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ON THE MARGINAL PRODUCT OF LABOUR

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by

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Abstract

On the occasion of the one hundred years' anniversary an unsettled dispute between John Hobson and Alfred Marshall is revived. The paper argues that Marshall misunderstood Hobson who by claiming that the choice of technique is endogenous arrives at the conclusion that marginal labour is an indeterminate and hence inoperational concept.

Key words: marginal product, endogenization of technology, externality.
JEL: B3, B4, D1, D2, D3.

A pent-up controversy on marginalism[1]

Introduction

Time and again fundamental methodological issues dive out to the surface of the literature, often triggered by some controversial contribution. After some dispute - very often in a rather passionate form - the issue is dropped again either because of an authoritative refusal of the heresy or because of a realization that an unsolvable enigma has been the core of the discussion.

The issue to be scrutinised below was concerned with the fundamental problem of representing labour as a factor input by mathematical tools; the dispute on this ran for several decades around the year 1900[2].

The dispute about marginal product of labour

In the wake of the adoption of 'marginalization' as a core instrument in economic theory in the second half of the nineteenth century many reservations -and mistakes- emerged; one of them was forwarded by the 'heretic' John Hobson in a series of writings. While Hobson's under-consumption theory is rather well-known, i.a. because of its rehabilitation by Keynes in his GT, Hobson's critical view on the marginal labour theory was never generally accepted. At least partly, my explanation of this is that Professor Alfred Marshall by an unchallenged authority succeeded in suppressing the Hobson criticism; in the sequel, however, I shall argue that Hobson was right and that Marshall and Edgeworth misunderstood Hobson to the effect that since then a fundamental and well-founded criticism has been tacitly bottled-up.

The argument against marginal labour theory of value, raised by John Hobson, may be presented by the following quotation from Hobson [1909]:

> "A very simple instance of co-operative group of labourers, in a business where the capital element may be ignored, will serve to expose the fallacy. In a primitive fishery let us say that one man fishing alone could make a catch of ten; a two-man group a catch of twenty-two; a three-man group of thirty-seven; a four-man group of sixty; a five-man group of seventy-two. Here a four-man group is evidently the most advantageous, and that fishery would be worked upon this basis. Now in this business the fourth man ranks as the marginal worker.

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[1] The author is thankful to Gunnar Thorlund Jepsen, Philipp Schröder, Lars Muus and Peter Skott for helpful comments; the usual disclaimer applies.

[2] A second, more recent example with methodologically strong similarities is the so-called capital controversy between Cambridge, UK (Joan Robinson) and Cambridge, US (Robert Solow), cf. e.g. Harcourt [1972].

> His presence or his withdrawal from the group makes a difference of twenty-three fish, ie the difference between thirty-seven and sixty. Is that to be taken as representing his separate product? It ought, upon the lines of Professor Clark´s reasoning. But it cannot be so taken. For the fourth fisher is no more productive than the other three, and if the fourth fisher´s product is twenty-three, the product of all four together must be ninety-two. But, as we saw, it was sixty. His so-called separate product, therefore, must be fifteen, and not twenty-three. It is impossible to take what his presence adds, or his absence from the group removes, as measuring his individual productivity or as determining his wage.”[3]

What in fact Hobson is describing here is the indeterminacy of marginal individual products in a situation with interdependencies or externalities between labourers, cf. below.

This point is underlined by the following quotation:

> “So with the case of the “marginal shepherd,” the tenth man whom a farmer calculates it is just worth his while to employ because he can get him for the price of twenty sheep a year, and he will just save that number by his work. Of course no farmer really plans his farm this way. If he comes to employ ten shepherds instead of nine or eleven, it is because he reckons that ten will give the best division of labour, and will, as a co-operative whole, enable him to get the most out of his farm. If the employment of a tenth shepherd means twenty more sheep per annum than the employment of nine, it cannot be maintained, that twenty sheep form the separate product of the tenth shepherd, but only that a ten group is more productive by twenty sheep than a nine group.”

No doubt, what Hobson was thinking of was the “.. best division of labour...” and further he argues:

> “If all the labourers in a community had full knowledge of their capacities, full power to educate them, full knowledge of every labour market, full freedom to enter any, full access to the ranks of the employers in case they had ability, and to the requisite capital or credit; if all capital were equally free to dispose itself to the best advantage, and the owners of land and ability were equally free and intelligent-in that event the whole of the industrial power would organise itself in business units of the most productive size and character in the several industries.”

And finally:

[3] The fact that Hobson uses an S-formed production function is of no influence on the arguments to follow.

> "For the product, after all, will be the product of the cooperative complex of units of the several factors, and there will be no means of splitting it into separate products attributable to each. In other words, the theory of productivity, marginal or even average, as a determinant of the distribution of that part of the product, which remains as "surplus" over and above necessary expenses, has no validity."

This heretic stance by Hobson was well-known, already in the 1890's and opposed by Alfred Marshall e.g. in his 'Principles of Economics', 4. ed. 1898. In defence of marginalism Professor Alfred Marshall in his famous footnote felt called for to write (8. ed. 1920, book V, ch. 8)

> "Mr. J. A. Hobson is a vigorous and suggestive writer on the realistic and social sides of economics: but, as a critic of Ricardian doctrines, he is perhaps apt to underrate the difficulty of the problems which he discusses. He argues that if the marginal application of any agent of production be curtailed, that will so disorganize production that every other agent of production be working to less effect than before; and that therefore the total resulting loss will include not only the true marginal product of that agent, but also a part of the product due the other agents: but he appears to have overlooked the following points: (1) There are forces constantly at work tending so to readjust the distribution of resources between their different uses, that any maladjustment will be arrested before it has gone far: and the argument does not profess to apply to exceptional cases of violent maladjustment. (2) When the adjustment is such as to give the best results, a slight change in the proportions in which they are applied diminishes the efficiency of that adjustment by a quantity which is very small relatively to that change-in technical language it is of "the second order of smalls"-; and it may therefore be neglected relatively to that change. (In pure mathematical phrase, efficiency being regarded as a function of the proportions of the agent; when the efficiency is at its maximum, its differential coefficient with regard to any one of these proportions is zero.) A grave error would therefore have been involved, if any allowance had been made for those elements which Mr. Hobson asserts to have been overlooked. (3) In economics, as in physics, changes are generally continuous. Convulsive changes may indeed occur, but they must be dealt with separately: and an illustration drawn from a convulsive change can throw no true light on the processes of normal steady evolution. In the particular problem before us, this precaution is of special importance: for a violent check to the supply of any one agent of production, may easily render the work of all other agents practically useless; and therefore it may inflict a loss out of all proportion to the harm done by a small check to the supply of that agent when applied up to that margin, at which there was doubt whether the extra net product due to a small additional application of it would be remunerative. The study of changes in complex quantitative relations is often vitiated by a neglect of this consideration to which Mr. Hobson seems to be prone; as indeed is instanced by his remarks on a "marginal shepherd" in The Industrial System, p. 110. See Professor

> Edgeworth´s masterly analyses of the two instances mentioned in this note, Quarterly Journal of Economics, 1904, p. 167; and Scientia, 1910, pp. 95-100."

Marshall's first reference is to F. Y. Edgeworth´s Economic Journal article in 1904. which i.a. contains the following text:

> "Mr. J. A. Hobson´s criticism of this doctrine exemplifies the difficulty of treating the more abstract parts of Political Economy without the appropriate mathematical conceptions. An elementary discipline in the differential calculus would have corrected the following passage and its context: "In order to measure the productivity of the last dose of labor, let us remove it. The diminution of the total product may be 8 per cent. This 8 per cent, according to Marshall´s method, we ascribe to the last dose of labor. If now, restoring this dose of labor, we withdrew the last dose of capital, the reduction of the product might be 10 per cent. This 10 per cent is regarded as the product of the last dose of capital. Similarly, the withdrawal of the last dose of land might seem to reduce the product by 10 per cent. What would be the effect of a simultaneous withdrawal of the last dose of each factor? According to Marshall´s method, clearly 28 per cent. But is this correct?" *The Economics of Distribution*, p. 146. Quite correct, if in the spirit of the differential calculus we understand by dose an increment as small as possible, not as large as the objector pleases! He goes on: "Put the same experiment upon its broadest footing, and the overlapping fallacy becomes obvious. Take the labor, capital, and land as consisting of a single dose each; now withdraw the dose of labor, and the whole service of capital and land disappears. Is the destruction of the whole product a right measure of the productivity of the labor-dose alone?" (loc.cit., p. 47.) Imagine an analogous application of the differential calculus in physics, "put upon its broadest footing," an objector substituting x wherever a mathematician had used dx or Δx!"

Marshall and Edgeworth on the one side and Hobson on the other side talked about different phenomena. Apparently, it did not occur to Marshall and Edgeworth that the 'production function' could be endogenous in the sense that the technique preferred could switch with the environment e.g. the team of labourers (homogeneous or not).

Formally, the Marshall and Edgeworth-reasoning took for granted that output X followed from some function

$$X = F(K, l_1, l_2 ... l_H) = F(K, \sum_{1}^{H} l_h) \qquad \textbf{(1)}$$

l_h representing individual homogeneous labourers' contributions, i.e. $l_1 = l_2 = ... l_H$, cf. the example with the substitutable (additive) shepherds.

It is completely off the point when Edgeworth patronizes Hobson and teaches him simple differential calculus. Hobson's reasoning follows quite a different line. Formally, his argument could be presented in the following way:

Assume that there exists an *optimal production technology* for each input combination viz. so that

$$X^{(1)} = G_1 (K, l_1)$$

$$X^{(2)} = G_2 (K, l_2)$$

$$X^{(12)} = G_{12} (K, l_1, l_2) \qquad \textbf{(2)}$$

$$\vdots$$

$$X^{(1 \ldots H)} = G_{1 \ldots H} (K, l_1, l_2, \ldots l_H)$$

Then, of course, the marginal product of say l_2 cannot in the general case be uniquely identified by $X^{(12)} - X^{(1)}$; a competing estimate would for instance be $X^{(32)} - X^{(3)}$ etc. In short *the choice of technology is endogenous*[4] or in a different parlance, Hobson refutes the idea of the existence of a universal production function defined in some additive labour input.[5] [6]

As an extremely simple example of this phenomenon think of two identical persons who are not able alone to lift a stone; together they can - then who does the work? Hobson would say that they contributed by one half each.

We shall elaborate on a slightly more general example in the next section where even homogeneous workers are not additive in the time domain either.

Collaboration with externalities or interdependency between workers

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[4] During the capital controversy Joan Robinson refuted the existence of a unique (aggregate) production function by endogenizing the choice of technique.

[5] In short, Hobson did not accept a differentiable X-function in additive l_i's.

[6] Of course the general formulation $X=F(K, l_1,l_2,\ldots l_H)$ may be open to differentiation, viz.

$$\frac{\partial X}{\partial l_i} = F_{l_i}$$

This infinitesimal increase, however, normally can only be defined in the (labour) time dimension!

Let us look at two workers A and B who are equally capable with respect to catching rabbits. By a proper definition of units we can imagine that the two persons are able to catch one rabbit per time unit. Thus in figure 1 the connecting line from C to D traces the collective prey - as drawn 7 rabbits - if A and B use varying working hours. In C the working time of B is 7 and that of A zero etc.

At point E the prey is still 7 and both A and B are working on a half time basis[7], but not collaborating (autarchy). If, however, they collaborate and use a different hunting technique, by realizing the time inputs of E’ together they are still able to catch 7 rabbits. Both persons save time but get the same collective output as before.

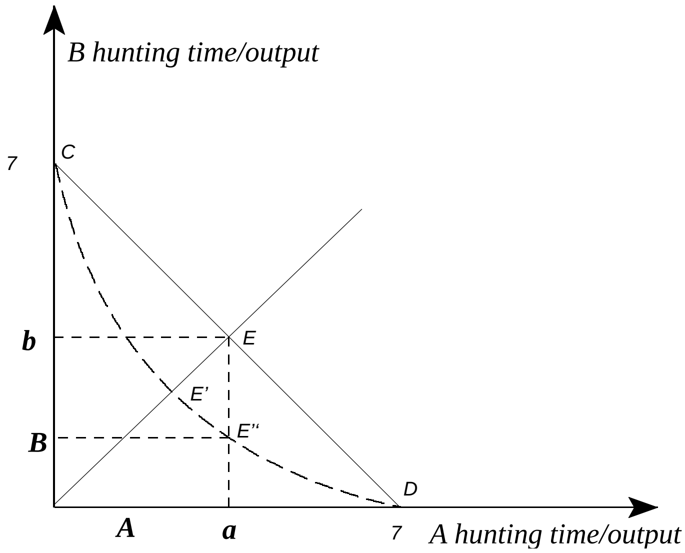
The dashed curved line (passing E') represents the time saving possibilities, brought about by collaborating some of the time. At the endpoints of the curve of course no collaboration takes place; where an origo-beam (45-degree) crosses the dashed line, they collaborate all the time (but for less than half a day at E’ as exemplified here).

If the line CE'D is uniformly closer than the CD-line to origo all the way, we shall characterize the phenomenon by the term *super-additivity.*

The methodological problem here is the following. Switch of technology (collaboration in some wide sense) makes it possible to realize the point E' and E", leading to the same (collective) catch. If A and B have decided to share only the collective output equally, the problem arises, who shall harvest the collaborative gains with respect working hours. In E" A gains nothing; B, however, can cut his working period by EE". Obviously, the problem is symmetric and the implication is that we end up in an *indeterminacy problem*; it is impossible to point at an apriori unique solution; the settlement of the input time distribution must be referred to a two-sided bargaining process.

What is worse from a methodological - and epistemological - point of view, is that the marginal product is indeterminate.

[7] In fact in this ‘additive’ world the point D could be interpreted as the outcome of two persons on a full time basis that have only half the ability of A, i.e. the input axes could be read as the input (A/n)*n*t where A represents the reference person and 1/n measures the fractional ability of substitutable persons; t is the working hours. If n=2, we thus claim that two persons of half the capability of A can substitute A i.e. x(A)=x(½A,½A)=x(½A+½A).



Interdependent hunters

To realize this, let us assume that a firm until now has hired A on a full time basis to catch 7 rabbits (point D). Now A wants to switch to half-time working hours and the company hires B on a half-time basis also, but only to find that output is now more than 7 rabbits, say 10, corresponding to the collaborating at point E (the reason why super-additivity seems to be a proper term for the phenomenon).

What is the marginal product of B: 6½ (10-3½ from A alone(**a**))? 3½ (**b**), i.e. the non-collaborative output potential or simply 5=10/2 as suggested by Hobson. This was the core problem, brought out by Hobson, but overruled by Marshall and Edgeworth.

Edgeworth claims that it is illegitimate to remove x instead of dx, in this case corresponding to removing e.g. B from the labour force. That, however, is not a valid objection because in general the labour force does not consist of 'many' individuals, each rendering only a 'marginal' contribution to labour input. Epistemologically, the rock-bottom problem remains as long as there exist interdependencies between labourers resulting in endogenization of technique. To exclude externalities the super-additivity possibility must be ruled out by postulating that CE'D coincides with CD or that workers are perfect substitutes (the elasticity of substitution is infinite). Mathematically, what happens is that the foundation of differentiability is not sustainable in the sense that

$$X'(L) = \lim_{h \to 0} \frac{X(L+h)}{h} \qquad \textbf{(3)}$$

tells what the output effect would be if one individual representing h (autarchic) labour units were added to the labour force; the production function/technology changes and the (+)-operator simply does not apply or in mathematical terms: the G-function is non-differentiable in L.

To elaborate further on the effect of positive externalities between two persons, collaborating in the same job let us postulate that together two equally able persons, are represented by the autarchic labour measures:

$$L_A = L_B = \overline{L} \qquad \textbf{(4)}$$

and further that the joint production of the output x follows from the CES-like function

$$X = (\alpha L_A)^\gamma + ((1-\alpha) L_B)^\gamma \qquad \textbf{(5)}$$

Here α and $(1-\alpha)$ represent the time shares by which A and B respectively render their (identical) services; as is easily verified in autarchy both persons will produce the same output, i.e. when the potentials are equal as assumed by (3).

γ is a parameter that must be less than one for the function to produce super-additivity; if γ is equal to one, time sharing by α has no effect, corresponding to simple additivity between the two separate inputs Y, i.e. $X'_\alpha = 0$; in a different parlance the elasticity of substitution between the two inputs in that case is infinite or finally we have a simple production function that is homogeneous of the first degree in (both) inputs (CRS).

If on the other hand γ is less than one, the elasticity of substitution is finite and we have a case of decreasing returns to scale in L's, but exhibiting super-additivity between its constituents, $X'_\alpha \neq 0$.

The construct, thus, has some bearing on the ordinary Cobb-Douglas function in K and L if the L-part of the CD-function is represented by our x from formula (4); decreasing returns to scale in L, but with the extension that the constitution of the aggregate L is open to mixing of different sources[8].

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[8] In fact, in the normal CD case simple additivity in L is implicitly assumed in the sense that $X = K^\alpha(\alpha\, \overline{L}_1 + (1-\alpha)\, \overline{L}_2)^\gamma$ for all $(0 \leq \alpha \leq 1)$, when $\overline{L}_1 = \overline{L}_2$.

Let us assume that $\bar{L}$ is equal to 100; γ=0.5 and further that a firm wants to produce the autarchic (γ=1) output of 10; then in engaging labour to produce this output it has the possibility of exploiting the mutual beneficial externality between A and B; the total wage bill is composed by

$$W=(\alpha L_A+(1-\alpha)L_B)w \qquad \textbf{(6)}$$

where w is the wage rate and W the total wage bill; then it is straightforward to verify that to minimize wage costs α should be set equal to 0.25 for both persons i.e. less than ½. For w=1 the autarchic wage bill would have been 100 monetary units; the optimal solution reduces this to 50 monetary units. Conversely, if the firm engages both persons on a half time basis, total output would be 14.14 > 10 as required.

In this illustration the firm cashes the positive externality effect; in a collective team situation the distribution of this benefit between A and B (and the firm for that matter) is indeterminate unless further assumptions are imposed[9].

Conclusion

It follows from the discussion above that the kernel of the Marshall-Hobson-dispute was the question of the existence of a measure of homogeneous (additive) labour - or not.

Apparently, the status of Professor Alfred Marshall was so high that his refusal of Hobson´s heresy bottled-up a fruitful criticism raised about one century ago! Nevertheless, Hobson deserves to be rediscovered to give access to a much richer perception and modelling of real phenomena exhibiting externalities. The naive homogeneous labour concept is not reconcilable with optimization of labour as input in a production process in general.

[9] The problem demonstrated here could be interpreted in a different setting; imagine that the iso-catch-curve of figure 1 were an iso-quant of a Cobb-Douglas-production function so that the two axis represented K and L respectively; then to define a marginal product of an aggregate of K and L, denoted e.g. I (input) would be as impossible as that of defining it on an aggregate L.

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