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Torben M. Andersen

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INSTITUT FOR ØKONOMI

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 350
8000 AARHUS C - ☎ 89 42 11 33 - TELEFAX 86 13 63 34

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STAGGERED WAGE-SETTING AND OUTPUT PERSISTENCE

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Torben M. Andersen

CEPR
and
Department of Economics
University of Aarhus
8000 Aarhus C, Denmark

Abstract
Persistent real effects of nominal shocks is considered in an intertemporal macromodel with capital accumulation and staggered nominal wage contracting. By placing the contracting process in a setting of imperfect competition it becomes possible to avoid the standard problem that intertemporal business cycle models generate plausible business cycle fluctuations only by assuming implausible large labour supply elasticities. It is shown that nominal shocks can have persistent real effects and that nominal propagation via staggered nominal wage contracts substantially strengthens the persistency in the adjustment process for real output caused by real mechanisms like capital accumulation.

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1. Introduction

What role do nominal demand shocks play for business cycles? An important question of wide policy implications, but despite a continuous stream of new theoretical and empirical analyses, it remains an open question on which it is difficult to reach consensus.

For nominal shocks to play any real role it is necessary to break the classical neutrality result, that is some nominal rigidity is required. A voluminous literature has explored the extent to which imperfect information and adjustment costs can generate nominal stickiness of aggregate importance (for a survey see e.g. Andersen (1994)). While the literature first explored the basic mechanisms in partial models there has recently been a growing literature placing these aspects in a general equilibrium setting and thereby putting the issue in a more genuine business cycle perspective.

Finding that nominal shocks can have important impact effects is only half of the story. Persistency in output adjustment is a crucial property of observed business cycle fluctuations and a convincing theory attributing an important role to nominal shocks needs also to be consistent with persistency in output fluctuations.

If a (unanticipated) monetary shock has an impact effect it would be propagated as real shocks via the (real) propagation mechanism running through capital accumulation, intertemporal substitution and various kinds of adjustment lags or costs. That is, if output is temporary high, this will activate these mechanisms no matter whether the impulse is real or nominal in origin. This is brought out by the analysis in e.g. Bénassy (1995). This raises, however, a severe problem as the internal propagation mechanism is rather weak in the dynamic macromodels which hitherto have been analysed (see e.g. Cogley and Nason (1995)). Accordingly, the processes for real shocks have been specified in such a way as to include substantial persistence, that is, one has to bring in external sources of dynamics in order to replicate observed output dynamics. However, this will not do the trick for monetary shocks given the important difference between nominal and real shocks arising from the fact that it is only the unanticipated part of the former which have real effects, while even fully anticipated real shocks have real effects[1]. This reflects that any model based on first principles has the classical neutrality property as a long-run property. Though this still leaves open how decisions depend on anticipations and how agents form their anticipations and what information they have access to.

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[1] Disregarding super non-neutralities. Attempts to quantify e.g. the inflation tax (see e.g Cooley and Hansen (1995)) has shown this to have a very modest effect.

Considering the impulse response functions to nominal shocks reveal that monetary shocks do not contribute in any significant way to output dynamics as they only have a temporary output effect (see e.g. Hairault and Portier (1993), Cooley and Hansen (1995), Ohanian et al.(1996)). This reflects that the internal propagation mechanism is too weak, a problem both for theories stressing real and nominal shocks as driving the business cycle. There is thus a need to consider propagation mechanisms further.

Could nominal inertia in itself be an important propagation mechanism? In a seminal paper Taylor (1980) argued that staggered wage setting could account for the observed persistency in output fluctuations in the US even for contracts lasting for as short as one year. The framework can be seen as a modified Philips curve in which nominal wage setting depends not only on an excess demand variable (captured by an output gap measure) but also on past and future wage rates to capture that current wages overlap with wages set in the past as well as with wages to be set in the future. This model has motivated a huge literature exploring the consequences of staggering of wages and prices (see e.g. Blanchard (1983,1986)). There is also a growing literature introducing staggered nominal contracts in explicit dynamic macromodels (Chari et al. (1996), Yun (1996), Erceg (1997), Jeanne (1997) and Ascari (1997)).

The explanatory power of staggered wage and price setting has recently been questioned on two grounds. First, it has been pointed out that the Taylor model cannot explain inflation inertia and that is has the implausible implication that a credible disinflation programme can be implemented at no output costs (Fuhrer and Moore,1995). Secondly, it has been pointed out that the elasticity of labour supply with respect to the real wage is critical to the persistency result and for plausible values of this parameter the model is not capable of generating persistency(Chari et al, 1996).

In Andersen (1997) it is shown that there is a qualitative difference between the dynamic implications of price and wage staggering. While the former need not generate persistency, the latter always does so. The basic difference between the two is that a price staggering model implies that business cycles are movements up and down the labour supply curve, while wage staggering essentially makes it movements up and down the labour demand curve. As empirical evidence generally finds labour supply to be fairly inelastic (see eg Pencavel (1986)), it is inherently difficult to base a business cycle story on movements up and down the labour supply curve. A primary aim of this paper is thus to build an explicit intertemporal model with wage staggering which does not have to rely on a fairly elastic labour supply. Based on this the analysis proceeds to investigate how real and nominal propagation mechanisms interact.

The paper is organized as follows: Section 2 sets up the intertemporal model with staggered wage contracts, section 3 solves the model for a specific process for the money supply, while section 4 interprets the dynamic properties of the system. Finally, section 5 offers a few concluding comments.

2. An Intertemporal Model with Wage Staggering[2]

There is one type of output produced by competitive firms by use of labour and capital. Households supply labour and there are N types of labour. The output price is denoted P and the wage for labour of type j is denoted W(j) (j = 1... N).

The economy has two assets, one nominal (money) with zero nominal return and one real (capital) with real rate of return r_t. Money is needed for transactions purposes and thus provides a liquidity service. Using the equivalence result of having liquidity costs in the budget constraint or real balances in the (indirect) utility function allows a considerable simplification (Feenstra (1986)).

Households

The representative household has a utility function given by

$$E\sum_{t=0}^{\infty}\rho^{t}\left[\ln C_{t}+\theta\ln\frac{M_{t}}{P_{t}}-\psi L_{t}\right] \tag{1}$$

where ρ is the subjective time preference, C denotes consumption, M money holdings, P the price level and L working time (≤ L). It follows that labour is supplied inelastically at the (real) reservation price ψ.

The temporary budget constraints read

$$C_{t}+\frac{M_{t}}{P_{t}}+I_{t}=\frac{W_{t}}{P_{t}}L_{t}+r_{t}\left(I_{t-1}+q_{t-1}K_{t-1}\right)+\frac{M_{t-1}}{P_{t}}+\frac{T_{t-1}}{P_{t}} \tag{2}$$

The LHS of (2) shows that available resources are absorbed by consumption, holdings of money or real investments (I_t). The RHS shows that resources arise from labour income, the return on real capital investments, initial money holdings and transfers from the government. It is assumed that transfers depend on initial money holdings such that

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[2] The model structure builds on Bénassy (1995).

$$M_{t-1} + T_{t-1} = \mu_t M_{t-1}$$

where μ_t has the interpretation as one plus the period t growth rate of the money stock.

Maximizing the expected utility taking prices, the rates of return and income as given, we get

$$\frac{1}{C_t} = \lambda_t$$

$$\lambda_t = \rho E_t \left(\lambda_{t+1} r_{t+1} \right)$$

$$\lambda_t = \frac{\theta P_t}{M_t} + \rho E_t \left(\lambda_{t+1} \frac{\mu_{t+1} P_t}{P_{t+1}} \right)$$

where λ_t is the Lagrange multiplier associated with the temporary budget constraint (2).

Firms

The representative price-taking firm produces subject to a Cobb-Douglas technology

$$Y_t = Z_t K_t^{\alpha} L_t^{\beta} \qquad 0 < \alpha < 1 \qquad 0 < \beta < 1 \qquad \alpha + \beta \leq 1$$

where Z_t is a technology parameter, K_t the capital stock and L_t a composite labour input defined as

$$L_t = \prod_{j=1}^{N} L_t(j)^{\frac{1}{N}}$$

where $L_t(j)$ is the input of labour type j (j = 1,2 ....N).

All profits are distributed to households on top of wage income and the return to real capital. The capital stock in period t+1 is related to the capital stock in period t and investments (I_t) as

$$K_{t+1} = K_t G\left(\frac{I_t}{K_t} \right) \qquad G' > 0 \quad G'' < 0$$

where the G(·) captures adjustment costs (cf Hercowitz and Sampton (1991)). Assuming the following simple form for the adjustment cost function

$$G\left(\frac{I_t}{K_t}\right) = \left(\frac{I_t}{K_t}\right)^{\delta} \qquad 0 \le \delta \le 1$$

we get[3]

$$K_{t+1} = K_t^{1-\delta} I_t^{\delta}, \quad 0 < \delta < 1$$

The firm is a wage and price-taker. To characterize its decisions, it is useful first to consider the minimum costs at which a quantity L_t of the composite labour input can be acquired. This can be written

$$W_t L_t$$

where

$$W_t \equiv N \prod_{j=1}^{N} W_t(j)^{\frac{1}{N}}$$

The expression W_t thus has the convenient interpretation as the wage rate for the composite labour input. Solving for the profit maximizing level of capital and composite labour input, we find

$$r_t = \alpha Z_t K_t^{\alpha-1} L_t^{\beta}$$

$$\frac{W_t}{P_t} = \beta Z_t K_t^{\alpha} L_t^{\beta-1}$$

For later reference it is useful to note that the elasticity of labour demand type j wrt its own wage rate is given as

$$\frac{\partial L_t}{\partial W_t(j)} \frac{W_t(j)}{L_t(j)} \equiv \eta = \frac{N}{\beta - 1} < -1$$

---

3 Alternatively, if $K_{t+1}=(1-\lambda)K_t+I_t$ where λ is the rate of depreciation then $\ln K_{t+1} = \ln((1-\lambda)K_t(1+z_t))$ where $z_t = I_t/(1-\lambda)K_t$. Hence, $\ln K_{t+1} = \ln(1-\lambda)+\ln K_t + \ln(1+z_t)$. If we make a linearization of $\ln(1+z)$ at its steady state value we have $\ln(1+z) = \delta \ln z$ where $\delta = \partial \ln(1+\bar{z})/\partial \ln \bar{z}$ and $\bar{z} = \lambda/1-\lambda$, hence $\ln K_{t+1} = (1-\delta)\ln K_t + \delta \ln I_t$.

We also note that the price of capital in terms of current output

$$q_t = \frac{\partial K_{t+1} / \partial K_t}{\partial K_{t+1} / \partial I_t} = \frac{1-\delta}{\delta}\frac{I_t}{K_t}$$

and

$$r_t = \frac{\partial Y_t}{\partial I_{t-1}} = \alpha\delta\frac{Y_t}{I_{t-1}}$$

Wage Setting

As noted there are N different types of labour. The attributes of workers underlying these differences are exogenously given, and it is not possible by endogenous actions to change type. There is an equal number of workers of each type, and they are all organized in craft specific unions which have the power to determine the nominal wage rate. Unions set wages for T-periods and do so in a staggered fashion. In particular, in each period t a fraction N/T of unions choose a wage rate applying for T-periods. The unions are indexed such that unions h = {1, ..., N/T} set wages in periods 0, T, 2T etc., unions h = {N/T+1, ..., 2N/T} set wages in periods 1, T+1, 2T+1 etc, and so on for the N unions.

To derive the optimal nominal wage rule, consider first the optimal nominal wage which the utilitarian union would set if it could adjust the nominal wage freely in any period. The utility gain when a union worker gets a job is the indirect utility of the wage income received $v\left(\frac{W_t}{P_t}\right)$ less the disutility of work, i.e.

$$\left(v\left(\frac{W_t(h)}{P_t}\right) - \psi\right)L_t(h)$$

Maximizing this expression subject to the labour demand function, we get

$$W_t^*(h) = P_t\psi\frac{\eta}{1+\eta} \equiv P_t\theta \tag{3}$$

Each union thus aims at a real wage target θ. This is consistent with the empirical finding that there is no systematic cyclical variations in real wages (see Abraham and Haltiwanger, 1995).

Due to the N-period contract structure, unions cannot achieve their optimal wage (3) in each period, as the nominal wage has to be the same over the contract period. We assume that the loss function can be approximated by

$$\Im_t = E_{t-1}\sum_{j=0}^{T-1}\rho^j\left(\ln\overline{W}_t(h) - \ln P_{t+j} - \ln\theta\right)^2$$

that is, the present value of the squared sum of the deviation between the (log of) actual and the optimal real wage rate. E_{t-1} denotes expectations conditional on the information available at the end of period t-1. Minimizing this loss function yields

$$\ln \overline{W}_t = \ln\theta + \sum_{j=0}^{T-1} \phi_j E_{t-1} \ln P_{t+j}$$

where

$$\phi_j \equiv \rho^j \left[\sum_{i=0}^{T-1} \rho^j \right]^{-1}$$

and

$$\Sigma\phi_j = 1$$

The union index has been dropped as all unions setting wages for T periods at the start of period t set the same wage due to the symmetry imposed on the model.

Since unions can be identified by the period in which they set wages, we have

$$\ln W_t = \ln\theta + \frac{1}{T}\sum_{s=0}^{T-1} \ln \overline{W}_{t-s}$$

$$= \ln\theta + \frac{1}{T}\sum_{s=0}^{T-1}\sum_{j=0}^{T-1} \phi_j E_{t-s-1} \ln P_{t-s+j}$$

Equilibrium

The equilibrium condition for the goods market reads

$$Y_t = C_t + I_t$$

and for the money market

$$M_t = \mu_t M_{t-1}$$

and employment is demand determined given the wage set by unions.

We find, cf appendix A, that consumption can be written

$$C_t = (1 - \alpha\rho\delta) Y_t$$

Total investment becomes

$$I_t = \alpha\rho\delta Y_t$$

and total real money holdings

$$\frac{M_t}{P_t} = \frac{\theta(1-\alpha\rho\delta)}{1-\rho} Y_t$$

We can thus summarize the model (in log form) as

$$y_t = z_t + \alpha k_t + \beta l_t$$

$$w_t - p_t = z_t + \alpha k_t - (1-\beta) l_t$$

$$k_{t+1} = (1-\delta) k_t + \delta y_t$$

$$p_t = m_t - y_t$$

$$w_t = \frac{1}{T} \sum_{s=0}^{T-1} \sum_{j=0}^{T-1} \phi_j E_{t-s-1} p_{t-s+j}$$

where all constants have been eliminated, and the convention that $x \equiv \ln X$ has been used.

3. Shocks and Output Dynamics

Before specifying processes for the shocks, it is useful to condense the model by first solving it for a given wage rate which yields

$$y_t = z_t + \alpha k_t + \beta \left(m_t - w_t \right) \tag{10}$$

$$k_t = (1-\delta) k_{t-1} + \delta y_{t-1} \tag{11}$$

or combining the two

$$y_t = z_t + \beta \left(m_g - w_t \right) + \alpha \delta y_{t-1} + (1-\delta) k_{t-1}$$

$$= z_t + \beta \left(m_t - w_t \right) + \alpha \delta y_{t-1} + \alpha \delta (1-\delta) y_{t-2} \tag{12}$$

$$+ \alpha \delta (1-\delta)^{s-1} y_{t-s} + \alpha (1-\delta)^{s} k_{t-2}$$

Equation (12) reveals several important aspects concerning output dynamics. Impact effects can be generated by both real (z_t) and nominal shocks (m_t-w_t). The latter arises only to the extent that changes in the money supply are not fully reflected in nominal wages (classical neutrality is broken). A positive money stock will be expansionary if wages are less than fully responsive to this change ($\partial w_t/\partial m_t < 1$). There is an internal propagation mechanism generated by capital accumulation and the persistency parameter is $\alpha\delta$. That is, even if real and nominal shocks are transitory, there will be persistency in output adjustment. Persistency is strengthened the higher the elasticity of output wrt to capital α, and the more current investments add to the capital stock (δ).

The interesting aspect here is how wage staggering supplements this propagation mechanism, that is, on top of the (real) propagation mechanism generated by capital accumulation we add a (nominal) propagation mechanism driven by staggered wage-setting. To this end it is useful to note that the wage equation[4]

$$w_t = \frac{1}{T}\sum_{s=0}^{T-1}\sum_{j=0}^{T-1}\phi_j E_{t-s-1} p_{t-s+j}$$

by use of the money market equilibrium condition can be written

$$= \frac{1}{T}\sum_{s=0}^{T-1}\sum_{j=0}^{T-1}\phi_j E_{t-s-1}\left(m_{t-s+j} - y_{t-s+j}\right)$$

4. Persistency in the Adjustment Process to Nominal Shocks

To proceed we need to specify a process for the shocks impinging on the economy. We disregard real shocks (z = 0) and focus only on the nominal shocks where it is assumed that the specific process for the money supply

$$m_t = m_{t-1} + \varepsilon_t$$

where ϵ_t is idd N(0, σ_ε^2). It is assumed that $\left\{\varepsilon_{t-j}\right\}_{j=1}^{\infty} \subseteq I_{t-1}$.

(i) One-period contracts

Consider first the case where all wage contracts are one-period contracts (T=1) implying that contract renewal has to be synchronized. In this case

$$w_t = E_{t-1}\left(m_t - y_t\right)$$

Using this in (10) we have

$$y_t = \alpha k_t + \beta\left(m_t - E_{t-1}m_t + E_{t-1}y_t\right)$$

This already suggest that wage contracts may modify the propagation process, since expectations based on past information influence current output.

4 Using that lnEX=ElnX-1/2 Var (lnX), cf Aitchison and Brown (1957).

Given the process for $m_{t,}$ it follows that we can find a solution of the form

$$y_t = \frac{\alpha(1-\delta)}{1-\beta}k_{t-1} + \frac{\alpha}{1-\beta}\delta y_{t-1} + \beta\varepsilon_t$$

We recover the persistency parameter as

$$\frac{\alpha\delta}{1-\beta} > \alpha\delta$$

arising from the propagation running via capital accumulation and one period contracts. This shows that one-period contracts imply that nominal shocks can have real effects, and that the dynamic properties of the system are changed.

It is quite common to introduce nominal rigidities by assuming that the nominal wage (price) is predetermined at the expected value of the Walrasian wage (cf eg Bénassy (1995)). In the present setting focussing only on nominal shocks this would be equivalent to

$$w_t = E_{t-1}m_t$$

and output would become

$$y_t = \alpha k_t + \beta\varepsilon_t$$

$$= \alpha\delta y_{t-1} + \alpha(1-\delta)k_{t-1} + \beta\varepsilon_t$$

This type of anticipatory wage-setting does not change the dynamic properties of the system which is determined by the real propagation mechanisms ($\alpha\delta$).

The wage rule applied here which follows explicitly from a consideration of the incentives of wages-setters make the wage conditional on expected values of endogenous variables and this affects the dynamic properties. We find that one-period nominal wage contracts strengthen the propagation mechanism as

$$\frac{\alpha\delta}{1-\beta} > \alpha\delta$$

(ii) Two-period contracts

The nominal wage rule can in the case of two-period contracts be written

$$\ln\overline{w}_t = \ln\theta + \phi E_{t-1}p_t + (1-\phi)E_{t-1}p_{t+1}$$

where

$$\phi = (1+\rho)^{-1} \geq \frac{1}{2} \ .$$

The aggregate period t wage level thus reads (ignoring constants)

$$w_t = \frac{1}{2}\left(\phi E_{t-2}p_{t-1} + (1-\phi)E_{t-2}p_t\right)$$

$$+ \frac{1}{2}\left(\phi E_{t-1}p_t + (1-\phi)E_{t-1}p_{t+1}\right)$$

Using this, the equilibrium output in period t can be written as

$$y_t = a k_t + b\left[e_t + \frac{1}{2}e_{t-1} + \frac{1}{2}\left(f E_{t-2} + (1-f)E_{t-2}y_t + f E_{t-1}y_t + (1-f)E_{t-1}y_{t+1}\right)\right] \quad (4)$$

For later reference, it is worth pointing to the property of (4) that higher expected future levels of output tend to increase current output. The reason is that higher expected output other things being equal tend to lower prices, and thus in turn wages which for a given level of nominal demand tend to boost activity.

To solve this model, use the undetermined coefficients method and conjecture a solution of the form

$$y_t = \pi_0 k_{t-1} + \pi_1 y_{t-1} + \pi_2 \varepsilon_t + \pi_3 \varepsilon_{t-1} \quad (5)$$

Given (5) we have

$$E_{t-2}y_t = \pi_0 k_{t-1} + \pi_1 E_{t-2}y_{t-1}$$

and

$$E_{t-2}y_{t-1} = \pi_0 k_{t-2} + \pi_1 y_{t-2} + \pi_3 \varepsilon_{t-2} = y_{t-1} - \pi_2 \varepsilon_{t-1}$$

which inserted in (4) yields

$$y_t = \alpha(1-\delta)k_{t-1} + \alpha\delta y_{t-1} + \beta\left(\varepsilon_t + \frac{1}{2}\varepsilon_{t-1}\right)$$

$$+\frac{1}{2}\beta\Big[\big(\phi + (1-\phi)\pi_1\big)\big(y_{t-1} - \pi_2\varepsilon_{t-1}\big) + (1-\phi)\pi_0 k_{t-1}$$

$$+\big(\phi + (1-\phi)\pi_1\big)\big(y_t - \pi_2\varepsilon_t\big) + (1-\phi)\pi_0 k_t\Big]$$

or

$$\left(1-\frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\right)y_t = \alpha(1-\delta)k_{t-1} + \alpha\delta y_{t-1} + \beta\left(\varepsilon_t + \frac{1}{2}\varepsilon_{t-1}\right)$$

$$+\frac{1}{2}\beta\Big[\big(\phi + (1-\phi)\pi_1\big)\big(y_{t-1} - \pi_2\varepsilon_{t-1}\big)$$

$$+(1-\phi)\pi_0 k_{t-1} - \big(\phi + (1-\phi)\pi_1\big)\pi_2\varepsilon_t + (1-\phi)\pi_0\big((1-\delta)k_{t-1} + \delta y_{t-1}\big)\Big]$$

Hence consistency with (5) is ensured for the following values of the π-coefficient

$$\pi_0 = \left[1-\frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\right]^{-1}\left[\alpha(1-\delta) + \frac{1}{2}\beta\pi_0(1-\phi)(2-\delta)\right]$$

$$\pi_1 = \left[1-\frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\right]^{-1}\left[\alpha\delta + \frac{1}{2}\beta\Big(\big(\phi + (1-\phi)\pi_1\big) + (1-\phi)\pi_0\delta\Big)\right]$$

$$\pi_2 = \left[1-\frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\right]^{-1}\left[\beta - \frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\pi_2\right]$$

$$\pi_3 = \left[1-\frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\right]^{-1}\left[\frac{1}{2}\beta - \frac{1}{2}\beta\big(\phi + (1-\phi)\pi_1\big)\pi_2\right]$$

Given a solution to (5) it is useful to write the dynamic output equation more compactly as

$$A(L)y_t = B(L)\varepsilon_t \qquad (6)$$

where

$$A(L) \equiv 1 - A_1 L - A_2 L^2$$

$$B(L) \equiv B_0 - B_1 L - B_2 L^2$$

$$A_1 \equiv 1 - \delta + \pi_1$$

$$A_2 \equiv \delta\pi_0 - (1-\delta)\pi_1$$

$$\beta_0 \equiv \pi_2$$

$$B_1 \equiv -(1-\delta)\pi_2 + \pi_3$$

$$B_2 \equiv -(1-\delta)\pi_3$$

In the appendix A proof is given that for α+β < 1, the system is stable and that

$$A_1 > 0$$

$$A_2 < 0$$

and

$$A_1 + A_2 < 1$$

Output is seen to follow an ARMA (2,2) process in the innovations to the money stock. Multi-period contracts as captured here by two-period contracts thus imply a much more complicated adjustment pattern than implied solely by propagation via capital accumulation.

Comparing the cases of one- and two-period contacts, we find an important difference as the latter adds the dynamics arising from overlapping contracts. Both models display persistency, but the two-period contract model has more persistency as implied by the fact that the lagged output term has a higher weight, ie

$$\pi_1 > \frac{\alpha\delta}{1-\beta}$$

This shows that propagation by staggered wage contracts adds to the propagation already implied by real mechanisms. An increase in current output implies a higher capital stock and thus a larger future production potential. Other things being equal this leads to a lower price level and therefore wages which for a given nominal level of demand has an independent expansionary effect. Via the staggering of wage contracts the effect is spread out over time and thereby it interacts with the real propagation mechanism to produce more persistency.

As a new aspect relative to one-period contracing the multiperiod contracting model has a second order autoregressive effect which is negative. This is so because a high period t-2 output reflects an innovation to the money stock which after two periods will be built into all wage contracts. A period t-2 expansion in output reflects a positive nominal shock and thus leads to nominal wage increases which tend to lower output.

It is an implication that nominal wages are less responsive than prices to the nominal shock, ie

$$\frac{\partial w_t}{\partial \varepsilon_t} < \frac{\partial p_t}{\partial \varepsilon_t}$$

reflecting that prices take a larger burden of adjustment over the business cycle than wages.

Notice that the money market equilibrium condition implies

$$1 = \frac{\partial p_t}{\partial m_t} + \frac{\partial y_t}{\partial m_t}$$

That is, a (permanent) change in the money supply will be reflected in either prices or quantities. Having considered how the burden of adjustment falls on activity, we can conclude that persistent real effects of the nominal change will be matched by persistent effects on prices, ie there will also be persistency in the price process.

Consider next the limiting case of constant returns to scale ($\alpha+\beta = 1$). In this case we find (cf appendix B) that the output process has a unit root since

$$A_1 + A_2 = 1$$

where

$$A_1 > 1$$

$$A_2 < 0$$

that is output changes come to follow a first order autoregressive process, ie

$$y_t - y_{t-1} = -A_2\left(y_{t-1} - y_{t-2}\right) + B(L)\varepsilon_t$$

where the autoregressive term ($-A_2$) is positive.

It is illustrative for the role of wage staggering as a propagation mechanism to consider the special case where $\delta = 1$. This is the case where the propagation via capital accumulation plays its largest role as $(\alpha\delta)$ is increasing in δ (for $0 \leq \delta \leq 1$) and in the absence of staggered wage contracts, we find[5]

$$y_t = \alpha y_{t-1} + (1-\alpha)\varepsilon_t$$

With two period staggered wage contracts we get (see appendix)

$$y_t = y_{t-1} + (1-\alpha)\varepsilon_t$$

We find thus a very strong persistency effect as output comes to follow a random walk. The key to the understanding of this result is that the wage rule implies that nominal wages respond proportionally to price changes so as to keep the real wage unchanged and together with the constant returns to scale assumption this implies that the capital-labour ratio is constant. With one period contracts a nominal shock would have a temporary effect on output and wages will subsequently change in proportion to the price change. However, with two-period wage staggering the wage level is always tied to the past level of prices. In this case it is possible that a nominal shock does not release any price changes and thus a wage response which eventually will neutralize the real effects of the nominal shock. Since the capital stock moves proportionally to output, so will employment for a given real wage, and it follows that output will follow a random walk provided that nominal wages and thus prices do not change. The fact that wage staggering includes a backward looking element in wage formation makes this possible.

5. Concluding Remarks

The present analysis has demonstrated that wage staggering can potentially have qualitative implications for persistency properties via its interplay with real propagation mechanisms. In particular it is important that this does not rely on implausible large labour supply elasticities.

This finding also suggests that nominal shocks may play a more important role for business cycle fluctuations than usually asserted within explicit intertemporal macromodels. Wage staggering implies not only that nominal shocks have an important effect, but also that they can have persistent effects.

The model presented here is clearly too stylized to be reliable for empirical investigations. However, indication of important quantitative effects is readily available from simple numerical examples. Set the parameters at the following reasonable values $\alpha = 0.4$, $\delta = 0.1$ and $\phi = 0.5$

[5] Note that for $\delta = 0$ implying that the capital stock is fixed and propagation is constrained to run via wage formation, we find $A_1 = 1$ and $A_2 = 0$, ie the random walk property.

it follows that the persistency parameter in the real model without wage contracts is 0.04, with one period wage contracts it is 0.08, while with two period staggered wage contracts it becomes 0.36. That is, persistency is increased by a factor close to 10 when comparing the model without wage contracts with the model with two periods staggered wage contracts.! Clearly, this does not prove anything, but it suggests strongly that nominal propagation mechanisms may contribute significantly to generate persistency.

The analysis has been based on an exogenously postulated staggering structure. The basic point is that if contract renewal is not coordinated in time, it will be an important source of persistency. As there is no automatic coordination device in a decentralized economy, this seems to be an integral part of resource allocation. This is also confirmed by theoretical models endogenizing the timing structure. Asyncronization may thus naturally arise because it facilitates dissemination of costly information, enhances market power, or allows a more appropriate adjustment to shocks (see Ball and Cecchetti (1988), Maskin and Tirole (1988), Ball and Romer (1989), Fethke and Policano (1984), Freja (1993) and Cahuc and Kempf (1997).)

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Appendix A: Derivation of Demand Equations

From the first order conditions of the household maximization problem and the definition of the return to capital we get

$$\frac{1}{C_t} = \rho E_t\left(\alpha\delta \frac{Y_{t+1}}{C_{t+1}}\frac{1}{I_t}\right)$$

Using that $Y_t = C_t + I_t$, yields

$$\frac{I_t}{C_t} = \alpha\delta\rho + \alpha\delta\rho E_t\left(\frac{I_{t+1}}{C_{t+1}}\right)$$

Hence,

$$\frac{I_t}{C_t} = \frac{\alpha\delta\rho}{1-\alpha\delta\rho}$$

from which it follows that

$$C_t = (1-\alpha\delta\rho)Y_t$$

and

$$I_t = \alpha\delta\rho Y_t$$

From the first order conditions we also have

$$\frac{M_t}{P_t C_t} = \theta + \rho E_t\left(\frac{M_{t+1}}{P_{t+1}C_{t+1}}\right)$$

which implies

$$\frac{M_t}{P_t C_t} = \frac{\theta}{1-\rho}$$

and therefore

$$\frac{M_t}{P_t} = \frac{\theta}{1-\rho}c_t = \frac{\theta(1-\alpha\delta\rho)}{1-\rho}Y_t$$

Appendix B

(i) Proof that $\pi_1 < \frac{2}{\beta}\frac{1-\frac{\beta}{2}\phi}{1-\phi}$

Note that

$$1-\frac{\beta}{2}\left(\phi+(1-\phi)\pi_1\right)=0$$

for

$$\pi_1=\hat{\pi}\equiv\frac{2}{\beta}\frac{1-\frac{\beta}{2}\phi}{1-\phi}$$

Assume that $\pi_1=\hat{\pi}_1$ implying that

$$\pi_0=-\left[\frac{\beta}{2}(1-\phi)(2-\delta)\right]^{-1}\alpha(1-\delta)<0$$

The solution for π_1 is found from the definition of the π -coefficients as

$$\pi_1\left[1-\frac{\beta}{2}\left(\phi+(1-\phi)\pi_1\right)\right]-\frac{1}{2}\beta(1-\phi)\pi_1=\alpha\delta+\frac{\beta}{2}\phi+(1-\phi)\pi_0\delta \qquad \text{(A-1)}$$

Note that the left hand side of this expression has the property that it equals

$$-\frac{1}{2}b(1-f)\hat{p}_1 \qquad \text{for} \quad p_1=\hat{p}_1$$

and that it is decreasing in π_1 evaluated for $\hat{\pi}_1$, ie

$$\frac{\partial}{\partial\pi_1}\left(\pi_1\left(1-\frac{\beta}{2}\left(\phi+(1-\phi)\pi_1\right)\right)-\frac{1}{2}\beta(1-\phi)\pi_1\right)$$

$$=1-\frac{\beta}{2}\left(\phi+(1-\phi)\right)-\beta(1-\phi)\pi_1<0 \qquad \text{for } \pi_1=\hat{\pi}_1$$

Moreover,

$$\left.\frac{\partial \pi_0}{\partial \pi_1}\right|_{\pi_1=\hat{\pi}_1} = \frac{\alpha(1-\delta)\frac{\beta}{2}(1-\phi)}{\left(-\frac{\beta}{2}(1-\phi)(2-\delta)\right)^2} > 0$$

Evaluating the LHS of (A-1) for $\pi_1 = \hat{\pi}_1$, we get

$$-\left(1-\frac{\beta}{2}\phi\right)$$

and the RHS is

$$\alpha\delta + \frac{\beta}{2}\phi - \frac{2\alpha}{\beta}\frac{\delta(1-\delta)}{2-\delta}$$

Since

$$-\left(1-\frac{\beta}{2}\phi\right) < \alpha\delta + \frac{\beta}{2}\phi - \frac{2\alpha\delta(1-\delta)}{\beta(2-\delta)}$$

it follows that there exists a solution for $\pi_1 < \hat{\pi}_1$ since the LHS is decreasing in π_1 and the RHS is increasing.

(ii) Proof that $\pi_0 + \pi_1 \leq 1$

From the definition of π_0 and π_1 we have

$$\left(\pi_0 + \pi_1\right)\left(1-\frac{1}{2}\beta\left(1+(1-\phi)\pi_1\right)\right) = \alpha + \frac{1}{2}\beta\left(\phi + \pi_0(1-\phi)\right)$$

which can be rewritten as

$$\left(\pi_0 + \pi_1 - 1\right)\left(1-\frac{1}{2}\beta\left(1+(1-\phi)\pi_1\right)\right) = \alpha - 1 + \frac{1}{2}\beta\left(1+\phi+\left(\pi_0+\pi_1\right)(1-\phi)\right)$$

or

$$\left(\pi_0 + \pi_1 - 1\right)\left(1-\frac{1}{2}\beta\left(\phi+(1-\phi)\pi_1\right)\right) = \alpha + \beta - 1$$

since

$$1-\frac{1}{2}\beta\left(\phi+(1-\phi)\pi_1\right) > 0 \quad \text{for} \quad \pi_1 < \hat{\pi}_1$$

It follows that

$$\text{sign}(\pi_0 + \pi_1 - 1) = \text{sign}(\alpha + \beta - 1)$$

hence,

$$\pi_0 + \pi_1 < 1 \quad \text{for} \;\; \alpha + \beta < 1$$

$$\pi_0 + \pi_1 = 1 \quad \text{for} \;\; \alpha + \beta = 1$$

(iii) Proof that $\delta\pi_0$ - (1-δ)$\pi_1 < 0$.

From the definition of π_0 and π_1 we have

$$\delta\pi_0 - (1-\delta)\pi_1 =$$

$$\left[1 - \frac{\beta}{2}\left(\phi + (1-\phi)\pi_1\right)\right]^{-1} \frac{1}{2}\beta\left[\delta\pi_0(1-\phi) - (1-\delta)(1-\phi)\pi_1 - (1-\delta)\phi\right]$$

or

$$\left(\delta\pi_0 - (1-\delta)\pi_1\right)\left(1 - \frac{\beta}{2}(1-\phi)\left(\pi_1 - 1\right)\right) = -\frac{1}{2}\beta(1-\delta)\phi$$

since

$$1 - \frac{\beta}{2}(1-\phi)\left(\pi_1 - 1\right) > 0 \quad \text{for} \;\; \pi_1 < \hat{\pi}_1$$

it follows that

$$\delta\pi_0 - (1-\delta)\pi_1 < 0$$

(iv) Solution to the dynamic output equation

From the dynamic output equation we have

$$A(L) = 1 - A_1 L - A_2 L^2$$

where

$$A_1 \equiv 1 - \delta + \pi_1$$

$$A_2 \equiv \delta\pi_0 - (1-\delta)\pi_1$$

Hence, the eigenvalue is found as

$$\lambda = \frac{A_1 \pm \sqrt{A_1^2 + 4A_2}}{-2A_2}$$

Since $A_2 < 0$, it follows that the stable root is given by

$$\lambda = \frac{A_1 - \sqrt{A_1^2 + 4A_2}}{-2A_2}$$

Stability requires $\lambda > 1$ or

$$\frac{A_1 - \sqrt{A_1^2 + 4A_2}}{-2A_2} > 1$$

which in turn can be written

$$A_1 + A_2 < 1$$

Using that

$$A_1 + A_2 = 1 - \delta\left(1 - \pi_0 - \pi_1\right)$$

It follows that the system is globally stable for $\alpha+\beta < 1$ implying $\pi_0 + \pi_1 < 1$. Since $A_2 < 0$ it follows that $A_1 > 0$.

(v) $A_1 + A_2 = 1$ for $\alpha + \beta = 1$

From above it follows immediately that $\pi_0 + \pi_1 = 1$ for $\alpha + \beta = 1$, and hence it follows that $A_1 + A_2 = 1$. Notice that $\delta\pi\text{-}(1\text{-}\delta)\pi_1$ remains negative. Hence $A_2 < 0$ and $A_1 > 1$.

(vi) $\delta = 1$

Inserting in the expression for π_0 it follows directly that $\pi_0 = 0$. Using this in the expression for π_1, we find that it has a solution $\pi_1 = 1$. Inserting we find

$$A_1 = 1$$

$$A_2 = 0$$

$$B_0 = \frac{1-\alpha}{\alpha}$$

$$B_1 = \frac{1}{2}\frac{1-\alpha}{\alpha}\left(1 - \frac{1-\alpha}{\alpha}\right)$$

$$B_2 = 0$$

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