of income taxation (the dynamic Ramsey problem) calls taxrate. Accordingly, temporary increases in public expenditures c revenue would optimally be accommodated by running a public deficit. developed this result for an income tax in a partial model with exogeno it has later been cast in a general equilibrium setting by Lucas an

The timing of taxes should also take into account the possible ways deficits interfere with market failures (the dynamic Pigou problem). On important role here is the fact that the public budget may serve a impinging on the economy. Thereby, the public budget may stabilize e private consumption providing an insurance or stabilization functio

The recent literature on the welfare state has pointed out that pub taxation in many cases serve an insurance function to the extent that t on the state of nature (see eg Sandmo (1991) and Sinn (1995)). In the ca private insurance market there may be a welfare case for such contin present discussion it is particularly noteworthy that taxes and tran

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[4] Bayomi and Eichengreen (1995) conclude based on simulations with the MULTIMOD mode restraints may have severe consequences for macroeconomic volatility.

implicit insurance function (See eg Varian (1980)). Redistribution progressive taxation may be associated with efficiency gains to the ext sector provides diversification possibilities for idiosyncratic sho available via private markets. In a macroeconomic context it has also that taxation may affect the impact of idiosyncratic shocks and the precautionary savings (Barsky, Mankiw and Zeldes (1986)). To deal with idiosyncratic shocks there is however neither a need nor a welfare gain from runni budget as the question is to design a transfer scheme from "lucky" to (Fremling and Lott (1994)).[5] This is an atemporal problem and from this it inferred that budget deficits as such provide no insurance.

This conclusion needs not hold in the case of aggregate shock diversification possibilities for such shocks exists in a closed econom present in an open economy. An important channel for risk diversifi international capital markets. By running deficits or surpluses the gove these markets to attain social insurance of aggregate shocks. A balan is effectively a constraint on the ability of the public sector to u may mean nothing if capital markets are perfect and the public sect using capital markets better than the private sector. However, amble that capital markets are not perfect and private agents are not abl markets fully.[6] Under such circumstances restrains on public budgets may consequences.

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[5] Croushore (1996) endogenizes labour supply to analyse how insurance of ideosyncrati savings and labour supply decision.

[6] An important example of this is the failure for private agents to fully divers international capital markets, see Lewis (1996).

We explore this issue in an open economy with fluctuations driv (productivity) shocks. The focus is thus on the interplay between inc come taxation. The optimal design of the income taxation system to f of public expenditures[7] is considered by taking account of both the insuranc the distortionary effects of taxation and by comparing it to the tax system arising case of a balanced budget. This makes it possible both to evalu consequences of budget restraints and the implications for macroeconomi also analysed how the financing rules for the government affects th public consumption.

The analysis is performed by use of a model for a small open economy generations. This is a convenient way by which to formulate a ful intertemporal general equilibrium model in which there is a capital mark creating a role for social insurance. By the very nature of this s imperfection in the sense that no private market can be set up to diversify this risk reason being that this should be diversified among different generati means by which current generations can extract resources from yet ur and no mechanisms by which the latter can ensure that resources are (the problem of insurance at zero age). However, the government may and we analyse how this work in a small open economy with liberaliz capital models. This provides a simple way by which to model capital ma over, it highlights the relationship between budget deficits and markets.

Possibilities for diversification of aggregate risk have in an open analysed by Aizenman (1981). The idea is that the balance of paymen

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[7] Barro(1979) considers how distortionary taxes should be smoothed to finance variati driven by eg wars.

absorbed and changes in the stock of international reserves can be aggregate shocks and smooth consumption so as to increase welfare. The market, and the scope for diversification is determined by the size of Gordon and Varian (1988) show how the government can implement a t scheme between different generations alive at a given period so as allocation between generations in a way which implies welfare improvem cases, the capital market and the public budget play no role. Moreov exogenous, and the issue of distortionary taxation does not arise.

The paper is organized as follows: section 2 develops a small c generations economy with liberalized capital movements. Section 3 deve insurance implications of a balanced budget regime and regimes allow imbalances by considering the case of exogenous production, while sect distortionary taxation by endogenizing production. Finally, sectio concluding comments.

2. A Small Open Overlapping Generations Economy

Consider a small open economy producing a commodity which is a perfect internationally traded goods being traded at a price P (in domestic c market. The exchange rate is fixed, and there are no restrictions on ac capital markets implying that the rate of interest equals the world

Households

The population is constant, and individuals live for two periods. Th ($c_{1,t}$) and old ($c_{2,t+1}$) and work only as young ($l_{1,t}$). Moreover, they obtain utility

access to a public good available in the economy[8]. The utility for the represen
household is given by

$$u\left(c_{1,t}, c_{2,t+1}\right) - v\left(l_{1,t}\right) + s(g)$$

where

$$\frac{\partial u}{\partial c_{j,t+j-1}} > 0 \qquad \frac{\partial^2 u}{\partial c^2_{j,t+j-1}} < 0 \qquad j = 1, 2$$

$$\frac{\partial v}{\partial l_{1,t}} > 0$$

$$\frac{\partial s}{\partial g} > 0 \qquad \frac{\partial^2 s}{\partial g^2} < 0$$

The consumer problem can conveniently be analysed in two steps, name
sidering the consumption decision given income and second by consider
supply decision to generate income. Households inherit ownership of
entitled to profit income generated by firms.

For a given disposable income level I, the consumption problem is to m
of consumption subject to the budget constraint

$$c_{1t} + \left(1 + r_t\right)^{-1} c_{2t+1} = I_t / P_t \equi i_t$$

where r_t denotes the real rate of interest, and i_t real income.

The consumption while young and old can now be stated

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[8] This good may yield utility either as young, old or both. This does not matter a
exogenous to the agent and there is no uncertainty concerning the supply of the publ

$$c_{1t} = c_1(1 + r_t, i_t)$$

$$c_{2t+1} = c_2(1 + r_t, i_t)$$

The utility of consumption following from the optimal consumption dec
marized by the indirect utility function

$$U(1 + r_t, i_t) \qquad (1)$$

where

$$\frac{\partial U}{\partial i_t} > 0; \quad \frac{\partial^2 U}{\partial i_t^2} < 0$$

The real rate of interest is exogenous due to the small open econo
since the focus here is on income variability, we simplify and assu
The real disposable income[9] is given by

$$i_t = (1 - \tau_t)(w_t l_t + \pi_t) \qquad (2)$$

where w_t is the real wage rate, π_t real profits, and τ_t the tax rate applying to income.

Given (2), the labour supply decision is easily found as the soluti
problem

$$\max_{l_t} U\left((1 - \tau_t)(w_t l_t + \pi_t)\right) - v(l_t)$$

The labour supply decision is characterized by the following condit

$$(1 - \tau_t) w_t U'(i_t) = v'(l_t) \qquad (3)$$

[9] Notice that this formulation presumes that the only form of taxation is income taxa
possible to tax eg capital income, but this is disregarded to focus on the interplay be
income taxation.

Firms

All firms are price and wage takers and produce subject to a produc

$$y_t = a_t f(l_t) \; ; \qquad f^{/} > 0, \; f^{//} \leq 0$$

where l_t is labour input, a_t is an indicator for productivity. The labou decision of the firms is characterized by the condition

$$a_t f^{/}(l_t) = w_t \qquad (4)$$

Note that the production decision is taken under full certainty, ie t known. This also implies that it is inconsequential whether profits accrue in t+1 as long as there is perfect information and perfect capital mar

Shocks

Since the focus is social insurance, we want to rule out transfers/red periods (generations) which is motivated by changes in the perceptio permanent income for the economy[10]. It is therefore convenient to specify a p the productivity variable a_t such that it does not induce shifts in the percei income. This requires that the expected present value of the shock i that

$$E_t \sum_{j=0}^{\infty} (1+r)^{-j} a_{t+j} = \text{constant} \; \forall t$$

This condition is fulfilled by the following process

$$a_t - \bar{a} = -(1+r)(a_{t-1} - \bar{a}) + \nu_t \qquad (5)$$

where $\bar{a}$ is the permanent level of a, and ν_t is iid having a symmetric density fu $f(\nu)$ with support on $[\underline{\nu}, \bar{\nu}]$. This specification implies that there will be g

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[10] It is well-known that changes in permanent income may be a reason for redistributio Fatás (1997).

states, but it is ex ante uncertain which generation will be lucky unlucky.

Note that for a more general process for the shock variable, (5) can transfers across generations which can be justified on pure insurance as pure redistribution.

Government

The government supplies a public good g which is financed by an inc

The real value of the primary public budget in period t is

$$b_t = \tau_t y_t - g \tag{6}$$

The public sector has - as the private sector - access to the inter and the real debt level d develops according to

$$d_t = b_t + (1+r)d_{t-1}$$

The initial debt level is assumed to be zero, ie $d_{t-1} = 0$

We shall consider different budgetary regimes for the public secto continuously balanced budget, ie

$$b_t = 0 \quad \forall t \tag{7}$$

implying that the intertemporal solvency condition is automatically f regime allows for budget imbalances within the constraint set by the i which we operationalize by imposing that the expected budget bala[11)]

[11] This is a more strict condition than needed to have a sustainable debt level for Chang (1990).

$$E_t b_t = 0 \quad \forall\, t$$

which is sufficient to ensure that the expected level of debt is bo

$$E_t d_{t+j} < \bar{d}, \forall\, t, j > 0$$

This regime cooresponds to the argument often made in policy debates should be balanced over the business cycle. We consider both how t schemes operate to finance a given level of public expenditures, and optimal level of public consumption.

Equilibrium Conditions

The labour market is competitive and the equilibrium condition read

$$l_t^d = l_t^s \tag{8}$$

As the good produced is traded internationally, there is no product condition. The trade balance tb_t in period t reads

$$tb_t = y_t - c_t - g \tag{9}$$

where c_t is total private consumption in period t, ie the sum of consum old given by

$$c_t = c_{1t} + c_{2t} \tag{10}$$

3. Exogenous Production

To clarify the mechanisms through which the budget can provide social insurance, useful to start by considering the case with exogenous production. assumed to be supplied inelastically (l = 1, v(1) = 0) and producti that y = a (f(1) = 1).

Assume that the level of public consumption is given and the problem
this. If the budget is required to be balanced period by period it fol
that the tax rate has to be

$$\tau(a_t) = \frac{g}{a_t}$$

that is, the tax rate moves countercyclical

$$\frac{\partial \tau(a_t)}{\partial a_t} = -\frac{g}{a_t^2} < 0 \qquad (11)$$

In periods with high production, the given level of public consumpti
by a low tax rate and vice versa in states of nature with low produ

The utility to the generation born in period t can thus be written

$$U(a_t - g)$$

implying that the ex ante or expected utility to a member of any genera

$$EU(a_t - g)$$

With a balanced budget it follows that the public sector does not u
capital market. Clearly this may impy a welfare loss as such markets c
of smoothing the tax burden and thereby allowing a diversification of
production. One possibility for achieving this would be to choose
avoiding that taxes vary with the state of nature equal to

$$\tau = \frac{g}{Ea_t} \qquad (12)$$

In this case utility of a period t generation becomes

$$U\left(a_t - g\frac{a_t}{Ea_t}\right)$$

and the expected utility can be written

$$EU\left(a_t - g\frac{a_t}{Ea_t}\right)$$

Clearly generations are better off in terms of expected utility und constant tax rate as compared to balance budget system as is seen by noting expected after-tax income is the same

$$E\left(a_t - g\frac{a_t}{Ea_t}\right) = Ea_t - g$$

but its variance is lower in the constant tax-rate regime, ie

$$Var\left(a_t - g\frac{a_t}{Ea_t}\right) < Var(a_t - g)$$

Itfollows that the expected utility is higher in the constant tax balanced budget regime

$$EU\left(a_t - \frac{a_t}{Ea_t}g\right) > EU(a_t - g)$$

Itis easily seen how this policy works by considering how the public the state of nature, ie

$$b_t = \left(\frac{a_t}{Ea_t} - 1\right)g$$

In bad (good) states, there is a budget deficit (surplus). The pu international capital market to smooth the tax burden by letting tax-payments be when income is low and vice versa. Notice that this is not attainable due to the fact that the shock is an aggregate and thus non-diversi given generation and due to the limited possibilities for private households to such risk in the international capital market due to their fixed li[12]

It is easily checked that this policy is feasible as

$$E_t d_{t+1} = E_t\left((1+r)b_t + b_{t+1}\right) = E_t\left(\frac{g}{\bar{a}}\nu_{t+1}\right) = 0$$

Although holding a constant tax rate does achieve some insurance, it the optimal tax policy in the sense of being the best way of financ public expenditures so as to maximize expected utility across generat there exists a tax policy which will remove all risk and thereby consumption level for all generations. This can be accomplished by function

$$\tau(a) = \frac{g}{a_t} + \left(1 - \frac{Ea_t}{a_t}\right) \qquad (13)$$

It is easily verified that it implies that

$$a_t\left(1-\tau(a_t)\right) = Ea_t - g$$

and that

$$Var\left(a_t\left(1-\tau(a_t)\right)\right) = Var\left(Ea_t - g\right) = 0$$

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[12] A direct transfer scheme between generations would attain some diversification, s (1988). However, this cannot be decentralized as a market outcome.

and this policy is moreover consistent with the budget constraint.

Notice that the optimal policy implies that the tax rate becomes proc
is high when income is high and vice versa, ie

$$\frac{\partial \tau(a_t)}{\partial a_t} > 0$$

This provides an argument for a progressive taxation system which
sensitivity of the public budget to the business cycle situation (moves
thereby provides social insurance. It is also noted that the progress
size of the public sector as

$$\frac{\partial}{\partial g}\left(\frac{\partial \tau(a_t)}{\partial a_t}\right) < 0$$

It is worth stressing that it is an implication of the optimal tax p
even if lump-sum taxation is feasible, it is not optimal to use this
it is unconditional and therefore achieves no diversification.

Having considered the optimal tax policy as given by (13) in the case
balances are allowed, it is natural to question the extent to which a
imbalances affects the optimal level of public consumption. Budget n
as instrumental to the objective of reducing the relative size of
consider public consumption to be of a type which cannot easily be chan
infrastructure etc.) and it is thus most plausible to consider public
consumption before the state of nature is known. The optimal level of
maximizing expected utility including the value of public goods in the
budget regime (indexed by B) is determined by the condition

$$EU'(a_t - g_B) = s'(g_B)$$

while it under the optimal tax rule (13) (indexed by D) in the absence balance rule reads

$$U'\left(Ea_t - g_D\right) = s'\left(g_D\right)$$

It follows that

$$g_D \gtrless g_B \quad \text{for } U''' \gtrless 0$$

This shows that the institutional rules on the mode of financing in g optimal level of public consumption even when the level of public decided before the veil of ignorance is lifted. It is in particularly generally the case that the balanced budget regime delivers the lowes consumption.

4. Endogenous Production

The preceding analysis disregarded the distortionary effects of taxat duction level to be exogenous. This may be critical as the distortiona may inflict with the insurance effects in a non-trivial way. We con present section by allowing for endogenous production[13].

It is useful to start by considering in more detail how activity and state of nature for a given tax rate. Next we consider the different finances.

Equilibrium employment can be written as a function of the variable $\hat{a}$ which might be termed the after tax value of the state of nature variable a, ie (S

$$l = e\left(\hat{a}\right) \qquad \hat{a} \equiv a\left(1 - \tau\left(a\right)\right) \qquad (14)$$

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[13] On the other hand - the positive effects of insurance on private incentives are neglected by a that all private decisions are contingent on a, see eg Sinn (1995).

and

$$\mathrm{sign}\, e'(\hat{a}) = \mathrm{sign}(1 - R_U) \qquad R_U \equiv -\frac{U''(i)\, i}{U'(i)}$$

To simplify the notation, the time index is suppressed. Note that R_U is the measure of
relative risk aversion for the indirect utility function U. Attent
where the labour supply function is upward sloping which follows if

$$R_U < \frac{1}{\gamma} \; ; \quad \gamma \equiv \frac{wl}{\pi + wl}$$

An upward sloping labour supply function and a linear ($f'' = 0$) production te
are sufficient conditions to ensure that equilibrium employment is

Using (14) we can summarize the utility of consumption and the disut
equilibrium as a function of â (see appendix (ii)), ie

$$V(\hat{a}) \equiv \underset{l}{\mathrm{argmax}}\, U(i) - v(l)$$

where (see appendix (ii))

$$V'(\hat{a}) = U' \cdot f > 0 \qquad (15)$$

One important and surprising finding is that although the underl
function is characterized by risk aversion, this does not generally a
utility as

$$V''(\hat{a}) \gtrless 0 \quad \text{for } R_U \gtrless R_U^{*} = \frac{\hat{a} f' e'}{f + \hat{a} f' e'}$$

The reason is that the marginal utility of $\hat{a}$ is given as the product of t marginal utility of consumption $(U^{/})$ and the production level (f). Hence, eve increase in $\hat{a}$ increases consumption and thus lowers the marginal utility $(U^{//} < 0)$, this may be counteracted by a increase in production $(f^{/}e^{/} > 0)$.

Balanced Budget

Consider first the case of a balance budget regime where the tax-rate the budget condition

$$\tau a f\left(e\left(a\left(1-\tau\right)\right)\right) = g \tag{16}$$

implying that

$$\frac{\partial \tau}{\partial a} = -\frac{\tau f + \tau a^{2} f^{/} e^{/}\left(1-\tau\right)}{af - \tau a^{2} f^{/} e^{/}} \gtrless 0$$

The tax rate may thus move pro- or countercyclically. Notice that a rate implies that the effects of variations in a are amplified and v pro-cyclically.

To see more clearly the effects at stake here, we start by noting th creasing in a $(e^{/} > 0)$, then the tax rate always move counter-cyclical in budget regime. For the tax to move procyclically, it is necessary th induces such a large fall in employment that total income moves co Clearly, this is an extreme case and the assumption that labour supply combined with a linear production technology $(f^{//} = 0)$ is sufficient to rule it out.

Social Insurance

We shall prove that there always exists a budgetary system which dom
budget regime. Let $\tau(a)$ be the tax-function consistent with a balanced budget, cf. (16).
Consider then an alternative tax-function $\tilde{\tau}(a)$ given by

$$\tilde{\tau}(a) = \tau(a) + \epsilon\left(\frac{Ea}{a} - 1\right)$$

Under this tax-function expected utility is

$$EV\left(a\left(1 - \tau(a) - \epsilon\left(\frac{Ea}{a} - 1\right)\right)\right)$$

and it is easily proven that (see appendix)

$$\left.\frac{\partial EV}{\partial \epsilon}\right|_{\epsilon=0} \neq 0 \qquad \text{for } V'' \neq 0$$

That is, unless the indirect utility function is linear in â, there
which yields higher expected utility than the balanced budget regim

The direction in which deviations from the balanced budget should go is seen fro
fact that

$$\left.\frac{\partial EV}{\partial \epsilon}\right|_{\epsilon=0} < 0 \quad \text{for } V'' < 0$$

That is, if there is risk aversion with respect to variations in â, th
$(\epsilon = 0)$ can be dominated by a regime which has a lower tax rate in bad states of n
and vice versa $(\epsilon < 0)$, ie

$$\tilde{\tau}(a) \gtrless \tau(a) \quad \text{for } a \gtrless Ea$$

This means that there will be a budget deficit (surplus) in bad (go

If the indirect utility function displays risk-preference, $V'' > 0$, we t variati
get that

$$\left.\frac{\partial EV}{\partial \epsilon}\right|_{\epsilon=0} > 0 \quad \text{for } V'' > 0$$

and hence deviations from the balanced budget regime should go in
making taxes move countercyclically, ie

$$\tilde{\tau}(a) \lesseqgtr \tau(a) \quad \text{for } a \gtreqless Ea$$

which means that there will be a budget surplus (deficit) in bad (g

Full Insurance

Even with endogenous production, it is possible to design a tax syste
the level of public consumption g and at the same time attain full
private consumption risk free. Consider the possibility of designing a

$$\hat{a} = a(1-\tau) = \kappa$$

where κ is the highest obtainable constant level of $\hat{a}$, which requires

$$\tau = 1 - \frac{\kappa}{a}$$

To check whether this can be consistent with budget balance on aver
revenue in state a is

$$(a-\kappa)f(e(\kappa))$$

The expected value of which is

$$(Ea-\kappa)f(e(\kappa)) = g$$

Hence κ is defined by this relation and can be attained with full ce[14]

Optimal Taxation

Next we have to consider the optimal tax policy to see if it entails what extent it is influenced by the distortionary and insurance eff optimal tax-policy solves the following problem[15]

$$\max_{\{\tau(a)\}} EV(a-a\tau(a)) \tag{17}$$

subject to

$$E\,aR(a,\tau(a)) = g \tag{18}$$

where

$$R(a,\tau(a)) \equiv \tau(a)f(e(\hat{a}))$$

that is, $aR(a,\cdot)$ denotes the revenue attained in state a for a tax rat $\tau(a)$.

The first-order condition to the problem given in (17) and (18) can

$$\frac{V_{\hat{a}}'(a-a\tau(a))}{R_{\tau}'(a,\tau(a))} = \lambda \tag{19}$$

---

[14] A solution exists provided g is not too large.

[15] Notice that ex-ante the expected level of productivity is the same for all generati the specification of a process for the shock implying a constant expected permanent time.

where λ is a Lagrange-multiplier associated with the constraint (18)

$$aR_\tau^{/}(a,\tau(a)) = a\left[f(e(\hat{a})) + \tau(a) f^{/} e^{/}(\hat{a})(-a)\right]$$

Note that $aR_\tau^{/}$ is the marginal revenue effect of changing the tax rate a

$$aR_\tau^{/}(a,\tau(a)) < af(e(\hat{a})) \quad \text{for } e^{/} > 0$$

which reflects the distortionary effects of taxes.

For two different states of nature, a_1, a_2 ($a_1 \neq a_2$), we find from (19) that the optimal policy implies

$$\frac{V_{\hat{a}}^{/}(a_1 - a_1\tau(a_2))}{R_\tau^{/}(a_1,\tau(a_1))} = \frac{V_{\hat{a}}^{/}(a_2 - a_2\tau(a_2))}{R_\tau^{/}(a_2,\tau(a_2))} \tag{20}$$

Condition (20) says that the optimal tax structure ensures that the private consumption relative to the "marginal tax revenue" must be equa nature.

Full insurance requires that

$$a_1 - a_1\tau(a_1) = a_2 - \tau_2(a_2) \qquad \forall\ a_1, a_2\ (a_1 \neq a_2)$$

That full insurance is not in general implied by the optimal tax stru from (20) as it would require that $R_\tau^{/}(a_1,\tau(a_1)) = R_\tau^{/}(a_2,\tau(a_2))$ $\forall\ a_1, a_2\ (a_1 \neq a_2)$. A condition which is not generally fulfilled.

Notice that in the case where $f^{/}e^{/} = 0$, i.e. taxes are non-distortionary, it fol $R_\tau^{/}(a_1\tau(a)) = f(e(\hat{a}))$ and hence full insurance is optimal. Notice, that thi

with the finding in section 3 where production was exogenously giv
taxation therefore by assumption did not have any distortionary eff

Hence, when taxes are distortionary, it is inoptimal via the public b
insurance although it is a feasible option.

To consider in more detail the properties of the optimal tax policy,
that sufficient conditions for procyclical tax rates (progressive t

$$\frac{\partial \tau(a)}{\partial a} > 0$$

are that (i) agents are risk averse ($V'' < 0$), (ii) tax-distortions are increasing i
rate ($R''_{\tau\tau} < 0$) and (iii) the tax-distortion is lower in good states of n ($R''_{\tau a} > 0$).

According to the "tax-smoothing" principle, the optimal policy is a
(Barro (1979)).[16] This result takes into account only the distortionary ef
By also including the insurance effects of taxation, we find that a
in general optimal although it relative to the balanced budget case
insurance. Under plausible assumptions the optimal tax rate is procy

Notice that even in the case where agents effectively are risk-neut ($V'' = 0$), the
optimal tax policy is not a constant tax rate as

$$\operatorname{sign}\left(\frac{\partial \tau}{\partial a}\right)\Bigg|_{V''=0} = -\operatorname{sign}\left(\frac{R''_{\tau a}}{R''_{\tau\tau}}\right)$$

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[16] In Andersen and Dogonowski (1998) we show that an explicit modelling of tax distort
intertemporal substitution in labour supply does not support a constant tax rate as m

Hence only if the distortionary effects of taxation are independent $(R''_{ta} = 0)$ of t

does it follow that the optimal policy is a constant tax rate.

Finally, it should be pointed out that even by allowing for lump-sum t
optimal to fully finance public expenditures by this non-distorti
appendix (iv)). This shows that the insurance effect at the margin i
outweigh the distortions of income taxation.

Macroeconomic Stability

The financing regime for public expenditures has implications for
volatility. For output we find

$$\epsilon_{ya} = 1 + \eta_y \epsilon_{\hat{a},a}$$

where

$$\epsilon_{xy} \equiv \frac{\partial x}{\partial y} \frac{y}{x}$$

$$\eta_y \equiv f' e' \frac{\hat{a}}{f}$$

We find that output is more sensitive to the state of nature under a
(indexed by B) than under the optimal tax structure[17] (indexed by D)

$$\epsilon_{ya}\Big|_B > \epsilon_{ya}\Big|_D$$

[17] See Röell and Sussman (1997) for a case where taxes provide implicit insurance, but
structure is not stabilizing.

if $e' > 0$ (output and employment is increasing in the state of nature

optimal taxes are non regressive $\epsilon_{ga} > 0$. This is consistent with the empirical f

Gali (1994). As should be expected, this also lowers the sensitivit

$$\epsilon_{ca}\big|_B > \epsilon_{ca}\big|_D$$

It is also easily verified that both private and public net-savings a

variable in this case. This implies that the trade balance moves procyc

in accordance with stylized empirical facts (see eg Backus and Keho

Optimal Public Consumption

Finally, we consider the optimal level of public consumption under a

(g_B) rule and under the optimal tax policy (g_D). As for the case with exogenous

production, we find that there is no unambiguous relation between the t

appendix (vii)), ie

$$g_D \gtrless g_B$$

It may surprise that public consumption is not generally larger in the

regime as the budget balance restriction is lifted. Although this eff

an opposite effect from the fact that providing insurance may inc

marginal value of private consumption.

5. Concluding Remarks

Policy restrictions on public deficits means limitations on the possi

sector to use international capital markets for intertemporal smooth

inflict with the insurance or stabilizing effects of "automatic stabili

budgets.[18])

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[18] In a European perspective the insurance or stabilizing aspects of the public budge
as there is no federal budget to compensate for the loss of fiscal flexibility in me

Solving for the optimal tax policy we find that it under plausible as both the tax rate (progressive taxation) and the primary public budget and moreover this also produces macroeconomic stability.

This insurance argument relies on a capital market imperfection imply sector has diversification possibilities to aggregate shocks which are not fu to the private sector. While this possibility easily arises in an economy with an inoperative bequest motive, we think of this as an i modelling an aspect which goes beyond the specificities of int diversification of shocks.

The present analysis has not dealt with the political decision proces may influence debt policy and lead to a deficit bias (see eg Alesin The present argument that there are welfare gains from allowing public budget imbalance suggests that there is a traditional rules vs. discretion p extent that there is a political deficit bias (see eg Corsetti and

are implemented strictly in the Economic and Monetary Union.

Appendix

(i) Equilibrium Employment

Using the conditions determining labour supply and demand (3), (4), equilibrium employment from the relation

$$(1-\tau)af'(l)U'\big((1-\tau)af(l)\big) - v'(l) = 0 \qquad \text{(A-1)}$$

This gives equilibrium employment as an implicit function of $\hat{a} \equiv a(1-\tau)$, ie

$$l = e(\hat{a})$$

Differentiation of (A-1) yields

$$e'(\hat{a}) = \frac{\frac{l}{\hat{a}}\left(R_U - 1\right)}{\frac{f''l}{f'} - R_U\gamma + R_v}$$

where

$$\gamma \equiv \frac{wl}{\pi + wl}, \; R_v \equiv \frac{-v''(l)l}{v'(l)}, \quad R_U \equiv \frac{-U''(i)i}{U'(i)}$$

From the second order condition to the household optimization proble

$$-R_U\gamma + R_v < 0$$

Hence, given that $f'' < 0$, it follows that

$$\text{sign}\, e'(\hat{a}) = \text{sign}\left(1 - R_U\right)$$

From the labour supply function (3), we find

$$\frac{\partial l}{\partial w} = \frac{\frac{l}{w}\left(1 - R_U\gamma\right)}{R_U\gamma - R_v}$$

For labour supply to be increasing in the wage rate, we require

$$R_U < \frac{1}{\gamma}$$

(ii) The Indirect Utility Function V(â)

Since

$$i = (1-\tau)(\pi + w\,l) = (1-\tau)af(l)$$

and

$$l = e(\hat{a})$$

we can write the sum of utility of consumption and disutility of lab

$$V(\hat{a}) \equiv U(\hat{a}f(e(\hat{a}))) - v(e(\hat{a}))$$

We find by use of the first order condition (3) that

$$V'(\hat{a}) = U'f$$

and

$$V''(\hat{a}) \equiv U''f^2 + f'e'U'(1-R_U)$$

We have that $V''(\hat{a}) < 0$ if

$$U''f^2 + f'e'U'(1-R_U) < 0$$

or

$$-R_U\frac{f}{\hat{a}} + f'e'(1-R_U) < 0$$

which can be rewritten

$$R_U > \frac{\hat{a}f'e'}{f+\hat{a}f'e'} \equiv R_U^*$$

Similarly, $V'' > 0$ if

$$R_U < R_U^*$$

(iii) Derivation of Expected Utility wrt ϵ.

We have that

$$\frac{\partial EV\left(a\left(1-\tau(a)-\epsilon\left(\frac{Ea}{a}-1\right)\right)\right)}{\partial\epsilon} = EV'\left(a\left(1-\tau(a)-\epsilon\left(\frac{Ea}{a}-1\right)\right)\right)\left(-\left(\frac{Ea}{a}-1\right)\right)$$

and hence

$$\left.\frac{\partial EV\left(a\left(1-\tau(a)-\left(\frac{Ea}{a}-1\right)\right)\right)}{\partial\epsilon}\right|_{\epsilon=0} = EV'(a(1-\tau(a)))\left(-\left(\frac{Ea}{a}-1\right)\right)$$

Next we shall prove that $\left.\frac{\partial EV}{\partial\epsilon}\right|_{\epsilon=0} < 0$ for $V'' < 0$.

By the symmetry of the density function h around the mean Ea, it fol $\forall\lambda > 0$
for which $Ea-\lambda$ and $Ea+\lambda$ belong to the support of h(a) that

$$h(Ea-\lambda) = h(Ea+\lambda) \qquad \forall\ \lambda \in \left[0,\bar{\lambda}\right]$$

As $V'' < 0$ we have

$$V'((Ea-\lambda)(1-\tau(Ea-\lambda))) > V'((Ea+\lambda)(1-\tau(Ea+\lambda)))$$

or

$$V'((Ea-\lambda)(1-\tau(Ea-\lambda)))h(Ea-\lambda)(-\lambda) < V'((Ea+\lambda)(1-\tau(Ea+\lambda)))h(Ea+\lambda)(-$$

Hence,

$$V'((Ea-\lambda)(1-\tau(Ea-\lambda)))h(Ea-\lambda)(-\lambda) + V'((Ea+\lambda)(1-\tau(Ea+\lambda)))h(Ea+\lambda)(\lambda) < 0$$

from which is follows that

$$EV'(a(1-\tau(a)))(-(Ea-a))$$

$$= \int_0^{\bar{\lambda}}\Big[V'((Ea-\lambda)(1-\tau(Ea-\lambda)))h(Ea-\lambda)$$

$$+ V'((Ea+\lambda)(1-\tau(Ea+\lambda)))h(Ea+\lambda)\Big] < 0$$

Using the same procedure, it can be proved that

$$\left.\frac{\partial EV'\left(a\left(1-\tau(a)-\epsilon\left(\frac{Ea}{a}-1\right)\right)\right)}{\partial\epsilon}\right|_{\epsilon=0} > 0 \quad \text{for } V'' > 0$$

(iv) Progression of the Optimal Tax System with Endogenous Productic

The first order condition characterizing the optimal tax system read

$$V_{\hat{a}}'(a - a\tau(a)) = \lambda R_{\tau}'(a, \tau(a))$$

when transformed by log

$$\log V_{\hat{a}}'(a - a\tau(a)) = \log \lambda + \log R_{\tau}'(a, \tau(a))$$

where $\log \lambda$ is fixed.

We take the derivative with respect to a and get

$$\frac{V_{\hat{a}\hat{a}}''}{V_{\hat{a}}'}\left(1 - \tau - a\frac{\partial \tau}{\partial a}\right) = \frac{1}{R_{\tau}'}\left\{R_{\tau a}'' + R_{\tau,\tau}''\frac{\partial \tau}{\partial a}\right\}$$

If we solve for $\frac{\partial \tau}{\partial a}$, we get

$$\frac{\partial \tau}{\partial a} = \frac{\frac{V_{\hat{a}\hat{a}}''}{V_{\hat{a}}'}(1 - \tau) - \frac{R_{\tau a}''}{R_{\tau}'}}{\frac{R_{\tau\tau}''}{R_{\tau}'} + \frac{V_{\hat{a}\hat{a}}''}{V_{\hat{a}}'}a}$$

If risk aversion $(V'' < 0)$, $R_{\tau a}'' > 0$ and $R_{\tau\tau}'' < 0$, then $\frac{\partial \tau}{\partial a} > 0$.

If risk neutrality $(V'' = 0)$ then $\text{sign}\left(\frac{\partial \tau}{\partial a}\right) = -\,\text{sign}\left(\frac{R_{\tau a}''}{R_{\tau\tau}''}\right)$

If risk seeking $(V'' > 0)$, then the sign $\left(\frac{\partial \tau}{\partial a}\right)$ is ambiguous.

(v) Risk Neutrality and a Constant Tax Rate

The optimal tax system is given by

$$\lambda = \frac{V'(\hat{a})}{R_\tau(a,\tau)} = \frac{U'f}{f-\tau f'e'a} = \frac{U'}{1-\frac{\tau}{1-\tau}\frac{e'\hat{a}}{a}}$$

Sufficient conditions for a constant optimal tax rate are that i) $U''=0$, ii) f has constant elasticity wrt e, and iii) the employment function e has an pendent of a.

The employment elasticity is given as

$$\frac{\partial e}{\partial \hat{a}}\frac{\hat{a}}{e} = \frac{-1}{\frac{f''e}{f'} + R_v}$$

and for this to be independent of a it is required that f has a cons R_v is independent of a. This will be the case if v belongs to the CR

(vi) Non-optimality of Pure Lump-sum Taxation

If we introduce a lump-sum tax T, the problem for the optimal tax sy

$$\max_{T,\tau(a)} \left[E\,v\left(a(1-\tau),T\right)\right]$$

$$\text{s.t.} \quad g = E\left[a\tau H\left(a(1-\tau),T\right) + T\right]$$

where

$$H\left(a(1-\tau),T\right) = f\left(e\left(a(1-\tau),T\right)\right)$$

The first order conditions for the optimal choice of τ and T will be

$$\hat{\lambda} = \frac{V_1^{/}\left(a(1-\tau),T\right)}{H\left(a(1-\tau),T\right) - a\tau H_1^{/}\left(a(1-\tau),T\right)} \qquad \text{(B-1)}$$

$$\hat{\lambda} = \frac{-E\left[V_1^{/}\left(a(1-\tau),T\right)\right]}{E\left[a\tau H_2^{/}\left(a(1-\tau),T\right)+1\right]} \qquad \text{(B-2)}$$

where λ is the Lagrange multiplier to the problem.

We will show that $\tau = 0$ is inconsistent with the conditions (B-1) and (B-2

optimal tax structure.

The first order condition for optimal labour supply requires $\tau = 0$ equation (3) and

condition (B-1) $\lambda = U^{/}(i)$, where income $= a \cdot if(e(a,T)) - T = i(a,T)$ is a func

a, and therefore not constant for all possible values of a, as requ

So we have a contradiction, pure lump-sum taxation is not optimal.

(vii) The Optimal Level of Public Consumption with Endogenous Produc

When solving for the optimal public consumption (and the optimal tax

regime (indexed by D) the problem reads

$$\max_{g,\tau} E\left[V\left(a(1-\tau)\right)\right] + s(g)$$
$$s.t.\ g = E\left[aR(a,\tau)\right]$$

The shadow price of one extra unit of the public good λ^D, measured in terms of utility

of the household may be expressed as

$$\lambda_D = \frac{E\left[aV'\left(a\left(1-\tau^D\right)\right)\right]}{E\left[aR_\tau\left(a,\tau^D\right)\right]}$$

For the balanced budget regime (indexed by B) the problem reads

$$\max_g E\left[V\left(a\left(1-\tau^B\right)\right)\right] + s(g)$$
$$\text{s.t. } g = aR\left(a,\tau^B\right)$$

and the shadow price λ_B reads

$$\lambda_B = E\left[\frac{aV'\left(a\left(1-\tau^B\right)\right)}{aR_\tau\left(a,\tau^B\right)}\right]$$

We will next identify the condition for $\lambda_B \lesseqgtr \lambda_D$.

We know that $s'(g) = \lambda$ in an optimum such that

$$g_D \gtreqless g_B \quad \text{for } \lambda_D \lesseqgtr \lambda_B$$

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