

# Economics Working Papers

2016-10

On the equivalence of buyer and seller proposals within canonical matching and pricing environments

John Kennes and Daniel le Maire



DEPARTMENT OF ECONOMICS AND BUSINESS ECONOMICS AARHUS UNIVERSITY

# On the equivalence of buyer and seller proposals within canonical matching and pricing environments<sup>\*</sup>

John Kennes<sup>†</sup>and Daniel le Maire<sup>‡</sup>

October 25, 2016

#### Abstract

This paper considers equilibrium proposals by either buyers or sellers in the canonical 'urnball' matching market. The proposals can either be posted prices announced by buyers; posted prices announced by sellers; or announcements by sellers (or by buyers) to entertain price proposals, as in auctions. We derive the expected revenue equivalence of these different modes of proposing in this canonical trading environment.

Keywords: Matching, pricing, frictions JEL codes: D44, C78, D83

## 1 Introduction

Decentralized offer making is an essential driver of exchange in a multitude of market places. Casual observation suggests that, depending on the market context, offers can be made by either buyers or sellers. For example, in many market places - the labor market in particular - the buyers of labor (firms) play a strategy of proposing prices to sellers (workers). The non-trivial strategic reasoning of such buyers is that by proposing higher prices, they will have to pay the workers more for their services but such offers are more likely to be accepted by any particular worker (Mortensen 2003, pp. 16-22). In other market places, the identity of the offerer is clearly that of the seller. For example, sellers might advertise offers to trade at particular posted price to buyers. Knowing the posted prices of sellers, a buyer might then strategically decide to choose one of the sellers offering to trade at a higher posted price as a means to economize on the problem of choosing a seller without sufficient inventories (Peters 1984).

The nature of what is proposed in a market place might also be quite different from a posted price price. For example, in some market places, sellers offer buyers the opportunity to participate in an auction. This alternative pricing mechanism asks buyers, in turn, to make price proposals to the seller. The seminal equilibrium theory of decentralized offers to auction is McAfee's (1993) model of competing auctions. In a one shot game, McAfee's analysis offers a key insight that each competing auctioneer sets a reserve price equal to zero in equilibrium, because a strategy of no reserve price maximizes the individual seller's expected revenue by optimally encouraging

<sup>\*</sup>We thank Dale Mortensen, Benoit Julien and Ian King for helpful comments and suggestions. All errors are our own.

<sup>&</sup>lt;sup>†</sup>Department of Economics and Business Economics, Aarhus University, E-mail:jkennes@econ.au.dk

<sup>&</sup>lt;sup>‡</sup>Department of Economics, University of Copenhagen, Email: Daniel.le.Maire@econ.ku.dk

the possibility of multiple bidders. Therefore, equilibrium pricing in McAfee's model is equivalent to simply assuming a Bertrand competition between any group of buyers who are stochastically allocated to any particular seller.

The sequence of moves in the matching games described by Mortensen, Peters, and McAfee are depicted in figure 1.

Matching game	1. Pre-assignment stage	2. Assignment stage	3. Pricing/matching stage
Mortensen (2003)	Buyers post prices		Seller selects highest offer
Peters (1984)	Sellers post prices	Buyers choose a seller	Offer extended to one buyer
McAfee (1993)	Sellers post reserves		Highest bidder wins

Figure 1: Timing of the different proposal games

The three games have important differences over (i) what is proposed in the pre-assignment stage and (ii) how the goods and or services are priced and allocated after the assignment stage. Note, in Peters (1984) game, the seller's offer is extended to only one buyer by a simple protocol of randomization. Furthermore, in addition to these three canonical models, we will also characterize a fourth pricing and matching game where the buyers post reverse auctions with reserve prices in the first stage and the sellers bid in the final stage.

The goal of this short paper is to contrast and compare the implications of these different models of offer making. We begin by briefly discussing some common features of the underlying trading environments. We then demonstrate that the different pricing models generate identical expected revenues for both the sellers and the buyers.

## 2 A simple urn-ball market

A key unifying element of the matching games considered here is a simple 'urn-ball' matching environment. The basic idea is that one group of agents is described as 'balls' and another group of agents are described as 'urns'. The urn-ball matching environment describes a frictional assignment involving a large number of m balls and a large number of n urns, where the balls are assumed to be randomly assigned to the urns. An illustrative example of a random assignment of balls to urns is given in figure 2. The key order statistic is the number of x balls that are assigned to each urn.



Figure 2: A random urn-ball assignment

According to the theory of the Poisson distribution, the distribution of balls (i.e. what Mortensen calls 'offers') is

$$\Pr\left\{X=x,\lambda\right\} = \frac{\exp\left(-\lambda\right)\lambda^x}{x!}, \text{ where } \lambda = \frac{m}{n}$$
(1)

Following Mortensen (2003), we assume (large) fixed numbers of m firms (balls) and n workers (urns). We also follow Mortensen and assume that each firm is restricted to make only one offer to a worker and that each worker can accept only one offer from a firm. Furthermore, we assume that a match between a worker and a firm produces y(1) units of output, an unmatched workers gets y(0) < y(1) units of utility, and an unmatched firm produces no output. Therefore, in this static model of assignment, match surplus is simply y(1) - y(0).

Since matching is always pairwise, the expected number of matches is given by the 'urn-ball' matching function,

$$\chi(m,n) = n \left(1 - \exp\left(-m/n\right)\right).$$

This matching function displays constant returns to scale, is increasing in both arguments, and has an elasticity with respect to the number of firms equal to

$$\eta_m \equiv \frac{\chi_1(m,n)m}{\chi(m,n)} \\ = \frac{\lambda \exp(-\lambda)}{1 - \exp(-\lambda)}.$$
(2)

and an elasticity with respect to the number of workers,  $\eta_n$ , equal to  $1 - \eta_m$ .

Note, Hosios (1990) uses the elasticity of a matching technology to characterize a sharing rule (the Hosios rule) that rewards firms a fraction  $\eta_m$  of match surplus and workers the remaining share. For a broad class of pairwise matching environments with well behaved, constant returns to scale matching technologies, Hosios (1990) demonstrates that the Hosios rule is both necessary and sufficient to give both sides of the matching market a private expected return equal to the marginal social return of their participation. The urn-ball matching technology is, of course, only a special example of the broader set of matching technologies, which are considered by Hosios. However, the urn-ball matching environment is of special interest due to the explicit microfoundations of the equilibrium matching technology, which centers on the problem of coordination frictions.

#### 2.1 Mortensen (2003) pricing and matching

As discussed in the previous subsection, Mortensen (2003) considers an economy composed of fixed numbers of identical employers and identical job seeking workers. Time consists of a single period and all workers are unemployed initially. In his baseline model, employers possess a linear technology relating the number of workers employed to output. Both workers and employers are expected income maximizers. Frictions exists in the sense that no worker knows the wage paid by any employer at the beginning of the period. Although each employing firm has an incentive to inform workers of its wage, its capacity to do so is limited. Specifically, each firm randomly mails a single offer to a single worker. Once informed of this wage, each worker applies for the highest paying offers received. Each employer, realizing that workers can receive more than one offer, sets a wage taking into account similar incentives of the other competing employers to hire the worker.<sup>1</sup>

The offers of the *m* buyers are randomly allocated to the *n* sellers. Knowing this, each of the identical firms chooses to post a wage that it will submit as an offer to the worker. We let F(w) denote the distribution of posted offers by buyers. Using equation (1), the probability that a worker is offered a wage less than *w* is given by

$$P(F(w),\lambda) = \sum_{x=0}^{\infty} \Pr\{X = x\} F(w)^{x} = \exp(-\lambda [1 - F(w)])$$
(3)

We state proposition 1 of Mortensen (2003).

**Proposition 1.** (Mortensen 2003) Any equilibrium market distribution of offers, represented by the cdf F(w) is continuous, has connected support is bounded below by y(0), and has upper support less than y(1).

Proposition 1 follows from the claim that the buyers must play a mixed strategy. This is driven by the fact that a firm always has an incentive to undercut by a small number any common wage if that common wage is between y(0) and y(1). The value of deviating in this way from a common wage reduces slightly the revenue in the event that the firm is a monopoly (X = 1) and lets the firm extracts all revenue in the event that there is head to head competition with other competing employers  $(X \ge 2)$ . Mortensen shows also that the distribution of offers is of full support, since firms facing no competing offers in a range below their posted wage will always do better to lower the wage. Finally, Mortensen shows that lowest wage offer is offer is exactly y(0) because this offer will be accepted if and only if the worker has one offer (X = 1).

We use Proposition 1 to construct equations for the distribution of offers and the expected payoffs of workers and firms. The expected profit of the firm offering the lowest wage is given by

$$\pi (y (0)) = (y (1) - y (0)) \Pr \{X = 0, \lambda\}$$

since  $\exp(-\lambda)$  is the probability that the firm will not face a competitor if its offer is randomly assigned to a worker. In the mixed strategy equilibrium, a firm earns the same expected revenue from each possible choice of posted wage offer. Thus

$$\pi\left(w\right) = \pi\left(y\left(0\right)\right)$$

The firm offering wage w will have its offer accepted with probability  $\exp(-\lambda [1 - F(w)])$ . Therefore, payoffs in the mixed strategy equilibrium are given by

$$(y(1) - w) \exp(-\lambda [1 - F(w)]) = (y(1) - y(0)) \exp(-\lambda)$$

By simple manipulation of this expression, the distribution of offers is given by the cdf

$$F(w) = \frac{1}{\lambda} \log\left(\frac{y(1) - y(0)}{y(1) - w}\right)$$
(4)

 $<sup>^{1}</sup>$ Mortensen's elegant characterization of this problem is closely related to the work of Butters (1977) and Burdett and Judd (1983)

where the upper support is

$$\bar{w} = (1 - \Pr\{X = 0, \lambda\}) y(1) + \Pr\{X = 0, \lambda\} y(0)$$

The expected payoff of the buyer (firm) in Mortensen's model,  $\pi^{Mortensen}$ , is obviously

$$\pi^{Mortensen} = (y(1) - y(0)) \Pr \left\{ X = 0, \lambda \right\}.$$

A little more algebra (see notes at the end of this paper) allows us to determine the expected wage of a worker. The expected payoff of a worker in Mortensen's model,  $u^{Mortensen}$ , is given by

$$u^{Mortensen} = y(0) + (1 - \Pr\{X = 0, \lambda\} - \Pr\{X = 1, \lambda\})(y(1) - y(0))$$

The key trade-off facing the buyer in Mortensen's pricing and matching game is between the buyer's price and probability of trade. By posting a higher wage, the firm is more likely to hire a worker, but the firm then earns less profits on each worker hired. Interestingly, the expected payoff of the worker above their outside option is simply the probability that the worker has multiple offers time the economic surplus generated by the worker-firm match. Armed with this result, we can anticipate the connection between the expected payoffs of this model of equilibrium pricing with the expected payoffs of auctions.

#### 2.2 Peters (1984) pricing and matching

Peters (1984) considers a game where proposals are made by individual sellers.<sup>2</sup> His model has price competition among sellers when there are capacity constraints and buyers have limited ability to visit sellers. Peters gives sufficient conditions under which the buyers' equilibrium varies continuously with the prices charged by sellers. Capacity constraints are used to guarantee the existence of (mixed strategy) equilibria for the pricing game played by sellers. Following the convention set by Mortensen, we call sellers workers and firms buyers.

In Peters model, a seller of labor (worker) advertises - via a posted wage - the expected value a buyer (firm) will obtain by choosing her location. The key assumption is that buyers make rational inferences about the expected queue lengths for each advertised posted wage. Therefore, a seller who advertises a wage  $\tilde{w}$  with an expected queue length  $\tilde{\lambda}$  has expected utility

$$u\left(\tilde{\lambda},\tilde{w}\right) \equiv \left(1 - \Pr\left\{X=0,\tilde{\lambda}\right\}\right)\tilde{w} + \Pr\left\{X=0,\tilde{\lambda}\right\}y\left(0\right)$$
(5)

while a buyer who chooses to approach this seller has expected utility

$$\pi(\tilde{w},\tilde{\lambda}) \equiv \frac{\left(1 - \Pr\left\{X = 0, \tilde{\lambda}\right\}\right)}{\tilde{\lambda}} \left(y\left(1\right) - \tilde{w}\right).$$
(6)

where  $\left(1 - \Pr\left\{X = 0, \tilde{\lambda}\right\}\right) / \tilde{\lambda} = \chi\left(\tilde{\lambda}, 1\right) / \tilde{\lambda}$  is the probability that this buyer (firm) is chosen by the seller (worker).

<sup>&</sup>lt;sup>2</sup>See also Montgomery (1991) and Burdett Shi and Wright (2001) for a related characterizations of this problem. Since each seller is a submarket, we can think of the buyers as choosing to participate in one of many possible submarkets with potentially different wages and queue lengths. Moen (1997) extends this argument to a larger class of matching environments, which includes matching functions that are not necessarily 'urn-ball'.



Figure 3: Equilibrium of the Peters (1984) pricing and matching game

We solve for the equilibrium posted prices in Peters' model using the so-called *market utility* property.<sup>3</sup> The equilibrium is characterized by two conditions:

1. The seller chooses to post a wage  $\tilde{w}$  and expected queue length  $\tilde{\lambda}$  in order to maximize her expected revenue,  $u\left(\tilde{\lambda}, \tilde{w}\right)$ , subject to the constraint that this seller offers a buyer (firm) an expected utility  $\pi(\tilde{w}, \tilde{\lambda})$  equal to the buyer's market utility,  $\bar{\pi}$ . The solution to the seller's program is given by:

$$\tilde{w}^*, \tilde{\lambda}^* = \underset{\tilde{w}, \tilde{\lambda}}{\operatorname{arg\,max}} u\left(\tilde{\lambda}, \tilde{w}\right)$$

s.t.

$$\pi(\tilde{w}, \tilde{\lambda}) \geq \overline{\pi};$$

2. The 'equilibrium' market utility is given by

$$\overline{\pi} = \pi(w^P, \lambda)$$

where  $(w^P, \lambda) = (\tilde{w}^*, \tilde{\lambda}^*)$  are the equilibrium posted wage and queue length, respectively.

The seller's (worker's) problem involves a trade-off between asking for a higher wage  $\tilde{w}$  and asking for a larger queue of buyers  $\tilde{\lambda}$ . We use figure 3 to illustrate that the seller's expected utility  $u\left(\tilde{\lambda}, \tilde{w}\right)$  is increasing and concave with respect to  $\tilde{w}$  and  $\tilde{\lambda}$  and that the buyer's expected utility  $\pi\left(\tilde{\lambda}, \tilde{w}\right)$  is decreasing with level sets that are concave to the origin. We also use this diagram to

 $<sup>^{3}</sup>$ See McAfee (1993), for example.

illustrate the unique equilibrium for  $w^P = \tilde{w}^*$  given that  $\tilde{\lambda}^* = \lambda$ .

The functional form for the unique solution for the equilibrium posted wage in Peter's pricing and matching game,  $w^P$ , is given by

$$w^{P} = (1 - \eta_{m}) \left( y \left( 1 \right) - y \left( 0 \right) \right)$$
(7)

where  $\eta_m$  elasticity with respect to the number of firm, which we defined earlier in equation (2).<sup>4</sup> Therefore, in Peters' model, with homogeneous buyers and sellers, there is a unique price for labor services.<sup>5</sup> By contrast, in Mortensen's model, the wage distribution is given by the function F(w)with upper and lower support, y(0) and  $\bar{w}$ , respectively. We also note that  $w^P$  gives a division of match surplus, which is equivalent to the previously described Hosios rule.

We substitute the unique equilibrium sellers' posted wage  $w^P$  into equations (5) and (6) to get expressions for the expected payoffs of buyers and sellers. With a little extra manipulation, these expected payoffs can be expressed as follows,

$$\begin{aligned} \pi^{Peters} &= \pi(w^{P}, \lambda) \\ &= (y(1) - y(0)) \Pr \{X = 0, \lambda\} \\ u^{Peters} &= u(w^{P}, \lambda) \\ &= y(0) + (1 - \Pr \{X = 0, \lambda\} - \Pr \{X = 1, \lambda\}) (y(1) - y(0)). \end{aligned}$$

where the functional form for  $\Pr \{X = x, \lambda\}$  is given by equation (1). We can then compare the expected payoffs of the buyers and the sellers in the Peters and the Mortensen pricing and matching games. We have the following equalities:

$$\pi^{Peters} = \pi^{Mortensen}$$
  
 $u^{Peters} = u^{Mortensen}$ .

Therefore, the expected payoffs of buyers and sellers are the same in both the Peters and Mortensen pricing and matching games.

### 2.3 McAfee (1993) pricing and matching

Now consider the auction game of McAfee (1993). Since sellers compete with respect to reserve prices, the distribution of buyer valuations is endogenous. However, using an argument similar to the previous subsection, it is straightforward to demonstrate that an auction with efficient reserve is the optimal mechanism from each seller's point of view. In other words, the buyers simply bid for the workers services subject to an auction with no reserve price.<sup>6</sup> Therefore, in equilibrium sellers hold identical auctions without reserve prices and buyers randomize over the sellers they visit.

 $<sup>^{4}</sup>$ Hosios (1990) notes that equilibrium prices in Peters (1984) satisfies this pricing rule. However, Hosios (1990) also makes a more well known insight that this 'Hosios rule' is not generally satisfied if pricing is determined by Nash bargaining, which is a key assumption of the benchmark model of equilibrium unemployment developed by Pissarides (2000).

 $<sup>{}^{5}</sup>$ It is crucial here that the posted prices direct the search of the buyers. If not, sellers would simply have an incentive to post a price of zero. This is, of course, a special example of the Diamond (1971) paradox.

<sup>&</sup>lt;sup>6</sup>For a related characterization of this efficient equilibrium reserve price, see Julien, Kennes and King (2000). Julien, Kennes and King (2007) compare the 'Mortensen' rule (which is implemented by an auction) to the Hosios rule.

The equilibrium bidding function of each firm is to bid a wage equal to the next highest valuation of the worker's services. Thus

$$w(X) = \begin{cases} y(1) & \text{if } X \ge 2\\ y(0) & \text{if } X = 1 \end{cases}.$$
 (8)

Therefore, if the bidder is alone at this auction, the bidder bids the seller's continuation value y(0), and if the bidder has a competitor, the bidder offers the entire surplus of the match to the seller. The equilibrium payoffs and probabilities of buyers and sellers are characterized by the simple bidding function, described by equation (8), and the probabilities that a seller has X offers (and the probabilities that a buyer has X competitors), described by equation (1). Here, the expected payoff of the buyer in McAfee's game of pricing and matching with auctions,  $\pi^{McAfee}$ , is given by

$$\pi^{McAfee} = \Pr \{ X = 0, \lambda \} (y(1) - y(0));$$

and the expected payoff of the worker,  $u^{McAfee}$ , is given by

$$u^{McAfee} = y(0) + (1 - \Pr\{X = 0, \lambda\} - \Pr\{X = 1, \lambda\})(y(1) - y(0))$$

Therefore, we have established the expected revenue equivalence of the different modes of proposing in the canonical urn-ball trading environment. This is summarized by the following equalities:

$$\pi^{McAfee} = \pi^{Mortensen}$$
$$= \pi^{Peters}$$
$$u^{McAfee} = u^{Mortensen}$$
$$= u^{Peters}$$

The distribution of wages in the auction model is, however, different than either the buyer or price posting models. In particular, we have a simple two step wage distribution that is characterized by equation (8) with probabilities for X = 1 and  $X \ge 2$ , which are described by equation (1).<sup>7</sup>

### 2.4 Auctions by buyers

Finally, for completeness, we briefly discuss the case where auctions are conducted by the buyers in this frictional environment. The analysis of this game is a special case of Mortensen (2003) pricing and matching. In particular, since the buyer (the auctioneer) knows that he will meet only one seller, the buyer will set a reserve price in this reverse auction that effectively becomes his posted price. The seller (the worker) will then decide to participate in the auction of the buyer (firm) that asks the lowest reserve price. Then, applying proposition 1 and solving for the equilibrium prices as in Mortensen's model, we find a distribution of reserve prices equivalent to the distribution of posted prices, which are given by the distribution function, equation (4), with identical upper and lower supports. Again, this mode of proposals gives the same expected payoffs as the previously

 $<sup>^7\</sup>mathrm{See}$  Kultti (1999) on the equivalence of auctions and seller price posting.

analyzed modes of price proposals.

## 3 Conclusions

This paper has compared different modes of price proposals in a static urn-ball matching market with homogenous buyers and sellers. We also state, without demonstrating here, that it is straightforward to extend the equivalence results to related urn-ball matching markets with heterogenous buyers and sellers.<sup>8</sup> Furthermore, we state, without demonstrating here, that the equivalence results are robust to related matching environments where buyers and sellers continue to seek matches in a series of subsequent matching rounds.<sup>9</sup>

We believe that one use of our comparative analysis of the canonical urn-ball environments is to offer a starting point for problems of market design where the market designer seeks to explore conditions for which the equivalence between the different modes of price proposals breaks down. A couple of important examples of such research applications are Eeckhout and Kircher (2010) and Einav, Farronato, Levin and Sundaresan (2016).<sup>10</sup>

Another use of our analysis is to organize related theoretical research that offers descriptive and normative results for each of these pricing and matching frameworks in isolation. For example, we also state without demonstrating here, that the properties of sorting and constrained efficiency in the seller posting model analyzed by Shimer (2005) are also applicable to the other pricing and matching models subject to similar assumptions about production functions, coordination frictions and distributions of buyers and sellers. This is useful for extensions such as the normative analysis of matching with a continuum of different buyer and seller types, where the competing auction model is, arguably, a more natural and tractable assumption about equilibrium pricing (Kennes and le Maire 2016).<sup>11</sup>

Finally, our results might be of interest for empirical analysis where the matching environment is an important part of the empirical identification strategy. For example, Abowd, Kramarz, Perez-Duarte and Schmutte's (2014) derive an empirically testable model of labor market sorting using a static urn-ball framework with seller price posting.<sup>12</sup> According to our equivalence result, their identification strategy is robust to alternative theories of pricing such as Mortensen (2003).

## References

 John M. Abowd, Francis Kramarz, Sébastien Pérez-Duarte and Ian M. Schmutte, (2014), "Sorting Between and Within Industries: A Testable Model of Assortative Matching," NBER Working Papers 20472, National Bureau of Economic Research, Inc.

<sup>&</sup>lt;sup>8</sup>Mortensen (2003) describes some of the basic properties of this pricing and matching problem (page 16-22) with respect to the buyer posting case. See also Shi (2002) and Shimer (2005) regarding the case of seller price posting with heterogenous agents. See also Kennes and le Maire (2016) regarding the case of competing auction equilibrium with heterogenous buyers and sellers.

 $<sup>^{9}</sup>$ See Kennes and le Maire (2016) for a dynamic analysis of McAfee's (1993) competing auction environment with heterogenous buyers and sellers.

 $<sup>^{10}</sup>$ Einav, Farronato, Levin and Sundaresan (2016) simply assume a fixed cost of conducting an auction. Eeckhout and Kircher (2010) consider the case where there is a restriction on how many offers can be entertained by the seller (i.e. the size of the 'urn' is limited).

<sup>&</sup>lt;sup>11</sup>Note that the two models have different implications regarding the stochastic distribution of prices.

 $<sup>^{12}</sup>$ A related theoretical analysis of sorting using auctions in a dynamic trading environment is described by Kennes and le Maire (2016).

- [2] Burdett, Kenneth and Kenneth Judd, (1983), "Equilibrium Price Dispersion," Econometrica, 51, issue 4, pp. 955-69.
- [3] Burdett, Kennth, Shouyong Shi and Randall Wright, (2001), "Pricing and Matching with Frictions," Journal of Political Economy, 109, no. 5, pp. 1060-1085.
- [4] Butters, G. R. (1977): "Equilibrium Distributions of Sales and Advertising Prices," Review of Economic Studies, 44(3), pp. 465–491.
- [5] Diamond, P (1971), "A Model of Price Adjustment," Journal of Economic Theory 3, pp.156-168.
- [6] Hosios, Arthur J., (1990) "On the Efficiency of Matching and Related Models of Search and Unemployment," The Review of Economic Studies Vol. 57, No. 2, pp. 279-298
- [7] Eeckhout, Jan and Philipp Kircher, (2010), Sorting versus screening: Search frictions and competing mechanisms, Journal of Economic Theory, 145, issue 4, pp. 1354-1385
- [8] Einav, Liran, Chiara Farronato, Jonathan Levin and Neel Sundaresany, (2016) Auctions versus Posted Prices in Online Markets, Stanford University working paper
- [9] Julien, Benoit, John Kennes and Ian King, (2000), "Bidding for Labor," Review of Economic Dynamics, vol. 3(4), pp. 619-649.
- [10] Julien, Benoit, John Kennes and Ian King, (2007), "The Mortensen rule and efficient coordination unemployment," Economics Letters, 90(2), pp. 149-155.
- [11] Kennes, John and Daniel le Maire (2016) "Competing Auctions of Skills," Economics Working Papers 2016-02, Department of Economics and Business Economics, Aarhus University.
- [12] Kultti, Klaus, 1999. "Equivalence of Auctions and Posted Prices," Games and Economic Behavior, Elsevier, vol. 27(1), pp. 106-113, April.
- [13] McAfee, R. Preston, (1993), Mechanism Design by Competing Sellers, Econometrica, 61, issue 6, pp. 1281-1312
- [14] Montgomery, James D., (1991), Equilibrium Wage Dispersion and Interindustry Wage Differentials, The Quarterly Journal of Economics, 106, issue 1, pp. 163-179
- [15] Mortensen, Dale T., (2003) Wage dispersion: why are similar workers paid differently? Cambridge, Mass, MIT Press.
- [16] Pissarides, C (1990/2000), Equilibrium Unemployment Theory, Basil Blackwell; second edition, MIT Press.
- [17] Peters, Michael, (1984), Bertrand Equilibrium with Capacity Constraints and Restricted Mobility, Econometrica, 52, issue 5, pp. 1117-27
- [18] Shi, Shouyong, (2002), A Directed Search Model of Inequality with Heterogeneous Skills and Skill-Biased Technology, Review of Economic Studies, 69, issue 2, pp. 467-491

[19] Shimer, Robert, (2005), The Assignment of Workers to Jobs in an Economy with Coordination Frictions, Journal of Political Economy, 113, issue 5, pp. 996-1025

#### Notes

A solution for the expected payoff of workers in Mortensen's pricing and matching game is as follows:

- 1. The expected firm profit of a given firm is given by:  $[y(1) y(0)] \exp(-\lambda)$
- 2. Total firm profits:  $[y(1) y(0)] \exp(-\lambda) m$
- 3. Total firm profits per worker:  $[y(1) y(0)] \exp(-\lambda) m/n = [y(1) y(0)] \lambda \exp(-\lambda)$
- 4. The expected worker productivity (including home production):  $\exp(-\lambda) y(0) + [1 \exp(-\lambda)] y(1)$

 $\begin{aligned} u^{Mortensen} &= \text{The expected worker productivity} - \text{Total firm profits per worker} \\ &= \exp(-\lambda) y(0) + [1 - \exp(-\lambda)] y(1) - [y(1) - y(0)] \lambda \exp(-\lambda) \\ &= y(0) + [y(1) - y(0)] [1 - \exp(-\lambda) - \lambda \exp(-\lambda)] \end{aligned}$ 

## **Economics Working Papers**

2015-23:	John Kennes, Daniel Monte and Norovsambuu Tumennasan: Dynamic Matching Markets and the Deferred Acceptance Mechanism
2015-24:	Anders Rosenstand Laugesen: Extensive Margins of Offshoring and Exporting
2015-25:	John Kennes and John Knowles: Liberalization of Birth Control and the Unmarried Share of Births - Evidence from Single Mothers in the Marriage Market
2015-26:	Benjamin Friedrich: Trade Shocks, Firm Hierarchies and Wage Inequality
2015-27:	Kenneth Lykke Sørensen: Active Labor Market Programs and Reservation Wages: Its a Hazard
2016-01:	Alexander K. Koch and Julia Nafziger: Correlates of Narrow Bracketing
2016-02:	John Kennes and Daniel le Maire: Competing Auctions of Skills
2016-03:	Mette Trier Damgaard and Christina Gravert: The hidden costs of nudging: Experimental evidence from reminders in fundraising
2016-04:	Sylvanus Kwaku Afesorgbor and Renuka Mahadevan: The Impact of Economic Sanctions on Income Inequality of Target States
2016-05:	Martin Paldam and Erich Gundlach: Jumps into democracy: The transition in the Polity Index
2016-06:	Erich Gundlach and Martin Paldam: Socioeconomic transitions as common dynamic processes
2016-07:	Rune V. Lesner: Testing for Statistical Discrimination based on Gender
2016-08:	Rune V. Lesner: The Long-Term Effect of Childhood Poverty
2016-09:	Sylvanus Kwaku Afesorgbor: Economic Diplomacy in Africa: The Impact of Regional Integration versus Bilateral Diplomacy on Bilateral Trade
2016-10:	John Kennes and Daniel le Maire: On the equivalence of buyer and seller proposals within canonical matching and pricing environments