A Reconsideration of the Empirical Evidence on the Asymmetric Effects of Money-Supply shocks: Positive vs. Negative or Big vs. Small?^{*}

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Abstract:

In this paper we reconsider the empirical evidence on the possible asymmetric effects on aggregate output of changes in nominal demand. Many versions of such asymmetries are common in the theoretical literature. The particular versions of asymmetries that we will concentrate on are the following. The first is the traditional Keynesian asymmetry which is concerned with different effects of positive and negative changes in money-supply stating that the former, for various reasons, are neutral, and the latter have real effects. Earlier empirical studies have found evidence in favour of this hypothesis but we argue that the results are not robust to changes in the money supply process. Using an alternative empirical technique, we argue that the evidence points instead towards asymmetric effects of big and small changes in nominal demand, i.e. that there is a non-linear relationship between nominal demand changes and output changes. The empirical results imply that big shocks are neutral but small shocks have real effects. The findings indicate the relevance of menu-cost models but also imply that the traditional Keynesian asymmetry has less empirical support.

Key words: Nominal Demand, Asymmetric effects, Regime Switching

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1. Introduction

The purpose of the present paper is to present new time series evidence on the possible asymmetric effect on output of changes in nominal demand. We shall investigate, using quarterly data for the US, whether a change in aggregate demand, represented by money-supply, has a different effect depending on whether it is positive or negative, and whether it is big or small. Asymmetries such as these arise in a number of different macroeconomic theories and can hypothetically have strong implication for issues such as the conduct of policy and the "costs" of changes in nominal demand. Because the available empirical evidence for such effects is not particularly overwhelming, further investigation is warranted.

One popular asymmetry hypothesis says that positive and negative changes in nominal demand have different effects. This asymmetry is usually associated with the traditional Keynesian notion of upwardly flexible - downwardly sticky wages. This implies that positive money-supply shocks should be neutral, whereas negative money supply shocks should be non-neutral. Such an asymmetry can also be identified in other environments, and, as discussed by Cover (1992), it can also arise in a model with a particular form of rationing. More recently, a number of papers in the New-Keynesian tradition have attempted to provide microfoundations for this asymmetry, and to derive it in models with optimizing agents. Ball and Mankiw (1994) look at a model in which firms can costlessly set prices every second period, but subject to a menu-cost if they wish to change prices between periods. They show that in an inflationary environment, the traditional Keynesian asymmetry will indeed occur. The explanation for this can be understood as follows. With "core" inflation, a small negative change in demand can lead firms to take no action since inflation automatically leads to a downward change in relative prices, even with unchanged nominal prices. Hence, if the "optimal" price does not differ too much from the preset price, it may not be individual rational to pay the cost associated with the menu-cost of changing prices between periods. A positive change in nominal demand, on the other hand, is much more likely to trigger price adjustment because of the existence of inflation that magnifies the change in demand in case of no change in prices. Tsiddon (1993) finds a similar result in a model with similar features.

A second version of asymmetric effects, which can also be related to the New Keynesian literature, but differs from the traditional Keynesian asymmetry, is that big and small shocks to nominal demand can have different effects. The implication of this is that there is a non-linear relationship between nominal demand changes and their effects on output. In models with menu-costs, such as Blanchard and Kiyotaki (1987) and many others,

a strategy of keeping prices constant so as to avoid the menu-cost can be optimal in response to a small change in nominal demand. The reason for this is that the loss associated with this strategy are of second order. Hence "small" shocks, as defined relative to the size of the menu-costs, are non-neutral. However, in the face of big changes in nominal demand, both first-order and second-order costs will be incurred if the strategy of keeping prices fixed is followed, and hence firms are likely to find it optimal to neutralize demand changes by adjusting nominal prices. Therefore, the prediction of such models is that small shocks have real effects, but big shocks are neutral. Naturally, the distinction between these two types of shocks is conditional upon a number of things, including the size of the menu-cost associated with changing prices, and on the size of real rigidities (see Ball and Romer, 1990).

A third, and somewhat different, version of asymmetric effects is the distinction between anticipated and unanticipated money-supply changes associated with the New Classical literature. The formulation of the asymmetry in this case is that while anticipated shocks are neutral, unanticipated shocks have real effects. This view, motivated by the theoretical work of Lucas (1972, 1973, 1975) and others, was examined empirically in a number of papers including Barro (1977,1978), Barro and Herchowitz (1980), Boschen and Grossman (1982) and Mishkin (1982). While the earlier studies found empirical support for the hypothesis, the later studies identified real effects even from anticipated shocks. This version of asymmetric effects will not be the primary concern of the present paper, but it shall be included in the analysis while we focus on testing for the presence of the two former versions of asymmetry.

There is some empirical evidence that supports the traditional Keynesian version of asymmetric effects. DeLong and Summers (1988) and Cover (1992) both present empirical evidence for the US in favour of this asymmetry even though their results differ slightly. The methodology in both of these papers resembles the techniques used for testing the New Classical asymmetry and consists of jointly estimating a money-supply relation and an aggregate output relation. The anticipated and/or unanticipated shocks from the aggregate money-supply equation feed into the output relation, which is tested for asymmetric effects of positive and negative shocks. Delong and Summers (1988) investigate annual US data for a relatively long sample period and find that negative unanticipated shocks have a larger absolute effect on output than unanticipated positive shocks. The magnitude of this difference is found to be somewhat sensitive to the sample period. Cover (1992) performs essentially the same exercise on quarterly US data for the period 1947-1987, and finds that positive

unanticipated shocks to money supply are neutral, while negative unanticipated shocks have real effects. Moreover, this finding is claimed to be robust to alternative specifications of the money-supply and output relations. Karras (1995) carry out a similar analysis for a sample of 18 European countries and reach the conclusion that there is very strong support for the asymmetry hypothesis: negative (positive) money-supply shocks have significant (insignificant) output effects. A parallel conclusion holds if the analysis uses interest rates rather than money-supply. This evidence seems to be supportive of the second version of asymmetric effects, thus providing an empirical basis for the conventional Keynesian models based on rigidities.

Here we will reconsider the empirical evidence using an alternative technique to analyze the asymmetry propositions. In our analysis we try to distinguish between the first two versions of asymmetric effects so as to test whether one or both of them have empirical support. Initially, however, we shall reconsider the previous seemingly-affirmative evidence in favour of the traditional Keynesian asymmetry. For this purpose we will use the same data and the same methodology as in Cover (1992). We argue that the evidence in favour of the asymmetry proposition is less robust than it may seem and that the results are sensitive to the existence of a change in the time series behaviour of money-supply coinciding with the end of the Volcker regime. Essentially, the change in monetary policy that occurred in 1983 produced a large negative outlier in the residuals of the money-supply process, which has important bearings for the results since the asymmetry result does not have empirical support if the change in economic policy is controlled for. Alternatively, if one reestimates moneysupply and uses a specification that is insensitive to this change, the asymmetry result does not have strong empirical support. Therefore, the empirical evidence for the US does not seem supportive of the traditional Keynesian asymmetry.

The new procedure that we propose consists of estimating a money-supply process in which one allows for changes-in-regime. We shall be using the regime-switching model of Hamilton (1988) appropriately modified to allow for lagged exogenous variables that are not allowed to switch between regimes. We model the money-supply as a regime-switching process that can have changes in its mean and in the variance of the innovations to the process¹. We first consider whether the distinction between anticipated and unanticipated

¹Garcia and Schaller (1995) also apply the Hamilton (1989) Markov-Switching methodology to investigate the question of asymmetric effects of monetary policy. They focus, however, on whether the effects of monetary policy depend on the state of

money-supply shocks seems important. Our results imply that only unanticipated moneysupply shocks have real effects, and the remainder of the analysis is then carried out in terms of unanticipated money-supply shocks.

Because of the regime-switching technique, we can distinguish between four different shocks to money supply: "big positive shocks", "big negative shocks", "small positive shocks" and "small negative shocks". This allows us to test simultaneously for the existence of the two versions of asymmetric effects that are our primary interest, i.e. whether there is evidence of the traditional Keynesian asymmetry and/or the asymmetry due to different effects of big and small shocks. The traditional Keynesian asymmetry can be tested by assessing whether the coefficients on the positive shocks are zero, and that the coefficients on the negative shocks are significantly different from zero. The other asymmetry hypothesis can be examined by testing whether the coefficients on the big shocks are zero, and whether the coefficients on the small shocks are significantly different from zero. Naturally, one may also consider a hybrid case in which the hypothesis is that only small and negative shocks have significant effect on aggregate output; testing this hypothesis is a more restrictive test of the models that are built on menu-costs, and includes features that make the traditional Keynesian asymmetry occur.

From the analysis of US data we find unambiguous results that seem to be quite strong: the important distinction is between big and small shocks, with strong evidence that big shocks are neutral, but small shocks have real effects. Furthermore, we cannot reject that the coefficients in the aggregate output equation on the "small positive" and the "small negative" shocks are identical. Hence there is empirical support for the asymmetry which says that big shocks are neutral and small shocks are non-neutral, but no support for the more traditional Keynesian asymmetry version. These results are different from those obtained in previous studies and are not due to differences in the data. The implication of these results is that the time series evidence seems to lend support to the menu-cost literature.

The remainder of the paper is organized as follows. In the next section the results of Cover (1992) are reexamined. Section 3 is devoted to the new methodology and the application of this procedure. Section 4 summarizes and draws some conclusions.

the economy, i.e. on whether there is a relationship between the phase of the economy and the output effects of monetary policy.

2. Asymmetric Effects of Positive and Negative Shocks to Money Supply?

The hypothesis that positive and negative shocks to money-supply have asymmetric effects on output has been examined on US data by Delong and Summers (1988) and by Cover (1992). The methodology used in both of those papers can be seen as extensions of the procedure used for testing the New Classical theories in Barro (1977,1978) and, in particular, Mishkin (1982). Two relationships are estimated simultaneously. The first of these is meant to capture money-supply. From this relationship one can obtain estimates of the anticipated shocks to money-supply and the unanticipated money-supply shocks (i.e. the residuals). These shocks feed into the next equation meant to capture aggregate output.

DeLong and Summers (1988) include anticipated shocks as well as unanticipated shocks. They find the anticipated shocks to be significant in the prewar and postwar periods but insignificant in the pre-depression period, and that negative shocks to unanticipated money growth have larger effects than positive unanticipated shocks. Cover (1992) experiments with a number of different specifications for the money-supply process, two different aggregate output equations, and with including lagged values of the money-supply shocks in the output equation. The results are reasonably robust. Positive money supply shocks are neutral but negative money-supply shocks have real effects. Karras (1995) apply a similar technique to an analysis of data for 18 European countries and find very strong support for the asymmetry hypothesis. He also finds that this result is robust to a respecification of the system in terms of looking at the interest rate rather than the money-supply.

Here we will reexamine the empirical evidence of Cover (1992). Following the methodology outlined above, the following two equations are estimated simultaneously:

$$\Delta m_{t} = \Phi (L) \Delta m_{t-1} + \Theta x_{t-1} + \varepsilon_{t}$$
(1)

$$\Delta \mathbf{y}_{t} = \mathbf{\psi} \mathbf{z}_{t} + \boldsymbol{\beta}^{+} \boldsymbol{\varepsilon}_{t}^{+} + \boldsymbol{\beta}^{-} \boldsymbol{\varepsilon}_{t}^{-} + \boldsymbol{\xi}_{t}$$
(2)

where Δ is the first-difference operator, m_t is the log of the M1, $\Phi(L)$ is a lag-polynomial, Θ is a vector of parameters, x_{t-1} is a vector of exogenous regressors, y_t is the log of real output (GNP), Ψ is a parameter-vector, and z_t is a vector of regressors. z_t can include lagged values of dependent and independent variables. ε_t^+ and ε_t^- are the positive and negative parts of ε_t from equation (1) and are defined as:

$$\boldsymbol{\varepsilon}_{t}^{+} \equiv \max(0, \boldsymbol{\varepsilon}_{t}), \qquad \boldsymbol{\varepsilon}_{t}^{-} \equiv \min(0, \boldsymbol{\varepsilon}_{t})$$
⁽³⁾

Equation (1) is to be thought of as a money-supply process and equation (2) as an aggregate output equation. The asymmetry hypothesis is a test on whether β^+ equals β^- ; rejection of this restriction together with β^+ being insignificantly different from zero and β^- significantly different from zero, supports the hypothesis.

Cover (1992) experiments with a number of different specifications of the moneysupply equation and with two different version of the output equation. The money-supply process is specified as either in Barro and Rush (1980), as in Mishkin (1982) modified slightly, or as an "Optimal" money-supply. In the "Barro-Rush" specification, the vector of regressors (x_{t-1}) includes a constant, the unemployment rate, and the contemporaneous real federal expenditure to normal expenditure. In the "Modified Mishkin" specification, x_{t-1} contains a constant, lagged changes in money-supply, lagged changes in the treasury-bill rate, and lagged values of the federal government's budget surplus. The "Optimal" specification includes various elements of the above variables as well as lagged values of the changes in the monetary base. The two different specifications of the output equation have in common that z_t includes a constant, lagged change in real output, $\varepsilon^+(t)$, and $\varepsilon^-(t)$ but differ in whether the change in the t-bill rate is included or not.

We start by reconsidering the specification of money-supply. In table 1 we reproduce Cover's (1992) single equation estimates of the "Modified Mishkin" and "Optimal" money-supply processes². Columns (1) and (2) list the parameter estimates and specification tests. There are two basic problems with both of these specifications. The first is evident from figures 1A and 1B that plot the residuals of these equations. One notes from these plots that the residuals of both specifications have a very big negative observation in the first quarter of 1983. This is caused by a large outlier in the growth of M1 for this quarter which coincides with the end of the interest rate targeting of the Volcker regime. Columns (3) and (4) of table 1 report the regression results of reestimating (1) and (2) by introducing a dummy to partial out the effect of the change in monetary policy. The dummy is highly significant in both cases.

The important question is whether the outlier invalidates the asymmetry results. Table

²We also tried to replicate the results of the "Barro-Rush" specification, but were unable to do so.

3 reports the results of estimating the output equation estimated jointly with the moneysupply equation for the "Modified Mishkin" specification (the results for the "optimal" specification were identical and are not reported). We modify the procedure slightly by restricting the parameters on the money-supply shocks to be positive³. Column 1 reports the results of using the original specification of this relation, and the results are basically identical to those reported by Cover (1992)⁴. Column 2 reports the results of using the shocks from the money-supply process when a dummy is included. There are very clear signs of asymmetries in the original formulation since positive shocks are insignificant, negative shocks are highly significant, and one can easily reject that the coefficients on these two shocks are equal. When the dummy is introduced, these results change radically. First, the significance-level of the negative shock drops dramatically, and second one cannot reject the hypothesis that the coefficients are equal. Hence, this preliminary analysis brings into question whether the empirical support of this particular asymmetry hypothesis is robust.

An alternative technique to including a dummy variable is to reestimate the money supply processes such that it is robust to the presence of the outlier in money growth. In order to do so we apply a general-to-specific technique and initially include up to six lagged values of the change in M1, the change in the monetary base, the inflation rate, the output growth rate, the change in the t-bill rate, the unemployment rate, and the federal government's budget surplus. Testing downwards leads to the results given in table 2.

This relationship is robust to the outlier in the money growth rate and a dummy on the observation in 1983.1 is no longer significant. One may now ask whether using this specification affects the asymmetry result. This is answered in the third colun of table 3 which reports the results of estimating the alternative money-supply process and the output equation (2) simultaneously. The results are basically identical to the results obtained when the "Modified Mishkin" is augmented with a dummy. This is true for both the asymmetry results and for the parameter estimates of the remaining variables. With respect to the asymmetry results we note that the negative shocks are not nearly as significant as in the "Modified Mishkin" specification, and one cannot reject that the coefficients on the positive

³This parameter restriction is needed to ensure consistency of the results. Imagine e.g. that the coefficient on the negative shock is positive while that on the positive shock is negative. In this case the two shocks cannot be distinguished.

⁴The small differences in the parameter values may be due to the use of different optimization procedures.

and negative shocks are identical, i.e. there are no clear signs of asymmetric effects.

To sum up, the finding that there are asymmetric effects from positive and negative unanticipated shocks to money-supply (that is, to nominal demand) is not a very robust feature of the data.

3. A Generalized Test for Asymmetric Effects of Money-Supply Shocks.

We now wish to extend the basic methodology in order to simultaneously test, and discriminate between, the two major versions of asymmetric effects. We apply a method that allows us to divide the money-supply changes into big and small shocks in addition to the division into positive and negative shocks. As we will show below, this can be accomplished by using a regime-switching methodology in which we allow for changes in the mean growth rate of money-supply and in the variance of innovations to money-supply.

Another advantage of using the changes-in-regime model is that it allows us to address another problem with the money-supply specifications analyzed in the previous section. If one inspects the specification tests of the money-supply relationships (see tables 1 and 2⁵), then one notes that the tests for heteroscedasticity turns out to be significant even in the cases where a dummy is introduced in the "Modified Mishkin" regression and in the "general-to-specific" specifications. Figure 1C plots the rolling variance of the residuals from the four different specifications of the money-supply equation investigated in the previous section. These variances were computed as the variance of each of the residual-series in a 4-year window. These plots suggest that there are changes in the variance of the money-supply shocks over the sample. In particular, the evidence points towards a lower variance in the early part of the sample than in the later part of the sample.

One possible reason for this is that the growth rate of M1 is subject to changes-inregime. A plot of this time series is produced in figure 2 and the plot seems to suggest that the moments of the growth rate of M1 change behaviour over the sample. To be precise, there are signs that the mean of the growth rate and its variance are lower in the earlier part of the sample. It is, however, not that clear where the change precisely occurs, and for this reason we allow the data itself to identify the change.

⁵These are the results of the single equation OLS estimates of the various moneysupply processes. We do not report the results of the same tests on the money-supply processes estimated in the simultaneous equation procedure, but the parameter estimates are very close to the single-equation results.

3.1 Modelling Money-Supply with Changes-in-Regime

One way to deal with such a problem is to apply the discrete-state regime-switching technique of Hamilton (1989) to the modelling of money-supply. Such a technique has been used widely to characterize movements that arise when the moments of the variables under scrutiny change behaviour over time, see e.g. Hamilton (1988, 1989, 1991), Phillips (1991), Sola and Driffill (1994), and Ravn and Sola (1995). The basic elements of the method is described extensively in Hamilton (1994). One big advantage of using this procedure is, as will become clear shortly, that we are able to test for the presence of the various versions of asymmetric effects, and to distinguish between them. Garcia and Schaller (1995) also apply a switching-regime method to investigate the issue of asymmetric effects of monetary policy. The question they address, however, is somewhat different to the one we will look at. In their analysis they model output as a regime switching process and test whether monetary policy have different effects depending on the current state of the economy. They find that, in line with sticky-price models (such as Ball and Mankiw, 1994, and Tsiddon, 1993) interest rates have greater output effects during recessions and that interest rates have substantial effects on the probability of a switch in output.

According to the regime-switching methodology, a time series is modelled as having discrete changes in its unconditional mean and/or variance. The variable that dictates the changes-in-regime is modelled as an unobservable discrete-valued state variable, s_t , that takes on the values 0 or 1. We also add to the switching regression a set of conditioning variables which are not subject to changes in regime. With this modification, we estimate a money-supply equation that allows for changes in mean and variance. This leads us to the following specification of money-supply:

$$(\Delta m 1_{t} - \mu(s_{t})) = \Phi(L) (\Delta m 1_{t-1} - \mu(s_{t-1})) + \Theta x_{t-1}' + \sigma(s_{t}) \eta_{t}$$
(4)

. .

where $\Phi(L)$ is a lag-polynomial, Θ is a vector of parameters, x_{t-1} is a vector of de-meaned exogenous variables⁶ defined as $x-\mu_x$, $\mu(s_t)$ is a state-dependent mean, s_t is the discrete-valued state variable that can take on the values 0 and 1, and η is an i.i.d N(0,1) error term.

The money-supply process can now have two different means, μ_0 and μ_1 with associated variances σ_0^2 and σ_1^2 . In the practical application these are estimated as $\mu_0+\Delta\mu s_t$ and $\sigma_0+\Delta\sigma s_t$. It is assumed that the (unobserved) states are generated by a two-state Markov-

⁶We de-mean the non-switching exogenous variables so that $\mu(s_t)$ can be interpreted as the unconditional mean of money-growth.

process. Let π_{ij} be defined as P(s_t=i|s_{t-1}=j), i,j=0,1, then the probability transition matrix is given as:

$$\Pi = \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix}$$

where each of the transition probabilities are restricted to be non-negative and belong to the unit interval. The columns of Π sums to one.

In this approach we have not allowed for regime-switching in the exogenous variables. To allow these variables to have changes-in-regime will require either to impose that they all switch simultaneously with money-supply (see e.g. Sola and Driffill, 1994) or that each variable is allowed to switch independently (see e.g. Ravn and Sola, 1995). The first approach is applicable when the variables are closely related as would be the case for example for interest rates of bonds of different maturities⁷, but does not seem applicable in the present analysis. The second approach has the disadvantage that the increase in the number of states quickly makes it intractable⁸.

3.2 Empirical Results

3.2.1 Single Equation Results

Before moving on to the estimation of output and money-supply simultaneously, it is instructive initially to perform a single-equation estimation of the money-supply equation with changes-in-regime approach. This exercise will reveal whether this specification is a suitable way to model the data.

The results of this regression are listed in table 4. The results are encouraging since both the change in the mean of the process and the change in the variance of the process are significant. The estimates suggest that there is a low-mean low-variance regime where the

⁷In the case of the term structure, there are, of course, very strong theoretical reasons why the state variables that dictate the changes-in-regime would be closely connected.

⁸Hamilton, 1994, propose the use of specification test as a means to assess whether (i) the number of states is well chosen, and (ii) whether the proposed specification is valid. If either (i) or (ii) are not valid, then the regression residuals should show signs of misspecification.

mean is around 0.8 percent per quarter and the standard deviation around 0.4 percent, and a high-mean high-variance regime where the mean is around 1.7 percent per quarter and the variance around 0.8. This implies that the means and innovation variance in the "high" state (state 1) of money-supply are estimated to be roughly twice the corresponding numbers in the "low" state (state 0). Note also that both regimes are estimated as being quite persistent since the diagonal elements of the transition matrix are both in excess of 0.98⁹.

A plot of the estimated probabilities of being in regime 1, that is the regime where the mean and the variance are both high, is shown in figure 3^{10} . The filter divides the sample very clearly into the two regimes and the estimates imply that money growth and the variance of money growth were low from the start of the sample until 1967. From 1967 to 1987.4 the probability of being in the regime with high mean and high variance is practically equal to one, with the exceptions from this being the last three quarters of 1976 and the final three observations. The final, and important, point to note from table 3 is that the residuals are now homoscedastic and the model seems well specified¹¹.

3.2.2 Setting Up the Tests for Asymmetric Effects

For our purposes the primary interest is to test the asymmetry hypotheses. This means that we, as in section 2, estimate the money-supply process and the output process jointly. In order to make the results comparable with those of section 2, the specification of the output equation is maintained.

The first thing we look at is whether there is a difference in how anticipated and unanticipated money-supply shocks affect output. The purpose of this analysis is to find out whether the proceeding investigation should focus on unanticipated shocks, as in section 2, or whether anticipated shocks should be added as well. Table 5 reports the resulting output equation when both anticipated and unanticipated money-supply shocks are included. The results imply that anticipated shocks are insignificant whereas unanticipated shocks are

⁹The expected time in each regime (whence the process is in either of these) is given by the inverse of one minus each of the diagonal elements.

¹⁰One should, however, keep in mind that the simultaneous equations system not necessarily implies the same probabilities and, hence, does not need to divide the sample into the same regimes.

¹¹The only sign of misspecification is that the first lag of the change in the monetary base is no longer significant. We decided, however, to keep this variable in the model in order to make the results directly comparable to those of section 2.

significant at any conventional level of confidence. The data therefore seems to lend support to the neutrality of expected changes in money-supply. Therefore, we shall concentrate the following analysis around asymmetric effects of *unanticipated* changes in money-supply.

As discussed above, this framework allows us to test for the presence of asymmetric effects of (i) positive and negative money-supply shocks, and (ii) asymmetric effects of big and small changes in nominal demand. To do this we divide the money-supply shocks into four different categories. First, we make a distinction between "big" (e_t^B) and "small" money-supply shocks (e_t^S). We do this by defining "big" ("small") shocks as innovations to the money-supply process in the regime where the variance of the innovations is "high" ("low"). This is because the absolute size of the shocks will also, on the average, be higher (lower) when the variance is higher (lower). This follows from the assumption that $\dot{\eta}$ is an i.i.d N(0,1)-process. Next, we divide each of these into their positive and negative parts, as in the previous section.

The division into "big" and "small" shocks is done in the following fashion. Consider first the expected money-growth in period t given the realized state at time t-1. Expected money-growth is given as:

$$\mathbf{E}_{t-1} \Delta m_{t} = (\mu_{0} + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_{0}) + \Theta x_{t-1}) \quad \text{if} \quad s_{t-1} = 0$$

$$E_{t-1} \Delta m_t = (\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1}) \quad \text{if} \quad s_{t-1} = 1$$

The residuals (i.e. unexpected money-supply shocks) in these two cases can then be defined as:

$$\varepsilon_{0t} = \Delta m_t - [(\mu_0 + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_0) + \Theta x_{t-1})] \sim N(0, \sigma_0^2)$$

$$\varepsilon_{1t} = \Delta m_t - [(\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1})] \sim N(0, \sigma_1^2)$$

Recall, however, that the states are unobservable for the econometrician and hence the above distinction requires one to draw inference on the regimes. We do this by using the estimates of the probabilities of being in each of the two regimes. Let $P(s_{t-1}=i)$ be the (estimated) probability conditional on information available at time t-1 that the state is equal to "i" at time t-1 using the (modified) Hamilton filter. Then we can define these two shocks

in the following manner:

$$e_t^{S} \equiv P(s_{t-1} = 0) * (\Delta m_t - [\mu_0 + \Delta \mu \pi_{11} + \Phi (\Delta m_{t-1} - \mu_0) + \Theta x_{t-1}])$$
(5)

$$e_t^{B} \equiv P(s_{t-1} = 1) * (\Delta m_t - [\mu_0 + \Delta \mu \pi_{22} + \Phi (\Delta m_{t-1} - (\mu_0 + \Delta \mu)) + \Theta x_{t-1}])$$
(6)

Next, each of these two shocks can be divided into their positive and negative parts, which we denote by + (positive) and - (negative), using the same technique as in the previous section. Accordingly, we end up with four money supply shocks, $\{e^{B_{+}}_{t}, e^{B_{-}}_{t}, e^{S_{+}}_{t}, e^{S_{+}}_{t}\}$. This construction allows us to test for the presence of asymmetric effects using the following procedure.

We estimate jointly the money-supply equation (5) and the following version of the output equation:

$$\Delta y_{t} = \psi z_{t} + \beta^{B^{+}} \varepsilon_{t}^{B^{+}} + \beta^{B^{-}} \varepsilon_{t}^{B^{-}} + \beta^{S^{+}} \varepsilon_{t}^{S^{+}} + \beta^{S^{-}} \varepsilon_{t}^{S^{-}} + \xi_{t}$$
(7)

The initial regression estimates these relationships, (4) and (7), imposing no parameter restrictions. We will call this for **Case 1**. At this point one can look at the significance of each of these shocks as a check on signs of asymmetric effects. The real tests, however, are carried out in a sequential manner using LR-tests. This means that we impose parameter restrictions on the coefficients on the money-supply shocks, $\{e_{t}^{B_{+}}, e_{t}^{B_{-}}, e_{t}^{S_{+}}, e_{t}^{S_{-}}\}$. The first set of restrictions we impose is that:

Case 2: $H_0: \beta^{B_+} = \beta^{B_-} = \beta^{S_+} = \beta^{S_-}$

Testing Case 2 against Case 1 is a test of the absence of any asymmetry and can be performed as LR-test which is χ^2 -distributed with 3 degrees of freedom under the null. If these restrictions are rejected, the tests for the two versions of asymmetric effects are carried out by imposing a number of different parameter restrictions.

Consider first the case where one wishes to test for the asymmetry that positive and negative money-supply shocks have different effects (and that the former are neutral). This hypothesis can be tested for in two steps. First, according to this hypothesis, it should not matter whether a given money-supply shock is "big" or "small". Hence, we estimate the system imposing the following restrictions:

Case 3: $H_0: \beta^{B_+} = \beta^{S_+} \text{ and } \beta^{B_-} = \beta^{S_-}$

Testing Case 3 against Case 1 (a LR-test which is χ^2 -distributed with 2 degrees of freedom) constitutes the first part of this hypothesis. The second part of the hypothesis is that

positive shocks are neutral. Hence, we then impose that:

Case 4: $H_0: \beta^{B_+} = \beta^{S_+} = 0 \text{ and } \beta^{B_-} = \beta^{S_-}$

Testing Case 4 against Case 3 is then a test of whether positive shocks are neutral. If these tests are passed, and the coefficient on the negative shocks is significantly positive, then the data supports the traditional Keynesian asymmetry hypothesis.

The other asymmetry hypothesis can be tested in a similar manner. The hypothesis is that only small money-supply shocks have real effects, and a priori, there is no reason that positive and negative shocks should have different effects. Hence, we first estimate the following case:

Case 5: H_0 : $\beta^{B_+} = \beta^{B_-}$ and $\beta^{S_+} = \beta^{S_-}$

Testing Case 5 against Case 1 (a LR-test which is χ^2 -distributed 2 degrees of freedom under the null), constitutes the first part of the hypothesis. The second part imposes that "big" shocks are neutral:

Case 6: $H_0: \beta^{B_+} = \beta^{B_-} = 0 \text{ and } \beta^{S_+} = \beta^{S_-}$

Again, we test this specification against Case 5, and if the test is passed (and the coefficient on the "small" shocks is positive and significant) then the hypothesis is backed by the data.

A last case to consider is that the relevant hypothesis is a hybrid version in which only "small negative" shocks have real effects. This could be the case, if e.g. the menu-cost model was augmented with features of models that imply the existence of the traditional Keynesian asymmetry. We can also test such a hybrid version in any of the two sequences outlined above. This can be performed by imposing:

Case 7: $H_0: \beta^{B_+} = \beta^{S_+} = \beta^{B_-} = 0$

The validity of this restriction can be tested against either Case 4 if this case is passed or against Case 6 if this case is passed.

Before proceeding to discuss the results, it is worthwhile to relate our procedure to alternative methods used in the literature. The main difference between our approach and other applications is the definition of "big" and "small" shocks. Demery (1993) make the distinction between "small" and "big" shocks by defining "big" shocks as those (money-supply) residuals with an absolute value that exceeds two standard errors, and small shocks

as those which do not¹². This definition corresponds to viewing "big" shocks as outliers and the procedure is not robust to structural changes in the mean and the variance. Furthermore, as we shall now make clear, such an approach may result in an incorrect identification of the shocks.

To see this consider the following four hypothetical cases for the behaviour of the time series for money-supply: (i) No structural change in the mean and no change in the variance but a few outliers in the money-supply shocks (for example 5 percent). In this situation it is valid to define big shocks as outliers. (ii) No changes in the mean but changes in the variance of money-supply shocks. The latter method will now estimate the sample standard deviation as the average of the standard deviation in the two regimes. This implies that a portion of the "big" shocks are wrongly categorized as "small" shocks and as a result, the shocks are wrongly identified. (iii) No changes in the variance of money-supply but changes in the mean. In this case, by definition there is no real distinction between "big" and "small" shocks, but the above method <u>will</u> most likely separate the residuals in "big" and "small" shocks. (iv) Changes in the mean and the variance of the money-supply shocks. In this case the above approach to distinguishing between "big" and "small shocks will suffer from the problems discussed in (ii) and (iii) simultaneously.

Provided that we identify the correct number of states¹³, our proposed technique will give the correct separation in <u>all</u> four cases. In case (i) the two methods will give the same answer. In cases (ii) and (iv), we will separate the sample into the two variances, "big" and "small", and this allow us to test for the presence of asymmetric effects. In case (iii) the approach should indicate that there is only a change in the mean growth rate but no change in the variance, and hence that the test for asymmetric effects is ill-defined. Hence, we believe that our methodology is more natural.

3.2.3 Results

We will now move on to test the asymmetry hypotheses on the US data using the technique

¹²Caballero and Engel (1993) apply a similar strategy when testing for asymmetries in the price-adjustments of firms in the face of nominal rigidities and changes in demand.

¹³In the empirical application we have checked the number of states in two ways. First, as discussed as above, by means of misspecification tests. Secondly, we estimated a model that allowed for three states but the data is only separated into two regimes.

outlined above. The first step is to estimate jointly the two relationships, (4) and (7), imposing no parameter restrictions in (7). The results of this (the output equation) are reported in table 6, column 1. The first thing to note is that the model seems well-specified since there are no longer any signs of serial correlation or ARCH-effects. There are noticeable differences in the level of significance of the various money-supply shocks. The only single component that is significant at conventional levels of confidence is the small positive shock. Big positive shocks are at the other extreme, entering with a zero coefficient. Negative shocks, these being either big or small, are less significant than the small positive shocks. Hence, one may suspect that there is some version of asymmetric effects of money-supply in the data.

To investigate this, we first estimate a version of the system imposing the parameter restriction of Case 2, i.e. imposing that all components of the money-supply shocks enter with the same parameter. The results are given in the second column of table 6. The specification tests are still satisfactory, and the estimates imply that the unanticipated money-supply shock is significant in the output equation. However, the LR-test indicates that the parameter restrictions are rejected and hence, that the various components of these shocks have different effects. Therefore we proceed to look into whether the data supports any of the two asymmetry hypotheses discussed in the previous sections.

First, we will estimate the model imposing the restrictions of Case 3. This restriction forms the first step in testing for the presence of the traditional Keynesian asymmetry and imply that "big" and "small" shocks enter with the same coefficients, i.e. that the only important distinction is between positive and negative money-supply shocks. The results of this are given in column 3 of table 5. Again, the specification tests are satisfactory. The parameter estimates now imply that negative shocks enter with a coefficient of around 0.92 and that these are highly significant. Positive shocks enter with a coefficient close to zero and are insignificant. Hence, the t-ratios support the hypothesis. The probability of observing the value of the LR-test under the null, however, is only 4.4% and there is therefore not much evidence in favour of the restrictions. One further piece of evidence that points towards rejection of the hypothesis is that the "small positive" money-supply shock, which in the unrestricted specification (Case 1) was the single most significant component, is now

insignificant. Hence, one would tend to reject this specification¹⁴.

The other alternative to be tested is whether "big" and "small" money-supply shocks have asymmetric effects on output. The first step in testing this is to impose the restrictions under Case 5, i.e. that it is irrelevant whether the shocks are positive or negative. The results are given in column 5 of table 6. Under this specification only "small" shocks are significant and they enter with a coefficient of 0.95. "Big" shocks are insignificant at any level of confidence and enter with a coefficient very close to zero. Furthermore, the restrictions imposed on the output equation cannot be rejected using the LR-test which has a probability value as high as 25.1%. This implies evidence in favour of the hypothesis that "big" and "small" money-supply shocks have different output effects. Notice also that any test would lead one to prefer Case 5 to Case 3 as the preferred simplification of the unrestricted model. Not only is the likelihood much higher for Case 5, but an encompassing test (see e.g. Mizon and Richard (1986)), a technique that can be used for testing non-nested hypotheses, would also accept Case 5 and reject Case 3 as appropriate simplifications of Case 1.

Given this, it still needs to be tested whether "big" shocks are neutral. Hence, we impose this extra condition and estimate the model under the restrictions of Case 6. The results, given in column 6 of table 6, imply that this restriction cannot be rejected and probability value for the LR-test is as high as 85.3%.

Before going into more details about the results, we will look at the last hypothesis to be tested. The last case to be looked at is whether a hybrid model in which only small negative money-supply shocks is the relevant model. This model can be thought to represent a model of the menu-cost type augmented with features that lead to the traditional Keynesian asymmetry. This is our Case 7 and the model is nested under the model estimated for Case 6 and we therefore carry out the LR-test for that specification. The results are given in column 7 of table 5. The results are very clear and imply that the Case 7 is strongly rejected (the probability value for the LR-test is 0.09%).

The evidence is therefore that the data supports the hypothesis that there are asymmetric effects of money-supply shocks on output in the US. However, contrary to previous results, we do not find evidence in favour of the hypothesis that negative shocks have real effects and that positive shocks are neutral. The hypothesis that we find to be supported by

 $^{^{14}}$ Given that Case 3 is rejected we do not wish to interpret the LR-test of Case 4 against Case 3 since one can no longer be sure that the LR-test statistic is χ^2 -distributed.

the data is that "big" shocks are neutral but "small" shocks have real effects. This asymmetry would be implied by models in which firms face a menu-cost of changing their nominal prices. It may be of interest to inspect the results of our estimates of the money-supply equation for the joint estimation with the restricted output equation. The parameter estimates are summarized in table 7. The estimated money-supply equation is quite well-specified. Most of the variables enter significantly and the tests for autocorrelation and ARCH-effects are insignificant. The estimates imply, as in the single equation results, that there is low-money-growth regime in which the variance of the innovations is also low. Both the mean of money-growth and the standard deviation of innovations are approximately twice as big in the "high" regime as in the "low" regime. Note also that both means and both standard deviations are highly significant.

Figure 6 plots the estimated probability of being in the regime where the innovation variance and the mean money-growth are "high". We estimate the state with low mean and low variance to have dominated from the start of the sample up to around 1967. The state with high mean growth and high innovation variance then takes over in 1968 and dominates most of the period up to around 1975 and again from the second quarter of 1978 to 1987. The state with low mean and low innovation variance dominates most of the intermediary period of 1975-1978. In interpreting these estimates it should be kept in mind that we include additional conditioning variables and the results do therefore not relate to standard univariate estimates of the money-supply process.

To sum up, the data gives strong support to the existence of an asymmetry which says that small changes in nominal demand have real effects but big changes in nominal demand are neutral. The results also imply that, contrary to previous research, the traditional Keynesian asymmetry cannot be supported by the data.

4. Summary and Conclusion

In this paper we have taken a fresh view of the empirical evidence of asymmetric effects of changes in nominal demand on output. Such asymmetries have under various specifications been incorporated in a number of macroeconomic theories. We have identified three different versions of asymmetries. The first is associated mainly with the New Classical literature and states that anticipated money-supply shocks should be neutral but unanticipated shocks should have real effects. This theory was examined in a number of papers in the late 1970's and the early 1980's, and the evidence was mixed.

We have concentrated on two other types of asymmetric effects. The first of these, which dates back to the traditional Keynesian literature, is concerned with asymmetric effects from positive and negative shocks to money-supply, and says that negative nominal demand shocks should have real effects, and that positive shocks to nominal demand should be neutral. This hypothesis has found empirical support mainly in Cover (1992), but also to some extent in DeLong and Summers (1988), although these latter authors restate it in terms of larger effects of negative shocks. The other version of asymmetric effects implies that big and small shocks to nominal demand have different effects, the former should be neutral and the latter should have real effects. This hypothesis can be based on theories in which price-setting firms face menu-costs associated with changing prices. Such models have also been augmented in order to account for the second version of asymmetry.

Our results are two-fold. First, we argue that previous evidence in favour of the different effect of positive and negative money-supply shocks is not very robust. This is because the asymmetry hypothesis is rejected when the specification of money supply is altered. Next, we applied an alternative empirical technique that allowed us to distinguish between all three asymmetry versions. The results were clear and implied that unanticipated small changes in money-supply are non-neutral whereas big unanticipated shocks and anticipated shocks are neutral. This conclusion lends empirical support to models that include menu-costs, but it does not support refinements of these in which one tries to explain the traditional Keynesian asymmetry version.

5. References

Ball, Laurence, and N. Gregory Mankiw, 1994. Asymmetric Price Adjustment and Economic Fluctuations. *The Economic Journal*, 104 (March), 247-261.

Ball, Laurence and David Romer, 1990. Real Rigidities and the Non-Neutrality of Money. *Review of Economic Studies*, 57, 183-203.

Barro, Robert J., 1977. Unanticipated Money Growth and Unemployment in the United States. *American Economic Review*, 67, 101-115.

Barro, Robert J., 1978. Unanticipated Money, Output, and the Price Level in the United States. *Journal of Political Economy*, 86, 549-580.

Barro, Robert J. and Zvi Herchowitz, 1980. Money Stock Revisions and Unanticipated Money Growth. *Journal of Monetary Economics*, 6, 257-267.

Blanchard, Olivier J., and Stanley Fischer, 1989. Lectures on Macroeconomics. MIT Press, Cambridge, Massachusetts.

Blanchard, Olivier J., and Nobuhiro Kiyotaki, 1987. Monopolistic Competition and the Effects of Aggregate Demand. *American Economic Review*, 77, 647-77.

Boschen, John and Herschel I. Grossman, 1982. Tests of Equilibrium Macroeconomics Using Contemporaneous Monetary Data. *Journal of Monetary Economics*, 10, 309-333.

Caballero, Ricardo J., and Eduardo M.R.A. Engel, 1993. Microeconomic Rigidities and Aggregate Price Dynamics. *European Economic Review*, 37, 697-717.

Cover, James Peery, 1992. Asymmetric Effects of Positive and Negative Money-Supply Shocks. *The Quarterly Journal of Economics*, 1261-1282.

DeLong, J. Bradford, and Lawrence H. Summers, 1988. How Does Macroeconomic Policy Affect Output? *Brookings Papers on Economic Activity* (2), 433-480.

Demery, David, 1993. Asymmetric Effects of Money on Real Output: Evidence for the UK. Mimeo, University of Bristol.

Garcia, Rene and Huntley Schaller, 1995. Are the Effects of Monetary Policy Asymmetric? Reproduced, Universite de Montreal.

Hamilton, James D., 1988. Rational Expectations Econometrics Analysis of Changes in Regime: An Investigation of the Term Structure of Interest Rates. *Journal of Economic Dynamics and Control*, 12, 385-423.

Hamilton, James D., 1989. A New Approach to the Economic Analysis of Non-Stationary Time Series and the Business Cycle. *Eonometrica*, 57, 357-384.

Hamilton, James D., 1991. Analysis of Time Series subject to Changes in Regime. *Journal of Econometrics*, 45, 39-70.

Hamilton, James D., 1994. Time Series Analysis. Princeton University Press, Princeton, New Jersey.

Karras, Georgios, 1995. Are the Output Effects of Monetary Policy Assymmetric? Evidence From a Sample of European countries. Reproduced, University of Illinois.

Lucas, Robert E. Jr., 1972. Expectations and the Neutrality of Money. *Journal of Economic Theory*, 4, 103-124.

Lucas, Robert E. Jr., 1973. Some International Evidence on Output-Inflation Tradeoffs. *American Economic Review*, 63, 326-334.

Lucas, Robert E. Jr., 1975. An Equilibrium Model of the Business Cycle. *Journal of Political Economy*, 83(6), 1113-1144.

Mishkin, Fredric C., 1982. Does Anticipated Policy Matter? An Econometric Investigation. *Journal of Political Economy*, 90, 22-51.

Mizon, Grayham E., and Jean-Francois Richard, 1986. The Encompassing Principle and its Application to Testing Non-Nested Hypotheses. *Econometrica*, 54, 657-678.

Phillips, Kerk L., 1991. A Two-Country Model of Stochastic Output with Changes in Regime. *Journal of International Economics*, 31, 121-142.

Ravn, Morten O., and Martin Sola, 1996. Stylized Facts and Regime Changes - are Prices Procyclical? Forthcoming, *Journal of Monetary Economics*, 36(3).

Sola, Martin, and E. John Driffill, 1994. Testing the Term Structure of Interest Rates from a Stationary Markov-Switching VAR. *Journal of Economic Dynamics and Control*, 18, 601-628.

Tsiddon, Daniel, 1993. The (Mis)behaviour of the Aggregate Price Level. *Review of Economic Studies*, 60(4), 889-902.

Appendix: The Filter

It is assumed that one of the variables included in the filter is governed by a scalar state variable. The other variable(s) is(are) not allowed to switch and is(are) de-meaned. The filter involves the following five steps.

<u>Step 1.</u> Let y and x be the variables that are observed and let s be the unobserved state variable. Calculate the density of the m past states and the current state conditional on the information included in y_{t-1} , x_{t-1} and all past values of y and x:

$$p(s_{t}, s_{t-1}, s_{t-m} | y_{t-1}, y_{t-2}, y_{0}, x_{t-1}, x_{t-2}, x_{0})$$

$$= p(s_{t} | s_{t-1}) p(s_{t-1}, s_{t-2}, s_{t-m} | y_{t-1}, y_{t-2}, y_{0}, x_{t-1}, x_{t-2}, x_{0})$$
(A.1)

where $p(s_t|s_{t-1})$ is the transition probability matrix of the states which are assumed to follow a Markov process. As in all subsequent steps, the second term on the right-hand-side is known from the preceding step of the filter. In the present case $p(s_{t-1},s_{t-m}|y_{t-1},y_0,x_{t-1},x_0)$ is known from the input to the filter which in turn represents the result of the iteration at date t-1 (from step 5 described below).

Initial values for the parameters and the initial conditions for the Markov process are required to start the filter. The unconditional distribution, $p(s_m, s_{m-1}, s_0)$ has been chosen for the first observation.

<u>Step 2.</u> Calculate the joint conditional density of y_t and (s_t, s_{t-1}, s_{t-m}) ,

$$p(y_{t}, s_{t}, s_{t-1}, s_{t-m} | y_{t-1}, y_{0}, x_{t-1}, x_{0})$$

$$= p(y_{t} | s_{t}, s_{t-m}, y_{t-1}, y_{0}, x_{t-1}, x_{0}) p(s_{t}, s_{t-m} | y_{t-1}, y_{0}, x_{t-1}, x_{0})$$
(A.2)

where we assume that

$$p(y_t | s_{t}, s_{t-m}, y_{t-1}, y_0, x_{t-1}, x_0)$$

$$=\frac{1}{(2\pi)^{.5}(\sigma_0 + \Delta \sigma s_t)} \exp(-(2(\sigma_0 + \Delta \sigma s_t)^2)^{-1} u_{s_t}^2)$$

where

$$\mathbf{u}_{\mathbf{s}_{t}} \equiv \mathbf{y}_{t} - (\boldsymbol{\mu}_{0} + \Delta \boldsymbol{\mu} \mathbf{s}_{t}) - \boldsymbol{\phi}(\mathbf{L})(\mathbf{y}_{t} - (\boldsymbol{\mu}_{0} + \Delta \boldsymbol{\mu} \mathbf{s}_{t})) - \boldsymbol{\theta}(\mathbf{L})\mathbf{x}_{t}$$

It should be noted that $p(y_t|s_{t},s_{t-m},y_{t-1},y_0,x_{t-1},x_0)$ involved (s_t,s_{t-m}) which is a vector that can take on 2^{m+1} values.

<u>Step 3.</u> Marginalize the previous joint densities with respect to the states which gives the conditional density from which the (conditional) likelihood function is calculated:

$$p(y_{t}|y_{t-1}, y_{0}, x_{t-1}, x_{0}) = \sum_{s_{t}=0}^{1} \sum_{s_{t-1}=0}^{1} \cdots \sum_{s_{t-m}=0}^{1} p(y_{t}, s_{t}, s_{t-m}|y_{t-1}, y_{0}, x_{t-1}, x_{0})$$
(A.3)

<u>Step 4.</u> Combining the results from steps 2 and 3, calculate the joint density of the state conditional on the observed current and past realizations of y.

$$p(s_{t}, s_{t-1}, .., s_{t-m} | y_{t}, .., y_{0}, x_{t}, .., x_{0}) = \frac{p(y_{t}, s_{t}, s_{t-1}, .., s_{t-m} | y_{t-1}, .., y_{0}, x_{t-1}, .., x_{0})}{p(y_{t} | y_{t-1}, .., y_{0}, x_{t-1}, .., x_{0})}$$
(A.4)

Step 5. The desired output is then obtained from:

$$p(s_{t}, s_{t-1}, .., s_{t-m+1} | y_{t}, .., y_{0}, x_{t}, .., x_{0}) = \sum_{s_{t-m}=0}^{1} p(s_{t}, s_{t-1}, .., s_{t-m} | y_{t}, .., y_{0}, x_{t}, .., x_{0})$$
(A.5)

The output of step 5 is used as an input to the filter in the next iteration. Estimates of the parameters are calculated by maximizing the sample likelihood, which can be calculated from step 3.

Table 1. Single Equation OLS estimates of Money supply Processes,1948.1-1987.4

variable	(1) Modified Mishkin	(2) Optimal	(3) Mod. Mishkin with dummy	(4) Optimal with dummy
Constant	0.381 (0.120)	-0.183 (0.413)	0.302 (0.077)	-0.294 (-0.396)
$\Delta m 1_{t-1}$	0.319 (0.081)	0.176 (0.084)	0.302 (0.078)	0.183 (0.081)
$\Delta m l_{t-2}$	0.147 (0.086)	0.121 (0.086)	0.218 (0.084)	0.208 (0.086)
$\Delta m 1_{t-3}$	-0.039 (0.087)	-0.125 (0.074)	-0.062 (-0.084)	-0.118 (0.071)
$\Delta m 1_{t-4}$	0.199 (0.083)	0.150 (0.070)	0.190 (0.080)	0.153 (0.066)
Δb _{t-1}	-	0.177 (0.116)	-	0.180 (0.110)
Δb_{t-2}	-	0.161 (0.115)	-	0.075 (0.113)
Δtbr_{t-1}	-0.397 (0.063)	-0.383 (0.055)	-0.448 (0.062)	-0.439 (0.054)
Δtbr_{t-2}	0.055 (.074)	-	0.038 (0.071)	-
Δtbr_{t-3}	-0.068 (0.076)	-	-0.048 (0.073)	-
Δtbr_{t-4}	0.064 (0.957)	-	0.038 (0.064)	-
u _{t-1}	-	0.293 (0.245)	-	0.378 (0.236)
fs _{t-1}	-0.194 (0.785)	-0.423 (0.437)	-0.337 (0.756)	-0.292 (0.418)
fs _{t-2}	-1.142 (1.170)	-	-1.004 (1.073)	-
fs _{t-3}	0.978 (1.103)	-	1.091 (1.059)	-
fs _{t-4}	-0.433 (0.753)	-	-0.496 (0.723)	-
Δy_{t-1}	-	0.061 (0.061)	-	0.029 (0.060)
d1983.1	-	-	-2.829 (-0.781)	-2.941 (0.774)
R ²	0.473	0.491	0.514	0.534
S.E.	0.754	0.741	0.724	0.709
Q(1)	0.0007 [0.979]	0.000 [0.979]	0.047 [0.828]	0.101 [0.750]
Q(10)	1.845 [0.997]	2.107 [0.996]	1.962 [0.992]	3.720 [0.959]
QQ(1)	9.790 [0.002]	7.432 [0.006]	23.839 [0.000]	25.525 [0.000]
QQ(10)	44.118 [0.000]	47.439 [0.000]	59.125 [0.000]	63.005 [0.000]

Notes: $\Delta m1$ is the log difference of M1, Δb is the log difference of the monetary base, Δtbr is the log difference of the t-bill rate, fs is the federal government's budget surplus, u is the unemployment rate, and Δy is the log difference of output. Numbers in parentheses below parameter estimates are standard errors. Q(x) (QQ(x)) is a Box-Pierce test for autocorrelation in the standardized residuals (squared standardized residuals) of order x with the corresponding probability below the test value.

Variable		Test-Statistic	
$\Delta m 1_{t-1}$	0.219 (0.080)	R ²	0.533
u _{t-2}	0.412 (0.242)	S.E.	0.706
fs _{t-4}	-1.574 (0.753)	Q(1)	0.048 [0.828]
fs _{t-5}	2.835 (1.076)	Q(10)	4.927 [0.896]
fs _{t-6}	-1.750 (0.707)	QQ(1)	17.821 [0.000]
Δb_{t-1}	0.218 (0.062)	QQ(10)	45.226 [0.000]
Δb_{t-5}	0.347 (0.098)		
Δb_{t-6}	-0.194 (0.100)		
Δy_{t-2}	0.113 (0.055)		
Δy_{t-6}	0.132 (0.059)		
Δtbr_{t-1}	-0.429 (0.052)		
Δtbr _{t-3}	-0.177 (0.056)		
Δtbr_{t-3}	-0.150 (0.056)		
Constant	-0.441 (0.428)		

Table 2. General-to-Specific Money Supply

Variable	(1) Modified Mishkin	(3) Modified Mishkin with dummy	(5) General-to- Specific
Constant	0.732	0.609	0.619
	(0.176)	(0.190)	(0.177)
Δy_{t-1}	0.318	0.330	0.333
	(0.071)	(0.072)	(0.073)
Δtbr _t	0.286	0.286	0.293
	(0.066)	(0.067)	(0.064)
Δtbr_{t-1}	0.074	0.083	0.077
	(0.070)	(0.071)	(0.070)
ϵ^{+}_{t}	0.079	0.214	0.252
	(0.238)	(0.270)	(0.241)
ε _t	0.705	0.476	0.557
	(0.275)	(0.339)	(0.32 <u>5</u>)
d1983.1	-	-2.261 (0.806)	-
σ _M	0.731	0.696	0.680
σ,	0.922	0.941	0.934
Log likelihood	-378.7825	-374.388	-367.131
Pos=Neg (LR)	8.695	0.205	0.344
	[0.003]	[0.651]	[0.558]
Q(1)	0.277	0.135	0.141
	[0.599]	[0.714]	[0.707]
Q(10)	6.929	7.292	8.564
	[0.732]	[0.698]	[0.574]
QQ(1)	3.771	1.701	3.006
	[0.052]	[0.192]	[0.083]
QQ(10)	13.167	9.843	8.490
	[0.200]	[0.454]	[0.581]

Table 3. Output Equation Estimates, ML Estimates.

Notes: See notes to Table 1. Numbers in square brackets are probabilities. d1983.1 is the dummy that enters the money supply equation. σ_M is the standard deviation of the residual in the estimated money-supply process. σ_y is the standard deviation of the residual in the output equation.

Variable		Variable		Test-stat.
$\Delta m 1_{t-1}$	0.257 (3.566)	μ ₀	0.773 (0.130)	Q(1)=0.295 [0.587]
u _{t-2}	0.403 (2.131)	Δμ	0.934 (0.235)	Q(10)=9.522 [0.483]
fs _{t-4}	-1.160 (-1.908)	σ_0	0.417 (0.041)	Q(20)=16.117 [0.709]
fs _{t-5}	2.254 (2.675)	Δσ	0.368 (0.076)	QQ(1)=2.265 [0.132]
fs _{t-6}	-1.234 (-2.336)	π_{11}	0.985 (0.015)	QQ(10)=16.12 [0.096]
Δb_{t-1}	0.078 (0.846)	π_{22}	0.989 (0.013)	QQ(20)=27.22 [0.129]
Δb_{t-5}	0.141 (1.780)			
Δb_{t-6}	-0.181 (-2.038)			
Δy_{t-2}	0.144 (3.446)			
Δy_{t-6}	0.115 (2.395)			
Δtbr_{t-1}	-0.405 (-7.475)			
Δtbr_{t-3}	-0.199 (-3.452)			
Δtbr_{t-3}	-0.144 (-2.555)			

Table 4. Estimates of Money Supply Equation with Changes in Regime

Table 5.	Output	Equation	ML	Estimates	From	Simultaneous	Estimation	of Money	and
Output									

Vari- able	Constant	Δy_{t-1}	Δtbr_t	Δtbr_{t-1}	ϵ^{AN}_{t}	$\epsilon^{\text{UNAN}}_{t}$
Esti- mate	0.341 (2.001)	0.335 (4.596)	0.284 (4.270)	0.129 (1.576)	0.142 (1.150)	0.329 (2.607)
Teststat.	σ_{v}	Q(1)	Q(10)	QQ(1)	QQ(10)	Log lik.
Value	0.944	0.055 [0.815]	9.219 [0.512]	1.509 [0.219]	9.091 [0.524]	-376.115

Notes: See notes to table 1. $\epsilon^{AN}(t)$ is the anticipated change in money-supply, $\epsilon^{UNAN}(t)$ is the unanticipated money-supply shock.

Variable	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
Constant	0.552 (0.154)	0.528 (0.097)	0.792 (0.144)	0.804 (0.118)	0.457 (0.102)	0.452 (0.101)	0.511 (0.093)
Δy_{t-1}	0.318 (0.080)	0.333 (0.073)	0.307 (0.072)	0.305 (0.072)	0.337 (0.073)	0.338 (0.073)	0.328 (0.073)
Δtbr _t	0.285 (0.063)	0.291 (0.066)	0.297 (0.065)	0.297 (0.066)	0.275 (0.064)	0.274 (0.064)	0.268 (0.067)
Δtbr_{t-1}	0.088 (0.093)	0.084 (0.072)	0.067 (0.070)	0.066 (0.070)	0.088 (0.069)	0.088 (0.069)	0.078 (0.070)
e ^{S+} t	0.867 (0.301)	0.381 (0.122)	0.021 (0.154)	-	0.949 (0.258)	0.966 (0.250)	-
e ^{S-} t	0.008 (0.005)	0.381 (0.122)	0.918 (0.272)	0.937 (0.244)	0.949 (0.258)	0.966 (0.250)	1.040 (0.513)
e ^{B+} t	0.000 (0.005)	0.381 (0.122)	0.021 (0.154)	-	0.034 (0.166)	-	-
e ^{B-} t	-0.496 (0.301)	0.381 (0.122)	0.918 (0.272)	0.937 (0.244)	0.034 (0.166)	-	-
σ,	0.901	0.938	0.914	0.914	0.915	0.915	0.955
Q(1)	0.388 [0.534]	0.170 [0.680]	0.344 [0.558]	0.334 [0.563]	0.434 [0.510]	0.451 [0.502]	0.343
Q(10)	8.490 [0.588]	9.427 [0.399]	9.127 [0.520]	9.133 [0.520]	8.760 [0.555]	8.624 [0.568]	11.999
QQ(1)	1.229 [0.268]	1.831 [0.176]	3.072 [0.080]	3.108 [0.078]	1.719 [0.190]	1.817 [0.178]	2.926
QQ(10)	8.624 [0.568]	8.737 [0.557]	11.444 [0.324]	11.519 [0.319]	7.792 [0.649]	7.789 [0.650]	8.908
Log likelih.	-349.59	-354.56	-352.71	-352.72	-350.97	-350.99	-356.53
LR-test	-	9.951 ¹⁾ [0.019]	6.254 ²⁾ [0.044]	0.122 ³⁾ [0.912]	2.766 ⁴⁾ [0.251]	0.034 ⁵⁾ [0.853]	11.094 ⁶⁾ [0.00]

Table 6. Output Equation ML Estimates, only Unanticipated Shock.

1) LR-test of Case 2 vs. Case 1

2) LR-test of Case 3 vs. Case 1

3) LR-test of Case 4 vs. Case 3

4) LR-test of Case 5 vs. Case 1

5) LR-test of Case 6 vs. Case 5

6) LR-test of Case 7 vs. Case 6

Variable		Variable		Test-stat.
$\Delta m 1_{t-1}$	0.209 (0.074)	μ ₀	0.748 (0.181)	Q(1)=0.0.071 [0.790]
u _{t-2}	0.301 (0.184)	Δμ	0.914 (0.075)	Q(10)=6.818 [0.743]
fs _{t-4}	-1.014 (0.570)	σ₀	0.435 (0.039)	Q(20)=13.205 [0.868]
fs _{t-5}	2.248 (0.787)	Δσ	0.385 (0.083)	QQ(1)=0.877 [0.349]
fs _{t-6}	-1.267 (0.495)	π_{11}	0.968 (0.019)	QQ(10)=12.213 [0.271]
Δb_{t-1}	0.175 (0.093)	π ₂₂	0.979 (0.021)	QQ(20)=26.460 [0.151]
Δb_{t-5}	0.160 (0.076)			
Δb_{t-6}	-0.103 (0.079)			
Δy_{t-2}	0.133 (0.041)			
Δy_{t-6}	0.060 (0.045)			
Δtbr_{t-1}	-0.410 (0.053)			
Δtbr _{t-3}	-0.190 (0.059)			
Δtbr_{t-3}	-0.126 (0.056)			

Table 7. Estimates of Money-Supply Equation with Changes in Regime: JointlyEstimated with Output under Case 6











