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Equivalence of Canonical Matching Models*

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Abstract

This paper offers expected revenue and pricing equivalence results for canonical models of pricing and matching. The equivalence of these models is centered on the assumption that there are large numbers of buyers and sellers and the assignment of buyers within a submarket of sellers is random. Therefore, the distribution of buyers to sellers is approximated by the Poisson distribution. The list of canonical matching models includes the models developed by Burdett and Judd (1983), Shimer (2005), and McAfee (1993). In the Burdett and Judd (1983) model, buyers post prices and the equilibrium features price dispersion because identical buyers play mixed strategies. In the Shimer (2005) model, sellers post a vector of prices corresponding to different buyer types. In equilibrium, all identical buyers pay the same price. In the McAfee (1993) model, equilibrium pricing is determined by simple second price auctions. McAfee's model also features price dispersion, because the number of bidders at each auction is stochastic.

Key words: Directed search, price dispersion, competing auctions, Poisson distribution

JEL classification codes: D47, D83, J64

1 Introduction

Decentralized offer making is an essential driver of exchange in a multitude of market places. Casual observation suggests that, depending on the market context, offers can be made by either buyers or sellers. For example, in many market places - the labor market in particular - the buyers of labor (firms) play a strategy of proposing prices to sellers (workers). The non-trivial strategic reasoning of such buyers is that by proposing higher prices, they will have to pay the workers more for their

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services but such offers are more likely to be accepted by any particular worker (Mortensen 2003, pp. 16-22). In other market places, the identity of the offerer is clearly that of the seller. For example, sellers might advertise offers to trade at particular posted price to buyers. Knowing the posted prices of sellers, a buyer might then strategically decide to choose one of the sellers offering to trade at a higher posted price as a means to economize on the problem of choosing a seller without sufficient inventories (Peters 1991).

The nature of what is proposed in a market place might also be quite different from a posted price price. For example, in some market places, sellers offer buyers the opportunity to participate in an auction. This alternative pricing mechanism asks buyers, in turn, to make price proposals to the seller. The seminal equilibrium theory of decentralized offers to auction is McAfee's (1993) model of competing auctions. In a one shot game, McAfee's analysis offers a key insight that each competing auctioneer sets a reserve price equal to zero in equilibrium, because a strategy of no reserve price maximizes the individual seller's expected revenue by optimally encouraging the possibility of multiple bidders. Therefore, equilibrium pricing in McAfee's model is equivalent to simply assuming a Bertrand competition between any group of buyers who are stochastically allocated to any particular seller.

In this paper, we seek to compare the properties of these market games under the assumption that matching is subject to the common problem of coordination frictions. We assume that there are large numbers of heterogenous buyers (firms) and sellers (buyers). We assume that assignment of buyers within any submarket of sellers (a group of identical sellers) is random. We also assume that the participation choice of each buyer into each submarket is directed.

A key point of comparison is with the social planning solution of such markets. We provide a simple characterization of this planning solution. This problem is simplified by the assumption that each buyer type faces a common opportunity cost, which is exogenous. We then take from Shimer (2005) the proof that the planner's objective function is strictly concave. Our first main result is that the three pricing games are expected revenue equivalent. Revenue equivalence is found by comparing the decentralized payoffs with the expected payoffs needed to implement the planner's solution. In each market game, we find that the equilibrium is constrained efficient.

It is well known that all three of these models give different predictions about the nature of equilibrium wage dispersion when buyers and sellers are homogenous. For example, if there is a single buyer type, then the model of Shimer (2005) predicts that each identical seller posts a common wage. This result is also found in Burdett, Shi and Wright (2001) who consider a related model with identical buyers. Mortensen (2003) considers a version of the Burdett-Judd (1983) model where the assignment of buyers to sellers is subject to identical coordination frictions as in the models of Shimer (2005) and Burdett, Shi and Wright (2001). As in the original Burdett-Judd (1983) model, Mortensen (2003) finds that each buyer plays a mixed strategy. In order to be clear about the different roles of buyers and sellers, we refer to the buyers as workers, the sellers as firms, and the prices as wages. Therefore, this model predicts wage dispersion in a labor market where the buyers (firms) and the sellers (workers) are homogenous. We can also use a competing auction model to account for wage dispersion in markets with coordination frictions. However, in this case, workers expect only two possible wages: A high wage if there are multiple bidders and a low wage if there is a single bidder.

In this paper, we derive some common features of wages for the three matching models when buyers and sellers are heterogenous. We find that higher firm types always pay similar workers higher expected wages than lower types. This result is driven by the fact that higher firm types have higher opportunity costs and that equilibrium wages are used to ensure that higher firm types are always selected by workers whenever the worker has multiple offers. Our second main result is that the posted wage vector in the seller posting game gives a wage for each buyer type, which is bounded by the highest wage and lowest wage posted in the buyer posting game. A corollary of this result is that the two posting models give identical predictions about the distribution of wages if the number of buyer types becomes large.

The remainder of the paper is organized as follows. We first characterize a canonical urn-ball matching environment and we solve the social planning problem. We then establish the properties of the alternative market games. We establish efficiency and expected revenue equivalence and we compare the nature of wage dispersion within each of these environments. The final section concludes.

2 The Environment

The labor market consists of a large number of workers who wish to sell labor and a large number of firms that seek to buy labor in order to produce output. A group of identical workers in this labor market is referred to as a submarket. The firms are heterogenous. We let $k \in \{0, 1, 2, ..., K\}$ denote the firm's type and we assume that y_k is the output of a type k job when it is matched to a worker. We assume that each entrant firm can hire one worker and that each worker can be employed at only one firm. We assume that higher firm types are more productive than lower type firms. The number of firms in each worker submarket is determined by free entry. We assume that the opportunity cost of a type k entrant firm is c_k. We let $c = (c_0, ..., c_K)$. We use λ_k to denote the number of type k jobs (relative to the number of workers) that enter a worker submarket. We let $\lambda = (\lambda_0, \lambda_1, ..., \lambda_K)$. We assume that the opportunity cost of the lowest available job type is always zero, $c_0 = 0$. We refer to y_0 as the worker's home production output.

2.1 Urn-ball matching

A worker must receive an offer from a job in order to become employed. In each submarket the entrant firms are assumed to be randomly assigned to workers.

Definition 1. We define the following:

$$\Lambda_k \equiv \sum_{j=k}^{K} \lambda_j \tag{1}$$

Thus $\lambda_k = \Lambda_k - \Lambda_{k+1}$. We refer to Λ_1 as the ratio of entrant firms (not including home production opportunities) to workers. The value of λ_0 is in general infinite, because c_0 is zero, which implies Λ_0 is in general infinite.

By the Poisson limit theorem, the assignment of firms to workers follows a Poisson distribution. Moreover, as we show in the subsequent models, a worker is employed at a type k firm if the worker does not receive an offer from any firm type $j \in \{k+1, ..., K\}$ firm and it receives at least one offer from a type k firm. Therefore, if we abuse the order statistic notation and let $y_{(1)}$ denote the *highest* valuation firm, the probability that the worker is hired by a type k firm is given by:

$$\Pr\left(y_{(1)} = y_k \mid \lambda\right) = e^{-\Lambda_{k+1}}\left(1 - e^{-\lambda_k}\right) \tag{2}$$

in which case the matched firm-worker pair produces output y_k. The term $e^{-\Lambda_{k+1}}$ is the probability that the worker does not get an offer from a type $k+1$ firm or higher, and the term $1 - e^{-\lambda_k}$ is the probability that the worker gets one or more offers from a type k firm.

The expected output of each worker in a submarket is the product of the hiring probabilities and the output that a matched worker-firm pair produces, summed across firm types. Subtracting the opportunity costs of the entrant firms, gives the expected net output of each worker in a submarket:

$$\begin{aligned} Y(\lambda) &= \sum_{k=0}^{K} \Pr\left(y_{(1)} = y_k \mid \lambda\right) y_k - c_k\lambda_k \\ &= \sum_{k=0}^{K} e^{-\Lambda_{k+1}}\left(1 - e^{-\lambda_k}\right) y_k - c_k\lambda_k \end{aligned} \tag{3}$$

3 Planning solution

The planner's problem is to maximize the expected net output of each worker in a submarket by choosing non-negative quantities of employer types, which in turn dictates the probability of trade:

$$Y\left(\lambda^{*}\right)=\max_{\lambda\geq 0}Y\left(\lambda\right) \tag{4}$$

Shimer (2005) proves the strict-concavity of the objective function. Hence a unique, global maximum exists which is characterized by the first-order condition:

$$e^{-\Lambda_k}y_k-\sum_{j=0}^{k-1}e^{-\Lambda_{j+1}}(1-e^{-\lambda_j})y_j-c_k\leq 0 \text{ and } \lambda_k\geq 0 \text{ for all } k$$

with complementary slackness:

$$\lambda_k\left[e^{-\Lambda_k}y_k-\sum_{j=0}^{k-1}e^{-\Lambda_{j+1}}(1-e^{-\lambda_j})y_j-c_k\right]=0,\quad \forall k$$

Therefore, any entry $\lambda_k>0$ satisfying the first-order condition imply that:

$$e^{-\Lambda_k}\left[y_k-\sum_{j=0}^{k-1}e^{-(\Lambda_{j+1}-\Lambda_k)}(1-e^{-\lambda_j})y_j\right]=c_k \tag{5}$$

4 Buyer posting

A decentralized solution to the problem of firm entry in the labor market is to assume that each firm commits to a wage offer before the firm is assigned to any particular worker. Given that the worker may receive more than one offer, we know from the analysis of Burdett and Judd (1983) that identical firms will play a mixed strategy equilibrium in which some firms offer higher wages while other firms offer lower wages. In this section, we solve for this mixed strategy equilibrium for the case where the multiplicity of offers is determined by an urn-ball matching function. This problem generalizes an example, which is given in Mortensen (2003, pp. 16-22).[1] We use this problem to characterize the distribution of equilibrium prices when firms are heterogenous and to compare decentralized firm entry with the solution to the planners problem.

The Burdett-Judd pricing game with urn-ball matching is a stage game that unfolds as follows:

[1] We call this model a buyer posting model. However, we acknowledge that Butters (1977) would call buyers in our model, sellers. In his model, the distribution of seller posted price advertisements to buyers is urn-ball and each buyer selects the best posted price offer.

1. Firms enter a submarket. Let $\lambda_k \geq 0$ denote the entry of each firm type $k = 0, 1, ..., K$. Entry is free. Therefore, a firm enters if the expected profit is greater than their opportunity cost and stays out otherwise.

2. Each entrant firm posts a wage.

3. The assignment of firms to workers in a submarket is random.

4. Each worker chooses to work for the firm that offers the highest wage

We solve for the equilibrium wages at stage 2 after the entry decision is determined. This solution considers the pricing decision of firm types $k = 1, ..., K$ given λ. We use these prices to characterize the expected payoffs of each firm type given λ. The free entry assumption is then used to characterize equilibrium firm entry.

We start out with establishing a hierarchy with respect to wages and profits in productivity. Strictly higher productivity firms yield a strictly higher profit and they offer wages no smaller than the type just below them. Mortensen (2003) characterizes the two firm case, which generalizes to K types. Let $P(w)$ denote the probability that a worker accepts a wage w. $P(w)$ is increasing in w, as workers always choose the highest offer.

Lemma 1. *For firm types $k \in \{1, 2, \ldots, K\}$:*

$$y_k > y_{k-1} \Rightarrow \pi(y_k) > \pi(y_{k-1}), \quad k = 2, \ldots, K$$

$$y_k > y_{k-1} \Rightarrow w_k \geq w_{k-1}, \quad k = 2, \ldots, K$$

Proof. The proposition is proven using mathematical induction. Proposition 2 in Mortensen (2003) verifies the starting case for two firm types $(1, 2)$.

Assume true for types $\{1, 2, \ldots, k-1\}$, and let w_k^* denote a profit-maximizing wage offer for a type k firm.

Then:

$$\begin{aligned}
\pi_k(w_k^*) &= P(w_k^*)(y_k - w_k^*) \\
&\geq P(w_{k-1}^*)(y_k - w_{k-1}^*) \\
&> P(w_{k-1}^*)(y_{k-1} - w_{k-1}^*) = \pi_k(w_{k-1}^*) \\
&\geq P(w_k^*)(y_{k-1} - w_k^*)
\end{aligned}$$

That higher wages are offered is evident by subtracting the fourth line from the first, and the third line from the second, yielding: $P(w_k^*)(y_k - y_{k-1}) \geq P(w_{k-1}^*)(y_k - y_{k-1})$. □

As the urn-ball matching technology follows a Poisson distribution, we can uniquely pin down $P(w)$, as done in Mortensen (2003), letting x denote the number of offers a given worker receives, the probability that w is the highest offer, and thus the offer accepted by the worker is given by:

$$P(w) = \sum_{x=0}^{\infty} F(w)^x \frac{e^{-\Lambda_1}\Lambda_1^x}{x!} = e^{-\Lambda_1[1-F(w)]} \tag{6}$$

where $F(w)$ is the economy-wide cumulative distribution function of wages offered.

Mortensen (2003) provides the important elementary proof that the equilibrium distribution of wage offers is generally continuous, has connected support, and is bounded below by y_0. The proof follows the general insight given by Burdett and Judd (1983): There cannot be a mass of identical firms choosing a common wage, because each of these labor buyers gains a strictly higher probability of trade by any arbitrarily small increase in their wage offer. The distribution of offers has full support, since firms facing no competing offers in a range below their posted wage will always do better to lower their wage. And, the lowest wage offered by the lowest type firm $k \geq 1$ is y_0, because this offer will be accepted if and only if the worker has only one offer, which is type $k \geq 1$. The outside option of the worker is to work in home production and receive an income of y_0. We find

Lemma 2. *The distribution function of wages is given by*

$$\begin{aligned} F(w) &= \frac{1}{\Lambda_1} \log\left(\tfrac{y_k - y_0}{y_k - w}\right) & k &= 1 \\ F(w) &= \frac{1}{\Lambda_1} \log\left(\tfrac{y_k - \underline{w}_k}{y_k - w}\right) + \frac{\sum_{j=1}^{k-1}\lambda_j}{\Lambda_1} & k &= 2, ..., K \end{aligned}$$

where the wage w is paid by firm k such that $w \in [\underline{w}_k, \overline{w}_k]$ with $\underline{w}_k$ and $\overline{w}_k$ being, respectively the minimum and maximum wage paid by firm k.

Proof. $k = 1$ is proven in Mortensen (2003).

For $k \geq 2$ we note that all firms are profit maximizing and that the wage hierarchy established by lemma 1 implies that $F(\underline{w}_k) = F(\overline{w}_{k-1}) = \frac{\sum_{j=1}^{k-1}\lambda_j}{\Lambda_1}$. Let $w \in [\underline{w}_k, \overline{w}_k]$:

$$\pi_k(w) = \pi_k(\underline{w}_k)$$

$$\Updownarrow$$

$$e^{-\Lambda_1[1-F(w)]}(y_k - w) = e^{-\Lambda_1[1-F(\underline{w}_k)]}(y_k - \underline{w}_k)$$

$$\Updownarrow$$

$$e^{\Lambda_1 F(w)} = e^{\sum_{j=1}^{k-1}\lambda_j}\left(\frac{y_k - \underline{w}_k}{y_k - w_k}\right)$$

$$\Updownarrow$$

$$F(w) = \frac{1}{\Lambda_1} ln\left(\frac{y_k - \underline{w}_k}{y_k - w_k}\right) + \frac{\sum_{j=1}^{k-1}\lambda_j}{\Lambda_1}$$

□

The following lemma classifies highest offered wage by type k employer:

Lemma 3. *The highest wage offered by a type k employer is:*

$$\overline{w}_k = (1 - e^{-\lambda_k})y_k + e^{-\lambda_k}\underline{w}_k,\ \forall k = 1, 2, \dots, K$$

Proof. Solving

$$F(\overline{w}_k) = \frac{1}{\Lambda_1} ln\left(\frac{y_k - \underline{w}_k}{y_k - \overline{w}_k}\right) + \frac{\sum_{j=1}^{k-1}\lambda_j}{\Lambda_1} = \frac{\sum_{j=1}^{k}\lambda_j}{\Lambda_1}$$

$$\Updownarrow$$

$$\frac{1}{\Lambda_1} ln\left(\frac{y_k - \underline{w}_k}{y_k - \overline{w}_k}\right) = \frac{\lambda_k}{\Lambda_1}$$

$$\Updownarrow$$

$$y_k - \overline{w}_k = e^{-\lambda_k}(y_k - \underline{w}_k)$$

$$\Updownarrow$$

$$\overline{w}_k = (1 - e^{-\lambda_k})y_k + e^{-\lambda_k}\underline{w}_k$$

□

Lemmas 1 through 3 makes evident that the hierarchy in productivity results in a hierarchy of wages, as a result a given type firm will never be able to outcompete a higher type firm and will always outcompete a lower type firm. Hence, the only competition that the firm cares about is that of similar type firms, therefore the intuition behind the within-type distribution of wages is identical to that of the homogeneous firm case presented in Mortensen (2003).

Corollary 1. *The highest wage offered by a type k employer can be rewritten as:*

$$\overline{w}_k = y_k - \sum_{j=1}^{k} e^{-(\Lambda_j - \Lambda_{k+1})}(y_j - y_{j-1})$$

where y_0 is outside option.

Proof. Recursively substituting $\underline{w}_k = \overline{w}_{k-1}$ into the equation from Lemma 1. □

Corrolary 1 implies that the highest wage offered by a type k employer is equal to the productivity of the employer less an amount equal to the expected output the worker would produce in the next best available job.

Lemma 4. *The expected profit for an employer of type k is given by:*

$$\begin{aligned}
\pi_k &= e^{-\Lambda_{k+1}}(y_k - \overline{w}_k) \\
&= \sum_{j=1}^{k} e^{-\Lambda_j}(y_j - y_{j-1}) \\
&= e^{-\Lambda_k}\left(y_k - \sum_{j=0}^{k-1} e^{-(\Lambda_{j+1} - \Lambda_k)}\left(1 - e^{-\lambda_j}\right) y_j\right)
\end{aligned}$$

Proof. Given K employer types, the profit for any given type k is given by:

$$\begin{aligned}
\pi_k &= e^{-\Lambda_{k+1}}(y_k - \overline{w}_k) \\
&= e^{-\Lambda_{k+1}}(y_k - (y_k - \sum_{j=1}^{k} e^{-(\Lambda_j - \Lambda_{k+1})}(y_j - y_{j-1})) \\
&= \sum_{j=1}^{k} e^{-\Lambda_j}(y_j - y_{j-1}) \\
&= e^{-\Lambda_k} y_k - \left(\sum_{j=0}^{k-1} e^{-\Lambda_{j+1}} y_j - \sum_{j=0}^{k-1} e^{-\Lambda_j} y_j\right) \\
&= e^{-\Lambda_k} y_k - \sum_{j=0}^{k-1} e^{-\Lambda_{j+1}}(1 - e^{\lambda_j}) y_j \\
&= e^{-\Lambda_k}\left(y_k - \sum_{j=0}^{k-1} e^{-(\Lambda_{j+1} - \Lambda_k)}(1 - e^{-\lambda_j}) y_j\right)
\end{aligned}$$

In the fourth line we utilize that $e^{-\Lambda_0} = 0$, which follows from the assumption that $\Lambda_0 = \infty$ when $c_0 = 0$. $\square$

The first equality of lemma 4 shows the connection between profits and the highest wage of each firm type. The second equality of lemma 4 establishes the connection between profits and the market tightness variable Λ_j. Most importantly, the third equality of lemma 4 provides us equivalence with the first-order condition of the social planning solution, in (5). Hence, the following proposition holds true:

Proposition 1. *Entry of firms is equivalent to λ^* of the social planning solution. The expected income of a worker is $Y(\lambda^*)$*

5 Seller posting

A second decentralized solution to the problem of firm entry in the labor market is to assume that the search decisions of buyers are directed by the posted wage commitments of sellers (Peters 1991). Our analysis of this 'directed search' problem closely follows the model of Shimer (2005). However, in accordance with our urn-ball matching technology, we assume that the workers are 'urns' who post wages and that the firms are 'balls' and are directed to individual workers. The directed search pricing game is different from the Burdett-Judd pricing game in two important respects. The first difference is that the equilibrium wage decision of the wage setters is a pure strategy. The second difference is that the wage setting decision involves the choice of a vector of wages rather than a single wage by each wage setter. The directed search game is a stage game that unfolds as follows.

1. Workers advertise posted wages. A submarket is created by a set of workers advertising a common wage.
2. Firms enter the submarkets. Let $\lambda_k \geq 0$ denote the entry of each firm type $k = 0, 1, ..., K$.
3. Entrant firms are Poisson assigned within each submarket of workers.
4. The worker then selects the highest available wage opportunity.

If identical workers each post a vector of wages corresponding to each job type, $w = (w_0, ..., w_K)$, the expected profit of an entrant type k firm to this submarket is given by

$$\pi_k = e^{-\Lambda_{k+1}} \frac{\left(1 - e^{-\lambda_k}\right)}{\lambda_k} \left(y_k - w_k\right) \tag{7}$$

where $\frac{(1-e^{-\lambda_k})}{\lambda_k}$ denotes the probability that an individual firm is chosen out of the type k firms present. The workers' expected wage in this submarket is given by

$$W(\lambda, w) = \sum_{k=0}^{K} e^{-\Lambda_{k+1}} \left(1 - e^{-\lambda_k}\right) w_k$$

The workers' problem is to choose the submarket that maximizes the workers expected wage:

$$\{w^*, \lambda^*\} = \arg\max_{\lambda, w} W(\lambda, w) \tag{8}$$

such that

$$\pi_k \geq c_k$$

and

$$\lambda_k \geq 0$$

for all k. The worker problem is simplified by the fact that the profit constraint is always binding. Therefore, we substitute $\pi_k = c_k$ and equation (7) into the wage setter's problem (8) and write

$$\begin{aligned} W(\lambda, w(\lambda)) &= \sum_{k=0}^{K} e^{-\Lambda_{k+1}} \left(1 - e^{-\lambda_k}\right) y_k - c_k \lambda_k \\ &= Y(\lambda) \end{aligned} \tag{9}$$

The worker then maximizes (9) with respect to λ. Therefore, the wage setter's problem is equivalent to the planner's problem given by (4). The solution is given by the same first-order condition. Since the objective function is strictly concave, a maximum exists and is unique. In equilibrium, all workers solve the same problem with the same solution. We summarize the results as follows.

Proposition 2. *Entry of firms is equivalent to λ^* of the social planning solution. The expected income of a worker is $Y(\lambda^*)$*

The next proposition establishes that there is a hierarchy of the seller posted wages vector for each firm type and that this hierarchy is bounded by the hierarchy of the buyer posted wages, which we solved in the previous section.

Proposition 3. *The wage found in the seller posting (SP) game for a type k firm (denoted w_k^{SP}) is between the lower and upper bound for type k in the buyer posting (BP) pricing game. That is $\underline{w}_k^{BP} < w_k^{SP} < \overline{w}_k^{BP}$.*

Proof. We start with proving that $w_k^{SP} < \overline{w}_k^{BJ}$:

$$w_k^{SP} = y_k - e^{\Lambda_{k+1}} \frac{\lambda_k}{(1 - e^{-\lambda_k})} c_k < y_k - e^{\Lambda_{k+1}} c_k = \overline{w}_k^{BP}$$

$$\Updownarrow$$

$$1 < \frac{\lambda_k}{(1 - e^{-\lambda_k})}$$

Which holds for $\lambda_k > 0$. Hence, with entry of a type k firm in a local labor market $w_k^{SP} < \overline{w}_k^{BP}$. Next, we show that $w_k^{SP} > \underline{w}_k^{BP}$:

$$\underline{w}_k^{BP} = y_k - e^{\Lambda_k} c_k < y_k - e^{\Lambda_{k+1}} \frac{\lambda_k}{(1 - e^{-\lambda_k})} c_k = w_k^{SP}$$

$$\Updownarrow$$

$$\frac{e^{-\lambda_k} \lambda_k}{(1 - e^{-\lambda_k})} < 1$$

□

Hence, because lemma 1 provides us with a monotonicity result in the Burdett-Judd framework, proposition 4 provides us with strict monotonicity in the directed search framework.

An interesting implication of proposition 3 is that the equilibrium distribution of prices in the buyer and seller posting models converges as the number of firm types offered in equilibrium approaches a continuum. In this case, Shimer's (2005) model of first degree price discrimination and Burdett and Judd's (1983) model of equilibrium price dispersion become observationally equivalent with respect to trade and prices.

6 Second price auctions

A final decentralized solution to the problem of firm entry in the labor market is to assume that each worker auctions his individual labor services by a simple second price auction (McAfee 1993). In this case, the highest valuation firm will win the worker's services at a wage equal to the available output of the second highest valuation firm.[2] Firm entry into this labor market is then modeled by the following stage game.

1. Firms enter the matching market.

[2]If sellers do not know buyer valuations, McAfee (1993) proves that sellers adopt a second price auction with a reserve price equal to the sellers outside option.

2. Entrant firms are Poisson assigned to workers.

3. Firms bid for the worker according to a standard second price auction.

The equilibrium of this game is solved by first considering the expected payoffs of the firms given an entry level λ. We then solve for entry by imposing the free entry condition that entry occurs up until the point that the payoff from entry equals the firm's opportunity cost. As in Kultti (1999), we will establish revenue equivalence between the expected revenue of sellers in the second price auction equilibrium and the expected revenue of sellers in a seller posting equilibrium. Our analysis extends the equivalence result to the seller posting model where sellers post a vector of prices and to the buyer posting model where the buyers post prices with mixed strategies.

The expected payoff of a firm is easy to characterize. A type k firm does not hire a worker if there is a best competing buyer of type $j > k$. If the type k firm faces a best competing buyer of the same type then the firm earns zero revenue, because the wage is then equal to the output of the firm. If the firm faces a best competing buyer of type $j < k$, then the type k firm earns profit $y_k - y_j$. We compute the expected profit by summing over each possible event where (2) gives the probability distribution over y_j. We have:

$$\begin{aligned}\pi_k &= \sum_{j=0}^{K} \max\left[0, (y_k - y_j)\Pr\left(y_{(1)} = y_j \mid \lambda\right)\right] \\ &= e^{-\Lambda_k}\left[y_k - \sum_{j=0}^{k-1} e^{-(\Lambda_{j+1}-\Lambda_k)}\left(1 - e^{-\lambda_j}\right) y_j\right]\end{aligned}$$

By the free condition entry $\pi_k = c_k$, and the entry condition is identical to the first-order condition of the social planner in equation (5). Hence, we have the following proposition:

Proposition 4. *If workers auction labor by second price auction, entry of firms is equivalent to* λ^* *of the social planning solution. The expected income of a worker is* $Y(\lambda^*)$.

Since the workers wage is determined by a second price auction, the expected payoff of the worker is given by

$$Y(\lambda) = \sum_{j=0}^{K} y_j \Pr\left(y_{(2)} = y_j \mid \lambda\right)$$

where $\Pr\left(y_{(2)} = y_j \mid \lambda\right)$ is the probability that the second highest valuation firm is y_j. By substitution we have

$$\sum_{j=0}^{K} y_j \Pr\left(y_{(2)} = y_j \mid \lambda^*\right) = \sum_{j=0}^{K} y_j \Pr\left(y_{(1)} = y_j \mid \lambda^*\right) - c_j \lambda_j^* = Y\left(\lambda^*\right)$$

Therefore, the payoffs of seller can be expressed as ether a function of the expected probability distribution of the second highest buyer type at each seller or as a function of the expected probability distribution of the highest buyer type at each seller less the opportunity costs of entrant buyers.

He, Kennes and le Maire (2018) establish that equilibrium wages offered by higher type firms stochastically dominate the wages offered by lower type firms. This result follows from the fact that higher firm types will always outbid all lower firm types and the distribution of competing bidders is independent of each bidder type. Furthermore, higher type firms offer wages over a larger range of wages. Therefore, all three decentralized pricing mechanisms considered in this paper share the common prediction that the expected wage of each worker is monotonically increasing with respect to the firm type employing the worker.

7 Conclusions

Since buyers have common opportunity costs and are directed to submarkets, the main results of this paper extend naturally to the case of heterogenous sellers. This includes constrained efficiency, expected revenue equivalence, and the monotonicity of each workers' expected wages with respect to firm types. Therefore, if the assignment of jobs in each submarket of workers is urn-ball, models of sorting share a number of implications about wages and employment that are independent of the standard canonical wage setting mechanisms. Examples and applications of related models of heterogenous sellers include Burdett, Shi and Wright (2001), Shimer (2005), Albrecht, Gautier and Vroman (2014), and He, Kennes and le Maire (2018).

If there is a continuum of firm types offered in equilibrium, the buyer posting and seller posting models also feature identical distributions of prices. The mechanisms behind this common distribution of prices are very different. In the seller posting model, each seller perfectly price discriminates by offering each buyer a type specific price. By contrast, in the buyer posting model, prices are dispersed even though each price setter chooses to post a single price. The prices are disperse in the buyer posting model, because the seller simply selects the best posted price of buyers and these available posted prices are uncertain. Therefore, although the pricing models share common predictions about equilibrium prices and assignment, the pricing technology used to achieve these outcomes are different.

We have assumed that workers are always sellers and that firms are always buyers (See, for example, Hall 1979, Mortensen 2003, Julien Kennes and King 2000). However, other job matching models, such as Burdett, Shi and Wright (2000) and Shimer (2005), make the assumption that firms are sellers. The choice of who is the seller within an urn-ball matching environment is not neutral. For example, trade-offs related to such choices are developed in Shi and Delacroix (2018) and Julien, Kennes and King (2006). Our analysis can be used to complement and enlarge the discussion of pricing mechanism selection in such markets.

The revenue equivalence results are based on pricing mechanisms that give expected equilibrium payoffs that also implement constrained efficient market outcomes. Of course, not all pricing mechanisms are revenue equivalent when matching is urn-ball. For example, McAfee and Lu (1996) demonstrate that auctions are an evolutionary stable strategy over bargaining. Furthermore, it is well known that wage bargaining is generally constrained inefficient in the context of frictional matching environments (Hosios 1990). Our analysis also does not feature all possible frictions that are relevant to such markets. For example, Camera and Selcuk's (2009) consider an urn-ball matching model where sellers cannot commit to a selling mechanism. In this case, they find that bargaining is the equilibrium and that prices are disperse.

Trade-offs between auctions and seller price posting can be introduced by adding other sorts of frictions. Some of this work is derived from an important benchmark model of Kultti (1999), which demonstrates that seller posting and auctions are equivalent if buyers and sellers are homogenous. If buyers are heterogenous and sellers do not know the buyer valuations, then McAfee (1993) derives the prediction that the equilibrium selling mechanism is a second price auction. However, in the Einav, Farronato, Levin and Sundaresan (2018) model, sellers may choose to use posted prices in equilibrium as a means to avoid the higher transaction costs of a more complicated selling mechanism such as auctions. Less studied is the choice between whether buyers or sellers should post prices.

The nature of the meeting technology will also play an important role for the equilibrium choice of pricing mechanisms. For example, in the Eeckhout and Kircher (2010) model, sellers will choose to post a single price instead of auction if meetings are pairwise. We can speculate that a similar concern may also impact the choice between buyer posting and seller posting. For example, in the Diamond (1971) model, all meetings are bilateral and wage posting by buyers (firms) leads to a degenerate equilibrium outcome with all firms posting a wage of zero. This adverse outcome mitigates the advantage of pricing mechanism simplicity, which is an obvious feature of a buyer posting market over a seller posting market because the latter market type requires the posting of vectors of prices. In this case, the transaction costs associated with complicated mechanisms and the properties of the matching technology could both be used as a means to model the trade-offs associated with the choice of buyer price posting and seller price posting mechanisms.

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