
Economics

Working Papers

2020-01

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Managerial Overconfidence and Self-Reported Success*

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January 2020

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I consider the optimal contract for an overconfident manager in a principal-agent model with moral hazard where the contract is written on the earnings of the firm. Overconfidence causes the manager to overestimate his ability to affect the outcome of the firm. Overconfidence first reduces cost of agency, and if the level of overconfidence is significant enough, it causes the manager to wager on his wrong beliefs. The accounting system obscures the outcome of the manager's effort, which attenuates the effect of significant overconfidence and decreases the principal's profit. Inducing the manager to truthfully communicate his self-observed success allows the principal to directly contract on the cause of disagreement, the manager's effect on firm outcome. This reduces the risk premium for a slightly overconfident manager and emphasizes the wager effect for a significantly overconfident manager. The value of communication is first decreasing in overconfidence for a slightly overconfident manager and then increasing in overconfidence for a significantly overconfident manager.

Keywords: Overconfidence; Moral hazard; Communication; Disclosure

JEL Classification: D83; D86; D91; M41

*I would like to thank Hans Frimor, Qiang Guo, Jens Josephson, Marrku Kaustia, Alexander Koch, Julia Nafziger and James Ohlson as well as Southern Danish University, the participants at the Three Star Conference 2019 and at the Young Scholars Nordic Finance Workshop 2019 for helpful comments and constructive criticism. I would especially like to thank Leonidas Enrique de la Rosa for guidance throughout the process of writing this paper.

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1 Introduction

Overconfidence is a well-documented trait in managers (Blavatskyy (2009), Puri and Robinson (2007)), and acknowledging this when designing the optimal contract allows the principal to reap a profit (Goel and Thakor (2008), de la Rosa (2011)). Overconfidence means that the manager has exaggerated beliefs with regards to his ability to affect the outcome of the firm. In reality, the outcome of the firm is not observable to the principal and the board. The observable measure is accounting earnings, which is a noisy measure of the outcome of the firm, as well as the manager's presentation of the firm's economics. As the overconfident manager has exaggerated beliefs in his ability to make the firm succeed, the ability of the accounting system to correctly present this together with the possibility of managerial communication turns out to be an important factor in the optimal contract.

This paper explores the consequence of managerial communication when the manager is overconfident. To this end I use a principal-agent model with moral hazard to explore how the benefits of contracting with an overconfident manager are affected by the noisiness of the accounting system and managerial communication. Managerial overconfidence first reduces the cost of agency, and if the level of overconfidence is significant enough, it causes the manager to wager on his wrong beliefs. The accounting system obscures the actual economic outcome of the firm which attenuates the significantly overconfident manager's desire to wager against the principal. To circumvent this, I analyze a contract written on the overconfident manager's self-reported success. Introducing communication allows the observation of the actual success or failure of the firm to be incorporated into the contract. This is important as the exaggerated beliefs of the manager concern his ability to make the firm succeed. The value of communication is decreasing with overconfidence for a slightly overconfident manager, but increasing with overconfidence for a significantly overconfident manager.

I contribute to the literature by studying the benefits of the overconfident manager communicating unverifiable information to the principal. Dye (1983), Gigler and Hemmer (2001),

and Şabac and Tian (2014) all show that when the rational manager observes the true outcome of the firm, the principal is at least as well off using this information. With a significantly overconfident manager, this effect is emphasized as communication makes the manager's overconfidence affect the contract directly and increases the manager's desire to wager against the principal. As overconfidence increases within the range of significant overconfidence, the value of communication increases. However, when the manager is only slightly overconfident, communication, overconfidence and earnings quality all reduce the agency of contracting the manager, and as these effects crowd out each other, the value of communication decreases in the range of slight overconfidence. The results with a significantly overconfident manager are in line with Hribar and Yang (2016) and Libby and Rennekamp (2012), where Libby and Rennekamp (2012) find that overconfident managers issue more frequent management forecasts and Hribar and Yang (2016) who find that they are more precise.

In a contract setting with moral hazard, overconfidence is purely beneficial (de la Rosa (2011)). This result is a consequence of the partial equilibrium structure, where only the optimal contract is considered. Other strands of the literature find that overconfidence has both negative and positive effects. Malmendier and Tate (2005) find that an overconfident CEOs distorts investments, and Hsu et al. (2017) find that conservatism in the accounting system can mitigate this behavior, so that the combination of accounting conservatism and managerial overconfidence produces higher cash flows. Bouwman (2014) finds that overconfident CEOs are associated with higher earnings smoothing. Similarly, Schrand and Zechman (2012) find that a positive bias in the financial statement from a manager might lead to increased biases if, contrary to the manager's belief, everything starts going downwards. Although the present paper does not consider any investment decision, issues of truthful communication does relate earnings management. If one thinks of managerial communication as the non-audited part of the financial statement or possibly the areas which are more difficult to audit, then the result of this paper suggests that earnings management can be

useful for signaling the true underlying value of the firm to the principal, and this is increasingly value enhancing as the manager's overconfidence grows within the range of significant overconfidence.

To analyze the consequence of communication in case of an overconfident manager, I apply a principal-agent model with moral hazard. Here I use the framework of de la Rosa (2011), where an overconfident manager can choose two levels of effort, that can either lead to the success or failure of the firm. Overconfidence shows its face as the principal and the manager disagree about the manager's ability to make the firm succeed conditional on high effort. de la Rosa (2011) finds two effects of overconfidence in the optimal contract. In case of slight overconfidence, a less powerful contract is needed to induce high effort. When the manager becomes significantly overconfident, the moral hazard problem disappears, and the manager wants to wager against the principal on the success of the firm.

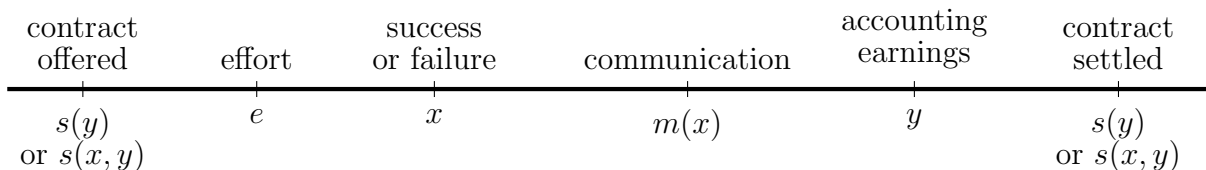
I include communication by extending de la Rosa (2011) with the framework from Gigler and Hemmer (2001). Here the principal cannot directly observe the outcome of the firm, but only the accounting earnings. Accounting earnings are a noisy measure of the true outcome of the firm, and increasing precision increases the principal's profits from contracting with the manager. When the manager is slightly overconfident, earnings quality, overconfidence and communication all decreases the agency cost of hiring the manager. However, the decrease in agency cost is a concave function in earnings quality and overconfidence, and thus earnings quality and overconfidence are substitutes, that the value of overconfidence is decreasing in earnings quality and vice-versa. For a significantly overconfident manager earnings quality emphasizes the wager effect, thus making the two compliments. Communication reduces the risk on the manager in the state of the world with low outcome, decreasing the cost of agency. When the manager is more overconfident within the range of slight overconfidence, agency cost is lower, and thus the value of communication goes down. When the manager is significantly overconfident, communication allows the contract to directly incorporate the effect of disagreement between the manager and the principal, thus emphasizing the effect of

overconfidence and increasing the wager effect between the principal and the manager. As the wager effect grows with overconfidence, which is beneficial to the principal, the value of communication increases with overconfidence in the range of significant overconfidence.

The paper proceeds as follows. In section 2 I present the model. In section 3 I analyze the setting where accounting earnings are the only contractible information. In section 4 I extend the framework with the possibility of the manager reporting his observation of the firm's outcome. In section 6 I conclude.

2 Model

Figure 1: **Time line**



The manager is offered a contract and after observing failure or success, $x \in \{x^H, x^L\}$, he can communicate his observation to the principal.

In this setting a risk-neutral principal hires a manager to supply unobservable effort. The manager is risk and effort averse. He makes a choice that affects the future outcome of the firm and that is costly for him. This could be laying off unnecessary workers, reducing unprofitable investments, or implementing a new product. The manager can choose either high or low effort, $e \in \{l, h\}$, which with a certain probability leads to either a high economic value of the firm x^H or low economic value of the firm x^L such that $x^H > x^L$. The manager is the only one who observes the true economic condition of the firm after his chosen effort level.

The principal and the manager have heterogeneous beliefs with regards to the probability of the outcome given the manager's effort, but agrees on the probability outcome conditional on no effort. They are both aware of the disagreement but agree to disagree. The principal believes that the probability of success of the firm given high effort is p_h . The manager

instead believes that it is \tilde{p}_h . I assume $0 < p_h < 1$ and $0 < \tilde{p}_h < 1$ ¹ and the manager is said to be overconfident overall if $\tilde{p}_h > p_h$. I will assume this to be the case throughout the paper. It is worth noticing that the optimal contract does not change irrespective of whether it is the principal or the manager who holds the correct beliefs, only the welfare implications changes.

I focus on a one-period model which means that I abstract away from managerial learning. It might be reasonable to assume that if the disagreement between the manager and the principal is sufficiently large, then after repeating the contract a few times, the manager might readjust his belief as regards his own ability. However, I believe that this setting will capture the essence of contracting with an overconfident manager, and settings also exists where the contract is short-term and not renegotiated, and thus they are well-described by this model. One of such settings could be the case of a private equity firm² who after buying majority of a target firm hires a manager to run it for a short period prior to selling their part again. Here one could reasonably assume that there would be no opportunity for the manager and the principal to resolve their disagreement, and therefore the contracting environment is well-described by the assumptions in this setting.

I assume that $(x^H - x^L) \times (p_h - p_l)$ is sufficiently large such that it is always optimal for the principal to induce high effort. The manager's utility is additively separable in money and effort and is given by

$$U(s(\cdot)) - c(e). \tag{1}$$

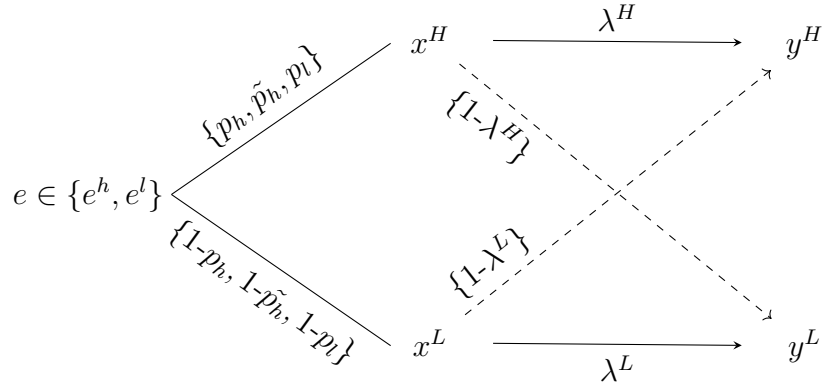
The manager's utility is continuous and twice continuously differentiable with $U' > 0$ and $U'' < 0$ so that the manager is risk averse. Given a choice of effort for the manager, $e \in \{l, h\}$, his personal cost is given by $c(l) = 0$ and $c(h) = c$. The manager's outside utility is independent of overconfidence and is given by \bar{U} .

The principal neither observes effort nor outcome. She observes a report that can either

¹This is to avoid the trivial contract with infinite punishment.

²I would like to thank Marrku Kaustia for suggesting this example.

Figure 2: **Probability structure**



- e = productive effort;
- x^H = success;
- x^L = failure;
- y^H = high accounting earnings;
- y^L = low accounting earnings;
- p_h = principal's belief of success conditional on high effort;
- \tilde{p}_h = manager's belief of success conditional on high effort;
- p_l = probability of success conditional on low effort;
- λ^H = probability of high accounting earnings when the firm succeed; and
- λ^L = probability of low accounting earnings when the firm fails.

show success, y^H , or failure, y^L . The probability of the report showing success when the underlying value is high is given by λ^H , and in the case of a report showing failure with low underlying value it is λ^L . It is assumed that $\lambda^H > 1 - \lambda^H$ and $\lambda^L > 1 - \lambda^L$. The intuitive interpretation of this report is the financial statement, but due to accounting recognition criteria it might not give a true view of the economic performance.

3 Reported earnings

I begin by considering the case where the principal only receives the mandatory report and thus excludes communication. I will use this as the case for comparison when I include communication.

The optimal contract written on accounting earnings that implements high effort is given

as the solution to the following "earnings-only" program

$$\min_{s(y)} p_h [\lambda^H s(y^H) + (1 - \lambda^H) s(y^L)] + (1 - p_h) [\lambda^L s(y^L) + (1 - \lambda^L) s(y^H)], \quad (2)$$

subject to the individual rationality constraint

$$\begin{aligned} & \tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\ & + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c \geq \bar{U}, \end{aligned} \quad (3)$$

and the incentive compatibility constraint

$$\begin{aligned} & \tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\ & + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c \geq \\ & p_l [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\ & + (1 - p_l) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))]. \end{aligned} \quad (4)$$

Here the principal's objective is to minimize remuneration costs in relation to the manager while ensuring that (i) the contract provides the manager the minimum benefit before he walks away, the individual rationality constraint, and (ii) that the manager decides to put in high effort, the incentive compatibility constraint.

Lemma 1. *If the manager is only slightly overconfident, the earnings-only contract will be defined by the binding individual rationality constraint and the incentive compatibility constraint.*

The optimal solution to the problem where reported earnings are the only performance

measure is given by:

$$U(s(y^H)) = \bar{U} + c - \frac{c\tilde{p}_h}{(\tilde{p}_h - p_l)} + \frac{c\lambda^L}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}, \quad (5)$$

$$U(s(y^L)) = \bar{U} + c - \frac{c\tilde{p}_h}{(\tilde{p}_h - p_l)} - \frac{c(1 - \lambda^L)}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}. \quad (6)$$

All proofs are found in the appendix.

As the manager becomes more overconfident, he believes in his ability to make the firm succeed increases. This relaxes the incentive compatibility constraint and decreases the necessary power of the contract needed to induce high effort from the manager as he becomes increasingly willing to perform high effort.

Corollary 1. *If the manager is slightly overconfident, the earnings-only contract exhibits power of incentives that decreases in the degree of overconfidence:*

$$\frac{d(U(s(y^H)) - U(s(y^L)))}{d\tilde{p}_h} < 0, \quad (7)$$

and that decreases in accounting precision:

$$\frac{d(U(s(y^H)) - U(s(y^L)))}{d\lambda^i} < 0 \text{ for } i = \{L, H\}, \quad (8)$$

but the reduction in the power of incentives from an increase in earnings quality decreases in overconfidence:

$$\frac{\partial^2 (U(s(y^H)) - U(s(y^L)))}{\partial \lambda^H \partial \tilde{p}_h} > 0 \quad (9)$$

The more overconfident the manager is, the less will the incentive compatibility constraint bind. I will show that for a sufficiently high degree of overconfidence, the incentive constraint will be slack:

Lemma 2. *If the manager is significantly overconfident, the solution will be defined by the*

binding individual rationality constraint.

The manager is significantly overconfident if

$$\begin{aligned} & \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L) U'(\bar{s}(y^H))}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L U'(\bar{s}(y^L))} \\ & > \frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L}. \end{aligned} \quad (10)$$

where $\bar{s}(y^L)$ and $\bar{s}(y^H)$ are the payments that satisfy the optimal earnings-only contract for the slightly overconfident manager.

The expression reflects the optimal risk-sharing between the principal and the manager taking into account the heterogeneity in beliefs and the noise in the accounting system. If the manager is slightly overconfident, the principal cannot increase the power of the contract without breaking the incentive compatibility constraint. However, when the inequality is satisfied, an increase in power which satisfies the individual rationality constraint does not break the incentive compatibility constraint. This means that only the individual rationality constraint will be binding. The expression of significant overconfidence includes the precision of recognizing gains as well as recognizing losses, which means that the quality of the accounting system affects the amount of overconfidence necessary for the moral hazard problem to disappear.

Lemma 3. *The optimal risk-sharing rule under overconfidence is given by:*

$$\frac{1}{U'(s(y^H))} = \phi \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}, \quad (11)$$

$$\frac{1}{U'(s(y^L))} = \phi \frac{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L}, \quad (12)$$

where ϕ is the Lagrange multiplier from the individual rationality constraint. Together with

$$\begin{aligned} & \tilde{p}_h [\lambda^H U (s(y^H)) + (1 - \lambda^H)U (s(y^L))] \\ & + (1 - \tilde{p}_h) [\lambda^L U (s(y^L)) + (1 - \lambda^L)U (s(y^H))] - c = \bar{U}, \end{aligned} \quad (13)$$

these equations define the optimal contract.

Equations (11) and (12) define the optimal risk-sharing rule. As the manager is significantly overconfident, I do not need to take into account the moral hazard problem. However, even though I can ignore the moral hazard problem, optimal risk-sharing cannot be reached as the principal and the manager want to wager against each other on the manager's ability to bring the firm success. If first-best risk sharing would have been reached, then the ratio of marginal utility between the manager and the principal in equations (11) and (12) would be equal. The heterogeneity in beliefs between the manager and the principal leads to the manager wanting to wager on his ability to make the firm succeed against the principal. The precision of the accounting system's ability to correctly display success, λ^H and λ^L , emphasizes this effect, as there is less noise to obscure the disagreement.

Corollary 2. *If the manager is significantly overconfident and has preferences such that $\frac{1}{U'(s)} \left(-\frac{U''(s)}{U'(s)} \right)$ is a non-decreasing function, the earnings-only contract exhibits power of incentives that increases in the degree of managerial overconfidence:*

$$\frac{d (U(s(y^H)) - U(s(y^L)))}{d\tilde{p}_h} > 0, \quad (14)$$

and which increases in accounting precision:

$$\frac{d (U(s(y^H)) - U(s(y^L)))}{d\lambda^i} > 0 \text{ for } i = \{L, H\}. \quad (15)$$

Under slight overconfidence, increasing either the precision of the accounting system or managerial overconfidence reduces the power of the contract necessary to induce high effort.

This translates into lower risk applied to the manager and therefore lowered required payments, which leads to a profit increase for the principal. When the overconfidence reaches into the significant overconfidence interval, the wager effect takes over, and managerial overconfidence increases the power of the contract. Similarly, increasing the accounting system's ability to correctly display success and failure increases the power of the optimal contract with significant overconfidence.

Proposition 1. *In the earnings-only contract the principal's profit is increasing in overconfidence, both in the interval of slight and significant overconfidence.*

The gain from overconfidence is decreasing in earnings quality for slight overconfidence as the two effects are substitutes, and increasing in overconfidence for a significantly overconfident manager as the effects are compliments.

4 Communication

I now consider the case where the manager will communicate the true underlying economic condition of the firm to the principal. A clear interpretation of the manager's communication would be managerial earnings forecast, but it could also be any other means of voluntary disclosure prior to the release of the financial statement.

This problem differs from the previous problem in several ways. I now allow the principal to contract on non-verifiable information in the financial statement, and this non-verifiable information reflects the true economic condition of the firm, $x \in \{x^H, x^L\}$. If the principal could also observe the true economic outcome of the firm, she would never write a contract on the financial statement, since the economic performance is a sufficient statistic for the financial report with respect to the manager's choice of effort. Given that the manager is the only one who observes the economic performance, I still include the mandatory report as contracting variable to discipline the manager to report truthfully about the underlying economic condition. This is reflected through two truth-telling constraints. The first con-

straint enforces that the manager reports truthfully when observed economic performance is high. The second enforces the manager to report truthfully when economic performance is low.

I define this as the "communication" program.

$$\begin{aligned} \min_{s(\tilde{x}, y)} p_h [\lambda^H s(\tilde{x}^H, y^H) + (1 - \lambda^H) s(\tilde{x}^H, y^L)] \\ + (1 - p_h) [\lambda^L s(\tilde{x}^L, y^L) + (1 - \lambda^L) s(\tilde{x}^L, y^H)], \end{aligned} \quad (16)$$

subject to the individual rationality constraint

$$\begin{aligned} \tilde{p}_h [\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^H, y^L))] \\ + (1 - \tilde{p}_h) [\lambda^L U(s(\tilde{x}^L, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^L, y^H))] - c \geq \bar{U}, \end{aligned} \quad (17)$$

the incentive constraint

$$\begin{aligned} \tilde{p}_h [\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^H, y^L))] \\ + (1 - \tilde{p}_h) [\lambda^L U(s(\tilde{x}^L, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^L, y^H))] - c \geq \\ p_l [\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^H, y^L))] \\ + (1 - p_l) [\lambda^L U(s(\tilde{x}^L, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^L, y^H))], \end{aligned} \quad (18)$$

the truth-telling constraint with success

$$\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^H, y^L)) \geq \lambda^H U(s(\tilde{x}^L, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^L, y^L)), \quad (19)$$

and the truth-telling constraint with failure

$$\lambda^L U(s(\tilde{x}^L, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^L, y^H)) \geq \lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H)). \quad (20)$$

Lemma 4. *The solution to the truthful communication problem under slight overconfidence will be defined by the binding individual rationality constraint, the incentive compatibility constraint and the truthful reporting with failure constraint.*

The optimal solution to the communication problem under slight overconfidence is given by:

$$\begin{aligned} U(s(\tilde{x}^H, y^H)) &= \bar{U} + c - \frac{c\tilde{p}_h}{(\tilde{p}_h - p_l)} + \frac{c\lambda^L}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}, \\ U(s(\tilde{x}^H, y^L)) &= \bar{U} + c - \frac{c\tilde{p}_h}{(\tilde{p}_h - p_l)} - \frac{c(1 - \lambda^L)}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}, \\ U(s(\tilde{x}^L, y^H)) &= U(s(\tilde{x}^L, y^L)) = \bar{U} + c - \frac{c\tilde{p}_h}{(\tilde{p}_h - p_l)}. \end{aligned}$$

Similar to the earnings-only contract, increasing overconfidence relaxes the incentive compatibility constraint and reduces the power of the contract needed to induce high effort.

If we look at the individual rationality constraint, the utility required to satisfy the opportunity cost of the manager (the outside opportunity) is now written on the known success of the firm. Notice that the expected utility when the firm is a success can be written as:

$$E[U(x^H)] \equiv E[\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H)U(s(\tilde{x}^H, y^L)) | x^h]. \quad (21)$$

Corollary 3. *If the manager is slightly overconfident, the communication contract exhibits power of incentives that is decreasing in the degree of overconfidence:*

$$\frac{d(E(U(\tilde{x}^H)) - U(\tilde{x}^L))}{d\tilde{p}_h} < 0, \quad (22)$$

and that is decreasing in accounting precision:

$$\frac{d(E(U(\tilde{x}^H)) - U(\tilde{x}^L))}{d\lambda^i} < 0 \text{ for } i \in \{L, H\}. \quad (23)$$

The effect of overconfidence on the power of incentives is decreasing in earnings quality, and vice-versa,

$$\frac{d^2 (E(U(\tilde{x}^H)) - U(\tilde{x}^L))}{d\lambda^i d\tilde{p}_h} > 0 \text{ for } i \in \{L, H\}. \quad (24)$$

As in the "earnings-only" contract, there will be a threshold of overconfidence at which the wager effect between the manager and the principal starts to dominate, and the incentive compatibility constraint becomes slack.

Proposition 2. *The solution to the communication problem with a significantly overconfident manager is defined by the binding individual rationality constraint and the truth-telling with failure constraint.*

The manager is significantly overconfident if

$$\begin{aligned} & \frac{\tilde{p}_h (\lambda^H - (1 - \lambda^L))}{\lambda^H [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h) \lambda^L] - (1 - \lambda^H) [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h) (1 - \lambda^L)] \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^H, y^L))}} \\ & \times \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} > \frac{p_h}{(1 - p_h)}. \end{aligned} \quad (25)$$

where $\bar{s}(\tilde{x}^H, y^H)$, $\bar{s}(\tilde{x}^H, y^L)$ and $\bar{s}(\tilde{x}^L)$ are the optimal payments in the communication contract for a slightly overconfident manager.

The new threshold for significant overconfidence deviates from the previous one. First, looking on the right side, there is no noise with regards to the principal's beliefs. This is due to the fact that she can directly observe the outcome of the firm by communicating with the manager. It is worth noticing that $\frac{U'(s(\tilde{x}^H, y^H))}{U'(s(\tilde{x}^L))} > \frac{U'(s(y^H))}{U'(s(\tilde{y}^L))}$, as these are based on the optimal utility in the slight overconfidence case where $U'(s(y^H)) = U'(s(\tilde{x}^H, y^H))$ and $U'(s(\tilde{y}^L)) > U'(s(\tilde{x}^L))$.

If I let $\lambda^H \rightarrow 1$, the expression approaches one similar to that without noise seen in de la Rosa (2011), $\frac{\tilde{p}_h}{(1 - \tilde{p}_h)} \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} > \frac{p_h}{(1 - p_h)}$, as having the manager communicate failure makes everything perfectly informative. Letting $\lambda^L \rightarrow 1$ yields the expression $\frac{\lambda^h \tilde{p}_h}{(1 - \tilde{p}_h)} \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} >$

$$\frac{p_h}{(1-p_h)}.$$

Lemma 5. *The optimal risk-sharing rule under overconfidence and communication is given by:*

$$\frac{1}{U'(s(\tilde{x}^H, y^H))} = \phi_1 \frac{\tilde{p}_h}{p_h} - \phi_2 \frac{1}{p_h} \frac{(1 - \lambda^L)}{\lambda^H}, \quad (26)$$

$$\frac{1}{U'(s(\tilde{x}^H, y^L))} = \phi_1 \frac{\tilde{p}_h}{p_h} - \phi_2 \frac{1}{p_h} \frac{\lambda^L}{(1 - \lambda^H)}, \quad (27)$$

$$\frac{1}{U'(s(\tilde{x}^L))} = \phi_1 \frac{(1 - \tilde{p}_h)}{(1 - p_h)} + \phi_2 \frac{1}{(1 - p_h)}, \quad (28)$$

where ϕ_1 is the Lagrange multiplier from the individual rationality constraint, and ϕ_2 is the Lagrange multiplier from the truth-telling with failure constraint. Together with

$$\begin{aligned} & \tilde{p}_h [\lambda^H U(s(\tilde{x}^H, y^H)) + (1 - \lambda^H) U(s(\tilde{x}^H, y^L))] \\ & + (1 - \tilde{p}_h) U(s(\tilde{x}^L)) - c - \bar{U} = 0, \end{aligned} \quad (29)$$

and

$$U(s(\tilde{x}^L)) = \lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H)), \quad (30)$$

these equations define the optimal contract.

The addition of communication to the contract has novel effects. The precision of the accounting with regards to success, λ^H , moves the payoff between $U(s(\tilde{x}^H, y^H))$ and $U(s(\tilde{x}^H, y^L))$, whereas the precision of the accounting system with regards to failure, λ^L , moves the payoff in low state as a weighted average according to the truth-telling constraint from equation (30).

One can write out the Borch rule on high outcome as the weighted average between

equations (26) and (27):

$$\frac{1}{U'(s(\tilde{x}^H))} \equiv \lambda^H \frac{1}{U'(s(\tilde{x}^H, y^H))} + (1 - \lambda^H) \frac{1}{U'(s(\tilde{x}^H, y^L))} = \phi_1 \frac{\tilde{p}_h}{p_h} - \phi_2 \frac{1}{p_h}. \quad (31)$$

The effects of the precision of the accounting system and managerial overconfidence are separated, as seen by observing that the relations are independent of each other in the first order constraint. This means that the topic of disagreement, that is, the manager's ability to affect outcome, is incorporated directly into the contract, without being obscured by noise in the accounting system. The heterogeneity in beliefs enters the solution to the problem through the Lagrange multiplier from the individual rationality constraint, ϕ_1 , whereas the noise of the accounting system now affects the contract through the Lagrange multiplier from the truth-telling with failure constraint, ϕ_2 . Hence the optimal risk-sharing rule in the communication contract is now directly comparable to the optimal risk-sharing rule in de la Rosa (2011), where outcome is directly contractible. I use equations (31) and (28) to compare the power of the "communication" contract with the contract without noise in de la Rosa (2011). The difference in the "communication" contract is an additional term which accounts for the truth-telling with failure constraint reducing the manager's utility with success and increasing the managers utility with failure. This ensures that truth-full communication can be implemented.

Increasing managerial overconfidence has two distinct effects. First, as observed in the first term of the Borch rule, (31) and (28), it increases the manager's marginal utility with success and decreases it with failure. As the truth-telling constraint is binding, the principal cannot provide the manager with as powerful incentives as if the true outcome of the firm had been observable. This is seen when we compare the solution from Lemma 5 with the optimal risk-sharing rule in de la Rosa (2011) and comes from the Lagrange multiplier, ϕ_2 , from the truth-telling constraint. The way this affects the optimal contract is that the observed earnings, y^H and y^L , must serve to verify the manager's communication, and thus

discipline him if the two signals disagree. Like in the case with slight overconfidence, this is implemented with the payment $s(\tilde{x}^H, y^H)$ being larger than $s(\tilde{x}^L)$, which again is larger than $s(\tilde{x}^H, y^L)$. Writing the manager's expected utility conditional on the success of the firm gave the expression in equation (21), which puts the largest weight on $s(\tilde{x}^H, y^H)$. Similarly, the payment conditional on the failure of the firm is determined by equation (30), which puts the largest weight on $s(\tilde{x}^H, y^L)$. To keep the truth-telling incentives while increasing the power of the contract, the payment $s(\tilde{x}^H, y^H)$ must increase, whereas the payment of $s(\tilde{x}^H, y^L)$ must decrease.

Similar to the "earnings-only" contract, the power of incentives in the optimal contract is increasing in both precision and overconfidence.

Corollary 4. *If the manager is significantly overconfident and has preferences such that $\frac{1}{U'(s)} \left(-\frac{U''(s)}{U'(s)} \right)$ is a non-decreasing function, the communication contract exhibits power of incentives that increases in the degree of managerial overconfidence:*

$$\frac{d(E(U(\tilde{x}^H)) - U(\tilde{x}^L))}{d\tilde{p}_h} > 0, \quad (32)$$

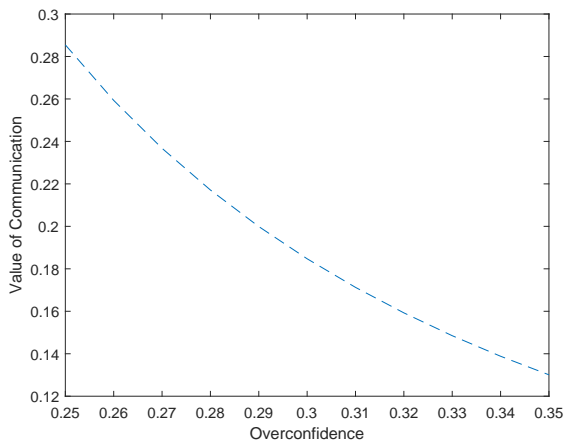
and in the precision of the accounting system:

$$\frac{\partial (E(U(\tilde{x}^H)) - U(\tilde{x}^L))}{\partial \lambda^i} > 0 \text{ for } i \in \{L, H\}. \quad (33)$$

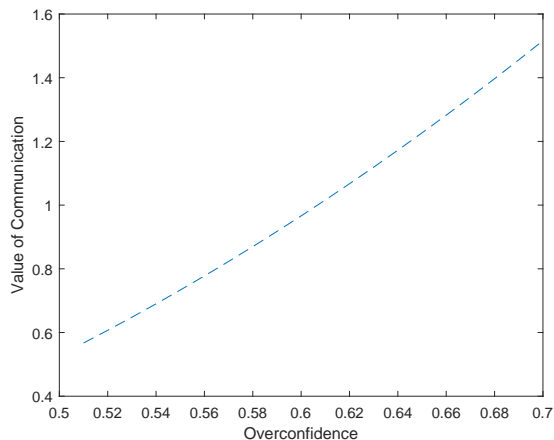
While the qualitative effects of precision and overconfidence on the optimal contract is the same in the "earnings-only" and "communication" problems, the mechanism differs. In the "communication" contract, the noise from the accounting system is separated from the heterogeneity in beliefs by communication, and this is implemented through the truth-telling constraint. This differs from the "earnings-only" contract, where noise in the accounting system decreased the effect of heterogeneity in beliefs on the optimal contract. The effect of noise in the accounting system in the "communication" contract now indirectly affect the power of the contract by increasing the tightness of the truth-telling constraint.

Figure 3: Overconfidence and the value of Communication.

(a) Slight Overconfidence and the Value of Communication.



(b) Significant Overconfidence and the Value of Communication.



The graphs display the value of communication, defined as the difference between the cost of hiring the manager with and without communication, scaled by the mean cost of hiring the manager without communication for the different levels of overconfidence within the range of slight overconfidence or significant overconfidence, respectively.

For the numerical solution, the following functional forms and parameterizations are used: $U(s(\cdot))$ is assumed to be $1 - \exp(-s)$, $p_h = 0.4$, $p_l = 0.15$, $c = 0.1$, $\bar{U} = 0$ and $\lambda^H = \lambda^L = 0.75$.

Proposition 3. *In the communication contract, the principal's profit is increasing in overconfidence, both in the interval of slight and significant overconfidence.*

The gain from overconfidence is decreasing in earnings quality for slight overconfidence as the two effects are substitutes, and increasing in overconfidence for a significantly overconfident manager as the effects are compliments.

5 The value of communication

In this section I explore how accounting characteristics affect the value of communication in the optimal contract. While a closed-form solution for the value of communication in the case of slight overconfidence is well defined, sadly, I must rely on numerical analysis in the case of significant overconfidence. Hence, I will rely on negative exponential utility for the functional form, which is in line with the assumptions required for Corollaries 2 and 4.

First, overconfidence in the range of slight overconfidence decreases the agency cost of contracting. The gain from communication comes from the reduced noise of earnings in the case of payment with low outcome, thus reducing the risk-premium which is forced upon the manager.

Lemma 6. *The value of communication under slight overconfidence is given by:*

$$\Delta S_{slight} = (1 - p_h)\{E[G(U(s(y))) | x^L] - G(E[U(s(\tilde{x}, y)) | x^L])\} \geq 0 \quad (34)$$

where $G(\cdot)$ is the inverse of the manager's utility function.

As overconfidence increases within the range of slight overconfidence, the agency cost decreases, which reduces the gains from communication. This relationship is shown in figure 3a.

Proposition 4. *The value of communication when contracting with a slightly overconfident manager is decreasing in the level of overconfidence.*

Looking at figure 3b we note a positive relationship between the level of overconfidence for a significantly overconfident manager and the value of communication. This comes from the ability of the communication contract with a significantly overconfident manager to allow the principal to directly contract on the heterogeneity in beliefs. This causes an increase in the optimal power of incentives in the contract, in comparison with the "earnings-only" contract, thus increasing the principal's benefit of contracting with a significantly overconfident manager. The gain from communication becomes larger the more overconfident the manager is, thus leading to the conclusion that the value of communicating with a significantly overconfident manager is increasing in overconfidence. In the setting with a significantly overconfident manager, the reduction in the cost of contracting with the manager, caused by including communication, is not a tractable parameter, and I therefore have to rely on numerical analysis for the following conclusion.

Conclusion 1. *The value of communication when contracting with a significantly overconfident manager is increasing in the level of overconfidence.*

Let us now assume that the manager is not able to directly observe the success of the firm. In this case the manager's self-report is no longer a sufficient statistic as regards his effort choice, and hence communication is not necessarily valuable. Şabac and Tian (2014) study this setting and find that incorporating soft information into the optimal contract creates tension between creating incentives for effort and maintaining incentives for truthful communication. This happens as principal want to provide high enough payment with success such that the manager performs high effort. However, if the power of the incentives is too high, the manager will be tempted to misreport in the case of failure, tightening the truth-telling constraint.

With an overconfident manager, I conjecture that this tension is less problematic as increasing the managerial overconfidence level within the interval of slight overconfidence alleviates the moral hazard problem and relaxes the incentive compatibility constraint and thus reduces the tension. Now, heuristically,³ having the manager become significantly overconfident means that the incentive compatibility constraint does not bind, which completely removes the tension coming from the incentive compatibility constraint, However, it instead creates tension from the wager effects as the manager wants increasingly large incentives to bet against the principal.

6 Conclusion

In this paper I develop a model that incorporates managerial overconfidence and communication into a principal-agent model with moral hazard. To accommodate unverifiable communication of the firm's economic outcome, I introduce accounting earnings as the public signal, which is a noisy signal of the economic outcome. Both with and without communica-

³The existence of a level of overconfidence for which the incentive compatibility constraint will not bind is not ensured in this setting.

tion, I find a threshold where overconfidence goes from decreasing agency cost to cause side betting between the principal and the manager, which is an extension of the result in de la Rosa (2011).

In the case of slight overconfidence, with and without communication, the precision of accounting earnings and overconfidence decreases agency cost, but they act as substitutes. In the case of significant overconfidence, with and without communication, an increase in overconfidence causes the principal and the manager to wager more against each other. The precision of the accounting system and overconfidence work as complements, as the higher the precision of the accounting system, the larger the effect of overconfidence on the wager effect.

Communication in the case of slight overconfidence reduces the agency cost of hiring the manager. As overconfidence increases within the range of slight overconfidence, the agency cost of hiring the manager decreases, thus reducing the potential effect of communication. This leads to the value of communication being decreasing in overconfidence. As the manager's overconfidence grows into the threshold of significant overconfidence, communication allows the principal to directly observe the cause of disagreement, that is, the manager's effect on outcome, such that the accounting system no longer obscures the effect of overconfidence. Communication thus increases the power of the contract, which in turn increases the principal's profit. As communication emphasizes the effect of the heterogeneity in beliefs, the value of communication is increasing in overconfidence for a significantly overconfident manager.

The results in this paper provides insight and a theoretical foundation for the empirical studies on managerial overconfidence and disclosure, such as Bouwman (2014), Hribar and Yang (2016) and Libby and Rennekamp (2012). There are a few limitations of this model. First, this model does not capture any downside of managerial overconfidence. Secondly, it only describes a setting where there is no possibility of the overconfident manager learnings from prior experiences. I leave these topics for further research.

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Appendix

Proof of Lemma 1. I say the manager is slightly overconfident if

$$\begin{aligned} & \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L) U'(\bar{s}(y^H))}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L U'(\bar{s}(y^L))} \\ & \leq \frac{(q + v)\lambda^H + (1 - (q + v))(1 - \lambda^L)}{(q + v)(1 - \lambda^H) + (1 - (q + v))\lambda^L}. \end{aligned} \quad (35)$$

This means that the principal cannot make a change in payments that makes them better off, without breaking the incentive compatibility constraint. More specifically, the change she would make is $d\bar{s}(y^H) < 0$ and $d\bar{s}(y^L) > 0$, which means that such a decrease in the incentives of the manager would break in the incentive constraint. Therefore, the incentive constraint of the manager must be binding in the case of slight overconfidence.

Solving the two equations for two unknowns yields the solution. □

Proof of Corollary 1. The incentive constraint in the case of slight overconfidence can be written as:

$$(\tilde{p}_h - p_l) [\lambda^H U(s(y^H)) + (1 - \lambda^H)U(s(y^L)) - \lambda^L U(s(y^L)) - (1 - \lambda^L)U(s(y^H))] = c, \quad (36)$$

by rearranging the terms

$$\begin{aligned}
(\tilde{p}_h - p_l) [(\lambda^H + \lambda^L - 1) (U(s(y^H)) - U(s(y^L)))] &= c, \\
[(U(s(y^H)) - U(s(y^L)))] &= \frac{c}{(\tilde{p}_h - p_l)} \frac{1}{(\lambda^H + \lambda^L - 1)}, \tag{37}
\end{aligned}$$

the derivatives are then given by

$$\frac{d}{d(\tilde{p}_h - p_l)} (U(s(y^H)) - U(s(y^L))) < 0, \tag{38}$$

$$\frac{d}{d\lambda^H} (U(s(y^H)) - U(s(y^L))) < 0, \tag{39}$$

$$\frac{d}{d\lambda^L} (U(s(y^H)) - U(s(y^L))) < 0. \tag{40}$$

and the cross derivative is given as

$$\frac{\partial}{\partial \lambda^H \partial (\tilde{p}_h - p_l)} (U(s(y^H)) - U(s(y^L))) > 0 \tag{41}$$

$$\frac{\partial}{\partial \lambda^L \partial (\tilde{p}_h - p_l)} (U(s(y^H)) - U(s(y^L))) > 0. \tag{42}$$

□

Proof of Lemma 2. This proof is analogous to the proof of Proposition 3 in de la Rosa (2011).

I will show that the principal can make a change in payments that makes them better off and that will not break the incentive compatibility constraint.

I begin by total differentiating the manager's binding individual rationality constraint

$$\begin{aligned}
&\tilde{p}_h [\lambda^H U'(\bar{s}(y^H)) d\bar{s}(y^H) + (1 - \lambda^H) U'(s(y^L)) d\bar{s}(y^L)] \\
&+ (1 - \tilde{p}_h) [\lambda^L U'(\bar{s}(y^L)) d\bar{s}(y^L) + (1 - \lambda^L) U'(\bar{s}(y^H)) d\bar{s}(y^H)] = 0. \tag{43}
\end{aligned}$$

This yields

$$\begin{aligned}
& d\bar{s}(y^L) \\
&= \frac{(1 - \tilde{p}_h - \tilde{p}_h)\lambda^L - \tilde{p}_h\lambda^H - (1 - (\tilde{p}_h + \tilde{p}_h))}{-\lambda^H\tilde{p}_h - \lambda^L(\tilde{p}_h + \tilde{p}_h - 1) + \tilde{p}_h + \tilde{p}_h} \\
&\times \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} d\bar{s}(y^H).
\end{aligned}$$

Now I need to find the total differentiated principal cost minimization in order to find a change in payments that makes the principal better off:

$$\begin{aligned}
& d\bar{s}(y^L) [p_h(1 - \lambda^H) + (1 - p_h)\lambda^L] \\
&+ d\bar{s}(y^H) [p_h\lambda^H + (1 - p_h)(1 - \lambda^L)] < 0,
\end{aligned} \tag{44}$$

and substitute in our expression for the $d\bar{s}(y^L)$

$$\begin{aligned}
& \frac{(1 - \tilde{p}_h - \tilde{p}_h)\lambda^L - \tilde{p}_h\lambda^H - (1 - (\tilde{p}_h + \tilde{p}_h))}{-\lambda^H\tilde{p}_h - \lambda^L(\tilde{p}_h + \tilde{p}_h - 1) + \tilde{p}_h + \tilde{p}_h} \\
&\times \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} d\bar{s}(y^H) [p_h(1 - \lambda^H) + (1 - p_h)\lambda^L] \\
&+ d\bar{s}(y^H) [p_h\lambda^H + (1 - p_h)(1 - \lambda^L)] < 0.
\end{aligned} \tag{45}$$

Algebraic manipulation leads to:

$$\begin{aligned}
& \left[\frac{\tilde{p}_h\lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L) U'(\bar{s}(y^H))}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L U'(\bar{s}(y^L))} \right. \\
& \left. - \frac{p_h\lambda^H + (1 - p_h)(1 - \lambda^L)}{[p_h(1 - \lambda^H) + (1 - p_h)\lambda^L]} \right] d\bar{s}(y^H) > 0,
\end{aligned} \tag{46}$$

and rewriting this I get the expression that defines the threshold between slight overconfidence and significant overconfidence:

$$\begin{aligned}
& \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L) U'(\bar{s}(y^H))}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L U'(\bar{s}(y^L))} \\
& \geq \frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L}.
\end{aligned} \tag{47}$$

For this level of overconfidence, increasing the power of the contract makes the principal better off while satisfying the incentive compatibility constraint.

The incentive constraint is not binding. This leaves the principal with the following minimization problem:

$$\min_{s(y)} p_h [\lambda^H s(y^H) + (1 - \lambda^H) s(y^L)] + (1 - p_h) [\lambda^L s(y^L) + (1 - \lambda^L) s(y^H)], \tag{48}$$

subject to the individual rationality constraint

$$\begin{aligned}
& \tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\
& + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c \geq \bar{U}.
\end{aligned} \tag{49}$$

□

Proof of Lemma 3. I can then write up the Lagrangian for the constrained optimization problem:

$$\begin{aligned}
L &= p_h [\lambda^H s(y^H) + (1 - \lambda^H) s(y^L)] + (1 - p_h) [\lambda^L s(y^L) + (1 - \lambda^L) s(y^H)] \\
& - \phi [\tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\
& + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c - \bar{U}].
\end{aligned} \tag{50}$$

where ϕ is the Lagrange multiplier from the IR constraint. This gives me the first-order

conditions:

$$\begin{aligned}
\frac{\partial L}{\partial s(y^H)} &= p_h \lambda^H + (1 - p_h)(1 - \lambda^L) - \phi \tilde{p}_h \lambda^H U'(s(y^H)) \\
&\quad - \phi(1 - \tilde{p}_h)(1 - \lambda^L) U'(s(y^H)) = 0, \\
\frac{\partial L}{\partial s(y^L)} &= p_h(1 - \lambda^H) + (1 - p_h)\lambda^L - \phi \tilde{p}_h(1 - \lambda^H) U'(s(y^L)) \\
&\quad - \phi(1 - \tilde{p}_h)\lambda^L U'(s(y^L)) = 0, \\
\frac{\partial L}{\partial \phi} &= \tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\
&\quad + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c - \bar{U} = 0.
\end{aligned}$$

Solving these equation for the Borch rule yields the solution. \square

Proof of Corollary 2. This proof follows the proof of Proposition 3 in de la Rosa (2011). I begin by total differentiating the manager's binding individual rationality constraint with respect to \tilde{p}_h :

$$\begin{aligned}
&(\lambda^H + \lambda^L - 1) [U(s(y^H)) - U(s(y^L))] \\
&+ [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)] \frac{dU(s(y^H))}{d\tilde{p}_h} \\
&+ [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L] \frac{dU(s(y^L))}{d\tilde{p}_h} = 0.
\end{aligned} \tag{51}$$

Given $[U(s(y^H)) - U(s(y^L))] > 0$, it follows immediately that $\frac{dU(s(y^H))}{d\tilde{p}_h} < 0$ or $\frac{dU(s(y^L))}{d\tilde{p}_h} < 0$

I now move on to take the total derivative of the optimal risk-sharing rule with respect to \tilde{p}_h :

$$\begin{aligned}
&\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \frac{d}{d\tilde{p}_h} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) \\
&+ \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \frac{d}{d\tilde{p}_h} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) = 0.
\end{aligned} \tag{52}$$

Given that both $\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L}$ and $\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))}$ are positive, and that

$$\begin{aligned} & \frac{d}{d\tilde{p}_h} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) \\ &= \frac{\lambda^H + \lambda^L - 1}{\lambda^L + \tilde{p}_h(1 - \lambda^H - \lambda^L)^2} > 0, \end{aligned} \quad (53)$$

it must be so that

$$\frac{d}{d\tilde{p}_h} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) < 0. \quad (54)$$

Now assume that $\frac{dU(s(y^L))}{d\tilde{p}_h} \geq 0$. Since $\frac{dU(s(y^L))}{d\tilde{p}_h}$ and $\frac{dU(s(y^H))}{d\tilde{p}_h}$ cannot both be positive, then $\frac{dU(s(y^H))}{d\tilde{p}_h} < 0$, but this means that (54) is violated, which is a contradiction. This means that $\frac{dU(s(y^L))}{d\tilde{p}_h} < 0$.

If $\frac{dU(s(y^H))}{d\tilde{p}_h} \geq 0$, then $\frac{d}{d\tilde{p}_h} (U(s(y^H)) - U(s(y^L)))$ follows immediately.

Now if both $\frac{dU(s(y^L))}{d\tilde{p}_h}$ and $\frac{dU(s(y^H))}{d\tilde{p}_h}$ are negative, taking into account that $\frac{dU'(s)}{d\tilde{p}_h} = \frac{U''(s)}{U'(s)} \frac{du(s)}{d\tilde{p}_h}$, I can write (54) as:

$$\frac{1}{U'(s(y^H))} \left(-\frac{U''(s)}{U'(s)} \right) \frac{du(s(y^H))}{d\tilde{p}_h} - \frac{1}{U'(s(y^L))} \left(-\frac{U''(s(y^L))}{U'(s(y^L))} \right) \frac{du(s(y^L))}{d\tilde{p}_h} > 0. \quad (55)$$

Given $U(s(y^H)) > U(s(y^L))$, by assumption

$$\frac{1}{U'(s(y^H))} \left(-\frac{U''(s)}{U'(s)} \right) \geq \frac{1}{U'(s(y^L))} \left(-\frac{U''(s(y^L))}{U'(s(y^L))} \right). \quad (56)$$

This means that I can write

$$\begin{aligned} & \frac{1}{U'(s(y^H))} \left(-\frac{U''(s)}{U'(s)} \right) \frac{du(s(y^H))}{d\tilde{p}_h} - \frac{1}{U'(s(y^L))} \left(-\frac{U''(s(y^L))}{U'(s(y^L))} \right) \frac{du(s(y^L))}{d\tilde{p}_h} \\ & \leq \frac{1}{U'(s(y^H))} \left(-\frac{U''(s)}{U'(s)} \right) \left(\frac{du(s(y^H))}{d\tilde{p}_h} - \frac{du(s(y^L))}{d\tilde{p}_h} \right). \end{aligned} \quad (57)$$

Since the left-hand side is strictly positive I have showed that $\frac{d}{d\tilde{p}_h} (U(s(y^H)) - U(s(y^L)))$.

To analyze how the optimal power of the contract changes with accounting precision, I start by total differentiating the individual rationality constraint with regards to λ^H :

$$\begin{aligned} & \tilde{p}_h [U(s(y^H)) - U(s(y^L))] + [\tilde{p}_h \lambda^H + (1 - \lambda^L)(1 - \tilde{p}_h)] U'(s(y^H)) d\lambda^H \\ & + [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L] U'(s(y^L)) d\lambda^H = 0. \end{aligned} \quad (58)$$

Given $[U(s(y^H)) - U(s(y^L))] > 0$, it follows immediately that $\frac{dU(s(y^H))}{d\lambda^H} < 0$ or $\frac{dU(s(y^L))}{d\lambda^H} < 0$.

I now move on to take the total derivative of the optimal risk-sharing rule. with respect to λ^H

$$\begin{aligned} & \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \frac{d}{d\lambda^H} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) \\ & + \frac{d}{d\lambda^H} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \\ & - \frac{d}{d\lambda^H} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) = 0, \end{aligned} \quad (59)$$

where

$$\frac{d}{d\lambda^H} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) = \frac{\tilde{p}_h}{(\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L)^2} > 0, \quad (60)$$

and

$$\frac{d}{d\lambda^H} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) = \frac{p_h}{(\lambda^L + (1 - \lambda^H - \lambda^L)p_h)^2} > 0, \quad (61)$$

From the relation for a significant manager in Proposition 2 I see that

$$\begin{aligned} & \frac{d}{d\lambda^H} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \\ & - \frac{d}{d\lambda^H} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) > 0, \end{aligned} \quad (62)$$

as

$$\tilde{p} \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} > (\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))}, \quad (63)$$

and

$$\tilde{p}_h \gg p_h, \quad (64)$$

which means that

$$\frac{d}{d\lambda^H} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) < 0. \quad (65)$$

Now assume that $\frac{dU(s(y^L))}{d\lambda^H} \geq 0$. Since $\frac{dU(s(y^L))}{d\lambda^H}$ and $\frac{dU(s(y^H))}{d\lambda^H}$ cannot both be positive, then $\frac{dU(s(y^H))}{d\lambda^H} < 0$, but then (65) is violated.

Next, I want to find the effect of accounting precision with firm failure on the power of the contract. Total differentiating the individual rationality constraint with regards to λ^L :

$$\begin{aligned} & (1 - \tilde{p}_h) [U(s(y^L)) - U(s(y^H))] + [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)] \frac{dU(s(y^H))}{d\lambda^L} \\ & + [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L] \frac{dU(s(y^L))}{d\lambda^L} = 0. \end{aligned} \quad (66)$$

Given $[U(s(y^L)) - U(s(y^H))] > 0$, it follows immediately that $\frac{dU(s(y^H))}{d\lambda^L} > 0$ or $\frac{dU(s(y^L))}{d\lambda^L} > 0$.

$$\begin{aligned} & \frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \frac{d}{d\lambda^L} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) \\ & + \frac{d}{d\lambda^L} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \\ & - \frac{d}{d\lambda^L} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) = 0, \end{aligned} \quad (67)$$

where

$$\frac{d}{d\lambda^L} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) = -\frac{(1 - p_h)}{(p_h(1 - \lambda^H) + (1 - p_h)\lambda^L)^2} < 0, \quad (68)$$

and

$$\frac{d}{d\lambda^L} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) = -\frac{(1 - \tilde{p}_h)}{(\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L)^2} < 0. \quad (69)$$

From the relation for a significant overconfidently manager in Proposition 2:

$$\frac{d}{d\lambda^L} \left(\frac{\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)}{\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L} \right) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} - \frac{d}{d\lambda^L} \left(\frac{p_h \lambda^H + (1 - p_h)(1 - \lambda^L)}{p_h(1 - \lambda^H) + (1 - p_h)\lambda^L} \right) > 0, \quad (70)$$

as

$$(1 - \tilde{p}) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} < (\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)) \frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))}, \quad (71)$$

which means that

$$\frac{d}{d\lambda^L} \left(\frac{U'(\bar{s}(y^H))}{U'(\bar{s}(y^L))} \right) < 0. \quad (72)$$

so the power increases in λ^L □

Proof of Proposition 1. The principal's minimization problem is given by:

$$\min_{s(y)} p_h [\lambda^H s(y^H) + (1 - \lambda^H) s(y^L)] + (1 - p_h) [\lambda^L s(y^L) + (1 - \lambda^L) s(y^H)] \quad (73)$$

s.t

$$\begin{aligned} & \tilde{p}_h [\lambda^H U(s(y^H)) + (1 - \lambda^H) U(s(y^L))] \\ & + (1 - \tilde{p}_h) [\lambda^L U(s(y^L)) + (1 - \lambda^L) U(s(y^H))] - c \geq \bar{U} \end{aligned} \quad (74)$$

Using the duality between profit maximization and cost minimization I twist the problem around so as to specifically analyze profit. Using the envelope theorem and differentiating the principal' profit with regards to managerial overconfidence yields:

$$\frac{\partial \pi}{\partial \tilde{p}_h} = \phi \left[[\lambda^H - (1 - \lambda^L)] U(s(y^H)) - [\lambda^L - (1 - \lambda^H)] U(s(y^L)) \right] > 0. \quad (75)$$

The next step is to analyze how earnings quality affects the gain from overconfidence:

$$\frac{\partial^2 \pi}{\partial \tilde{p}_h \partial \lambda^H} = \phi \left[U(s(y^H)) - U(s(y^L)) \right] > 0. \quad (76)$$

and similar for λ^L . □

Proof of Lemma 4. That these three constraints will be binding conditional on slight overconfidence follows directly from Lemma 2 in Gigler and Hemmer (2001).

To understand that the incentive compatibility constraint will bind for a slightly overconfident manager, then consider the following change in payment. The principal cannot make a payment that satisfies i) the individual rationality constraint, ii) the truth-telling constraint, and iii) that decreases the cost of contracting with the manager with breaking the incentive compatibility if the degree of managerial overconfidence satisfies the following constraint:

$$\frac{\tilde{p}_h (\lambda^H - (1 - \lambda^L))}{\lambda^H [\tilde{p}_h (1 - \lambda^H) + (1 - \tilde{p}_h) \lambda^L] - (1 - \lambda^H) [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h) (1 - \lambda^L)] \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^H, y^L))}} \times \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} < \frac{p_h}{(1 - p_h)}.$$

Using the individual rationality, incentive compatibility and truth-telling with failure constraints and solving for the three unknowns yields the Lemma. □

Proof of Corollary 3. The incentive compatibility constraint can be rewritten to:

$$\lambda^H U(s((x^H, y^H))) + (1 - \lambda^H) U(s((x^H, y^L))) - U(s((x^L))) = \frac{c}{(\tilde{p}_h - p_l)}. \quad (77)$$

The derivative with regards to overconfidence:

$$\frac{d \frac{c}{(\tilde{p}_h - p_l)}}{d(\tilde{p}_h - p_l)} < 0. \quad (78)$$

To analyze precision one can use the binding truth-telling with failure constraint into the incentive compatibility constraint, and by moving around I get the following expression:

$$[U(s((x^H, y^H))) - U(s((x^H, y^L)))] = \frac{c}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}. \quad (79)$$

This yields:

$$\frac{d \frac{c}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}}{d\lambda^H} < 0, \quad (80)$$

$$\frac{d \frac{c}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}}{d\lambda^L} < 0. \quad (81)$$

And similarly

$$\frac{d^2 \frac{c}{(\tilde{p}_h - p_l)(\lambda^H + \lambda^L - 1)}}{d\lambda^L d\tilde{p}_h} > 0. \quad (82)$$

□

Proof of Proposition 2. I will show that for a sufficiently overconfident manager, the incentive compatibility constraint in the communication problem will not bind.

Total differentiating the manager's binding individual rationality constraint with respect to the payments yields

$$\begin{aligned} & \tilde{p}_h [\lambda^H U' (s(\tilde{x}^H, y^H)) d\bar{s}(x^H, y^H) + (1 - \lambda^H) U' (s(x^H, y^L)) d\bar{s}(x^H, y^L)] \\ & + (1 - \tilde{p}_h) U' (s(x^L)) d\bar{s}(x^L) = 0. \end{aligned} \quad (83)$$

I also total differentiate the binding TR constraint:

$$U' (s(\tilde{x}^L)) d\bar{s}(x^L) = \lambda^L U' (s(\tilde{x}^H, y^L)) d\bar{s}(x^H, y^L) + (1 - \lambda^L) U' (s(\tilde{x}^H, y^H)) d\bar{s}(x^H, y^H). \quad (84)$$

Inserting $U' (s(\tilde{x}^L)) d\bar{s}(x^L)$ from TR into the individual rationality constraint yields:

$$\begin{aligned} & \tilde{p}_h [\lambda^H U' (s(\tilde{x}^H, y^H)) d\bar{s}(x^H, y^H) + (1 - \lambda^H) U' (s(x^H, y^L)) d\bar{s}(x^H, y^L)] \\ & + (1 - \tilde{p}_h) (\lambda^L U' (s(\tilde{x}^H, y^L)) d\bar{s}(x^H, y^L) + (1 - \lambda^L) U' (s(\tilde{x}^H, y^H)) d\bar{s}(x^H, y^H)) = 0. \end{aligned} \quad (85)$$

Isolating $d\bar{s}(x^H, y^L)$:

$$d\bar{s}(x^H, y^L) = - \frac{\left[\tilde{P}r(x^h | e) \lambda^H + \left(1 - \tilde{P}r(x^h | e) \right) (1 - \lambda^L) \right] U' (s(\tilde{x}^H, y^H))}{\left[\tilde{P}r(x^h | e) (1 - \lambda^H) + \left(1 - \tilde{P}r(x^h | e) \right) \lambda^L \right] U' (s(\tilde{x}^H, y^L))} d\bar{s}(x^H, y^H). \quad (86)$$

Inserting this back into the TR yields:

$$\begin{aligned} d\bar{s}(x^L) &= \left[(1 - \lambda^L) - \lambda^L \frac{\left[\tilde{P}r(x^h | e) \lambda^H + \left(1 - \tilde{P}r(x^h | e) \right) (1 - \lambda^L) \right]}{\left[\tilde{P}r(x^h | e) (1 - \lambda^H) + \left(1 - \tilde{P}r(x^h | e) \right) \lambda^L \right]} \right] \\ & \times \frac{U' (s(\tilde{x}^H, y^H))}{U' (s(\tilde{x}^L))} d\bar{s}(x^H, y^H). \end{aligned} \quad (87)$$

Now I need to find the principals' total differentiated cost minimization:

$$p_h [\lambda^H d\bar{s}(x^H, y^H) + (1 - \lambda^H) d\bar{s}(x^H, y^L)] + (1 - p_h) d\bar{s}(x^L) < 0, \quad (88)$$

and substitute in my expression for the $d\bar{s}(x^L)$ and $d\bar{s}(x^L)$:

$$\begin{aligned}
& p_h \left[\lambda^H d\bar{s}(x^H, y^H) - (1 - \lambda^H) \frac{\left[\tilde{P}r(x^h | e)\lambda^H + \left(1 - \tilde{P}r(x^h | e)\right)(1 - \lambda^L) \right]}{\left[\tilde{P}r(x^h | e)(1 - \lambda^H) + \left(1 - \tilde{P}r(x^h | e)\right)\lambda^L \right]} \right. \\
& \quad \left. \times \frac{U'(s(\tilde{x}^H, y^H))}{U'(s(\tilde{x}^H, y^L))} d\bar{s}(x^H, y^H) \right] \\
& + (1 - p_h) \left[(1 - \lambda^L) - \lambda^L \frac{\left[\tilde{P}r(x^h | e)\lambda^H + \left(1 - \tilde{P}r(x^h | e)\right)(1 - \lambda^L) \right]}{\left[\tilde{P}r(x^h | e)(1 - \lambda^H) + \left(1 - \tilde{P}r(x^h | e)\right)\lambda^L \right]} \right] \\
& \quad \times \frac{U'(s(\tilde{x}^H, y^H))}{U'(s(\tilde{x}^L))} d\bar{s}(x^H, y^H) < 0. \tag{89}
\end{aligned}$$

Algebraic manipulation leads to the requirement that

$$\begin{aligned}
& \left(\frac{\tilde{p}_h \lambda^H - (1 - \lambda^L) \tilde{p}_h}{\left[\lambda^H [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L] - (1 - \lambda^H) [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)] \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^H, y^L))} \right]} \right. \\
& \quad \left. \times \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} - \frac{p_h}{(1 - p_h)} \right) d\bar{s}(x^H, y^H) > 0. \tag{90}
\end{aligned}$$

I need

$$\begin{aligned}
& \frac{\tilde{p}_h \lambda^H - (1 - \lambda^L) \tilde{p}_h}{\left[\lambda^H [\tilde{p}_h(1 - \lambda^H) + (1 - \tilde{p}_h)\lambda^L] - (1 - \lambda^H) [\tilde{p}_h \lambda^H + (1 - \tilde{p}_h)(1 - \lambda^L)] \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^H, y^L))} \right]} \\
& \quad \times \frac{U'(\bar{s}(\tilde{x}^H, y^H))}{U'(\bar{s}(\tilde{x}^L))} > \frac{p_h}{(1 - p_h)} d\bar{s}(x^H, y^H) \tag{91}
\end{aligned}$$

for the manager to be significantly overconfident.

For this level of overconfidence, the principal can change the optimal payments of the contract to make them better off, without breaking the incentive compatibility constraint. This means that the incentive compatibility constraint is slack.

Now for managers who are significantly overconfident, I can rewrite the problem as the incentive compatibility constraint is not binding.

This yields:

$$\min_{s(x,y)} p_h [\lambda^H s(\tilde{x}^H, y^H) + (1 - \lambda^H) s(x^H, y^L)] + (1 - p_h) s(\tilde{x}^L), \quad (92)$$

subject to the individual rationality constraint:

$$\begin{aligned} & \tilde{p}_h [\lambda^H U(s((x^H, y^H))) + (1 - \lambda^H) U(s((x^H, y^L)))] \\ & + (1 - \tilde{p}_h) U(s(\tilde{x}^L)) - c \geq \bar{U}, \end{aligned} \quad (93)$$

and truth-telling constraint with failure constraint:

$$U(s(\tilde{x}^L)) \geq \lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H)). \quad (94)$$

□

Proof of Proposition 2. Provided that the heterogeneity in beliefs is sufficiently large such that the manager's incentive compatibility constraint is not binding, ($\tilde{p}_h \gg p_h$), I now show that the principal's problem in case of overconfidence and communication is defined by the binding individual rationality constraint and the truth-telling in case of lower economic earnings constraint.

In this proof, I follow the proof of Gigler and Hemmer (2001) and show that it holds without a binding incentive compatibility constraint. First, notice that, as shown in Gigler and Hemmer (2001), if truth-telling with success is binding then $U^L \equiv U(s(\hat{x}^L, y^H)) = U(s(\hat{x}^L, y^L))$, and truth-telling with failure is binding, then $U^H \equiv U(s(\hat{x}^H, y^H)) = U(s(\hat{x}^H, y^L))$.

The solution to the problem has constraint truth-telling with success slack and truth-telling with failure binding. To show this, suppose that truth-telling with success binds. Then I know that truth-telling with failure is slack, and that $U^H \equiv U(s(\hat{x}^H, y^H)) = U(s(\hat{x}^H, y^L))$. I can then rewrite the incentive compatibility and truth-telling with failure constraints such

that:

$$\begin{aligned}
U^H &> \frac{c}{\tilde{p}_h - v} + \lambda^L U(s(\hat{x}^L, y^L)) + (1 - \lambda^L) U(s(\hat{x}^L, y^H)), \\
U^H &< \lambda^L U(s(\hat{x}^L, y^L)) + (1 - \lambda^L) U(s(\hat{x}^L, y^H)).
\end{aligned} \tag{95}$$

which generates a contradiction and therefore proves that the truth-telling with success constraint is slack.

The solution to the problem has the individual rationality constraint binding and the incentive compatibility slack. The individual rationality must be binding due to the standard argument. □

Proof of Lemma 5. I can write up the Lagrangian for an overconfident manager with communication as:

$$\begin{aligned}
L &= p_h [\lambda^H s(\tilde{x}^H, y^H) + (1 - \lambda^H) s(x^H, y^L)] \\
&+ (1 - p_h) [\lambda^L s(x^L, y^L) + (1 - \lambda^L) s((x^L, y^H))] \\
&- \phi_1 [\tilde{p}_h [\lambda^H U(s((x^H, y^H)) + (1 - \lambda^H) U(s((x^H, y^L)))] \\
&+ (1 - \tilde{p}_h) [\lambda^L U(s((x^L, y^L)) + (1 - \lambda^L) U(s((x^L, y^H)))] - c - \bar{U}] \\
&- \phi_2 [\lambda^L U(s(\tilde{x}^L, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^L, y^H)) \\
&- \lambda^L U(s(\tilde{x}^H, y^L)) - (1 - \lambda^L) U(s(\tilde{x}^H, y^H))].
\end{aligned} \tag{96}$$

where ϕ_1 is the Lagrange multiplier from the IR constraint, and ϕ_2 is the Lagrange multiplier

from the TT_L constraint. This yields the first-order conditions:

$$\begin{aligned} \frac{\partial L}{\partial s(\tilde{x}^H, y^H)} &= p_h \lambda^H - \phi_1 \tilde{p}_h \lambda^H U' (s(\tilde{x}^H, y^H)) ds(\tilde{x}^H, y^H) \\ &+ \phi_2 (1 - \lambda^L) U' (s(\tilde{x}^H, y^H)) ds(\tilde{x}^H, y^H) = 0, \end{aligned} \quad (97)$$

$$\begin{aligned} \frac{\partial L}{\partial s(x^H, y^L)} &= p_h (1 - \lambda^H) - \phi_1 \tilde{p}_h (1 - \lambda^H) U' (s((x^H, y^L))) ds(x^H, y^L) \\ &+ \phi_2 \lambda^L U' (s(\tilde{x}^H, y^L)) ds(\tilde{x}^H, y^L) = 0, \end{aligned} \quad (98)$$

$$\begin{aligned} \frac{\partial L}{\partial s(\tilde{x}^L)} &= (1 - p_h) - \phi_1 (1 - \tilde{p}_h) U' (s((x^L))) ds(\tilde{x}^L) \\ &- \phi_2 U' (s(\tilde{x}^L)) ds(\tilde{x}^L) = 0, \end{aligned} \quad (99)$$

$$\begin{aligned} \frac{\partial L}{\phi_1} &= \tilde{p}_h [\lambda^H U (s((x^H, y^H))) + (1 - \lambda^H) U (s((x^H, y^L)))] \\ &+ (1 - \tilde{p}_h) U (s((x^L))) - c - \bar{U} = 0, \end{aligned} \quad (100)$$

$$\frac{\partial L}{\phi_2} = (U (s(\tilde{x}^L)) - \lambda^L U (s(\tilde{x}^H, y^L)) - (1 - \lambda^L) U (s(\tilde{x}^H, y^H))) = 0. \quad (101)$$

Solving these yields the Lemma. □

Proof of Corollary 4. Using the truth-telling constraint the individual rationality constraint can be rewritten as:

$$\tilde{p}_h E(U(s(x^H))) + (1 - \tilde{p}_h) U (s((x^L))) - c - \bar{U} = 0. \quad (102)$$

Taking the total derivative with regards to \tilde{p}_h yields:

$$E(U(s(x^H))) - U (s((x^L))) + \tilde{p}_h E(U'(s(x^H))) d\tilde{p}_h + (1 - \tilde{p}_h) U' (s((x^L))) d\tilde{p}_h = 0. \quad (103)$$

It is known that $E(U(s(x^H))) - U (s((x^L))) > 0$. I then need to determine $E(U'(s(x^H))) d\tilde{p}_h = \lambda^H U' (s(\tilde{x}^H, y^H)) d\tilde{p}_h + (1 - \lambda^H) U' (s(x^H, y^L)) d\tilde{p}_h$ and $U' (s((x^L))) d\tilde{p}_h = \lambda^L U' (s(\tilde{x}^H, y^L)) d\tilde{p}_h + (1 - \lambda^L) U' (s(\tilde{x}^H, y^H)) d\tilde{p}_h$, so I need to find the sign of the change in $E(U'(s(x^H))) d\tilde{p}_h$ and $U' (s((x^L))) d\tilde{p}_h$.

Notice that written this way, the problem mimics the proof of Corollary 2.

Letting $\lambda^H \rightarrow 1$ and $\lambda^L \rightarrow 1$, the Borch rule from Lemma 3 takes the form:

$$\frac{1}{U'(s(y^H))} = \phi \frac{\tilde{p}_h}{p_h}, \quad (104)$$

$$\frac{1}{U'(s(y^L))} = \phi \frac{(1 - \tilde{p}_h)}{(1 - p_h)}, \quad (105)$$

which was used to sign $U'(s(x^y))d\tilde{p}_h$ and $U'(s(y^L))d\tilde{p}_h$.

In the rewritten version of Lemma 5, the Borch rule takes the form:

$$\frac{1}{E[U'(s(\tilde{x}^H))]} = \phi_1 \frac{\tilde{p}_h}{p_h} - \phi_2 \frac{1}{p_h}, \quad (106)$$

$$\frac{1}{U'(s(\tilde{x}^L))} = \phi_1 \frac{(1 - \tilde{p}_h)}{(1 - p_h)} + \phi_2 \frac{1}{(1 - p_h)}. \quad (107)$$

The only difference between these two versions is the second terms that depend on the Lagrange multiplier from the truth-telling constraint.

Now assume that the truth-telling constraint forces overconfidence to not increase the power of the contract. Taking the total derivative yields:

$$\left(\frac{1}{E[U'(s(\tilde{x}^H))]} \right) d\tilde{p}_h = \left(\phi_1 \frac{\tilde{p}_h}{p_h} \right) d\tilde{p}_h - (\phi_2) d\tilde{p}_h \frac{1}{p_h} \quad (108)$$

$$\left(\frac{1}{U'(s(\tilde{x}^L))} \right) d\tilde{p}_h = \left(\phi_1 \frac{(1 - \tilde{p}_h)}{(1 - p_h)} \right) d\tilde{p}_h + (\phi_2) d\tilde{p}_h \frac{1}{(1 - p_h)}. \quad (109)$$

From the proof of 2 it is known that $\left(\phi_1 \frac{\tilde{p}_h}{p_h} \right) d\tilde{p}_h > 0$ and $\left(\phi_1 \frac{(1 - \tilde{p}_h)}{(1 - p_h)} \right) d\tilde{p}_h < 0$.

If the truth-telling constraint breaks the relation from 2, then

$$\left(\phi_1 \frac{\tilde{p}_h}{p_h}\right) d\tilde{p}_h - (\phi_2) d\tilde{p}_h \frac{1}{p_h} < 0 \quad (110)$$

$$\left(\phi_1 \frac{(1 - \tilde{p}_h)}{(1 - p_h)}\right) d\tilde{p}_h + (\phi_2) d\tilde{p}_h \frac{1}{(1 - p_h)} > 0, \quad (111)$$

which means that $(\phi_2) d\tilde{p}_h$ must not just be positive, it must be sufficiently positive for this to hold.

However, using the IR and TR constraints:

$$\begin{aligned} & [\lambda^H U(s((x^H, y^H))) + (1 - \lambda^H) U(s((x^H, y^L)))] - [\lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H))] \\ & + \tilde{p}_h [\lambda^H U'(s((x^H, y^H))) d\tilde{p}_h + (1 - \lambda^H) U'(s((x^H, y^L))) \tilde{p}_h] \\ & + (1 - \tilde{p}_h) [\lambda^L U'(s(\tilde{x}^H, y^L)) \tilde{p}_h + (1 - \lambda^L) U'(s(\tilde{x}^H, y^H)) \tilde{p}_h], \end{aligned} \quad (112)$$

since

$$\begin{aligned} & [\lambda^H U(s((x^H, y^H))) + (1 - \lambda^H) U(s((x^H, y^L)))] \\ & - [\lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H))] > 0, \end{aligned} \quad (113)$$

either $U'(s((x^H, y^H))) d\tilde{p}_h < 0$ or $U'(s((x^H, y^L))) d\tilde{p}_h < 0$, or both, will be below 0.

As the payments must incentivize high effort, we know from equations (26) and (27) which define the Borch rule conditional on high observed outcome of the firm, that:

$$\frac{1}{U'(s(\tilde{x}^H, y^H))} > \frac{1}{U'(s(\tilde{x}^H, y^L))}, \quad (114)$$

so that

$$\phi_2 \frac{1}{p_h} \left(\frac{\lambda^L}{(1 - \lambda^H)} - \frac{(1 - \lambda^L)}{\lambda^H} \right) > 0. \quad (115)$$

Total differentiating this expression yields:

$$\left(\frac{1}{U'(s(\tilde{x}^H, y^H))} - \frac{1}{U'(s(\tilde{x}^H, y^L))} \right) d\tilde{p}_h = \phi_2 d\tilde{p}_h \frac{1}{p_h} \left(\frac{\lambda^L}{(1 - \lambda^H)} - \frac{(1 - \lambda^L)}{\lambda^H} \right). \quad (116)$$

So that if $\phi_2 d\tilde{p}_h > 0$, then the power of the contract is increasing in \tilde{p}_h , which yields a contradiction.

To get the intuition from this, equation (21) shows that the expected utility of the manager conditional on the firm being a success is a weighted average of the payments conditional on the public report and communicating success, however the largest weight is on $s(\tilde{x}^h, y^H)$.

From the truth-telling constraint, the outcome of the manager conditional on low output is also a weighted average of the payment conditional reporting success and the accounting earnings, however the largest weight is on $s(\tilde{x}^h, y^L)$. So if $s(\tilde{x}^h, y^H)$ increases, then the conditional expected power of the contract increases.

This means that as overconfidence increases the power of the contract conditional on either success or failure, the truth-telling constraint forces the payments conditional on success to become farther apart.

To analyze how the effect of precision affects the optimal contract, consider the following:

If the payments $s(\tilde{x}^H, y^L)$ and $s(\tilde{x}^H, y^H)$ that satisfy the optimal contract are kept fixed, then the manager's utility conditional on observing success or failure can be written as:

$$U(\tilde{x}^L) = \lambda^L U(\bar{s}(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(\bar{s}(\tilde{x}^H, y^H)), \quad (117)$$

$$E[U(x^H)] := [\lambda^H U(\bar{s}(x^H, y^H)) + (1 - \lambda^H) U(\bar{s}(x^H, y^L))]. \quad (118)$$

The power of the contract can then be written as:

$$\begin{aligned} E [U(x^H)] - U(\bar{s}(\tilde{x}^L)) &= \lambda^H U(\bar{s}(x^H, y^H)) + (1 - \lambda^H) U(\bar{s}(x^H, y^L)), \\ &\quad - \lambda^L U(\bar{s}(\tilde{x}^H, y^L)) - (1 - \lambda^L) U(\bar{s}(\tilde{x}^H, y^H)). \end{aligned} \quad (119)$$

This yields the partial derivatives:

$$\frac{\partial E [U(x^H)] - U(\tilde{x}^L)}{\partial \lambda^H} = U(\bar{s}(x^H, y^H)) - U(\bar{s}(x^H, y^L)) > 0, \quad (120)$$

$$\frac{\partial E [U(x^H)] - U(\tilde{x}^L)}{\partial \lambda^L} = U(s(\bar{s}^H, y^H)) - U(\bar{s}(x^H, y^L)) > 0. \quad (121)$$

□

Proof of Proposition 3. I want to minimize the principal' payment to the manager given high effort:

$$\min_{s(x,y)} p_h [\lambda^H s(\tilde{x}^H, y^H) + (1 - \lambda^H) s(x^H, y^L)] + (1 - p_h) s(\tilde{x}^L), \quad (122)$$

subject to the individual rationality constraint:

$$\begin{aligned} \tilde{p}_h [\lambda^H U(s((x^H, y^H)) + (1 - \lambda^H) U(s((x^H, y^L)))] \\ + (1 - \tilde{p}_h) U(s(\tilde{x}^L)) - c \geq \bar{U}, \end{aligned} \quad (123)$$

and truth-telling with failure constraint:

$$U(s(\tilde{x}^L)) \geq \lambda^L U(s(\tilde{x}^H, y^L)) + (1 - \lambda^L) U(s(\tilde{x}^H, y^H)). \quad (124)$$

Again using the duality of cost minimization and profit maximization, in optimum I can use the envelope theorem to get:

$$\frac{\partial \pi}{\partial \tilde{p}_h} = \phi_1 [\lambda^H U(s((x^H, y^H)) + (1 - \lambda^H) U(s((x^H, y^L)) - U(s(x^L))]. \quad (125)$$

From the binding truth-telling with failure constraint, I know that:

$$U(s(x^L)) = (1 - \lambda^L) U(s((x^H, y^H))) + \lambda^L U(s((x^H, y^L))). \quad (126)$$

Inserting that and moving around yields:

$$\frac{\partial \pi}{\partial \tilde{p}_h} = \phi_1 [[\lambda^H - (1 - \lambda^L)] U(s((x^H, y^H))) - [\lambda^L - (1 - \lambda^H)] U(s((x^H, y^L)))] > 0 \quad (127)$$

and

$$\frac{\partial^2 \pi}{\partial \tilde{p}_h \partial \lambda^H} = \phi_1 [U(s((x^H, y^H))) - U(s((x^H, y^L)))] > 0. \quad (128)$$

The welfare gain from the combined increase of overconfidence and precision is given from the comparative statics in 3. □

Proof of 6. This proof follows that of Proposition 1 in Gigler and Hemmer (2001).

The expected cost of hiring the manager in the "earnings-only" contract is given by:

$$\begin{aligned} E[G(U(s(y)))] = & \tilde{p}_h [\lambda^H G(U(s(y^H))) + (1 - \lambda^H) G(U(s(y^L)))] \\ & + (1 - \tilde{p}_h) [\lambda^L G(U(s(y^L))) + (1 - \lambda^L) G(U(s(y^H)))] , \end{aligned} \quad (129)$$

and in the "communication" contract

$$\begin{aligned} E[G(U(s(\tilde{x}, y)))] = & \tilde{p}_h [\lambda^H G(U(s(y^H))) + (1 - \lambda^H) G(U(s(y^L)))] \\ & + (1 - \tilde{p}_h) G[\lambda^L (U(s(y^L))) + (1 - \lambda^L) (U(s(y^H)))] . \end{aligned} \quad (130)$$

This leads to the change in payment from the "communication" contract to the "earnings-

only” constraint

$$\Delta S_{slight} = (1 - p_h) \{E[G(U(s(y))) | x^L] - G(E[U(s(\tilde{x}, y))] | x^L)\} \geq 0, \quad (131)$$

due to convexity of the risk-averse manager’s inverse utility function $G(\cdot)$. \square

Proof of Proposition 4. First, notice that the difference between $E[G(U(s(y))) | x^L] - G(E[U(s(\tilde{x}, y))] | x^L)$ is decreasing in managerial overconfidence \tilde{p}_h .

The expression in Lemma 6 can be rewritten as

$$(1 - \lambda^L) [G(U(s(y^H))) - G(E[U(s(\tilde{x}, y))] | x^L)] \quad (132)$$

$$+ \lambda^L [G(U(s(y^L))) - G(E[U(s(\tilde{x}, y))] | x^L)] \geq 0, \quad (133)$$

where

$$(1 - \lambda^L) [G(U(s(y^H))) - G(U(s(x^L)))] \geq -\lambda^L [G(U(s(y^L))) - G(U(s(x^L)))] , \quad (134)$$

due to the convexity of $G(\cdot)$.

Notice from Lemma 1 and 4 that

$$\frac{\partial U(s(y^L))}{\partial \tilde{p}_h} > \frac{\partial U(s(x^L))}{\partial \tilde{p}_h} > \frac{\partial U(s(y^H))}{\partial \tilde{p}_h}, \quad (135)$$

which means that

$$\frac{\partial \Delta S_{slight}}{\partial \tilde{p}_h} < 0, \quad (136)$$

as the difference between $G(U(s(y^H))) - G(U(s(x^L)))$ becomes smaller, thus reducing the value of communication as there is less difference between the two payments, reducing the effect of Jensen’s Inequality. \square

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