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The Optimal Extraction of Non-Renewable Resources Under Hyperbolic Discounting[*]

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Abstract. In this paper, we investigate the effects of a declining social discount rate (SDR) on the optimal extraction of non-renewable resources and economic growth. For this purpose, we introduce time-consistent hyperbolic utility discounting into models of resource extraction. First, we investigate a small model of pure resource extraction holding constant the magnitude of discounting for hyperbolic and exponential discounting. We show that resource use is more conservative under hyperbolic discounting resulting in a permanently higher resource stock. Second, we introduce hyperbolic discounting into the seminal Dasgupta-Heal-Solow-Stiglitz (DHSS) model and derive analytically that positive long-run consumption growth requires a lower rate of technological progress under hyperbolic discounting. We show numerically that resource use is more conservative under hyperbolic discounting in the medium- and long-run.

Keywords: *hyperbolic discounting; social discount rate; non-renewable resource extraction; Dasgupta-Heal-Solow-Stiglitz model*

JEL: *Q30; C60; H30*

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1. Introduction

The appropriate choice of the social discount rate is of tremendous importance for the evaluation of policies with long-run impacts. This is particularly true for environmental policies such as the optimal extraction of nonrenewable resources or the optimal reduction of greenhouse gas emissions, for which costs and benefits are distributed over a very long time horizon and decisions today affect the welfare of generations in the distant future. The choice of the appropriate social discount rate dwarfs the impact of all other factors on these types of optimal environmental policies.

While much of the discussion has focused on the size of the social discount rate (SDR), a debate about whether the SDR should be constant or declining has also been flourishing. Recently, a panel of leading economists in environmental economics came to the conclusion that the SDR should be declining (Arrow et al., 2014). In short, a declining SDR is motivated by uncertain but positively correlated shocks to the future SDR (Gollier, 2012). Thereby, either the growth rate of consumption as a component of the SDR can be uncertain (see Gollier, 2012, for a survey) or the SDR itself (e.g., Weitzman, 1998, 2001, 2007).

We take up these insights and introduce a declining social discount rate into macroeconomic models with optimal extraction of non-renewable resources. We then investigate how hyperbolic discounting affects optimal resource extraction and economic growth in comparison to exponential discounting. In order to isolate the impact of the discounting *method* on resource extraction and growth, we hold the *magnitude* of discounting constant for both discounting methods by assuming that for both discounting methods an infinite constant income stream has the same net present value (Myerson et al., 2001).

For our analysis we have to distinguish between the social discount rate, δ, applying to future monetary values (of consumption or output) and the utility discount rate, ρ, applying to future instantaneous utility. Both rates are connected through the Ramsey formula $\delta = \rho + \sigma g_C$, where σ denotes the inverse of the intertemporal elasticity of substitution, and g_C is the rate of consumption growth. In our macroeconomic model, consumption growth, g_C, is endogenous and therefore also the SDR, δ. This implies that we cannot introduce a declining SDR directly. For example, for constant consumption growth and a constant intertemporal elasticity of substitution σ, the SDR declines if the utility discount rate ρ declines. In cases in which the growth rate of consumption increases (and σ is constant), the utility discount rate ρ has to decline sufficiently fast to achieve a declining SDR. We assume that the utility discount rate is declining hyperbolically, resulting in a declining social discount rate if the consumption growth rate does not increase too rapidly.

For introducing a declining SDR caused by a declining utility discount rate we face a fundamental problem. Either decisions of the social planner are time-inconsistent, or the planner's preferences are only weakly stationary. Weak stationarity in this context means that two social planners only face the same preferences at the point in time when they initially optimize and decide on the optimal policy path, or when the time-span between initial time of optimization and the time of re-optimization is the same for both planners. This is reflected by the fact that preferences depend on calendar time and not only on the distance between planning and decision time as for exponential discounting. We follow Pezzey (2004) and Strulik (2020) and introduce a time-consistent hyperbolic discount function which is multiplicatively separable in decision time t_0 and pay-off time t and therefore leads to time-consistent decision making (see Burness, 1976; Drouhin, 2009, 2020). These types of preferences are only weakly stationary.

In order to gain a basic understanding of how hyperbolic discounting affects optimal resource extraction, we first compare hyperbolic and exponential discounting in a simple model of resource extraction, which we can solve analytically. Contrary to what one might expect, resource depletion is not faster with hyperbolic discounting compared to exponential discounting. For empirically realistic values of the intertemporal elasticity of substitution for consumption, we find that the extraction rate is always lower under hyperbolic discounting and, consequently, the resource stock is always higher under hyperbolic discounting.

This seemingly counter-intuitive result can be explained by the fact that even though the discount rate is initially higher for hyperbolic discounting, it drops below the level of the discount rate for exponential discounting at some point and remains lower thereafter. Since complete depletion of the resource in finite time can never be optimal, the social planner is inevitably more patient under hyperbolic discounting from some point in time onward. This also leads to a more conservative resource use from the beginning. In other words, the forward looking social planner anticipates his high future patience already in the initial years and chooses low extraction rates in order to allow for high consumption in the distant future.

We then compare resource extraction and economic growth for both discounting methods in the seminal Dasgupta-Heal-Solow-Stiglitz (DHSS) model (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974). We show analytically, that a (much) lower rate of technological progress is needed in order to achieve a positive long-run growth rate of consumption for hyperbolic discounting compared to exponential discounting. Again, this is caused by higher patience of the social planner in the long-run, which results in a more conservative resource use and higher accumulation of capital.

In order to solve the model numerically, we calibrate it with US data. The model predicts that the resource extraction rate is higher for hyperbolic discounting compared to exponential discounting in the initial years, but declines below that of exponential discounting after about 25 years and converges towards a lower long-run ratio asymptotically. Therefore, resource use is also more conservative in the DHSS model in the medium- and long-run.

Our paper is most closely related to Pezzey (2004), who introduces hyperbolic discounting into the DHSS model. His paper belongs to a strand of literature analyzing conditions under which a maximin solution and a classical utilitarian solutions exist and are non-trivial (see e.g. Mitra, 1983; Pezzey, 2004; or Asheim et al., 2007).[1] Our paper is related to this strand by an equivalence result, i.e. under specific parameter restrictions our model is equivalent to the models in these papers. This implies that our results with respect to resource extraction of a hyperbolic discounting economy can be interpreted as the solution of either a maximin or classical utilitarian social planner, depending on the exact restriction of parameter values. We do not elaborate on the equivalence result and instead leave it for future research, because our focus is the comparison of a resource extracting economy with a hyperbolic discounting social planner and an exponential discounting social planner, which to our knowledge has not been analyzed in this strand of the literature so far.

In Section 2, we introduce hyperbolic discounting and apply it to a simple model of resource extraction. We then proceed with introducing hyperbolic discounting into the DHSS model, which we analyze in Section 3. Section 4, we calibrate it to the US and solve it numerically. In Section 5, we conclude, and we collect the formal derivations of the DHSS model in A.

2. The Simplified Model

Before turning to the more complex DHSS model, we first study the impact of the social planner's discounting method on optimal resource extraction and consumption in a simple resource model,

[1] Mitra (1983) finds that quasi-arithmetic population growth is consistent with non-trivial maximin or "optimal" classical-utilitarian programs. Pezzey (2004) finds that there is a feasible development path that is "as if" optimal with respect to hyperbolic utility discounting and quasi-arithmetic technological progress and obtains a maximin path as a special case. Asheim et al. (2007) provide closed-form solutions to both the maximin and the classical utilitarian problem with quasi-arithmetic population growth and show the equivalence to constant savings rates.

for which we can provide an analytical solution. In the simple model, technology and population are constant. Furthermore, we assume that there is no physical or human capital, and that final output is produced from extracted resources only.

2.1. **The social planner.** A social planner maximizes households' welfare U understood as utility from consumption $u(c(\tau))$ experienced over an infinite time horizon and discounted to the present using the discount function $D(t_0, t, \tau)$,

$$U = \int_t^\infty D(t_0, t, \tau) u(c(\tau)) d\tau \,. \tag{1}$$

For the discount function D we distinguish between three different notations of time. Time t_0 denotes the initial planning time, which we assume to be fixed. Time $t \geq t_0$ denotes the actual planning time. It only deviates from t_0 if the social planner reoptimizes at a later point in time $t > t_0$. Finally, the discount function includes the decision time $\tau \geq t$ for which forward-looking decisions have to be made.

2.2. **Discounting.** For assessing how the discounting method affects resource extraction and consumption, we solve the model for both discount functions: exponential discounting and hyperbolic discounting. For exponential discounting we get

$$D_e(t_0, t, \tau) = \mathrm{e}^{-\bar{\rho}(\tau - t)} \,. \tag{2}$$

For hyperbolic discounting we follow Mazur (1987) and Strulik and Trimborn (2018), and choose the discount function[2]

$$D_h(t_0, t, \tau) = \left(\frac{1 + a(t - t_0)}{1 + a(\tau - t_0)} \right)^b \qquad a \geq 0, \quad b > 1 \,. \tag{3}$$

Since the exponential discount function is only affected by the distance between decision time and actual planning time $\tau - t$, preferences are independent from the actual point in time when they are made and decisions are time-consistent (Strotz, 1956).

In contrast, two different social planners with the hyperbolic discount function (3) only have the same discount factor if the difference between initial planning time and actual planning time $t - t_0$ is the same. In other words, when two social planners start planning at a different initial time t_0,

[2]With the normalization $t = t_0 = 0$ the discount function is the same as in Pezzey (2004) and Strulik (2020).

they only face the same discount function if the same amount of time has elapsed between initial planning t_0 and replanning t. This implies that preferences are only weakly stationary (see Strulik and Trimborn, 2018). Note that decisions are nonetheless time-consistent. When the social planner reoptimizes at a later point in time $t > t_0$ he choose the same allocation for all future points of time $\tau > t$ compared to the initial plan made at t_0.

The discount factor is 1 for both discounting methods at time t and decreases when the decision time τ moves further into the future. However, the speed at which both discount functions decline differs due to the functional form. This can best be seen by focussing on the discount rate, which captures the speed at which the discount function declines. The discount rate is the (negative) growth rate of the discount function. To simplify the notation we normalize $t_0 = 0$ and get that

$$-\frac{\mathrm{d}D_e/\mathrm{d}\tau}{D_e} = \rho$$

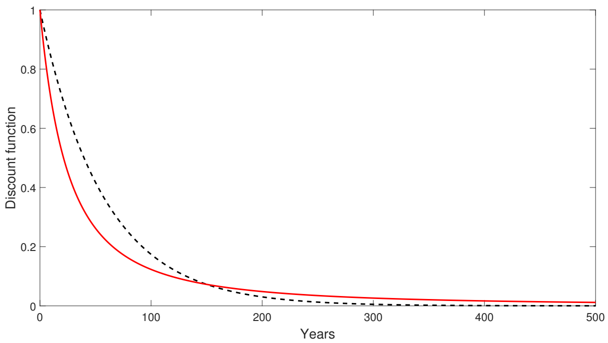
$$-\frac{\mathrm{d}D_h/\mathrm{d}\tau}{D_h} = \frac{ab}{1+a\tau} .$$

Figure 1a shows the discount functions for exponential discounting (dashed line) and hyperbolic discounting (solid line). By construction, both discount functions start at 1 and decline as the time horizon increases. The hyperbolic discount function declines more steeply at first but is less steep at points in time in the distant future. Consequently, both functions intersect. The difference between both discount functions can also be seen by inspecting the discount rate (see Figure 1b). It is constant for exponential discounting. In contrast, the hyperbolic discount rate declines over time and converges to zero. If both discount functions are normalized, as we suggest in the next subsection, the hyperbolic discount rate is initially higher than for exponential discounting.

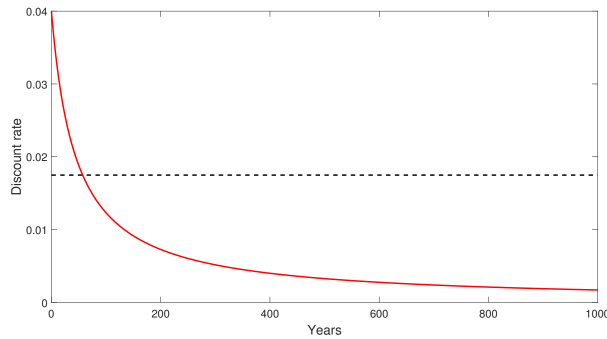
2.3. **Normalization.** In order to isolate the impact of the discounting method on resource extraction we apply the equivalent present-value approach (Myerson et al., 2001). The idea is to choose parameter values such that the present value of a constant stream of, for example, utility is the same for both discounting methods. In this way, any observed differences are not caused by differences in the magnitude of discounting, but by the discounting method itself. If we normalize $t_0 = t = 0$ we get

$$\int_0^\infty \mathrm{e}^{-\rho\tau} d\tau = \int_0^\infty \left(\frac{1}{1+a\tau}\right)^b d\tau .$$

FIGURE 1. Discount function and discount rates



(A) Discount function for exponential discounting (dashed line) and hyperbolic discounting (solid line).



(B) Discount rate for exponential discounting (dashed line) and hyperbolic discounting (solid line).

Solving the equation analytically provides the parameter restriction

$$\rho = a(b-1)\,. \tag{4}$$

2.4. **Resource extraction and production.** Output Y is produced from extracted resources R with a one-to-one production technology, $Y = R$. Since there is no savings and capital accumulation, all output is consumed, $C = R$.

The resources are extracted from a non-renewable resource stock S and the initial resource stock S_0 is given. Hence, the extraction equation is

$$\dot{S} = -R, \quad S(0) = S_0\,.$$

2.5. **Model solution and comparison.** We parameterize the utility function $u(\cdot)$ with a CIES utility function with the inverse of intertemporal elasticity of substitution denoted by σ. The social planner solves

$$U = \int_t^\infty D(t_0, t, \tau)\frac{C(\tau)^{1-\sigma} - 1}{1-\sigma} d\tau$$
$$\text{s.t. } \dot{S} = -R, \quad S(0) = S_0\,.$$

We normalize $t_0 = t = 0$ and solve the model for exponential and hyperbolic discounting. For exponential discounting we get a constant extraction ratio $R/S = \rho/\sigma$, i.e.

$$\frac{\dot{S}}{S} = -\frac{R}{S}$$

$$\frac{\dot{R}}{R} = -\frac{\rho}{\sigma} \,.$$

The optimal path of resource extraction is

$$R_e(\tau) = \frac{\rho}{\sigma} S_0 \mathrm{e}^{-\frac{\rho}{\sigma}\tau} \,. \tag{5}$$

For hyperbolic discounting optimal extraction is given by

$$\frac{\dot{S}}{S} = -\frac{R}{S}$$
$$\frac{\dot{R}}{R} = -\frac{1}{\sigma}\left(\frac{ba}{1+a\tau}\right) \,,$$

and solving for the time path of R gives

$$R_h(\tau) = \frac{a(b-\sigma)}{\sigma} S_0 (1+a\tau)^{-\frac{b}{\sigma}} \,. \tag{6}$$

for $b > \sigma$.

We now insert the normalizing equation $\rho = a(b-1)$ and get

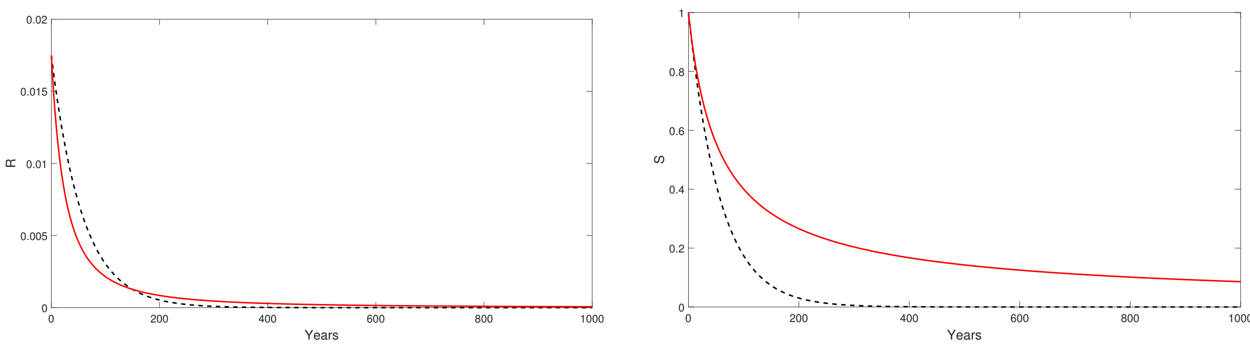
$$R_e(\tau) = \frac{a(b-1)}{\sigma} S_0 \mathrm{e}^{-\frac{a(b-1)}{\sigma}\tau} \tag{7}$$
$$R_h(\tau) = \frac{a(b-\sigma)}{\sigma} S_0 (1+a\tau)^{-\frac{b}{\sigma}} \,. \tag{8}$$

Note that in both cases it is optimal to stretch extraction over an infinite time horizon such that extraction rates are always positive and converge to zero asymptotically. This results from our assumption that resources are essential for consumption.

For $\tau = 0$ both discount factors are normalized to 1 and hence $R_e(0) > R_h(0)$ for $\sigma > 1$ and $R_e(0) = R_h(0)$ for $\sigma = 1$. Since Chetty (2006) argues that the intertemporal elasticity of substitution has to be equal or less than 1 implying that $\sigma \geq 1$ we focus our analysis on this case. To illustrate the impact of the discounting method on resource extraction we show the optimal extraction R and the resource stock S for the border case of $\sigma = 1$.

Figure 2a shows optimal resource extraction over time. It starts at the same level for both discounting methods but declines faster for hyperbolic discounting. At some point in time τ^* both lines intersect and from there on resource extraction is higher for hyperbolic discounting. This inversion in resource extraction follows from the fact that for both discounting methods it is optimal to deplete the entire resource stock eventually. If $R_e > R_h$ would hold for all time, resources would not be exhausted even in infinite time for the case of hyperbolic discounting. In the case of $\sigma > 1$, the extraction path for hyperbolic discounting starts even lower compared to exponential discounting, but otherwise the pattern of resource extraction is identical to the case of $\sigma = 1$.

Figure 2. Resource extraction in the simple model



(a) Optimal paths of resource extraction for exponential discounting (dashed line) and hyperbolic discounting (solid line).

(b) Optimal resource stock for exponential discounting (dashed line) and hyperbolic discounting (solid line).

The insights from the extraction paths can be used to infer how the resource stock evolves over time. Figure 2b shows the evolution of the resource stock for both discounting methods. The resource stock under hyperbolic discounting is always larger compared to that of exponential discounting. The reason is that extraction is always lower under hyperbolic discounting for the initial periods and larger afterwards. In other words, resource use will be more conservative under hyperbolic discounting. We summarize the results for the simple model.

Result 1. *Consider the simple resource extraction model with the normalizing assumption that exponential discounting and hyperbolic discounting both yield the same net present value of a constant utility stream. In this case, resource extraction is more conservative under hyperbolic discounting compared to exponential discounting when the intertemporal elasticity of substitution is in the empirically realistic range, i.e.* $\sigma \geq 1$. *In this case* $S_h(\tau) > S_e(\tau)$ *for all* $\tau > 0$ *holds.*

The intuition for this result is driven by the observation that hyperbolic discounting in comparison to exponential discounting does not imply that the social planner is more impatient per se. Since we hold constant the magnitude of discounting by our normalization, a social planner who uses hyperbolic discounting is more impatient initially but less impatient at later points in time as measured by the instantaneous discount rate. This means that even when the immediate discount rate is high initially, the social planner who uses hyperbolic discounting hesitates to choose high extraction rates, because he anticipates that he will be much more patient at later points in time. He saves resources for the future, when the weight on utility is higher compared to the social planner who uses exponential discounting. This results in a more conservative resource use for the entire extraction plan.

3. The DHSS Model

We now turn to the more complex model with capital accumulation and study how the discounting method of the social planner affects resource extraction and long-run economic growth in the seminal Dasgupta-Heal-Solow-Stiglitz (DHSS) model (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974). We assume constant population growth at rate n and constant exogenous technological progress at rate g. Output is produced with labor, capital and resources with a Cobb-Douglas production technology, implying that resources are essential for production. The social planner maximizes the discounted stream of utility over an infinite time horizon by choosing the optimal path of consumption and resource extraction.

3.1. **The social planner's solution.** In contrast to the simplified model presented in Section 2 we now use the more general discount function

$$D(t_0, t, \tau) = \mathrm{e}^{-\rho(\tau - t)} \cdot \left(\frac{1 + a(t - t_0)}{1 + a(\tau - t_0)} \right)^b . \tag{9}$$

For this discount function the discount rate is

$$-\frac{\frac{\partial D(t_0,t,\tau)}{\partial \tau}}{D(t_0,t,\tau)} = \rho + \frac{ab}{(1 + a\tau)} . \tag{10}$$

The general discount function nests two special cases. For $a = 0$ and $\rho > 0$ the discount function collapses to the exponential discount function with the constant discount rate ρ (see also equation (2)). For $\rho = 0$ and $a > 0$ the discount function reduces to the hyperbolic discount function (3), for which the discount rate declines and converges towards zero. We denote this second case 'pure' hyperbolic discounting from now on. We denote the general discount function with $\rho > 0$ and $a > 0$ 'mixed' discounting. It exhibits a declining discount rate that converges to ρ asymptotically.

This allows us not only to compare the limiting cases of pure exponential and pure hyperbolic discounting, but also to compare pure exponential discounting with 'mixed' discounting. A small but positive discount rate ρ is supported by Stern (2007) who argues that a small but positive long-run discount rate accounts for the possibility of humanity being extinguished by an exogenous event.

The social planner's maximisation problem now reads

$$\max_{C,R} \int_t^\infty \frac{C^{1-\sigma}-1}{1-\sigma} \mathrm{e}^{-\bar{\rho}(\tau-t)} \left(\frac{1+a(t-t_0)}{1+a(\tau-t_0)}\right)^b d\tau \,,$$

with CIES utility function and σ denoting the inverse of the elasticity of intertemporal substitution. The social planner takes two constraints into account. The first constraint is the capital accumulation equation which describes the evolution of capital K over time:

$$\dot{K} = AK^\alpha L^{1-\alpha-\gamma} R^\gamma - C - \delta K, \qquad 0 < \alpha, \gamma < 1 \,,$$

where we assume the production function is Cobb-Douglas and has constant returns to scale. The elasticity of capital in final output production is denoted by α, and the elasticity of resources in output production is denoted by γ. The size of the labor force L and technology A are assumed to grow exogenously at rate n and g, respectively, i.e., $\frac{\dot{L}}{L} = n$ and $\frac{\dot{A}}{A} = g$ hold. The initial values of the capital stock $K(0) = K_0$, the labor force $L(0) = L_0$, and technology $A(0) = A_0$ are given.

We assume that the parameter restriction $(1-\alpha)\bar{\rho} > (1-\sigma)(g + (1-\alpha-\gamma)n)$ holds, ensuring finite lifetime utility. For reasonable parameter values this restriction is always fulfilled.

The second constraint describes the evolution of the resource stock S:

$$\dot{S} = -R, \qquad S(0) = S_0 \,.$$

Naturally, resource extraction and the resource stock have to be non-negative, i.e., $R \geq 0$ and $S \geq 0$ have to hold.

Setting $t_0 = 0$, the social planner's optimization problem reads

$$\max_{C,R} \int_t^\infty \frac{C^{1-\sigma}-1}{1-\sigma} \mathrm{e}^{-\bar{\rho}(\tau-t)} \left(\frac{1+at}{1+a\tau}\right)^b d\tau \tag{11}$$

$$s.t. \; \dot{K} = AK^\alpha L^{1-\alpha-\gamma} R^\gamma - C - \delta K$$

$$\dot{S} = -R\,.$$

3.2. **Model solution and scaling.** The dynamic system (derived in A) reads:

$$\frac{\dot{K}}{K} = AK^{\alpha-1}L^{1-\alpha-\gamma}R^{\gamma} - \frac{C}{K} - \delta = \frac{Y}{K} - \frac{C}{K} - \delta \tag{12a}$$

$$\frac{\dot{S}}{S} = -\frac{R}{S} \tag{12b}$$

$$\frac{\dot{C}}{C} = \frac{\alpha AK^{\alpha-1}R^{\gamma} - \delta - \bar{\rho} - \frac{ba}{1+at}}{\sigma} = \frac{\alpha\frac{Y}{K} - \delta - \bar{\rho} - \frac{ba}{1+at}}{\sigma} \tag{12c}$$

$$\frac{\dot{R}}{R} = -\frac{\alpha}{1-\gamma}\frac{C}{K} + \frac{(1-\alpha)}{1-\gamma}\delta + \frac{1}{1-\gamma}(g + (1-\alpha-\gamma)n) \tag{12d}$$

The variables of system (12) are either converging to infinity or to zero. For analyzing the system we have to convert it into a stationary form such that the variables converge to finite, interior values. However, because the model, and hence system (12), is non-autonomous, it does not exhibit a balanced growth path in finite time. This means we cannot apply the usual procedures which are applied to conventional growth models for converting the system into a stationary form (see Trimborn, 2018). Therefore, we extend a procedure called scale-adjustment, which is frequently used to analyze models of balanced growth, to systems without balanced growth.

The idea of scale-adjustment is to slow down the growth of variables by the respective balanced growth rate such that the resulting system is stationary (see Trimborn, 2018). Assume that variable X grows with constant rate g_X in the long-run. Then the scale-adjusted variable x is defined as $x := X \cdot e^{-g_X t}$.

For our model, the growth rate of X may change over time. In fact, the dynamic system converges asymptotically to that of the standard DHSS model with discount rate $\bar{\rho} \geq 0$, implying that the growth rates also converge towards that of the standard model asymptotically. In order to facilitate the analysis of the model, we aim at scaling the model with scaling factors that are close to the solution under study. For this purpose, we extend the definition of scale-adjustment and introduce time dependent scaling factors according to

$$x := X \cdot e^{-\int_0^t g_X(\tau)d\tau}\,. \tag{13}$$

For deriving the asymptotic and time-dependent growth factors for scale-adjustment, we proceed as follows. We first derive ratios to which the variables converge asymptotically. Then, we insert these ratios into system (12) for deriving asymptotic growth rates of the model variables. Finally, having derived asymptotic growth rates, we apply scale adjustment to define variables that are stationary in the long-run (analogous to scale-adjustment in Trimborn, 2018) and convert the dynamic system into its scale-adjusted counterpart.

We find that, in the long-run, two groups of variables grow with the same rate. The first group consists of output, Y, consumption, C, and capital, K, and the second group consists of extraction rates, R, and the resource stock, S. We use this insight for deriving the level to which the ratios Y/K, C/K, and R/S converge asymptotically.

By denoting the growth rate of variable X by g_X, we get from the production function

$$(1-\alpha)g_K = \gamma g_R + g + (1-\alpha-\gamma)n\,. \tag{14}$$

Substituting $\frac{\dot{K}}{K}$ and $\frac{\dot{R}}{R}$ into this equation we get

$$(1-\alpha)\frac{Y}{K} = \frac{1-\alpha-\gamma}{1-\gamma}\frac{C}{K} + \frac{1-\alpha}{1-\gamma}\delta + \frac{1}{1-\gamma}(g + (1-\alpha-\gamma)n)\,. \tag{15}$$

Equating equations (12a) and (12c) gives a second relation for Y/K and C/K,

$$\frac{Y}{K} - \frac{C}{K} - \delta = \frac{\alpha\frac{Y}{K} - \left(\bar{\rho} + \frac{ba}{1+at}\right) - \delta}{\sigma}\,. \tag{16}$$

Equating (15) and (16) and solving for $\frac{Y}{K}$ and $\frac{C}{K}$ yields the following asymptotic relations

$$\lim_{t\to\infty}\frac{Y}{K} = \lim_{t\to\infty}\left[\frac{1-\alpha-\gamma}{(1-\alpha-\gamma+\sigma\gamma)\alpha}\left(\bar{\rho}+\frac{ba}{1+at}\right) + \frac{1}{\alpha}\delta + \frac{\sigma(g+(1-\alpha-\gamma)n)}{\alpha(1-\alpha-\gamma+\sigma\gamma)}\right]\,, \tag{17}$$

$$\lim_{t\to\infty}\frac{C}{K} = \lim_{t\to\infty}\left[\frac{(1-\gamma)(1-\alpha)}{\alpha(1-\alpha-\gamma+\sigma\gamma)}\left(\bar{\rho}+\frac{ba}{1+at}\right) + \frac{1-\alpha}{\alpha}\delta + \frac{(\sigma-\alpha)(g+(1-\alpha-\gamma)n)}{\alpha(1-\alpha-\gamma+\sigma\gamma)}\right]\,. \tag{18}$$

Equating g_R and g_S and substituting for C/K results in

$$\lim_{t\to\infty}\frac{R}{S} = \lim_{t\to\infty}\left[\frac{1-\alpha}{1-\alpha-\gamma+\sigma\gamma}\left(\bar{\rho}+\frac{ba}{1+at}\right) - \frac{(1-\sigma)(g+(1-\alpha-\gamma)n)}{1-\alpha-\gamma+\sigma\gamma}\right]\,. \tag{19}$$

By substituting the limits of equations (17) and (18) into equation (12a) we get

$$\lim_{t\to\infty}\frac{\dot{K}}{K} = \lim_{t\to\infty}\frac{\dot{C}}{C} = \lim_{t\to\infty}\frac{\dot{Y}}{Y} = \lim_{t\to\infty}\left[-\frac{\gamma}{1-\alpha-\gamma+\gamma\sigma}\left(\bar{\rho}+\frac{ba}{1+at}\right) + \frac{(g+(1-\alpha-\gamma)n)}{1-\alpha-\gamma+\gamma\sigma}\right] \tag{20}$$

Substituting equation (19) into equation (12b) gives

$$\underset{t\to\infty}{lim}\frac{\dot{R}}{R}=\underset{t\to\infty}{lim}\frac{\dot{S}}{S}=\underset{t\to\infty}{lim}\left[-\frac{1-\alpha}{1-\alpha-\gamma+\sigma\gamma}\left(\bar{\rho}+\frac{ba}{1+at}\right)+\frac{(1-\sigma)(g+(1-\alpha-\gamma)n)}{1-\alpha-\gamma+\sigma\gamma}\right]. \tag{21}$$

From these equations we define the scaling factors for the variables according to

$$g_K^*(t)=g_C^*(t)=\left[-\frac{\gamma}{1-\alpha-\gamma+\gamma\sigma}\left(\bar{\rho}+\frac{ba}{1+at}\right)+\frac{(g+(1-\alpha-\gamma)n)}{1-\alpha-\gamma+\gamma\sigma}\right], \tag{22a}$$

$$g_R^*(t)=g_S^*(t)=\left[-\frac{1-\alpha}{1-\alpha-\gamma+\sigma\gamma}\left(\bar{\rho}+\frac{ba}{1+at}\right)+\frac{(1-\sigma)(g+(1-\alpha-\gamma)n)}{1-\alpha-\gamma+\sigma\gamma}\right]. \tag{22b}$$

Using equations (22) we introduce the scale-adjusted variables k, c, r, s, and derive the system

$$\frac{\dot{k}}{k}=\frac{y}{k}-\frac{c}{k}-\delta-g_K^* \tag{23a}$$

$$\frac{\dot{s}}{s}=-\frac{r}{s}-g_S^*s \tag{23b}$$

$$\frac{\dot{c}}{c}=\frac{\alpha\frac{y}{k}-\delta-\left(\bar{\rho}+\frac{ba}{1+at}\right)}{\sigma}-g_C^* \tag{23c}$$

$$\frac{\dot{r}}{r}=-\frac{\alpha}{1-\gamma}\frac{c}{k}+\frac{(1-\alpha)}{1-\gamma}\delta+\frac{1}{1-\gamma}(g+(1-\alpha-\gamma)n)-g_R^* \tag{23d}$$

with $y=k^\alpha r^\gamma$.

3.3. **Normalization.** Just like in the simplified model, we apply the equivalent present-value approach (Myerson et al., 2001) to compare exponential and hyperbolic discounting. This means that the parameters have to be chosen such that the present value of a constant utility stream is the same for both discounting methods:

$$\int_{t_0}^{\infty}\mathrm{e}^{-\rho(t-t_0)}dt=\int_{t_0}^{\infty}\mathrm{e}^{-\bar{\rho}(t-t_0)}\left(\frac{1+at_0}{1+at}\right)^b dt\,. \tag{24}$$

Setting $t_0=0$ and solving this equation for ρ yields:

$$\rho=\frac{\bar{\rho}}{\Gamma(1-b,\frac{\bar{\rho}}{a})\mathrm{e}^{\frac{\bar{\rho}}{a}}(\frac{\bar{\rho}}{a})^b}, \tag{25}$$

where $\Gamma(\cdot,\cdot)$ is the incomplete gamma function. This equation, while more complex than its counterpart in the simplified model (4), is numerically solvable.

3.4. **Long-run growth and sustainability.** We now focus on long-run economic growth and derive the conditions under which the social planner chooses a consumption path exhibiting positive growth asymptotically. We then compare these conditions for both discounting methods.

From $\lim_{t\to\infty} g_C^*(t) \overset{!}{\geq} 0$ we derive the condition $\gamma\bar{\rho} < (g + (1-\alpha-\gamma)n)$ for positive growth of total consumption. Converting this into a relation for positive per-capita consumption growth in the long-run gives

$$\gamma\bar{\rho} < g - (\alpha+\gamma)n\,. \tag{26}$$

For pure exponential discounting, this equation turns into $\gamma\rho < g - (\alpha+\gamma)n$ with a large ρ. For pure hyperbolic discounting, the equation reduces to $(\alpha+\gamma)n < g$ and, because $\alpha+\gamma < 1$ holds, it is always fulfilled if $g > n$ holds. If $g = n = 0$ holds, consumption is asymptotically constant at some positive level under pure hyperbolic discounting.

We now investigate the parameter restrictions under which positive consumption growth is possible for mixed discounting. Results for pure exponential and pure hyperbolic discounting then follow immediately by setting $\bar{\rho} = 0$ and $\bar{\rho} = \rho$, respectively. For the analysis we focus on the growth rate of technology, g, and the population growth rate n. First, we derive the minimum rate of technological progress needed for positive consumption growth under both discounting methods, and second, we derive the maximum rate of population growth which can be supported with a positive growth rate of consumption at the same time.

Focusing on the growth rate of technology we get

$$g > \gamma\bar{\rho} + (\alpha+\gamma)n\,. \tag{27}$$

Plugging in $\bar{\rho} = \rho$ and $\bar{\rho} = 0$ for both discounting methods shows that under hyperbolic discounting a (much) smaller (i.e., by $\gamma\rho$) rate of technological progress is needed for supporting positive per-capita consumption growth than under exponential discounting. If population growth is zero, any positive growth rate of technology leads to positive per-capita consumption growth asymptotically under hyperbolic discounting.

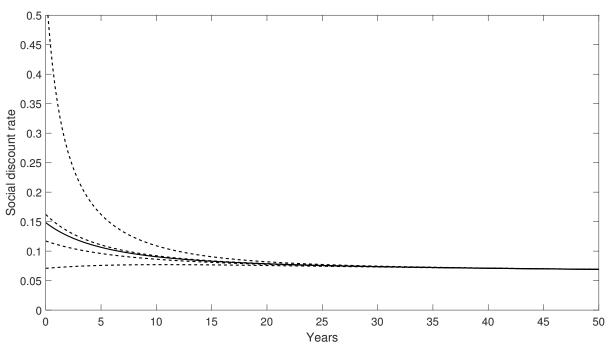
For focussing on population growth we rearrange the condition to

$$n < \frac{g - \gamma\bar{\rho}}{\alpha+\gamma} \tag{28}$$

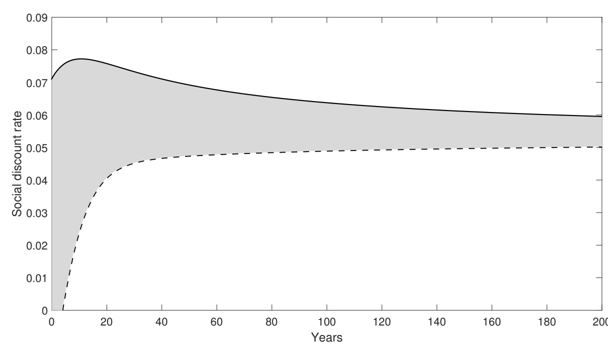
Because $\bar{\rho}$ enters the right hand side with a negative sign, a much higher population growth rate can be supported for hyperbolic discounting compared to exponential discounting (i.e., the difference is $\gamma\rho/(\alpha+\gamma)$).

Since these are only asymptotic results and the optimal consumption path chosen by the social planner is endogenous in our model, we take a closer look at the social discount rate δ. The social discount rate is connected to the growth rate of consumption via the modified Ramsey rule ($\delta = \bar{\rho} + \frac{ab}{(1+a\tau)} + \sigma g_C$) and is thus also endogenous in our model. The left panel of Figure 3 shows the social discount rate for different initial values K_0 and S_0. While the SDR is monotonically declining for most combinations of initial values, there are some paths for which the SDR rises in the first years.

FIGURE 3. Social discount rate



(A) Social discount rate for $K_0 = S_0 = 1$ (solid line) and four points on a circle around this point (dotted lines)



(B) Social discount rate and consumption growth rate for $K_0 = 1.75$ and $S_0 = 1$. The shaded area indicates the utility discount rate.

In the right panel of Figure 3 we focus on one exemplary path for a SDR which initially rises and only declines afters some years, and decompose it according to the modified Ramsey rule. The solid line depicts the social discount rate. The dotted line shows the second part of the Ramsey formula, σg_C, and the shaded area indicates the utility discount rate $\bar{\rho} + \frac{ab}{(1+a\tau)}$. Even though the utility discount rate declines monotonically, the steeply increasing growth rate of consumption dominates the shape of the SDR in the first years. However, after a few years, when most of the rapid convergence subsided, the declining utility discount rate dominates and the SDR is declining.

4. Quantitative analysis

4.1. **Parametrization.** In this section, we parameterize the model to the US. The capital share in the Cobb-Douglas production function is set to $\alpha = 0.3$, which is in line with literature suggesting a plausible range for the capital share of 0.3 to 0.4 (e.g. Maddison, 1987; Englander and Gurney, 1994). The output elasticity of resources, γ, is set to 0.046, which is in line with previous studies (see Acemoglu and Rafey, 2018, Hassler and Krusell, 2012 and Golosov et al., 2014). The depreciation rate, δ, is set to 0.08. The growth rates of population and technology are set to $n = 1\%$ and $g = 1.5\%$, respectively.

For the calibration of the inverse of the elasticity of intertemporal substitution, σ, we use a meta-analysis of 2735 published estimates of the elasticity of intertemporal substitution by Havranek et al. (2015). They find the average value of the elasticity of intertemporal substitution to be 0.5, and thus we set $\sigma = 2$ accordingly. In order to calibrate the discount rate for the model with exponential discounting, ρ, we use the assumption that the average growth rate of consumption is 1.6%. From equation (20) we get, in conjunction with the other parameter values, that $\rho = 2.4\%$. For calibrating the preference parameters for hyperbolic discounting, a, b and $\bar{\rho}$ we follow Stern (2007) and set $\bar{\rho}$ to 0.1% to account for the possibility of humanity being extinguished by an exogenous event. The parameters a and b have to be chosen such that the present value of a constant utility stream is the same for both discounting methods, see equation (24). However, this condition is not sufficient to get unique values for a and b. Therefore, we use values which additionally provide a good fit to the responses Weitzman (2001) obtained from asking over two thousand PhD-level economists "what real interest rate should be used to discount over time the (expected) benefits and (expected) costs of projects being proposed to mitigate the possible effects of global climate change". The forward rates he suggested are presented in Table 1. Using equation (24) and Table 1, we obtain $b = 2$ and $a = 0.02$.

Table 1. Forward Discount Rate Schedule, Source: Weitzman (2001)

Time Period	**Name**	**Marginal Discount Rate (Percent)**
Within years 1 to 5 hence	*Immediate* Future	4
Within years 6 to 25 hence	*Near* Future	3
Within years 26 to 75 hence	*Medium* Future	2
Within years 76 to 300 hence	*Distant* Future	1
Within years more than 300 hence	*Far-Distant* Future	0

For the calibration of K_0 we take the estimate from FRED, Federal Reserve Bank of St. Louis (2020) which amounts to 56 trillion dollars. The initial resource stock is calibrated by adding up the worth of the United States' proved crude oil (EIA, 2020b) and natural gas (EIA, 2020c) reserves which yields a worth of approximately 5 trillion dollars. We normalise S_0 to unity and scale K_0 accordingly, yielding $S_0 = 1$ and $K_0 = 11$. All parameter values are collected in table 2.

TABLE 2. Parameter values used for numerical simulation

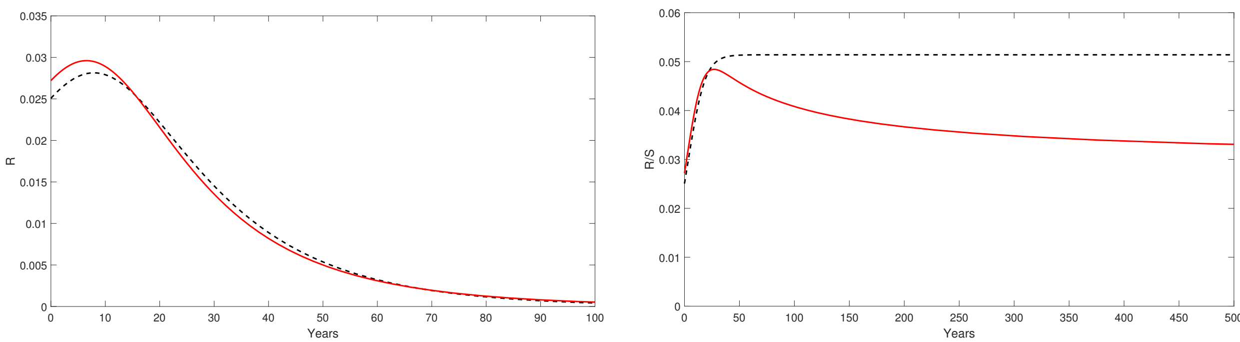
Parameter	**Description**	**Value**
α	Output elasticity of capital	0.3
β	Output elasticity of labour	0.6
γ	Output elasticity of resources	0.1
δ	Depreciation rate	0.08
σ	Inverse of the intertemporal elasticity of substitution	2
g	Growth rate of the level of technology	0.015
n	Growth rate of population	0.01
ρ	Discount rate (exponential discounting)	0.024
a	Parameter for the discount rate (hyperbolic discounting)	0.021
b	Parameter for the discount rate (hyperbolic discounting)	2
$\bar{\rho}$	Parameter for the discount rate (hyperbolic discounting)	0.001

4.2. **Resource extraction under hyperbolic and exponential discounting.** We now turn to the quantitative solution of the DHSS model and show that resources are exploited more conservatively under hyperbolic discounting compared to exponential discounting. For that purpose we solve the model numerically using the Relaxation algorithm Trimborn et al. (2008). The method does not use local approximation around the steady state but solves for the exact adjustment path globally, up to a user specified error. Therefore, it is particularly useful for solving highly non-linear adjustment paths far away from the steady state, just as for the model at hand. For the solution we employ the method of (Trimborn, 2013) to ensure that non-negativity constraints for resource extraction and investment hold.

The method can only be applied to stationary systems. This is why we proceed in two steps. In a first step, we employ the Relaxation method to solve transitional dynamics of the scale adjusted system (23) given the initial conditions. In a second step, we convert the scale adjusted variables back into their original counterparts by using equation (13).

Figure 4a shows the resource extraction path for hyperbolic discounting (solid line) and exponential discounting (dashed line). In the initial period, resource extraction is higher for hyperbolic discounting, but after about 16 years, extraction under exponential discounting overtakes and remains higher than under hyperbolic discounting until about year 70. From then on, extraction under hyperbolic discounting is again higher for the remaining time.

FIGURE 4. Comparison of resource extraction levels



(A) Resource extraction for exponential (dashed line) and hyperbolic (solid line) discounting

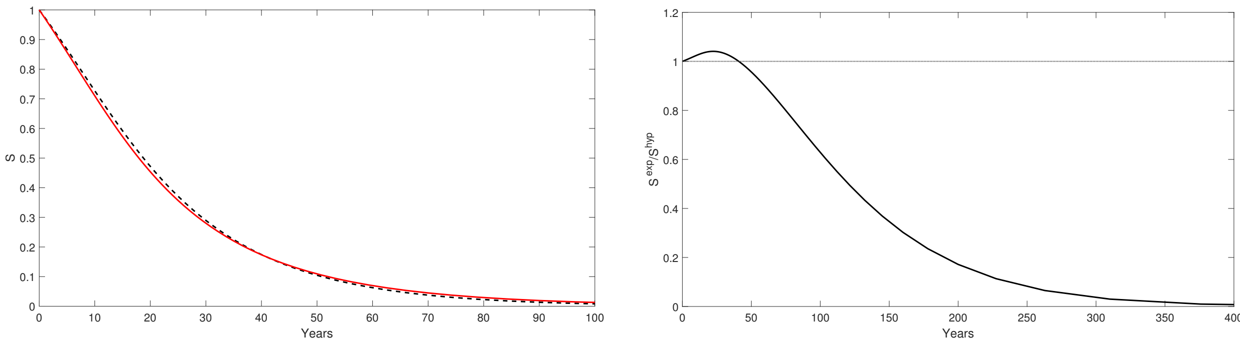
(B) Resource extraction rate for exponential (dashed line) and hyperbolic (solid line) discounting

The race of resource exploitation under both discounting methods can be also illustrated by focusing on the extraction *rate*, R/S, which we show in Figure 4b. While the extraction rate is initially higher under hyperbolic discounting (solid line), it is soon overtaken be the extraction rate under exponential discounting (dashed line). From the intersection point onwards, both extraction rates follow strikingly different paths and converge towards different levels asymptotically. From equation (19) we can derive that the rate converges towards 5.1% under exponential discounting and towards 3.0% under hyperbolic discounting, i.e. asymptotically the rate is only 58% of that under exponential discounting.

Finally, we compare the evolution of the resource stock under both discounting methods. Figure 5a shows the evolution of the resource stock for hyperbolic discounting (solid line) and exponential discounting (dashed line). By construction, both stocks start at 1. Caused by initially higher extraction under hyperbolic discounting, the resource stock is lower compared to exponential discounting in the first years. After 22 years, the gap starts narrowing and after about 42 years, the ranking switches and the stock remains higher for hyperbolic discounting thereafter.

The different evolution of the resource stock under both discounting methods can also be illustrated by focusing on the ratio of both stocks. We show the ratio of the stock between exponential discounting and hyperbolic discounting in Figure 5b. The ratio starts at 1, peaks in year 12 and

Figure 5. Comparison of resource stocks



(A) Resource stock for exponential (dashed line) and hyperbolic (solid line) discounting

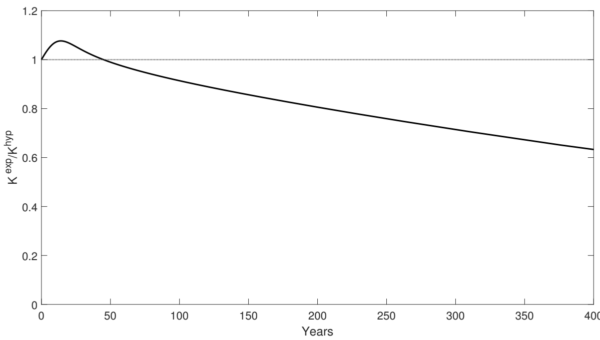
(B) Ratio of resource stock with exponential and hyperbolic discounting S^{exp}/S^{hyp}

declines monotonically afterwards. From year 60 onwards, the ratio is below 1, indicating that the stock is higher under hyperbolic discounting.

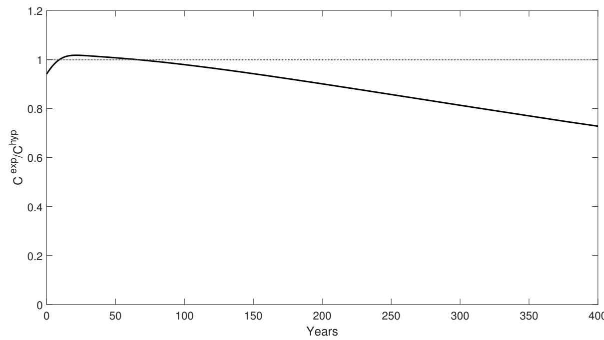
To summarize, the overall picture is that resource extraction is faster in the initial years under hyperbolic discounting compared to exponential discounting. However, in the medium- and long-run, resource use is more conservative under hyperbolic discounting, leading to a higher resource stock in the medium- and long-run. The intuition behind this finding is driven by the different weight that both discounting methods give to instantaneous utility over time. Initially, the social planner using hyperbolic discounting is very impatient compared to the social planner using exponential discounting, measured by the high initial discount rate. Consequently, resource extraction is faster in the first years under hyperbolic discounting. This changes, however, after a few years when the hyperbolic discount rate declines to low levels and the social planner employing hyperbolic discounting becomes very patient. Resource extraction rates slow down and converge towards a much lower long-run level.

The different impact of both discounting methods can also be seen by focusing on the second intertemporal decision of the social planner: the accumulation of capital. Figure 6a shows the ratio of the capital stock for both discounting methods. By construction, it starts at 1. Since the savings rate of the social planner using exponential discounting is higher initially, the ratio increases but peaks already after about 15 years. Thereafter, the savings rate of the planner using hyperbolic discounting is higher and the ratio declines monotonically. Just like for resource extraction, the social planner using hyperbolic discounting only behaves less conservatively in the initial years and more conservatively thereafter.

Figure 6. Comparison of capital stock and consumption



(a) Ratio of capital stock with exponential and hyperbolic discounting K^{exp}/K^{hyp}



(b) Ratio of consumption with exponential and hyperbolic discounting C^{exp}/C^{hyp}

Finally, we compare consumption under both discounting methods. The overall picture is that, maybe surprisingly, consumption is higher under hyperbolic discounting, except for a short period between year 10 and year 65. This results from the fact that the higher savings rate and lower resource extraction under hyperbolic discounting soon allow for a higher level of consumption.

4.3. **Sensitivity Analysis.** Because we can only solve the DHSS model numerically, we cannot provide a formal proof for our results. Therefore, we counter the uncertainty associated with the choice of parameters with a sensitivity analysis. In order to do so, we vary the values of parameters ceteris paribus and check how these changes affect the depletion of the resource stock S, the extraction rate $\frac{R}{S}$ and the initial resource extraction R_0. More precisely, we analyse how the changes affect the key figures $S_{intersect}$ and $\frac{R}{S}_{intersect}$, which indicate the time it takes until the lines of the resource stock and the extraction rate intersect for exponential and hyperbolic discounting. Additionally, we will check how changes in the parameter values affect the ratio of initial resource extraction, R_0^{exp}/R_0^{hyp}. Table 3 shows $S_{intersect}$, $\frac{R}{S}_{intersect}$ and R_0^{exp}/R_0^{hyp} for each scenario.
In our first scenario, we increase the value of the output elasticity of resources γ from 0.046 to 0.06 ($\gamma+$) and decrease it to 0.03 ($\gamma-$). The effect on our key figures is minimal, $S_{intersect}$ and $\frac{R}{S}_{intersect}$ both change by approximately one year while the ratio of initial resource extraction does not change at all.

TABLE 3. Sensitivity analysis

Scenario	$\mathbf{S_{intersect}}$	$\mathbf{\frac{R}{S}}_{\mathbf{intersect}}$	$\mathbf{R_0^{exp}/R_0^{hyp}}$
-	40.93	22.33	0.92
$\gamma+$	39.90	21.91	0.92
$\gamma-$	42.07	22.80	0.92
$\sigma+$	48.64	26.44	0.93
$\sigma-$	27.38	15.23	0.93
$g+$	43.51	23.45	0.92
$g-$	34.54	19.34	0.93
$n+$	47.14	24.92	0.92
$n-$	36.96	20.51	0.93
$a+$	22.98	13.33	0.94
$a-$	105.81	50.69	0.92
$b+$	30.07	17.17	0.94
$b-$	56.37	28.99	0.90
$\frac{K}{S}-$	45.66	21.77	0.91

Next, we vary the value of the inverse of the elasticity of intertemporal substitution, σ. An increase of σ to 2.5 increases $S_{intersect}$ by almost 8 years, a decrease of σ to 1.5 decreases $S_{intersect}$ by more than 13 years. $\frac{R}{S}_{intersect}$ changes by 3 and 7 years, respectively. The ratio of initial resource extraction is similar to the benchmark case for both scenarios.

Changes in the growth rate of technology g and the growth rate of population n have similar effects on the key figures as they only appear together in the expression $(g + \beta n)$ in the dynamic system (23a)-(23d). Varying g between 0.01 and 0.02 and n between 0.005 and 0.2 leads to changes in $S_{intersect}$ and $\frac{R}{S}_{intersect}$ of up to 7 years. In these scenarios, too, the changes in R_0^{exp}/R_0^{hyp} are minimal.

We next investigate the effects of changes in the parameters a and b, which determine the discount function in the hyperbolic model. For this analysis we stick to our calibration strategy and re-calibrate ρ such that the normalization for both discounting methods keeps holding. Increasing a drastically to 0.03, which corresponds to an increase in ρ from 2.4% to 3.3%, significantly reduces both $S_{intersect}$ and $\frac{R}{S}_{intersect}$ to 22.98 and 13.33 years, respectively. A decrease in a to 0.01 (yielding $\rho = 1.25\%$) increases $S_{intersect}$ and $\frac{R}{S}_{intersect}$ to 105.81 and 50.69 years, respectively. The impact of changes in b is smaller. Varying b between 1.75 and 2.25 leads to changes in $S_{intersect}$ and $\frac{R}{S}_{intersect}$

of up to 16 years. The ratio of initial resource extraction is similar to the benchmark case for all scenarios.

Finally, we analyze changes in the initial capital-resource ratio $\frac{K_0}{S_0}$. Since we only consider proved resource reserves in our benchmark scenario, we only decrease the initial capital-resource ratio in our sensitivity analysis. In particular, we recalculate the initial capital-resource ratio taking estimates of technically recoverable resource reserves (EIA, 2020a) instead of proven reserves. This changes the initial capital-resource ratio drastically from approximately 11 to less than 2. Nevertheless, changes in our indicators are small, with changes in $S_{intersect}$ of around 5 years. $\frac{R}{S}_{intersect}$ changes by less than one year and R_0^{exp}/R_0^{hyp} hardly changes at all. Considering the large difference in initial capital-resource ratio, the sensitivity of the results with respect to the initial capital-resource ratio seems to be rather small.

Concluding, it can be said that the results are robust with respect to changes in some of the key parameters such as the output elasticity of resources, γ, and the initial capital-resource ratio $\frac{K_0}{S_0}$. Other parameters, such as a and b, do impact the quantitative results quite significantly when changed drastically. However, none of the parameter alterations changed the qualitative results of the model. Initial resource extraction is always higher with hyperbolic discounting, which means that the resource stock declines faster in the short-run. In the medium-term, the level of resource extraction in the model with hyperbolic discounting drops below the level of resource extraction in the model with exponential discounting. Therefore, the resource stock declines slower in the model with hyperbolic discounting and remains higher after a unique point of intersection, $S_{intersect}$. Initially, the extraction rate is also higher in the model with hyperbolic discounting, but falls below the extraction rate of the model with exponential discounting after $\frac{R}{S}_{intersect}$.

5. Conclusion

In this paper, we proposed a time-consistent method of hyperbolic utility discounting implying a declining social discount rate and introduced it into macroeconomic models of resource extraction. We compared the implications for growth and resource extraction between hyperbolic and exponential discounting holding constant the magnitude of discounting for both discounting methods. We found that in a small model of pure resource extraction, for a reasonable intertemporal elasticity of consumption, hyperbolic discounting leads to a more conservative resource use resulting in lower extraction rates at all times and a higher resource stock at all times. The intuition for this result is that the forward looking social planner anticipates from the beginning that the discount rate will be much lower in the future giving a high weight to future generations' utility. This leads to an already low optimal rate of resource extraction in the initial years.

We also introduced hyperbolic discounting into the Dasgupta-Heal-Solow-Siglitz model. First, we showed analytically that compared to exponential discounting, a (much) lower rate of technolgical progress is necessary under hyperbolic discounting to achieve positive consumption growth in the long-run. Second, we calibrated the model to the US and showed that resource extraction is more conservative in the medium- and long-run. Again, this can be motivated by the social planner's foresight concerning future low discount rates.

Our analysis complements papers asking for a declining social discount rate (Arrow et al., 2014). It contradicts the intuition that the initially higher discount rates in comparison with exponential discounting might lead to much faster resource extraction, thereby neglecting the need of future generations. Instead, we showed that resource use is more conservative in the medium- and long-run implying that long-run growth is higher.

Although we focused on extraction of non-renewable resources, a similar mechanism might apply to climate change. Since most of the damages of climate change occur in the far future, a social planner using hyperbolic discounting rates those damages at a higher weight than a social planner using exponential discounting, and will probably advocate stronger measures against climate change (see e.g. Karp, 2005 for a theoretical analysis). We leave this analysis for future research.

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Appendix A. Solution of DHSS model

In the case of mixed discounting, the present value Hamiltonian given by

$$H = \mathrm{e}^{-\bar{\rho}t}\left(\frac{1}{1+at}\right)^{b}\frac{C^{1-\sigma}-1}{1-\sigma} + \lambda_K(AK^{\alpha}L^{\beta}R^{\gamma} - C - \delta K) - \lambda_S R + \lambda_A g A + \lambda_L n L\,.$$

The first order conditions (FOCs) are given by

$$\frac{\partial H}{\partial C} = 0 \Leftrightarrow \mathrm{e}^{-\bar{\rho}t}\left(\frac{1}{1+at}\right)^{b} C^{-\sigma} - \lambda_K = 0 \tag{29a}$$

$$\frac{\partial H}{\partial R} = 0 \Leftrightarrow \gamma\lambda_K AK^{\alpha}L^{\beta}R^{\gamma-1} - \lambda_S = 0 \tag{29b}$$

$$\frac{\partial H}{\partial K} = \alpha\lambda_K AK^{\alpha-1}L^{\beta}R^{\gamma} - \lambda_K\delta = -\dot{\lambda_K} \tag{29c}$$

$$\frac{\partial H}{\partial S} = 0 = -\dot{\lambda_S} \tag{29d}$$

$$\frac{\partial H}{\partial A} = \lambda_K K^{\alpha}L^{\beta}R^{\gamma} + \lambda_A g = -\dot{\lambda_A} \tag{29e}$$

$$\frac{\partial H}{\partial L} = \beta\lambda_K AK^{\alpha}L^{(}\beta-1)R^{\gamma} + \lambda_L n = -\dot{\lambda_L} \tag{29f}$$

together with the initial conditions and the four transversality conditions:

$$\lim_{t\to\infty} \lambda_K K = 0 \tag{30a}$$

$$\lim_{t\to\infty} \lambda_S S = 0 \tag{30b}$$

$$\lim_{t\to\infty} \lambda_A A = 0 \tag{30c}$$

$$\lim_{t\to\infty} \lambda_L L = 0\,. \tag{30d}$$

The Keynes-Ramsey rule. In order to derive the Keynes-Ramsey rule, we take the logarithm of equation (29a) and differentiate the result with respect to time:

$$\frac{\dot{\lambda_K}}{\lambda_K} = -\frac{ba}{1+at} - \bar{\rho} - \sigma\frac{\dot{C}}{C}\,. \tag{31}$$

Dividing equation (29c) by λ_K yields

$$-\frac{\dot{\lambda_K}}{\lambda_K} = \alpha AK^{\alpha-1}R^{\gamma} - \delta\,. \tag{32}$$

Combing equation (31) and equation (32) we get the Keynes-Ramsey rule

$$\frac{\dot{C}}{C} = \frac{\alpha A K^{\alpha-1} R^{\gamma} - \delta - \bar{\rho} - \frac{ba}{1+at}}{\sigma}\,. \tag{33}$$

The $\dot{R}$-equation. In order to get the $\dot{R}$-equation, we combine the growth rates of λ_S and λ_K. Equation (32) gives us the growth rate of λ_K. Equation (29b) establishes a relation between the two costate variables λ_K and λ_S. The equation can be converted to growth rates by taking the logarithm of each side and their derivatives with respect to time yielding

$$\frac{\dot{\lambda_S}}{\lambda_S} = \frac{\dot{\lambda_K}}{\lambda_K} + g + \alpha\frac{\dot{K}}{K} + \beta n + (\gamma - 1)\frac{\dot{R}}{R}\,. \tag{34}$$

By substituting the growth rates of λ_K and λ_S and K into equation (34) and solving this equation for $\frac{\dot{R}}{R}$ we get

$$\frac{\dot{R}}{R} = -\frac{\alpha}{1-\gamma}\frac{C}{K} + \frac{(1-\alpha)}{1-\gamma}\delta + \frac{1}{1-\gamma}(g + \beta n)\,. \tag{35}$$

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