Reconsidering the Case for Monetary Exchange

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Abstract
The literature on media of exchange has a standard example of indirect exchange being dominated by direct exchange. This case occurs under the assumption of additive transaction costs. In particular, if for an agent who is exchanging good i for good j, the cost of sale (s) and purchase (b) are represented by $C_i^s + C_j^b$ we find that introducing good m as a medium of exchange will rise his total transaction costs to $C_i^s + C_m^b + C_m^s + C_j^b$. Hence, the agent prefers direct exchange. From here it is frequently concluded that other costs (or settings e.g. incomplete information) have to be examined in order to explain the use of money (m). However, the present paper exemplifies that if double coincidences of wants are absent it can be beneficial to introduce money already for additive transaction costs. The additive transaction costs form represents the physical characteristics of goods. Hence, the paper demonstrates that the physical properties of media of exchange matter. We show under what conditions commodity money might emerge and when it will be dominated by fiat money in this setup. Finally we consider the possibility of agent specific transaction costs and resulting trading posts, again we find that a certain fiat money can dominate such arrangement.

JEL classification: D70, E49

Key words: Media of exchange, fiat money, additive transaction cost.

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1. In preparing this paper I benefited from discussions with Claus Vastrup, Ebbe Yndgaard, and Per Svejstrup Hansen.

1. Introduction

Two of the central issues in monetary economics are (1) why agents would rely on the use of apparently worthless paper money[2], and (2) why indirect exchange via media of exchange is preferred to direct exchange. This paper features a model addressing the latter issue, in particular we examine why certain modes and media of indirect exchange are preferred to others. Hereby it will be shown that the physical characteristics of money (low transport costs, cognizability) - which are usually in the literature represented by additive transaction costs - matter. Even though in textbook definitions of money these physical features have received a great deal of attention formal explanations of their importance have been rare.[3] The present model fills this gap, in particularly we show that - once double coincidences of wants[4] are absent - the physical characteristics of media of exchange might suffice in order to rationalize their use.

One branch of the literature on the second issue aims to focus on the way in which money can make exchange less costly 'an oil to the economic machine'.[5] However, when examining the cost of exchange there is a standard example in the literature which shows that for additive transaction costs direct exchange dominates indirect exchange Jones (1976, p.761), Oh (1989, p.103) and Ostroy and Starr (1990, p.40).

The example is the following: The cost (C_{ij}) to an agent who trads i for j is divided into costs of sale (s) and costs of purchase (b); in particular, $C_{ij} = C_i^s + C_j^b$, where $C^s, C^b > 0$. Facilitating this via indirect exchange instead, by invoking a good m the total cost of exchange would be $C_{im} + C_{mj} = C_i^s + C_m^b + C_m^s + C_j^b$ which is obviously greater that the cost associated with direct exchange.

$$C_{im} + C_{mj} = C_i^s + C_j^b + C_m^b + C_m^s > C_i^s + C_j^b = C_{ij} \qquad (1)$$

Since monetary exchange is per definition indirect it is concluded that the use of media of exchange could never be beneficial if transaction costs were only additive. For example Ostroy and Starr (1990, p.40) conclude:

> "Because arguments based on the physical characteristics of commodities or on transport costs typically have this additive form, they will have a limited role in

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2. Similarly, it is a puzzle why an agent would accept a commodity as a medium of exchange (commodity money) that might give him less utility than the good he gave up in exchange.

3. A notable exception is Kiyotaki and Wright (1989) who present a matching model - see below - in which the storability of goods (also an intrinsic physical feature) matters for their ability to become media of exchange.

4. A double coincidence of wants occurs when agent A wants to trade good 1 for good 2 and meets agent B who wants to trade good 2 for good 1.

5. For a compleat discussion see Niehans (1969, pp.706-708) or Jones (1976, pp.757-759).

> explaining the function of a medium of exchange as well as its spontaneous emergence."

Hence, the authors proceed to explain the use of media of exchange via other types of cost or restrictions on the environment. For example search costs, the time needed to find an agent willing to accept what one got on offer, here a common medium of exchange rapidly increases the probability of a match, see Jones (1976) or an extension of Jones model in Oh (1989).

Jones (1976, p.775) proposes on the underlying theme of this branch of models:

> "(...), the approach suggests that a very common good would emerge as a first commodity money in a barter economy. The important point is that this commonness is a market characteristic of goods rather than an intrinsic physical characteristic such as portability, divisibility or cognizability."

A more complex environment of matching and the resulting use of media of exchange is analysed in Kiyotaki and Wright (1989).

The analysis of this paper will show that - contrary to the above claimed - physical characteristics of goods, which are represented by additive transaction costs, might already be sufficient in order to rationalize the use of media of exchange. Hence, in relation to the quotation by Ostroy and Starr, this paper concludes that a setup of additive transaction costs suffices to explain the function of money. Additionally, this setup gives also insight into how certain modes and media of exchange might emerge within the economy.
The paper derives conditions on the relative size of transaction costs, stating when commodity and fiat money respectively might emerge. Whereas the search cost models have to rely on limited information in order to motivate the use of money, the agents in our setup know a great deal about the economy. In particular they know who has the good they want and who wants the good they have - still the use of media of exchange might be beneficial.

The model depends crucially on the absents of double coincidences of wants. However, this is no peculiarity in the context of monetary models. Also the models of Jones (1976) and Oh (1989) are driven by the absence of double coincidences of wants, without explicitly naming it. Agents do not know who got the good they want and who demands what they got. Hence, their models consider an agents subjective probability of encountering a trader that wants to trade i for j represented by $p_i p_j$. Where p_i (p_j) is the probability of dealing in i (j), which is set to be equal for both buy and sell probabilities. The number of expected encounters before a double coincidence of wants occurs (given that I want to trade j for i) is $1/p_ip_j$. Now their equation (5) states that indirect exchange (using good n) will be beneficial if $1/p_ip_j > 1/p_ip_n + 1/p_np_j$, since waiting is costly. As p_ip_j goes towards zero, the condition will be fulfilled for some medium of exchange n. Of course is $1/p_ip_j$ nothing but a measure of the degree of absents of double coincidences of wants; as the expression rises the possibility of a double coincidence shrinks, not because the required trade partner does not exist but because he can not be found.

Also Ostroy and Starr (1974) depend on the absents of double coincidences of wants. Opposed to both the Jones type model and the present model they do not utilise transaction costs, but rather examine what trading rule can bring about efficient exchange. Agents only know who got the good they want and who desirers the good they have, but do not know how the chain of consistent desires with no double coincidences is composed, i.e. agents do not understand what intermediate good will be acceptable to their trade partners. Hence, they highlight how common media of exchange can circumvent the huge informational requirements of barter trade, that emerge as soon as double coincidences of wants are absent.

By explicitly using additive transaction costs and the absents of double coincidences of wants, the present model merges the literature on transaction costs as in Jones (1976) and Oh (1989), with the examination of indirect exchange in the tradition of Wicksell or in Ostroy and Starr (1974).

The present approach is not intended to be a more realistic description of the benefits of using money, in fact I would argue that the search cost models are preferable in terms of realism. However, the model proofs that already in one of the simplest environments possible - namely by using additive transaction costs - the introduction of media of exchange might be beneficial. Moreover, since additive transaction costs are usually taken to represent the physical properties of goods, we end up at a model that illustrates that the physical features of money matter.

Section 2 describes the setup of the present model, states the core assumptions and defines the central terms, in particular what type of costs the examined additive transaction costs might be. Section 3 proceeds with a simple example that falsifies the claim of the inferiority of indirect exchange which was given in inequality (1). We follow up with a generalisation in section 4, propositions on the conditions for the beneficial use of media of exchange are derived. Section 5 discusses the emergence of common media of exchange. In section 6 we extend the present model considering disequilibrium prices; further the possibility of trading posts and agent specific transaction costs are examined, in particular we will state when fiat money can dominate an otherwise superior trading post arrangement. Finally section 7 concludes the paper.

2. The Model Setup

The model features agents and goods of a similar specification as in Jones (1976) and Oh (1989). However, agents in our setup have full information, and the assumptions on transaction costs and exchange differ.

The agents or traders of the model produce each a different good (one unit per trading round). Agents utility functions depend on the ultimately consumed good, the good that would maximise their utility is produced by one of the other agents. Hence, there is the need for exchange. Alternatively, we could assume that in each trading round agents exchange towards a different good, thus composing their optimal bundle over several rounds. Goods are measured (produced and desired) in units such that the price of goods in terms of some abstract numeraire is equal to 1. Thus, the only possible exchange is of the nature one unit for one unit. Agents production and desires are such that market clearing will prevail at given prices. This setting amounts to Walrasian auctioneer prices.

At the beginning of a trading round the agents have a trade proposal T_{ij} which denotes their desire to trade i for j. Within a trading round there are several trading instants, which each consist of one or more bilateral matches of pairs of agents. Since the agents in the model know the T_{ij}'s of other agents they can willingly aim at other agents. Exchange can only be executed at quid pro quo. That is, when ever an agent A gives up a good to agent B he needs to be compensated by B immediately. The first trade instants will be cases of double coincidence of want, in particular such that an agent with T_{ij} meets an agent with T_{ji}. After all double coincidences of wants are exhausted circular chains of no-double coincidences of wants will prevail. That the remaining exchanges always can be decomposed into one or more circular chains is formally shown by Ostroy and Starr (1974, pp.1100-1). The shortest chain would be of the nature T_{ij}, T_{jk}, T_{ki}. This resembles Wicksell's popular example of A wanting to trade wheat for fish, B wanting to trade fish for timber and C wanting to trade timber for wheat. Larger circles might remain; we denote the number of agents involved in a particular chain by n.

Exchange is costly. In particular the transaction costs are of an additive nature. Also are the costs to one agent not revenue to any other agents, we think of them as executed in terms of time or effort. One part of the cost is associated with the sales action and one with the buying action. Hence, the trade proposal T_{ij} has the associated basic transaction cost C_{ij} which in turn is given by C_i^s and C_j^b such that $C_{ij} = C_i^s + C_j^b$. The different C_i^s and C_i^b are known to all agents for all goods i.

Agents take prices and transaction costs as given. Utility maximisation ensures that the sequence of meeting other traders and engaging in exchange minimises total transaction costs. However, here emerges the problem that a trader might not want to participate in exchange after all, at the resulting transaction costs. The standard solution to this conflict is to assume that agents expectations of encountered transaction costs are ex-ante correct.

Finally, the model allows for the introduction of fiat money m. Following Laidler's (1992) definition in the new Palgrave dictionary, fiat money has two features, it is government issued and represents no claim on either a commodity or an asset. Hence in our model, fiat money is a good which gives no utility to any of the agents.[6] One unit is handed out to every agent by a government. The government enforced budget constraint demands that by the end of a trading round each agent must end up at the same m balance that they started with. Put differently, only between trading instants may agents accumulate or deteriorate their holding of m. Negative m holdings (credit) are not permitted. As mentioned in the introduction and as discussed by Laidler the puzzle about fiat money is how its value is determined, why agents adjoin value to intrinsically valueless paper.[7] The present paper does not discuss how the value of fiat money is obtained. To be explicit: This model proposes **no** theory of price formation!

What type of transaction costs might be described by the additive form? Costs of transporting goods to and from the location where another trader is encountered. Costs of information exchange, were agents have to describe the good they offer or describe what they desire. Inspection cost might be a classic interpretation: agents have to verify what good it is they offer and/or invest effort into examining the good they are about to purchase. This might include cost of weighting and measuring, in general costs associated with assessing the value of goods. Note that here the cognizability (mentioned in the above quotation) of goods comes in. We can also reinterpreat storage costs into the additive form from above. In a different setting Niehans (1978, pp.113) shows how (within period) storage costs can motivate the use of media of exchange. In particular, in order to impose the additive transaction cost form we assume that exchange (trade instants) would occur inbetween periods while production and consumtion would occure in the middle of each period. Like this the initial owner and the final consumer would only endure half a period of storage costs, while agents that use a good in intermediate exchange encounter a full period of storage costs.

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6. In fact it is not important whether or not the good m gives utility, since it can due to the budget constraint never be consumed anyway.

7. Laidler (1992) points out that commodity money and credit money really feature the same puzzle, by exchanging for values higher than their intrinsic value.

In all the examples the physical features of goods matter for the size of the cost.[8]

3. A Simple Example

We turn now to a simple example which shows that in the above setup with additive transaction costs fiat-monetary exchange might be preferred to non-fiat-monetary exchange. Hence, the conclusion derived from inequality (1), namely that monetary exchange will not be beneficial in settings of additive transaction costs is falsified.

The pitfall of the analysis in (1) is that we have not considered the costs endured by the other traders involved. In particular we have only considered the situation of trader i. As soon as we have a circular chain of traders with no double coincedents of wants the situation changes. Say we are after the first instants of trade left with a circle of no-double coincidences of wants of size n=3.

Agent 1	**Agent 2**	**Agent 3**
T_{ij}	T_{jk}	T_{ki}

Remember that traders can only meet bilaterally, and have to deal at quid pro quo. For example one possible pattern of resolving the situation would be that agent 2 first purchases a good he does not want in order to exchange it for the good he wants in a later instant. Any agent could take on this role via simple relabelling.

We get the following cost pattern for non-monetary bilateral exchange of this kind:

C_{ij} $\qquad$ $C_{ji} + C_{ik}$ $\qquad$ C_{ki}

Which gives:

(*) $\quad = C_i^s + C_j^b \qquad = C_j^s + C_i^b + C_i^s + C_k^b \qquad = C_k^s + C_i^b$

Agent 2 has purchased good i in order to sell it on to agent 3.

If we now introduce fiat money m in the above mentioned fashion the situation changes. For

8. Note that most of the suggested transaction costs have a time dimension. In general the important thing about money is time - as pointed out by Keynes. However, the literature on indirect exchange does not model this dimension explicitly - the present model continues in this tradition.

example assume agent 1 meets agent 2 (A1, A2). They have a j match (C_j^b, C_j^s) which they can cover by a m transaction (C_m^s, C_m^b). Note that Agent 2 now has more m than at the beginning of the round. The costs of this trade for each agent are:

$= C_j^b + C_m^s$ $\qquad$ $= C_j^s + C_m^b$ $\qquad$ $= 0$

Now we match (A2, A3). They will exchange k (C_k^b, C_k^s) via m (C_m^s , C_m^b). The costs of this exchange are:

$= 0$ $\qquad$ $= C_k^b + C_m^s$ $\qquad$ $= C_k^s + C_m^b$

Now agent 3 got an additional unit m. Finally we match (A3, A1), they have a double coincidence of want. Agent 3 will buy good i from agent 1, paying with his extra unit of m; the markets are cleared. The costs of this exchange are:

$= C_i^s + C_m^b$ $\qquad$ $= 0$ $\qquad$ $= C_i^b + C_m^s$

It is now obvious that we can represent the costs of monetary exchange by:

$C_{im} + C_{mj}$ $\qquad$ $C_{jm} + C_{mk}$ $\qquad$ $C_{km} + C_{mi}$

Which gives:

(**) $= C_i^s + C_j^b + C_m^b + C_m^s$ $\qquad$ $= C_j^s + C_k^b + C_m^b + C_m^s$ $\qquad$ $= C_k^s + C_i^b + C_m^b + C_m^s$

The cost terms in (**) do cover any sequence of traders meeting, given that they whenever they find a match of non-monetary goods (i.e. one wants the good the other got) facilitate this by using m.

When will monetary exchange (involving fiat money m) be preferred to non-monetary exchange? Whenever the sum of cost terms in (*) is greater than the sum of cost terms in (**), this is the case if the condition,

$$\frac{1}{3}(C_h^s + C_h^b) > (C_m^s + C_m^b) \qquad \forall\ h = i,j,k. \qquad (2)$$

is fulfilled.

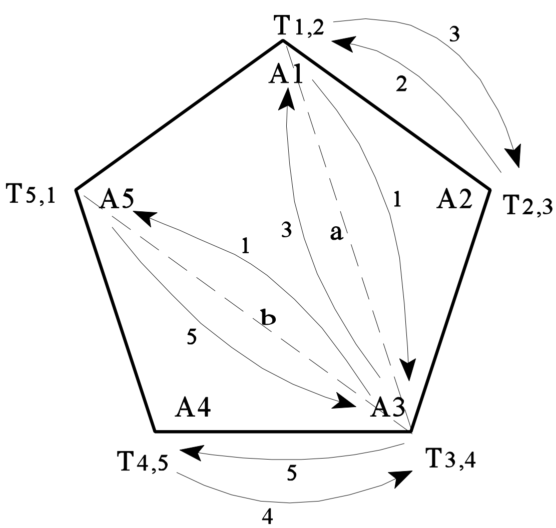
This condition says: if the transaction cost of using money are less than 1/3 of the transaction costs of using any of the goods, then monetizing the economy will be beneficial. This result emerged because we had an initial situation with no double coincidences of wants, and imposed a quid pro quo condition.[9]

4. Generalisation

We turn now to consider the conditions for monetary exchange in circular chains of no-double coincidences of wants of size n. By deriving such conditions we have to note that the economy we describe is only ever partially monetized, since the first trade instants of double coincidences are cases of pure barter. This appears to be somewhat natural, since a model where the exchange of a meeting between T_{ij} and T_{ji} would be facilitated with money, is considered odd.

The circular chain consists of agents A_i where i = 1, ..., n. Agents are labelled such that A_i got a trade proposals $T_{i,i+1}$, and the corresponding basic transaction cost $C_{i,i+1} = C_i^s + C_{i+1}^b$ for i = 1, ..., n-1. Agent n is described by $T_{n,1}$ and $C_{n,1} = C_n^s + C_1^b$.[10]

Figure 1



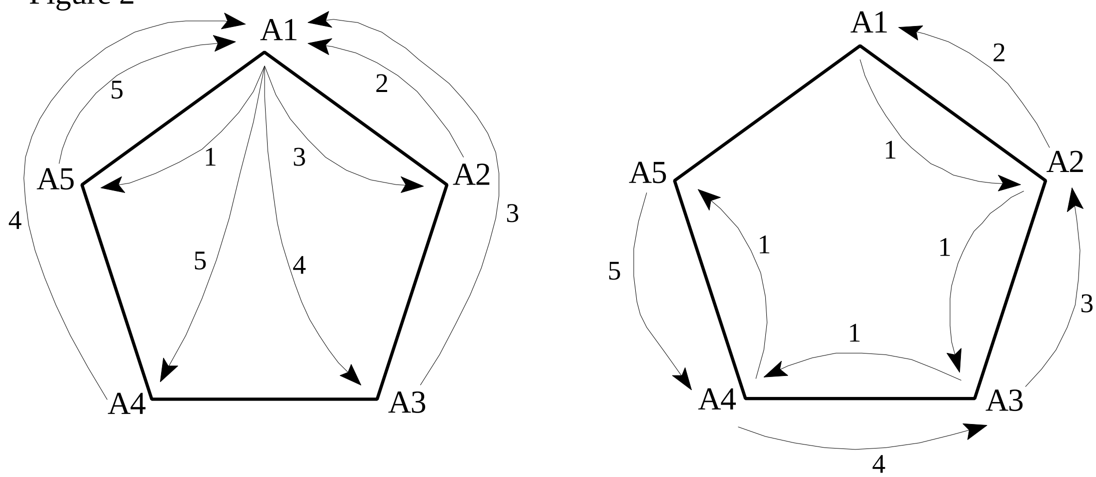
9. Note, that if we only considered the first or third column of the above example we would end up at inequality (1), the example of Jones (1976), Oh (1989) and Ostroy and Starr (1990), were the use of media of exchange would never be beneficial if transaction cost are only additive.

10. Even if we had predetermined agent and good labels, for example such that a chain T_{13}, T_{34}, T_{42}, T_{21} is obtained, then we can obviously relabel in the proposed fashion. Agent and good 3 become 2; agent and good 4 become 3; agent and good 2 become 4.

For now the agents are not furnished with money m. There are obviously a large number of ways in which the chain might be collapsed.[11] Consider the example of n=5, shown in figure 1.
One way of solving this circle (while still enforcing quid pro quo) is that first A1 meets A3, agent 1 gives good 1 in exchange for good 3 (a). With good 3 agent 1 can now deal with agent 2. They got a double coincidence of want. Agent 3 can sell good 1 to agent 5 were he gets payed with good 5 (b) which agent 4 is willing to exchange into good 4. Alternatively agent 3 could sell good 1 to agent 4 who than sells it on to agent 5.

In the following we are only interested in two special modes of solving the circular chain. Agent facilitated exchange and good facilitated exchange. The two forms of exchange are illustrated for n=5 in figure 2.

Figure 2



2a: Agent Facilitated Exchange

2b: Good Facilitated Exchange

With agent facilitated exchange we denote the case where an agent A_k (A_1 in panel 2a) gives away his good k to agent A_{k-1} (A_5 in panel 2a) receiving good k-1. Now agent k exchanges k-1 with A_{k-2}, and so forth. Finally agent k deals with agent k+1, giving him good k+2 which is the good A_{k+1} wants, in exchange A_k receives good k+1 which is exactly what he desires. The chain is collapsed via agent k taking in turn n-2 goods in intermediate exchange. The total transaction costs (TC) of agent k facilitated exchange are:

$$TC_a^k = \sum_{i=1}^{n} (C_i^s + C_i^b) + \sum_{i=1; i \neq k, k+1}^{n} (C_i^s + C_i^b) \qquad (3)$$

11. Niehans (1969) takes a closer look at this issue, also Ostroy and Starr (1974) formalise on the issue of what kind of trading procedure can collapse such chains.

Note that if k=n then k+1=1, due to the circular setup.

Goods facilitated exchange on the other hand implies that agent k (A_1 in panel 2b) gives good k to A_{k+1} (A_2 in panel 2b) receiving in exchange good k+1 which is the good he desires. Agent k+1 (who now got good k) turns to A_{k+2} in order to receive good k+2 while paying with k. Like this, the trade continues, each time using good k (good 1 in figure 2 panel 2b), until agent k-2 receives good k and exchanges it with agent k-1. Note that now k-2 and k-1 have a double coincidences of wants. So all agents except agent k and k-1 did use good k in intermediate exchange.[12] The total transaction costs (TC) of good k facilitated exchange are:

$$TC_g^k = \sum_{i=1}^{n} (C_i^s + C_i^b) + (n-2)(C_k^s + C_k^b) \tag{4}$$

Good k in the good facilitated exchange is in fact a medium of exchange in particular we talk of it as commodity money. On the other hand can agent facilitated exchange be taken as a representation of a trading post driven exchange. Note also that in the n=3 case of section 2 both modes of exchange and their total cost appear identical.

Considering the way in which contact between agents is made we observe that in agent facilitated exchange the order of contact among agents is in an anti-clockwise fashion, while for goods facilitated exchange the contacts are made in a clockwise fashion (figure 2). In a different setup with given trade flow directions Krugman (1980) finds a similar distinction for payment flows in a n=3 case.

We find now the good k for which,

$$(C_i^s + C_i^b) > (C_k^s + C_k^b) \qquad \forall\ i=1,...,n;\ i \neq k \tag{5}$$

is fulfilled, i.e. k is the good with the lowest transaction cost. Such good k will exist for every circular chain of no double coincidences of wants.[13] It is now obvious that $TC_g^k < TC_a^i$ (i = 1, ..., n). Any agent facilitated exchange will have higher total transaction costs than the good k facilitated exchange. Hence, commodity money of the type specified in (5) is preferred to any trading post arrangement. We return to this point in section 6.

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12. Note that if k=1 then k-1=n.

13. Note that stating condition (5) is only made possible due to the additive nature of transaction costs in our model.

It is now possible to state:

> *Proposition 1:* In an economy of the kind described in section 2, featuring additive transaction costs in exchange and **no** fiat money, a good facilitated exchange using the commodity money k which fulfills (5) is the cheapest exchange possible that can solve the circle n of no-double coincidences of wants.

Proposition 1 can easily be shown to be true. A circle of n agents needs at least n-1 bilateral contacts to clear (due to the quid pro quo requirement). Hence, there are 2(n-1) cost terms C_{ij}. However, n of these terms are basic standard cost terms[14] of the form $C_{i,i+1}$ (these are accounted for in the first term of the RHS in (3) and (4)). Finally, we are left with n-2 other cost terms which will be minimised by using k, since k is the good with the lowest transaction costs.

When we now turn to consider the use of fiat money, proposition 1 allows us to focus solely on the comparison between fiat money and commodity money k fulfilling (5). In particular we ask when will it be cheaper to use fiat money instead of commodity money k. First let's determine the cost of monetary exchange using money m.
Once the agents in a circular chain are equipped with money m in the manner described in section 2, and use money m to facilitate the ultimate exchange, the structure of costs is different. In particular has each agent i in addition to the basic transaction costs terms C_i^s and C_{i+1}^b, specific cost terms C_m^s and C_m^b. A number of exchanges[15] using m will have the same minimized total cost structure. The total cost (TC) of fiat money m facilitated exchange are:

$$TC_f^m = \sum_{i=1}^{n} (C_i^s + C_i^b) + n(C_m^s + C_m^b) \tag{6}$$

For example can any agent i go to agent i+1 and buy the good i+1 in exchange for m. But A_i will also be looking for a buyer to his good i, since he needs to earn a monetary unit m in order not to violate his budget constraint at the end of the trading round. This buyer is A_{i-1}.

In terms of figure 3 we can see that money m will flow one step in a clockwise direction while

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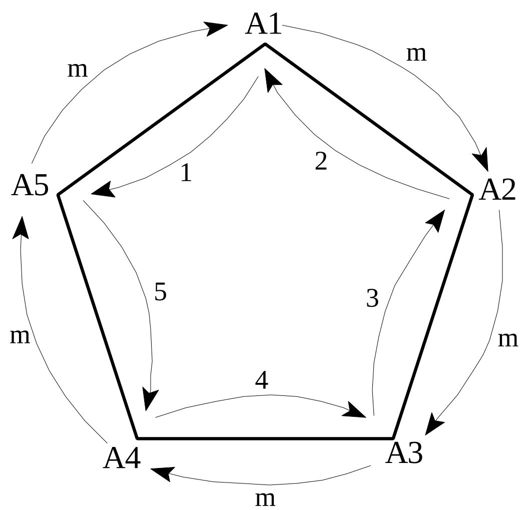
14. The standard cost terms correspond to the basic costs that each agent must assume simply by getting rid of what he is endowed with and by receiving what he ultimately wants. These costs can never be circumvented.

15. There are a number of possible exchanges that will minimize total transaction costs - in particular the chain could start to collapse at any of the agents or simultaneously at several points. Note that in all forms of exchange discussed here (agent-, good- or commodity money-, fiat money-facilitated exchange) there are a host of inefficient exchanges possible, for example could agents always exchange back and forth any good; thus exploding the total transaction costs.

the goods flow each one step in anti-clockwise direction. Agents can contact their neighbours in what ever fashion, and in part simultaneously, a feature the good facilitated exchange did not have. Also we note that in fact only one unit m given to one of the agents could facilitate the clearing of the circle pretty much in the fashion of good facilitated exchange defined above, and would still have the total cost TC_f^m given in (6).[16]

Figure 3



It is now obvious that fiat money will be beneficial to use if $TC_f^m < TC_g^k$ for good k fulfilling (5). This leads to the following proposition.

Proposition 2a: An economy of the kind described in section 2 featuring additive transaction costs in exchange will benefit from the introduction of fiat money m if

$$\frac{(n-2)}{n}\;(C_k^s + C_k^b) \;>\; (C_m^s + C_m^b) \qquad (7)$$

is fulfilled for good k fulfilling (5).

Proposition 2a follows directly from (4) and (6) and states that a fiat money will only ever benefit

16. Lets say agent i is the only one endowed with a unit of m; at first he would purchase good i+1 with m, at the end of the trading round he would sell his good i for m to agent i-1, such that i dose not violate his budget constraint.

an economy if the transaction costs of using this money are less than the cost that using any of the goods as a medium of exchange would cause. In particular does condition (7) (which is the general form of condition (2) in the example of section 3) say that m has to have transaction costs associated that are less than the fraction ((n-2)/n) of the transaction costs of the lowest cost good k.[17] This difference in the terms in (4) and (6) namely the (n-2) and (n) part stems from the fact that in the commodity money case there are two agents that deal anyway in k, the agent that is endowed with it (A_k) and the agent that desires it (A_{k-1}). Once the economy is monetised with m they also endure each the extra cost terms $C_m^s + C_m^b$.

The statement of proposition 2a, namely that the economy will benefit from monetization if fiat money is cheaper to use than goods is intuitively compelling.[18] It corresponds nicely to some of the physical features of modern fiat money: Extremely low transport cost. Easy cognizability and verification of value and kind. Low storage cost as long as there is low inflation. From the last point it is easy to see that barter trade and commodity money will reemerge in a high inflation environment!

Putting proposition 2a differently we can restate:

> *Proposition 2b:* For a good k fulfilling (5) and fiat money m let there be an ε>0 such that
>
> $$C_k^s + C_k^b = C_m^s + C_m^b + \varepsilon \ .$$
>
> Then for any ε there will always exist some n^* such that for all - good k including - circular chains of no double coincidences of wants with $n>n^*$ condition (7) is true.

2b follows directly from proposition 2a. Proposition 2b states that if there exists a fiat money (m) that is marginally cheaper to use than the lowest transaction cost commodity money, then for a large enough circle n it will be beneficial to use money m. So for larger economies - where hence the possibility of large circular chains of no double coincidences of wants exists - it will be more probable to find fiat monetization.

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17. Note that this analysis only considers the privately endured costs of transaction. There is also an argument of the social costs of tying goods down as media of exchange, hence keeping them from being consumed. Again, utility-less fiat money reduces this social costs. However, this argument is not pursued in the present model.

18. Also note, that since money m can be used simultaneously by agents, a chain of size n is collapsed within at most 3 trade instants, while a commodity money driven exchange requires n-1 instants. However, since the time dimension is of no concern in the present paper, we ignore this additional advantage of using fiat money.

Also note that from condition (7) it follows that for a two person economy fiat monetisation will never be beneficial.[19]

5. Emergence of Money

While we so far only considered the question of when it will be beneficial to use money we have also to address the issue of whether the agents would at all want to participate in a monetized exchange and how a common medium of exchange might emerge.

The underlying problem is that it appears odd to impose a trade desire T_{ij} on agents without them knowing the transaction costs to be endured - since these depend on the form of exchange. Also, by moving from one mode of exchange to another, some agents might endure higher cost than in the initial mode. In particular it is obvious that any agent in a chain n can by starting a goods driven exchange minimise his individually endured costs even so his good might not fulfill (5) and hence be collectively undesirable. Similarly, once fiat money is introduced, the agents k and k-1 (for k fulfilling (5)) that now face each $C_m^s + C_m^b$ higher transaction costs might not wish to accept it.

There are two responses: Firstly, we have to see that also in the basic solution to any situation of no double coincidences of wants will some agents be exposed to extra transaction cost if quid pro quo is imposed - i.e. our base case suffers already from the question of why agents at all would participate. That is why we assumed agents expectations of encountered transaction costs to be ex-ante correct.

Secondly, we have stated conditions in the above text that are of the Kaldor-Hicks criterion type. Hence, in principle the gainers of moving from one form of exchange to another could compensate the losers. However, it would be disturbing to assume that this compensation is actually executed in a setting where transaction is costly - had we a means of compensating with zero transaction costs our whole problem would vanish. Expanding on the notion of the Kaldor-Hicks principle, we might reinterpret the exercise of section 4 as the maximisation of a social welfare function:

$$\max_{x,y} W = -TC_x^y$$

subject to the desired allocation being obtained. The social planner can decide upon the mode x and the means y of exchange.

The solution imposed above is jointly cost minimizing - nothing more. From this discussion it

19. This is so since in a two person economy there can be no absents of double coincidence of wants.

should become clear that a model that wished to tackle these issues more explicitly has to consider the bargaining structure of each trade encounter.

Still we can make some considerations on the emergence of common media of exchange. One condition for an exchange to occur must be that both parties agree. In the present model agents do know the different $C_i^s+C_i^b$ of the goods, hence they also know which good k fulfills (5). If any other agent i (i≠k) tries to sell his good i on to A_{i+1} (who got what A_i wants) then agent i+1 would not except i but demand k. Also agent k him self will be interested to trigger the collapse of the chain, since if he let any other agent do it he would get two extra cost terms (from the then circulating good). It is easy to imagine how the information on which good is the cheapest to use could derive from experience in a repeated setup.
How about the emergence of fiat money? Obviously all agents but two will willingly use fiat money that fulfills condition (7) in proposition 2a if the alternative exchange was goods facilitated via good k fulfilling (5). The two critical agents are k and k-1 who actually get higher transaction costs than before. However, the collapse of the chain can happen simultaneously at a multitude of places, no activity by k and k+1 is needed at first. Finally, agent k dealing with A_{k+1} will find that k+1 desires good m, while A_{k-1} can not get offered k by A_{k-2} but only gets offered m in exchange for his good k-1. This implies that a chain n can never be partially monetized. In fact, with fiat money the agents of a chain can pre-commit them self to using the good m, and like this force agent k and k-1to do the same.
In general it applies that once an agent meets the agent that has the good he desires, he will find that this trader prefers k (m - except for trader k-1) in return; where k (m) fulfills (5) ((7)). Hence commodity money (fiat money) emerges via its general acceptability.

The fiat money of our model is government imposed, however some agent, that has for some reason high credibility, could start a credit money by writing out an IOU, assuming that the associated transaction costs fulfill (7). The IOU would travel through the chain (clockwise) and solve it. The problem here is how to keep other agents from doing the same? Since otherwise the last trade instants would be pure IOU clearing. Finally, we should note that the verification of IOUs might be difficult for rising n hence they actually have transaction costs so high that they violate (7) and will not be used in large circular chains.

6. Extensions

In this section we consider two extensions of the model. First we consider what would happen if the prices in fiat monetary terms were distorted. Second we examine the implications of agent specific transaction costs. Hereby, the case of an agent facilitated exchange that dominates commodity money is considered.

How would wrong - non Walrasian - money prices affect the model?[20] In particular we want to see if - once fiat money quoted prices are in disequilibrium - commodity money trade will reemerge. In general this type of models, with imposed relative prices equal to 1 do not allow for this sort of experiment, nevertheless we can discuss the issue. A distortion in prices would for example be if at one point in the circular chain good i would have a price of 2m and not 1m. Hence, A_{i-1} would expand his individual transaction cost terms by an extra C_m^s, ending up at $C_{i-1}^s + C_i^b + C_m^b + C_m^s + C_m^s$ while A_i has individual transaction cost terms $C_i^s + C_{i+1}^b + C_m^b + C_m^b + C_m^s$. Now there are two things happening here.

Firstly, one unit of m is permanently transferred from A_{i-1} to A_i. Hence, both their budget constraints are violated. If the budget constraints are binding the fiat money facilitated exchange is no longer possible, however there is no reason why a commodity money could not be used. But lets instead say that excess money holdings are permitted and that A_{i-1} is in each trading round endowed with a new unit of m. Then A_i collects over time more and more m with which he can not purchase anything - representing the situation of a monetary overhang.

Secondly, there occurred extra transaction costs $C_m^s + C_m^b$, that might for sufficiently distorted prices result in a violation of the condition for fiat money efficiency. In particular it might turn out that $TC_f^m > TC_g^k$ for k fulfilling (5); hence commodity money k would be preferred.

This discussion corresponds with the observation that price distorted economies experience an increase in barter trade.

The second issue I want to rise is the possibility of agent specific transaction costs. The model of this paper has besides the assumption of additive transaction costs also imposed that transaction costs are associated with goods and not with agents. However, if transaction costs are associated with for example verification and cognizability then these cost might differ across agents rather than across goods. Some agents might have better (and cheaper) opportunities, to represent the goods they want to sell or to asses what they get offered, than others.

If we design the perfect mirror image case to the above, we have to assume that dealing with any of the goods triggers some fixed cost for the different agents: say $A_i^s + A_i^b$, is the cost of sale and buy that agent i has when dealing with any good.

If we now find agent k such that

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20. We can think of this situation as a centrally planned economy were wrong money prices are imposed.

$$(A_i^s + A_i^b) > (A_k^s + A_k^b) \qquad \forall\ i=1,...,n;\ i \neq k$$

is fulfilled, then it is obvious that agent k is the preferred trading post. This trade pattern will emerge, given that the other traders can compensate k, for his extra costs! We can also see, that no fiat money could improve on this arrangement - since fiat money would also trigger the costs $A_i^s + A_i^b$ for all the agents.

However, this interpretation of agent specific transaction costs appears odd, it seems more natural to assume that goods vary in the size of transaction costs they trigger. Hence, we suggest, that even so transaction costs are agent specific, they also vary among goods. In particular is the ranking of all goods according to the associated transaction costs identical for all agents.[21]

We can still find an agent k who is the preferred trading post. There is an agent k for whom

$$\sum_{i=1;i\neq j,j+1}^{n} (C_i^s + C_i^b)_{A_j} > \sum_{i=1;i\neq k,k+1}^{n} (C_i^s + C_i^b)_{A_k} \qquad \forall\ j=1,...,n;\ j \neq k \tag{8}$$

applies (the condition derives directly from the second term of the RHS in (3)); then agent k facilitated exchange is cheaper than any other agent facilitated exchange that collapses the circular chain of no double coincidences of wants. For example imagine a government (k) that exploits some informational advantage (or the possibility to impose some tax to compensate for its expenses) in order to function as a clearing house.

However, this arrangement might still be dominated by a goods facilitated exchange. So in order for the agents facilitated exchange around agent k to dominate any commodity money[22] we also require:

$$\sum_{i=1;i\neq g,g-1}^{n} (C_g^s + C_g^b)_{A_i} > \sum_{i=1;i\neq k,k+1}^{n} (C_i^s + C_i^b)_{A_k} \qquad \forall\ g=1,...,n \tag{9}$$

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21. For example, if good j is the most expensive good in transaction for one agent it is also the most expensive good for all others.

22. There might still be some mixed mode of exchange using both agent and goods facilitated exchange patterns in order to solve the circle, which is cheaper than the trading post around k.

This condition follows from the RHS of (8) and the second term of the RHS in (4). Note that on the LHS in (9) we can not simple say (n-2) but have to sum the individual cost terms for all agents using g except those two that anyway would have dealt in g (g and g-1).

We can now state:

> *Proposition 3:* If transaction costs are additive, agent specific and ranked identically by size for all agents, then a trading post arrangement around agent k fulfilling condition (8) and (9) will always be preferred to any other trading post arrangement or any form of commodity money. But a fiat money m which fulfills
>
> $$\sum_{i=1;i\neq k,k+1}^{n} (C_i^s+C_i^b)_{A_k} \;>\; \sum_{i=1}^{n} (C_m^s+C_m^b)_{A_i} \qquad (10)$$
>
> would dominate the trading post arrangement around k.

The first sentence of proposition 3 follows straight from (8) and (9) since this is how we designed the two conditions. Sentence two can easily be shown to be true: The total cost (TC) of collapsing the circular chain with agent specific transaction cost will always be of the form:

$$TC_x^y \;=\; \sum_{i=1}^{n} (C_i^s+C_{i+1}^b)_{A_i} \;+\; \Omega.$$

For the fiat money m (x=f, y=m) the Ω part will be the RHS of inequality (10), while for the trading post arrangement around k (x=a, y=k) the Ω term will be the LHS of (10).

Proposition 3 thus says that even though trading post arrangements - basically exchange shops - might emerge in the economy where the cost of transaction vary for individuals a fiat money fulfilling (10) can still be beneficially introduced. So even though commodities might not be able to serve as media of exchange a suitable fiat currency might.[23] An additional advantage of the fiat currency, is that the transaction costs are more evenly distributed; recall that in the trading post arrangement around agent k, this agent would encounter n-2 extra cost terms.

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23. Of course if (9) was not fulfilled for some g then we had a commodity money that would dominate the trading post around k.

7. Conclusion

In order to explain the use of money or how it might emerge we need some friction in the economy. This can be incomplete information, time constraints, transaction costs, etc.

The present paper has shown that already in a setup of additive transaction cost media of exchange might be beneficially introduced. This contradicts part of the existing literature. Also, usually far more complicated settings are needed in order to explain the use of money. Moreover, since additive transaction costs are usually taken to represent the physical characteristics of goods, we arrive at a formal exposition of the importance of the intrinsic features of media of exchange.

The model hinges on the absents of double coincidences of wants - however, this is nothing new for a model wanting to explain the use of money. So does Jones (1976) and Oh (1989) depend on the degree of absents of double coincidences, or does Ostroys and Starrs (1974) examination of media of exchange utilise chains of no double coincidences of wants. In fact even Samuelson's consumption loans model (1958) features the same situation.[24]

Further we note that as an endogen feature of our model the economy displays a Clower constraint; once a fiat money (m) and a size (n) fulfilling proposition 2a and 2b are present - goods do not buy goods within the circle of no double coincidences of wants.

Summarizing propositions 1-3: We showed that if a good has the lowest transaction costs then it is the obvious candidate for a commodity money. Fiat money will be beneficial, if the transaction costs of buying and selling money are lower than the transaction cost of buying and selling the goods - in particular the commodity money good. Large economies are more likely to use fiat money. Even for agent specific transaction costs fiat money might dominate the otherwise cheapest trading post arrangements.

In short, the economy will be monetized if money is cheaper to use than goods. And this happens already for additive transaction costs. Thus physical characteristics of money - easy transport, cognizability, low cost storage - can motivate its use. Ergo the paper has shown us what we knew all along: that money is designed as it is because it is supposed to be used as it is.

24. The economy with overlapping generations never ends. To see the correspondence to the circular chains discussed here let instead the last generation meet the first.

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