Quantity Adjustment Costs and Price Rigidity

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Abstract

It is by now well known that the presence of small lump sum costs of adjusting prices can lead to nominal price rigidity. However, the argument of adjustment costs could equally well be applied to quantities, thereby leading to rigid quantites. This paper examines the importance of adjustment costs on both prices and quantites in a dynamic monopoly model, where the firm has to decide whether it uses price or quantity or both as the adjustment mechanism to shocks which can be permanent or temporary. It is shown that there is more downward rigidity than upward, and that this asymmetry is enhanced when the quantity adjustment costs consist of linear adjustment costs on top of the lump-sum. Furthermore, prices seem to bear a relatively larger proportion of the adjustment burden when the shock is 'large', and vice versa for quantities when the shock is 'small'. Finally, the firm may be rationed even when adjustment is carried out.

Keywords: Adjustment costs, price rigidity, rationing

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1) INTRODUCTION

In recent years a widespread explanation for price rigidity has been lump-sum costs of changing nominal prices, the socalled menu costs. As the name suggests these costs are considered to be small, but shown to have real effects in Mankiw (1985) and Blanchard and Kiyotaki (1987) among others. The key point is that the loss of not adjusting prices to a given shock is only of second order smallness for the individual agent, whereas the aggregate effect is of first order. This also indicates that because it is not optimal for the single price setter to change its price, the price level may be rigid, and the shock (for instance a demand shock) may affect output causing real effects on welfare. The welfare consequenses of menu costs have been dealt with quite extensively. For an thorough and very good overview see Andersen (1994).

However, the effect on output might be an implication of there being no adjustment costs on quantities. If there were costs associated with changing quantities, output might not respond to a (demand) shock. Therefore, there is a bias against not adjusting prices in the menu cost models, as only price adjustment is thought to be costly. Hence it seems of vital importance to integrate both price and quantity adjustment costs, since it is by no means obvious at all that adjustment costs on quantities should be zero. Surprisingly, this has received almost no attention in literature with Andersen (1995) as the only exception, to my knowledge. He demonstrates how a single firm will choose optimally between adjusting either the price or the quantity or both in a static model with lump-sum costs of adjusting both the price and the quantity. It turns out that the mode of adjustment is not just the trivial one that bears the lowest costs of adjustment, other interesting implications emerge as well, such as rationing and asymmetric adjustment to positive and negative shocks.

However, in Andersen (1995) the adjustment costs for quantities are only of a lump-sum nature. This appears odd in the context of quantities. If for instance, you need to increase production, it seems natural that the costs of doing so increases with the size of production in excess of the ordinary marginal costs[1]. For instance, over-time payment to the employees or new capital acquisition if you initially were at maximum capacity. Thus, in this paper the model by Andersen (1995) is extended by incorporating quantity adjustment costs that have both a linear and a lump-sum component. Furthermore, the model is dynamic, allowing for an analysis of the firm's response to both permanent and transitory shocks. These extensions alternate the conclusions reached in Andersen (1995) concerning the mode of adjustment. One of the major new insights from incorporating linear adjustment costs is that prices seem to bear a relatively larger burden of adjustment when the shock is 'large', and quantities a relatively larger share when the shock is 'small'. This has the important implication that for small shocks prices are sticky and the nominal rigidity has real effects; and this effect is an even larger effect than in the traditional menu cost models. Furthermore, the dynamic aspect contributes to interesting findings concerning the mode of adjustment. In particular, it will be demonstrated that even if it is more costly to adjust quantities, price rigidity may still be observed. Also, if the quantity adjustment costs involve a linear component on top of the lump-sum costs, price rigidity becomes more pronounced, even though price changes are now relatively cheaper. Thus, some of the critique levelled at the traditional menu costs models saying that there is an inherited bias against price stickiness, may in some cases not be justified.

These features relating to the dynamic aspect of the present paper, does by assumption not occur

[1] The effects of different adjustment cost functions on quantities have been dealt with quite extensively in the investment and labour economics literature. See e.g Nickell (1986), Abel (1990).

in Andersen (1995). However, the important findings concerning rationing and the asymmetric adjustment to positive and negative shocks remain.

The paper is organised as follows. In section 2 the basic model is set up. In section 3 we discuss the case of a permanent shock, in order to be able to study effects that pertain to the cost structure and not necessarily to the model being dynamic. Then we proceed to the dynamic case in section 4 where a temporary shock is considered. Finally section 5 concludes and discusses limitations of the results obtained.

2) THE MODEL

Consider a monopolist producing subject to fixed marginal costs c and facing a linear demand curve given as d(p) = a - bp. The profit maximising price and quantity is:

$$\bar{p} = \frac{a+bc}{2b} \qquad\qquad \bar{q} = \frac{a-bc}{2}$$

Now assume that there is a shock to demand u, after the firm has decided on its price and production ($\bar{p}$, $\bar{q}$). The demand shock is completely unanticipated when hitting the economy but fully known ex post (Knightean uncertainty). After observing this shock the monopolist might want to alter either its price, its quantity, both, or leaving both unchanged. There are, however, costs associated with adjusting. The price adjustment costs are assumed to be lump-sum and of size α while the quantity adjustment costs consist of both a linear and a lump-sum part. These costs are given by $k\cdot|q-\bar{q}| + \beta$, where k is a constant. We will impose the restriction $k < c$. This implies that the marginal adjustment cost is less than the marginal production cost which *a priori* seems like a reasonable assumption.

The firm maximises the present discounted value of profits taking into account the demand shock, when it is costly to adjust both the price and the quantity. If the shock is transitory, it is assumed to occur at the beginning of period one and only lasting for that period. This allows us without any loss of generality to consider only two periods, as the second period can be thought of as the present discounted value of future profits from period one[2]. We will however, start with a permanent shock as this allows us to spell out some general results independent of the shock being permanent or transitory.

3) A PERMANENT DEMAND SHOCK

Consider a permanent shock. Since it by assumption is completely unanticipated and permanent, it suffices to analyse only one period because all future periods can be condensed into a single period. The new demand function is $d(p) = a - bp + u$, where u represents shifts in demand. The firm has four strategies pertaining to this new demand situation. Either adjusting both the price and the quantity (AA), adjusting the price leaving the quantity unchanged (AD), adjusting the quantity leaving the price unchanged (DA), or keeping both the price and the quantity fixed at $\overline{p}$ and $\overline{q}$ which it has inherited from previous periods (DD). To each of these strategies is an associated profit which is calculated in the appendix[3].

[2] This is only true if the discount factor is constant. This is certainly going to be correct in this paper, as there for simplicity is no discounting at all.

[3] The appendix is available from the author upon request

In this section we will merely show a few calculations in order to emphasise some results that stem from the adjustment cost structure and do not relate to the dynamic aspect. In that way it is possible to separate the contributions from this paper, without being to repetitious.

The general profit maximisation problem after the shock has occured now reads:

$$(1) \qquad \max_{q}\ 1/b(a - q + u)q - cq - k\cdot|q - \bar{q}| - \beta - \alpha$$

From the FOC we obtain the profit maximising price and quantity, p^{AA} and q^{AA}:

<u>u > 0</u>

$$(2a) \qquad p^{AA} = \frac{a+u+b(c+k)}{2b} \qquad q^{AA} = \frac{a+u-b(c+k)}{2}$$

<u>u < 0</u>

$$(2b) \qquad p^{AA} = \frac{a+u+bc-bk}{2b} \qquad q^{AA} = \frac{a+u-bc+bk}{2}$$

with the following profit in both cases:

$$\Pi^{AA} = \frac{(a+u-b(c+k))(a+u-bc+bk)}{4b} - k\frac{(|u|-bk)}{2} - \alpha - \beta$$

It is useful to compare this to the maximisation outcome in the case where there is no linear adjustment cost component (k=0 in (1)). This yields

$$(3) \qquad p^{*} = \frac{a+u+bc}{2b} \qquad q^{*} = \frac{a+u-bc}{2}$$

Comparing (2a,b) with (3) we observe $p^{AA} > p^{*}$ and $q^{AA} < q^{*}$ for u > 0 and vice versa for u < 0.

This implies that the linear adjustment cost part on quantities tends to reduce the profit maximising output and increase the price, relative to the lump-sum cost case.[4] The intuition is clear. Assume that the lump-sum costs on prices are not prohibitive. Then, if it does not matter how much you change the price, but matter how much you change the quantity, then it is obviously cheaper to change prices a lot relative to changing quantities.

In order to demonstrate how rationing might emerge in this kind of model, the maximisation problem for the strategy where neither the price nor the quantity is adjusted, is shown below. Consider the case where a positive shock hits the economy ($u > 0$). If the price is kept unchanged then $p^{DD} = \bar{p}$ and actual sales is $\bar{q}$. Even though the monopolist could produce more to the new improved demand conditions, he does not, due to the adjustment costs. Hence, consumers are rationed due to insufficient supply (Classical regime). Profit is then: $\Pi^{DD} = (\bar{p} - c)\bar{q}$. If instead there is a negative shock ($u < 0$) and $p^{DD} = \bar{p}$, then actual sales is $y = (a - bc - 2u)/2$ whereas production is still given by $\bar{q}$. In this case the firm is rationed due to insufficient demand (Keynesian regime). Here the profit is $\Pi^{DD} = py - c\bar{q}$. So extending the traditional menu cost models allowing for costly quantity adjustment, provides a rationale for rationing. The case for rationing will only exist for small shocks though, as will be demonstrated below, since the strategy of keeping both the price and the quantity fixed cannot be an equilibrium mode of adjustment if the shock is large, as the costs of not adjusting become too large, then. This has the significant policy implication that if demand management is pursued, it should be sizeable stimuli to demand. Otherwise, it might cause rationing on either side of the market.

[4] This result generalises with convex adjustment costs as well.

The profit functions for each of the four scenarios can be found as in Andersen (1995) with correction made for the linear quantity adjustment costs, and they are stated below. The expressions are the profit net of the strategy (DD) such that whenever these expressions are positive, the corresponding strategy is profitable compared to doing nothing. If they are negative, the firm does not adjust neither of its choice variables.

CASE I: adjustning both the price and the quantity (AA)

$u > 0$ $\Pi_+^{AA} = u^2 + u(2a - 2bc- 2bk) + (bk)^2 - 4b(\alpha + \beta)$

$u < 0$ $\Pi_-^{AA} = u^2 + u(2bk - 4bc) + (bk)^2 - 4b(\alpha + \beta)$

CASE II: adjusting the price leaving the quantity unchanged (AD)

$u > 0$ $\Pi_+^{AD} = u(2a - 2bc) - 4b\alpha$

$u < 0$ $\Pi_-^{AD} = u(-4bc) - 4b\alpha$

CASE III: adjusting the quantity leaving the price unchanged (DA)

$u > 0$ $\Pi_+^{DA} = u(2a - 2bc - 2bk) - 4b\beta$

$u < 0$ $\Pi_-^{DA} = u(4bk - 4bc) - 4b\beta$

The importance of including linear and not merely lump-sum costs can be examined from the above profit functions. First, it should be noted that the case of $k=0$ corresponds exactly to Andersen (1995). Now consider the case where $\beta = 0$ whereas $k > 0$. That is, there is only linear quantity adjustment costs. Then it is easy to see that the strategy (DA) will always yield positive profits compared to doing nothing, such that there will always be adjustment of the quantity for any (positive or negative) sufficiently small shock.[5] If the shock is sufficiently large, it can be

[5] This result generalises with convex adjustment costs as well.

shown that the price will be adjusted together with the quantity[6]. Hence, if there is no lump-sum component in the quantity adjustment costs, non-adjustment will never prevail even for the tiniest shock.

Assume that k and β are strictly positive. Then it can be seen from the profit functions that there will exist an interval where no adjustment is optimal (if u = 0, then the profit is negative for α,β >0). As the shock increases, adjustment will eventually be optimal. The mode of adjustment will be either AD or DA depending on which bears the lowest costs. If the shock is sufficiently large, AA appears as the equilibrium mode of adjustment. Thus, by including a lump sum component in the quantity adjustment costs, a zone of inaction emerges for small shocks. In that zone rationing might be the outcome as explained above. Hence, a necessary condition for rationing, which in this case corresponds to non-adjustment, is the lump-sum part.

Another feature that pertains to the cost structure and not necessarily to the dynamic structure, is the asymmetric adjustment to positive and negative shocks, causing more downward rigidity than upward. If β = 0, then the mode of adjustment is symmetric as can be seen by comparing strategy (AA) to (DA) for positive and negative shocks. For asymmetric adjustment to emerge a fixed quantity adjustment cost is again required. This is immediately apparent by comparing (AA) to (DA)/(AD) when both k and β are positive. The fact that asymmetric adjustment occurs is a feature that has been part of Keynesian economists way of thinking, but has been difficult to establish formally unless one relies on some underlying inflation for creating this asymmetry (see e.g. Tsiddon (1991).

[6]There is a possibility that the strategy AD can be an equilibrium before AA. This depends on the relative of size of the parameters, and the feature will show up in the next section.

From this analysis it should be evident that the specific cost structure in Andersen (1995) accounts for many of his results. However, just an infinitesimal amount of lumpiness in the quantity adjustment costs is sufficient to restore many of the Keynesian features. Let us now turn to the dynamic aspect and investigate what this gives of new insights to the analysis.

4) EQUILIBRIUM MODE OF ADJUSTMENT FOR A TEMPORARY SHOCK

Now the previously unanticipated, but known to be temporary, shock hits demand, changing it to $d(p) = a - bp + u$ in period 1 whereas it in period 2 is known to be $d(p) = a - bp$ again. Now the firm has to decide whether it wants to change its price, its quantity, neither or both in period one and then if it wants to change them back again in period 2. The monopolist has the same four different strategies in period one as it had in the permanent case. That is, it can chose (AA), (AD), (DA) or (DD). To each of these period one strategies correspond period two strategies. If the firm has chosen to adjust both its choice variables, it then again faces the same four different alternatives in period 2, as it had at the beginnig of period one. If it has decided only to adjust its price, then in period two the monopolist has three modes of adjustment: readjusting the price, leaving the price unchanged and instead adjust the quantity, or doing nothing. Similarly for the case where only the quantity is adjusted in period one. Finally, if the firm did not adjust in period one , it is not going to do so in period two either. This gives us the following eleven strategies, where A denotes adjust, D denotes don’t adjust, and the first character is for the price, the second for the quantity.

Period 1	Period 2
p q	p q
	A A
A A	A D
	D A
	D D
	A D
A D	D A
	D D
	D A
D A	A D
	D D
D D	D D

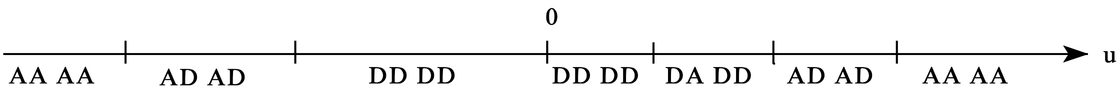
The profit functions to each of the eleven different strategies are calculated in the appendix. For ease of exposition and comparability amongst strategies in this section, the discount factor is assumed identically equal to one, without any loss of generality as long as we were to work with a constant discount factor. Each of the profit functions depends on the parameters in the demand function, the cost functions, and the size of the shock. That is $\Pi = \Pi(a,b,c,k,\alpha,\beta,u)$. This can easily get cumbersome without any restrictions on parameters and along the way some restriction are made. Naturally, these restrictions diminish the generality of the results, but from the outset it is hard to have strong opinions on different parameter constellations, and the configurations chosen bring out the possible mode of adjustment which can arise. The proofs of all statements and results are shown in the appendix. In the following, only the results and intuition wil be stated. We wil consider three scenarios starting with a direct extension of Andersen (1995) into a dynamic model. That is $k = 0$. We then proceed to the opposite case, so to say, where $\beta = 0$, while $k > 0$, and finally we consider the more general case where we assume $\alpha = \beta > 0$, $k > 0$. The symmetry requirement concerning α and β is not essential as will be demonstrated later.

CASE I: ONLY LUMP-SUM COSTS ON BOTH PRICES AND QUANTITES (k = 0)

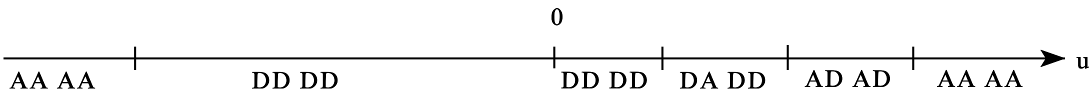
Result 1

Assume $k = 0$, $\alpha = \beta > 0$, $a > 5bc$

If $\alpha < 2bc^2$ then the mode of adjustment is:



If $\alpha > 2bc^2$ then the mode of adjustment is:



The strategies AD AD and DA DA are identical up to the adjustment costs

This result is an extension of the static model of Andersen (1995) into a dynamic, and therefore it clearly spells out the importance of making the model dynamic. However, some of the results remain the same. In particular, that adjusting prices or quantities have the same profit consequences up to the adjustment costs, when symmetric strategies are considered. As in his model we also choose one of the strategies as a single dominant mode, noticing that which one it is depends on the relative adjustment costs. Here we choose AD AD.

What we learn from extending the model from the static to the dynamic case, is that many strategies can be ruled out as equilibrium modes of adjustment. We observe essentially the same adjustment modes when it is taken into account that this is a two period model. However, and importantly, the mode of adjustment where only the quantity is changed in the first period, is an equilibrium when demand is sufficiently high ($a > 5bc$). If demand were less such that $a < 5bc$, then DA DD is no longer a possibility, and the zone of inaction would increase compared to the situation where a > 5bc. If the shock is negative, DA DD is not an equilibrium strategy, either. This is can be seen by examining what would happen if it did not adjust. Then in period one, there would be a loss of $c(\bar{q} - q_1^{DA})$ since the firm would produce too much. In period two there would be a gain of not adjusting of $\bar{p}(\bar{q} - q_1^{DA})$. Since $\bar{p} > c$, doing nothing is more profitable than the strategy DA DD when the shock is negative.

As we saw in the previous section, rationing emerges for small shocks where neither the price nor the quantity is changed. Result 1 shows that rationing can also prevail even when adjustment is carried out. The strategy DA DD implies rationing in the second period for the firm. It has committed itself to too high a production relative to the second period price. Hence, there is insufficient demand (Keynesian regime).

That DA DD is an equilibrium mode of adjustment turns out not to depend critically on the assumption of equal adjustment costs for prices and quantities. It can be shown that as long β is not much larger than α, DA DD is still an equilibrium. Hence, the conclusion reached by Andersen (1995):"if costs of changing prices are larger than changing quantites ($\beta > \alpha$), the case of keeping the price fixed and letting quantities bear the burden of adjustment will never appear"; is not a valid statement here. The intuition is that by pursuing the strategy DA DD the firm has the period two profit maximising price and actual sales are the profit maximising quantity. It just has too large a production in period two compared to actual sales. If the firm were to use the same strategy with price adjustment instead (AD DD), it would perceive a greater profit loss, since it would still have too high a production and furthermore a price that is not period two profit maximising. So even if it is more expensive to change the quantity than the price, quantity adjustment might still be observed if the shock is not too large and positive.

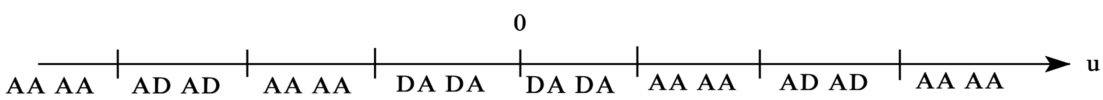
We note that the asymmetry between positive and negative shocks, causing more downward rigidity than upward, carries over to the dynamic setting. It can be shown that it requires a larger negative than positive shock (in absolute value) for the strategy AD AD to be an equilibrium. Hence prices are more downward inflexible than upward. This rigidity is much more pronounced when the lump-sum adjustment costs are relatively high ($\alpha > 2bc^2$), as AD AD is never an equilibrium for negative shocks. It should also be pointed out that the zone of no adjustment is in itself asymmetric.

<u>CASE II: NO LUMP-SUM COSTS IN QUANTITY ADJUSTMENT ($\beta = 0$)</u>

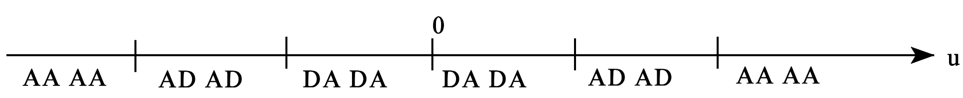
<u>Result 2</u>

Assume $\beta=0$, $a > 5bc$, $k < c/2$.

i) if $\alpha \in [3/8bk^2 ; bk^2]$ then the mode of adjustment is:



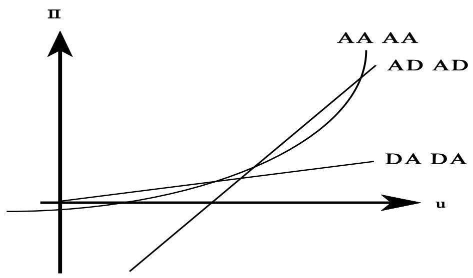
ii) if $\alpha \in]bk^2 ; 3bk^2]$ then the mode of adjustment is



Result 2 has some very interesting implications. We already know from section 3 that if the costs of adjusting quantities are of linear form, there will always be adjustment of some form to any strictly positive or negative shock. Furthermore, for small shocks the mode of adjustment is by quantities whereas it for larger shocks is by prices. If the shock is large enough, we observe changes in both price and quantity. What causes this asymmetry between adjustment by means of quantities or prices is of course the linear adjustment costs on quantities. For small shocks it is cheaper to use quantitities as the adjustment mechanism, whereas for a large shock the firm has to pay much more to change the quantity, since it is per unit, compared to price adjustment which is lump-sum.

Which of the two equilibrium strategy profiles (i) or ii)) that prevails depends on the size of the price adjustment costs. If these are relatively small ($\alpha \in [3/8bk^2 ; bk^2]$), we encounter the strategy AA AA in between adjusting either the quantity or the price. This may be difficult to explain intuitively, but part of the explanation is due to the menu costs beeing small relative to situation ii), since the range of shocks that makes DA DA an equilibrium strategy in situation i) is less than the range of shocks for which it is an equilibrium strategy in situation ii). Hence it is profitable to use AA AA for relatively small shocks when the price adjutment costs are relatively modest. Of course if the shock is sufficiently large there is full adjustment, i.e adjustment of both the price and the quantity, if the adjustment costs are not prohibitive. The reason for this peculiar adjustment pattern can be seen from the figure below, where the profit profiles are drawn for the involved strategies. Since DA DA and AD AD are linear with different intercepts and slopes and AA AA is quadratic, the situation depicted below is a possibility depending on AA AA's intercept with the y-axis.

Fig.1   illustration of result 2 i)



It should also be noticed that if $\alpha < 3/8bk^2$ then AA AA is not an equilibrium any longer, and we have the adjustment mode corresponding to result 2 i) without AA AA. In this case the unit quantity adjustment cost is high relative to the menu costs, so the monopolist will only use price adjustment even though the shock is large. To observe $\alpha > 3bk^2$ is not likely if the price adjustment costs are to be small, as they are usually thought to be. For k=c and middle sizes demand(a=6bc) it would require α to be a bit less than 20% of initial profit. That is not likely. Last but not least; the boundaries between the different strategies are symmetric for positive and negative shocks. A feature we know from section 3 does not relate specifically to the dynamic structure but the cost structure. So the findings in Andersen (1995) showing more downward inflexibility are mitigated in this case including only linear adjustment costs.

In result 2 it was assumed that $k < c/2$. If $c/2 \leq k < c$ then result 2 differs in merely one respect; the strategy DA DA is no longer viable for negative shocks, and this strategy should be replaced by “DD DD”, indicating that there is no adjustment for small negative shocks. The intuition is clear. Pursuing the strategy DA DA the firm adjusts back and forth and pays adjustment costs both ways. If it did not adjust, it would have to pay the marginal costs c per unit for too high a production. By adjusting the firm invokes the linear adjustment costs in both directions, that is

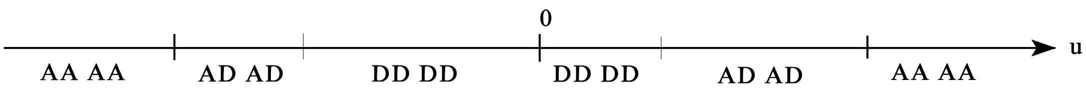
2k per unit. Since k > c/2 this is more expensive than doing nothing. Then it also follows that symmetry is broken. For small shocks there is more downward inflexibility than upward, but it is in quantities not in prices that this asymmetry occurs. However, prices are also rigid. The rigidity is just symmetric, since encountering the strategy AD AD requires an equal sized positive or negative demand shock.

CASE III: LUMP-SUM AND LINEAR COSTS ($\alpha = \beta > 0$, $k > 0$)

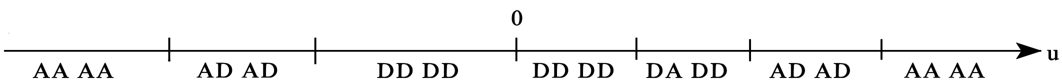
Result 3

Assume $\alpha = \beta > 0$, and $\alpha \in [3/16bk^2\ ;\ \bar{\alpha}]$, $0 < k < c$

if $5bc < a < 5bc + 4bk$, then the mode of adjustment is



if $a > 5bc + 4bk$, then the mode of adjustment is



Where $\bar{\alpha}$ is the upper level that still permits positive profits when adjusting[7]

The above result is a combination of the two previous and it enables us to study the effect of the cost structure in a dynamic setting. Let us first notice that we never observe quantities as the single adjustment mode if demand is relatively low. This is of course due to the assumption that

[7] If $\alpha < 3/16bk^2$ then AA AA is not an equilibrium since the linear quantity adjustment costs are large relative to the menu costs. Hence it becomes to expensive to use quantities at all.

$\alpha = \beta$ so that it is always more expensive to change the quantity than the price, since on top of the lump-sum quantity costs there are also the linear, and since demand is low the relative importance of the quantity adjustment costs is significant. It can be shown that if $\beta/\alpha \in [(1-(8bk)/(2a-2bc)); 1]$ such that the price adjustment costs are greater than the lump-sum quantity adjustment costs, we still observe price adjustment instead of quantity adjustment as long as $k>0$ and demand is low.

The second point to observe is that whether there is a linear quantity adjustment costs present or not, do not affect the equilibrium strategies, if demand is big ($a > 5bc+4bk$) (compare with result 1). The reason, that the market needs to be large for DA DD to be an equilibrium when $k > 0$ is, that when the firm has to pay the unit adjustment costs too, the relative of importance of these costs declines when demand is big, and hence the loss of not adjusting back in the second period has minor importance.

Even though the strategies are the same, independent of the linear costs, the consequences are not. When $k > 0$, more rigidity is created than when $k = 0$. If we compare result 1 and 3 it can be shown that all the boundaries between the different strategies are larger when the linear cost term is present. This has the interesting implication that when the costs of changing quantities increase, **prices** become more rigid (it requires a larger shock in abolute value to make DD DD and AD AD equilibria). Hence, introducing quantity adjustment costs does not necessarily lead to prices being more flexible. Instead the price rigidity can become more distinct. This indicates that the critique pointed at the traditional menu cost models, saying that there is a bias against quantity adjustment, is not vindicated by inclusion of quantity adjustment costs, in some cases[8].

5) CONCLUSION

Incorporating linear quantity adjustment costs into traditional menu cost models and at the same time analysing a dynamic model, yields new and interesting results compared to Andersen (1995). Considering a permanent shock to demand which allow us to focus on the importance of the adjustment cost structure, we note the following important findings: i) in order to have an asymmetry in adjustment, it is necessary that quantity adjustment considerations are taken into account, since traditional menu cost models do not predict this asymmetry (see bottom of p.8, though). Besides, the cost structure for quantities must involve some lumpiness, since without lump-sum costs, the adjustment modes are symmetric, ii) inclusion of linear adjustment costs gives rise to two new insigths comparing with Andersen (1995) a) there is always going to be adjustment of one kind or the other for any size of the shock when there are no lump-sum quantity costs , b) prices bear a relatively larger burden of adjustment when the shock is 'large', while quantities bear a relatively larger burden when the shock is 'small', iii) as long as there is any lump-sum cost component in the quantity adjustment costs, rationing will always prevail for 'small' shocks.

The dynamic aspect, here captured by analysing the optimal modes of adjustment to a temporary shock, does also yield new and striking results: i) A direct extension of the static model with only lump sum costs in Andersen (1995) to a dynamic turns out to give the conclusion that; even if

[8] Of course, there is less quantity adjustment than in the case without quantity adjustment costs, but that is not the point. If you attempt to comply with the critique by including quantity adjustment costs, the present analysis shows that the interaction between price and quantity adjustment costs can make price rigidity stronger.

it is more expensive to change the quantity than the price, we might still observe quantity adjustment. This is in stark contrast to the conclusions reached by Andersen (1995). ii) Rationing can also be an issue when adjustment is actually carried out, and not only for small shocks where there is total inaction. iii) The asymmetric adjustment behaviour to positive and negative shock, creating more downward inflexibility than upward, survives in the dynamic model. It does not even carry over, it is strengthened. In result 3 it was shown that when the costs of changing quantities increase (inclusion of the linear cost term), prices become more rigid. Hence, introducing quantity adjustment costs does not necessarily lead to prices being more flexible. Instead the price rigidity can become more distinct. This indicates, that the critique levelled at the traditional menu cost models, saying that there is a bias against quantity adjustment, is, in some cases, not vindicated by inclusion of quantity adjustment costs.

However, this model is carried out with a linear demand function and it would useful to examine whether the findings carry over with more general demand specifications. We do not have any suspicions why they should not. Finally the generality of the conclusions would be greatly improved, if they continue to hold for anticipated shocks as well, such that the firm can adapt its price and output decisions to events known to come. A conjecture is that it does not change the results. The same modes of adjustment will still be optimal, since the firm does not have an incentive to smooth the adjustment because there is no convexity in the cost structure. If there was some convexity it is likely to change the findings. Also an interesting topic for future research.

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APPENDIX

THE PROFIT FUNCTIONS

The monopolist maximises its two period profit, where it for simplicity and without loss of generality is assumed that the discount factor is identically equal to unity. The firm maximises profits for each period, taking the period one strategy as given when it maximises its period two profit. Let us consider each period at a time, starting with period one. Subscripts i=1,2 denote periods and j=+,- denote whether the shock is positive or negative. The initial price and quantity is given by:

$$\bar{p} = \frac{a+bc}{2b} \qquad \bar{q} = \frac{a-bc}{2}$$

<u>Period 1 (A A)</u>

From p.4 we obtain

<u>u > 0</u>

$$p_{1+}^{AA} = \frac{a+u+b(c+k)}{2b} \qquad q_{1+}^{AA} = \frac{a+u-b(c+k)}{2}$$

<u>u < 0</u>

$$p_{1-}^{AA} = \frac{a+u+bc-bk}{2b} \qquad q_{1-}^{AA} = \frac{a+u-bc+bk}{2}$$

with the following profit in both cases:

$$\Pi_{1j}^{AA} = \frac{(a+u-b(c+k))(a+u-bc+bk)}{4b} - k\frac{(|u|-bk)}{2} - \alpha - \beta$$

period 1 (A D)

This is independent of u.

$q_1 = \bar{q}$ and p_1 = (a + bc + 2u)/2b

$$\Pi_{1j}^{AD} = (\frac{a+bc+2u}{2b} - c)(\frac{a-bc}{2}) - \alpha$$

period 1 (D A)

This is independent of u.

$p_1 = \bar{p}$

With respect to q_1 we need to be a bit careful. By inserting $\bar{p}$ into the demand function, the maximum that can be sold is $\hat{q}$ = (a-bc+2u)/2. Only if $\bar{p}$ > (c + k) will $\hat{q}$ be sold, if the shock is positive. We will assume this to be the case. If the shock is negative, clearly the firm would not adjust any further than $\hat{q}$. This gives the following profit:

$$\Pi_{1j}^{DA} = (\frac{a+bc}{2b} - c)(\frac{a-bc+2u}{2}) \ - k|u| \ - \ \beta$$

period 1 (D D)

This is derived on p.4 and 5. The two profit functions are:

$$\Pi_{1+}^{DD} = (\frac{a+bc}{2b} - c)(\frac{a-bc}{2})$$

$$\Pi_{1-}^{DD} = (\frac{a+bc}{2b})(\frac{a-bc+2u}{2}) \ - \ c(\frac{a-bc}{2})$$

Now we turn to period two, and consider the different strategies given the strategy chosen in period one.

period 2 (A A), given period 1 (A A)

$p_2 = \overline{p}$ and $q_2 = \overline{q}$

$$\Pi_{2j}^{AA\ AA} = \left(\frac{a+bc}{2b} - c\right)\left(\frac{a-bc}{2}\right) - k\left(\frac{|u|-bk}{2}\right) - \alpha - \beta$$

period 2 (A D), given period 1 (A A)

$q_2 = q_1$ and $p_2 = (a-u+b(c+k))/2b$

$$\Pi_{2+}^{AA\ AD} = \left(\frac{a-u+b(c+k)}{2b} - c\right)\left(\frac{a+u-b(c+k)}{2}\right) - \alpha$$

$$\Pi_{2-}^{AA\ AD} = \left(\frac{a-u+bc-bk}{2b} - c\right)\left(\frac{a+u-bc+bk}{2}\right) - \alpha$$

period 2 (D A), given period 1 (A A)

$p_2 = p_1$

For a positive shock the price is too high relative to demand so the maximum that can be sold is $q_2 = (a-u-b(c+k))/2$ and this will be done if $p_2 > (c+k)$ which we assume. If the shock is negative, the price is too low relative to demand and there is no problem in selling q_2.

$$\Pi_{2+}^{AA\ DA} = \left(\frac{a+u+b(c+k)}{2b} - c\right)\left(\frac{a-u-b(c+k)}{2}\right) - ku - \beta$$

$$\Pi_{2-}^{AA\ DA} = \left(\frac{a+u+bc-bk}{2b} - c\right)\left(\frac{a-u-bc+bk}{2}\right) + ku - \beta$$

Period 2 (D D), given period 1 (A A)

$p_2 = p_1$ and $q_2 = q_1$

If the shock was positive actual sales is $\hat{q}$ = (a-u-b(c+k)) < q_2 so that there is too high a production relative to demand. If he shock was negative there is no problem in selling q_2.

$$\Pi_{2+}^{AA\ DD} = \left(\frac{a+u+b(c+k)}{2b}\right)\left(\frac{a-u-b(c+k)}{2}\right) - c\left(\frac{a+u-b(c+k)}{2}\right)$$

$$\Pi_{2-}^{AA\ DD} = \left(\frac{a+u+bc-bk}{2b}-c\right)\left(\frac{a+u-bc+bk}{2}\right)$$

Period 2 (A D), given period 1 (A D)

$q_2 = q_1 = \bar{q}$ and $p_2 = \bar{p}$

$$\Pi_{2j}^{AD\ AD} = \left(\frac{a+bc}{2b}-c\right)\left(\frac{a-bc}{2}\right) - \alpha$$

period 2 (D A), given period 1 (A D)

$p_2 = p_1$ and q_2 = (a-bc-2u)/2

$$\Pi_{2j}^{AD\ DA} = \left(\frac{a+bc+2u}{2b}-c\right)\left(\frac{a-bc-2u}{2}\right) - k|u| - \beta$$

period 2 (D D), given period 1 (A D)

$q_2 = q_1 = \bar{q}$ and $p_2 = p_1$

Again, if the shock was positive the firm's sale is less than its production, whereas there is no problem for the firm if the shock was negative. The consumers are rationed, though.

$$\Pi_{2+}^{AD\ DD} = \left(\frac{a+2u+bc}{2b}\right)\left(\frac{a-2u-bc}{2}\right) - c\left(\frac{a-bc}{2}\right)$$

period 2 (D A), given period 1 (D A)

$$\Pi_{2-}^{AD\ DD} = \left(\frac{a+2u+bc}{2b}-c\right)\left(\frac{a-bc}{2}\right)$$

$p_2 = p_1 = \bar{p}$ and $q_2 = \bar{q}$

$$\Pi_{2j}^{DA\ DA} = \left(\frac{a+bc}{2b}-c\right)\left(\frac{a-bc}{2}\right) - k|u| - \beta$$

period 2 (A D), given period 1 (D A)

$q_2 = q_1$ and $p_2 = (a+bc-2u)/2b$

$$\Pi_{2j}^{DA\ AD} = \left(\frac{a+bc-2u}{2b}-c\right)\left(\frac{a-bc+2u}{2}\right) - \alpha$$

period 2 (D D), given period 1 (D A)

$p_2 = p_1 = \bar{p}$ and $q_2 = (a-bc+2u)/2$

Again, the firm is rationed if the shock is positive and the consumers are rationed if the shock is negative.

$$\Pi_{2+}^{DA\ DD} = \left(\frac{a+bc}{2b}\right)\left(\frac{a-bc}{2}\right) - c\left(\frac{a-bc+2u}{2}\right)$$

$$\Pi_{2-}^{DA\ DD} = \left(\frac{a+bc}{2b}-c\right)\left(\frac{a-bc+2u}{2}\right)$$

period 2 (D D), given period 1 (D D)

$p_2 = p_1 = \bar{p}$ and $q_2 = q_1 = \bar{q}$

$$\Pi_{2j}^{DD\ DD} = \left(\frac{a+bc}{2b}-c\right)\left(\frac{a-bc}{2}\right)$$

Now, the total profit consists of the two single period profits added together. Doing this and considering profits net of the strategy DD DD, we obtain the following profit functions expressing whenever the particular strategy is optimal compared to doing nothing.

$\underline{u > 0}$

Strategy		profits	
AD DD	$-4u^2$	$+ u(2a - 6bc)$	$- 4b\alpha$
DA DD		$u(2a - 6bc - 4bk)$	$- 4b\beta$
AA DD		$u(2a -6bc -4bk)$	$- 4b(\alpha + \beta)$
AD AD		$u(2a - 2bc)$	$- 8b\alpha$
AD DA	$-4u^2$	$+ u(2a - 6bc - 4bk)$	$- 4b(\alpha + \beta)$
DA DA		$u(2a - 2bc - 8bk)$	$- 8b\beta$
DA AD	$-4u^2$	$+ u(2a - 6bc - 4bk)$	$- 4b(\alpha + \beta)$
AA AD		$u(2a - 2bc)$	$- 4b(2\alpha + \beta)$
AA DA		$u(2a - 2bc - 8bk)$	$- 4b(\alpha + 2\beta)$
AA AA	u^2	$+ u(2a - 2bc - 4bk) + 3(bk)^2$	$- 8b(\alpha + \beta)$

$\underline{u < 0}$

AD DD		$u(2a - 6bc)$	$- 4b\alpha$
DA DD		$u(2a - 6bc + 4bk)$	$- 4b\beta$
AA DD	$2u^2$	$+ u(2a - 6bc + 2bk)$	$- 4b(\alpha + \beta)$
AD AD		$u(-4bc)$	$- 8b\alpha$
AD DA	$-4u^2$	$+ u(-8bc + 4bk)$	$- 4b(\alpha + \beta)$
DA DA		$u(-4bc + 8bk)$	$- 8b\beta$
DA AD	$-4u^2$	$+ u(-8bc + 4bk)$	$- 4b(\alpha + \beta)$
AA AD		$u(-4bc)$	$- 4b(2\alpha + \beta)$
AA DA		$u(-4bc + 8bk)$	$- 4b(\alpha + 2\beta)$
AA AA	u^2	$+ u(-4bc + 4bk) + 3(bk)^2$	$- 8b(\alpha + \beta)$

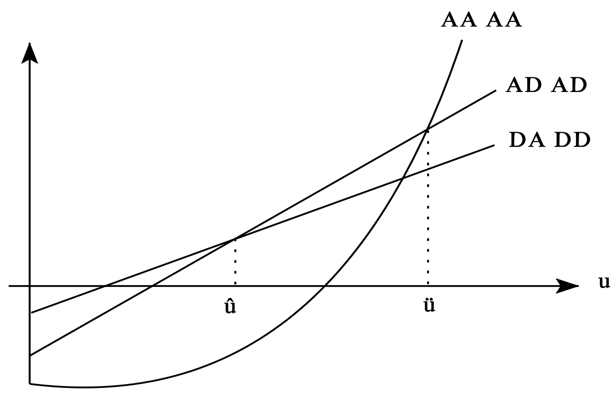
EQUILIBRIUM STRATEGIES

To find the equilibrium mode of adjustment, the different profit functions must be compared. Let us begin by eliminating those strategies that are dominated by others. A direct comparison reveals that as long as $\alpha > 0$, DA DD dominates AA DD and DA DA dominates AA DA for $u > 0$. For $u < 0$ we can see that DA DA dominates AA DA. Besides, AD DD and DA DD have negative profit associated with them. It can also be shown by comparing AD AD with AD DA, and DA DA with DA AD that the symmetric adjustment mechanism is preferred if α is not much larger than β, and both α and β in itself are not large. This is hard to imagine given the notion of small menu costs. This applies for any shock. This is as far as we can get without going into the three cases from the paper.

CASE I: $k = 0$, $\alpha = \beta > 0$

$u > 0$

We observe that DA DA dominates AA AD, that DA DD dominates AD DD, and that DA DA and AD AD are identical up to the adjustment costs. We choose to work with AD AD as the single dominant mode. This leaves us with three strategies to compare: AD AD, DA DD, and AAAA. Start with AD AD and DA DD and define û as the point where these are equal. This gives: $\hat{u} = \alpha/c$. Then we need to check which of the two cuts $\Pi = 0$ first. It can be shown by comparison that if $a > 5bc$ DA DD dominates AD AD for $u < \alpha/c$. If $a < 5bc$ DA DD is not an equilibrium mode of adjustment. Finally consider AA AA. This is a quadratic and we need to check whether it cut DA DD to the left of α/c or AD AD to the right. Define ü as the point where AD AD = AA AA. This gives us $\ddot{u} = (8b\alpha)^{½}$. Now check if $\ddot{u} > \hat{u}$. That implies $8bc^2 > \alpha$. This is certainly the case unless the menu costs are extremely high. This gives us the following picture



u < 0

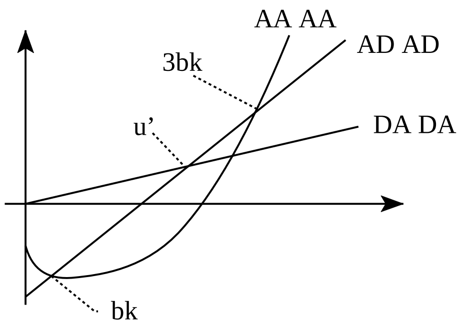
AD AD dominates AA AD. AD AD and DA DA are again identical up to the adjustment costs. Notice DA DD does never yield positive profit. Then we have to compare AD AD, AA DD and AA AA. Start with AD AD and AA AA. They are equal at $ü = -(8b\alpha)^{½}$ AD AD is zero at $û = -2\alpha/c$ and $|û| < |ü|$ if α is not very large. Finally, consider AA DD = AD AD and compare with ü. This reveals that $|ü| < |u(\text{AA DD} = \text{AD AD})|$. This gives us result 1. The asymmetry can be seen by comparing $û_+$ with $|û_-|$, and the symmetry by comparing $ü_+$ with $|ü_-|$

CASE II: $\beta = 0$, $k > 0$

u > 0

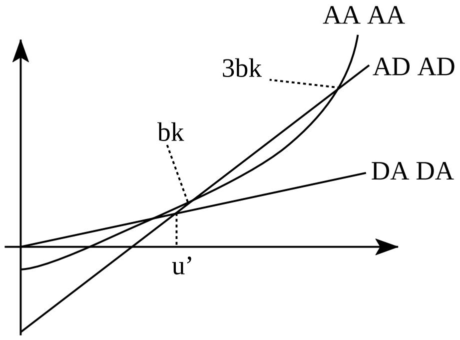
Here DA DA dominates DA DD. AD AD and AA AD are identical. For ease of comparability to the other cases, we choose to work with AD AD. The reason they are identical is because there is no discounting, otherwise AD AD would have dominated. This leaves us with four strategies: AD AD, DA DA, AD DD, and AA AA.

DA DA starts at origo while AD AD has a negative intercept but is steeper than DA DA. Hence, DA DA dominates to the left of u', where u' is the point where AD AD = DA DA. $u' = \alpha/k$. Now check whether AD DD can ever dominate. If it cuts AD AD to the left of u' it can never dominate as it is an inverted parabola. The condition is $bk < bc + \alpha/k$ which is clearly fulfilled as $k < c$. All we need to analyse now is when AA AA dominates. For this to be an equilibrium at all, it is required that $3(bk)^2 - 8b\alpha < 0$. This gives us $\alpha > 3/8bk^2$. Compare AD AD and AA AA. They are equal at bk or 3bk. This gives us three possibilities:



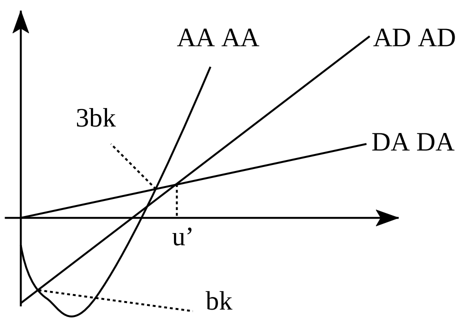
$\alpha/k > bk$

$\alpha/k < 3bk$



$\alpha/k < bk$

$\alpha/k < 3bk$



$\alpha/k > bk$

$\alpha/k > 3bk$

Rearranging these conditions give us result 2 for the positive part.

u < 0

This situation is symmetric to the case where u > 0, except that we need to consider the strategy AA DD. So except this strategy everything yield the same as for u > 0. Since $\partial AADD/\partial u|_{u=0} > 0$ and $\partial AAAA/\partial u|_{u=0} < 0$ we only need to compare the intercept of the two curves for the most negative u. Setting AA DD = AA AA, using the requirement that $q = (a-bc+2u)/2 > 0$ and comparing the two resulting u-values, it turns out that AA DD can never be an equilibrium. Hence, nothing changes to the situation where u > 0, and result 2 follows.

CASE III: $\alpha = \beta > 0$, $k > 0$

u > 0

AD AD dominates both AA AD and AA DA which follows immediately from comparing the profit functions. AD AD and DA DA are identical up to the adjustment costs and we choose to work with AD AD as the dominant mode of adjustment. With a bit more work it can be shown that AD AD dominates AD DD. To show this, find the positive u that equates AD AD and ADDD, and substitute this into the expression for AD DD. The resulting value will be negative unless α is very large, which is unlikely. Hence, AD DD can never have dominated AD AD. That leaves three strategies: DA DD, AD AD, and AA AA.

Compare AD AD with DA DD. There are two possibilities; either AD AD or DA DD cuts $\Pi=0$ first. $u^*|_{DA\ DD\ =\ 0} = 4b\beta/(2a-6bc-4bk)$ and $u^{**}|_{AD\ AD\ =\ 0} = 8b\beta/(2a-2bc)$. For $u^* < u^{**}$ we need $a>5bc+4bk$. Otherwise we do not observe DA DD as the adjustment mechanism. Now equate AD AD and DA DD. This gives $u'' = \alpha/(c+k)$. Finally AA AA cuts to the right of u”, and we require $3(bk)^2-8b(\alpha+\beta) > 0$. This yield the positive part of result 3.

u < 0

Here we know that DA DD cannot be an equilibrium. Direct inspection implies that AD AD dominates AA AD. Then we are left with three strategies AD AD, AA DD, and AA AA. Comparing AA DD and AA AA and applying the same argument as in case II for u < 0, we observe that AA DD cannot be an equilibrium. Thus, only AD AD and AA AA are equilibria. This completes result 3.

The asymmetry follows easily from comparing the relevant interceptions. Also that going from case I to case III yield more rigidity can be seen without difficulty by comparing boundaries.