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IN A TWO-SECTOR MODEL:
KALDOR'S MATTIOLI LECTURES

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Growth and Stagnation in a Two-Sector Model:

Kaldor's Mattioli Lectures *

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Abstract
Kaldor's Mattioli lectures analyse a two-sector model with increasing returns to scale (IRS) in industry and decreasing returns (DRS) in agriculture. This review article shows that (i) with IRS in industry a long-run equilibrium growth path with strictly positive growth rates may exist even if agriculture is subject to DRS; (ii) the industrial sector is the `engine of growth' if agricultural investment is determined passively by available saving; (iii) if one introduces a separate agricultural investment function both positive and negative agricultural supply shocks may lead to stagnation, thus vindicating Kaldor's emphasis on commodity price stabilization.

JEL classification: O41

Key words: Kaldor, two-sector model, engine of growth, price stabilization

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Section 1: Introduction

The 1984 Mattioli lectures at Italy's Bocconi University were given by Nicholas Kaldor. After a 12 year delay these five lectures on "The Causes of Growth and Stagnation in the World Economy" have now been published by Cambridge University Press. The lectures, which are wide-ranging and very stimulating, address some of the central themes in Kaldor's work, and in addition to the lectures the volume includes comments from some of the participants as well as a biographical essay by Anthony Thirlwall and a bibliography compiled by Ferdinando Targetti.

The lectures touch on many issues but the dominant theme concerns the determination of world economic growth. The first two lectures, which discuss different theoretical approaches, prepare the ground, and the third lecture on sectoral balance gives a sketch of the two-sector model underlying Kaldor's own views. Lecture four looks at international differences in the levels and growth rates of income, and lecture five presents an interpretation of post-war developments and a set of policy recommendations.

The interaction between an industrial sector with increasing returns to scale and an agricultural sector with decreasing returns is central to Kaldor's position, and the analysis of the sectoral issues in lecture three is, I think, the most interesting part of the lectures. It is also, however, incomplete and in some ways unsatisfactory. The assumptions are described verbally, and the implications of the model are derived and illustrated graphically using figures that look deceptively simple. But the basic model is not set out formally and it is sometimes hard to follow the argument. Following a brief overview of the lectures in section 2, I shall therefore set up a simple two-sector model which incorporates Kaldor's main assumptions. Several formalisations of Kaldorian ideas have appeared over the last 12 years. The model draws on these existing formalisations but also includes a couple of Kaldorian elements that, to my knowledge, have been ignored by the literature.

Section 3 presents the model and analyses its implications for equilibrium growth. The analysis of equilibrium growth paths, however, may be a poor guide to the problems of the real world. One such problem - the instability of prices for primary commodities - may, Kaldor argues, affect not just primary good producers but also reduce the overall rate of growth in the world economy. Section 4 addresses this issue and relates the model to Kaldor's position on `engines of growth'. A final section summarises the main conclusions.

Section 2: The Mattioli lectures

Kaldor's opening lecture focuses on problems and limitations of Walrasian general equilibrium theory. Some of the detailed criticisms may be wide of the mark (for instance the claim that uniqueness of equilibrium is central to GE theory) but it is hard to disagree with the thrust of the argument. General equilibrium theory is essentially timeless even when extended to include dated commodities; it ignores disequilibrium behaviour, the role of professional intermediaries and quantity signals; and assumptions of constant preferences, a given number of goods and given technical knowledge provide a poor basis for understanding the dynamics of capitalist development.[1]

The second lecture gives an overview of different approaches to growth theory. Kaldor stresses one of the key points in his position since the late 1960s. It is a mistake, he argues, to assume that the labour supply constitutes a binding constraint on economic growth. The world economy and most, if not all, regional and national economies have large amounts of hidden unemployment. Moreover, a tendency to labour shortages in any particular location can be met

1 `New' growth theory, which took off a couple of years after Kaldor's lectures and which in some ways follows the methodology of general equilibrium theory, may seem to contradict this view; see Skott and Auerbach (1995) for a discussion of the connection between new growth theory and theories of cumulative causation and uneven development.

by migration, as indeed it has been on many occasions in the past.

This line of argument leads to a criticism of Keynesian one-sector models. The presence of hidden unemployment and induced technical progress implies, Kaldor argues, that there is no such thing as a Harrodian `natural' rate of growth determined independently of the demand for labour. Another problem with Harrod's analysis relates to Kaldor’s work from the 1950s on growth and distribution. The saving rate should not be seen as exogenously given but as dependent on the distribution of income. Kaldor no longer combines this determination of the saving rate with an assumption of full employment (as in the models from the 1950s) but both the importance of retained earnings as a source of finance and the connection between the share of investment in output and the profitability (and viability) of capitalist production remain valid, he argues.

Intersectoral balance is the theme of the third lecture which presents an outline of a two-sector model with agriculture and industry. The two sectors, which depend on each other both as markets for output and as suppliers of essential inputs, differ in a number of ways. Agriculture has hidden unemployment and produces consumption goods which are sold in competitive markets; industry produces investment goods and is characterized by imperfect competition and mark-up pricing. Agriculture is land-based and has decreasing returns to labour and capital; industry uses only labour and capital and has increasing returns to scale.

The presentation of the model is followed by a description of some possible extensions and an analysis of the effects of agricultural supply shocks. Even favourable shocks in the form of an increase in agricultural supply can, it is argued, lead to a slump, and the analysis leads to the advocacy of an international buffer-stock mechanism to stabilize the prices of primary commodities.

Spatial aspects and international differences in growth performance are the topics of the fourth lecture. Unlike land-based activities in the primary sector, industry and services tend to cluster geographically. External scale and agglomeration economies are the obvious explanations for this phenomenon, but scale economies of this kind are excluded by both neoclassical trade theory and the Ricardian theory of comparative advantage. This exclusion forms the basis for Kaldor's criticism of traditional trade theory. As in the case of the criticisms of general equilibrium theory in lecture 1, some of the detailed arguments presented by Kaldor seem less than fully convincing, and trade theory has undergone significant change since 1984. The recent changes, however, confirm Kaldor's position: free trade may cause a polarization of incomes rather than factor price equalization in the presence of scale economies. Moderate and selective protection, Kaldor argues, may be essential to address this problem and to get industrialization started; high and indiscriminate protection on the other hand can foster inefficiency and prevent a country from breaking into world markets.

The lecture closes with a brief discussion of the foreign trade multiplier. If the balance of payments constraint is binding and the terms of trade remain roughly constant, the growth rate of an economy can be determined by the growth of its exports divided by the income elasticity of demand for imports. Empirically, this relation, sometimes referred to as `Thirlwall's law', gives a good fit but the key question then concerns the determinants of an economy's export growth and income elasticity of import demand. Kaldor's answer in the lectures is the "innovative ability and the adaptive capacity of its manufacturers" (p.69). This may be true, but unfortunately does not take us much further, and the overall analysis of cumulative processes in the lecture adds little to Kaldor's earlier writings (e.g. Kaldor (1970)).

The fifth and final lecture discusses policy implications and offers an interpretation of postwar developments. Strong growth of manufacturing demand was, Kaldor argues, the key factor behind the `golden age' that lasted until about 1973. This growth in demand was fuelled by the outflow of US dollars under the Bretton Woods system in combination with rapidly increasing world trade and, in some countries, active expansionary policy. The rough stability in the terms of trade between primary commodities and industrial goods, a stability made possible partly by active price support and buffer stock programmes, acted as an important permissive factor.

The end of the golden age came when inflationary pressures built up in the late 1960s, partly at least under the influence of wider social developments like May 1968 in France. These developments were followed by the breakdown of the Bretton Woods system and a dramatic rise in commodity prices in general and oil prices in particular. The OPEC surplus created a demand deficiency in the mid 1970s as did disastrous monetarist policies from the late 1970s. The policies of Mrs. Thatcher in Britain, in particular, come in for scathing attack. North Sea oil, Kaldor argues, freed Britain from balance of payments constraints and provided a unique opportunity for Britain to pursue a policy of expansion. Instead, however, deflationary policies produced a large balance of payments surplus and a huge increase in domestic unemployment while, internationally, the British policies severely aggravated the balance of payments for other European countries and deepened the recession in Europe.

Kaldor's programme for recovery - designed for the situation in 1984 but sadly relevant for Europe in the late 1990s - has a coordinated fiscal expansion as an important element. This expansion, Kaldor argues, should be combined with balance of payments targets and, if necessary to avoid balance of payments problems, measures to regulate international trade. Low interest rates and international buffer stocks to reduce the volatility of commodity prices make up two other elements of the package, leaving labour markets and bargaining structure as a final problem. Kaldor's analysis of wage formation anticipates later work on corporatism and bargaining structure. Systems of "sectional collective bargaining" (that is, intermediate levels of centralization) tend to have strong inflationary tendencies, Kaldor argues. A centralized system with continuous consultation between workers, management and government may overcome this problem, but Kaldor acknowledges that the creation of this kind of system is a difficult task.

The discussion that follows the lectures has interesting contributions from a number of Italian economists covering issues ranging from Kondratieff cycles and the limitations of balanced growth models (Lombardini, Filippini and Pasinetti) to the need for monetary reform (Sylos Labini), and the bibliography and Thirlwall's biographical essay round off the volume nicely.

Section 3: Equilibrium growth in a two-sector model

Kaldor's verbal analysis usually seems persuasive but, as pointed out by Pasinetti, his ideas and thoughts "often go beyond even the very formal model he is trying to present" (p.103). In the Mattioli lectures his analysis of the constraints on world economic growth and the engine of growth raises several questions.

At an overall level it is difficult to reconcile his view of manufacturing as the engine of growth with a model in which industrial employment is determined by the agricultural surplus. Yet, on p. 40 we are told that "the manufacturing sector provides the true dynamic element - the fundamental `engine of growth' of an economy" while on p. 43 "the total amount of corn sold by the agriculturalists determines the total amount of employment" in industry. A distinction between the analysis of uneven development across countries and the determination of the path of the world economy as a whole could offer a possible reconciliation of these statements. Kaldor (1979, p. 290) makes this kind of argument, suggesting that primary production determines world economic growth while industrialization may be critical for the performance of an individual country. This interpretation, however, finds little support in the Mattioli lectures.[2]

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2 One may note that in a later paper Kaldor (1986a) plays down the constraints from primary products. He argues that with price stabilization "it is highly probable that the long-term rate of growth of output of primary commodities would be sufficiently enhanced to equal or to exceed the requirements arising from any *feasible* rate

Another critical question is the determination of the rate of land-saving technical progress in agriculture. This type of technical progress is, Kaldor argues, a precondition for long-term economic growth, but the causes of land-saving technical progress in Kaldor's model are not entirely clear. There is some ambiguity, in particular, on whether the rate of progress is exogenously given or determined by the requirements of the industrial sector. On p. 47 Kaldor argues that “the critical factor in continued economic growth is the persistence or continuance of *land-saving innovations*”, and there is nothing in the presentation of the model to suggest that the rate of innovation is itself endogenous. In lecture 2, however, both land- and labour-saving technical progress are considered endogenous: “Necessity is the mother of invention - as the proverb goes. Many of the momentous technological changes occurred in response to need created by scarcities. The greatly increased scarcity of wood in 18th-century Europe due to rapid deforestation, partly caused by the growth of ship-building, led to the invention of producing coke out of coal ... Such examples could be multiplied almost ad infinitum.” (p.25)

The role of increasing returns in industry raises additional questions. Can the presence of increasing returns in industry offset decreasing returns in agriculture? What does an equilibrium growth path look like in an economy with decreasing returns in agriculture and increasing returns in industry? And what is the link between the formal two-sector model and Kaldor's stress on the negative effects of unstable commodity prices?

Kaldor addresses these questions in the Mattioli lectures but the lack of formalisation means that the analysis lacks precision at key points. The lack of formalisation may reflect the fact that the model was still "on the drawing board" (Kaldor (1978, pxxii)). Alternatively, it could be a reflection of Kaldor’s well-known skepticism with respect to the use of mathematics, a

of growth of industrial output" (p. 195) and the physical limits on growth "have continued to be set by the availabilities of labour in the advanced industrial countries" (p.197). This conclusion seems at odds with both the Mattioli lectures and most of Kaldor's other writings after the mid 1960s.

skepticism which is also raised explicitly at several points in these lectures. The problems and ambiguities that beset the analysis in this lecture, however, demonstrate that sometimes formal techniques can be very useful, and the rest of this section will be devoted to a formalisation of Kaldor's verbal argument.

3.1 A formal model

A Kaldorian specification of technology is relatively straightforward. There is decreasing returns to capital and labour in agriculture while industry is subject to increasing returns.

A simple specification of the agricultural production function is given by

$$A = T\, K_A^{\alpha}\, L_A^{\beta} \;\; ; \;\; \alpha+\beta \le 1 \tag{1}$$

where A, K_A, and L_A denote the agricultural output, capital stock and labour input, respectively, and where the productivity parameter T includes the productive input of land. For the time being T is assumed constant but in section 4 agricultural supply shocks will be modelled as shifts in the value of T.[3]

Agricultural producers, Kaldor argues, face atomistic competition and take prices as given. Profit maximisation then implies that

$$p_A\, \beta\, A/L_A = w_A \tag{2}$$

where p_A and w_A are the price of output and the (nominal) wage in agriculture.

Following most other formalisations of Kaldor's model, I shall assume that the product real

3 A similar specification is used by Thirlwall (1986). Thirlwall, however, assumes that the productivity parameter T is determined endogenously as an increasing function of the capital stock. Integrating his technical progress function and leaving out exogenous technical progress his analysis implies that

$$T = K_A^{\gamma} \;\; ; \;\; 0 \le \gamma < 1-\alpha-\beta \tag{*}$$

This extension, which Thirlwall interprets as the effect of capital accumulation on land-saving technical progress, does not affect the analysis. Substituting (*) into (1) yields a new but qualitatively identical Cobb-Douglas production function: there is still decreasing returns to scale, and the only difference compared to (1) is a re-labelling of the parameters with $\alpha+\gamma$ now taking the place of α in equation (1).

wage (ω_A) is constant in agriculture.[4] Substituting a constant product real wage into (2), we get

$$L_A = (\beta T K_A^{\alpha+\gamma}/\omega_A)^{1/(1-\beta)} \tag{3}$$

and, using (1), the agricultural output is given by

$$A = \mu\, K_A^{\rho} \quad ; \rho \leq 1 \tag{4}$$

where $\mu = T^{1/(1-\beta)}(\beta/\omega_A)^{\beta/(1-\beta)}$ and $\rho = \alpha/(1-\beta)$. The wage share in agriculture is constant and equal to β. Thus, if Π_A is the sum of agricultural rent and profit we have

$$\Pi_A = (1-\beta)\, p_A\, A \tag{5}$$

Equations identical or very similar to (4)-(5) appear in several formalisations of Kaldor's model (although usually it is assumed that $\rho=1$) and the two equations can be derived in a number of ways using different underlying models.[5] It should be noted that $\rho<1$ corresponds to the case of decreasing returns to capital and labour.

Turning to industry, I assume a modified Leontief technology and disregard labour hoarding. Thus, if M is output and q_M denotes labour productivity, industrial employment L_M is

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4 Molana and Vines (1989) justify this approach by assuming that agricultural producers can draw on hidden unemployment from a self-contained, low-income, subsistence sector. Targetti (1985), Thirlwall (1986) and Dutt (1992) also assume a constant marginal product of labour in agriculture and get equations similar to equation (4) below. They take the marginal product to be zero, however, and thus implicitly assume a different agricultural production function (the marginal product in (1) is strictly positive for all finite levels of employment).

Skott and Larudee (1997) take a different approach in their analysis of a closed economy with perfect sectoral mobility of labour. They assume that agricultural employment is determined as the residual, that agriculture is `traditional' and that the endogenously determined agricultural `wage' is equal to the value of the value of the average product.

5 Kaldor stresses the importance of land-saving as opposed to labour-saving technical progress. With a Cobb-Douglas formulation there is no distinction between these two types of technical change. As an alternative to equation (1), however, the production function can be of a modified Leontief type. Possible specifications could be:

$$A = \min \{T\, K_A^{\alpha}, L_A\} \tag{*}$$

or

$$A = \min \{J, L_A\} \quad ; \quad \hat{J} = h(\hat{K}_A) \tag{**}$$

The variable T in (*) represents the amount of land while J in (**) can be interpreted as land in efficiency units, land-saving technical change being determined by a technical progress function: the growth rate of efficiency land ($\hat{J}$) is an increasing function of the growth rate of the capital stock ($\hat{K}_A$).

Both (*) and (**) assume a strict complementarity between labour and the combined input of capital and land. Retaining the assumption of a fixed product real wage ω_A, equation (4) can be derived from (*) or (**) if $\omega_A<1$ (if $\omega_A>1$ unit labour cost exceeds the price p_A and A=0). The wage share in this case is ω_A and in order to get equation (5) the parameter β should be equal to ω_A.

given by

$$L_M = M/q_M \qquad (6)$$

The maximum output-capital ratio is assumed constant but labour productivity changes over time, and I shall use the following simple relation

$$\hat{q}_M = f(\hat{K}_M) \;\; ; \;\; 0 \leq f' \qquad (7)$$

where carets are used to denote growth rates, $\hat{q}_M = dq_M/dt\ 1/q_M$. One of the persistent themes in Kaldor's writings since the late 1950s has been the presence of dynamic increasing returns to scale, and this is captured by equation (7). If the actual output-capital ratio is constant, the rate of growth of output will be equal to the accumulation rate $\hat{K}_M$, and the equation is identical to Verdoorn's law; with a constant rate of employment it describes Kaldor's 1957 technical progress function.[6]

In accordance with Kaldor's description of the asymmetry between agricultural and industrial pricing one may assume that the agricultural price p_A adjusts freely to clear the market for agricultural goods. Industrial goods, on the other hand, are subject to a constant mark-up on variable cost and hence - given the specification of technology - a constant profit share π_M

$$(p_M M - w_M L_M)/(p_M M) = \pi_M = \pi_M^* \qquad (8)$$

Investment decisions are based on expectations of future demand, and along the equilibrium growth path these expectations will be fulfilled. This definition of a long-run equilibrium growth path corresponds to Harrod's `warranted growth'. It is, in modern parlance, a growth path with rational expectations. With fulfilled expectations, the actual level of capacity

6 As is well known, Kaldor's specification of the technical progress function as

$$\hat{q}_M = f(\hat{K}_M - \hat{L}_M)$$

is mathematically equivalent to a Cobb-Douglas production function if f(.) is linear. But unlike equation (7) this mathematical specification fails to capture Kaldor's verbal argument if employment is determined endogenously: Kaldor's specification implies that reductions in the level of employment have the same effect on productivity as increases in the capital stock; see Skott (1989, chapter 7) for further discussion of this issue.

utilisation in industry must be equal to the level that firms consider optimal; if, say, utilisation were above the optimal level, firms would increase the rate accumulation in order to reach the optimal level. The rate of accumulation, in other words, can only be constant if

$$M/K_M = u = u^* \tag{9}$$

where u* denotes the value of the output-capital ratio when utilisation is at the desired level.[7]

Equation (9) may not look like it, but in fact it represents the equilibrium-growth investment function. The equation implies that the equilibrium rate of accumulation in industry becomes perfectly elastic at u=u*. A simple disequilibrium specification of an investment function with this long-run property is

$$dg_M/dt = dg_M^*/dt + \lambda\,(u - u^*) \tag{10}$$

where $g_M=\hat{K}_M$ and $g_M^*=\hat{K}_M$ denote the actual and the equilibrium rates of accumulation. Equation (10), which says that accumulation rates will be increasing relative to the equilibrium rate whenever actual utilisation exceeds the desired level, captures the claim that a lasting discrepancy between actual and desired utilisation rates will lead to changes in the rate of accumulation.

In agriculture there is atomistic competition and price taking. The capital stock will therefore always be fully utilised. But if there is capital mobility and free entry into the competitive A-sector, it seems reasonable to assume that capital will flow into (out of) agriculture when the profit rate exceeds (falls short of) the risk-adjusted real rate of interest and that in long-run equilibrium the two rates should be equal. The steady growth condition therefore becomes

$$r_A = r^* \tag{11}$$

where r* denotes the risk-adjusted real rate of interest and r_A is the agricultural rate of profit (net

7 For present purposes the desired rate of utilization may be taken as exogenously given. This, however, is merely a simplifying assumption. The crucial point is that the utilization rate should not be treated as an accommodating variable in the analysis of long-run equilibrium growth (see Committeri (1986), Skott and Auerbach (1988) and Skott (1989a) for further discussion; Amadeo (1986), Dutt (1990) and Lavoie (1995) present an opposing view).

of rent, R),

$$r_A = (\Pi_A\text{-}R)/(p_M K_A) = \alpha p_A A/(p_M K_A) = (\alpha/(1\text{-}\beta))\Pi_A/(p_M K_A) \tag{12}$$

Note that since, by assumption, the industrial sector is imperfectly competitive and may be subject to barriers to entry, there is no need for profit-rate equalisation; the industrial profit rate may exceed the agricultural profit rate.

It would be tempting to follow Kaldor's specification of the sectoral demand structure and assume that the agricultural good is a pure consumption good and the industrial good a pure investment good. But as shown below (see footnote 17) this specification would fail to capture an important aspect of Kaldor's argument in favour of price stabilization for primary commodities. I shall assume therefore that consumption is directed towards both A and M goods and, furthermore, that the two goods are complements: the elasticity of substitution between the two goods is bounded below 1. To simplify the exposition all agents choose the same composition of consumption (there are no income effects; utility functions are homothetic) and the agricultural good is a pure consumption good.

With these assumptions, the equilibrium conditions for the two sectors can be written

$$p_A\, A = \phi(p)\, [w_A\, L_A + (1\text{-}s_A)\, \Pi_A + w_M\, L_M + (1\text{-}s_M)\, \Pi_M] \tag{13}$$

$$p_M M = p_M g_M K_M + p_M g_A K_A + (1\text{-}\phi(p))[w_A L_A + (1\text{-}s_A)\Pi_A + w_M L_M + (1\text{-}s_M)\Pi_M] \tag{14}$$

where $p=p_A/p_M$ is the agricultural terms of trade, s_A and s_M are the saving rates out of agricultural and industrial profits (there is no saving out of wage income), and $\phi(p)$ is the share of agricultural goods in the industrial sector's total expenditure on consumption. Complementarity between the two goods implies that $\phi'(p)>0$.

3.2 Equilibrium

Using equations (4), (5), (8), (9) and (11)-(14) it can be shown (see Appendix A) that if there is decreasing returns in agriculture (that is, $\rho<1$) the equilibrium growth path has the following properties:[8]

$$k(t) = K_M(t)/K_A(t) = k(p(t)) \;\; ; \;\; k'<0 \tag{15a}$$

$$p(t) = p(K_A(t)) \;\; ; \;\; p'>0 \tag{15b}$$

$$K_A(t) \rightarrow \infty \text{ for } t \rightarrow \infty \tag{15c}$$

$$p(t) \rightarrow \infty \text{ for } t \rightarrow \infty \tag{15d}$$

$$k(t) \rightarrow k^* = s_A(1-\beta)r^* / [\alpha(1-s_M\pi_M^*)u^*] \quad \text{for } t \rightarrow \infty \tag{15e}$$

$$\hat{K}_M(t) \rightarrow g^* = u^*s_A((1-\beta)/\alpha)r^* / [s_A((1-\beta)/\alpha)r^* + (1-s_M\pi_M^*)u^*] \quad \text{for } t \rightarrow \infty \tag{15f}$$

Equations (15a)-(15b) say that ratio of the capital stocks, K_M/K_A, is an decreasing function of the agricultural terms of trade, and that the terms of trade increases as a function of the agricultural capital stock. The four remaining equations describe the asymptotic outcome as t goes to infinity: both the agricultural capital stock and the agricultural terms of trade go to infinity ((15c)-(15d)); the ratio of the two capital stocks converges to a finite and positive constant ((15e)), and the accumulation rate is constant asymptotically ((15f)).

In the derivation of these results no use is made of the equations (6)-(7) that describe the evolution of labour productivity in the industrial sector. Moreover, the value of ρ - the degree of returns to scale in agriculture - does not appear in the expression for the asymptotic values of the growth rate and the composition of the capital stock (and does not influence the equilibrium values of k(t) and g(t) for finite values of t either). It might appear, therefore, that as long as the output capital ratio in industry is kept constant the returns to scale in the two sectors are of no importance for the existence and properties of the equilibrium growth path. This conclusion is not correct.

The model is intended for an analysis of the world economy with industrial and agricultural

8 The case where $\rho=1$ implies that both p and k will be constant along the equilibrium growth path.

goods produced in the North and South, respectively. Migration between South and North is limited, and no restrictions have been imposed, therefore, on the ratio of real wages in the two sectors. Even in the absence of labour mobility, however, the consumption real wage in the industrial sector must remain above some minimum level for the system to be viable,[9] and this is where the production technology in both sectors becomes critical.

By assumption the two goods are complements with a less-than-unit elasticity of substitution, and the constraint on the consumption real wage therefore implies that both w_M/p_M and w_M/p_A must be bounded above some strictly positive values $(w_M/p_M)^{min}$ and $(w_M/p_A)^{min}$, respectively. Using equation (8) we get

$$w_M/p_M = q_M\,(1-\pi_M^*) \tag{16}$$

and since q_M is increasing the asymptotic behaviour of w_M/p_M causes no problems.

Dividing both sides of (16) by p_A/p_M and using equations (11)-(12) and (4), the real wage in terms of agricultural goods can be written

$$w_M/p_A = q_M\,(1-\pi_M^*)\,\alpha\,\mu\,K_A^{\rho-1}/r^* \tag{17}$$

For w_M/p_A to be bounded above $(w_M/p_A)^{min}$ the asymptotic growth rate of $q_M K_A^{\rho-1}$ must be non-negative. Algebraically, this condition becomes

$$\lim\,[\hat{q}_M(t) - (1-\rho)\,\hat{K}_A(t)] = f(g^*) - (1-\rho)\,g^* \geq 0 \tag{18}$$

where the common asymptotic growth rate g^* of K_A and K_M is determined by (15f). Equation (18) shows that in the presence of decreasing returns in agriculture (that is, $\rho<1$) it is essential that the industrial sector be subject to increasing returns. Moreover, the degree of increasing returns must be sufficient to offset the decreasing returns in agriculture. If, for instance, the technical progress function is linear, $f(g)=\theta g$, the viability condition reduces to the simple expression $\theta \geq 1-\rho$. In this

9 A constant product real wage in agriculture has already been embodied in the specification of equations (3)-(5) and since $p=p_A/p_M \rightarrow \infty$ the consumption real wage will be increasing asymptotically and exceed the product real from some t onwards.

simple case the returns to scale in industry and agriculture are $1+\theta$ and ρ, respectively, and the condition says that the `average returns to scale' must be greater than or equal to one.

3.3 Passive agricultural investment

The above specification of the model has included an independent investment function for the agricultural sector. It could be argued that agriculture is different from industry in this respect and that agricultural investment is constrained by the availability of agricultural saving.[10] Passive agricultural investment of this kind implies the replacement of (11) by the equation

$$p_M\, I_A = s_A\, \Pi_A \qquad (11')$$

In this case there is no capital mobility between the sectors and the equilibrium condition $p_M I_M = S_M = s_M \Pi_M$ must hold for the industrial sector. Hence, equilibrium growth requires that

$$\hat{K}_M(t) = s_M\, r_M = s_M\, u^*\, \pi_M^* = g^* \text{ for all t} \qquad (19)$$

The growth rate of the agricultural capital stock then is given by (see Appendix B)

$$\hat{K}_A(t) = k(t)\, [\phi(p(t))(1-s_M\pi_M^*)u^*s_A(1-\beta)]/[1-\phi(t)(1-s_A(1-\beta))] \qquad (20)$$

Asymptotically for $t\to\infty$ the price ratio p(t) goes to infinity and - since the substitution elasticity is bounded below one - the expenditure share $\phi(p(t))$ converges to one. Using (19)-(20) these asymptotic results imply that (see Appendix B)

$$k(t) \to s_M\pi_M^*/(1-s_M\pi_M^*) \text{ for } t\to\infty \qquad (21a)$$

$$\hat{K}_A(t) \to g^* \quad \text{for } t\to\infty \qquad (21b)$$

Thus, the equilibrium growth rate and the asymptotic composition of the capital stock are determined entirely by the industrial sector in this case.

Section 4: Sectoral balance, engines of growth and the benefits of price stabilization

10 Kaldor makes this argument on p.43 in the Mattioli lectures.

In the Mattioli lectures Kaldor presents the model using a diagram in which the growth rates in agriculture and industry are depicted as functions of the terms of trade, the intersection between the two curves determining the equilibrium. The shapes and positions of the two curves are constant over time if there are constant returns to scale in both sectors and no technical progress. Kaldor attempts to deal with the interesting cases of non-constant returns and technical progress by shifting the curves but, as pointed out be the editors of the Mattioli-volume, this analysis is unsatisfactory.

The formalisation in section 3 shows that a Kaldorian model with non-constant returns to scale is capable of positive rates of long-run equilibrium growth and that asymptotically the capital stocks grow at the same rate in the two sectors. Differences in the returns to scale imply that the rates of output growth will be different (industry having the higher growth rate) and that the agricultural terms of trade and the real wage in the industrial sector will be steadily increasing.[11] These results are similar to those obtained by Canning (1988) in a different set-up, but most other formalisations of Kaldor's theory impose constant returns to scale in industry in the formal analysis.[12]

The search for the `engine of growth' is a recurrent theme in Kaldor's writing. The Kaldorian model in section 3 implies that when there is an independent investment function for the agricultural sector neither sector can be regarded as the sole engine of growth: the equilibrium growth rate in (15f) is determined by behavioural parameters relating to both sectors. An increase

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11 An increasing value of the agricultural terms of trade may seem suspect empirically. A respecification in which the consumption shares satisfy Engel's law could neutralize (or reverse) the trend in the terms of trade but would complicate the model significantly.

12 This is the case, for instance, in Targetti (1985), Thirlwall (1986), Molana and Vines (1989) and Dutt (1992).

in the saving rate in either sector, for instance, will raise the growth rate.[13]

Passive agricultural investment leads to a different result. Provided the viability condition is satisfied, the industrial sector can be considered the engine of growth in this case:[14] the growth rate, which is given by the simple Kaldor-Pasinetti equation $g=s_M u^* \pi_M^*$, is determined exclusively by industrial-sector parameters. Note that the growth equation should be read from right to left with pricing (π_M^*), investment (u^*) and saving behaviour (s_M) determining the equilibrium rate of growth. Thus, as before, an increase in the saving rate raises the equilibrium growth rate.

Passive agricultural investment does not change the fact that primary production may impose a limit on the rate of growth, and this is where the need for `land-saving innovations' comes in. The limit appears in the model through the viability constraint (18). However, equation (18) shows that if capital is a substitute for land (or if capital accumulation causes land-saving technical change, cf. n. 5), the viability constraint cannot be expressed exclusively in terms of agricultural technology. The effective primary-good constraint also depends on industrial technology and on the interaction between the two sectors.[15]

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13 This Harrodian result may seem puzzling from a Kaleckian `stagnationist' perspective: there is no `paradox of thrift' in this model. The specification of the investment function accounts for these contrasting results. Stagnationist models assume that investment is relatively insensitive to variations in utilization both in the short and the long run. Harrodian models accept short run insensitivity (which is required for the stability of short run equilibrium) but assume that investment responds strongly to long-run deviations of utilization from the desired level. From this Harrodian perspective the basic flaw in the stagnationist story is the assumption that the short run relation between contemporaneous values of the rates of utilization and accumulation carries over to the long run, that is, that there are no lagged effects of past utilization on accumulation.

Harrodian models of this kind tend to reproduce Harrod's instability result: the warranted growth path is likely to be locally asymptotically unstable. Local instability, however, need not imply unbounded divergence. Instead, non-linearities - in combination with local instability - may produce a pattern of fluctuations around the warranted path. This is the case, for instance, in Skott (1989a).

14 There is a similarity here with Findlay's (1980) model. Unlike Findlay, however, the Kaldorian model has an independent investment function for the industrial sector and there is no assumption of full employment in the North. The growth rate therefore is not pinned down by exogenous technical progress and the growth rate of the labour supply in the North.

15 Without the substitutability between capital and land, an exogenous rate of land-saving technical progress may be a binding constraint on long-term growth. Skott and Larudee (1997) analyse a model of this kind with increasing returns in industry and a demand structure that satisfies Engel's law. Thirlwall (1986) and Molana and

The analysis of the long-run equilibrium growth path has ignored so far the implications of fluctuations in the output of primary commodities. Fluctuations of this kind which may arise from the effects of the weather on the size of the harvest or from unexpected discoveries of new primary resources, have implications for economic growth. One of the central conclusions in the Mattioli lectures (and in Kaldor (1976)) is that both positive and negative supply shocks in the primary sector can have negative effects, but for different reasons.

A negative output shock, Kaldor argues, will tend to increase the agricultural terms of trade and put downward pressure on the industrial real wage. Workers respond by higher nominal wage demands which feed into rising prices of industrial goods. Inflation, in turn, causes policy-makers to adopt contractionary demand policies, and these policies affect industrial production, the rate of capacity utilization drops and investment is depressed. In the Mattioli lectures this mechanism is discussed briefly on p. 89 and in a reply to Lombardini on pp. 119-20. Kaldor (1976) presents a more detailed analysis, and Kanbur and Vines (1986) formalise the argument.

That negative supply shocks and contractionary policy may harm growth is probably not surprising and, like Kaldor in the Mattioli lectures, I shall largely ignore this aspect of the argument. More interesting are the paradoxical effects of a positive agricultural output shock. Far from stimulating economic growth, Kaldor argues, a positive shock has final effects on industrial utilization and growth that are similar to those of a negative shock, but the mechanism is very different. The increased agricultural supply will lead to a deterioration of the agricultural terms of trade, a decline in agricultural incomes and reduced agricultural demand for industrial goods. In a sector with mark-up pricing and output adjustment this reduction in demand causes a

Vines (1989) also get land as a binding constraint when they assume that the available land grows at a fixed rate. They assume substitutability between capital and land, however, and in their models the constraint could also be overcome by introducing increasing returns in industry.

contraction in output and, as the utilization rate of capital drops, a decline in the rate of accumulation. (This scenario is described on pp. 50-54 in the Mattioli lectures.)

In terms of the model, shocks to agricultural output can be represented as changes in the parameter T in equation (1) and hence in μ in equation (4). Does an increase in μ have the negative effects described by Kaldor? In the case with passive agricultural investment the answer is no. Fluctuations in μ are reflected in these agricultural terms of trade, but the fluctuations in the terms of trade serve to offset any influence of agricultural output on the demand for industrial goods. This can be shown formally.

By assumption the relative price p adjusts so as to clear the market for agricultural goods, and substituting (13) into (14) the market clearing condition for industrial goods can be written

$$S_M = u\, p_M s_M \pi_M^* K_M = p_M g_M K_M + (p_M I_A - S_A) = p_M I_M + (p_M I_A - S_A) \qquad (22)$$

With passive agricultural investment we have $S_A = p_M I_A$ and (22) reduces to

$$u\, s_M \pi_M^* = g_M \qquad (23)$$

Equation (23) describes a short-run condition for market clearing. It assumes mark-up pricing ($\pi_M = \pi_M^*$) but does not require that $u=u^*$. Along the equilibrium growth path we have $u=u^*$ (and (23) becomes identical to (19)) but in the short run the rate of accumulation is the independent variable determining actual utilization (cf. equation (10)).

Neither μ nor the terms of trade have any effects on the rate of accumulation in the industrial sector. The short-run equilibrium condition (23) and the general investment function (10) form a self-contained dynamic system: substituting (23) into (10) yields an autonomous, one-dimensional differential equation (with a Harrodian unstable equilibrium). This conclusion makes perfect intuitive sense. A passive agricultural sector which spends all that it earns cannot be a source of fluctuations in autonomous demand. It is for this reason that section 3 focussed mainly on the case with a separate agricultural investment function. Kaldor's demand-based argument for the growth reducing effects of positive agricultural supply shocks is inconsistent with his assumption (on p. 43) of passive investment.[16]

In the case of active investment, agricultural supply shocks may affect industrial activity through their influence on $p_M I_A - S_A$. The formal analysis is somewhat complicated but the derivations are given in the Appendix C while Appendix D analyses the stability of the short-run equilibrium (stability is important since quantity adjustment in the industrial sector and price adjustment for agricultural goods must bring the economy to a position of short run equilibrium if the analysis is to be meaningful). The results in the two appendices can be given a simple intuitive interpretation.

Industrial investment can be taken as predetermined in the short run (equation (10)), and as shown by (22) the utilization rate is an increasing function of $p_M I_A - S_A$. A positive supply shock, an increase in μ, causes a decline in the agricultural terms of trade and with an elasticity of substitution of less than one, this decline is translated into a lower profit rate, r_A and hence a decline in agricultural accumulation.[17] The outcome for the industrial sector depends on whether agricultural investment responds more or less than saving to the decline in profitability. Kaldor's argument - "steel producers will find that their sales are restricted by `effective demand'" (p. 51; see also Kaldor 1976, p. 218) - is based on the view that investment will decrease more than saving.[18]

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16 Dutt (1992) makes the same point in his criticism of Thirlwall (1986).

17 The decline in the profit rate is caused by a fall in the agricultural share of total consumption expenditure. If the industrial good is a pure investment good then, by assumption, all consumption is directed towards agriculture, and the industrial growth rate becomes invariant with respect to changes in agricultural productivity. Algebraically, this result follows from equation (C8) by setting $\phi(p)=1$ and $\phi'=0$.

18 Destabilizing speculation in the commodity markets is cited by Kaldor as an important factor behind this `excess sensitivity' of investment. The argument is challenged by Tabellini in the discussion following the lectures, but in order to examine this aspect of Kaldor's theory one would need to include the activity of speculators explicitly in the model, something that is beyond the present paper.

It may be justified in many cases to assume that agricultural investment reacts more strongly than saving to fluctuations in profitability but Kaldor himself reverses the `excess sensitivity' assumption in the case of oil: the OPEC price increase caused a drain on aggregate demand precisely because the OPEC members failed to increase their spending in line with the rise in oil revenues. A distinction could be made, perhaps, between broad-based changes in the general level of commodity prices and large variations in the price of a commodity that is geographically concentrated and that dominates the economy of relatively high-income countries. Nevertheless, the oil example shows that excess sensitivity of investment in the agricultural sector is an empirical hypothesis that does not always hold.

It should be noted, finally, that it is the interaction between the degree of substitution of the two goods in consumption and the relative sensitivities of investment and saving that produce the effects of agricultural supply shocks on industrial growth. Following Kaldor it has been assumed that agricultural and industrial goods are complements. If one reverses this assumption, a positive supply shock will raise rather than reduce the agricultural profit rate, and in this case a high sensitivity of investment to variations in profitability will stimulate industrial demand and growth while a low sensitivity of investment (relative to saving) implies that positive supply shocks depress the demand for industrial goods.

Section 5: Concluding comments

The Mattioli lectures illustrate both the strengths and weaknesses of Kaldor's post-1966 work on economic growth. The lectures identify and analyse important real problems in a way that is often provocative and always insightful. But the logical structure of the sectoral and distributional interactions is sometimes so complex that most people will need a formal model to check the logical consistency of the argument. Kaldor provides a suggestive sketch of a model but it is merely a sketch. It fails to capture the richness of his verbal argument, and his presentation and analysis of the model leaves many questions unanswered.

The formalisation in this paper largely supports Kaldor's argument but it also points to inconsistencies and hidden assumptions that need to be satisfied in order to generate Kaldor's conclusions. Thus, it turned out that the presence of decreasing returns to scale in agriculture is compatible with positive long-run growth but only if the industrial sector has increasing returns to scale and the average returns to scale, loosely speaking, are non-decreasing. This result might seem to suggest that the industrial sector is the engine of growth. This conclusion, however, is only warranted if agricultural investment is determined passively by the available agricultural saving. Furthermore, with passive agricultural investment, supply shocks in the agricultural sector have no effects on the industrial growth rate. It therefore seems hard to maintain both that supply shocks cause stagnation and that industry is the sole engine of growth.

If one introduces a separate agricultural investment function both positive and negative supply shocks may cause stagnation. The mechanism is different for the two types of shocks, and the paradoxical negative effects of positive agricultural shocks depend on the interaction between the elasticity of substitution of the agricultural and industrial goods in consumption and the sensitivity of agricultural saving and investment to changes in agricultural profitability. Kaldor's advocacy of commodity price stabilization therefore may well be justified but his argument relies on particular assumptions that may or may not hold empirically.

These results have been derived for a particular model. The model has limitations and one may challenge its assumptions, both on empirical grounds and as representations of Kaldor's verbal argument. What seems beyond doubt, however, is that Kaldor's ideas and insights deserve further exploration.

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Appendix A: Properties of the equilibrium growth path

Equations (4), (5), (8) and (13) imply that

$$[1 - \phi(p)(1-s_A(1-\beta))]\ p\ \mu\ K_A^{\rho} = \phi(p)\ (1-s_M\pi_M^*)\ u^*\ K_M \tag{A1}$$

and equations (4), (11) and (12) can be combined to give

$$\alpha\ p\ \mu\ K_A^{\rho-1} = r^* \tag{A2}$$

or

$$p(t) = (r^*/(\alpha\mu))\ K_A(t)^{1-\rho} = p(K_A(t))\ ,\ \ p'>0 \tag{A3}$$

Equations (A1) and (A2) and the long-run investment function (9) can be used to derive an expression for the equilibrium ratio of the capital stocks in the two sectors

$$k(t) = K_M/K_A = [1-\phi(p(t))(1-s_A(1-\beta)]r^*/[\alpha\phi(p(t))(1-s_M\pi_M^*)u^*] = k(p(t))\ ,\ \ k'<0 \tag{A4}$$

where the sign of the derivative k' follows from the properties of the expenditure share $\phi(p)$: $\phi(0)=0$, $\phi'(p)>0$ and $\phi(p)\rightarrow 1$ for $p\rightarrow\infty$.

Equation (A3) implies that $p(t)\rightarrow\infty$ if $K_A(t)\rightarrow\infty$ and using (A4) it then follows that

$$k(t) \rightarrow k^* = s_A(1-\beta)r^*\ /\ [\alpha(1-s_M\pi_M^*)u^*] \quad \text{for } t\rightarrow\infty \tag{A5}$$

The common asymptotic growth rate, g*, of the capital stocks can be found from the equilibrium condition for the industrial good which can be written (using (14), (A5) and $\phi(p)\rightarrow 1$) as

$$M = g^*\ K_M\ (1 + [(1-s_M\pi_M^*)u^*]/[s_A((1-\beta)/\alpha)r^*]) \tag{A6}$$

or

$$g^* = u^* s_A((1-\beta)/\alpha)r^*/[s_A((1-\beta)/\alpha)r^* + (1-s_M\pi_M^*)u^*] \tag{A7}$$

In order to establish the results in (15a)-(15f) we still need to prove that K_A must go to infinity as $t\rightarrow\infty$. Assume the opposite, that is, assume that there is some $\kappa<\infty$ such that for all t_0 there exists a $t>t_0$ with $K_A(t)<\kappa$. Now let t_n be a sequence of t values with $t_n>n$ and $K_A(t_n)<\kappa$. Then, since $I_A+I_M=S_A+S_M$ and since both sectors have a positive saving rate, the industrial capital stock $K_M(t_n)$ must grow without limits, $K_M(t_n)\rightarrow\infty$ for $n\rightarrow\infty$. Using the equilibrium condition (A1) it then follows that

$$[1 - \phi(p(t_n))(1-s_A(1-\beta))]\ p(t_n)/\phi(p(t_n)) \to \infty \quad \text{for } n\to\infty \tag{A8}$$

The first term in square brackets is bounded between $s_A(1-\beta)$ and 1. (A8) therefore requires that $p/\phi(p)\to\infty$, and $p/\phi(p)=(p_A/p_M)/[p_AC_A/(p_AC_A+p_MC_M)=(p_A/p_M)+(C_M/C_A)$ is increasing in p with $p/\phi(p)\to\infty$ for $p\to\infty$. But equation (A3) shows that if $p(t_n)\to\infty$ then $K_A(t_n)$ must also go to infinity. We have therefore reached a contradiction; K_A must go to infinity as $t\to\infty$.

Appendix B: Equilibrium growth with passive agricultural investment

Equation (11') implies that

$$\hat{K}_A(t) = s_A\ (1-\beta)\ \mu\ p\ K_A^{\rho-1} \tag{B1}$$

or

$$p\mu K_A^{\rho} = \hat{K}_A\ K_A\ /\ (s_A(1-\beta)) \tag{B2}$$

Substituting (B2) into (A1), which does not depend on agricultural investment behaviour, we get

$$\hat{K}_A(t) = [\phi(p)(1-s_M\pi_M^*)u^*\ s_A(1-\beta)]/[1-\phi(p)(1-s_A(1-\beta))]\ \ K_M/K_A \tag{B3}$$

Equation (A1) also implies that p is an increasing function of k and K_M. To see this, first rewrite (A1) as

$$G(p) = [1-\phi(p)(1-s_A(1-\beta))]\ p/\phi(p) = (1-s_M\pi_M^*)\ u\ k^{\rho}\ K_A^{1-\rho}/\mu \tag{B4}$$

G(p) is strictly increasing (using the definition of p and $\phi(p)$, the ratio $p/\phi(p)$ can be rewritten as $p/\phi(p) = [p_A/p_M] / [p_AC_A/(p_AC_A+p_MC_M)] = p + C_M/C_A$, where C_M and C_A denote real consumption of industrial and agricultural goods). Thus, in long-run equilibrium with u=u* we have

$$p = G^{-1}((1-s_M\pi_M^*)u^*k^{\rho}K_M^{1-\rho}/\mu) = H(k,K_M)\ \ ;\ \ H_k>0,\ H_{K_M}>0 \tag{B5}$$

Substituting (B5) into (B3) and using (19) yields

$$\hat{k} = s_M\pi_M^*u^* - F(k,H(k,K_M)) = s_M\pi_M^*u^* - \psi(k,K_M)\ \ ;\ \ \psi_k>0,\ \psi_{KM}>0 \tag{B6}$$

The capital stock in the industrial sector grows exponentially at a constant rate so $K_M\to\infty$, and using the definition of ψ it is readily seen that

$$\psi(k,K_M) \rightarrow (1-s_M\pi_M^*)u^* \quad \text{for } k>0 \text{ and } K_M\rightarrow\infty \tag{B7}$$

Asymptotically, the movement of k is therefore determined by

$$\hat{k} = s_M\pi_M^*u^* - (1-s_M\pi_M^*)u^*\, k \tag{B8}$$

Equation (B5) describes a stable first order differential equation and $k\rightarrow k^*=s_M\pi_M^*/(1-s_M\pi_M^*)$.

Appendix C: Short run effects of agricultural supply shocks

Using (13) the industrial market-clearing condition, equation (14), can be written

$$g_M\, K_M + K_A\, (g_A - s_Ar_A(1-\beta)/\alpha) - p(A^D - A) = u\, s_M\, \pi_M^*\, K_M \tag{C1}$$

and when the agricultural market clears, this equation reduces to

$$g_M\, K_M + K_A\, (g_A - s_Ar_A(1-\beta)/\alpha) = u\, s_M\, \pi_M^*\, K_M \tag{C2}$$

It is assumed that industrial accumulation is predetermined in the short run (cf. equation (10)) but Kaldor's argument explicitly assumes that agricultural accumulation g_A responds to variations in profitability and that, indeed, agricultural investment may be more sensitive than agricultural saving to variations in profitability. Thus, let

$$d[g_A - s_Ar_A(1-\beta)/\alpha]/dr_A = \delta \tag{C3}$$

where the parameter δ captures the `excess sensitivity' of investment.

We are now in a position to derive the short-run effects of a change in μ (that is, of an agricultural supply shock). Short-run equilibrium requires the fulfilment of (A1), which can be rewritten as in (B4), and - using (C3) and (12) - total differentiation of (B4) and (C2) yields

$$G(p)\, d\mu + \mu\, G'\, dp = (1-s_M\pi_M^*)\, K_A^{-\rho}\, K_M\, du \tag{C4}$$

$$\delta\, \alpha\, [p\, d\mu + \mu\, dp] = s_M\pi_M^*\, K_A^{-\rho}\, K_M\, du \tag{C5}$$

Equations (C4)-(C5) imply that

$$dp/d\mu = [G - a\delta p] / [\mu(a\delta - G'] \tag{C6}$$

$$du/d\mu = b\, a\, \delta\, [G-pG'] / [a\delta-G'] \tag{C7}$$

where $a=[\alpha(1-s_M\pi_M*)]/[s_M\pi_M*]>0$ and $b=[(1-s_M\pi_M*)K_A^{-\rho}K_M]^{-1}>0$.

The industrial growth rate is predetermined but the change in g_M is inversely related to u (equation (10)). Thus, an increase in μ -- a positive supply shock -- has a negative effect on industrial growth if $du/d\mu<0$. To determine the sign of $du/d\mu$ we note first that (using (B4)) the term [G-pG'] can be written

$$G - pG' = p/\phi - p(1-s_A(1-\beta)) - p\ [1/\phi - p/\phi^2\ \phi' - (1-s_A(1-\beta))] = p^2/\phi^2\ \phi' \qquad \text{(C8)}$$

By assumption, the elasticity of substitution between industrial and agricultural goods is less than unity and hence $\phi'>0$. It follows that (G-pG') is positive.

The term ($a\delta$-G') can be signed using the stability conditions for the short run equilibrium. Unstable short-run equilibria are economically meaningless and - as shown in Appendix D - stability requires a negative value of ($a\delta$-G').

Combining these results it follows that positive agricultural supply shocks lead to falling industrial growth for $0<\delta$. This condition is also sufficient to ensure that a positive output shock causes a decline in the agricultural accumulation rate: a fall in u must be associated with a fall in r_A (use (C2) and (C3)) and hence in agricultural accumulation.

Appendix D: Stability of short-run equilibrium

Kaldor assumes that market clearing as achieved through price adjustment in the market for agricultural goods and quantity adjustment for industrial goods. A standard specification of the disequilibrium dynamics is given by

$$\hat{p} = \lambda_p\ (A^D - A)/A \qquad \text{(D1)}$$

$$\hat{u} = \lambda_u\ (M^D - M)/M \qquad \text{(D2)}$$

Using (4), (5), (8) and (13)-(14) this system can be written

$$\hat{p} = \phi(p)/p\ \lambda_p/A\ [u\ (1-s_M\pi_M*)\ K_M - G(p)\ \mu\ K_A^\rho] \qquad \text{(D3)}$$

$$\hat{u} = \lambda_u/M\,[\,g_M K_M + (g_A - s_A r_A(1-\beta)/\alpha)\,K_A - p\,(A^D - A) - u s_M \pi_M^*\,K_M\,] \tag{D4}$$

where G is defined as in (B4). If, as in appendix C, δ denotes the `excess sensitivity' of investment to changes in profitability, $\delta = d[g_A - s_A r_A(1-\beta)/\alpha]/dr_A$, the Jacobian for this two-dimensional system (calculated at an equilibrium with $\hat{p}=\hat{u}=0$) is given by

$$J(p,u) = \begin{pmatrix} p\,\partial\hat{p}/\partial p & p\,\partial\hat{p}/\partial u \\ -u\lambda_u/M(pA/\lambda_p\,\partial\hat{p}/\partial p - \delta K_A r_A/p) & -u\lambda_u/M(pA/\lambda_p\,\partial\hat{p}/\partial u + s_M\pi_M^* K_M) \end{pmatrix} \tag{D5}$$

Stability requires a negative trace and a positive determinant. The signs of the partial derivatives of $\hat{p}$ are unambiguous, $\partial\hat{p}/\partial p<0$ and $\partial\hat{p}/\partial u>0$. Thus, the trace condition is always satisfied

$$Tr = p\,\partial\hat{p}/\partial p - u\lambda_u/M\,(pA/\lambda_p\,\partial\hat{p}/\partial u + s_M\pi_M^* K_M) < 0 \tag{D6}$$

The determinant is given by

$$\begin{aligned} Det &= -\,u\,\lambda_u/M\,(s_M\pi_M^* K_M\,p\,\partial\hat{p}/\partial p + \delta\,r_A K_A\,\partial\hat{p}/\partial u) \\ &= -\,u\,\lambda_u/M\,p\,\partial\hat{p}/\partial p\,[s_M\pi_M^* K_M + \delta\,\alpha\,\mu\,K_A^{\rho}\,(\partial\hat{p}/\partial u)/(\partial\hat{p}/\partial p)] \end{aligned} \tag{D7}$$

At a short-run equilibrium with $\hat{p}=0$, the ratio $(\partial\hat{p}/\partial p)/(\partial\hat{p}/\partial u)$ of partial derivatives can be derived using (D3)

$$(\partial\hat{p}/\partial p)/(\partial\hat{p}/\partial u) = -\,G'\,\mu K_A^{\rho}/[(1-s_M\pi_M^*)K_M] \tag{D8}$$

and substituting (D8) into (D7) the stability condition Det>0 can be expressed

$$\delta < \delta^{max} = s_M\pi_M^*/(1-s_M\pi_M^*)\,G'\,\alpha \tag{D9}$$

The value of G'(p) depends on p, but using the definition of G it can be shown that $G'(p) \geq s_A(1-\beta)>0$ for all p. Thus, δ^{max} is bounded above zero for all p.

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