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Do Trade Unions Actually Worsen

Economic Performance?*

by

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Abstract

The answer to the question in the title of course depends on how we define economic performance. In an overlapping generations model we show that trade unions do worsen economic performance in the sense that we get unemployment, but it is quite likely that trade unions give rise to a higher growth rate than what would have been the case if the labour market were competitive. Therefore, in general the welfare implications of trade unions are ambiguous. One surprising result is that trade unions give rise to an unambiguous decrease in welfare if bargaining is over all variables affecting the bargaining parties (i.e. "efficient bargaining"), whereas welfare may increase if bargaining is only over a subset of the variables affecting the bargaining parties (e.g. training and wage).

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1. Introduction

Economists usually argue that trade unions are "bad guys" in the economy because they give rise to a wage above the market clearing level and as a result there is an efficiency loss. In this paper we examine whether this is the whole story concerning the effects of trade unions or whether there are other effects which are maybe more beneficial for the economy. In specific, we look at the growth implications of trade unions, and it turns out that when we take these growth implications into account, it is possible that the existence of trade unions give rise to a gain in social welfare.

If we look at the real world influence of trade unions, it has probably decreased somewhat in recent years (for instance in the US and the UK), but it is still the case that trade unions are very dominating at the labour markets in a lot of European countries (see e.g. OECD, 1994). What is maybe more disturbing for the standard argument against trade unions is that some of the countries where trade unions are most influential, such as Germany and Scandinavian countries, have not been doing significantly worse with respect to economic performance than the US and the UK. In specific, if we look at the growth rate, then it has for the last three decades been lower in the US than in most European countries (see e.g. Gordon, 1995).

The unemployment loss usually referred to when discussing the effects of trade unions is a static loss, and the purpose of this paper is to look into some of the dynamic effects of trade unions. One basic premise of the paper is that trade unions do not only care about wage and employment, but they also care about maintaining and improving the productivity of their members. In practise this is, for instance, seen as rules which seek to avoid that workers are worn-out too early, such as maximum working hours, safety rules and limitations for using piece-rate pay. It is also seen as a concern for conditions which seek to improve the human capital of workers such as a minimum amount of general training and schooling in apprenticeships, job rotation plans inside firms, and access to courses. In this paper we will focus on the training content in jobs, but it could be broadly interpreted as any means which seek to maintain or improve the level of general human capital of workers[1].

It is obvious that trade unions being concerned for, and having power to improve, labour productivity are not sufficient conditions for a potential welfare gain of trade unions relative to the competitive solution. If some working conditions, which would improve the future labour productivity, were sufficiently important, they would also be specified in optimal contracts at a competitive labour market. We need "somewhat more" for market power potentially improving social welfare, and this "somewhat more" could be that, in a competitive labour market, investments in future productivity are inefficiently low due to some externalities, and what we will assume in this paper is that there is a generational externality where the productivity of young

[1]We will focus at general human capital (contrary to specific human capital), because it is primarily with respect to general human capital that there is a potential conflict between firms and workers.

agents depend on the productivity of old agents.

Our model is an endogenous growth model, and we will assume that the growth engine in the economy is good and creative entrepreneurs. Moreover, we will assume that people only become good and creative entrepreneurs if they get some labour market experience and some general training inside firms. Our formal model is a two period overlapping generations model with endogenous growth, quite similar to the models in Prescott and Boyd (1987a,b). People are workers in the first period of their lives whereas they are either entrepreneurs or retired in the second period. An old agent only becomes entrepreneur if she gets a "good idea" (for instance for blue prints), and the probability of getting a good idea depends on the level of training received at the labour market when young. Moreover, the level of training also determines the productivity in case an old individual actually succeeds in becoming an entrepreneur. Therefore, training benefits young workers by increasing their expected income in the next period, but training is only supplied at a cost for the firms. In the real world, training would not only increase the productivity of a worker if she succeeds in becoming an entrepreneur but also if she remains at the labour market as a skilled worker. However, what is essential in our model is that there is a conflict between workers and firms concerning the amount of general training supplied by firms, and in order to simplify the model, we restrict attention to only two types of agents at the labour market: (young) workers and (old) entrepreneurs.

Since, in our model, workers care about the level of training, one obvious effect of more powerful trade unions is that they may be able to increase the level of training. This in turn would give rise to more productive entrepreneurs in future periods and, therefore, a higher growth rate. However, this is not the only effect of more powerful trade unions, and another is that entrepreneurs get a smaller share of the rent generated inside firms. Hence, the return from extra training may decrease implying that workers tend to prefer payment in terms of a higher wage instead of more training. This effect tends to decrease the level of training and in turn productivity and growth. Therefore, in general the implications for growth of more powerful trade unions is ambiguous. This we show by solving the model applying two different types of trade union models. First, we apply the "efficient bargaining model" (see e.g. McDonald and Solow, 1981) where firms and unions bargain over all variables affecting their pay-offs. In this model, it turns out that more powerful trade unions give rise to less training and a lower growth rate. Afterwards, we apply the maybe more realistic "right to manage model" (see e.g. Nickell and Andrews,1983) where firms unilaterally determine the size of employment. In this model, more powerful trade unions implies that the level of training, and in turn the growth rate, increases.

Since the bargaining power of trade unions affects growth as well as unemployment, our model has implications for how we should expect growth and unemployment to be correlated. In the "efficient bargaining model", more powerful trade unions give rise to a lower growth rate and a higher unemployment rate. Hence, an increase in the unemployment rate will be associated with a decrease in the growth rate. In the “right to manage model”, we get the opposite result, more powerful trade unions give rise to a higher growth rate as well as a higher unemployment rate. Hence, in the right to manage model there may be a trade off between dynamic efficiency (i.e. growth) and static inefficiency (i.e. unemployment). This also implies that the net welfare implications of powerful trade unions in general are ambiguous. In other words, the gain arising from a higher growth rate may dominate the loss due to higher unemployment.

With respect to related literature, there have been surprisingly few attempts to analyse potential links between growth and unemployment and to analyse whether labour market institutions have any implications for growth, but still, there are a few papers on these topics. Aghion and Howitt (1994) analyse how growth, which arises through the introduction of new technology, affects the equilibrium unemployment rate in a search model. They show that it depends on the importance of two types of effects. One is a capitalisation effect which implies that a higher growth rate makes it more attractive to start up new firms and create jobs. The other effect is termed the “creative destruction effect” which is that the duration of a job match becomes shorter because the speed by which old technology becomes unprofitable increases.

Laing, Palivos and Wang (1995) also consider the association between growth and unemployment in a search model. They assume that there is some schooling before entering the labour market, and the level of schooling has implications for productivity and growth. An important result is that, if the extent of matching frictions at the labour market increases, the unemployment rate increases and the return from extra schooling decreases. Hence, there is a simultaneous decrease in growth and increase in unemployment.

Another attempt to explain how labour market institutions may affect growth and employment is given in Blackburn and Hansen (1996). Their model is an extension of a Romer (1990) model where final goods are produced using a range of differentiated intermediate goods and a range of differentiated types of labour. Moreover, workers of the same type are organized in the same trade union. This implies that a decrease in the elasticity of substitution between different types of labour tends to increase the degree of imperfect competition in the labour market. This effect tends to decrease employment and growth. However, the increase in the variety of labour implies that the productivity of other inputs increases, and this effect tends to increase employment and growth. The net outcome for employment and growth is in general ambiguous.

The paper most closely related to our paper is Bean and Pissarides (1993). They set up a search model where firms and workers after a successful match bargain over the size of the wage, and the outcome of this bargaining is given by the asymmetric Nash bargaining solution. The model is an overlapping generations model where people are workers in the first period of their lives and capitalists in the second period. It is the accumulation of physical capital which gives rise to growth. It turns out that the relative bargaining strength of workers has implications for unemployment as well as growth. It is most likely that an increase in the bargaining strength of workers gives rise to an increase in unemployment (labour becomes more costly to use), but it is in general ambiguous whether the growth rate increases or decreases. This is so because income is redistributed from capitalists to workers, and it is workers who are young and do the savings. This effect tends to increase accumulation of capital[2]. On the other hand, the increase in unemployment reduces total income which in turn tends to give rise to lower savings and capital accumulation. Although, there are some similarities, our model differs in important respects from the model by Bean and Pissarides. First, in our model, it is not accumulation of physical capital, but accumulation of human capital through training inside firms, which gives rise to growth. This difference is important because the level of training inside firms becomes a potential variable for firms and trade unions to bargain over. Second, in our model workers are organized in trade unions whereas, in the Bean and Pissarides (1993) model, workers search and bargain as individuals.

The rest of the paper is organized as follows. In section 2, we present the basic model. In section 3, the model is solved under the assumption of "efficient bargaining", whereas the "right to manage model" is applied in section 4. In section 5, we look at the welfare implications of trade unions. Finally, we have some concluding remarks in section 6.

2. Model

The model is a two period overlapping generations model, and, in each period, M individuals are born. In the first period of their lives, people are workers, and in the second they are either entrepreneurs or they are retired.

Consumers

The utility of a representative young consumer is

$$U_t^y = c_t^y + \delta E_t c_{t+1}^o \tag{1}$$

where c_t^y is the consumption as young, δ is the subjective discount factor, and $E_t c_{t+1}^o$ is the expected consumption in the next period where this consumer will be old. Each young individual is endowed with one unit of time which she supplies inelastically to the labour market.

The utility of an old agent is simply given as the value of consumption

$$U_t^o = c_t^o \tag{2}$$

[2]In other words the saving rate of workers is higher than the saving rate of capitalist, and as the authors mention this is a sort of anti-Kaldorian result. It is argued that the result is maybe not that unrealistic if we take into account that most savings in industrialized countries occurs through pension funds.

We note that a retired agent will not have any income, and therefore $U_t^o = 0$. We assume that there is no credit market (and therefore no savings), but it can easily be shown that even if a credit market were allowed for, it would not change any of our results. This is so because utility functions are linear implying that utility from consumption is equal to the present value of all future income.

Firms

A firm is owned and managed by a single entrepreneur, and the production function for a representative firm is given as

$$y_t = \gamma_t^{1-\beta}\left(l_t\bar{\gamma}_t(1-x_t)\right)^{\beta}, \quad 0 \le \beta \le 1, \quad 0 \le x_t \le 1, \tag{3}$$

where l_t is employment, x_t is the training of each employee (per unit of time employed), γ_t is the productivity of the entrepreneur, and $\bar{\gamma}_t$ is the average productivity of all old agents. Hence, we assume that there is an externality from the old generation to the young, and we can imagine that this externality arises due to some initial schooling. We note that the firm produces goods as well as training, but training is not just a by-product of production as in learning by doing models (see e.g. Arrow, 1962 and Lucas, 1988). Instead, training reduces the amount of labour input for production, i.e. workers are "learning or doing" (see also Prescott and Boyd, 1987a,b).

The productivity of the entrepreneur is given as

$$\gamma_t = \bar{\gamma}_{t-1}Ax_{t-1}^{\nu}, \quad 0 \le \nu \le 1, \tag{4}$$

where A is a positive constant. Hence, a worker who gets more training also becomes a more productive entrepreneur in case she succeeds in the future to become an entrepreneur. Moreover, again we assume that there is an externality in productivity from one generation to the next.

The income of a representative entrepreneur, i.e. her profit is given as

$$\pi_t = y_t - w_t l_t, \tag{5}$$

where w_t is the wage rate, and we note that the price on the produced good has been normalized to one.

We assume that only old individuals with some labour market experience (and therefore production experience) are able to become entrepreneurs. This implies that an old individual who were unemployed while young does not have a chance of becoming an entrepreneur. Moreover, we assume that an old individual only becomes an entrepreneur if she gets "a good idea" (for blueprints), and in order to get a good idea it is necessary with some training as young. We will assume that the number of entrepreneurs (i.e. good ideas) "produced" by the economy is given as

$$N_t = \min\left[\bar{x}_{t-1}^{\varphi} L_{t-1}^{\alpha} M^{1-\alpha},\ L_{t-1}\right],\quad 0 \le \varphi \le 1,\ 0 \le \alpha \le 1, \qquad \textbf{(6)}$$

where N_t is the number of entrepreneurs, $\bar{x}_{t-1}$ is the average level of training in the previous period, and L_{t-1} is the aggregate employment in the previous period. This specification implies that the number of entrepreneurs cannot be higher than the potential number of entrepreneurs (i.e. L_{t-1}). Moreover, as long as the number of entrepreneurs is lower than the potential number, it is increasing in the potential number of entrepreneurs, in the training level of potential entrepreneurs, and in the number of consumers (i.e. the size of the economy)[3]. The specification in (6) implies that, if there is full employment (i.e. $L_{t-1} = M$), and all employed workers use all their time on training (i.e. $\bar{x}_{t-1} = 1$), the number of entrepreneurs becomes M.

It follows from the above specification that it is not necessarily all potential entrepreneurs who succeed in becoming entrepreneurs. We will assume that it is, to some extent, random, reflecting that it is, to some extent, random who get the good ideas. For a representative worker, the conjectured probability of becoming an entrepreneur is given as

$$p_t = \min\left[\left(\frac{x_{t-1}}{\bar{x}_{t-1}}\right)^{\rho} \frac{N_t}{L_{t-1}},\ 1\right]. \qquad \textbf{(7)}$$

As long as $p_t < 1$, a worker conjecture that she is able to increase the probability of becoming an entrepreneur by increasing her level of training relative to the average level of training (i.e. it is assumed that $\rho \ge 0$). If a worker becomes as the average, the conjectured probability of becoming an entrepreneur is equal to the total number of entrepreneurs relative to the potential number[4]. Moreover, we will assume that

$$\frac{\partial p_t}{\partial \bar{x}_{t-1}} \le 0, \qquad \textbf{(8)}$$

i.e. for a specific agent it is less likely that she becomes an entrepreneur if all other agents get more training. This condition is satisfied if $\rho \ge \varphi$.

Labour Market

We will assume that the labour market is either competitive, or there are firm

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[3]This specification implies that there is constant returns to scale (for a given average level of training) with respect to the "input" of potential entrepreneurs and consumers.

[4]This will be the probability in equilibrium. This is so because, if it is optimal to increase x_t for one agent, it will also be optimal for all other agents.

specific trade unions with bargaining power over one or more of the three variables of importance for trade union members, i.e.: wage (w_t), training (x_t) and employment (l_t). If a trade union only has bargaining power over a subset of these variables, the firm unilaterally determines the other variables after the outcome of the bargaining is known.

The institutional set up is assumed to be that, when there has been an agreement between a trade union and a firm, the firm does not hire workers at other conditions than what have been specified in the contract with the trade union. In other words, we rule out that unemployed workers underbid workers employed at conditions specified in the negotiated contract. It is well known that if underbidding is possible, it undermines the power of trade unions. We assume that the trade unions have solved that problem, for instance because the contract between a firm and a trade union is legally binding, and we have nothing new to add to that discussion (see e.g. Lindbeck and Snower, 1985, for some further discussion). The assumption of no underbidding also seems reasonable as the purpose of our analysis is to look at the implications of trade unions when trade unions do in fact have market power.

The trade unions are assumed to be utilitarian in the sense that they seek to maximize the expected utility for a certain group of workers (members). However, we will assume that a firm decides unilaterally who to employ among all workers seeking a job in the firm (i.e. we rule out closed shops), but no matter who the firm employs, the working conditions are as specified in the contract negotiated with the trade union. It now follows that no matter which share of the jobs in a firm will go to members of a specific trade union, the negotiating union in a firm cannot do better than seeking to maximize the aggregate utility raised from working in the firm, i.e. the objective of a representative firm is to maximize:

$$U_t^u = w_t l_t + l_t p_{t+1} \delta \pi_{t+1} - l_t \bar{U}_t \qquad \textbf{(9)}$$

The first term is simply the total wage income in the firm, the second term is the discounted expected value of income from becoming entrepreneurs in the next period (the expected number of employed who becomes entrepreneurs is the potential number (l_t) times the probability of becoming entrepreneur[5]), and the last term is the utility from alternative activities. In other words, $_t$ is the utility a worker would get if she tried to become employed somewhere else. In a first step in our analysis we will consider $_t$ to be exogenously given, but in a second step we close the model by assuming that $_t$ is the expected utility from seeking employment in the other firms in the industry. In this case, we get that:

[5]The unions know how this probability is determined but, since unions are firm specific, they consider the number of entrepreneurs (i.e. N_{t+1}) and the average level of training (i.e. $\bar{x}$) as given.

$$\bar{U}_t = \frac{L_t}{M}\left(\bar{w}_t + \delta\bar{p}_{t+1}\bar{\pi}_{t+1}\right) \qquad \textbf{(10)}$$

We assume that each firm is too small to have any significant influence on the industry as such. Therefore, L_t/M is the probability of getting employment somewhere else at the conditions specified in the market contract which are the wage, $\bar{w}_t$, and the discounted value of training, $\delta\bar{p}_{t+1}\bar{\pi}_{t+1}$. To get (10), it is implicitly assumed that a firm does not discriminate between the workers seeking a job in the firm (i.e. we use the assumption of no closed shops). In equilibrium all firms will be identical, and therefore training and wages will be the same in all firms.

We will assume that, in a steady state, $_t = \gamma$, where is time independent. This specification requires that there is a spill-over effect of productivity from the production sector to alternative employment. If this was not the case, either the production sector or the alternative sector would loose significance in the long run, since the relative productivity of the least productive sector would go to zero. Moreover, when the model is closed by using (10), it will, without any further assumptions, be the case that $_t = \gamma$.

With respect to the outcome of the bargaining between a firm and a trade union, it is assumed to be given by the asymmetric Nash-bargaining solution (see e.g. Binmore, Rubinstein and Wolinsky, 1986). In section 3, we solve the model in the case of efficient bargaining where the competitive solution will be given as a special case. Then, in section 4, we solve the model for the cases where the firm unilaterally determine employment, i.e. what in the trade union literature has been termed “right to manage models” (see e.g. Nickell and Andrews, 1983).

Social welfare

In an overlapping generations model, there is no obvious choice of a social welfare function. Should we only take into account the utility of the individuals presently alive, or should we also take into account the utility of still unborn individuals? We assume that social welfare is given as the present value of all future summarized utilities of entrepreneurs and workers, i.e. we assume a utilitarian welfare function. The discount factor is assumed to be equal to the subjective discount factor of the consumers (i.e. δ).Hence, by using (1) and (2), the welfare function becomes

$$W_t = \sum_{\tau=0}^{\infty} \delta^{\tau} \sum_{i=1}^{N_{t+\tau}} y_{i,t+\tau} \qquad \textbf{(11)}$$

where $y_{i,t+\tau}$ is the production in firm i in period $t+\tau$. Since the utility functions of workers as well as entrepreneurs are linear, the total utility raised in each period is equal to total production.

In equilibrium, production will be the same in all firms, and in most of the paper we will focus on a steady state equilibrium. A steady state requires that

$$\begin{aligned} N_t &= N_{t-1} = N \\ l_t &= l_{t-1} = l \\ x_t &= x_{t-1} = x \end{aligned} \qquad \textbf{(12)}$$

Using these conditions, it is easily seen that the (gross) growth rate becomes

$$\frac{N_t y_t}{N_{t-1} y_{t-1}} = \frac{\gamma_t}{\gamma_{t-1}} = A x^{\nu} \qquad \textbf{(13)}$$

i.e. the only variable which influences the growth rate is the level of training.

By using the conditions in (12), and (3), (4), and (6), in (11), we find that the steady state welfare becomes[6]

$$W_t = \frac{\gamma_t \left(x^{\varphi} (1-x)^{\beta} l^{\frac{\alpha}{1-\alpha}+\beta} \right)}{1 - \delta A x^{\nu}} M \qquad \textbf{(14)}$$

We note that the level of training affects welfare in three different ways. It gives rise to a higher growth rate and, therefore, a higher future production (if $\nu > 0$). It also gives rise to a bigger number of firms (if $\varphi > 0$) and by that a higher level of production in each period. Finally, a higher x implies higher training costs which tends to decrease production.

3. Efficient Bargaining

In the literature, the case where a firm and a trade union bargain over employment as well as wages has been termed efficient bargaining (see e.g. McDonald and Solow, 1981). When the pay-offs to the trade union and the firm are only affected by wage and employment, the outcome is efficient in the sense that it is not possible to make any of the two parties better off without making the other party worse off. In our model, the firm and the union are not only interested in wage and employment but also in training. Hence, efficient bargaining requires bargaining over training as well as employment and wage[7], and the outcome of the bargaining in the representative

[6] For the sum in (11) to converge, it must be the case that $\delta A x^{\nu} < 1$. Moreover, we have assumed that $N_t < L_{t-1}$ (see (6)). It will be confirmed below that this is in fact the case in the steady state.

[7] Note that the outcome is only efficient in the sense that, given what everybody else do, it is not possible for both parties to become better off.

firm is given as

$$x_t, l_t, w_t \;=\; \arg\;\max\left[w_t l_t + \delta p_{t+1}\pi_{t+1} l_t - \bar{U}_t l_t\right]^{\lambda}\pi_t^{1-\lambda} \qquad \text{s.t. } 0 \le x_t \le 1,\; l_t \ge 0,\; w_t \ge 0 \tag{15}$$

where λ $(1-\lambda)$ is the bargaining power of the trade union (firm). We assume that, if there is no agreement, the pay-offs to each of the two parties will be zero.

It is tedious, but relatively straightforward, to solve the bargaining problem given in (15), and details can be found in appendix 1. The problem is highly non-linear, and to find a solution, we make the further assumption that $\nu + \rho = 1$ which implies that the private return to training is constant. Now, by evaluating in a steady state, we get the following solution for a representative firm

$$\frac{x^{\rho}}{1-x} \;=\; \frac{\delta A\left(1-\left(\lambda+(1-\lambda)\beta\right)\right)}{\beta} \tag{16}$$

$$\tilde{w} \;=\; \frac{w_t}{\gamma_t} \;=\; \frac{\left(\lambda+(1-\lambda)\beta\right)\tilde{U}(1-x)}{\beta} \tag{17}$$

$$l \;=\; \left(\frac{\tilde{U}}{\beta}\right)^{\frac{1}{\beta-1}}(1-x)^{-1} \tag{18}$$

We note that the level of training, and therefore the growth rate of the economy, is decreasing in the bargaining power of trade unions (i.e. $\partial x/\partial\lambda < 0$). The intuition is the following. A firm and a union seek to maximize the total discounted pay-off to the two parties. Then this payoff is shared amongst the two parties according to their relative bargaining power. With respect to training, there is a cost in the present period and the pay-off is not achieved until the next period. In the next period this pay-off will be shared with trade union members in a future generation. Therefore, if the bargaining power of trade unions is high, the present generations only get a small share of the pay-off from extra training implying that the training level becomes relatively low. In the extreme, where trade unions have all the bargaining power, trade unions get all the total pay-off in each period. Hence, the present generations do not gain anything from using resources on training, and the training level (and the growth rate) will be zero.

Second, we see that the employment level in each firm is increasing in the level of training (i.e. $\partial l/\partial x > 0$). If there were no training (x=0), employment would be determined so that the marginal product of labour is equal to the cost of labour in terms of alternative payment (i.e.). In the case of training, workers get a share of their payment in the following period (in other firms), implying that the net cost of employment decreases. This in turn induces the firm and the union to agree on a higher employment level.

Finally, we see that the wage rate is proportional to the productivity of entrepreneurs, i.e. the increase in the wage rate from one period to the next is equal to the growth rate. Moreover, the wage level is increasing in the bargaining power of trade unions (i.e. $\partial\tilde{w}/\partial\lambda > 0$), and the reason is that, if trade unions become stronger, they also succeed in getting a bigger share of total pay-off.

By using (6), the number of entrepreneurs in the steady state becomes

$$N = x^{\frac{\varphi}{1-\alpha}} l^{\frac{\alpha}{1-\alpha}} M \tag{19}$$

Since x is decreasing in the bargaining power of trade unions, and l is increasing in x, it follows that the number of entrepreneurs is decreasing in the bargaining power of trade unions (i.e. $\partial N/\partial\lambda < 0$). With respect to aggregate employment, it is given as the number of firms times employment in each firm and since both are decreasing in the bargaining power of trade unions, aggregate employment will also be decreasing (i.e. $\partial L/\partial\lambda < 0$).

There is one special case which is particularly interesting, namely the case where trade unions have no bargaining power. This case is synonymous to the case of a competitive labour market. If the labour market is competitive, firms offer workers a contract which include wage and training and which gives rise to a utility level at least at the level associated with alternative employment. It is easily shown that the solution to this problem is identical to the solution given above when λ is equal to zero, and models of this type is solved in Hashimoto (1982) and Ravn and Sørensen (1996).

Using the above comparisons, it follows that, if trade unions have bargaining power, growth as well as employment will be lower than in the case of a competitive labour market. Moreover, if we vary the bargaining power of trade unions and by that get different combinations of unemployment (defined as M-L) and growth, these variables will be negatively correlated. We can think about this experiment as finding growth and employment in different countries which differ with respect to the bargaining power of trade unions, and we observe that a high unemployment rate is associated with a low growth rate.

Until now, in this section, the analysis has been based on the assumption that $_t$ is exogenously given. If we close the model by using that $_t$ is determined as in (10), we find (by using (3) - (7), 17 and (18)) that

$$\tilde{U} = \frac{\overline{U}_t}{\overline{\gamma}_t} = x^{\varphi(1-\beta)}\beta^{\alpha+\beta(1-\alpha)}(1-x)^{\alpha(\beta-1)}\big((\lambda+(1-\lambda)\beta) + \delta A x^{\nu}(1-(\lambda+(1-\lambda)\beta))\big)^{(1-\alpha)(1-\beta)} \quad \textbf{(20)}$$

Then, by using (18), (19) and (20), we find that aggregate employment becomes

$$L = \frac{\beta}{\lambda(1-\beta)(1-x) + \beta}M \quad \textbf{(21)}$$

We see that, if $\lambda = 0$, then $L = M$, and we have full employment. If λ increases, then x decreases, and we get unambiguously that L decreases. Hence, we still get a negative correlation between growth and unemployment[8].

4. Right to Manage Model

In this section, we will assume that firms and unions only bargaining over a subset of the variables affecting the two parties, and by appealing to realism we will assume that bargaining is over wage and training whereas employment is determined unilaterally by firms. In the literature, this structure is usually referred to as “the right to manage model” (see e.g. Nickell and Andrews, 1983), and by following the literature, we assume that bargaining is completed before firms determine the size of employment.

By maximizing profits (i.e. (5)), labour demand in the representative firm becomes

$$l_t = \left[\frac{\beta\gamma_t^{1-\beta}\overline{\gamma}_t^{\beta}(1-x_t)^{\beta}}{w_t}\right]^{\frac{1}{1-\beta}} \quad \textbf{(22)}$$

An increase in the wage rate and/or an increase in the level of training gives rise to higher cost of production and, therefore, lower employment.

Now, the outcome of the bargaining becomes

$$x_t, w_t = \arg\max\left[w_t l_t + \delta p_{t+1}\pi_{t+1}l_t - \overline{U}_t l_t\right]^{\lambda}\pi_t^{1-\lambda} \quad \textbf{(23)}$$
$$\text{s.t. } 0 \le x_t \le 0,\ w_t \ge 0,\ (22)$$

[8] The analysis above requires that N < L (see (6)), which, in the steady state, implies that $l > 1$ (i.e. there should be more than one worker in each firm). Using (18) this condition becomes that $(1-x)^{1-\beta} < \beta$, and by using (20) it is easily confirmed that this is in fact the case (assuming as in footnote 5 that $\delta A x^{\nu} < 1$).

This bargaining problem is solved in appendix 2, and the solution for the steady state becomes

$$\frac{x^{\rho}}{1-x} = \delta A\left[\beta^{-1}-1\right]\left[\frac{\lambda+(1-\lambda)\beta}{\beta}\right] \tag{24}$$

$$\tilde{w} = \frac{w_t}{\gamma_t} = \frac{(\lambda+(1-\lambda)\beta)\tilde{U}(1-x)}{\beta} \tag{25}$$

$$l = \left[\frac{\beta^2}{(\lambda+(1-\lambda)\beta)\tilde{U}}\right]^{\frac{1}{1-\beta}}\frac{1}{1-x} \tag{26}$$

We note that, if trade unions do not have any bargaining power (i.e. $\lambda = 0$), we get the competitive solution. Hence, the competitive solution is a special case of “the efficient bargaining model” as well as “the right to manage model”. Moreover, from (24) we see that, in “the right to manage model”, the level of training, and therefore the growth rate, is increasing in the bargaining power of trade unions ($\partial x/\partial\lambda > 0$). Even though (25) does not give a closed form solution for the wage rate it can be shown by using (24) in (25) that the wage rate is also increasing in the bargaining power of the trade unions. Hence, if trade unions become more powerful, they use this extra bargaining power to increase both types of payments. It follows (by using (22)) that employment in each firm decreases if the bargaining power of trade unions increases (i.e. $\partial l/\partial\lambda < 0$).

By using (24) and (26) in (19), the number of firms becomes

$$N = x^{\frac{\varphi-\rho\alpha}{1-\alpha}}\left[\left(\frac{\beta}{\tilde{U}}\right)^{\frac{1}{1-\beta}}\left[\frac{\beta}{\lambda+(1-\lambda)\beta}\right]^{\frac{\beta}{1-\beta}}\delta A\left[\beta^{-1}-1\right]\right]^{\frac{\alpha}{1-\alpha}}M \tag{27}$$

It is easily seen that it is ambiguous whether N is increasing or decreasing in the bargaining power of trade unions. The reason is that the decrease in l reduces the potential number of entrepreneurs, but the increase in x improves the creativity of potential entrepreneurs which tends to increase the number of firms. A sufficient condition for the number of firms to decrease (i.e. $\partial N/\partial\lambda < 0$) is that $\varphi < \rho\alpha$ implying that the creativity of entrepreneurs is not “too” important for their numbers.

By using (19), (24) and (26), total employment can be found to be

$$L = Nl = x^{\frac{\varphi-\rho}{1-\alpha}}\left[\left(\frac{\beta}{\tilde{U}}\right)^{\frac{1}{1-\beta}}\left[\frac{\beta}{\lambda+(1-\lambda)\beta}\right]^{\frac{\beta}{1-\beta}}\delta A\left[\beta^{-1}-1\right]\right]^{\frac{1}{1-\alpha}} M \quad \textbf{(28)}$$

Since it is assumed that $\rho > \varphi$ (and since $\partial x/\partial\lambda > 0$), it follows that total employment is decreasing in the bargaining power of trade unions (i.e. $\partial L/\partial\lambda < 0$).

From the reasoning above, it follows that, if trade unions have bargaining power, employment will be lower but growth will be higher than in the case of a competitive labour market. Moreover, an increase in the bargaining power of trade unions gives rise to a decrease in employment but an increase in growth. Hence, in this "right to manage model"there will be a positive relationship between unemployment and growth.

If we again use (10) to close the model, we find that (by using (4)-(7), (25) and (26))

$$\tilde{U} = x^{\varphi(1-\beta)}(1-x)^{\alpha(\beta-1)}\beta\left[1+\delta A x^{\nu}\left(\beta^{-1}-1\right)\right]^{(1-\alpha)(1-\beta)}\left[\frac{\beta}{\lambda+(1-\lambda)\beta}\right]^{\alpha+\beta(1-\alpha)} \quad \textbf{(29)}$$

Then, by inserting (29) into (28), and by using (24), we find the following two useful ways of expressing aggregate employment:

$$\begin{aligned} L &= \frac{x^{-\rho}\delta A\left[\beta^{-1}-1\right]}{1+\delta A x^{\nu}\left(\beta^{-1}-1\right)} M \\ &= \frac{\beta}{\lambda(1-\beta)(1-x)+\beta} M \end{aligned} \quad \textbf{(30)}$$

If $\lambda = 0$, we get that L = M and we have full employment. However, L is unambiguously decreasing in x implying that L is unambiguously decreasing in λ. Hence, more powerful trade unions implies that employment decreases but growth increases[9].

There is one natural possibility with respect to the set of bargaining parameters which we have not been considering above, namely the case where firms and trade unions only bargain over wages whereas firms determine employment and training. It is easily shown that this is a fatal case with respect to growth. As long as the training level is positive, an increase in the wage rate induces the firms to decrease training so that the full wage (i.e. the value of wage and training) is kept equal to the alternative payment (i.e. $_t$). This implies a zero net payment for the trade unions (i.e. $U_t^u = 0$),

[9]The analysis above also requires that $l > 1$ (i.e. there should be more than one worker in each firm, implying that N < L , see also (6)). By using (26) this condition can also be expressed as $(1-x)^{1-\beta} < \beta^2/(\lambda + (1-\lambda)\beta)$, and by using (29) this is easily seen to be the case (as in footnote 5, it is assumed that $\delta A x^{\nu} < 1$).

and in turn a zero Nash product. Therefore, trade unions choose to, and succeed in, increasing the wage above the level where there will be no training. In other words, no matter what is the bargaining power of the trade unions (as long as $\lambda > 0$), the growth rate will be zero (i.e. $x = 0$).

5. Welfare Implications of Trade Unions

Standard labour market theory would tell us that trade unions are a source of inefficiency and, therefore, the overall welfare implications of trade unions are negative. In our model things are different since we have two potential sources of inefficiency, namely the trade unions and the generational externality. The generational externality in itself tends to give rise to a growth rate which is too low relative to what maximizes the steady state welfare (i.e. (14))[10]. The interesting question then is whether it is possible that the existence of trade unions corrects the inefficiency due to the generational externality to such a degree that the overall welfare implications of trade unions are positive.

In the “efficient bargaining model” the welfare implications of trade unions are obviously negative. Compared to the competitive case, trade unions implies a lower growth rate as well as a higher unemployment rate. Hence, the existence of trade unions gives rise to a static loss, and it also worsens the dynamic loss due to the generational externality. In the “right to manage model” the welfare implications of trade unions are less obvious. Trade unions still give rise to unemployment (i.e. a static loss) but they also give rise to a higher growth rate (i.e. a dynamic gain). In general, we cannot determine which effect is dominating, and it is easily seen by some examples that social welfare will be increasing in the bargaining power of trade unions for some parameter values while it will be decreasing for other parameter values. Still it may be instructive with a specific example with “reasonable” parameter values.

Let us assume that $\varphi = \rho = \nu = 1/2$ so that extra training has some effect on the number of entrepreneurs as well as on the productivity of the entrepreneurs. Moreover, we will assume that $\delta A = 1$, and let us say that $A = 3$ while $\delta = 1/3$. If one period is 25 years (a “reasonable” level in an overlapping generations model for the labour market), the chosen value of δ implies a yearly subjective discount rate at 4.49%. The value of α we simply choose to be zero implying that the total number of “ideas” (i.e. the total number of entrepreneurs) only depends on the average level of training and the size of the economy (see (6)). Finally, let us choose $\beta = 2/3$ implying that the labour share of total income is 2/3 . Using these parameter values in (24), we find that

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[10] Since our main purpose is to analyse the long run implications of trade unions, we confine our attention to the steady state welfare.

$$x = 1 + \frac{1 - \sqrt{4\left(\frac{1}{2} + \frac{1}{4}\lambda\right)^2 + 1}}{2\left(\frac{1}{2} + \frac{1}{4}\lambda\right)^2} \tag{31}$$

Using this formula, the competitive solution ($\lambda = 0$) implies that x = 0.1715729, whereas the other extreme solution where trade unions have all the bargaining power ($\lambda = 1$) implies that x = 0.2864217. By using (13) and that one period is 25 years, the yearly growth rate in the competitive economy becomes 0.87% whereas, when $\lambda = 1$, it becomes 1.91%. Hence, the growth rate in the economy where trade unions have all the bargaining power is more than the double of the growth rate in the competitive economy. With respect to employment, it follows from (30) that

$$L = \frac{2}{\lambda(1-x) + 2}M \tag{32}$$

This expression confirms that in the competitive economy ($\lambda = 0$), there is full employment, while in the economy where trade unions have all the bargaining power (i.e. $\lambda = 1$ and x = 0.2864217), the unemployment rate (i.e. (M-L)/M) is 26.30%. In other words, in the economy where trade unions are strong, there is a higher growth rate but there is also a very high unemployment rate. Finally, by using (24), (26) and (29) in (14), and inserting the assumed parameter values, social welfare becomes

$$W_t = \gamma_t \frac{\frac{1}{2}^{\frac{2}{3}} x^{-\frac{1}{6}} (1-x)^{\frac{2}{3}} \left(1 + \frac{1}{2} x^{\frac{1}{2}}\right)^{-\frac{2}{3}}}{1 - x^{\frac{1}{2}}} \tag{33}$$

By inserting the specific values of x, we find that, in the competitive economy, $W_t = \gamma_t 1.12$, whereas in the economy where trade unions have all the bargaining power, $W_t = \gamma_t 1.14$. Hence, social welfare is marginally higher in the economy with strong trade unions than in the competitive economy. However, in this specific case the relationship between the bargaining power of trade unions and social welfare is actually non-monotonic. For small values of λ, W_t is decreasing in λ, whereas, for higher values of λ, W_t becomes increasing in λ . Hence, in our specific example trade unions may give rise to higher welfare than what is the case in the competitive economy, but only if trade unions are sufficiently strong. It should of course remain clear that this is only a simple illustrative example. For some parameter values, social welfare will be monotonously decreasing in the bargaining power of trade unions, whereas for other parameter values, it will be monotonously increasing in the bargaining power of trade unions. Therefore, the example only illustrates that the welfare implications of trade unions are not that clear as standard theory tells us.

6. Conclusion

In a simple overlapping generations model, we have analysed the growth and unemployment effects of trade unions. Our main conclusions are that, while trade unions always give rise to a higher unemployment rate than what would be the case in a competitive labour market, it is ambiguous how trade unions affect the growth rate. If the bargaining at the labour market is over all variables affecting the bargaining parties (i.e. "efficient bargaining"), trade unions give rise to a lower growth rate, and in this case we find that welfare is unambiguously lower than if the labour market were competitive. On the other hand, if bargaining is only over wage and training, which is probably the most realistic case, the growth rate will be higher than if the labour market were competitive. In this case it is possible that trade unions give rise to a net welfare gain.

It should be kept in mind, when interpreting our results, that the model used to get these results is very simple and very stylized. However, our purpose has not been to demonstrate that trade unions are always a curse or always a blessing for an economy, but we just want to point out that the effects of trade unions are maybe not as simple as "standard" theory tells us. In specific, it seems to be a reasonable assumption that trade unions care about the future productivity of their members, and if this is the case we have en essential ingredient for trade unions influencing the growth rate.

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Appendix 1

In this appendix, we solve the efficient bargaining model in section 3.To do this we need to know how the profit for the workers who become entrepreneurs in the next period depends on the training achieved in this period. For that purpose, let us assume that the profit in a firm in the next period is given as

$$\pi_{t+1} = \gamma_{t+1}\tilde{\pi}_{t+1} \quad \textbf{(34)}$$

where $\tilde{\pi}_{t+1}$ does not depend on x_t . We will confirm below that this is in fact the case. This implies that (it is assumed that $N_t < L_{t-1}$ (see (6)), but in footnote 8, it is confirmed that this is in fact the case in the steady state when we close the model by using (10))

$$p_{t+1}\delta\pi_{t+1} = x_t K_t \quad \textbf{(35)}$$

where K_t is the marginal discounted value of extra training (for a specific worker), and it is given as

$$K_t = \delta\bar{\gamma}_t A\tilde{\pi}_{t+1}\bar{x}_t^{-\rho+\varphi}L_t^{\alpha-1} \quad \textbf{(36)}$$

As we will see below, there is no interior value of x_t maximizing the Nash product given in (15). To handle this problem most easily, let us start by maximizing the Nash product with respect to w_t and l_t for a given value of x_t. The first order conditions become

$$\lambda\frac{1}{l_t} + (1-\lambda)\frac{\gamma_t^{1-\beta}\bar{\gamma}_t^{\beta}\beta l_t^{\beta-1}(1-x_t)^{\beta} - w_t}{\gamma_t^{1-\beta}\bar{\gamma}_t^{\beta}(l_t(1-x_t))^{\beta} - w_t l_t} = 0 \quad \textbf{(37)}$$

$$\lambda\frac{1}{w_t + x_t K_t - \bar{U}_t} - (1-\lambda)\frac{l_t}{\gamma_t^{1-\beta}\bar{\gamma}_t^{\beta}(l_t(1-x_t))^{\beta} - w_t l_t} = 0 \quad \textbf{(38)}$$

Solving (37) and (38), we get

$$l_t = \left(\frac{\beta \gamma_t^{1-\beta} \bar{\gamma}_t^{\beta} (1 - x_t)^{\beta}}{\bar{U}_t - x_t K_t} \right)^{\frac{1}{1-\beta}} \tag{39}$$

$$w_t = (\lambda + (1-\lambda)\beta) \left(\frac{\bar{U}_t - x_t K_t}{\beta} \right) \tag{40}$$

By using (3), (5), (9) and (35), we get that

$$\pi_t = \gamma_t (1 - (\lambda + (1-\lambda)\beta)) \beta^{\frac{\beta}{1-\beta}} \left(\frac{\bar{\gamma}_t (1 - x_t)}{\bar{U}_t - x_t K_t} \right)^{\frac{\beta}{1-\beta}} \tag{41}$$

$$U_t^{\,u} = \gamma_t ((\lambda + (1-\lambda)\beta) \beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}) \left(\frac{\bar{\gamma}_t (1 - x_t)}{\bar{U}_t - x_t K_t} \right)^{\frac{\beta}{1-\beta}} \tag{42}$$

We see that the pay-off to the firm as well as to the trade union is increasing in

$$\Psi_t = \frac{\bar{\gamma}_t (1 - x_t)}{\bar{U}_t - x_t K_t} \tag{43}$$

and

$$\frac{\partial \Psi_t}{\partial x_t} = \frac{\bar{\gamma}_t (K_t - \bar{U}_t)}{(\bar{U}_t - x_t K_t)^2} \tag{44}$$

The firm and the trade union would agree on increasing (decreasing) x_t if $K_t >$ $_t$ (K_t < $_t$. Since this would happen in all firms, x_t would increase (decrease). Hence, there are three candidates for an equilibrium: $x_t = 0$, $x_t = 1$ or $K_t =$ $_t$. It is easily seen that if $x_t \to 0$, then $K_t \to \infty$, and therefore $\partial \Psi_t / \partial x_t > 0$. Hence, $x_t = 0$ cannot be an equilibrium. Similarly, if $x_t \to 1$, then $K_t \to 0$, and therefore $\partial \Psi_t / \partial x_t < 0$. Hence, $x_t = 1$ cannot be an equilibrium, either. The only possibility for an equilibrium which is left is that

$$K_t = \bar{U}_t \tag{45}$$

In this case $\partial\Psi_t/\partial x_t = 0$ and no firm and union pair have incentives to change behaviour. This condition is basically a "zero profit condition" which implies that the return from extra training is equal to the cost of extra training. By using (6), (36), (39), (41), and the steady state conditions in (12), we find that for a representative firm

$$\frac{x^{\rho}}{1-x} = \frac{\delta A(1-(\lambda+(1-\lambda)\beta))}{\beta} \tag{46}$$

which is (16) in section 3.

By using (45) in (41), we find that

$$\pi_t = \gamma_t(1-(\lambda+(1-\lambda)\beta))\beta^{\frac{\beta}{1-\beta}}\tilde{U}^{\frac{\beta}{\beta-1}} \tag{47}$$

Hence, we see that $\tilde{\pi}_t = \pi_t/\gamma_t$ is in fact independent of x_t as assumed above.

If (45) is used in (39) and (40), and if we use that $\gamma_t = \gamma$ in the representative firm, we find that

$$\tilde{w} = \frac{w_t}{\gamma_t} = \frac{(\lambda+(1-\lambda)\beta)\tilde{U}(1-x)}{\beta} \tag{48}$$

$$l = \left(\frac{\tilde{U}}{\beta}\right)^{\frac{1}{\beta-1}}(1-x)^{-1} \tag{49}$$

which are equations (17) and (18) in section 3.

Appendix 2

In this appendix, we solve the Right to manage model from section 4. First, by inserting (3) and (22) into (5), we get that

$$\tilde{\pi}_t = \frac{\pi_t}{\gamma_t} = \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right]\tilde{z}_t^{\frac{\beta}{1-\beta}} \tag{50}$$

where

$$\tilde{z}_t = \frac{(1 - x_t)}{w_t}\overline{\gamma}_t \tag{51}$$

Hence, we see that the firm only cares about the value of $\tilde{z}_t$ and not on how $\tilde{z}_t$ is determined by specific values of x_t and w_t.

By using (6), (7), (9), (23) and (50), we find that

$$U_t^u = \gamma_t\left(\beta^{\frac{1}{1-\beta}}\tilde{z}_t^{\frac{\beta}{1-\beta}} + \beta^{\frac{1}{1-\beta}}\tilde{z}_t^{\frac{1}{1-\beta}}f(x_t)\right) \tag{52}$$

where[11]

$$f(x_t) = \left(\delta A x_t \overline{x}_t^{\varphi-\rho} L_t^{\alpha-1}\tilde{\pi}_{t+1} - \tilde{U}\right)\frac{1}{1-x_t} \tag{53}$$

We note that $f(x_t)$ does not depend on $\tilde{z}_t$. Hence, by solving the bargaining problem in (23) is equivalent to solving (note that π_t as well as U_t^u are proportional to γ_t)

$$\begin{aligned} \tilde{z}_t &= \arg\max\left[\tilde{\pi}_t\right]^{1-\lambda}\left[\frac{1}{\gamma_t}U_t^u\right]^{\lambda} \\ &\text{s.t. } \tilde{z}_t \geq 0,\ (50)-(53) \\ x_t &= \arg\max f(x_t) \end{aligned} \tag{54}$$

It is easily seen that

$$\text{sign } f'(x_t) = \text{sign}\left(\delta A \overline{x}_t^{\varphi-\rho} L_t^{\alpha-1}\tilde{\pi}_{t+1} - \tilde{U}\right) \tag{55}$$

where it has been assumed that $\tilde{\pi}_{t+1}$ does not depend on x_t. It will be confirmed below that this is in fact the case. Now, in all firms x_t will be increased if $f'(x_t) > 0$, and it will be decreased if $f'(x_t) < 0$. Hence, our only candidate for an equilibrium is that $\overline{x}_t$ (i.e. the level of training in the representative firm) is determined so that

$$\tilde{U} = \delta A \overline{x}_t^{\varphi-\rho} L_t^{\alpha-1}\tilde{\pi}_{t+1} \tag{56}$$

It is also easily shown that this equilibrium is stable in the sense that $f'(x_t)$ is

[11] We use the assumption that $\nu + \rho = 1$, and that $N_t < L_{t-1}$ which in footnote (9) is confirmed to be the case in steady state.

decreasing in x_t.

By using (56) in (53), it follows that

$$f(x_t) = -\tilde{U} \quad \textbf{(57)}$$

Now, by solving (54) with respect to $\tilde{z}_t$, we get that

$$\tilde{z}_t = \frac{\beta}{(\lambda + (1-\lambda)\beta)\tilde{U}} \quad \textbf{(58)}$$

By inserting (58) in (50), it follows that

$$\tilde{\pi}_t = \left[\beta^{\frac{\beta}{1-\beta}} - \beta^{\frac{1}{1-\beta}}\right]\left[\frac{\beta}{(\lambda + (1-\lambda)\beta)\tilde{U}}\right]^{\frac{\beta}{1-\beta}} \quad \textbf{(59)}$$

Hence, it is confirmed that $\tilde{\pi}_{t+1}$ does not depend on x_t (as assumed above).

By inserting (58) into (51), and by using that $\gamma_t = \gamma$, we find that

$$\tilde{w}_t = \frac{w_t}{\gamma_t} = \frac{(\lambda + (1-\lambda)\beta)\tilde{U}(1-x_t)}{\beta} \quad \textbf{(60)}$$

which is the expression given in (25). Now, by inserting (60) into (22), it follows that

$$l_t = \left[\frac{\beta^2}{(\lambda + (1-\lambda)\beta)\tilde{U}}\right]^{\frac{1}{1-\beta}} \frac{1}{1-x_t} \quad \textbf{(61)}$$

which is the expression given in (26). Finally, if we use the steady state condition (19), (59) and (61) in (56) (recalling that L = Nl), we get that

$$\frac{x^{\rho}}{1-x} = \delta A\left[\beta^{-1} - \right]\left[\frac{\lambda + (1-\lambda)\beta}{\beta}\right] \quad \textbf{(62)}$$

which is the expression given in (24).

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