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Anticipation of Future Consumption, Excessive Savings, and Long-Run Growth

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Anticipation of Future Consumption, Excessive Savings, and Long-Run Growth

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Abstract. In this paper, we provide a new theory that explains how the anticipation of future consumption leads to excessive savings. We introduce utility from anticipatory consumption into an otherwise standard endogenous growth model and assume that individuals cannot commit to future consumption plans. We show that the associated time-inconsistent decisions lead individuals to save more of their income than planned, and that this behavior increases growth but decreases welfare. We then modify the model to account for time-inconsistent decisions due to hyperbolic discounting, which in and of themselves would result in individuals saving less of their income than planned. By combining the assumptions of anticipatory consumption and hyperbolic discounting, we show that the excessive savings outcome is preserved for the benchmark calibration of the model. We also show that an alternative parameterization of the model exists where hyperbolically discounting individuals stick to their original consumption plan even though there are no commitment devices.

Keywords: Anticipated Consumption, Time-Inconsistency, Over-Saving, Endogenous Growth, Discounting.

JEL: D91, E21, O40.

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1. Introduction

Time-inconsistent consumption decisions are widely considered to be associated with present bias and insufficient savings (e.g. Angeletos et al., 2001). In this paper, we discuss an alternative theory of consumption that also explains a gap between long-term goals and short-term behavior, but which motivates individuals to save too much and consume too little. Specifically, we investigate the behavior of individuals who experience utility from anticipating future consumption in the framework of an otherwise standard model of endogenous growth. We assume that individuals cannot commit to future consumption plans and show that anticipatory consumption motivates individuals to continually revise their consumption plans and to save more than originally planned. We show that the time inconsistent behavior leads to higher growth, but reduces welfare.

The importance of anticipating future pleasant and painful events for current utility was emphasized by classical economists such as Bentham (1789), Marshall (1891), and Jevons (1905), but has been ignored in modern economics until Loewenstein (1987) presented a formal description and analysis. Loewenstein showed how the utility derived from the imagination of future consumption motivates individuals to postpone consumption activities, corroborated the conclusions with evidence from behavioral studies, and discussed how it may lead to excessive savings and the reversal of original consumption plans. This behavior (called 'reverse time inconsistency' by Loewenstein), occurs because a consumption activity postponed to some future date t happens to be less pleasurable than initially thought when t arrives, compared to postponing it further into the future and to experience the pleasure of anticipating it, a problem that may repeat itself indefinitely.

A couple of studies have discussed anticipatory utility of consumption in the context of long-run growth. Monteiro and Turnovsky (2016) showed that anticipation effects can be conveniently modeled using a reference stock of weighted future consumption levels analogously to the modeling of adaptation to past consumption (as introduced by Ryder and Heal, 1973; see also, among others, Carroll and Weil, 2000; Alvarez-Cuadrado et al, 2004; Strulik 2015a). Monteiro and Turnovsky compared adaptation and anticipation effects on adjustment dynamics for the neoclassical growth model. As a key difference, they pointed out that the baseline reference stock for adjustment effects is predetermined (by past levels of consumption), while for anticipation effects it is a choice variable (by planned future levels of consumption). Like Loewenstein, they mentioned that the free choice of future consumption levels is a potential source of time inconsistency but ignored this issue in their analysis, assuming that individuals can commit to their initial choices. Gomez and Monteiro (2020) applied this approach to the analysis of anticipatory utility in the basic endogenous growth model (also known as Ak model, based on Rebelo, 1991). Their most relevant result with respect to our study is that anticipatory utility introduces adjustment dynamics but leaves the steady-state savings rate and growth rate unaffected.

The assumption of commitment is certainly plausible for some consumption activities where commitment devices are available such as reserving a table at a restaurant, buying tickets for a specific event, or booking a holiday. In general, however, the assumption of commitment appears to be too strong. This is in particular the case in the context of long-run growth where households maximize a (dynastic) utility function with consumption flows extending into the infinite future. The question also arises as to what might determine the particular initial date on which all decisions are made. A more plausible assumption seems to be that individuals have the ability to change their consumption plans at any time. This is the assumption under which we investigate consumption decisions in the endogenous growth model. We show that it drastically changes the implications of anticipatory utility. Specifically, we show that the implied time-inconsistent consumption behavior leads to an increase of the steady-state savings rate and the long-run growth rate. Savings and economic growth are sub-optimally high in the sense that individuals would experience higher utility if they could commit to their consumption plans.

The feature of time-inconsistent consumption decisions relates our paper to the literature on hyperbolic discounting. Hyperbolic discounting creates a present bias that may lead to time-inconsistent excessive consumption choices and too low savings (see Laibson, 1996, 1998; Angeletos et al., 1999).[1] Barro (1999) integrated hyperbolic discounting into the neoclassical growth model and showed that exponential and quasi-hyperbolic discounting lead to structurally equivalent solutions. A stronger equivalence result has been shown in the context of the AK model. When discounting parameters are such that a discounted constant and infinite flow provides the same present value, the growth rates under hyperbolic and exponential discounting coincide (Strulik, 2015b). Here, we contribute to this literature by the combined consideration of hyperbolic discounting and anticipatory consumption. This gives individuals two reasons for time-inconsistent behavior. In and of itself, hyperbolic discounting would result in undersaving, while anticipatory consumption would result in oversaving (compared to planned behavior) and it is not a priori evident which mechanism dominates when both assumptions are considered simultaneously. We show that for the calibrated model the anticipation effect dominates and individuals save too much. Observing hyperbolic discouting and plan reversals

[1] As most of the related literature we focus on discount factors that are not multiplicatively separable in calender time and decision time such that naive individuals actually make time-inconsistent decisions (see Burness, 1976; Drouhin, 2020).

is therefore not sufficient to conclude that people care too little about the future and consume too much. To further corroborate this point, we show that an alternative parameterization of the model exists in which under- and oversaving effects balance each other such that individuals stick to original plans even though there are no commitment devices.

The paper is organized as follows. Section 2 introduces the benchmark model with anticipatory utility. In Section 3, we solve the model numerically and derive the main results. In Section 4, we set up and discuss the model variant with hyperbolic discounting. Section 5 concludes the paper.

2. The AK-Growth Model with Anticipatory Consumption Utility

We consider individuals who derive utility from consumption C and anticipated future consumption R. Specifically, an individual maximizes intertemporal utility

$$V = \int_{t_0}^{\infty} D(t,t_0)u(C(t),R(t))\mathrm{d}t, \qquad D(t,t_0) = e^{-\rho(t-t_0)} \tag{1}$$

with t denoting calender time and t_0 denoting decision time. The instantaneous utility function $u(C(t),R(t))$ has positive and diminishing marginal utility in C and R. For the benchmark model, we consider exponential discounting at the constant discount rate ρ. Following Loewenstein (1986) and the related literature (Monteiro and Turnovsky, 2016; Gomez and Monteiro, 2020; Gomez, 2021), anticipated consumption is modeled as a stock of future discounted consumption streams given by

$$R(t) = \beta \int_{t}^{\infty} C(s)e^{-\beta(s-t)}ds. \tag{2}$$

The parameter β captures the 'discount rate' of the anticipation stock. In the terminology of Loewenstein (1987) it captures the vividness of future consumption and the individual's preoccupation with the future. Differentiating (2) with respect to time yields

$$\dot{R}(t) = \beta(R(t) - C(t)). \tag{3}$$

The capital stock in the economy evolves according to

$$\dot{K}(t) = AK(t) - \delta K(t) - C(t), \tag{4}$$

where A is the productivity parameter and δ the depreciation rate. From the individual's intertemporal budget constraint we can show that

$$\int_{t}^{\infty} C(s)e^{(A-\delta)(s-t)}ds = K(t). \tag{5}$$

We require that $\beta > A - \delta$ implying that anticipated consumption is discounted at a higher rate as compared to the net marginal product of capital. Under this condition, anticipated consumption is consistent with the intertemporal budget constraint, i.e. $\int_t^\infty C(s)e^{(A-\delta)(s-t)}ds > R/\beta$. Note that for $A - \delta = \beta$, $K = R\beta$ would apply and the individual's utility function would amount to a direct dependence on the capital stock (Kurz, 1968; Zou, 1994; Bakshi and Chen, 1996). Further, we assume that $A - \delta > \rho$ such that there is a positive balanced growth rate. Thus, $\beta > A - \delta > \rho$ holds.

The individual maximizes intertemporal utility (1) subject to the constraints (3) and (4) and the initial condition $K(t_0)$. For better readability, we henceforth omit the calender time index t. The Hamiltonian associated with the maximization problem is given by:

$$H = u(C,R) + \lambda(AK - \delta K - C) + \mu\beta(R - C) \tag{6}$$

where λ and μ represent the shadow prices of capital and the anticipation stock, respectively. The optimality conditions are

$$u_C = \lambda + \beta\mu \tag{7}$$

$$\lambda(A - \delta) = \lambda\rho - \dot{\lambda} \tag{8}$$

$$u_R + \mu\beta = \mu\rho - \dot{\mu} \tag{9}$$

$$\mu(t_0) = 0 \tag{10}$$

$$\lim_{t\to\infty} \lambda K e^{-\rho(t-t_0)} = 0, \ \lim_{t\to\infty} \mu R e^{-\rho(t-t_0)} = 0 \tag{11}$$

where u_C and u_R denote marginal utilities with respect to consumption and anticipation, respectively.

Equation (7) states that the marginal utility of consumption equals the (weighted) shadow prices of capital and the anticipation stock. Equation (8) states the optimal evolution of the shadow price of capital, and equation (9) states the optimal evolution of the shadow price of the anticipation stock. In more detail, the left-hand side of equation (9) provides the value in terms of μ of postponing consumption by one unit of time. The value is composed of the marginal utility gain with respect to R and a higher accumulation of R by $\mu\beta$. The right-hand side gives the cost of postponing consumption. Future consumption is discounted giving the cost $\mu\rho$ and the future change in price of R must be taken into account by $\dot{\mu}$.

The condition $\mu(t_0) = 0$ in Equation (10) derives from the fact that the initial anticipation stock $R(t_0)$ can be chosen (see e.g. Kamien and Schwartz, 1971). This is the case since individuals decide about the future path of consumption, and hence, about $R(t_0)$ through equation (2). To understand the optimality condition, recall that μ denotes the value of one additional unit of anticipation, R, in terms of marginal utility. Intuitively, the individual determines $\mu(t_0)$ such that costs and benefits of the anticipation stock outweigh each other. If $\mu(t_0) > 0$ held, the anticipation stock would be evaluated with a positive price indicating that increasing the anticipation stock (i.e. consuming less now and planning more consumption for the future) would increase lifetime utility. The plan would not be optimal. A similar logic applies for $\mu(t_0) < 0$. Hence, $\mu(t_0) = 0$ has to hold for the optimal solution.

Condition (10) is the gateway for the time-consistency of consumption plans. In contrast to the related literature where the consumption plan is made once and for all, we allow individuals to revise their plans at any time. This means that at any new decision time t_0, individuals choose $\mu(t_0) = 0$ again. Formally, the reset of $\mu(t) < 0$ to $\mu(t_0)$ causes a revision of the original plan for $C(t)$, i.e. the time inconsistency of the original plan. Intuitively, current consumption is less attractive than originally thought since additional utility can be gained from postponing part of the planned consumption and experiencing utility from anticipation of the resulting increase of future consumption.

Equations (11) state the transversality conditions which are standard for infinite horizon optimality problems. We verify later when we calculate the planned steady state growth rate that under the given parameter restriction both transversality conditions hold.

Equation (9) implies

$$\dot{\mu} = (\rho - \beta)\mu - u_R. \tag{12}$$

From the parameter restriction $\beta > \rho$ together with $u_R > 0$, $\dot{\mu}(t_0) < 0$ holds. Since this implies that $\mu(t) < 0$ for $t = t_0$, $\dot{\mu}(t) < 0$ also holds for a neighborhood $t_0 + \varepsilon > t > t_0$. In other words, for the planned solution $\mu(t) < 0$ applies for all $t_0 + \varepsilon > t > t_0$. From this observation, we can deduce in which direction the planned solution deviates from the realized solution. Since $\mu(t_0) < 0$ for the original plan but $\mu(t_0) = 0$ for the revised plan, the shadow price of an additional unit of R turned out to be too low and increases with the revision of the plan. The individual thus reduces consumption below the originally planned level in order to increase the anticipation stock. Formally, this can be seen in (7) and noting that the evolution of λ is independent from plan revisions. The upgrade of μ implies an increase of U_C, i.e. a reduction of instantaneous consumption C.

Unfortunately, the solution of the AK growth model with time-inconsistent behavior is not analytically accessible. In the next section, we solve the calibrated model numerically and determine the impact of time-inconsistent behavior by comparing the planned and realized solution.

3. Model Solution and Main Results

3.1. **Parameterization.** In order to solve the model numerically, we employ the iso-elastic instantaneous utility function $u(C,R) = \frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}-1}{1-\sigma}$. The parameter σ represents the inverse of the intertemporal elasticity of substitution and γ governs the utility weight of anticipatory consumption. From the first order conditions we derive the following dynamic system (see Appendix):

$$\frac{\dot{K}}{K} = A - \delta - \frac{C}{K} \tag{13}$$

$$\frac{\dot{R}}{R} = \beta - \beta\frac{C}{R} \tag{14}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - (A - \delta) \tag{15}$$

$$\frac{\dot{\mu}}{\mu} = \rho - \beta - \gamma\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{\mu R} \tag{16}$$

$$\lambda + \beta\mu = (1-\gamma)\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{C}. \tag{17}$$

We next show that for the planned solution there exists a long-run balanced growth path (BGP), along which C, R, K, and Y grow at the common rate $g = \frac{A-\delta-\rho}{\sigma}$, and λ and μ decline at rate $-\sigma g$. From the capital accumulation equation we obtain that along the BGP:

$$\frac{C}{K} = A - \delta - g, \tag{18}$$

and hence C and K grow at the same rate g. Given that C grows at a constant rate, R grows at the same rate g. From $\dot{\mu}/\mu \equiv const.$ we conclude that $\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{\mu R}$ needs to be constant along the BGP. Taking logs of this expression, differentiating the result with respect to time and setting the time derivative equal to zero, we conclude that the growth rate of μ equals $\dot{\mu}/\mu = -\sigma g$ on the planned balanced growth path. Having fixed the relation between the balanced growth rates of C and μ, we see now from the algebraic equation that λ grows at rate $-\sigma g$. Since the growth rate of λ equals $\rho - (A-\delta)$, the long-run growth rate is given by $g = \frac{A-\delta-\rho}{\sigma}$. Finally, we verify that the transversality conditions are satisfied along the BGP.

While we can characterize the BGP of the planned solution analytically, for the realized solution we have to rely on numerical solution techniques. As explained above, the difference between the planned and the realized solution originates from the revision of the shadow price of μ. For the planned solution, the shadow price evolves according to Equation (9) and (10). In the realized solution, individuals are allowed to revise their plans at any time by applying the optimality condition $\mu(t_0) = 0 \; \forall t_0 = t$.

3.2. **Calibration.** To gain quantitative insights with respect to the effects of time-inconsistent behavior, we calibrate the parameters of the model. We calibrate the utility weight with respect to anticipation γ with reference to estimates from Vendrik (2013). Using data from the German Socio-Economic Panel for the years 1987–2007, Vendrik finds that a future increase in consumption has an impact on instantaneous utility through the increase of consumption directly and a utility gain from anticipation. He estimates the shares of the utility gains from instantaneous and anticipated consumption are 60% and 40%, respectively. Hence, we calibrate $\gamma = 0.4$. The anticipation parameter β affects the speed at which the planned solution converges towards the steady state but it does not have any impact on the size of initial consumption and, hence, on the time path of the realized solution. We choose as a benchmark $\beta = 0.2$ according to Schünemann et al. (2023) who estimate the anticipation parameter in the context of the anticipation of future health shocks. We address parameter uncertainty by providing a robustness analysis for the main results.

The remainder of the calibration is straightforward and follows standard procedure in quantitative growth economics. We set the interest rate $A - \delta$ to 7% as estimated by Jorda et al. (2019) for the average return to equity since industrialization. We set the inverse of the elasticity of intertemporal substitution with respect to the consumption aggregate to $\sigma = 2$ which reflects a mean value from the literature (Chetty, 2006; Havranek, 2015). Finally, we set $\rho = 0.03$ such that the implied planned long-run growth rate is 0.02, which is about the average growth rate of U.S. per capita income since 1950 and a benchmark value assumed in quantitative growth economics (e.g. Mankiw et al., 1992; Jones, 2022).

Finally, we normalize the initial capital stock to 1. This has no impact on the results, since the growth rates of equations (13) - (17) are invariant with respect to the scale of the economy (see Trimborn, 2018, for a formal definition of scale invariance). The calibration of the benchmark model is summarized in Table 1.

3.3. **Results.** The benchmark results are shown in Figure 1. Although the main comparison is between planned and realized behavior with anticipatory utility, it is helpful to also consider the solution

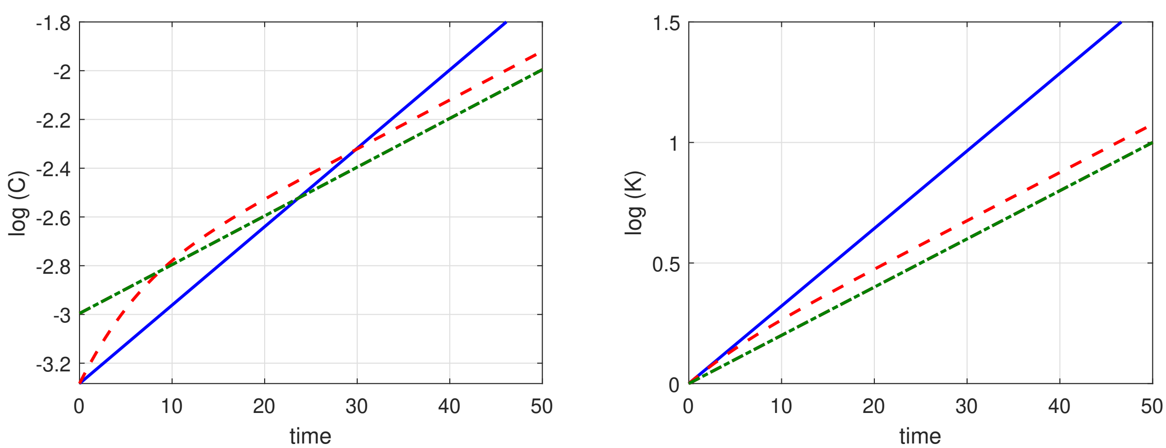
Table 1. Benchmark Calibration

Parameter	Value	Explanation	Source
γ	0.4	utility weight anticipation	Vendrik (2013)
β	0.2	anticipation discount rate	Schünemann et al. (2023)
σ	2.0	(inverse) intertemp. ela. of substitution	Chetty (2006), Havranek (2015)
$A-\delta$	0.07	productivity and depreciation	implied interest rate 7% (Jorda et al., 2019)
ρ	0.03	time discount rate	implied growth rate 2% (Jones, 2022)

of the associated standard AK growth model without anticipation (for $\gamma = 0$). In this and the following figures, the realized solution is shown by blue (solid lines), the planned solution is shown by red (dashed) lines, and the corresponding standard solution is shown by green (dash-dotted) lines.

Compare first the planned solution with that of the standard AK growth model. The introduction of anticipation leads to individuals initially consuming at a lower level, i.e. initially choosing a higher savings rate in order to increase the joy from anticipation of future consumption. This means that the growth rate of consumption is higher in the transition to the steady state. Once more capital is accumulated, individuals plan to save at the same rate as in the standard AK model in the long run. As time progresses, the red (dashed) trajectories and the green (dash-dotted) trajectories become parallel to each other. Due to higher growth rates in the transition, however, the planned solution features higher levels of consumption and capital on the BGP. The red (dashed) trajectories lie above the green (dash-dotted) trajectories.

Figure 1: Consumption (C) and Wealth (K) with Consumption Anticipation: Benchmark Case



Blue (solid) lines: realized solution; red (dashed) lines: planned solution; green (dash-dotted) lines: standard model ($\gamma = 0$).

Consider now the realized solution. As with the planned solution, individuals start with a relatively low level of consumption and a high saving rate, but fail to adjust and reduce the saving rate in the medium and long run. In fact, they keep delaying consumption in order to further experience the joy of anticipation. Therefore, long-run growth is higher than in the planned solution. The blue (solid) trajectories are steeper than the planned trajectories. On the BGP, the planned growth rate is 0.02 while the realized growth rate is 0.033. Growth, however, is suboptimally high. Realized intertemporal utility (or welfare) $V(t_0)$ is 4.5 % lower than planned.

3.4. **Sensitivity Analysis.** In this section we check the robustness of the results with respect to changes in key parameters of the model. To this end, we analyze the effects on the realized long-run growth rate and the welfare loss due to time-inconsistent behavior when varying separately i) the utility weight of anticipation (γ), ii) the discount rate of anticipation (β), iii) the interest rate (r), iv) and the planned long-run growth rate (g). Table 2 summarizes the results.

TABLE 2. Sensitivity Analysis

Case	Explanation	Realized growth rate	Δ Growth rate	Δ Welfare
1) benchmark	benchmark calibration	0.033	0.013	-4.49
2) $\gamma = 0.2$	lower utility weight of anticipation	0.026	0.006	-0.80
3) $\gamma = 0.6$	higher utility weight of anticipation	0.041	0.021	-16.0
4) $\beta = 0.1$	lower anticipation discount rate	0.033	0.013	-1.91
5) $\beta = 0.4$	higher anticipation discount rate	0.033	0.013	-5.63
6) $r = 0.05$	lower interest rate	0.028	0.008	-5.35
7) $r = 0.09$	higher interest rate	0.038	0.018	-3.63
8) $g = 0.01$	lower planned growth rate	0.025	0.015	-4.20
9) $g = 0.03$	higher planned growth rate	0.040	0.010	-4.82

Δ Welfare indicates deviations of realized lifetime utility from planned lifetime utility in percent. Δ Growth rate represents the difference in long-run growth rates between the realized and planned solution.

The first column of Table 2 shows the parameter that is varied in the sensitivity analysis and the second column explains the economic effect of the parameter change. The third column reports the growth rate in the realized solution and the fourth column calculates the difference in the growth rates between the realized and the planned solution. The last column reports the welfare loss from time-inconsistent behavior by determining the percentage deviation of lifetime utility in the realized solution from lifetime utility in the planned solution.

The first row of Table 2 reiterates the benchmark run with parameters as calibrated in Table 1. As we saw in the last subsection, time-inconsistent behavior leads to a realized growth rate of 0.033 which is 1.3 percentage points higher than the planned growth rate (0.020). This results in a welfare loss of 4.49%. Rows 2 and 3 show the results when varying the utility weight of anticipation γ. Since anticipation is the driving force behind time-inconsistent behavior, lowering γ from $\gamma = 0.4$ to $\gamma = 0.2$ and thus the relevance of anticipation decreases the deviation from the planned solution. Therefore, the difference between the planned (0.020) and realized growth rate (0.026) is reduced to 0.6 percentage points. As a result, welfare is reduced by only 0.80%. The opposite result holds true when increasing γ from $\gamma = 0.4$ to $\gamma = 0.6$. This experiment implies a realized growth rate of 0.041 and thus also a higher welfare reduction of 16%.

In rows 4 and 5 we halve and double the parameter β, respectively. As already mentioned in the calibration section, this parameter leaves the long-run growth rates unaffected. Therefore, the realized growth rate coincides with the one in the benchmark case. The parameter change, however, affects the welfare loss in the realized solution. The driving force behind this result is that β affects the anticipation stock. A higher β means that future consumption streams are discounted at a higher rate when building the anticipation stock. A higher depreciation rate of the anticipation stock β thus reduces lifetime utility, a feature that applies to both the planned and the realized solution. Our analysis shows that the welfare-reducing effect is larger for the realized solution. Increasing β from $\beta = 0.1$ to $\beta = 0.4$ increases the welfare loss from time-inconsistent behavior from 1.91% to 5.79%. In the planned solution, individuals factor in that delaying consumption has a smaller impact on the anticipation stock when β is higher. Therefore, they increase consumption in the short run which counteracts the reduction in welfare from a lower anticipation stock. In the realized solution, on the other hand, the consumption path remains unaffected by β.

Rows 6 and 7 illustrate the results when changing the interest rate from $r = 0.07$ to $r = 0.05$ and $r = 0.09$. Note that according to our calibration strategy, manipulating r requires recalibrating the parameters $A - \delta$ and ρ to preserve our calibration targets. Changing the interest rate to $r = 0.05$ ($r = 0.09$) implies $A - \delta = 0.05$ ($A - \delta = 0.09$) and $\rho = 0.01$ ($\rho = 0.05$). As can be seen from the table, moving from $r = 0.05$ to $r = 0.09$ increases the realized growth rate from 0.025 to 0.040 because the higher interest rate amplifies the suboptimally high savings in the realized solution. The welfare loss, on the other hand, decreases in r from 5.70% to 3.22%. A higher level of r implies a higher recalibrated time preference rate ρ, placing more emphasis on the near rather than the distant future when calculating lifetime utilities. Since the planned and realized solution exhibit different long-run growth rates, the paths of c and R diverge over time. Therefore, a higher time discount rate puts more weight on the time periods when the time paths of the realized and planned solution are closer together. This results in a lower welfare loss from time-inconsistent behavior.

Finally, in rows 8 and 9 the planned growth rate g is halved and doubled, respectively. Again, changing g requires recalibrating other parameters to maintain the calibration targets. For $g = 0.01$ ($g = 0.03$), the time preference rate changes to $\rho = 0.05$ ($\rho = 0.01$). Naturally, the realized growth rate increases in the planned growth rate from 0.025 to 0.04. Although the planned growth rate is tripled when moving from $g = 0.01$ to $g = 0.03$, the deviation from the respective realized growth rates changes rather mildly from 1.5 to 1.0 percentage points. The welfare loss changes moderately from -4.20% to -4.82%.

Summarizing, the sensitivity analysis shows the robustness of the main results. The time inconsistency involved as individuals anticipate future consumption motivates excessive savings leading to suboptimally high growth and welfare losses. The extent of these changes varies in intuitively plausible ways.

4. Anticipatory Consumption and Hyperbolic Discounting

4.1. **Introducing Hyperbolic Discounting.** As motivated in the Introduction, it is interesting to investigate anticipatory consumption when individuals discount the future hyperbolically. Since the present bias from discounting, taken for itself, leads to undersaving and anticipatory consumption to oversaving, it is a priori not clear which mechanism of time-inconsistency will prevail when both are considered simultaneously. To this end, we introduce a hybrid discount factor, which additively combines exponential discounting with the most general hyperbolic discounting function (Loewenstein and Prelec, 1992):

$$D(t, t_0) = \frac{\mathrm{e}^{-\bar{\rho}(t-t_0)}}{[1 + \alpha\phi(t - t_0)]^{\frac{1}{\phi}}},$$

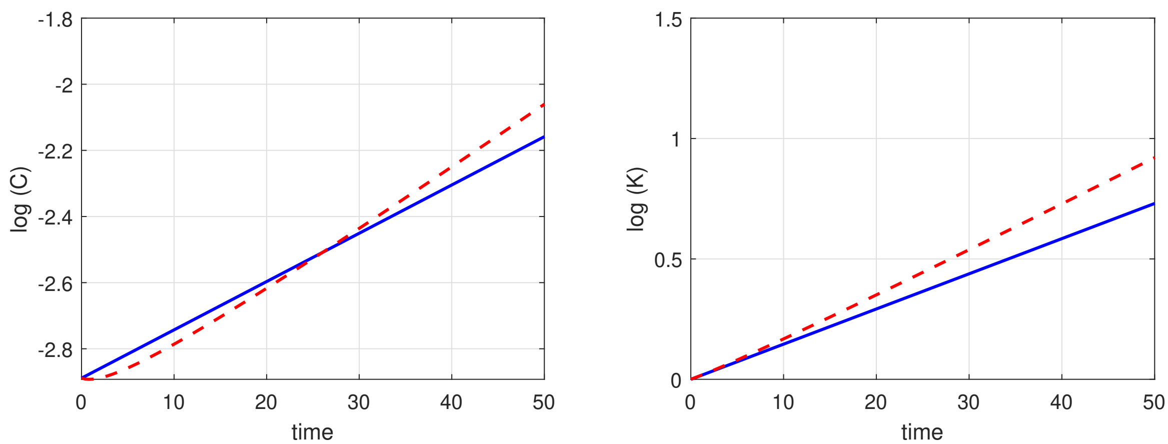
which replaces the exponential discount factor in (1). The parameter $\alpha > 0$ controls the initial discount rate and the parameter $\phi > 0$ controls the speed at which the discount rate declines. The implied discount rate is:

$$\rho(t, t_0) = \bar{\rho} + \frac{\alpha}{1 + \alpha\phi(t - t_0)}.$$

The presence of decision time t_0 in the discount rate indicates that consumption plans will be time-inconsistent. The rest of the model coincides with the benchmark model. We furthermore assume that $\bar{\rho} = 0.03$, implying that the planned discount rate converges to the discount rate of the benchmark model as the planning horizon t approaches infinity. This means that the hybrid discount factor is always lower than the exponential discount factor and that the 'equivalent present value' condition is not fulfilled (Myerson et al., 2001). For our analysis, however, this feature is irrelevant since we are not interested in a 'fair' comparison of exponential and hyperbolic discounting but in the impact of anticipatory consumption utility on actual consumption choices.

As a first plausible example, we consider $\alpha = 0.06$ and $\phi = 10$. These parameters imply an initial discount rate of 0.09, which declines quickly to 0.03 at a half-life of 1.5 years. The time path of the discount rate is shown in Figure A.1 of the Appendix. We first replicate the well-known undersavings result for the case of absent anticipation effects (for $\gamma = 0$). Aside from γ, we take the benchmark calibration from Table 1. The solution of the model is shown in Figure 2. We see that realized consumption (blue solid lines) lies above planned consumption (red dashed lines). As a result, the realized path of wealth grows at lower rate than planned (the solid blue line lies below the red dashed line in the $\log(K)$–panel on the right-hand side of Figure 2).

Figure 2: Consumption (C) and Wealth (K) with Hyperbolic Discounting

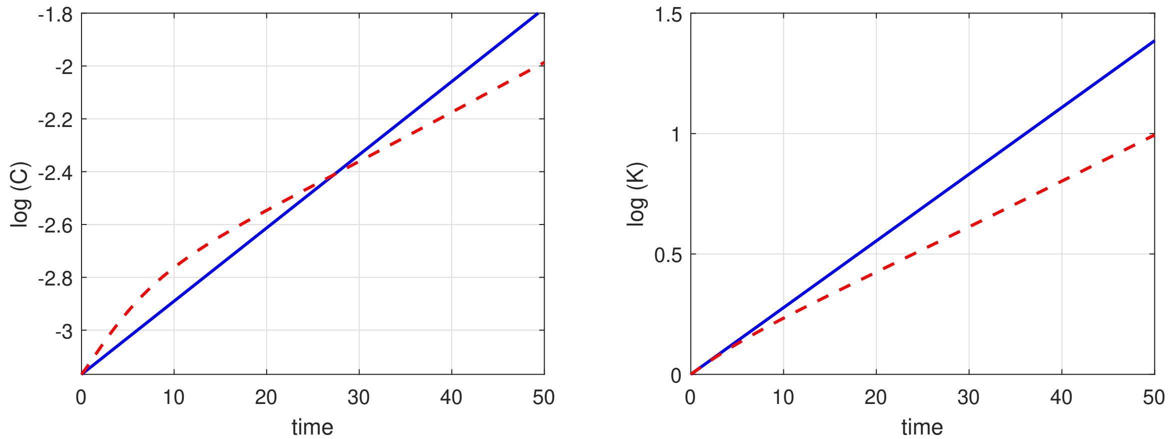


Blue (solid) lines: realized solution; red (dashed) lines: planned solution. No anticipation ($\gamma = 0$). Discount rate $\rho + \alpha(1 + \alpha\phi t)^{-1}$; $\alpha = 0.06$, $\phi = 10$. Other parameters as for benchmark model.

We next introduce anticipation of future consumption and set $\gamma = 0.4$ as calibrated above. Results are shown in Figure 3. The anticipation effect overturns the undersavings result. Realized consumption lies below planned consumption for the first 25 years, indicating oversavings. Afterwards, realized consumption is higher than planned due to excessive capital accumulation. Realized growth is higher than planned, which can best be seen by the steeper path of $\log(K)$ in the panel on the right-hand side.

As a first robustness check, we consider an even faster decline of the discount rate by setting $\phi = 100$. This means a decline with a half-life of 0.06 years and a discount rate of 0.038 after one year (see Figure A.1). It approximates the frequently discussed case of quasi-hyperbolic discounting (or $\beta\delta$-discounting, Laibson, 1998). Results shown in Figure A.3 in the Appendix corroborate the robustness of the excessive savings outcome. The robustness of the result is intuitive when one considers that

Figure 3: Hyperbolic Discounting and Consumption Anticipation



Blue (solid) lines: realized solution; red (dashed) lines: planned solution. Benchmark ($\gamma = 0.4$). Discount rate $\rho + \alpha(1 + \alpha\phi t)^{-1}$; $\alpha = 0.06$, $\phi = 10$. Other parameters as for benchmark model.

hyperbolic discounting generally motivates individuals to exaggerate whatever pleases them in the present and that individuals with anticipatory consumption prefer to delay some consumption in order to experience the present joy of anticipation.

4.2. **Sensitivity Analysis.** We now perform a sensitivity analysis for the extended model with hyperbolic discounting. The first row in Table 3 reiterates the benchmark run with the parameter values from Table 1 and $\phi = 10$ as well as $\alpha = 0.06$. Compared to the basic model without hyperbolic discounting, the realized growth rate reduces from 0.033 to 0.028 such that the deviation from the planned growth rate shrinks from 0.013 to 0.008. As discussed in the last subsection, hyperbolic discounting taken for itself leads to undersaving and thus partly compensates the oversaving motive stemming from anticipation. Since the realized solution is closer to the planned solution, the welfare loss reduces from 4.49% in the model without hyperbolic discounting to 2.19%.

Rows 2 to 9 show the results for the same sensitivity analysis as in the model without hyperbolic discounting in Table 2. Including hyperbolic discounting does not change the nature of this analysis. All results and their economic intuition carry over from the basic model to the extended model. As for the benchmark calibration, however, the magnitudes of growth rate differences and welfare losses from time-inconsistent behavior are smaller once hyperbolic discounting is added to the model. While in the basic model the growth rate differences (welfare losses) range from 0.06 to 0.021 (0.08% to 16%), in the extended model they range from 0.0032 to 0.018 (0.05% to 11.4%).

In rows 10 to 13 we vary the hyperbolic discounting parameters ϕ and α. Changing ϕ from $\phi = 10$ to $\phi = 5$ and $\phi = 15$, we observe that the realized growth rate and thus also the difference from the planned growth rate is higher the faster the discount rate converges to the discount rate of the basic

Table 3. Sensitivity Analysis with Hyperbolic Discounting

Case	Explanation	Realized growth rate	Δ Growth rate	Δ Welfare
1) benchmark	benchmark calibration	0.028	0.008	-2.19
2) $\gamma = 0.2$	lower utility weight of anticipation	0.0203	0.0003	-0.05
3) $\gamma = 0.6$	higher utility weight of anticipation	0.038	0.018	-11.4
4) $\beta = 0.1$	lower anticipation discount rate	0.028	0.008	-0.61
5) $\beta = 0.4$	higher anticipation discount rate	0.028	0.008	-3.13
6) $r = 0.05$	lower interest rate	0.024	0.004	-2.28
7) $r = 0.09$	higher interest rate	0.032	0.012	-1.88
8) $g = 0.01$	lower planned growth rate	0.020	0.010	-2.15
9) $g = 0.03$	higher planned growth rate	0.036	0.006	-2.20
10) $\phi = 5$	slower decline of discount rate	0.026	0.006	-1.64
11) $\phi = 15$	faster decline of discount rate	0.029	0.009	-2.54
12) $\alpha = 0.04$	lower initial discount rate	0.029	0.009	-2.60
13) $\alpha = 0.08$	higher initial discount rate	0.027	0.007	-1.89

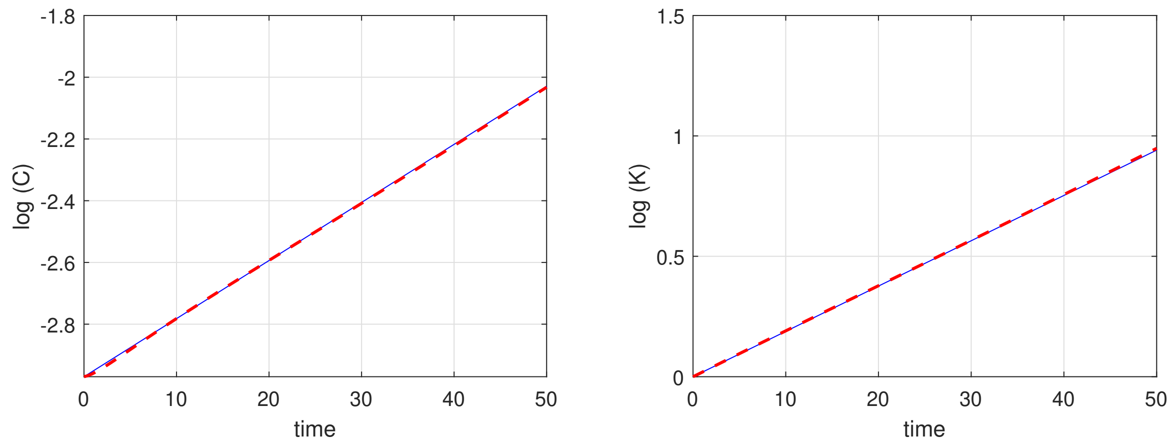
Δ Welfare indicates deviations of realized lifetime utility from planned lifetime utility in percent. Δ Growth rate represents the difference in long-run growth rates between the realized and planned solution.

model ($\bar{\rho} = 0.03$). The reason behind this result is that a faster convergence speed of the discount rate reduces the behavioral distortion from hyperbolic discounting. Thus, the realized growth rate from the extended model resembles more that of the basic model. The same intuition holds for the resulting welfare losses, which increase from 1.64% to 2.54% as ϕ goes from 5 to 15.

We observe the opposite when gradually increasing α from $\alpha = 0.04$ to $\alpha = 0.08$. The realized growth rate and the welfare loss decrease in α from 0.029 to 0.027 and from 2.60% to 1.89%, respectively. A higher α implies a higher initial discount rate and, for given convergence speed ϕ, a higher deviation from the basic model without hyperbolic discounting. Therefore, the dampening effect of hyperbolic discounting on the anticipation effect increases and brings the realized solution closer to the planned solution.

4.3. **Offsetting Time-Inconsistent Behaviors.** A more disaggregated view would allow individuals to experience utility from anticipation of some consumption activities (that dinner at the fancy restaurant) while other consumption activities are driven by short-run desires (that unspectacular lunch break taken to stave off hunger pangs). In the aggregated view, the parameter γ approximates this distinction of consumption activities. For $\gamma = 0$ all consumption activities are like lunch breaks while for $\gamma = 1$ all consumption activities are like fancy dinners. Since the undersavings result is obtained for $\gamma = 0$ and the oversavings result is obtained for the (calibrated) $\gamma = 0.4$, it seems to be obvious that there exists a value of γ where both effects cancel in their effect on aggregate consumption. Indeed we find this case when γ is 0.15 (and $\alpha = 0.06$ and $\phi = 10$), as shown in Figure 4. The solid blue and the red-dashed trajectories are basically superimposed, showing that planned and realized consumption decisions coincide (in order to make this more visible, the realized solution is represented by thinner lines).

Figure 4: Hyperbolic Discounting and Consumption Anticipation ($\gamma = 0.15$)



Blue (solid) lines: realized solution; red (dashed) lines: planned solution. Benchmark ($\gamma = 0.4$). Discount rate $\rho + \alpha(1 + \alpha\phi t)^{-1}$; $\alpha = 0.06$, $\phi = 10$. Other parameters as for benchmark model.

The result is remarkable because it shows that 'two wrongs can make it right', because the behavioral distortions exactly cancel out each other. The individuals suffer potentially through two gateways from time-inconsistent decisions and have no commitment devices but nevertheless stick to their original consumption and savings plan.

Conclusion

In this paper we introduced consumption anticipation in an otherwise standard model of endogenous growth and showed that consumption anticipation gives rise to time-inconsistent behavior. In contrast to previous studies, we abolished the assumption that individuals are able to commit to their initial plan and investigated the implications of time-inconsistent behavior. We solved the calibrated the model for planned and realized consumption and showed that the anticipation of future consumption leads to oversaving and thus to an inefficiently high long-run growth rate, thereby reducing welfare of the representative individual.

In a second step we introduced hyperbolic discounting as an additional source of time-inconsistent behavior. As is well established in the literature, hyperbolic discounting, taken for itself, leads to undersaving and therefore results in inefficiently low long-run growth rates. Combining consumption anticipation and hyperbolic discounting we showed that according to our benchmark calibration, the anticipation effect dominates the hyperbolic discounting effect such that hyperbolically discounting individuals save too much.

Interestingly, we found a reasonable parameter constellation in which the effects of time-inconsistent behavior cancel each other out such that the realized solution matches the initial plan. In other words, the combination of opposing inefficiencies created by time-inconsistent behavior can actually lead to a welfare-maximizing outcome.

Our framework could be extended and applied to other life-cycle decisions. One extension could analyze the effect of consumption anticipation in the context of a life cycle model with bequest motive. Our findings can help to explain oversaving in old age. Another potential application could examine the anticipation or fear of future health deficits. In this context, time-inconsistent behavior could lead to inefficiently high health care spending at the cost of consumption utility.

Appendix

Parameterized Model. The first-order conditions derived in Equations (7)-(9) read

$$(1-\gamma)\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{C} = \lambda + \beta\mu \tag{19}$$

$$\lambda(A-\delta) = \lambda\rho - \dot{\lambda} \tag{20}$$

$$\gamma\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{R} + \mu\beta = \mu\rho - \dot{\mu}. \tag{21}$$

Numerical Method for the solution of the AK model. We calculate the planned solution by scale-adjusting the dynamic system. Note, that variables K, C, and R grow with rate g at the steady state, while λ and μ grow with rate $-g$. Hence, the scale adjusted system reads (with keeping the designation of variables)

$$\frac{\dot{K}}{K} = A - \delta - \frac{C}{K} - g \tag{22}$$

$$\frac{\dot{R}}{R} = \beta - \beta\frac{C}{R} - g \tag{23}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - (A-\delta) + \sigma g \tag{24}$$

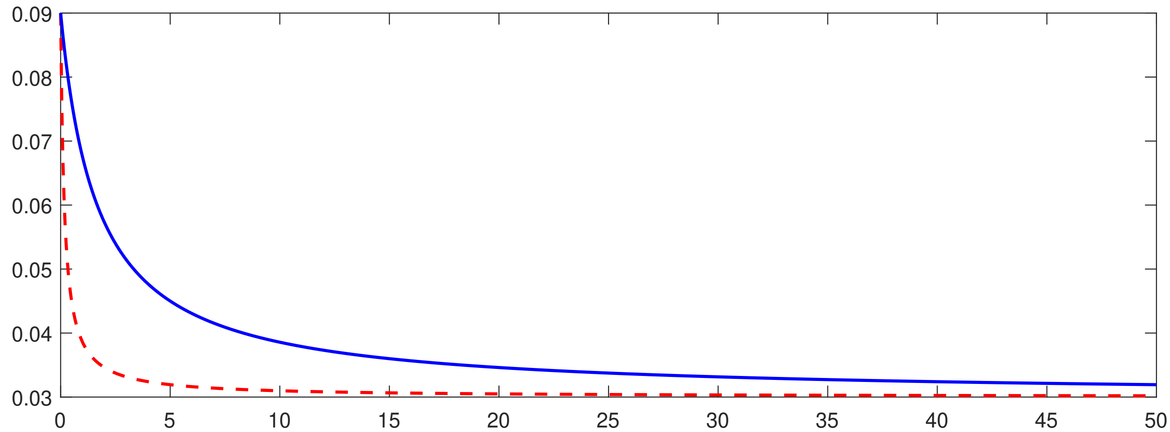
$$\frac{\dot{\mu}}{\mu} = \rho - \beta - \gamma\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{\mu R} + \sigma g \tag{25}$$

$$\lambda + \beta\mu = (1-\gamma)\frac{(C^{1-\gamma}R^{\gamma})^{1-\sigma}}{C} \tag{26}$$

Once, the planned solution is calculated we obtain the policy function of the realized solution by evaluating C/K at time 0. From this ratio we can calculate the realized growth rate.

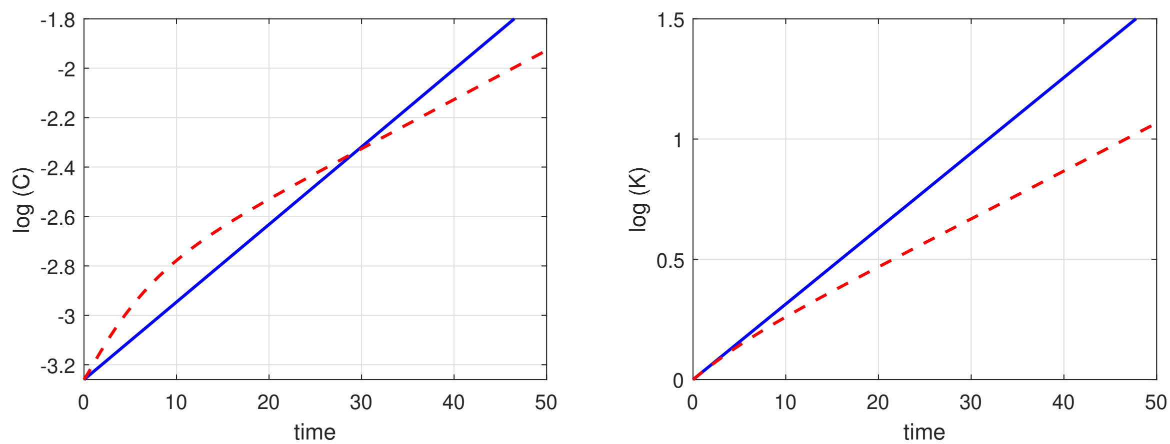
Further Results.

Figure A.1: Hybrid Hyperbolic Discount Rates



Hybrid Exponential–Hyperbolic Discount rates: $\rho + \alpha(1 + \alpha\phi t)^{-1}$. Parameters: $\rho = 0.03$, $\alpha = 0.06$ and $\phi = 10$ (blue solid lines) and $\phi = 100$ (red dashed lines).

Figure A.3: Consumption (C) and Wealth (K) with Hyperbolic Discounting and Consumption Anticipation ($\phi = 100$)



Blue (solid) lines: realized solution; red (dashed) lines: planned solution. Benchmark ($\gamma = 0.4$). Discount rate $\rho + \alpha(1 + \alpha\phi t)^{-1}$; $\alpha = 0.06$, $\phi = 100$. Other parameters as for benchmark model.

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