Economics Working Papers

2019-9

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Loss aversion and the zero-earnings discontinuity *

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August 7, 2019

Abstract

Prior literature suggests that the zero-earnings discontinuity is caused by earnings management. This makes sense if investors are naïve. We test for the possibility of investor naïveté and find that they are aware of firms performing earnings management around zero reported earnings and that there is no obvious gain of reaching zero reported earnings. We extend a signaling model to include loss-averse investors and we find that earnings management is not only rational, but in equilibrium, it is not possible for investors to deduce the correct value of firms' earnings around the discontinuity. Assuming our model generates the observed data, a loss-aversion coefficient of 1.2595 matches the discontinuity below zero reported earnings observed in the data simulated from the model and in the actual data. This loss-aversion coefficient is consistent with Tversky and Kahneman (1992), who find that losses are weighted roughly twice as heavily as gains.

Keywords: Discontinuity, Loss Aversion, Reporting, Signaling
JEL Classification: C70, D91, G41, M41

*We would like to thank Christian Hoffmann, Bjorn Jorgensen, Naomi Soderstrom, Ane Tamayo and the participants at the Third Annual Scandinavian Accounting Research conference for helpful suggestions and constructive comments.

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1 Introduction

In their seminal paper, Burgstahler and Dichev (1997) described the zero-earnings discontinuity. It has been discussed widely in the literature and has been both disputed and supported. It has led to a literature considering beating or meeting thresholds, and there have been various attempts to rationalize this behavior. We ask whether irrationality, in particular loss aversion on the part of investors, may lead to this particular behavior. We first provide evidence of the zero-earnings discontinuity and estimate the size of the insufficiency in the frequency of reported small negative earnings, as well as the excess of reports of zero or small positive values. We then find results that indicate that there is no obvious gain from reaching zero earnings and that the market anticipates earnings manipulation. Consistent with these results, we formulate an equilibrium model where the presence of loss-averse investors leads to earnings manipulation. We then estimate the size of the loss-aversion coefficient of the investors consistent with the empirically observed discontinuity in earnings within this model.

In a survey, Dichev et al. (2013) find that 99.4% of CFOs believe that earnings management happens in at least some firms. Several explanations for the discontinuity have been provided. Burgstahler and Chuk (2017) argue that the most likely explanation is still the desire to meet or beat expectations. Guttman et al. (2006) show that a partially pooling equilibrium can endogenously arise when the managers of the firms are able to save on unnecessary manipulation costs by pooling. Li (2014) shows that only the partially pooling equilibrium exists when earnings between periods are highly positively autocorrelated. A strand of the literature argues against earnings management being the cause of the earnings discontinuity. Hemmer and Labro (2019) show that a discontinuity in earnings can arise even without a reporting bias by letting earnings be a noisy measure of a real option that the manager can either keep or reject. Beaver et al. (2007) argue that discontinuity in the earnings distribution can be explained by asymmetric effects of income taxes and special items for firms either realizing a loss or a profit. Durtschi and Easton (2005) argue that it could be caused by deflation choices, sample selection and differences between characteristics of observations to the left and the right of zero reported earnings. Burgstahler and Chuk (2015) however argue that the research design by Durtschi and Easton (2005) obscures the evidence of the discontinuity by not accounting for the effect of firm size as a covariate, putting too much weight on small firms and not accounting for differences in the amount of earnings that can be managed for a cost lower than the benefit between firms of different sizes.

Our main contribution to the literature is to show that loss aversion is also able to explain the earnings discontinuity. Loss aversion has been shown to affect decision making. In a study of marathon completion times, for example, Allen et al. (2016) show that there is pattern of discontinuities around the hourly marks which is reminiscent of the zero-earnings discontinuity. They allude to runners' reference-dependent preferences to explain the pattern: the runner would experience going over the hourly mark as a loss. In our model, however, it is somebody else's reference dependence (the investors') which drives the decision maker's (the manager's) behavior. We are not the first to use reference dependence as an explanation for unexplained signaling behavior in capital markets. For example, Baker et al. (2015) are able to explain persistence in dividends as a decision taken by the manager to accommodate reference-dependent investors.

We start by analyzing the irregularities in reporting behavior around zero reported earnings. We find evidence of a discontinuity by using a density test by McCrary, McCrary (2008). We use a bunching estimator developed by Chetty et al. (2011) to quantify the lack of reported small negative earnings and the excess of zero and small positive earnings reported. We then move on to analyze the stock market response to earnings around zero. The naïve explanation for firms' decision to avoid reporting small negative earnings would be that there is a capital market gain from doing so if the market is unaware of this behavior. We do not expect this explanation to hold in equilibrium. We define the market reaction to the earnings announcement as the cumulated abnormal returns from an event study around the earnings announcement. We then use a regression discontinuity design to test whether there is a change in the market response to earnings around zero. To analyze whether the market is aware of the misreporting behavior, we use a research design closely related to Keung et al. (2010); we estimate earnings response coefficients for the intervals around zero to measure how much weight the market puts on earnings in these intervals. Investors seem to anticipate earnings management and therefore lend less credence to reported earnings at or just above zero. There is a large increase in the earnings response coefficient when we move outside of the earnings interval just around zero earnings in most years, which we interpret as investors being skeptical of the validity of earnings reports close to zero. We interpret this as investors being aware of managers performing earnings management to avoid reporting losses, in line with, e.g, Brown and Caylor (2005), Degeorge et al. (1999), Jia (2013) and Roychowdhury (2006). This result is similar to, and complements, the result by Keung et al. (2010) who find that investors reacts less to earnings surprises from firms which just beat expectations compared to firms which beat expectations slightly more. We then test and reject the hypothesis that firms which miss zero earnings suffer stock market losses and receive a gain by reaching reported earnings of zero. This differs from, e.g., Skinner and Sloan (2002), who find that missing the market's expectation leads to large decreases in stock price, Kothari et al. (2009), who find that the market reacts more to bad news compared to good news, and Bird et al. (2019), who focus on beating the market's expectation and find a direct benefit of earnings manipulation in terms of the stock price. An interpretation of this is that results between the different benchmarks and news on the stock price might not be generalizable. If there is no direct gain from reaching zero earnings, however, would there be any reason for managers to avoid reporting negative earnings?

To provide a possible explanation of earnings management, even if investors anticipate such practice so there are no direct gains from doing so, we extend Guttman et al. (2006) and show that only a partially-pooling equilibrium survives when we include loss aversion on the part of investors.[1] The larger the loss-aversion, the larger the discontinuity in earnings. Li (2014) achieves a similar result, however he base his model not on loss-aversion, but instead the strength and sign of the correlation of earnings between periods. A consequence of this is that when firms have weakly or negatively correlated earnings the equilibrium will not be characterize by pooling around the benchmark, whereas in our model, as long as investors are loss-averse, there will only ever be a partially-pooling equilibrium. We then use simulated method of moments to find the size of the loss-aversion coefficient that best matches the discontinuity in data simulated from our model to the empirically observed data and estimate it at 1.2595 (investors' experience of loss being more than twice as strong as the experience of a like-sized gain, close to the estimate of loss aversion found by Tversky and Kahneman (1992)). One could extend the model to include other reference points, in line with benchmark studies in the literature, such as analysts' forecasts (e.g Kasznik and McNichols (2002) and Li (2014)), last year's earnings (e.g Burgstahler and Dichev (1997)), or the manager's earnings forecast (e.g Kross et al. (2011)).

[1]Without loss aversion, a perfectly-separating equilibrium is possible.

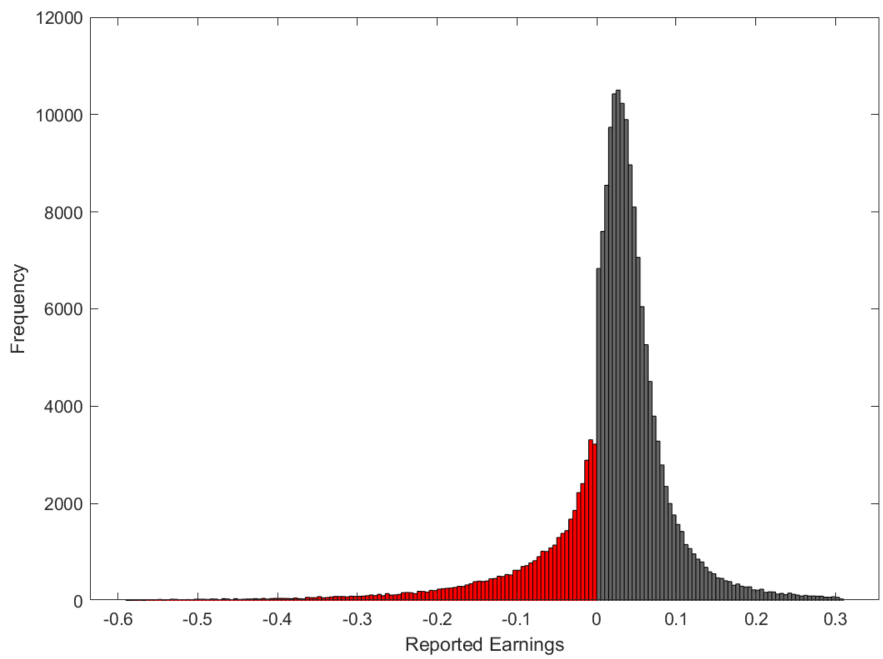
The remainder of the paper is organized as follows. We describe our data in Section 2. In Section 3, we analyze the distributional qualities around reported earnings of zero, which are consistent with earnings management. We test for market naïveté in section 4. We study a theoretical model of earnings management in the presence of loss-averse investors in section 5, which we then estimate in section 6. Section 7 concludes.

2 Data

For the empirical part of the paper we use data on all publicly traded firms in the united states from the second quarter in 1985 to the first quarter in 2018. We obtain the date of the quarterly earnings announcement and our accounting measures from the Compustat database, stock returns and prices from the CRSP database, and the consensus analyst forecasts on earnings per share and actual earnings per share from the I/B/E/S database. To link the databases together, we use the unique firm identifier in each database matched through linking tables provided by WRDS. We eliminate the lowest and highest percentile of earnings surprises, cumulated abnormal returns, prices, and net income scaled by assets. This leaves us with 189,598 firm-quarter observations.

In figure 1 we present the histogram of $NetIncome/Assets$, normalized quarterly earnings. Other measures of reported earnings, including reported earnings at the annual frequency, could be used. However, Burgstahler and Chuk (2017) argue that the discontinuity in reported earnings will be present in the data if the chosen measure of the firm's earnings is widely reported and used by stakeholders. We expect this to be the case for normalized quarterly earnings, which we will refer to as *reported earnings* in the remainder of this paper. The histogram shows what looks like a slightly skewed normal distribution at first glance. The only notable exception is just below zero, where there is a noticeable drop in the frequency of reported earnings. A common interpretation of this discontinuity, starting with Burgstahler and Dichev (1997), is that some firms that realize small negative earnings choose to report zero or small positive earnings. This zero-earnings discontinuity is the phenomenon which we will address empirically and theoretically throughout the paper.

Figure 1: **The distribution of quarterly earnings.**
This figure shows a histogram of the normalized quarterly earnings defined as $\frac{NetIncome}{Assets}$ for 189,598 earnings announcement of publicly trades firms in the united states from the second quarter in 1985 to the first quarter in 2018. The distribution is approximately normally distributed, with a large left tail and a discontinuity just below zero.



3 Evidence and magnitude of the discontinuity

We proceed by estimating the size of the drop in reported earnings just below zero, and of bunching at zero and small positive reported earnings. The first step in our analysis is to test whether the discontinuity in reported earnings is significant. To test for the discontinuity in reported earnings, we use a density test by McCrary (2008)[2], which is specifically designed to test for a discontinuity in the distribution of a variable, in our case in reported earnings, that could be caused by manipulation. Figure 2 depicts the distribution of earnings used in Mccrary's test. The t-statistic of the test is 45.2, which is well into the significant range, so we strongly reject the null hypothesis of no discontinuity (p-value below 0.00001). Furthermore, the test provides an estimate for the size of the discountinuity in reported earnings—measured as the log difference in height—of 68%, so the discontinuity is both statistically and economically significant.

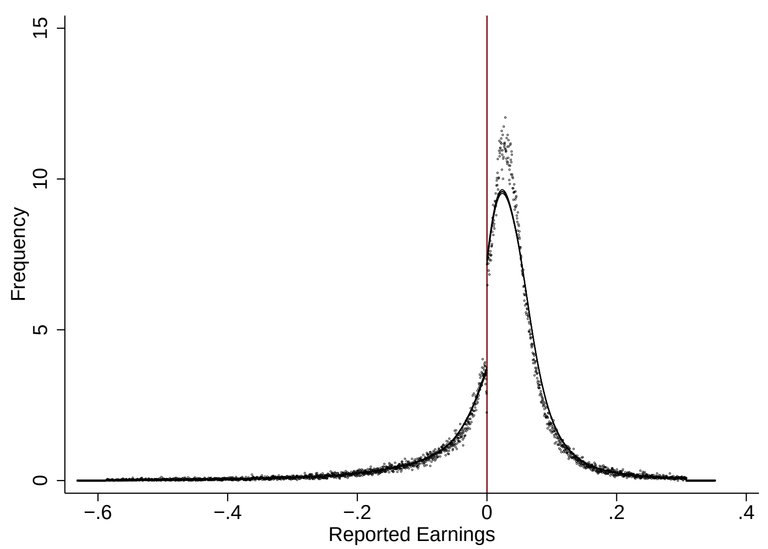
The next step is to estimate the number of firms reporting zero or small positive earnings, in excess of what one would expect if no earnings management took place, as well as the number of firms not reporting small negative earnings relative to that counterfactual. The underlying assumption is that true earnings are distributed smoothly. We follow the methodology of Chetty et al. (2011) in order to estimate the underlying distribution of true earnings. We do this by fitting a polynomial to the histogram of reported earnings excluding the observations near the discontinuity. We estimate the following equation:

$$C_j \times (1 + \mathbb{1}[j > R_h] \frac{\hat{B}_N}{\sum_{j=R_h+1}^{\infty} C_j}) = \sum_{i=0}^{q} \beta_i \times (Z_j)^i + \sum_{i=-R_l}^{R_h} \gamma_i \times \mathbb{1}[Z_j = i] + \epsilon_j, \quad (1)$$

where C_j is the number of firms reporting earnings in earnings bin j and Z_j is reported earnings relative to the discontinuity (calculated as standardized net income, $\frac{NetIncome}{Assets}$) of bin j (and each bin has width of 0.001, so the intervals of the histogram are given by $Z_j = -0.075, -0.074, ..., 0.1$). q is the order of the polynomial (we set $q = 11$), and R denotes the width of the region of the discontinuity (which is excluded). B_N is the insufficiency in the

[2] The test is similarly used by Bird et al. (2019) for testing whether the discontinuity around beating the market's expectation is caused by manipulation.

Figure 2: **Distribution used for McCrary's Density Test.**
To test for a discontinuity in the distribution of reported earnings we apply McCrary's density test. This figure shows the proposed density for reported earnings which is used to test for the presence of manipulation causing a discontinuity.



number of small negative reported earnings, and $1[j > R]\frac{\hat{B}_N}{\sum_{j=R+1}^{\infty} C_j}$ is a term that takes into account that all the firms that choose not to report small negative earnings have moved to report zero or small positive earnings instead, which shifts the counter-factual distribution to the right of the discontinuity upwards.[3]

One estimate of the counterfactual distribution can be found as the predicted value of the number of firms reporting earnings in each interval from the equation above by omitting the dummies estimated in the interval around the discontinuity:

$$\hat{C}_j = \sum_{i=0}^{q} \hat{\beta}_i \times (Z_j)^i.$$

The estimate of the insufficiency in small negative reported earnings is then given by

$$\hat{\beta}_N = \sum_{j=-R_l}^{R_h} C_j - \hat{C}_j = \sum_{i=-R}^{R} \hat{\gamma}_i.$$

We now build an estimate of the insufficiency in the mass of the reported small negative earnings relative to the average density of the counterfactual distribution between $-R_l$ and R_h:

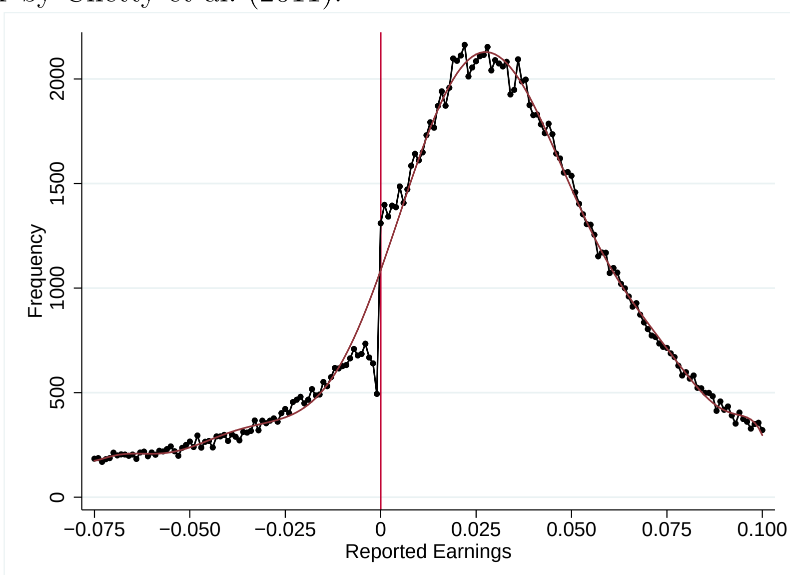
$$\hat{b} = \frac{\hat{B}_N}{\sum_{j=-R_l}^{R_h} \hat{C}_j/(R_l + R_h + 1)}.$$

To find a standard error of the estimate, we perform a parametric bootstrapping procedure. We draw from the vector of errors from estimating the counterfactual distribution, which gives a new set of the frequencies of reported earnings in each interval that we use to estimate new $\hat{b}^k$. We define the standard deviation from the distribution of $\hat{b}^k$ as the standard deviation of $\hat{b}$. This standard error reflects the misspecification errors from fitting the polynomial and not sampling errors, since reported earnings are perfectly observed.

First, we focus on the insufficiency in the number of reported small negative earnings. We define the area of small negative earnings as earnings

[3] One could argue that some firms will decide to manage earnings by a lesser amount, since they are already outside the range of beating zero earnings. Although this is a valid argument, we believe that the first-order effect stems from firms moving from small negative earnings to zero or small positive reported earnings.

Figure 3: **Estimated distribution of earnings.**
This figure combines a histogram of reported earnings for all publicly trades firms in the United States from the second quarter in 1985 to the first quarter in 2018, together with an estimated counterfactual distribution of the true underlying earnings. The counterfactual is found using the bunching estimator by Chetty et al. (2011).



in the range of $[-0.01, -0.001]$, which leads to an estimate of $\hat{b} = -2.831$ with a standard error of 0.144, which means that the observed number of firms with small negative reported earnings is $2,579$ fewer than implied by the counterfactual. The interpretation of the $\hat{b}$ coefficient is that in the range of reported earnings $[-0.01, -0.001]$ there is an insufficiency in the mass of reported earnings that is 2.831 times the average number of firms reporting earnings in those bins in the counterfactual distribution. This result is qualitatively robust to changes in q.[4]

Besides finding an estimate for how many firms decide to not report small negative earnings, we also estimate the excess mass of firms reporting zero or small positive earnings. In order to do this, we reverse the constraint that forces the excess mass of firms reporting zero or small positive earnings to come from below zero, i.e $\mathbb{1}[j < R_h]$. We analyze interval $[0, 0.01]$ and find that the excess mass in this area is 1.751 times the average frequency in this area estimated through the counterfactual, with a standard deviation of 0.2107, which implies that there are 2202 more firms reporting in this area than expected. There is difference between the insufficiency of firms reporting small negative earnings and the excess of firms reporting zero or small positive earnings suggests that some firms which realize small losses manipulate their way further into the profitable range and that other firms which realize small losses are aware that it might not be possible, or favorable, to incur the cost of reaching zero earnings and instead realize a larger loss, which will allow them to report higher earnings in the future.

There could be other discontinuities, besides the one around zero reported earnings; that seems to be the case when looking at the reported earnings compared to the counterfactual distribution in figure 3. In a study of marathon runners' completion times, Allen et al. (2016) show a histogram with discontinuities after each hour mark, so that the frequency of runners finishing at the hour mark or just before, is higher than number of runners finishing just after the hour mark. Allen et al. (2016) interpret this result as the marathon runners having reference-dependent preferences of beating the hour marks. This could also apply to managers who believe that their investors have a certain reference point that they want the firm to reach or surpass (other than zero earnings). One of the possible discontinuities is in

[4]We varied the order of the polynomial from 7 to 15 and we also relaxed the assumption that the excess mass of firms reporting small positive numbers had realized negative earnings, and vice versa, without qualitatively changing the results.

the interval $[0.018, 0022]$, where the excess mass is 0.1466 with a standard deviation of 0.0.0811. While only weakly significant, this is consistent with the explanation for why there were a difference between the number of firms failing to report losses and the excess number of firms presenting just positive earnings. The reason for this is that it provides evidence that firms try to reach higher earnings to beat some arbitrary reference-points from their investors, and firms who can not beat their prospective reference-point decide to realize losses instead to save up on accruals in order to meet or beat benchmarks in the future.

4 Market naïveté

We want to analyze the market's response to reported earnings around zero. If the discontinuity in earnings is caused by earnings management, then the market should be aware of this and respond accordingly. First, we want to investigate if the market is aware that firms with realized small negative earnings instead report zero or small positive earnings. We therefore test whether there is a change in the cumulated abnormal returns around the earnings announcement when firms' reported earnings reach zero. We hypothesize that if the market is aware of this behavior, they will put less weight on earnings surprises if the reported earnings are either zero or slightly positive.

4.1 Cumulated Abnormal Returns and the Earnings Response Coefficient

To measure the stock market reaction to the earnings announcement we use standard event study methodology. We define the day of the earnings announcement as the event day $t = 0$, and estimate the market model from day $t - 266$ to $t - 41$. The market model is defined as

$$R_{i,t} = \alpha_0 + \alpha_1 R_{M,t} + \epsilon_{i,t},$$

which is a regression of the return of firm i, $R_{i,t}$ on the market return, $R_{M,t}$. We use the estimates of α_0 and α_1 to find the abnormal returns as:

$$AR_{i,t} = R_{i,t} - (\hat{\alpha}_0 + \hat{\alpha}_1 R_{M,t}).$$

and we then define the cumulative abnormal returns as the sum of the abnormal returns from day $[-1, 1]$

$$CAR_{-1,1} = \sum_{t=-1}^{1} AR_{i,t}$$

A classical measure of the effect of earnings on stock returns is the earnings response coefficient. The idea is to find the effect on the surprise in returns from the surprise in earnings. We follow Easton and Zmijewski (1989), DeFond and Park (2001), Brown and Caylor (2005) and Keung et al. (2010) amongst others, and define the earnings surprise as:

$$SUE_t = \frac{EPS_t - E_{t-1}(EPS_t)}{P_{t-21}}.$$

Where SUE_t is the earnings surprise, EPS_t is the (reported) quarterly earnings per share, $E_{t-1}(EPS_t)$ is a proxy of the market's expectation of the reported earnings, where we use the consensus analyst forecast[5] and the earnings surprise is scaled by the stock market price 21 days prior, P_{t-21} .

The earnings response coefficient is the coefficient on the earnings surprise when we use the surprise:

$$CAR_{-1,1} = \beta_0 + \beta_1 SUE + \epsilon.$$

We use the earnings response coefficient as a measure of the market's assessment of earnings quality, as the higher the earnings response coefficient the more weight the market place on the firm's earnings.[6]

4.2 Test of market naïveté

We hypothesize that the market is aware of the misreporting behavior, be it window dressing activities or real earnings management.

To analyze this question, we follow a research design closely related to Keung et al. (2010). We define 14 indicator variables that denote the range

[5]We use the latest median analyst forecast.

[6]For further discussion on the interpretation of the earnings response coefficient as a measure of earnings quality, see Dechow et al. (2010).

of earnings reported by the firm. We base these indicator variables on the net income of the firm scaled by assets $\frac{NetIncome}{Assets}$:

$$
\begin{aligned}
I_{-7} \; if \; \frac{NetIncome}{Assets} &\in (-\infty, -0.08), \\
I_{-6} \; if \; \frac{NetIncome}{Assets} &\in [-0.08, -0.06), \\
I_{-5} \; if \; \frac{NetIncome}{Assets} &\in [-0.06, -0.04), \\
I_{-4} \; if \; \frac{NetIncome}{Assets} &\in [-0.04, -0.03), \\
I_{-3} \; if \; \frac{NetIncome}{Assets} &\in [-0.03, -0.02), \\
I_{-2} \; if \; \frac{NetIncome}{Assets} &\in [-0.02, -0.01), \\
I_{-1} \; if \; \frac{NetIncome}{Assets} &\in [-0.01, 0.00), \\
I_{1} \; if \; \frac{NetIncome}{Assets} &\in [0.00, 0.01), \\
I_{2} \; if \; \frac{NetIncome}{Assets} &\in [0.01, 0.02), \\
I_{3} \; if \; \frac{NetIncome}{Assets} &\in [0.02, 0.03), \\
I_{4} \; if \; \frac{NetIncome}{Assets} &\in [0.03, 0.04), \\
I_{5} \; if \; \frac{NetIncome}{Assets} &\in [0.04, 0.06), \\
I_{6} \; if \; \frac{NetIncome}{Assets} &\in [0.06, 0.08), \text{ and} \\
I_{7} \; if \; \frac{NetIncome}{Assets} &\in [0.08, \infty).
\end{aligned}
$$

As our test of the market perception of the earnings quality of the firms, we regress the cumulated abnormal return on indicator variables for the intervals around zero multiplied by the firms' earnings surprises. If the earnings surprise coefficient is larger on the I_2 indicator compared with that on the I_1 indicator, the results will support our hypothesis. This means that the market puts more weight on firms' reported earnings if they are further from zero.

We estimate the following equation:

$$CAR_{-1,1} = \gamma_0 + \gamma_1 RunUp + \sum_{i=1}^{14} \beta_i \times I_i \times SUE_i + \epsilon. \quad (2)$$

In addition to the index variables and SUE, we include $RunUp$, which is the sum of the abnormal returns between $[t-21, t-2]$ and controls for information leakage to the market, as an explanatory variable.

Table 1: **Earnings Response Coefficients around the zero earnings discontinuity.**
This table reports the estimates from $CAR_{-1,1} = \gamma_0 + \gamma_1 RunUp + \sum_{i=1}^{14} \beta_i \times I_i \times SUE_i + \epsilon$ where $CAR_{-1,1}$ is the cumulated abnormal returns, SUE is the the earnings surprise, I_i is 14 indicator dummies which depend on the intervals of earnings the firm reports, and $RunUp$ is abnormal return prior to the event which controls for information leakage. The regression is run on all publicly traded firms in the interval from the second quarter in 1985 to the first quarter in 2018.

	[19851, 20184]		[19851, 20021]		[20021, 20181]	
	Beta	t-stat	Beta	t-stat	Beta	t-stat
Constant	0.002	11.727	0.003	11.777	0.001	3.503
Run-up	-0.021	-16.969	-0.030	-19.989	-0.006	-2.650
$I_{-7} \times SUE$	0.160	20.111	0.144	14.123	0.182	14.479
$I_{-6} \times SUE$	0.218	9.986	0.181	6.725	0.275	7.656
$I_{-5} \times SUE$	0.239	12.185	0.179	7.803	0.367	10.308
$I_{-4} \times SUE$	0.256	9.195	0.140	3.991	0.424	9.539
$I_{-3} \times SUE$	0.257	10.336	0.205	6.718	0.334	8.158
$I_{-2} \times SUE$	0.287	10.771	0.260	7.482	0.324	7.931
$I_{-1} \times SUE$	0.290	9.914	0.251	6.731	0.334	7.368
$I_1 \times SUE$	0.304	12.890	0.235	8.643	0.475	10.757
$I_2 \times SUE$	0.420	13.439	0.283	8.162	0.854	13.065
$I_3 \times SUE$	0.467	12.683	0.284	6.967	1.052	13.592
$I_4 \times SUE$	0.526	12.521	0.336	6.846	0.939	12.256
$I_5 \times SUE$	0.327	10.225	0.179	4.939	0.711	11.493
$I_6 \times SUE$	0.311	7.440	0.195	3.943	0.529	7.136
$I_7 \times SUE$	0.244	8.315	0.209	4.735	0.272	6.880
$adjR^2$	0.011		0.010		0.017	
N	189598		105310		85882	

In table 1 we present the aggregate results for the period $[19851, 20184]$, and decomposed into the time periods $[19851, 20021]$ and $[20021, 20181]$. Looking at the earnings response coefficient for the time period $[19851, 20184]$, there is strong indication that the market discounts the value of earnings information when the firm reports earnings at zero or just above zero earnings, that is, in the interval I_1 with a coefficient of 0.304, compared to the interval above I_2 where the size of the coefficient is 0.420, this result aligns with the hypothesis that the market anticipates firms who realizes small negative earnings instead reports zero or small positive earnings, and therefore discounts the information provided. The earnings response coefficient increases again at interval I_3, which indicates that the market perceives the firm's earnings as being of higher quality when they are further into the black. Focusing on the time periods $[19851, 20021]$ and $[20021, 20181]$ we observe that the difference between size of the coefficient on $I_1 \times SUE$ and $I_2 \times SUE$ is much larger for the time period $[20021, 20181]$ compared to the prior period $[19851, 20021]$. It seems that it is the time period $[20021, 20181]$ which drives the results in the aggregate period $[19851, 20184]$, which suggests that the market becomes increasingly aware of the behavior of firms avoiding to report small losses[7].

The change in the earnings response coefficient from firms at or just above zero, I_1, to slightly higher earnings, I_2, indicates that the market is aware that earnings at or just above zero are of lower quality in the sense, that firms who report this have with high likelihood masked their true performance. The difference between the two increases as time passes and is especially large from the first quarter in 2000 and forwards which is consistent with investors adapting to the firms' behavior and the findings by Keung et al. (2010).

4.3 Market reaction to zero earnings

Our results in section 3 indicates that firms which realize small negative earnings decide to report zero or small positive earnings. The naïve explanation is that the market is unaware of this behavior and therefore rewards firms for reaching zero earnings. However, our results suggest that the market is aware of firms performing earnings management to reach zero or small positive earnings. The next step is to test whether there is a gain for reaching zero reported earnings. To do this we test if there is a change in the cumulated abnormal returns when firms report zero or small positive earnings. If there

[7] In the appendix we present results from further disaggregation of the time period into six different intervals.

is no reaction, then there is no obvious reason for reaching zero earnings. In addition to the test for a direct effect on the cumulated abnormal returns, we also test whether there is a change in the market reaction to earnings after meeting zero earnings. Our hypothesis predicts that the market should reward firms for providing positive earnings further away from zero, since the farther away from zero they report, the lower the risk of the firm having realized negative earnings. To test if the market responds differently to earnings just above zero, we apply a sharp regression discontinuity design[8]. We use reported earnings as the running variable in order to analyze the change in the cumulated abnormal returns at zero earnings. From the potential outcome framework, we can write the observed outcome on the cumulated abnormal return as

$$CAR_i = CAR_i(0) \times (1 - D_i) + CAR_i(1) \times D_i,$$

where $D_i = 1[ReportedEarnings_i \geq 0]$ denotes that the cumulated abnormal returns are attached to earnings greater than or equal to zero, and are "treated" in that sense. The variable of interest is the average treatment effect on CAR_i from reaching zero earnings. We define this as

$$\tau = E[CAR_i(1) - CAR_i(0) \mid ReportedEarnings_i = 0].$$

We then choose the bandwidth through the common mean squared error bandwidth selector, use a kernel with triangular weights, use robust nonparametric confidence intervals following Calonico et al. (2014), and include covariates following Calonico et al. (2018). This means that we estimate the following equation:

$$\begin{aligned} CAR_i = a &+ f_1(ReportedEarnings_i)D_i + \tau D_i \\ &- f_0(ReportedEarnings_i)(1 - D_i) + Z_i\tilde{\psi} + \epsilon_i. \end{aligned} \tag{3}$$

where $f_1(ReportedEarnings_i)$ is a local polynomial on the effect of reported earnings on the cumulated abnormal returns for reported earnings greater than or equal to zero, $f_0(ReportedEarnings_i)$ is a local polynomial on the effect of reported earnings on the cumulated abnormal returns for reported earnings below zero, and Z_i are our covariates, so that we get a covariate-adjusted regression discontinuity estimator. This allows us to estimate the average treatment effect on the cumulated abnormal returns when the firm reports zero earnings.

[8]See Imbens and Lemieux (2008) for an in depth analysis of theory and implementation of sharp regression discontinuity designs.

There is one assumption regarding the regression discontinuity design that needs to be mentioned and discussed in this case. The regression discontinuity design does not allow for perfect manipulation, and since we are focusing on earnings management, which by definition is manipulation of earnings within the legal flexibility of the accounting standards, we conjecture that reported earnings are a function of the true economic conditions and earnings management, $ReportedEarnings = Manipulation + EconomicPerformance$, and postulate that the contribution of manipulation to the reported earnings is small in comparison with the contribution from the true underlying economic performance. This means that the reported earnings are only affected by partial manipulation caused by earnings management and not by perfect manipulation. Partial manipulation is allowed in the regression discontinuity framework, and it has been successfully implemented in the case of partial manipulation see, e.g., DiNardo and Lee (2004) and Van der Klaauw (2002).

Using the regression discontinuity design allows us to find the causality, similar to a randomized trial, when we focus on the effect of reporting zero earnings on returns. Since the purpose of the event study is to find the affect on the stock market return caused by the earnings announcement, we are only left with the causal effect from the reporting of zero earnings on returns.

Figure 4 shows the scatter-plot that constitutes the starting point for our research design based on the regression discontinuity and regression kink methodology. At the edges of the scatter-plot there seems to be very little relation between the reported earnings and the cumulated abnormal returns around the earnings announcement.

When we move towards -0.2 and 0.2, some relation starts to form, with earnings below zero causing the cumulated abnormal returns to be just below zero, but the size of the cumulative abnormal returns does not seem to depend on the size of the loss. On the other side of zero there seems to be a small positive cumulated abnormal return. From 0 to 0.05 there seems to be a positive relation, where a small increase in earnings leads to a larger cumulative abnormal return. Interpreting the scatter-plot, there seems to be no large punishment from reporting negative earnings, nor is there a large gain from reporting zero earnings. The gain comes from reporting positive earnings that are still small in magnitude but seems to be outside the bunching interval. Earnings that are large in absolute magnitude do not seem to provide adequate information to be priced by the market, which could either

Figure 4: **Cumulated Abnormal returns around zero earnings.** This figure shows a regression discontinuity plot of the cumulated abnormal returns around reported earnings. The purple dots are a scatterplot presenting the mean of the cumulated abnormal returns related to a bin of reported earnings with width 0.0005, the data-driven choice of width using to find the amount of bins which mimic the variance. The black line to either side of zero earnings is two different fourth order polynomial which provides the best fit to the relation between reported earnings and the cumulated abnormal returns. The distance between the two separately fitted lines, unobservable in this case, show the discrete gain from meeting zero earnings, whereas the in increase in curvature after zero earnings shows the change market response for each reported earnings.

be because the market perceives them to be outliers or because other factors are more relevant in pricing these firms.

From figure 4 here seems to be no obvious reason, or at least very little reason, for firms to manage earnings upwards. If there was a reason behind this behavior, we should see a gain for the firms that avoid reporting negative earnings. To test this formally, we provide the results from a regression discontinuity design in table 2, where we consider a discontinuity in the cumulated abnormal return at zero earnings, at which point the firm is rewarded the treatment “positive earnings”.

Table 2: **Change in CAR at zero earnings.**
This table reports the estimates from changes in the cumulated abnormal returns when reported earnings equals zero using a regression discontinuity design approach. We include covariates and use a local-polynomial of second-order, triangular weights and end up having 100388 observations with non-zero weights.

	β	std. errors	t-stat
Conventional	0.00265	0.00126	2.1073
Robust			1.6123

Looking at the conventional standard errors in table 2, we find the estimate of the treatment to not be significant at a 1 percent level, and focusing on the robust standard errors we are much further away. The market does not (naïvely) reward the firm for reporting zero earnings. This shows that results relating to other benchmarks, such as meeting analyst expectations does not necessarily transfer to the zero earnings setting, as Bird et al. (2019) finds a significant jump using similar methodology in the market response to meeting analyst expectations. The significant discrete change in market price when reaching the earnings benchmark is fundamental for the estimation of structural model by Bird et al. (2019), and our results suggests that their model, where it is not assumed that the market is aware of manipulation, can not be extended to this setting.

Our next step is to use a regression kink design to see if there is a change in the stock-market response to earnings around zero we get the results in table 3. Here we find that the effect is statistically significant with both normal and robust standard errors.

There is no premium for reporting zero earnings, and there seems to be no, or only very small, punishment from reporting negative earnings. However, when the firm breaks the zero earnings ceiling, the market starts rewarding positive earnings. This is consistent with the market anticipating firms who realized negative earnings reporting zero or small positive numbers. As we move away from zero, while still being in the proximity of zero, the risk of the small positive earnings coming from a firm which realized negative earnings is decreasing, and therefore the market can reward the firm that likely had actually positive earnings.

Table 3: **Change in earnings response at zero earnings.**
This table reports the estimates from changes in the cumulated abnormal returns when reported earnings equal zero using a regression kink design approach. We include covariates and use a local-polynomial of order third-order, triangular weights and end up having 144155 observations with non-zero weights.

	β	std. errors	t-stat
Conventional	0.36878	0.11334	3.2536
Robust			2.8560

5 Loss aversion and the reporting equilibrium

We will now show that the presence of loss-averse investors can explain firms' reluctance to report small losses, even if investors are aware of this behavior and thus discount reports of zero or small positive earnings. We model a signaling game where a manager privately observes the true earnings of the firm and then has discretion to report earnings which may deviate from the truth. The market prices the firm based on the reported earnings with rational expectations, except that the investors exhibit loss aversion with respect to true earnings so that earnings below their reference point carry extra weight.

The presence of loss-averse investors implies that there cannot be a separating equilibrium, where investors perfectly deduce the firms' true earnings. Instead, there is only a partially-pooling equilibrium, where managers who observe small negative earnings (mis)report zero or small positive earnings.

5.1 Model

We model how the behavior of reporting zero or small positive earnings, instead of small negative earnings, can be an equilibrium when the market anticipates this. We build on the signaling model by Guttman et al. (2006)[9], in which a firm realizes true earnings x that are drawn from a normal distribution with mean x_0 and variance σ^2. The manager issues a report of the firm's earnings x^R, which may deviate from the truth. The manager's utility function is given by:

$$U_M(x, x^R) = \alpha P\left(x^R\right) - \beta\left(x - x^R\right)^2. \tag{4}$$

The first term in the manager's utility reflects his stock-price-based compensation. $\alpha \geq 0$ denotes the power of the manager's incentives, which determines his payoff for a given stock price; $P(x^R)$ is the representative investor's pricing function. The second term reflects the manager's cost of misreporting, which is a quadratic function of the deviation from the truth, $x^R - x$, scaled by $\beta > 0$ which denotes the cost of manipulation. The disutility from misreporting could stem from, e.g., psychic stress caused by lying or the expected punishment conditional on misreporting being discovered, which in turn would be affected by the governance quality of the firm, auditor quality, etc. In line with Fischer and Verrecchia (2000), note that β can also include the direct cost of suboptimal operational decisions, i.e., real earnings management. This interpretation is important, because academics have debated whether the zero-earnings discontinuity is caused by accrual earnings management. Dechow et al. (2003), for example, do not find any evidence of discretionary accruals around the discontinuity. Whether this means that there is no accruals manipulation or that the discretionary accrual models are not able to catch it, is up for interpretation. Roychowdhury (2006), however, finds evidence of real earnings manipulation. We have assumed true earnings to be normally distributed and a simple quadratic cost of earnings manipulation for tractability.

We assume that the representative investor is both loss averse and risk averse. She prices the stock based on all available information, the risk of the firm realizing earnings below zero, and the variance of the earnings. We use a functional form for the investor's utility based on Pasquariello (2014), which combines mean-variance preferences with reference-dependent preferences.

[9]Similar to Guttman et al. (2006), we restrict our attention to pure-strategy equilibria.

The investor's utility function is piecewise linear and given by:

$$V\left(z, x^R\right) \equiv V_{MV}\left(z, x^R\right) + V_{RD}\left(z, x^R\right), \tag{5}$$

where z denotes the investor's holdings of the firm. The first and second term reflect standard mean-variance preferences and reference-dependent preferences, respectively. We assume that the investor has a reference point of zero true earnings.[10]

$$V_{MV}\left(z, x^R\right) \equiv z\left(E\left[x \mid x^R\right] - P\left(x^R\right)\right) - z^2 \frac{r}{2} var\left[x \mid x^R\right], \text{ and} \tag{6}$$

$$V_{RD}\left(z, x^R\right) \equiv zE\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right). \tag{7}$$

In the mean-variance component, r denotes the investor's risk aversion. In the reference-dependent component, the investor's loss aversion is measured by γ, such that she incurs an additional utility loss of γ per unit of true earnings below zero.[11]

The investor needs to choose her holdings of the firm, z, which maximize her utility, and her optimization problem takes the form:

$$\begin{aligned}\max_z V\left(z, x^R\right) = z\left(E\left[x \mid x^R\right] - P\left(x^R\right)\right) - z^2 \frac{r}{2} var\left[x \mid x^R\right] \\ + zE\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right). \end{aligned} \tag{8}$$

The investor's optimal demand is given by the first-order condition of equation (8) with respect to z:

$$z^* = \frac{E\left[x \mid x^R\right] - P\left(x^R\right) + E\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right)}{r \times var\left[x \mid x^R\right]}. \tag{9}$$

The investor's demand is increasing in the expectation of true earnings and decreasing in price, loss aversion, risk aversion, and earnings variance. For the market to clear, the demand for the stock must equal the supply. The supply is exogenous and normalized to be equal to one. This gives us the equilibrium pricing function:

$$\begin{aligned}P^*\left(x^R\right) = E\left[x \mid x^R\right] + E\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right) \\ - r \times var\left[x \mid x^R\right]. \end{aligned} \tag{10}$$

[10] We also considered a reference point based on reported earnings, which leads to qualitatively similar results.

[11] For further reading on reference dependence and loss aversion, refer to Tversky and Kahneman (1992).

Inserting the price function from equation (10) into the manager's utility yields

$$U_M(x, x^R) = \alpha \left(E\left[x \mid x^R\right] + E\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right) \right. \\ \left. -r \times var\left[x \mid x^R\right]\right) - \beta\left(x - x^R\right)^2. \tag{11}$$

The manager faces a dilemma. On the one hand, he can manipulate the earnings report and thus affect his price-contingent compensation. On the other hand, he incurs a cost by doing so, and the marginal cost of manipulation is increasing in the extent of manipulation. The ratio $\frac{\alpha}{\beta}$ determines the weight of these incentives. The higher the ratio, the more inclined the manager is to manipulate the report. Furthermore, the manager knows that the loss-averse investor will punish him unduly for having true earnings below zero, so for earnings below zero, the relevant ratio is given by $\frac{\alpha(1+\gamma)}{\beta}$. This means that, *ceteribus paribus*, the manager will misreport more if the firm's realized earnings are below zero, as the manager's power of incentives increases from α to $(1+\gamma)\,\alpha$, when we think of them as a function of earnings.

5.2 The separating equilibrium

We now characterize the Perfect Bayesian Equilibria in this game. A reporting strategy for the manager is given by $\rho : \mathbb{R} \to \mathbb{R}$, which maps true earnings onto reports: $x^R = \rho(x)$. The pricing function $P : \mathbb{R} \to \mathbb{R}$ maps the manager's report into a market price. In a Perfect Bayesian Equilibrium, we need a reporting strategy ρ^* and a pricing function P^* such that:

1. The pricing function, P^*, is consistent with the reporting strategy, ρ^*, by applying Bayes' rule whenever possible, and

2. $\rho^*(x) \in \arg\max_{x^R} U_M(x, x^R)\ \forall x \in \mathbb{R}$.

The investor's reference point of zero true earnings leads to a piece-wise linear pricing function. We conjecture that this leads to two separate reporting functions, $\rho_g^*(x)$ and $\rho_l^*(x)$, one on either side of the investor's reference point of zero earnings.

We begin by finding a Perfect Bayesian Equilibrium for firms with realized earnings below zero and then one for firms with realized earnings greater than or equal to zero. We then paste these reporting functions together, and check whether this leads to a perfect Bayesian equilibrium which holds for all types of firms.

First, note that the manager having a truthful reporting function $\rho(x) = x \; \forall x$ cannot be an equilibrium. To see this, consider that the investor will adjust her belief and price the firm for a given report as $P(x^R) = x^R \; \forall \; x \geq 0$ and $P(x^R) = (1+\gamma)\, x^R$ for $x < 0$. If the manager reports truthfully, he gets a profit of αx for $x \geq 0$ and a $(1+\gamma)\,\alpha x$ for $x < 0$. Now, if the manager inflates his report by ϵ, he obtains $\alpha\,(x+\epsilon) - \beta\epsilon^2$ for $x \geq 0$ and $\alpha\,(1+\gamma)\,(x+\epsilon) - \beta\epsilon^2$ for $x < 0$. This means that there is a small enough deviation from the realized earnings which is beneficial for the manager: as long as $0 < \epsilon < \min\left\{\frac{\alpha}{\beta^2}, \frac{(1+\gamma)\alpha}{\beta^2}\right\}$, the manager is strictly better off reporting $x+\epsilon$ rather than x.

Similar to Guttman et al. (2006) and Li (2014), we begin our analysis by focusing on the conventional perfectly-separating equilibrium. We now state Lemma 1, which is similar to Proposition 1 in Guttman et al. (2006), with the caveat that we are splitting the universe of possible true earnings in two.

Lemma 1. *When restricting true earnings to $x \geq 0$, the manager's reporting function in a perfectly-separating equilibrium would be*

$$\rho^*_{gs}(x) = x + \frac{\alpha}{2\beta} \; \forall x \geq 0$$

and the market's pricing function given by

$$P^*_{gs}(x^R) = \; x^R - \tfrac{\alpha}{2\beta} \quad \textit{if } x \geq 0.$$

When restricting true earnings to $x < 0$, the manager's reporting function in a perfectly-separating equilibrium would be

$$\rho^*_{ls}(x) = x + \frac{(1+\gamma)\alpha}{2\beta} \; \forall x < 0$$

and

$$P^*_{gs}(x^R) = \; (1+\gamma)\,(x^R - \tfrac{(1+\gamma)\alpha}{2\beta}) \quad \textit{if } x < 0.$$

All proofs are relegated to the Appendix.

We have found the optimal reporting functions for a manager with true earnings larger or equal to zero, $x \geq 0$, and for managers with true earnings below zero, $x < 0$. We now need to paste these reporting functions together and see if this satisfies the conditions of a perfectly-separating equilibrium.

In the case of no loss aversion, $\gamma = 0$, we end up with the perfectly-separating equilibrium characterized by Guttman et al. (2006) in their Proposition 1. The presence of loss aversion breaks this equilibrium when earnings are close to the reference point. Consider a manager who realizes negative earnings, $x < 0$. He will suffer from the investor's reference dependence affecting the market price, and thus the manager will report $x^R = x + \frac{(1+\gamma)\alpha}{2\beta}$ $\forall x < 0$, biasing the report by a larger extent than if he had realized non-negative earnings. This means that for $\gamma > 0$, there will be some $x_+ > 0$ and $x_- < 0$ for which $x_+^R \leq x_-^R$, which implies that there cannot be a perfectly-separating equilibrium.

Proposition 1. *No perfectly-separating equilibrium exists when the investor is loss averse (i.e. $\gamma > 0$).*

The investor's reference dependence breaks the monotonicity between the manager's realized earnings and the following report when the realized earnings are in close proximity to the reference point. As the the loss aversion coefficient increases, the manager will misreport more when observing negative earnings. This result is similar to Proposition 1 stated by Li (2014). However, that result comes from an assumption regarding how earnings are correlated between periods, while in our case it stems from investors' loss aversion.

5.3 The partially-pooling equilibrium

We now conjecture an equilibrium in which it is optimal for the different types of managers to pool in a certain interval of realized earnings, $[a, b]$, which includes the investor's reference point. We assume that $a < 0$ and $b \geq 0$. The interpretation of this interval is that it will be optimal for managers who realize small negative earnings to manipulate earnings in order to avoid the (pricing) punishment for not reaching the investor's reference point. This yields the following reporting strategy:

$$\rho_p^*(x) = \begin{cases} b & \text{if } x \in [a, b]; \\ x + \frac{\alpha}{2\beta} & \text{if } x > b, \\ x + \frac{(1+\gamma)\alpha}{2\beta} & \text{if } x < a. \end{cases}$$

This is a modification of the separating equilibrium. Outside of the conjectured interval which includes the investor's reference point, the manager has reporting functions as in Proposition 1, so he signals the firms' true earnings.

Managers with earnings that fall inside the interval pool together and report the same earnings $x^R = b$. Similar to Guttman et al. (2006), the pricing function sees through manipulation outside of the pooling interval and exhibits rational expectations following the pooling report of b:[12]

$$P_p^*(x^R) = \begin{cases} (1+\gamma)\left(x^R - \frac{(1+\gamma)\alpha}{2\beta}\right) & \text{if } x^R < a + \frac{(1+\gamma)\alpha}{2\beta}, \\ x^R - \frac{\alpha}{2\beta} & \text{if } x^R > b + \frac{\alpha}{2\beta}, \\ E\left[x \mid x \in [a,b]\right] & \\ +\gamma E\left[x \mid x \in [a,0)\right] & \\ \times \Pr\left[x < 0 \mid x \in [a,b]\right] & \\ -r \times var\left[x \mid x \in [a,b]\right] & \text{if } x^R = b, \\ (1+\gamma)\, a & \text{if } x^R \in \left[a + \frac{(1+\gamma)\alpha}{2\beta}, b\right) \cup \left(b, b + \frac{\alpha}{2\beta}\right]. \end{cases}$$

A manager of type a must be indifferent between reporting $a + \frac{(1+\gamma)\alpha}{2\beta}$ and b, and a manager of type b must be indifferent between reporting $b + \frac{\alpha}{2\beta}$ and b. When we analyzed the empirical distribution of reported earnings in section 3, pooling seemed to take place at zero, or very close to zero. To apply our model to this setting, we start by letting $b = 0$. Recall the manager's utility function, from equation (11):

$$\begin{aligned} U_M\left(x, x^R\right) = \alpha \Big(& E\left[x \mid x^R\right] + E\left[\gamma x \mid x < 0, x^R\right] \times \Pr\left(x < 0 \mid x^R\right) \\ & -r \times var\left[x \mid x^R\right]\Big) - \beta\left(x - x^R\right)^2. \end{aligned}$$

Together with the equilibrium reporting strategy $\rho_p^*(x)$ and pricing function $P_p^*(x^R)$, the requirement that the manager must be indifferent between pooling and separating after observing earnings a or b leads to the following equation system:

[12] Just as in Guttman et al. (2006), there are some reports (close to b) that do not occur in equilibrium. Without loss aversion, their assumption that a "mistaken" report within the pooling interval that deviates from b comes from a manager playing the separating equilibrium is enough to sustain the pooling equilibrium. With loss aversion, however, we run into the same problem that broke the separating equilibrium: types to either side of the reference point would give the same report in a "separating" equilibrium, so we cannot use that to pin-point the off-equilibrium beliefs. Given the inability to place a given off-equilibrium report as above or below the reference point, we will assume that the investor is rather pessimistic when seeing a "mistaken" report, and believes it to come from the lowest type in the pooling range.

$$
\begin{aligned}
\alpha\left(a+\gamma a\right)-\beta\left(\frac{(1+\gamma)\alpha}{2\beta}\right)^2 &= \alpha\left(E\left[x \mid x\in[a,0]\right]+\gamma E\left[x \mid x\in[a,0)\right]\right. \\
&\quad \times \Pr\left(x<0 \mid x\in[a,0)\right) \\
&\quad \left.-r\times var\left[x \mid x\in[a,0)\right]\right)-\beta\left(-a\right)^2. \qquad (12) \\
-\beta\left(\frac{\alpha}{2\beta}\right)^2 &= \alpha\left(E\left[x \mid x\in[a,0]\right]+\gamma E\left[x \mid x\in[a,0)\right]\right. \\
&\quad \times \Pr\left(x<0 \mid x\in[a,0)\right) \\
&\quad \left.-r\times var\left[x \mid x\in[a,0)\right]\right) \qquad (13)
\end{aligned}
$$

Solving these two equations for the lower bound of the interval, a, and the expected value for a given report of zero earnings, $[x \mid x\in[a,0]]$, and dropping the positive solution, yields

$$
a_0^*=-\frac{\alpha}{2\beta}\left(1+\gamma+\sqrt{2\gamma^2+4\gamma+1}\right) \qquad (14)
$$

and

$$
E\left[x \mid x\in[a,0]\right]=\frac{-\frac{\alpha}{4\beta}+r\times var\left[x \mid x\in[a,0]\right]}{\left(1+\gamma\times\Pr\left(x<0 \mid x\in[a,0)\right)\right)} \qquad (15)
$$

From these two equations, we see that the size of the pooling interval is increasing in the investor's loss-aversion coefficient. We also see that the expected realized earnings are increasing in α and decreasing in β. The expected realized earnings are also increasing in the market's risk aversion and the variance of earnings.

Inserting the expected value of the realized earnings given a report of zero earnings into the price gives:

$$
P_p^*(0)=-\frac{\alpha}{4\beta}.
$$

Combining the price the investor pays for a report of zero earnings with the pricing function for realized earnings below zero and the pricing function for realized earnings greater than or equal to zero, yields the pricing function for the pooling equilibrium:

$$P_p^*(x^R) = \begin{cases} (1+\gamma)\left(x^R - \frac{(1+\gamma)\alpha}{2\beta}\right) & \text{if } x^R < a_0^* + \frac{(1+\gamma)\alpha}{2\beta}; \\ x^R - \frac{\alpha}{2\beta} & \text{if } x^R \geq \frac{\alpha}{2\beta}; \\ -\frac{\alpha}{4\beta} & \text{if } x^R = 0; \\ (1+\gamma)\, a_0^* & \text{if } x^R \in \left[a_0^* + \frac{(1+\gamma)\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right]. \\ \end{cases}$$

We can characterize a partially-pooling equilibrium, along the lines of Proposition 2 in Guttman et al. (2006)[13].

Proposition 2. *For a $\gamma > 0$ the reporting strategy*

$$\rho_p^*(x) = \begin{cases} 0 & \text{if } x \in [a_0^*, 0]; \\ x + \frac{\alpha}{2\beta} & \text{if } x > 0\ ; \\ x + \frac{(1+\gamma)\alpha}{2\beta} & \text{if } x < a_0^*. \end{cases} \tag{16}$$

combined with the pricing function

$$P_p^*(x^R) = \begin{cases} (1+\gamma)\left(x^R - \frac{(1+\gamma)\alpha}{2\beta}\right) & \text{if } x^R < a_0^* + \frac{(1+\gamma)\alpha}{2\beta}; \\ x^R - \frac{\alpha}{2\beta} & \text{if } x^R \geq \frac{\alpha}{2\beta}; \\ -\frac{\alpha}{4\beta} & \text{if } x^R = 0; \\ (1+\gamma)\, a_0^* & \text{if } x^R \in \left[a_0^* + \frac{(1+\gamma)\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right]. \\ \end{cases} \tag{17}$$

constitutes a Perfect Bayesian equilibrium.

We now allow the managers to pool at $b = \epsilon$ above zero ($\epsilon > 0$). This yields the following equations necessary for indifference at the extermes of

[13] Proposition 2 in Li (2014) yields a similar results for sufficiently strong correlation of earnings between the two periods.

the pooling interval:

$$
\begin{aligned}
\alpha\left(a+\gamma a\right)-\beta\left(\frac{\left(1+\gamma\right)\alpha}{2\beta}\right)^{2} &= \alpha\left(E\left[x \mid x\in[a,\epsilon]\right]+\gamma E\left[x \mid x\in[a,0)\right]\right. \\
&\quad \times \Pr\left(x<0 \mid x\in[a,\epsilon)\right) \\
&\quad \left.-r\times var\left[x \mid x\in[a,\epsilon)\right]\right)-\beta\left(\epsilon-a\right)^{2}. \qquad (18) \\
\alpha\left(\epsilon\right)-\beta\left(\frac{\alpha}{2\beta}\right)^{2} &= \alpha\left(E\left[x \mid x\in[a,\epsilon]\right]+\gamma E\left[x \mid x\in[a,0)\right]\right. \\
&\quad \times \Pr\left(x<0 \mid x\in[a,\epsilon)\right) \\
&\quad \left.-r\times var\left[x \mid x\in[a,\epsilon)\right]\right) \qquad (19)
\end{aligned}
$$

Solving these two equations for a and the expected value given a report of ϵ, again dropping the positive solution,[14] gives:

$$
a_{\epsilon}^{*}=\epsilon-\frac{\alpha\left(1+\gamma\right)+\sqrt{\alpha}\,\sqrt{\alpha(1+4\gamma+2\gamma^{2})-4\,\beta\,\epsilon\,\gamma}}{2\,\beta}
$$

and

$$
\begin{aligned}
E\left[x \mid x\in[a,\epsilon]\right] &= \epsilon-\gamma E\left[x \mid x\in[a,0)\right]\times\Pr\left(x<0 \mid x\in[a,\epsilon)\right) \\
&\quad +r\times var\left[x \mid x\in[a,\epsilon]\right]-\frac{\alpha}{4\,\beta} \qquad (20)
\end{aligned}
$$

When ϵ increases, the upper bound of the pooling interval increases and since the cost of manipulation is increasing in the deviation required, the lower-bound of the pooling interval, a, will also increase. As expected, we find that the expected economic earnings are increasing in ϵ. Inserting the expected value of earnings given a report of ϵ into the pricing function gives

$$
P_{p\epsilon}^{*}(\epsilon)=\epsilon-\frac{\alpha}{4\,\beta},
$$

where we see that the price increases in ϵ as well. We then write the investor's

[14] Recall that there cannot be an equilibrium that separates types around the reference point of zero. So ϵ must be close enough to zero such that one of the solutions leads to $a<0$.

pricing function when ϵ is the upper bound of the pooling interval as

$$P^*_{p\epsilon}(x^R) = \begin{cases} (1+\gamma) \times & \\ \left(x^R - \frac{(1+\gamma)\alpha}{2\beta}\right) & \text{if } x^R < a^*_\epsilon + \frac{(1+\gamma)\alpha}{2\beta}; \\ x^R - \frac{\alpha}{2\beta} & \text{if } x^R > \frac{\alpha}{2\beta} + \epsilon; \\ \epsilon - \frac{\alpha}{4\beta} & \text{if } x^R = \epsilon; \\ (1+\gamma)\, a^*_\epsilon & \text{if } x^R \in \left[a^*_\epsilon + \frac{(1+\gamma)\alpha}{2\beta}, \epsilon\right) \cup \left(\epsilon, \frac{\alpha}{2\beta} + \epsilon\right]. \end{cases}$$

From our model, we see that the investor's loss aversion leads to a discontinuity at the investor's reference point. This is similar to the phenomena observed empirically around reported earnings of zero. In our model the discontinuity comes from a partially-pooling equilibrium where it is optimal for managers who realized earnings below, or just above, the investor's reference point to pool together and report the same earnings number. In the direct application of this model, the reported earnings number would be zero or small positive earnings. The size of the pooling interval is increasing in the degree of loss aversion, and when the loss-aversion coefficient approaches zero, $\gamma \to 0$, the model converges to that of Guttman et al. (2006) where both a separating and partially-pooling equilibra exist. Our model is not restricted to analyze the zero-earnings discontinuity, but can also explain discontinuities around other possible reference points, such as reaching expected earnings, last year's earnings or having non-decreasing dividends.

6 Estimating the loss-aversion coefficient

Our empirical results indicate that the capital market is aware that firms avoid to report small negative earnings, and that there is no obvious gain of doing so for the firm. We then showed that this is rational behavior for the manager if the investors are loss averse. Assuming that our model is correct, what level of loss-aversion, γ, would best fit the estimated earnings manipulation?

We use structural estimation in order to estimate the parameters of the model. To achieve this we use simulated method of moments. The approach is similar to what is applied by others in the foray into structural estimation in the accounting literature (Bird et al. (2019); Beyer et al. (2019) and Zakolyukina (2018)). We simulate the reported earnings and the market response from the model and then match the moments of the simulated data with the actual data. The objective is to choose the parameters of the model, θ, to minimize the distance between the moments of the model-generated data, $\tilde{M}$, and the observed data moments, $\hat{M}$. This yields the objective function:

$$\min_{\theta} \ \tilde{M} - \hat{M}$$

We need to determine the mean and the variance of the (assumed normal) distribution of true earnings, x_0 and σ^2, the incentives of the manager, α, the cost of manipulation, β, and the investors' loss-aversion coefficient, γ.

To begin, we set the mean and variance of the true earnings equal to the mean and the variance of the observed reported earnings. We are assuming that the distribution of true earnings is very close to the distribution of the reported earnings, $x_0 = 0.0186$ and $\sigma^2 = 0.0875$, but that the reported earnings are a distortion of this distribution, close to zero, due to earnings manipulation. Accruals earnings management shifts accruals across periods to sculpt cash flows into the depicted earnings, but since accruals must reverse, reported earnings cannot deviate too much from the true earnings. Real earnings management can be quite costly, so we also expect that to not have a significant effect on the overall distribution of reported earnings.

When then use the equilibrium condition from (5.3):

$$P_p^*(0) = -\frac{\alpha}{4\beta}$$

to non-parametrically estimate the ratio between the manager's incentives and the cost of manipulation as the market's pricing of reported earnings of zero. To do this we identify reports which are close, but greater or equal, to zero, and then take the average value of the 25%, 50% and 75% percentile of the pricing of these reports.[15] The only variable left to estimate is the loss-

[15] We use the average of the percentiles to avoid biased estimates due the variance of prices to reports of zero earnings. The estimate is qualitatively robust to choices of different percentiles or using the mean of the prices to reports of zero earnings. In addition this we also used a parametric approach and estimated an OLS regression of the pricing of a report of zero on a constant, and found an estimate of $\frac{\alpha}{2\beta}$ of 0.0062 with a standard error of 0.0018, which is significant ($p < 0.01$). It is only the ratio $\frac{\alpha}{2\beta}$ that needs to be identified, not α and β individually as they only show up in the equilibrium conditions as a fraction. Empirically, α and β are specific to each firm-manager pair, and might be identifiable using the option and stock holdings of CEOs, α, and audit quality of the firm, β. However, this is outside the scope of this paper and Bird et al. (2019) finds that heterogeneity in the gain of manipulation, α in our setting, is not an important of explaining the discontinuity around meeting the markets' expectations.

aversion coefficient γ, which we do with the simulated method of moments estimator.

We start by simulating a data set constituted of reported earnings, x^R, and the stock price related to those earnings, $P(x^R)$, equal in size to the empirical data, using the calibrated values of x_0 and σ^2, the non-parametric estimate of $\frac{\alpha}{2\beta}$, a starting parameter of γ and the relations in Proposition 2. We simulate 1000 versions of the simulated data sets, so that we are able to reuse our actual data 1000 times.

From equation (14) we see that the pooling interval is increasing in γ, so we want to match the size of the discontinuity observed and the discontinuity from our model to identify the loss-aversion coefficient. Therefore, we calculate the difference between reported earnings just below zero $Obs^{x^R}_{[-0.015,0.000)}$ and reported earnings from zero to small positive earnings $Obs^{x^R}_{[0,0.015]}$, and then scale this difference by the number of reported earnings around zero $Obs^{x^R}_{[-0.015,0.015]}$:

$$M = \frac{Obs^{x^R}_{[-0.015,-0.000)} - Obs^{x^R}_{[0,0.015]}}{Obs^{x^R}_{[-0.015,0.015]}}.$$

The parameters γ, x_0, σ^2 and $\frac{\alpha}{2\beta}$, data moments and model moments are presented in Table 4.

Table 4: **The loss-aversion coefficient** This table reports the estimates of the parameters of the structural model using simulated method of moments. The main parameter of interest is the loss-aversion coefficient, γ.

Parameter	Estimate	Standard errors
x_0	0.0186	
σ^2	0.0077	
$\frac{\alpha}{2\beta}$	0.0054	
γ	1.2595	(0.0144)
Moment	Data moment	Model moment
$\frac{Obs^{x^R}_{[-0.015,-0.000)} - Obs^{x^R}_{[0,0.015]}}{Obs^{x^R}_{[-0.015,0.015]}}$	-0.2835	-0.2835
Number of observations	189598	

Our parameter of interest is γ where we find a coefficient of 1.2595,[16] so

[16]This estimate is found for the global minimum of the objective function, where the

investors are being hurt roughly twice as much by a loss compared to a like-sized gain. This estimate of the loss-aversion coefficient is consistent with the estimates of loss aversion found by Tversky and Kahneman (1992). We find the expected bias in reported earnings above the bunching interval as 0.0054, which is 0.70 of the standard deviation of reported earnings and the expected bias below the bunching interval as $\frac{(1+\gamma)\alpha}{2\beta} = 0.0122$. The interpretation of this is, that a loss-aversion coefficient of 1.2595 causes the bias for firms with true negative earnings to be 2.2595 times larger than for firms with true earnings which are greater or equal to zero.

The model prediction fits the results presented in Section 3, that managers are preferring to avoid reporting negative earnings, those in Section 4, that the market seems to be aware of earnings management, and yields an estimate for the loss-aversion coefficient which is in line with prior estimates in the literature. This makes us optimistic that reference-dependent preferences from the side of investors can help explain earnings management around possible reference points, zero earnings in particular, even though the market "knows" that such manipulation is taking place.

7 Conclusion

In this paper we document the insufficiency in the mass of firms reporting small negative earnings and the excess mass of firms reporting zero or small positive earnings. We find evidence that the market anticipates that firms with small negative earnings report zero or small positive earnings. The market does not reward firms for reporting zero earnings, but rewards firms with reported earnings outside of the bunching interval. We also find that the market discounts information from earnings when firms report zero or small positive earnings, compared to when firms provide slightly higher earnings.

As an explanation to why firms still perform earnings management even though investors correctly anticipate this, we consider the effects of investors' loss aversion in a model of earnings reporting. We show that if the firms are aware of the investors' loss aversion, the only equilibrium that survives is a partially-pooling equilibrium where firms with small negative earnings will report zero or small positive earnings, while without loss aversion there exists a perfectly-separating equilibrium. Using simulated method of moments we

deviation between the data and model moment only differs from zero on the eight decimal, and is therefore robust to changes in starting values.

find that the loss-aversion coefficient which provides the best fit between simulated data from the model and the observed data is 1.2595. This is in line with the loss-aversion coefficient estimated in the literature.

While we only focus on the threshold of zero earnings in this paper, our model can be generalized to other thresholds such as meeting or beating expectations and last year's earnings, and can be estimated to find the loss-aversion coefficient consistent with that behavior and the reporting bias it causes below and above the reference point. To get an improved estimate of the loss-aversion coefficient, the model could be extended to be a closer fit to empirically observed stylized facts. One could make the model dynamic, allow for heterogeneous incentive pay and cost of manipulation, as well as to include the extensions first seen in Fischer and Verrecchia (2000) such as making the manager's objective function unobservable for the market and including noise in the manager's observation of the firm's true earnings. There are promising avenues for further research on a topic where the literature has not reached a consensus.

References

Allen, E.J., Dechow, P.M., Pope, D.G., Wu, G., 2016. Reference-dependent preferences: Evidence from marathon runners. Management Science 63, 1657–1672.

Baker, M., Mendel, B., Wurgler, J., 2015. Dividends as reference points: A behavioral signaling approach. The Review of Financial Studies 29, 697–738.

Beaver, W.H., McNichols, M.F., Nelson, K.K., 2007. An alternative interpretation of the discontinuity in earnings distributions. Review of Accounting Studies 12, 525–556.

Beyer, A., Guttman, I., Marinovic, I., 2019. Earnings management and earnings quality: Theory and evidence. The Accounting Review, forthcoming .

Bird, A., Karolyi, S.A., Ruchti, T.G., 2019. Understanding the numbers game. Journal of accounting and economics, forthcoming .

Brown, L.D., Caylor, M.L., 2005. A temporal analysis of quarterly earnings thresholds: Propensities and valuation consequences. The Accounting Review 80, 423–440.

Burgstahler, D., Chuk, E., 2015. Do scaling and selection explain earnings discontinuities? Journal of Accounting and Economics 60, 168–186.

Burgstahler, D., Chuk, E., 2017. What have we learned about earnings management? integrating discontinuity evidence. Contemporary Accounting Research 34, 726–749.

Burgstahler, D., Dichev, I., 1997. Earnings management to avoid earnings decreases and losses. Journal of accounting and economics 24, 99–126.

Calonico, S., Cattaneo, M.D., Farrell, M.H., Titiunik, R., 2018. Regression discontinuity designs using covariates. Review of Economics and Statistics .

Calonico, S., Cattaneo, M.D., Titiunik, R., 2014. Robust nonparametric confidence intervals for regression-discontinuity designs. Econometrica 82, 2295–2326.

Chetty, R., Friedman, J.N., Olsen, T., Pistaferri, L., 2011. Adjustment costs, firm responses, and micro vs. macro labor supply elasticities: Evidence from danish tax records. The quarterly journal of economics 126, 749–804.

Dechow, P., Ge, W., Schrand, C., 2010. Understanding earnings quality: A review of the proxies, their determinants and their consequences. Journal of accounting and economics 50, 344–401.

Dechow, P.M., Richardson, S.A., Tuna, I., 2003. Why are earnings kinky? an examination of the earnings management explanation. Review of accounting studies 8, 355–384.

DeFond, M.L., Park, C.W., 2001. The reversal of abnormal accruals and the market valuation of earnings surprises. The accounting review 76, 375–404.

Degeorge, F., Patel, J., Zeckhauser, R., 1999. Earnings management to exceed thresholds. The Journal of Business 72, 1–33.

Dichev, I.D., Graham, J.R., Harvey, C.R., Rajgopal, S., 2013. Earnings quality: Evidence from the field. Journal of Accounting and Economics 56, 1–33.

DiNardo, J., Lee, D.S., 2004. Economic impacts of new unionization on private sector employers: 1984–2001. The Quarterly Journal of Economics 119, 1383–1441.

Durtschi, C., Easton, P., 2005. Earnings management? the shapes of the frequency distributions of earnings metrics are not evidence ipso facto. Journal of Accounting Research 43, 557–592.

Easton, P.D., Zmijewski, M.E., 1989. Cross-sectional variation in the stock market response to accounting earnings announcements. Journal of Accounting and economics 11, 117–141.

Fischer, P.E., Verrecchia, R.E., 2000. Reporting bias. The Accounting Review 75, 229–245.

Guttman, I., Kadan, O., Kandel, E., 2006. A rational expectations theory of kinks in financial reporting. The Accounting Review 81, 811–848.

Hemmer, T., Labro, E., 2019. Management by the numbers: A formal approach to deriving informational and distributional properties of unmanaged earnings. Journal of Accounting Research 57, 5–51.

Imbens, G.W., Lemieux, T., 2008. Regression discontinuity designs: A guide to practice. Journal of econometrics 142, 615–635.

Jia, Y., 2013. Meeting or missing earnings benchmarks: The role of ceo integrity. Journal of Business Finance & Accounting 40, 373–398.

Kasznik, R., McNichols, M.F., 2002. Does meeting earnings expectations matter? evidence from analyst forecast revisions and share prices. Journal of Accounting research 40, 727–759.

Keung, E., Lin, Z.X., Shih, M., 2010. Does the stock market see a zero or small positive earnings surprise as a red flag? Journal of Accounting Research 48, 91–121.

Van der Klaauw, W., 2002. Estimating the effect of financial aid offers on college enrollment: A regression–discontinuity approach. International Economic Review 43, 1249–1287.

Kothari, S.P., Shu, S., Wysocki, P.D., 2009. Do managers withhold bad news? Journal of Accounting Research 47, 241–276.

Kross, W.J., Ro, B.T., Suk, I., 2011. Consistency in meeting or beating earnings expectations and management earnings forecasts. Journal of Accounting and Economics 51, 37–57.

Li, W., 2014. A theory on the discontinuity in earnings distributions. Contemporary Accounting Research 31, 469–497.

McCrary, J., 2008. Manipulation of the running variable in the regression discontinuity design: A density test. Journal of econometrics 142, 698–714.

Pasquariello, P., 2014. Prospect theory and market quality. Journal of Economic Theory 149, 276–310.

Roychowdhury, S., 2006. Earnings management through real activities manipulation. Journal of accounting and economics 42, 335–370.

Skinner, D.J., Sloan, R.G., 2002. Earnings surprises, growth expectations, and stock returns or don't let an earnings torpedo sink your portfolio. Review of Accounting Studies 7, 289–312.

Tversky, A., Kahneman, D., 1992. Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and uncertainty 5, 297–323.

Zakolyukina, A.A., 2018. How common are intentional gaap violations? estimates from a dynamic model. Journal of Accounting Research 56, 5–44.

Appendix

Test of market naïveté in decomposed time-intervals

In table 5 we disaggregate the analysis of market naïveté into six different time periods. This is both meant as a robustness test and also to analyze the change in the difference between the earnings response coefficient in interval I_2 versus I_1. throughout time. First, note that the the coefficient on the earnings response coefficient in the interval I_2 is larger than the coefficient in I_1 for all time periods except for $[19911, 19963]$. Second, as indicated by the results in 4, the difference of the size of the coefficients between the intervals I_1 and I_2 is much larger from the second quarter of 2002 and onwards. This is especially pronounced in the time period $[20141, 20181]$. These results indicates that the market is becoming increasingly suspicious with regards to zero or small positive earnings.

Proof of Lemma 1. The proof is analogous to the proof of Proposition 1 in Guttman et al. (2006). Let ρ_s be a perfectly separable, continuously differentiable reporting strategy. As it is perfectly separable, it can be inverted $\phi_s = \rho_s^{-1}$. The pricing function consistent with ρ_s is $P_s = \rho_s$ for x. Since the equilibrium is perfectly separating $\gamma E[x \mid x < 0, x^R] Pr[x < 0 \mid x^R]$ reduces to $\gamma E[x \mid x < 0]\mathbb{1}[x < 0 \mid x^R]$ where $\mathbb{1}$ is an indicator variable which is equal to 1 if the realized earnings of the firm is below zero. For a manager with realized earnings $x \geq 0$ the utility is given by

$$U^M(x, x^R) = \alpha\phi_g(x^R) - \beta(x^R - x)^2 \tag{A.1}$$

The first-order condition with regard to the manager's report is

$$\frac{d}{dx^R}\phi_g(x^R) - \frac{2\beta}{\alpha}x^R + \frac{2\beta}{\alpha}x = 0 \tag{A.2}$$

In equilibrium $\phi_g(x^R) = x$, this leads to the following linear first-order differential equation.

$$\frac{d}{dx^R}\phi_g(x^R) = -\frac{d}{dx^R}\phi_g(x^R) + \frac{2\beta}{\alpha}x^R \tag{A.3}$$

All potentials of this solution are given by

$$\phi_g(x^R) = x^R - \frac{\alpha}{2\beta} + Ke^{\frac{2x^R\beta}{\alpha}} \tag{A.4}$$

Table 5: **Earnings Response Coefficients around the zero earnings discontinuity in separated time intervals.**

This table reports the estimates from $CAR_{-1,1} = \gamma_0 + \gamma_1 RunUp + \sum_{i=1}^{14} \beta_i \times I_i \times SUE_i + \epsilon$ where $CAR_{-1,1}$ is the cumulated abnormal returns, SUE is the the earnings surprise, I_i is 14 indicator dummies which depend on the intervals of earnings the firm reports, and $RunUp$ is abnormal return prior to the event which controls for information leakage. The regression is run on all publicly traded firms in 6 intervals from the second quarter in 1985 to the first quarter in 2018.

	[19852 19904]		[19911 19963]		[19964 20021]		[20022 20081]		[20082 20134]		[20141 20181]	
	Beta	t-stat	Beta	t-stat	Beta	t-stat	Beta	t-stat	Beta	t-stat	Beta	t-stat
Constant	0.001	2.754	0.002	6.236	0.004	9.747	0.002	3.934	0.001	1.178	0.000	0.133
Run-up	-0.020	-5.950	-0.035	12.024	-0.031	14.457	-0.014	-3.995	-0.004	-1.033	0.007	1.461
$I_{-7} \times SUE$	0.080	4.621	0.130	6.579	0.178	11.421	0.155	7.801	0.186	8.924	0.219	8.277
$I_{-6} \times SUE$	0.156	3.348	0.215	4.899	0.167	3.686	0.220	4.044	0.284	5.026	0.432	4.426
$I_{-5} \times SUE$	0.065	1.746	0.197	4.935	0.225	5.948	0.259	4.399	0.423	7.869	0.455	5.452
$I_{-4} \times SUE$	0.140	2.523	0.272	4.675	0.033	0.533	0.362	5.640	0.395	5.962	1.049	5.682
$I_{-3} \times SUE$	0.147	3.201	0.352	6.034	0.161	3.219	0.320	4.939	0.270	4.534	0.697	5.447
$I_{-2} \times SUE$	0.060	1.132	0.318	4.936	0.338	5.910	0.381	6.090	0.238	3.924	0.407	3.408
$I_{-1} \times SUE$	0.325	5.594	0.249	4.014	0.204	3.118	0.213	3.428	0.497	6.432	0.507	3.272
$I_1 \times SUE$	0.117	3.735	0.405	7.248	0.282	5.183	0.454	6.750	0.442	6.510	0.630	5.204
$I_2 \times SUE$	0.210	4.244	0.377	6.343	0.259	4.117	0.776	8.178	0.725	7.469	2.140	8.940
$I_3 \times SUE$	0.097	1.572	0.472	6.842	0.259	3.616	0.679	5.766	1.629	12.084	0.975	5.949
$I_4 \times SUE$	0.053	0.886	0.622	6.689	0.472	4.859	0.816	7.069	0.971	8.467	1.258	5.560
$I_5 \times SUE$	-0.036	-0.795	0.225	3.770	0.450	5.576	0.453	4.754	0.823	9.309	1.328	6.206
$I_6 \times SUE$	0.034	0.459	0.359	4.852	0.124	1.243	0.365	3.675	0.541	4.506	1.814	6.256
$I_7 \times SUE$	0.135	2.427	0.522	4.774	0.171	2.32	0.222	4.369	0.287	3.832	0.472	4.1309
$adjR^2$	0.006		0.014		0.011		0.012		0.025		0.023	
N	23568		36315		45427		36901		27752		19635	

where K is a constant, and we claim it to be 0. Suppose otherwise. First assume that $K > 0$, then as $\phi_g(x^R)$ is strictly convex and has a unique minimum at $x^R = -\frac{\alpha}{2\beta} ln \frac{\alpha}{2k\beta}$, which means that $\phi_g(x^R)$ has a lower bound, but we have defined x so that it can take on any value on the real line, which leads to a contradiction. Similar K cannot be below zero.

For a manager with $x < 0$

$$U^M(x, x^R) = \phi_l(x^R) - \beta(x^R - x)^2 \tag{A.5}$$

The first-order condition with regard to the manager's report is

$$\frac{d}{dx^R}\phi_l(x^R) - \frac{2\beta}{(1+\gamma)\alpha}x^R + \frac{2\beta}{\alpha}x = 0 \tag{A.6}$$

In equilibrium $\phi_s(x^R) = x$, this leads to the following linear first-order differential equation.

$$\frac{d}{dx^R}\phi_l(x^R) = -\frac{d}{dx^R}\phi_l(x^R) + \frac{2\beta}{(1+\gamma)\alpha}x^R \tag{A.7}$$

All potential solutions of this are given by

$$\phi_l(x^R) = x^R - \frac{(1+\gamma)\alpha}{2\beta} + Ke^{\frac{2x^R\beta}{(1+\gamma)\alpha}} \tag{A.8}$$

where K is a constant, and we claim it to be zero with the same argument as above. □

Proof of Proposition 1. For this to be a Perfect Bayesian equilibrium, we need $x^R_{x<0} < x^R_{x>0}$ for all x. Let us assume that it holds. Then for all $\gamma > 0$, $x_l < x_g$ and for some ϵ such that

$$\mid x_{x>0} - x_{x<0} \mid < \epsilon \tag{A.9}$$

this implies

$$x_{x<0} + \frac{(1+\gamma)\alpha}{2\beta} \geq x_{x>0} + \frac{\alpha}{2\beta} \tag{A.10}$$

which means that we do not have a separating equilibrium as there will be overlapping types for a given reported earnings in this interval.

We can find the distance between $x_{x<0}$ and $x_{x>0}$ for which $x^R_{x<0} \geq x^R_{x>0}$.

Since types $x < 0$ adds a bias to their report, which is $\frac{\gamma\alpha}{2\beta}$ larger than types $x > 0$, it is the types between $[-\frac{\gamma\alpha}{2\beta}, 0]$ that breaks the separating equilibrium. □

Proof of Proposition 2. Here we build upon on the proof of Proposition 2 in Guttman et al. (2006). In the interval $[a_0^*, 0]$, the necessary condition (12) ensures that a manger who realized earnings of a_0^* is indifferent between reporting $a_0^* + \frac{(1+\gamma)\alpha}{2\beta}$ and 0, and the necessary condition 13 ensures that a manager who realize earnings of 0 is indifferent between reporting $\frac{\alpha}{2\beta}$ and 0. We claim that ρ_p^* is an equilibrium strategy when applied to this interval.

By construction, the pricing function satisfies Bayes' rule on the equilibrium path given the manager's reporting strategy. ρ_{gs}^* is an equilibrium strategy for types $x \geq 0$ and ρ_{ls}^* is an equilibrium strategy for types $x \leq a_0^*$. The reporting strategy ρ_p^* paste these strategies together with the pooling strategy in the interval $[a_0^*, 0]$, which circumvents the problem encountered in Proposition (1). This means that we need to verify that it will not be optimal for managers in $\hat{x} \in (a_0^*, 0)$ to deviate away from the pooling interval nor to deviate and report anything that differs from zero inside the pooling interval, similarly we need to verify that no manager with true earnings outside the pooling interval wants to deviate and report earnings that fall inside the pooling interval.

Case 1 We begin by checking whether any types in $\hat{x} \in (a_0^*, 0)$ will find it optimal to deviate from reporting $\hat{x}^R = 0$. We first check on the left side of the interval, for a manager with type $\hat{x} = a_0^* + \epsilon$ for a small ϵ. We want to show that:

$$U^M(a_0^* + \epsilon, 0) \geq U^M\left(a_0^* + \epsilon, \hat{x}^R\right) \forall \hat{x}^R < a_0^* + \frac{(1+\gamma)\alpha}{2\beta} \tag{A.11}$$

Now, suppose that the manager of type $\hat{x} = a_0^* + \epsilon$ decides to report earnings below $a_0^* + \frac{(1+\gamma)\alpha}{2\beta}$. This will cause the manager to get an utility loss from manipulation of $\beta\left(a_0^* + \epsilon + \hat{x}^R\right)^2$. A manager of type $x = a_0^*$ is at least as good of by reporting 0 than by reporting anything below or equal to $a_0^* + \frac{(1+\gamma)\alpha}{2\beta}$. Since the manager of type $\hat{x} = a_0^* + \epsilon$ has to incur a cost of manipulation $\beta\left(\epsilon\right)^2$ higher than a manager of type a_0^*, it can not be optimal for the manager to deviate to below the pooling interval. We then check to see whether a manager of type $-\epsilon$ finds it beneficial to report to the right of the pooling interval:

$$U^M(-\epsilon, 0) \geq U^M\left(-\epsilon, \hat{x}^R > \frac{\alpha}{2\beta}\right) \forall \hat{x}^R > \frac{\alpha}{2\beta} \tag{A.12}$$

Now, suppose that the manager of type $\hat{x} = -\epsilon$ decides to report earnings above $\frac{\alpha}{2\beta}$. This will cause the manager to get an utility loss from manipulation of $\beta\left(\epsilon + \hat{x}^R\right)^2$. A manager of type $x = 0$ is at least as good of by reporting 0 than by reporting anything greater or equal to $\frac{\alpha}{2\beta}$. Since the manager of type $\hat{x} = a_0^* + \epsilon$ has to incur a cost of manipulation $\beta\left(\epsilon\right)^2$ higher than a manager of type 0, it can not be optimal for the manager to deviate to above the pooling interval. The last step is to to check whether a manager $\hat{x} \in (a_0^*, 0)$ finds it optimal to report $\hat{x}^R \in \left(a_0^* + \frac{(1+\gamma)\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right)$. So we want to verify that

$$
\begin{aligned}
&U^M(\hat{x}, 0) \geq U^M\left(\hat{x}, \hat{x}^R \neq 0\right) \\
&\forall \hat{x}^R \in \left(a_0^* + \frac{(1+\gamma)\,\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right), \hat{x} \in (a_0^*, 0)
\end{aligned}
\tag{A.13}
$$

Suppose that a manager of type $\hat{x} \in (a_0^*, 0)$ deviates from the pooling strategy of reporting $\hat{x}^R = 0$. This yields him the payoff $(1+\gamma)\,a_0^*$ which is the payoff that made type $\hat{x} = a_0^*$ indifferent between reporting 0 and reporting $a_0^* + \frac{(1+\gamma)\alpha}{2\beta}$. For a manager of type $x = a_0^*$ to report $x^R = 0$ incurs him a cost of $\beta(a_0^*)^2$, now since a manager of type $\hat{x} \in (a_0^*, 0)$ incurs a cost of reporting 0 which is strictly less than that, it can not be optimal for him deviate and report $\hat{x}^R \neq 0$ inside the pooling range.

We have now shown that it will not be optimal for any manager in the interval $[a_0^*, 0]$ to report outside the pooling interval. The next step is to show that no manager from outside the pooling interval will deviate and report 0 nor report any other value which belongs to the pooling interval.

Case 2 First, we will show that it is not optimal for any manager $\hat{x} = a_0^* - \epsilon$ to deviate and take part in the pooling interval. A manager of type $\hat{x} = a_0^* - \epsilon$ follows strategy ρ_{ls}^*, so we need to show that:

$$
U^M\left(a_0^* - \epsilon, a_0^* - \epsilon + \frac{(1+\gamma)\,\alpha}{2\beta}\right) > U^M\left(a_0^* - \epsilon, \hat{x}^R\right) \forall \hat{x}^R \in [a_0^*, 0] \tag{A.14}
$$

Now suppose that a manager of type $\hat{x} = a_0^* - \epsilon$ decides to report $\hat{x}^R \in \left(a_0^* + \frac{(1+\gamma)\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right)$. This yields him the payoff $(1+\gamma)a_0^*$ which makes type $x = a_0^*$ indifferent between reporting $a_0^* + \frac{(1+\gamma)\alpha}{2\beta}$ and 0. However, type $\hat{x} = a_0^* - \epsilon$ has an additional utility loss of $\beta(\epsilon)^2$, so this can not be optimal.

Similar, it can not be optimal for the manager to report 0. Second, we will show that it is not optimal for any manager $\hat{x} = \epsilon$ to deviate and take part in the pooling interval. A manager of type $\hat{x} = \epsilon$ follows strategy ρ^*_{gs}, so we need to show that:

$$U^M\left(\epsilon, \epsilon + \frac{\alpha}{2\beta}\right) > U^M\left(\epsilon, \hat{x}^R\right) \forall \hat{x}^R \in [a^*_0, 0] \tag{A.15}$$

Suppose that a manager of type $\hat{x} = \epsilon$ decides to report $\hat{x}^R \in \left(a^*_0 + \frac{(1+\gamma)\alpha}{2\beta}, 0\right) \cup \left(0, \frac{\alpha}{2\beta}\right)$. This yields him the payoff $(1+\gamma)a^*_0$, given that type $x = 0$ manager is indifferent between 0 and $\frac{\alpha}{2\beta}$, where $\frac{\alpha}{2\beta} > (1+\gamma)a^*_0$, this can not be optimal for a manager of type 0 as he has to incur an additional manipulation cost of $\beta(\epsilon)^2$. Now suppose that a manager of type $\hat{x} = \epsilon$ decides to report $x^R = 0$. This is the reported earnings which makes type $x = 0$ indifferent between reporting $\frac{\alpha}{2\beta}$ and 0. However, type $\hat{x} = \epsilon$ gets additional utility loss of $\beta(\epsilon)^2$, so this can not be optimal. $\square$

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