DEPARTMENT OF ECONOMICS

Working Paper

CONTRACT RENEWAL
UNDER UNCERTAINTY

Torben M. Andersen
Morten Stampe Christensen

Working Paper No. 1997-5

ISSN 1396-2426

UNIVERSITY OF AARHUS • DENMARK

INSTITUT FOR ØKONOMI

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 350
8000 AARHUS C - ☎ 89 42 11 33 - TELEFAX 86 13 63 34

WORKING PAPER

CONTRACT RENEWAL
UNDER UNCERTAINTY

Torben M. Andersen
Morten Stampe Christensen

Working Paper No. 1997-5

DEPARTMENT OF ECONOMICS

SCHOOL OF ECONOMICS AND MANAGEMENT - UNIVERSITY OF AARHUS - BUILDING 350
8000 AARHUS C - DENMARK ☎ +45 89 42 11 33 - TELEFAX +45 86 13 63 34

CONTRACT RENEWAL UNDER UNCERTAINTY

Torben M. Andersen
and
Morten Stampe Christensen
Department of Economics
University of Aarhus

September 1996

Abstract:
The incentive to call for contract renewal to adjust prices is considered from a bilateral perspective in a setting where changes in outside opportunities drive the incentive to renew contracts and costs preclude continuous renewal. A model encompassing several contract forms is formulated, and the existence of an equilibrium to the bilateral renewal game is established. Prices display inertia, and the incumbent contract is found to be more resistant to changes in outside opportunities, the larger the costs of contract renewal, the variability of outside opportunities and the lower the discount rate. The model is shown to match a number of empirical observations on contracts, and in a macroeconomic application of the model it is shown how nominal inertia may arise and why the rate of inflation and monetary uncertainty have real effects.

JEL: C72, D81, E31
Key Words: Contract renewal, stopping game, price adjustment.

We gratefully acknowledge comments from participants at the workshop "Imperfect Markets and Macroeconomics", November 1995 University of Copenhagen.

1. Introduction

Many transactions take place within a framework of long term contracts. However, actual contracts do not specify actions for all possible future contingencies and leave a significant part of the terms and duties to future determination (Carlton (1986)). The incentive to enter contractual relationships is thus primarily due to it being a means to establish procedures for adapting exchange and resolving disputes in the future (Crocker and Masten (1991)). Hence, rather than specifying the future terms of the transactions, the contract provides provisions for future adjustments. These can either be mechanical rules which link the terms of the contract to external developments like indexation of contract prices or be conditions for renegotiating or reopening parts of the terms of the contract.

One particular important aspect is how prices are determined in long term contracts, since fix price contracts of non-trivial duration are seldom observed. What are the incentives to change prices in long-term contracts, and is substantial inertia in adjustment bound to develop?

Case studies of markets for intermediary products and raw materials like e.g. natural gas, coal and petroleum coke (see e.g. Crocker and Masten (1991), Goldberg and Erickson (1987) and Joskow (1988)) find that transactions are settled by long-term contracts of a duration as long as fifty years. Although mechanical procedures for adjusting prices are used, one often encounter reopening clauses allowing for renegotiation of the terms of trade (eventually contingent on certain conditions being fulfilled). Carlton (1986) also finds a prevalence of long term contract in product markets as well as substantial inertia in price adjustment. In the labour market, contracts are usually of a duration between 1 and 5 years. Such long-term contracts usually have a fixed wage eventually allowing for mechanic (indexation) wage adjustments at fixed points in time. However, even such contracts often include reopening clauses allowing for wage adjustment to deal with exceptional cases (Vroman (1989)). Inertia in wage adjustment has been documented by Beaudry and DiWardo (1991) among others.

The case of unilateral contract renewal under uncertainty has been extensively analysed in so-called menu-cost models (see Sheshinski and Weiss (1977, 1983), Danziger (1983, 1984, 1987), Caplin and Spulber (1987) and Caplin and Leahy (1991)). The setting is a monopolist firm quoting a nominal price (implicit contract with customers). If market conditions change, price adjustment is only worthwhile if the gain from so doing outweighs the cost of changing the price and hence adjustments are only undertaken when the new optimal price deviates sufficiently from the initial price. This theory thus predicts that prices may remain sticky to "small" shocks while they are adjusted to "large" shocks. Assuming that the optimal price policy takes a (s,S)-form, it can be shown that inflation implies a downward inflexibility in price adjustment in the sense that prices tend to be adjusted more in the upward than in the downward direction (Tsiddon (1991)), and moreover hysteresis can arise (Dixit (1991)).

A parallel problem exists for labour market contracts when these cannot be made fully state contingent and contract renewal is costly. Under the maintained assumption that labour market contracts are of fixed duration, the focus has been on the determination of how uncertainty affects the optimal length of contracts assuming that either the firm or the workers (union) determine the terms of the contract (Gray (1978) and Dye (1985)). The fact that labour market contracts tend to be of fixed duration does not, as noted, prevent that the terms of the contract are renegotiated, and Danziger (1995) shows that reopening of labour contracts is a way to adapt to large shocks.

The present paper differs from the abovementioned literature by taking a bilateral approach explicitly considering the incentives both the buyer and the seller side have to reopen the contract to adjust e.g. the price. As most actual (explicit) contracts are characterized by both parties having a possibility of inducing contract renewal, it is relevant to consider the incentives to call for contract renewal from a bilateral point of view.

A bilateral approach is also taken by MacLeod and Malcomson (1993) in an analysis of investment incentives in the presence of long-term contracts. If contract negotiation implies surplus sharing, there may be insufficient incentives for (specific and general) investments[1)]. MacLeod and Malcomson (1993) consider cases where simple contract forms can overcome this problem and therefore imply efficient investment. The present paper differs from the abovementioned analysis by focusing on the problem of price adjustment rather than investment in long term contracts and specifically the problem is how costly contact renewal (switching costs) affects price adjustment under uncertainty.

Price adjustment in contracts where quantities cannot be changed (in the short run) constitutes an interesting starting case. Empirical evidence indicates that provisions for price adjustment are more widespread when the scope for quantity adjustment is modest (Crocker and Masten (1991)). Moreover, in this setting changing prices may be perceived to be a question of pecuniary redistribution and therefore essentially a zero-sum game (Williamson (1979)). The incentives underlying contract renewal should thus be symmetric in the sense that what one party gains, the other loses. This perception turns out to be misleading as reopening of contracts is

---

1) On this see also Holden (1995).

in general costly and therefore there are some frictions in renegotiating contracts. Contract renewal demanded by one party thus has a transaction cost externality on top of the pecuniary redistribution to the other party of the contract. This affects the incentives underlying contract renewal and gives the terms of the incumbent contract a special role, and may cause asymmetry in the incentive to adjust prices. Moreover the non-cooperative contract renewal game may have inefficiencies since an action by one party to call for contract renewal does not take into account the frictions inflicted on the opponent in terms of contract renewal costs.

The paper is organised in the following way. Section 2 sets up the basic problem of contract renewal under uncertainty when fully contingent contracts cannot be signed. The optimal renewal strategies are derived in section 3. Several extensions of this basic contract renewal problem are considered in section 4 allowing for asymmetric costs, quantity adjustment, fixed contract length, unlimited number of contract renewals and state dependent pay-off. A variety of contract forms are thus encompassed by the analysis, and in all cases price rigidity arises due to the role played by the incumbent price in a setting with uncertainty and transactions costs. The issue of price adjustment is particularly relevant for macroeconomics and some macroeconomic implications are considered in section 5, while section 6 concludes.

2. Contract Renewal under Uncertainty

Consider a contract between a principal (P) and an agent (A) stipulating a given flow of services or actions to be taken by the agent who in turn is compensated by a flow payment from the principal. At any point in time there is the possibility that either the principal or the agent may want to suggest a contract renewal - at a cost - because outside opportunities have changed, that is, the agent finds that he can receive a better compensation by shifting to another principal, or the principal perceives that he can replace the agent by another agent willing to accept a lower payment. Examples of contracts fitting this description are legio including employment contracts, tenant contracts, delivery contracts etc.

Let q denote the payment[2)] according to the incumbent contract and let w_t denote the outside opportunity (alternative price) available to the two parties to the contract. If the contract is renewed at time t, the new payment will be w_t. By specifying an exogenous outside opportunity, we avoid having to go into details about the bargaining procedure which allows us to focus on the implications of uncertainty and costs for contract renewal[3)]. The problem faced by the agent (principal) is when a costly contract renewal should be undertaken to change the current payment q (-q) to w_t ($-w_t$). A similar problem will arise if the incentive to call for contract renewal is driven by internal factors like e.g. changes in productivity[4)]. To focus on the incentives underlying price adjustment, the quantities transacted are assumed given (see section 4 for endogenous quantity determination).

If the outside opportunity was given deterministically, the contract renewal problem would be trivial and the timing of contract renewal could easily be determined. However, if the outside opportunities evolve stochastically, the question of contract renewal becomes non-trivial. Spe-

---

[2)] We express payments in present value terms, i.e. if the flow payment is q' then $q = \int_0^\infty e^{-\lambda s} q' ds = q'/\lambda$, where λ is the time-invariant discount rate. Similarly for costs.

[3)] Thereby we also leave out problems arising from attempts to exploit the market power arising from "switching costs", see Klemperer (1995).

[4)] An alternative interpretation of the model is to interpret w_t as a process driving the value of the output produced by the agent. In this case the analysis carries through if wages are settled in negotiations between the agent and principal such that the wage is a share η of productivity (ηw_t), leaving a share $(1-\eta)$ of output as profits ($(1-\eta)w_t$).

cifically, it is assumed that w_t evolves according to the following geometric Brownian motion process

$$dw_t = \mu w_t dt + \sigma w_t dz_t$$

where dz_t is the increment of a Wiener process, i.e.

$$E(dz_t) = 0 \qquad Var(dz_t) = dt$$

Obviously, if contract renewal is costless the contract would be renewed continuously with changes in outside opportunities.This is counterfactual. Contract renewal is costly and usually involves both fixed and variable costs.The latter arises through different channels: lawyers' fees are dependent on the contract sum, stamp duties are often based on the contract sum, the value of the time used to settle the contract (evaluated at the opportunity wage). These costs are here subsumed in contract renewal costs which are proportional to the new payment flow[5)]. Contract renewal costs are τw_t for both the agent and the principal. We assume for a start symmetric costs, and shall latter comment on the extent to which this assumption affects the results.

Both the principal and the agent have an infinite horizon with a discount rate λ. We assume $\lambda > \mu$ to rule out the trivial case where the parties to the contract are always better off by waiting and therefore never exercise the option to call a contract renewal (see e.g. Pindyck (1991)).

To see the mechanisms underlying contract renewal consider as a prelude to the general analysis in section 4 the special case where the contract can only be renewed once. The option of being able to renew the contract has a value to the contract parties. Assume that the agent demands a contract renewal when w_t reaches $c_A q$. The incentive for the agent to renew the contract is clearly one-sided, as it is only attractive if the outside opportunities are more favourable than the current payment ($c_A > 1$). Similarly assume that the principal demands a contract

[5)] Assuming that costs are independent of the payment flow causes problems in an infinite horizon model with drift in the payment flow since it implies counterfactually that the costs of contract renewal relative to the gain from renewal may approach zero.

renewal when w_t reaches $c_P q$. The incentive for contract renewal is clearly one sided arising when the opportunity wage of the agent is lower than the current payment ($c_P < 1$).

A contract renewal is called by the agent if $c_A q = w_t$ and by the principal if $c_p q = w_t$. Denote the point in time where the agent will call for contract renewal by T_A and similarly T_P for the principal. Both the agent and the principal are risk-neutral. Note that we rule out initial commitments to the payment flow (or its adaption) over the horizon of the contract for the simple reason that these would not in general be time-consistent given that a switch to the outside opportunity can be undertaken at any point in time (at a finite cost). It is therefore only relevant to consider the time-consistent contract renewal strategies.

Consider now the consequences to the agent of a contract renewal. The possibility of a contract renewal is an asset to the agent if the outside opportunity improves and the agent has the possibility of raising the payment. Oppositely, the possibility that the principal can call for contract renewal if outside opportunities deteriorate constitutes a liability to the agent as he will be worse off in this case. The expected value to the agent of a contract renewal when the outside opportunity is w_t, $x_A(w_t)$, can be expressed in terms of the sum of the expected value of the asset and the liability component of the option, i.e.

$$x_A(w_t) = x_A^a(w_t) + x_A^l(w_t)$$
$$= E\left(e^{-\lambda T_A}(c_A q - q - \tau c_A q) \middle| w_t \wedge T_A < T_P\right)$$
$$+ E\left(e^{-\lambda T_P}(q - c_P q - \tau c_P q) \middle| w_t \wedge T_P < T_A\right)$$

The value to the principal of the contract renewal opportunity can similarly be written

$$x_P(w_t) = x_P^a(w_t) + x_P^l(w_t)$$
$$= E\left(e^{-\lambda T_P}(c_P q - q - \tau c_P q) \middle| w_t \wedge T_P < T_A\right)$$
$$+ E\left(e^{-\lambda T_A}(q - c_A q - \tau c_A q) \middle| w_t \wedge T_A < T_P\right)$$

The expected value of the contract renewal option satisfies (cf. Karlin and Taylor (1981) section 15.3) the following second order differential equation

$$\frac{\sigma^2}{2} x_j^2 \frac{\partial^2 x_j}{\partial w^2} + \mu x_j \frac{\partial x_j}{\partial w} - \lambda x_j = 0, \quad j = A, P$$

This equation has a straightforward interpretation since the left hand side according to Ito's lemma gives the expected change in the value of the option to have a contract renewal while the right hand side gives the deterministic pay-off if the contract renewal is exercised immediately. Clearly these two forces have to balance to have a non-trivial solution to the contract renewal problem.

Solving this second order differential equation yields,

$$x_j(w_t) = m_j w_t^{\alpha} + n_j w_t^{\beta} \;; \qquad j = A,\ P \tag{1}$$

where

$$\alpha, \beta \equiv \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\lambda}{\sigma^2}} \gtrless 0$$

The parameters m_j and n_j are determined by the boundary conditions stating that at the time of contract renewal, the value of the contract renewal option is equal to the value of contract renewal (value matching condition). For the agent we have

$$x_A(c_A q) = [c_A(1-\tau) - 1]\, q \equiv \Gamma_1$$

$$x_A(c_P q) = [c_P(1-\tau) - 1]\, q \equiv \Gamma_2$$

and for the principal

$$x_P(c_A q) = [1 - c_A(1+\tau)]\, q \equiv \Gamma_3$$

$$x_P(c_P q) = [1 - c_P(1+\tau)]\, q \equiv \Gamma_4$$

Imposing these boundary conditions, (1) can be rewritten

$$x_A(w_t) = \frac{1}{\Delta}\left\{[\Gamma_1 c_P^{\beta} - \Gamma_2 c_A^{\beta}]\, q^{-\alpha} w_t^{\alpha} + [\Gamma_2 c_A^{\alpha} - \Gamma_1 c_P^{\alpha}]\, q^{-\beta} w_t^{\beta}\right\}$$

$$x_P(w_t) = \frac{1}{\Delta}\left\{[\Gamma_3 c_P^{\beta} - \Gamma_4 c_A^{\beta}]\, q^{-\alpha} w_t^{\alpha} + [\Gamma_4 c_A^{\alpha} - \Gamma_3 c_P^{\alpha}]\, q^{-\beta} w_t^{\beta}\right\}$$

where $\Delta \equiv c_A^{\alpha} c_P^{\beta} - c_P^{\alpha} c_A^{\beta} > 0$.

3. The Contract Renewal Game

There is a strategic interaction in contract renewal between the principal and the agent, since the action to demand contract renewal has consequences to the other part. We look for Nash-equilibria to this contract renewal game where each party decides on its optimal critical value c_j ($j = A, P$) given the critical value of the opponent. To this end we need

Lemma 1:

For any $c_P \in [0, 1]$, there exists a unique best response $c_A \in [1,\infty[$ for the agent, where

$$c_A \in \left[\max\left(1, \frac{\alpha\beta}{(\alpha-1)(\beta-1)(1-\tau)}\right), \frac{\alpha}{(\alpha-1)(1-\tau)}\right]$$

For any $c_A \in [1, \infty[$, there exists a unique best response $c_P \in [0, 1]$ where

$$c_P \in \left[\frac{\beta}{(\beta-1)(1+\tau)},\ \min\left(1, \frac{\alpha\beta}{(\alpha-1)(\beta-1)(1+\tau)}\right)\right]$$

Proof: See appendix A.

Existence of a Nash-equilibrium to the contract renewal game is ensured by

Proposition 1: A Nash-equilibrium (c_A^*, c_P^*) exists to the contract renewal game. A sufficient condition for uniqueness is absence of drift in the outside payment w_t ($\mu = 0$).

Proof: see Appendix B.

Figure 1 shows the reaction curves and illustrates the Nash equilibrium. As seen from the figure the strategic interaction in contract renewal is such that the more hesitant the agent is to demand contract renewal (the larger c_A), the more hesitant will also the principal be in calling a contract renewal (the smaller c_P). This shows a strategic complementarity in contract renewal decisions.

Comparing the non-cooperative outcome (c_A^*,c_P^*) to the cooperative outcome (c_A^C, c_P^C), it is found that the latter entails a larger region supporting the incumbent contract, i.e. $c_A^C > c_A^*$, $c_P^C < c_P^*$. The intuition is straightforward as there on top on the pecuniary redistribution is a transaction cost externality of calling a contract renewal. The party calling a contract renewal imposes transaction costs on the other party. When this externality is internalized, the region of inaction expands. Even though costs imply price inertia, prices may still be adjusted too frequently due to the interplay between uncertainty and renewal costs.

Figure 1.

Contains Data for

Postscript Only.

$$\frac{\alpha\beta}{(\alpha-1)(\beta-1)(1+\tau)}$$

$$\frac{\beta}{(\beta-1)(1+\tau)}$$

$$\frac{\alpha\beta}{(\alpha-1)(\beta-1)(1-\tau)} \qquad \frac{\alpha}{(\alpha-1)(1-\tau)}$$

The Nash equilibrium to the contract renewal game implies an interval $[c_P q, c_A q]$ of inaction in the sense that as long as the outside opportunity payment remains in this interval, none of the parties to the contract have an incentive to call a contract renewal. This interval of inaction resembles the [s, S]-rules imposed on unilateral adjustment problems as in e.g. the menu cost models. Although the implications are the same, the region of inaction follows here from a game between parties having opposite incentives concerning payment revisions.

It is noteworthy that the critical values for contract renewal are path independent as

Corollary 1: The equilibrium values of c_A^* and c_P^* are invariant to the realizations of the stochastic variable w_t, and the size of the incumbent payment flow q.

Proof: Follows from proof of proposition 1.

The actual payment displays, however, path-dependence. In the absence of contracts the spot market payment would equal the outside opportunity, while with a fixed payment long-term contract the payment would be constant over time. In the present setting with long-term contracts allowing for contract renewal the payment is determined by the highest (lowest) outside payment in the past if the payment has been revised upwards (downwards). In this sense extreme market conditions in the past come to determine payments due to the lock-in effect caused by costly contract renewal. In a study of labour contracts Beaudry and DiWardo (1991) find empirical support for wages being positively correlated with the best labour market conditions observed since the worker was hired.

The adjustment turns out to be asymmetric as

Corollary 2: The interval supporting the incumbent payment flow $[c_P q, c_A q]$ is not geometrically symmetric around q, i.e. $c_P c_A \neq 1$.

In the case of no drift in the payment flow w_t ($\mu = 0$), we have $c_P c_A = (1-\tau^2)^{-1} > 1$ and hence the non-adjustment region is rightward-skew, i.e.

$$c_P > \frac{1}{c_A}$$

Proof: Follows from Lemma 2 in Appendix B.

Although the contract renewal problem considered here is set up to be symmetric (payment changes are a zero-sum game and contract renewal costs are symmetric), it is striking that the region of no-adjustment is (geometrically) asymmetric.

As a point of reference it is noted that in the case of certainty the critical values would be $\bar{c}_A = (1-\tau)^{-1}$, $\bar{c}_P = (1+\tau)^{-1}$. Comparing these to the case of uncertainty without drift ($\mu = 0$) we find that the product of the critical levels are the same, i.e. $c_A c_P = \bar{c}_A \bar{c}_P$ (compare to the unilateral case, cf. e.g. Dixit (1991)), but the effect of uncertainty is to expand the range implying price inertia, i.e. $c_A > \bar{c}_A$, $c_P < \bar{c}_P$. The intuition is simply that uncertainty adds to the costs of adjusting prices since there is an option value of waiting.

Due to the complexity of the model it is not possible to obtain analytical results on how the contract renewal problem is affected by the drift parameter (μ), the variance (σ^2), the discount rate (λ) and the cost parameter (τ). Accordingly, numerical simulations have been undertaken, and they are reported in figure 2. All figures are based on a benchmark case where ($\mu = 0$, $\sigma^2 = 0.04$, $\lambda = 0.1$, $\tau = 0.05$), and one parameter is then changed in each experiment.

The simulations show that an increase in the cost parameter enlarges the band supporting the existing contract payment as the critical level increases for the agent, and decreases for the principal. It is worth pointing out that although both the agent and principal are assumed to be risk-neutral, the contract renewal problem is affected by risk. The reason is simply that the value of waiting to have a contract renewal depends on the variability of outside opportunities. It is found here that an increase in the variance enlarges the interval supporting the initial price. This reflects that the possibility of extreme values becomes larger, i.e. the value of waiting increases and therefore the interval supporting the initial contract payment expands. A higher discount factor reduces the gains from waiting and therefore the critical level decreases for the agents and increases for the principal. It is found that both critical values are increasing in the drift parameter.

The contract renewal problem implies that the point in time at which contract renewal will be called by one of the parties is stochastic. This allows us to consider the length of the con-tract in terms of the expected time to contract renewal which can be written

$$Y(w_t) \triangleq E[T|w_t]$$

where $T = T_A$ if $w_t = c_A q$ and $T = T_P$ if $w_t = c_P q$. Following Karlin and Taylor (1980) the expected time to contract renewal satisfies

$$\frac{1}{2}\sigma^2 w_t Y^{//} + \mu w_t Y^{/} + 1 = 0$$

and the boundary conditions are given by

$$\Upsilon\left(c_{A}\mathbf{q}\right) = \mathbf{0}$$

Figure 2.

Figure 2A. c_A and c_P as a function of σ^2

Contains Data for

Postscript Only.

Figure 2B. c_A and c_P as a function of μ.

Contains Data for

Postscript Only.

Figure 2C. c_A and c_P as a function of λ.

Contains Data for

Postscript Only.

Figure 2D. c_A and c_P as a function of the cost parameters τ.

Contains Data for

Postscript Only.

Figure 3 shows how the expected contract length depends on the parameters of the contract renewal problem. The expected contract length is decreasing in the discount rate (λ), increasing in the cost parameter (τ) and decreasing in the variance (σ^2). While the two first results are straightforward implications of the results found above on the critical values, the latter effect is not. A larger variability induces a larger interval supporting the initial payment, cf. above, and this tend to lengthen the time to contract renewal. However, the likelihood of having a contract renewal increases as the variance increases, and this tends to reduce the time to contract renewal. The latter effect dominates such that the expected contract length is decreasing in the variance. For the drift parameter we find a non-monotone relationship with the longest expected duration in the case of zero drift. This is intuitive as the drift term implies an underlying deterministic trend in outside opportunities which is bound to release a contract renewal.

These predictions are in accordance with empirical analysis of contract duration in labour markets in which it is found that contract length is decreasing in uncertainty, decreasing in inflation and increasing in contracting costs (Vroman (1989), Murphy (1992)).

Finally, it should be pointed out that these findings have implications for empirical work on long-term contracts. Finding that contracts are never or only rarely renewed cannot be taken as evidence that the terms of the contract are irrevocably fixed since it may reflect that no severe shocks have taken place so as to induce contract renewal. The contract renewal option works as an escape clause which is more relevant the more variable, the larger the absolute drift, the smaller the contract renewal costs and the more patient the parties to the contract are.

Figure 3.

Figure 3A. The expected length of a contract as a function of σ^2

Contains Data for

Postscript Only.

Figure 3B. The expected length of a contract as a function of μ.

Contains Data for

Postscript Only.

Figure 3C. The expected length of a contract as a function of λ.

Contains Data for

Postscript Only.

Figure 3D. The expected length of a contract as a function of τ.

Contains Data for

Postscript Only.

4. Extensions

The contract renewal problem considered so far builds on a number of simplifying assumptions and it is consequently of interest to analyse whether the model is robust to generalizations. This turns out to be the case, and we present the extensions of the basic model in order of increasing complexity.

(1) Contract renewal costs

The contract renewal costs have been assumed to be symmetric and proportional to the payment flow. It is easily shown that allowing for both a fixed (F_j) and a proportional (τ_i) component in renewal costs (j = A,P) possibly differing between the agent and the principal do not change anything qualitatively as the value of contract renewal for the agent becomes[6)]

$$x_A(w_t) = E\left[e^{-\lambda T_A}\left(\left(c_A(1-\tau_A)-1\right)q - F_A\right)\middle| w_t \wedge T_A < T_P\right) + E\left(e^{-\lambda T_P}\left(\left(c_P(1-\tau_A)-1\right)q - F_A\right)\middle| w_t \wedge T_P - T_A\right)$$

and the boundary conditions become

$$x_A(c_A q) = \left(c_A(1-\tau_A)-1\right)q - F_A$$
$$x_A(c_P q) = \left(c_P(1-\tau_A)-1\right)q - F_A$$

Similar reasoning applies to the principal, and it is easily seen that a solution of the form (1) can be found.

(2) Fixed Contract Length

As mentioned in the introduction, some contracts - notably in the labour market - have the property of being finite in length but allowing for renegotiating of the terms of the contract in unusual circumstances. Denote the point in time when the contract expires by S. The value of contract renewal to the agent is thus

6) It is easily verified that the model can also be modified such that contract renewal costs are solely born by the party demanding a contract renewal without changing anything qualitatively.

$$x_A(w_t) = E\left(e^{-\lambda T_A}\left(c_A q - q - \tau c_A q\right) \middle| w_t \wedge T_A < T_P \wedge T_A < S\right)$$
$$+ E\left(e^{-\lambda T_P}\left(c_P q - q - \tau c_P q\right) \middle| w_t \wedge T_P < T_A \wedge T_P < S\right)$$

and mutatis mutandis for the principal.

In this case the value of the option to renew the contract declines the closer we get to the point in time when the contract expires. The intuition is simply that the period in which the gains from contract renewal can be reaped gets smaller the shorter the remaining life time of the contract. The value of waiting is declining, but the period over which contract renewal costs can be regained is also declining. It is thus ambiguous whether the incentive to call for contract renewal decreases or increases the closer we get to S.

(3) Quantity Adjustment

A special and restrictive feature of the contract problem considered in section 2 is that only payments can be changed while quantities (implicitly) are assumed to be invariant. The problem can easily be modified to allow for quantity adjustment. To see this, return to the expression giving the expected value of contract renewal for the agent. Define an indirect utility function for the agent V(q) depending on the payment net of transactions costs, and similarly for the principal $\pi(q)$ then we get

$$x_A(w_t) = E\left(e^{-\lambda T_A}\left(V\left((c_A - \tau)q\right) - V(q)\right) \middle| w_t \wedge T_A < T_P\right) + E\left(e^{-\lambda T_P}\left(V\left((c_P - \tau)q\right) - V(q)\right) \middle| w_t \wedge T_P < T_A\right)$$

$$\mathbf{x_P(w_t) = E\left(e^{-\lambda T_P}\left(\pi\left((c_P - \tau)q\right) - \pi(q)\right) \middle| w_t \wedge T_P < T_A\right) + E\left(e^{-\lambda T_A}\left(\pi\left((c_A - \tau)q\right) - \pi(q)\right) \middle| w_t \wedge T_A < T_P\right)}$$

with boundary conditions changed accordingly.

Clearly, if $V_q^{/} > 0$ and $\Pi_q^{/} < 0$, the analysis carries through. The intuition of these conditions is that the agent should always be better off after an increase in payment taking into account quantity responses, while the principal should be worse off. A simple example fitting into this problem is a utility function for an agent reading u = y - d(e) where y is income (net of contract renewal costs) and d(e) is disutility of labour, i.e. $d_e^{/}(e) > 0,\ d_e^{//}(e) > 0$. Hence, effort supplied fulfil $d_e^{/}(e) = q(1-\tau)$ implying that $\frac{\partial e}{\partial q(1-\tau)} > 0$ and $u = q(1-\tau)e - d(e) \equiv V(q(1-\tau))$

where $V_q > 0$. Let profit be $\Pi = f(en)-q(en+\tau)$, where n is the number of agents employed and f is the production function ($f^{/} > 0,\ f^{//} < 0$), then it follows from profit maximization that $\Pi_q < 0$.

(4) Unlimited Contract Renewals Possibilities

So far it has been assumed that contract renewal only can take place once. An assumption which is clearly restrictive when agents have an infinite horizon as in the base model. We shall now show that the model can easily accommodate the case where the number of possible contract renewals is unrestricted and therefore potentially infinitely large.

Suppose that the agent has just undertaken a contract renewal yielding a payment q_1 and there is one last possibility for contract renewal with expected value x_A^1 (determined by (1)). The total value of the current payment and the expected value of the option for contract renewal to the agent is

$$\psi_A^1 = q_1(1-\tau) + x_A^1$$

The payment to the agent prior to the second to last contract renewal is denoted q_2. The total value to the agent of the current payment and the expected value of the option of two contract renewals is

$$\psi_A^2 = q_2 + x_A^2$$

where x_A^2 is the expected value of the option to have the contract renewed twice.

The problem of when to exercise the second to last contract renewal is when to replace ψ_A^2 by ψ_A^1. Using that x_A^1 is proportional to q_1, $x_A^1 = k_A^1 q_1$, we have $\psi_A^1 = q_1 (1-\tau+k_A^1) = q_1(1-\tau_A^1)$ where[7)] $\tau_A^1 \equiv \tau - x_A^1$. It is thus seen that the problem of when to exercise the second to last contract

[7)] Notice that this implies that although the underlying renewal costs are symmetric, the net cost of contract renewal differs between the agent and the principal, i.e. $\tau - x_A^1 \neq \tau - x_P^1$.

renewal is formally equivalent to the problem of when to exercise the last contract renewal, the only difference is a reinterpretation of the cost parameter. Applying this method recursively defining τ_A^n and x_A^n respectively as the cost parameter and the expected value of the option to have the n'th last contract renewal, it is immediately apparent that the problem can be applied to the case of infinite contract renewal possibilities.

In the limit we have

$$\tau_A^\infty = \tau_A - x_A^\infty$$

where x_A^∞ denotes the value of x_A evaluated for $\tau_A = \tau_A^\infty$. A similar equation holds for the principal. Solving for τ_A^∞ and τ_P^∞ yields

$$\tau_A^\infty = \frac{(1-c_A)\left(c_P^{\beta}-c_P^{\alpha}\right) + (1-c_P)\left(c_A^{\alpha}-c_A^{\beta}\right) - \tau_P \cdot \Delta}{c_A\left(c_P^{\beta}-c_P^{\alpha}\right) + c_P\left(c_A^{\alpha}-c_A^{\beta}\right) - \Delta}$$

$$\tau_P^\infty = \frac{(c_A-1)\left(c_P^{\beta}-c_P^{\alpha}\right) + (c_P-1)\left(c_A^{\alpha}-c_A^{\beta}\right) - \tau_A \cdot \Delta}{c_A\left(c_P^{\beta}-c_P^{\alpha}\right) + c_P\left(c_A^{\alpha}-c_A^{\beta}\right) - \Delta}$$

Notice that changes in μ will affect x_A^∞ and x_P^∞ and thereby c_A^∞ and c_P^∞. The monotone relationship between μ and c_A (c_P) found for the base model will therefore not necessarily hold in this case of unlimited contract renewal possibilities.

(5) State Dependent Pay-Offs

The contract problem considered above was a problem of replacing a deterministic payment q with a stochastic outside payment w_t. In general the pay-off is stochastic both under the existing contract and after contract renewal. We shall show that the model can be modified to cope with this situation.

Assume that the pay-off under the existing contract q is stochastic and evolves according to

$$dq_t = \mu_q q_t dt + \sigma_q q_t dz_q$$

while the outside opportunity evolves according to

$$dw_t = \mu_w w_t dt + \sigma_w w_t dz_w$$

The contract renewal problem is now when to replace the stochastic pay-off q_t $(-q_t)$ with the stochastic pay-off w_t $(-w_t)$.

For the agent we have that the expected value of the option to call for contract renewal can be written

$$x_A(q, w) \;=\; E\left(e^{-\lambda \tau_A}\left(w_t - q_t - \tau w_t\right)\middle| w_t, q_t \tau_A < \tau_P\right) \;+\; E\left(e^{-\lambda \tau_P}\left(w_T - q_T - \tau w_t\right)\middle| w_t, q_t \tau_P < \tau_A\right)$$

and similarly for the principal.

The boundary conditions are

$$x_A\left(q, c_A q\right) = q\left(c_A - 1 - \tau\right)$$

In appendix C it is shown that the expected value of the contract renewal option can be written

$$x_j\left(w_t, q_t\right) \;=\; m_j q_t \left(\frac{w_t}{q_t}\right)^{\alpha} \;+\; n_j q_t \left(\frac{w_t}{q_t}\right)^{\beta}$$

where m_j and n_j are determined from the boundary conditions. With these modifications the analysis from section 2 carries through.

5. Macroeconomic Implications

As the contract renewal game entails a region of no-action, the model holds the potential of explaining rigidity of real and nominal prices. The intuition is that there is an incentive for one of the parties to a call for contract renewal and thus adjustment of the payment only in case of "large" changes in the state of nature. Moreover, the model implies that the no-action region is asymmetric around the incumbent price, and hysteresis arises in adjustment as current payments are affected by past (extreme) market conditions.

The question of nominal rigidities has been devoted extensive attention in the literature. The model can easily be modified to address this question by assuming that the contract stipulates a nominal payment while the price level and therefore the real payment is stochastic. To consider this case, assume that w_t is the process driving the inverse of the price level and the real outside opportunity payment is constant and equal to one (the outside nominal payment is thus $\frac{1}{w_t}$). It is assumed that the outside opportunity is proportional to the price level to ensure that no nominal rigidities are built into the model by assumption. The contract offers a nominal payment q the real value of which is $r_t = qw_t$. The real payment offered by the contract evolves according to

$$dr_t = \mu_r r_t dt + \sigma_r r_t dz_r$$

The contract renewal problem is now when to replace the incumbent and stochastic real payment r_t with the deterministic outside real payment ($\equiv$ 1).

It is immediately apparent that the above-mentioned contract renewal problem fits into the set-up of section 4.5. Moreover, as section 4.3 shows how quantity adjustments can be incorporated, this implies that nominal neutrality is broken, i.e. nominal prices do not adjust instantaneously and quantities are affected by nominal changes. It is an immediate implication that nominal shocks can have persistent effects. The fact that the interval supporting the initial price is not (geometrically) symmetric around 1 (for $\mu = 0$) has a particularly interesting implication for the dynamic adjustment process. Assume that the price level increases so as to induce an upward payment adjustment from q to $c_A q$. Subsequently the price level has to fall to $c_P (c_A q)$ to induce a downward payment revision. Since $c_A c_P \neq 1$, it follows that there is a path dependence in the nominal payment in the sense that the nominal payment prevailing at a given price level (and thus the real payment) depends on the history of the price level. The path-dependence in nominal payments leads to paradoxical results as a temporary nominal expansion which induces an upward nominal wage adjustment can thus have a lasting contractionary effect by locking nominal wages at a high level. Oppositely, a temporary monetary contraction can have a lasting expansionary effect by locking nominal wages at a low level.

Another implication is that monetary uncertainty even in a setting with risk-neutral agents can have real effects since larger uncertainty increases the interval supporting the existing payment thereby strengthening nominal rigidities. At the same time expected contract duration falls and this induces more frequent payment adjustments.

One consequence of inflation - and possibly one of the reasons why it is considered to be a problem - is that it causes variability in prices. The real contract payment $r_t \equiv qw_t$ belongs to the interval $[c_A^{-1}, c_P^{-1}]$. It is of particular interest to consider how the real payment is affected by changes in the drift parameter μ_r as this corresponds to changes in the underlying rate of inflation. As the problem is set up, the real payment in the spot market is constant (normalized to unity) and thus unaffected by nominal changes.

In order to calculate the mean value of the real contract payment r,

$$E(r) = \int_{c_A^{-1}}^{c_P^{-1}} r\,h(r)\,dr$$

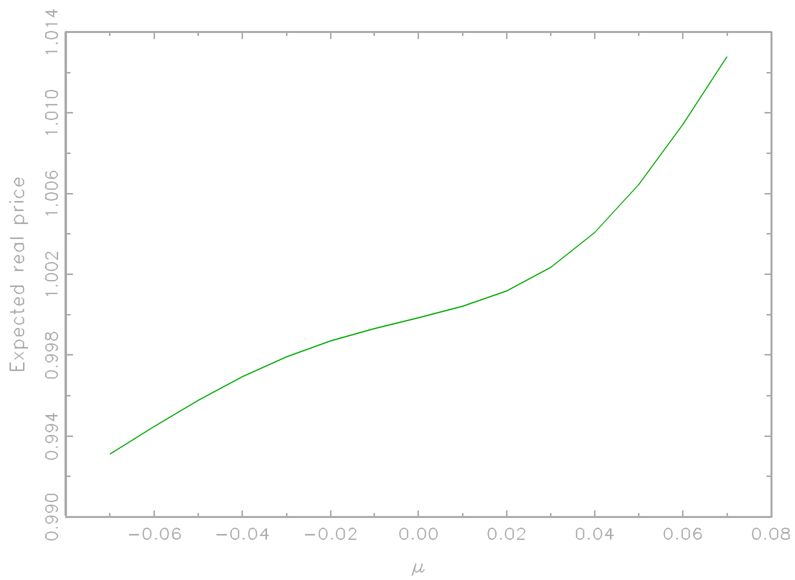
we need the steady state (ergodic) distribution h(r) of contract real payments over the interval $[c_A^{-1}, c_P^{-1}]$. This is derived in appendix D. The mean value of r would correspond to the aggregate deviation of real payments in an economy with an infinite number of payments settled by contracts of this form provided that the synchronization condition of Caplin and Spulber (1987) requiring that individual prices are distributed over the feasible interval $[c_A^{-1}, c_P^{-1}]$ according to the steady-state distribution h(r).

In figure 4, E(r) is plotted as a function of μ_r[8)], and it is immediately apparent that the mean value of the real payment is affected by the underlying nominal growth rate. The intuition for this result is quite simple. When a contract is renewed, the nominal price is set such that the real price equals 1. In the case of monetary expansion ($\mu_r < 0$) most nominal price adjustments will be upward and the real price of the contract will on average have been eroded until the contract is renewed thereby yielding $E[r] < 1$ and vice verse for monetary contraction ($\mu_r > 0$). Note that this result arises despite that price adjustment always reestablishes a relative price of one. This is an important difference to the unilateral menu cost models (see e.g. Sheshinski and Weiss (1977) in which case the average real price is also independent of the underlying nominal drift rate (Tsiddon (1993)).

[8)] Simulations performed for the case of unlimited contract renewals with $\sigma_r = 0.04$ and $\lambda = 0.01$, $\tau = 0.001$.

It should, however, be stressed that the expected relative price E(r) deviates from unity even in the absence of nominal drift ($\mu_r = 0$), E[r] cf. figure 4. In fact it can be shown that E[r] = 1 only if $c_A \cdot c_P = 1$[9], a condition which is only satisfied under special assumptions. This shows how uncertainty affect price adjustment in a bilateral setting.

Figure 4. Expected real price as a function of μ_r.



It is thus the case that while both the unilateral menu cost model and the bilateral contract renewal model cause money to be non-neutral, the latter also has that the underlying nominal drift rate matters (super non-neutrality) as well as effects of monetary uncertainty on relative prices.

6. Concluding Remarks

In a bilateral contract renewal problem in which the incentive to call for contract renewal is

---

[9] For $\mu_r = 0$, it can be shown that

$$E[r] = \frac{c_A \cdot c_P[\log(c_A) - \log(c_P)] - c_A \log(c_A) + c_P \log(c_P)}{\log(c_A) - c_P \log(c_A) - \log(c_P) + c_A \log(c_P)}$$

and it is easily verified that E[r] = 1 for $c_A c_P = 1$.

driven by changes in outside opportunities, it has been shown that payments display inertia. Contract renewal costs prevent continuous payment revisions and the incumbent contract payment comes to play a crucial role. This implies among other things path dependence in payment and nominal rigidities.

Although the findings have been shown to be robust to various modifications of the contract renewal problem, the model remains in a number of respects stylized. An important issue for future research would be to combine the question of payment adjustment with the problem of long term investment which is at the root of explaining why there is an incentive to enter contracts in the first place.

Appendix A: Proof Lemma 1

Deriving the optimal c_A (c_P) can be done in one of two ways

1) Solving the first order condition for the maximization of x_A (x_P).

2) Imposing a "smooth pasting" condition, see Dixit (1988). Define by ψ_A^1 the value of a portfolio consisting of the value of the old contract plus x_A and by ψ_A^0 the value of a portfolio consisting of the new contract. The smooth pasting condition states that

$$\left.\frac{\partial \psi_A^1}{\partial w}\right|_{w_t = c_A \cdot q} = \left.\frac{\partial V_A^0}{\partial w}\right|_{w_t = c_A \cdot q}$$

and similarly for the principal. Note that $\psi_A^0 = c_A(1-\tau_A)q$ and $\psi_P^0 = -c_P(1+\tau_P)q$, hence

$$\left.\frac{\partial \psi_A^1}{\partial w}\right|_{w_t = c_A \cdot q} = 1 - \tau_A \ , \quad \left.\frac{\partial \psi_P^1}{\partial w}\right|_{w_t = c_P \cdot q} = -1 - \tau_P$$

As both methods yields the same results, we stick to the first one as this is most intuitive.

Denote by FOC_A the derivative of $x_A(w_t)$ wrt. c_A and by FOC_P the derivative of $x_P(w_t)$ wrt. c_P. The agent's (principal's) choice of c_A (c_P) is found by equating FOC_A to zero which yields

The second order conditions evaluated at the points where the first order conditions are satisfied are given by

$$FOC_A \equiv c_A \Delta^{-2} \left[q^{1-\alpha-\beta} \left(c_P^{\alpha} q^{\alpha} w_t^{\beta} - c_A^{\beta} q^{\beta} w_t^{\alpha} \right) \right]^{-1}$$

$$SOC_A = - c_P^{\beta} c_A^{\alpha} \Delta \left[\alpha\beta + c_A (1-\tau_A)(\alpha - 1)(1-\beta) \right] < 0$$

The proof of lemma 1 proceeds by showing that

$$FOC_A\Big|_{c_A = \frac{\alpha\beta}{(\alpha-1)(\beta-1)(1-\tau_A)}} < 0 \;, \quad \frac{\alpha}{(\alpha-1)(1-\tau_A)} \geq 1 \qquad \textbf{(A1)}$$

$$+ \; c_A^{\alpha+\beta} \left[(\beta - \alpha)\left(1 - c_A(1+\tau_A)\right) \right] = 0$$

Hence, either $c_A^* = 1$ or

$$c_A^* \in \left[\frac{\alpha\beta}{(\alpha-1)(\beta-1)(1-\tau_A)} \;,\; \frac{\alpha}{(\alpha-1)(1-\tau_A)} \right]$$

in which case SOC_A assures us that we have a maximum. Furthermore, this is the unique value satisfying FOC_A as $SOC_A < 0$ for all values of c_A belonging to the above interval.

Similarly, it is shown that

$$FOC_P\Big|_{c_P = \frac{\beta}{(\beta-1)(1+\tau_P)}} < 0 \;, > 0 \;, \quad \frac{\beta}{(\beta-1)(1+\tau_P)} \geq 1 \qquad \textbf{(C1)}$$

Hence, either $c_P^* = 1$ or

$$c_P^* \in \left[\frac{\beta}{(\beta-1)(1+\tau_P)} \;,\; \frac{\alpha\beta}{(\alpha-1)(\beta-1)(1+\tau_P)} \right]$$

and the second order conditions ensure a unique maximum value.

To save space, we shall not prove each of these statements but ony the first as the proof of the remaining three follows the same procedure. To prove (A1) insert

$$c_A^1 = \frac{\alpha}{(\alpha-1)(1-\tau_A)}$$

in FOC_A to obtain

$$FOC_A\Big|_{c_A = c_A^1} = \psi_1(-1+\tau_A)c_P^{\alpha}\left[1 + c_A^{\alpha} c_P^{-\alpha} (\alpha - 1\left(1 - c_P(1-\tau_A)\right))\right]$$

where ψ_1 is a positive term. Define

$$x \equiv c_P(1-\tau_A) \quad , \quad x \in \left[0, \frac{\alpha}{\alpha-1}\right]$$

where the domain of x follows from $c_P \in [0,1]$ and $c' > 1$. The square bracket of $FOC_A\Big|_{c_A = c_A^1}$ may now be written as

$$b(x) \equiv 1 - \left(\frac{\alpha}{\alpha-1}\right)^{\alpha} x^{-\alpha}(\alpha-1)(1-x)$$

Now use that

$$b(0) = 1 \;, \quad b\left(\frac{\alpha}{\alpha-1}\right) = 0$$

and

$$b'(x) = (\alpha-1)^2\left(\frac{\alpha}{\alpha-1}\right)^{\alpha} x^{-\alpha-1}\left[x - \frac{\alpha}{\alpha-1}\right] < 0$$

to establish

$$b(x) > 0 \quad \forall \quad x \in \left[0, \frac{\alpha}{\alpha-1}\right]$$

Since $(-1+\tau_A) < 0$ we have

$$FOC_A\Big|_{c_A = c_A^l} < 0 \quad , \quad c_A^l = \frac{\alpha}{(\alpha - 1)\left(1 - \tau_A\right)} \geq 1$$

Appendix B: *Proof of Proposition 1*

The proof proceeds by considering what happens with the optimal choice of c_A (c_P) for extreme values of c_P (c_A). From lemma 1 and the first order conditions derived in the proof of lemma 1, it follows

By drawing the reaction curves in a (c_A, c_P) diagram using the above end points, it follows from the continuity of the curves that they must have at least one intersection. Hence, at least one Nash equilibrium exists. Uniqueness can be proven in the case of zero drift, i.e. $\mu = 0$ by using the following lemma:

$$\lim_{c_P \to 1} c_A = \hat{c}_A^{//} \leq \mathrm{Max}\left[1, \frac{\alpha\beta}{(\alpha - 1)(\beta - 1)\left(1 - \tau_A\right)}\right]$$

$$\lim_{c_A \to 1} c_P = \hat{c}_P^{//} \leq \mathrm{Max}\left[1, \frac{\beta}{(\beta - 1)\left(1 + \tau_P\right)}\right]$$

Lemma 2

If $\mu = 0$, and $FOC_A = 0$ for

then FOC_P also equals zero.

$$\left(c_A, c_P\right) = \left(c_A, \frac{1}{c_A\left(1 - \tau_A\right)\left(1 + \tau_P\right)}\right)$$

Proof

implying

Insert $c_P = \dfrac{1}{c_A\left(1 - \tau_A\right)\left(1 + \tau_P\right)}$ and $c_A\left(1 - \tau_A\right) = \dfrac{1}{c_P\left(1 + \tau_P\right)}$ in FOC_A to obtain $FOC_A = 0$

which can be rearranged to read

$$- \; c_A^{2\alpha-1}(1-\tau_A)^{\alpha-1}(1+\tau_P)^{\alpha-1}\left[(1-\alpha)\frac{1}{c_P(1+\tau_P)}+\alpha\right]$$

from which we get

$$- \; c_A^{2\alpha-1}(1-\tau_A)^{\alpha-1}(1+\tau_P)^{\alpha-1}c_P^{-1}(1+\tau_P)^{-1}\left[(1-\alpha)+\alpha\, c_P(1+\tau_P)\right]$$

Multiply with $c_P(1+\tau_P)$ to obtain

$$- \; c_A^{1-2\alpha}(1-\tau_A)^{-\alpha}(1+\tau_A)^{-\alpha}c_P^{-1}(1+\tau_P)^{-1}\left[(1-\alpha)(1+\tau_P)c_P-\alpha\right]$$

$$+ \; c_A^{1-\alpha}c_P^{\alpha}\left[(1-\alpha)(1+\tau_P)c_P+\alpha\right]c_P^{-1}(1+\tau_P)^{-1}$$

$$- \; c_A\left[\cdot\; c_A^{-1}(1+\tau_P)^{-1}\left[(1-2\alpha)\left(c_A(1+\tau_P)-1\right)\right]\right] \;=\; 0$$

$$+ \; c_A\left[(1-2\alpha)\left(c_A(1+\tau_P)-1\right)\right]c_A^{-1}(1+\tau_P)^{-1} \;=\; 0$$

It follows that $FOC_P = 0$. Q.E.D.

To prove a unique Nash equilibrium for $\mu = 0$ we observe that: i) At least one Nash equilibrium exists, ii) Furthermore it follows from the end points of the reaction curves that an odd number of equilibria exists, iii) Lemma 2 implies that whenever the reaction curve for c_A (or c_P) intersects the hyperbola $c_A = \dfrac{1}{c_P(1-\tau_A)(1+\tau_P)}$, a Nash equilibrium exists.

We now show that the reaction curve for c_A has at most two intersections with the hyperbola $c_A = \dfrac{1}{c_P(1-\tau_A)(1+\tau_P)}$, and it then follows from i) and ii) that there is only one intersection. and hence only one Nash equilibrium for

$$c_A \in \left[\frac{1}{1-\tau_A},\; \frac{\alpha}{\alpha-1}\frac{1}{1-\tau_A}\right) \quad \text{and} \quad c_P \in \left[\frac{\alpha-1}{\alpha}\frac{1}{1+\tau_P},\; \frac{1}{1+\tau_P}\right]$$

Proof

Insert

$$c_P \;=\; \frac{1}{c_A(1-\tau_A)(1+\tau_P)} \;\equiv\; \frac{1}{c_A\kappa}$$

in FOC_A.

Differentiate twice wrt. c_A to obtain

$$2\alpha(1-3\alpha+2\alpha^2)c_A^{-3-2\alpha}\kappa^{-1-2\alpha}\left(c_A^2\kappa+c_A^{4\alpha}\kappa^{2\alpha}\right)\left(1-c_A(1-\tau_A)\right)$$

which always is negative.

We conclude that FOC_A is concave in c_A along the hyperbola $c_A = \dfrac{1}{c_P(1-\tau_A)(1+\tau_P)}$ and therefore is zero at most twice.

Appendix C: State Dependent Pay-off

In this case the contract renewal option satisfies

$$\lambda x_j = \mu_w \, w \cdot \frac{\partial x_j}{\partial w} + \mu_q \cdot q \frac{\partial x_j}{\partial q} + \frac{1}{2}\left[\sigma_w^2 \, w^2 \frac{\partial^2 x_j}{\partial w^2} + \sigma_q^2 \cdot q^2 \cdot \frac{\partial^2 x_j}{\partial q^2} + 2\rho_{wq}\sigma_w\sigma_q wq \frac{\partial^2 x_j}{\partial w \partial q}\right]$$

where ρ_{wq} is the correlation coefficient between w and q.

Now guess that $x_j(q,w)$ is of the form

$$x_j(q,w) = k \cdot q^a w^b$$

Inserting in the second order differential equation, using that the boundary conditions imply a = 1-b and solving yields

$$b = \frac{1}{2} - \frac{\mu_w - \mu_q}{\sigma^2} + \sqrt{\left(\frac{\mu_w - \mu_q}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(\lambda - \mu_q)}{\sigma^2}}$$

Hence,

$$\boldsymbol{\beta} = \frac{1}{2} - \frac{\boldsymbol{\mu_w - \mu_q}}{\boldsymbol{\sigma^2}} + \sqrt{\left(\frac{\boldsymbol{\mu_w - \mu_q}}{\boldsymbol{\sigma^2}} - \frac{1}{2}\right)^2 + \frac{\boldsymbol{2(\lambda - \mu_q)}}{\boldsymbol{\sigma^2}}}$$

where $\sigma^2 = \sigma_w^2 + \sigma_q^2 - 2\rho_{wq}\sigma_w\sigma_q$ and $\mu_q < \lambda$.

Q.E.D.

Appendix D: Calculation of E(r)

Derivation of the ergodic distribution of X. Following Karling & Taylor (1981, p. 261), a stochastic process y, regulated in the interval [a,b] with return point y_0 when either a or b is reached has the following distribution:

$$h(y) = \frac{G(y_0, y)}{\int_a^b G(y_0,\xi)\, d\xi}$$

where

$$G(y_0,\xi) = \frac{2[S(b) - S(y_0)][S(\xi) - S(a)]}{[S(b) - S(a)] \cdot \sigma^2(\xi) \cdot s(\xi)} \quad a \le \xi \le y_0$$

$$G(y_0,\xi) = \frac{2[S(y_0) - S(a)][S(b) - S(\xi)]}{[S(b) - S(a)] \cdot \sigma^2(\xi) \cdot s(\xi)} \quad y_0 \le \xi \le b$$

where

$$S(x) = \int^x s(\eta)\, d\eta$$

$$s(x) = \exp\left[-\int^x \frac{2\mu(\xi)}{\sigma^2(\xi)} d\xi\right]$$

In our case $\mu(w) = \mu w$ and $\sigma^2(w) = \sigma^2 w^2$. We can now calculate E[r] as

$$E[r] = \int_{1/c_A}^{1/c_P} y\, h(y)\, dy$$

References

Beaudry, P. and J. DiWardo, 1991, The Effect of Implicit Contracts on the Movement of Wages over the Business cycle: Evidence from Micro Data, *Journal of Political Economy*, 99, 665-688.

Caplin, A. and J. Leahy, 1991, State-Dependent Pricing and the Dynamics of Money and Output, *Quarterly Journal of Economics*, CVI, 683-799.

Caplin, A.S. and D.F. Spulber, 1987, Menu Costs and the Neutrality of Money, *Quarterly Journal of Economics*, CII, 703-725.

Carlton, D.W., 1986, The Rigidity of Prices, *American Economic Review*, 76, 637-58.

Crocker, K.J. and S.E. Masten, 1991, Pretia Ex Machina? Prices and Process in Long-term Contracts, *Journal of Law and Economics*, XXXIV, 69-99.

Danziger, L., 1983, Price Adjustments with Stochastic Inflation, *International Economic Review*, 24, 699-707.

Danziger, L., 1984, Stochastic Inflation, and the Optimal Policy of Price Adjustments, *Economic Inquiry*, XXII, 98-108.

Danziger, L., 1987, On Inflation and Real Price Variability, *Economic Inquiry*, 25, 285-299.

Danziger, L., 1995, Contract Reopeners, *Journal of Labor Economics*, 13, 62-87.

Dixit, A.K., 1988, A Heuristic Derivation of the Conditions for Optimal Stopping of Brownian Motion, Working Paper, Princeton University.

Dixit, A.K., 1991, Analytical Approximations in Models of Hysteresis, *Review of Economic Studies*, 58, 141-15.

Dye, R.A, 1985, Optimal Length of Labor Contracts, *International Economic Review*, 26, 251-270.

Gray, J.A., 1978, On Indexation and Contract Length, *Journal of Poltical Economy*, 86,1-18.

Goldberg, V.P. and J.R. Erickson, 1987, Quantity and Price Adjustment in Long-Term Contracts: A Case Study of Petroleum Coke, *Journal of Law and Economics*, XXX, 369-398.

Holden, S., 1995, Renegotiation and the Efficiency of Investments, Working Paper, Department of Economics, University of Oslo.

Joskow, Paul L., 1988, Price Adjustment in Long-Term Contracts: The Case of Coal, *Journal of law and Economics*, XXXI, 47-83.

Karlin, S. and H.M. Taylor, 1980, *A Second Course in Stochastic Processes*, Academic Press, New York.

Klemperer, P., 1990, Competition when Consumers have Switching Costs - An Overview with Applications to Industrial Organization, Macroeconomics and International Trade, *Review of Economic Studies*, 62, 515-39.

McLeod, W.B. and J. Malcomson, 1993, Investments, Holdup and the Form of Market Contracts, *American Economic Review*, 83, 811-837.

Murphy, K.J., 1992, Determinants of contract Durations in Collective Bargaining Agreements, *Industrial and Labor Relations Review*, 45, 352-365.

Pindyck, R.S., 1991, Irreversibility, Uncertainty and Investment, *Journal of Economic Literature*, XXiX, 1110-1148.

Sheshinski, E. and Y. Weiss, 1977, Inflation and Costs of Price Adjustment, *Review of Economic Studies*, 44, 287-303.

Sheshinski, E. and Y. Weiss, 1983, Inflation and Costs of Price Adjustment, *Review of Economic Studies*, 50, 287-303.

Tsiddon, D., 1993, The (Mis) Behaviour of the Aggregate Price Level, *Review of Economic Studies*, 60, 889-902.

Vroman, S.B., 1989, Inflation Uncertainty and Contract Duration, *Review of Economics and Statistics*, 71, 677-681.

Williamson, O.E., 1979, Transactions-Cost Economics:The Governance of Contractual Relations, *Journal of Law and Economics*, XXII, 233-261.

Working Paper

1996-6	Philip J. Grossman, Panayiotis Mavros and Robert W. Wassmer: Public Sector Inefficiency in Large U.S. Cities.
1996-7	Amitava Krishna Dutt and Peter Skott: Keynesian Theory and the Aggregate-Supply/Aggregate-Demand Framework: A Defense.
1996-8	Peter Skott: Strukturledighed, Phillipskurver og dansk økonomisk-politisk debat.
1996-9	Peter Skott: The Crisis of Egalitarian Policy and the Promises of Asset-Based Redistribution.
1996-10	Per Svejstrup Hansen, Hans Peter Møllgaard, Per Baltzer Overgaard and Jan Rose Sørensen: Asymmetric Adjustment in Menu Cost Duopoly.
1996-11	Philipp J.H. Schröder: Reconsidering the Case for Monetary Exchange.
1996-12	Per Svejstrup Hansen: Quantitiy Adjustment Costs and Price Rigidity.
1996-13	Robert F. Engle and Svend Hylleberg: Common Seasonal Features: Global Unemployment.
1996-14	Svend Hylleberg and Adrian R. Pagan: Seasonal Integration and the Evolving Seasonals Model.
1997-1	Tom Engsted, Jesus Gonzalo and Niels Haldrup: Testing for Multicointegration.
1997-2	Lykke E. Andersen: Modelling the Relationship between Government Policy, Economic Growth, and Deforestation in the Brazilian Amazon.
1997-3	Alvaro Forteza: Overinsurance in the Welfare State.
1997-4	Alvaro Forteza: Multiple Equilibria in the Welfare State with Costly Policies.
1997-5	Torben M. Andersen and Morten Stampe Christensen: Contract Renewal under Uncertainty.