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Abstract

This paper studies the mechanism by which club memberships, activities, and rules are chosen in a decentralized economy. For this purpose, we develop a model of competing auctions for club goods. This is a dynamic game where each seller first chooses an auction format; next, each buyer chooses a seller; and, finally, each buyer bids for membership. If sellers are restricted to simple auctions without reserve prices, then the decentralized equilibrium is generally inefficient across a multitude of important margins. However, if the sellers can compete by a broader class of auctions - and where the anonymity of buyer strategies implies coordination frictions - the sellers generally choose negative reserve prices and the equilibrium is constrained efficient. The advertisement of a negative reserve price is equivalent to the advertisement of an amenity that augments the value of the club good. The function of this amenity is to ensure a critical mass of bidders at the seller's location. For example, if a firm is attempting to assemble a team to utilize a club good at its location, it will also choose to advertise commitment to an additional amenity, such as a set of free health club memberships, as a means to attract a critical mass of applicants.

Key words: clubs, coordination frictions, Mortensen tax, Hosios rule, competing auctions

JEL classification: D71, D44, J64, D85

1 Introduction

James Buchanan (1965) introduced a theory of clubs to explain the distribution of memberships across organizations. In this theory, the three essential characteristics of a club good are

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excludability, congestibility and divisibility. Excludability means that individuals who do not contribute to the activities of the club can be excluded from gaining the benefits of club membership. Congestibility means that consumption of the club good eventually becomes rivalrous as the number of members increases. Divisibility means that, once a club membership has reached its optimal size, individuals who do not join can choose to join another club with similar attributes. The famous Tiebout model (Tiebout 1956) is a competitive analysis of club goods that is complementary to Buchanan's theory. In this model, excludability, by means of choosing a location, offers a mechanism by which the tragedy of the commons can be solved without appealing to the existence of a central authority.

The present paper explores the nature of decision making by associations in the context of a decentralized market environment for club goods. However, unlike the existing theoretical literature on club goods equilibrium (Berglas 1976, Bewley 1981, Scotchmer 2006, Wooders 2012), an important assumption of our analysis is that agents face coordination frictions when they choose between different clubs. This simplifies the assignment and price determination problems of the market, because the sellers of club goods then act as local monopolists who compete in mechanisms, and, importantly, the buyer reaction functions to these mechanisms are well behaved (McAfee 1993). In this analysis, we focus on the problem of a seller choosing the reserve price of a membership auction, where bidding in the auction sets the terms of trade between new and existing members. This problem yields a simple dynamic game to study the problem of mechanism selection: first, the sellers simultaneously choose their reserve prices; next, the buyers simultaneously choose a seller; and then, finally, the buyers bid for the good.

In the case of private goods, this problem of mechanism design by competing sellers has a well-known solution. This is explored by McAfee (1993), Julien (1997), Peters and Severinov (1997), Julien, Kennes and King (2000), and more recently by Albrecht, Gautier, and Vroman (2012). The basic result of the private goods model is that competing sellers will typically choose a reserve price of zero. Therefore, in equilibrium, it is in each seller's self interest to make a commitment not to appropriate their local market power as a means of ensuring the implementation of the optimal auction. Doing otherwise will lead to a reduction in the number of bidders, which then leads to a corresponding reduction to the expected revenue of the seller.

In our competing auction model, the sellers of club goods generally choose negative reserve prices. The role of the negative reserve price is to ensure a critical mass of bidders. A negative reserve price has the natural interpretation of an amenity that is used to 'sweeten the pot' if the local market has too few buyers. The problem, which is addressed by the seller in this case, is that shortfalls in the number of bidders leads to average product bids rather than marginal product bids, where marginal products are actually higher than average products when there are only a few bidders (By contrast, a private good's marginal valuation can never exceeds its average valuation, because consumption is everywhere rivalrous.). Thus a simple auction equilibrium offers too little returns to the agents who are choosing between alternative club goods, and too high returns to the incumbent associations who are engaged in the auctioning of club memberships. The sellers solve this problem with negative reserve prices, which make the average product of each club higher for all possible realizations of the number of bidders.[1]

We use our model to highlight some important margins by which the economy might fail to deliver optimal decisions if the clubs compete by simple auctions without amenities. One of these important margins is the number of clubs. Our theory predicts that a market with simple auction clubs (clubs that do not compete on reserve prices/amenities) will be subject to too many clubs. We also find that the importance of amenity competition in the competing auction model increases if clubs and club users become more specialized. Therefore, a market with simple auction clubs will be subject to too little specialization. Moreover, the decentralized market will be inefficiently biased towards pairwise arrangements and away from multilateral arrangements. We also explore how crowding in a club - the capacity of a club to absorb multiple teams - affects the advertised amenities of the clubs in equilibrium. Here, we find organizations that have a capacity to absorb many teams will offer greater total amenities than a small club. Therefore, a market with simple auction clubs will result in organizations with less capacity than is optimal. Finally, we show that a failure to implement the optimal auctions can also lead to incorrect incentives for the distribution of club users across organizations.

Our theory also permits a dynamic analysis of club good assignment. To construct this dynamic analysis, we first develop the static model to include incumbent club members and we allow for each additional club member to add value to the club according to a general club production function. We solve the entry and exit decision of incumbent and prospective members given that each member always bids their marginal contribution to the club where feasible. After we solve this model, it is straightforward to extend our analysis to a dynamic setting where agents anticipate opportunities to match in future periods. In the dynamic model, temporary arrangements between the sellers and buyers of club goods are possible in the event that the

[1] A closely related idea is contained in Mortensen's (2009) analysis of island matching. He suggests that a similar problem might be solved by means of a tax on 'islands'.

number of members is less than optimal. For example, if the club goods are chessboards and the agents seek partners to enjoy a match (or organize a tournament), then the dynamic model permits an analysis of the search for optimal club memberships such that some agents wait for others to arrive. One implication of this waiting model is that increased patience acts to convexify the payoffs of alternative membership numbers and thus reduces the importance of amenity advertisements as a means of attracting a critical mass of applicants. The key idea is that a club member can profit from being an existing member and thus the sale (and resale) of club goods becomes more like the private goods model.

The paper is organized as follows. In section 2, we present a canonical 'marriage model' that is loosely based on the analysis of 'island matching' by Mortensen (2009). In section 3, we develop a general competing auction model where the total value of the club is a general function of club membership and each club can start with an arbitrary number of incumbent members. In section 4, we extend this model to a dynamic environment. The final section offers concluding remarks.

2 A 'marriage' model

> "What kind of a guy can you be if you sit down every evening playing cards with a fellow who sits down to play cards with a guy like you!" (T. Reik, 1962, Jewish Wit)

Consider a symmetric economy where two large equal sized populations of identical specialist agents (for lack of better terminology we call them 'men' and 'women') must be matched to a large number of club goods in order to produce output. Each club good is located on an island and we assume that any number of clubs can freely enter this environment provided that each club pays a cost c. We normalize the total size of the male and female populations to one, and we let n denote the mass of clubs that enter this economy, where $q = 1/2n$ denotes the ratio of men/women to clubs. Each club operates subject to a strict capacity such that the output of a club with m male and w female members is given by

$$y(e) = \begin{cases} y & if \quad e \geq 1 \\ 0 & if \quad e = 0 \end{cases} \tag{1}$$

where $e = min\,[m, w]$ is the feasible number of couples at the club.

Suppose that all of the men and the women play identical mixed strategies when choosing between clubs, and that they choose to visit all clubs with equal probability. Thus the probability that each type of agent (male or female) visits a club is given by $\pi = 1/n$. Therefore, assuming a large market, the probability that a club is not contacted by a male (or a female) is given by e^{-q} where $q = 1/2n$ is the expected queue length of men and women at each club. The total number of matches is then approximated by the following function

$$X(q) = \left(1 - e^{-q}\right)^2 /q. \tag{2}$$

This matching function is first increasing, and then decreasing, as the number of clubs increases. The matching rate for an island (with a club) is $X\left(q\right)/n = x(q) = (1 - e^{-q})^2$.

2.1 The Hosios rule

The social planner maximizes the number of matches times output subject to the costs of entry for islands.

$$S_p^0 = \max_{\{q\}} \left\{X\left(q\right)y - c/q\right\}. \tag{3}$$

implies

$$F\left(q^o\right) \equiv -yx\left(q^o\right)\eta_{X,q^o} = c$$

where $\eta_{X,q} \equiv \frac{\partial X(q)q}{\partial q X(q)} = \frac{2e^{-q}q - \left(1-e^{-q}\right)}{1-e^{-q}} < 0$ is required for interior solution.

Lemma. *Define* $\bar{q}$ *such that* $X'\left(\overline{q}\right)$ *which implies* $\eta_{X,q} \mid \bar{q} = 0$*. Then, for any* $q > \bar{q}$*:*$X'\left(Q\right) < 0$ *and* $\eta_{X,q} < 0$*. Otherwise,* $X'\left(q\right) > 0$ *and* $\eta_{X,q} > 0$*.*

The elasticity term in the first order condition also highlights the fact that a simple sharing rule whereby clubs are rewarded a share of output equal to $|\eta_{X,q}|$, (i.e. the ***'Hosios rule'***) is sufficient to guarantee the efficient entry of clubs (Hosios 1990).

2.2 A negative result: The Mortensen tax

A ***simple club auction*** sells club memberships according to an auction without a reserve price (Mortensen 2009). In particular, suppose that m men and w women are on the island such that the feasible number of couples at the club is $e = min\left[m, w\right]$. Then, if the price of the club membership is determined by a Bertrand competition between the couples, the bidding function for club membership is given by

$$b(e) = \begin{cases} y \quad if \quad e \geq 2 \\ 0 \quad if \quad e = 0, 1 \end{cases} \tag{4}$$

In this case, a club earns a positive return of y only if there are multiple couples. Otherwise, a single couple extracts all of the match surplus. Since the club only earns a return if there are more couples than the capacity of the club, the free entry condition for clubs is given by

$$V(q) = \left(1 - e^{-q} - qe^{-q}\right)^2 y = c \tag{5}$$

where $(1 - e^{-q} - qe^{-q})^2$ is the probability that two or more couples will bid for club membership. This free entry condition can be rewritten as follows, $\eta_{X,q} x(q) y + (qe^{-q})^2 y = c$. Now compare this expression to the first-order condition of the social planning problem. Since $(qe^{-q})^2$ is positive and $\eta_{X,q} x(q)$ is decreasing in q, the decentralized equilibrium has more club entry than the social planner's solution. Therefore, as in Mortensen (2009), the problem of coordination is not solved by an 'island' matching model with random matching and simple auctions. The optimal ***'Mortensen tax'*** that solves this problem of inefficiency is given by $V(q^o) - F(q^o) = t$ where

$$t = \left(q^o e^{-q^o}\right)^2 y \tag{6}$$

which, coincidently, is the same rule proposed by Mortensen (2009). The required Mortensen tax will be even larger if the couple is given only a fraction of the match surplus in the event of a ***pairwise*** (couple-club) meeting. Therefore, there does not exist a ***bilateral sharing rule*** (The 'Hosios rule' or otherwise), which can implement efficient allocations in this environment.

2.3 A positive result: Competing auctions and 'negative' reserve prices

Suppose that sellers can set reserve prices as in the standard competing auction model of private goods (McAfee 1993). Here we allow for the possibility that the reserve price is negative (such that the clubs offers an amenity to the buyers). The timing of this competing auction game is as follows: Stage 1. Simultaneous entry of clubs; Stage 2. Simultaneous posting of the amenities d at each club; Stage 3. Men and women view the posted amenities at each club and make simultaneous choices over which club to visit; Stage 4. Couples bid against each other for club membership.

The model is solved by backwards induction. In the last stage the set of feasible couples bids for entry to the club. If the seller advertises an amenity d, the bidding function of these couples is given by

$$b(e) = \begin{cases} y+d & if \quad e \geq 2 \\ 0 & if \quad e = 0,1 \end{cases} \tag{7}$$

If there is a single couple bidding, then this couple extracts the entire surplus of the match including the value of the amenity. However, if there are multiple couples bidding, the seller recoups the cost of the amenity. Suppose that all $n = 1/2q$ clubs in this economy offer an identical amenity d such that the expected queue length at each club is q. In this case, the expected return to a female (or male) choosing such a club is given by

$$F(q,d) = \left(1 - e^{-q} - qe^{-q}\right) e^{-q} (y+d) + qe^{-q}e^{-q} \left(\frac{y+d}{2}\right) \tag{8}$$

where $(1 - e^{-q} - qe^{-q})\, e^{-q}$ is the probability that there is one female and multiple males such that the female gets the entire surplus of a match, and $qe^{-q}e^{-q}$ is the probability that there is only one female but also only one male, which then gives an equal share of match surplus. Suppose that one club (of measure zero) deviates and offers an amenity d'. The men and women can then choose between visiting this seller and the non-deviating sellers. In a mixed strategy equilibrium, the expected returns at this clubs and the clubs offering d must be the same. Thus $F(q,d) = F(q',d')$. Using this equality, we derive the following implicit relationship between amenities d and d' and the expected queue at the deviating seller.

$$d' = d(q', q, d) = \frac{F(q,d)}{\left(1 - e^{-q'} - \frac{q'e^{-q'}}{2}\right) e^{-q'}} - y \tag{9}$$

Application of the implicit function theorem gives $\partial q'/\partial d' > 0$.[2] Therefore, the reaction function of the buyers to changes in the posted amenity is well behaved. In particular, if a deviating club posts a higher amenity than the other clubs, the deviating club owner gets a higher probability of trade. The payoff to the club if it offers amenities d and enjoys an expected queue length q is given b

$$R(q,d) = \left(1 - e^{-q} - qe^{-q}\right)^2 y - d\left(2qe^{-q}\left(1 - e^{-q}\right) - \left(qe^{-q}\right)^2\right) \tag{10}$$

where $(1 - e^{-q} - qe^{-q})^2$ is the probability of multiple couples such that the club receives all

[2] With similar result, in a large market, we could also assume that each club offers a market utility (Peters 1991). We use this convention in future sections.

of the surplus including the value of the posted amenity, and $2qe^{-q}(1-e^{-q}) - (qe^{-q})^2$ is the probability of one couple such that club offers a free membership and is not compensated for the amenity. Now consider the problem of a (potentially) deviating seller when all other sellers choose an amenity of d. This deviating club's problem is a choice of q' such that $q^* = \arg\max_{q'} R(q', d(q', q, d))$ where $d(q', q, d)$ is the well behaved reaction function of the buyers. The first-order condition of this seller's problem is given by

$$\left(1-e^{-q*}\right)e^{-q*}y - \left(1-e^{-q} - \frac{qe^{-q}}{2}\right)e^{-q}\left(y + d(q^*, q, d)\right) = 0 \tag{11}$$

which, given the properties of $d(q', q, d)$, has a unique interior solution. In equilibrium, the expected queue length of the deviating club is equal to the expected queue length of the non-deviating clubs. Thus $q^* = q$. We then use the first-order condition of the seller's maximization problem to solve for the equilibrium amenity, d^*. Hence, $d^* = \left(\frac{q^* e^{-q^*}}{2}\right) y / \left(1 - e^{-q^*} - \frac{q^* e^{-q^*}}{2}\right)$. This gives expected revenue to the seller in the competing auction equilibrium of $R(q^*, d^*) = yx(q^*)|\eta_{X,q}|$. The expected revenue of a seller is equivalent to what they would earn in the simple auction equilibrium if they were also forced to pay the Mortensen tax. Therefore, the competing auction equilibrium with advertised amenities for the club goods implements this tax.

2.4 The Mortensen tax and 'the division of labor'

One experiment is study the relation between the Mortensen tax and the complexity of club activities. Suppose that there are z technologies where the type z club technology can produce y_z units of output if it is comprised of a team (team is now used in place of a couple) consisting of at least one of each z specialist types (Berglas 1976). Therefore, if m_i is the number of type $i \in \{1, ..., z\}$ agents at a type z club, the number of feasible teams at the club is $e_z = \arg\min\{m_1, ..., m_z\}$ and the output of the club is given by

$$y_z(e_z) = \begin{cases} y_z & if \quad e_z \geq 1 \\ 0 & if \quad e = 0 \end{cases} \tag{12}$$

We say that a club has ***higher complexity*** if z is larger. Implicit in this definition of complexity is the fact that a larger team is required in order to utilize a larger range of specialist club members.

To study how clubs of different complexity operate in the market, we first consider how the market operates if all clubs are of a complexity z. In this case, the unit mass of agents will choose to be equally divided into z specialists groups. Therefore, if a mass of $n_z = 1/q$ firms enter this market, the queue length of each specialist type at each club is q/z. The probability that a club is filled with a complete team of specialists is given by $x(q \mid z) = \left(1 - e^{-q/z}\right)^z$ and the total number of clubs with complete specialist teams is given by $X(q \mid z) = x(q \mid z)/q$. The social planner's problem is to choose the optimal number of clubs $n_z^o = 1/q_z^o$ that maximizes the total number of clubs with complete specialists teams times their output subject to the costs of club entry.

$$S_p^0(z) = \max_q \{X(q \mid z)\, y_z - c_z/q\} \tag{13}$$

implies

$$F(q_z^o \mid z) \equiv -yx(q_z^o)\,\eta_{X,q_z^o} = c_z$$

where $\eta_{X,q}(z) \equiv \frac{\partial X(q|z)q}{\partial q X(q|z)} = -\left(\left(1 - e^{-q_z^o}\right)^z - z\left(1 - e^{-q_z^o}\right)^{z-1} q_z^o e^{-q_z^o}\right) / \left(q_z^o\right)^2 <$ is required for interior solution. In a decentralized economy with simple club auctions, the number of clubs is given by the following free entry condition

$$V(q \mid z) = \left(1 - e^{-q} - qe^{-q}\right)^z y_z = c_z \tag{14}$$

where $(1 - e^{-q} - qe^{-q})^z$ is the probability that multiple teams bidding for membership. The optimal Mortensen tax that solves the problem of excessive club entry is given by $V(q^o \mid z) - F(q^o \mid z) = t_z$ where

$$t_z = \sum \binom{z}{i} \left(\frac{q_z^o}{z} e^{-q_z^o/z}\right)^i \left(1 - e^{-q_z^o/z}\right)^{z-i} y_z \tag{15}$$

which is a generalization of the $(z = 2)$ marriage model. Note also that the Mortensen tax is zero if the club has complexity $z = 1$, which is the standard model of private goods (McAfee 1993).

Now consider the following experiment. Suppose that there is a constant measure of clubs and that agents can free choose to specialize in response to the complexity of clubs. Assume also that the social planner is indifferent between the alternative technologies. We can then use the formula for t_z to establish the following proposition.

Proposition 1. *The Mortensen tax increases if club complexity/agent specialization increases*

Proof. Suppose that we hold constant constrained efficient economic output, $S_p^0(z) = S$ and the total number of clubs $n_z = n$. If the social planner is indifferent between different technologies, then the value of output according to technology type z must satisfy

$$y_z(S,n) = \frac{\left(1 - e^{-q_1^o}\right)}{\left(1 - e^{-q_z^o}\right)^z} y_1$$

where $q_z^o = z/n$. The formula for how the Mortensen tax changes as a function of club complexity is then given by substituting $y_z(S,n)$ into the equation for t_z to get

$$t_z(S,n) = \sum \begin{pmatrix} z \\ i \end{pmatrix} \left(\frac{q_z^o}{z} e^{-q_z^o/z}\right)^i \left(1 - e^{-q_z^o/z}\right)^{z-i} y_z(S,n) \tag{16}$$

where $q_z^o = z/n$. The solution of this formula is to impose a higher Mortensen tax as the complexity of the club, z, increases. □

A corollary to this proposition is that an economic environment with simple club auctions and no Mortensen taxes will generally be subject to too little complexity (too little specialization) , because the club users are not rewarded sufficient incentives for choosing more complex clubs. Moreover, this result implies that the simple club auction equilibrium will be subject to too many bilateral arrangements (clubs of complexity one) and too few multilateral arrangements (clubs of complexity two or more). In this sense, both the division of labor and the frequency of multilateral arrangements is limited by the ability of the simple auction market to coordinate trade in the absence of Mortensen taxes. An alternative solution to this coordination problem is a competing auction equilibrium where clubs post amenities that are conditioned on the complexity of the club. This is a straightforward extension of our analysis in section 2.3.

2.5 The Mortensen tax and club capacity

Another area of interest is to study the relation between the Mortensen tax and the club's level of 'crowding' (Wooders and Scotchmer 1987). Here, we will simply measure a club's 'crowding factor' by the number of 'teams' that can function within the umbrella of a club. This does not affect the nature of specialization in the market, but it allows for intermediate levels of production to occur if the number of teams within the club is less than the crowding factor of the club. For example, suppose that there are two types of specialists and a club with a crowding factor k attempts to accommodate the activities of e dyadic teams (couples). The output of a type k club with e dyadic teams is given by

$$y(e \mid k) = \begin{cases} ky_k & if \quad e > k \\ ey_k & if \quad e \leq k \end{cases} \tag{17}$$

where k is the maximum number of dyadic teams within the club's umbrella. A club type is said to be less subject to crowding (of larger size) if this crowding factor k is larger.

Consider the decentralized equilibrium if all clubs are of size k. Let $x_k(q)$ denote the number of trades that are expected at each island if the clubs are of size k and the expected queue length at the island is q. With some manipulation, it is possible to show that an increase in the size of the club holding constant their number increases the expected number of dyadic teams by the amount $\triangle x_k(q)$ such that

$$x_{k+1}(q) = x_k(q) + \triangle x_k(q)$$

where

$$\triangle x_k(q) = \left(1 - e^{-q} - qe^{-q} - \frac{q^2}{2}e^{-q} - ... - \frac{q^k}{k!}e^{-q}\right)^2.$$

By some further algebraic manipulations, the marginal contribution of additional clubs to the total number of dyadic teams is given by

$$\frac{\partial x_{k+1}}{\partial q} = \frac{\partial x_k}{\partial q} + 2\left(1 - e^{-q} - qe^{-q} - \frac{q^2}{2}e^{-q} - ... - \frac{q^k}{k!}e^{-q}\right)\frac{q^k}{k!}e^{-q}$$

Now consider how a social planner chooses the optimal number of clubs if club capacity is k. The social planner's problem is to choose the optimal number of clubs $n_k = 1/q_k$ that maximizes the total number of clubs with complete specialists teams times their output subject to the costs of club entry.

$$S_p^0(k) = \max_{q_k}\left\{\frac{x_k(q_k)}{q_k}y_k - \frac{c_k}{q_k}\right\} \tag{18}$$

implies

$$F(q_k^o \mid k) \equiv -y_k x(q_k^o)\,\eta_{X,q_k^o} = c_k$$

where $\eta_{X,q_k^o} < 0$ is required for an interior solution. In a decentralized economy with simple club auctions, the number of clubs is given by the following free entry condition

$$V(q \mid k) = y_k \triangle_k = c_k \tag{19}$$

where $\triangle x_k(q)$ is the probability that a club has more dyadic teams than capacity. The optimal Mortensen tax that solves the problem of excessive club entry is given by $V(q_k^o \mid k) - F(q_k^o \mid k) = t_k$ where

$$t_k = \sum_{i=1}^{k} i \left(\frac{(q_k^o)^i e^{-q_k^o}}{i!} \right)^2 \tag{20}$$

Now consider the following experiment. Suppose that clubs produce the same output per user $y_k = y$ and that the cost of club creation is constant $c_k = c$. We can then ask how the Mortensen tax changes if there is an increase in the state of club technology, k.

Proposition 2. *The Mortensen tax per club converges to infinity as the club's capacity k increases.*

Proof. If k gets large, then $q = 1/n \approx k$. Therefore, the Mortensen tax can be written as

$$t_k = \sum_{i=1}^{k} i \left(\frac{k^i e^{-k}}{i!} \right)^2$$

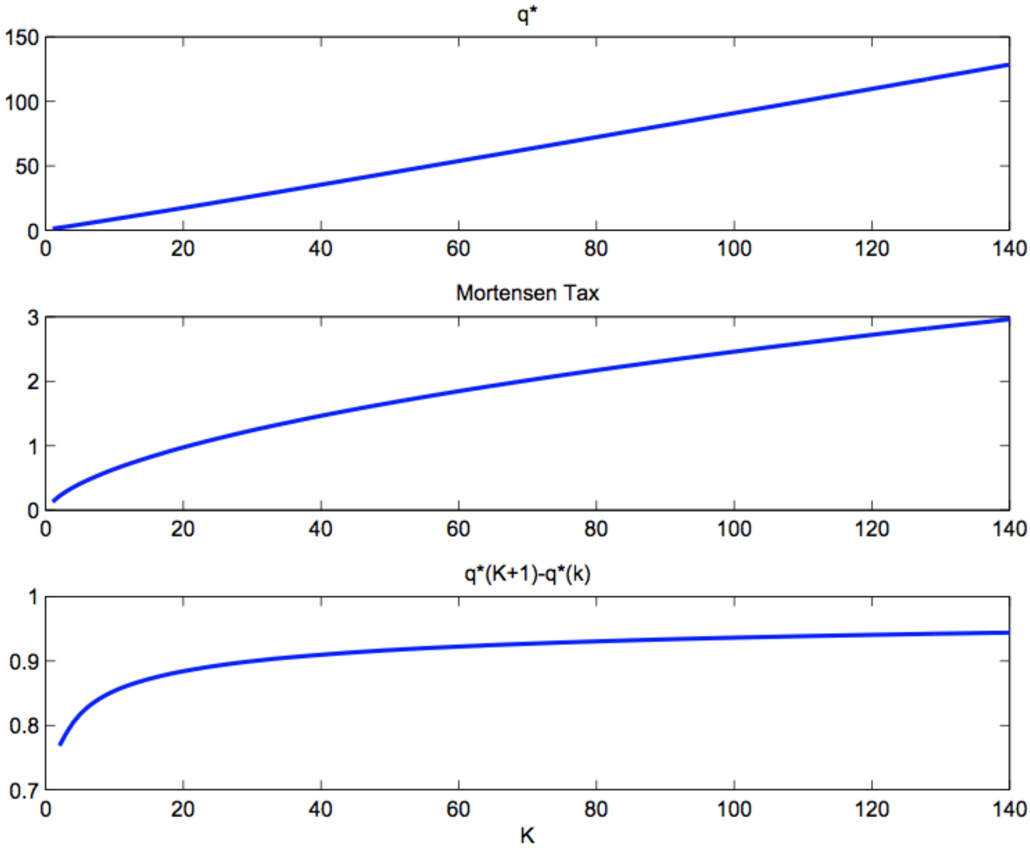
for large k. Using the Stirling approximation for $i!$, we solve t_k by

$$\begin{aligned} t_k &= \sum_{i=1}^{k} i \left(\frac{k^i e^{-k}}{i^i \sqrt{2\pi i}} e^i \right)^2 \\ &= \frac{1}{2\pi} \sum_{i=1}^{k} \left(\frac{k^{2i} e^{-2k}}{i^{2i}} e^{2i} \right) \\ &= \frac{1}{2\pi} \left(\frac{k^2 e^{-2k}}{1} e^2 + \frac{k^4 e^{-2k}}{2^4} e^4 + ... + \frac{k^{2(k-1)} e^{-2k}}{(k-1)^{2(k-1)}} e^{2(k-1)} + \frac{k^{2k} e^{-2k}}{k^{2k}} e^{2k} \right) \end{aligned}$$

Take the limit as $k \to \infty$, the last term goes to 1. The second to last goes to $e^2/e^2 = 1$... Hence the Mortensen tax goes to infinity. $\square$

As k gets large, the probability that the number of couples on one island is greater than k goes to zero. But since the queue length is lagging (i.e. $q \to k$) the probability remains sufficiently large so that the expected revenue goes to infinity. Hence, we need a very large tax to discourage entry. Figure 1 plots the Mortensen tax per unit of capacity and the total Mortensen taxes collected as a function of club capacity . Figure 1 illustrates that the tax as a fraction of the club's capacity is quickly decreasing. The figure also illustrates that the total quantity of Mortensen taxes is falling once the capacity is sufficiently large. In the limit as the capacity increases, the equilibrium converges to an environment with very few clubs (ultimately a single club) and that the total amount of Mortensen taxes as a fraction of economic output

Figure 1: The Mortensen tax and club capacity



approaches zero. In the limit as club span of control gets very large, the equilibrium is to have a single club. A Walrasian marketplace is the special case where the cost of operating this ultimate span of control club is zero.

2.6 The Mortensen tax and 'two-sided markets'

Suppose that the number of buyer of each type is asymmetric such that the planner must overcome a problem of two-sided markets (Rochet and Tirole 2003). In this case, we normalize the number of females to one and we let θ denote the number of males relative to females. Let q_m and q_f denote the average queue lengths of males and females at each club, respectively. Therefore, the number of matches is given by

$$X(q_f, q_m) = \frac{(1 - e^{-q_f})(1 - e^{-q_m})}{q_f} = \frac{x(q_f, q_m)}{q_f}$$

where $1/q_f$ is the number of clubs that enter, $x(q_f, q_m) \equiv (1 - e^{-q_f})(1 - e^{-q_m})$ is the probability that any particular club is filled, and $q_m/q_f = \theta$ is the number of males relative to females. The social planner's problem is to choose the optimal number of clubs $n = 1/q_f$ that maximizes the total number of clubs with complete specialists teams times their output subject to the costs of club entry.

$$S_p^0(\theta) = \max_{q_f} \left\{ \frac{x(q_f, q_f\theta)}{q_f} y - \frac{c}{q_f} \right\} \tag{21}$$

implies

$$F\left(q_f^o \mid \theta\right) \equiv -y_k x\left(q_f^o\right) \eta_{X,q_f^o} = c$$

where $\eta_{X,q_f^o} < 0$ is required for an interior solution. In a decentralized economy with simple club auctions, the number of clubs is given by the following free entry condition

$$V(q \mid \theta) = y\left(x(q_f, q_f\theta) - q_f \frac{\partial x}{\partial q_f}\right) = c, \tag{22}$$

The optimal Mortensen tax that solves the problem of excessive club entry is given by $V(q_k^o \mid k) - F(q_k^o \mid k) = t_\theta$ where

$$t_\theta = y q_f e^{-q_f} q_m e^{-q_m} \tag{23}$$

which is a generalization of our previous formula for a symmetric population of males and females. One complication, in this more general setting, is that we must reward males and females differently in order to ensure their participation. For example, if we solve the social planner's problem for a fixed number of clubs, and instead have entry of males and females, then we must reward a subsidy to men of $s_m = (1-\tau) y q_m q_f e^{-q_f} e^{-q_m}$ and a subsidy to women of $s_f = \tau y q_m q_f e^{-q_f} e^{-q_m}$ where the total value of this subsidy is equal to Mortensen tax, $s_m + s_f = t_\theta$ (The Mortensen tax is used to finance the subsidies). The following result proves that this is possible in an auction environment provided that there exists a weighted lottery, τ, for the right to be a contract proposer in the specific event that there is an equal number of males and females.

Proposition 3. *If there is an asymmetric number of males and females, the optimal proposer lottery, τ, is given by*

$$\tau = \frac{\frac{q_f}{q_m}\frac{\partial x}{\partial q_f}}{\frac{q_m}{q_f}\frac{\partial x}{\partial q_m} + \frac{q_f}{q_m}\frac{\partial x}{\partial q_f}}$$

Proof. Suppose that there is a posted amenity d such that the couples will get this additional surplus if there is only one couple. If the auction environment is to implement the social planners problem, the expected returns at the islands for males and females must equal the expected returns given by the planner. Thus

$$(y+d)\left(e^{-q_m}\left(1-e^{-q_f}\right)-(1-\tau)q_f e^{-q_f}e^{-q_m}\right)=ye^{-q_m}\left(1-e^{-q_f}\right)$$

$$(y+d)\left(e^{-q_f}\left(1-e^{-q_m}\right)-\tau q_m e^{-q_f}e^{-q_m}\right)=ye^{-q_f}\left(1-e^{-q_m}\right)$$

We can eliminate d and solve for the optimal lottery. □

Therefore, a two sided market can be implemented efficiently by a single Mortensen tax (buyer subsidy) provided that the outcome of Bertrand competition is modified by an optimal proposer lottery in the event of equal numbers of male and female bidders.

3 A 'roommate' model

"One's company, two's a crowd, and three's a party." Andy Wharhol

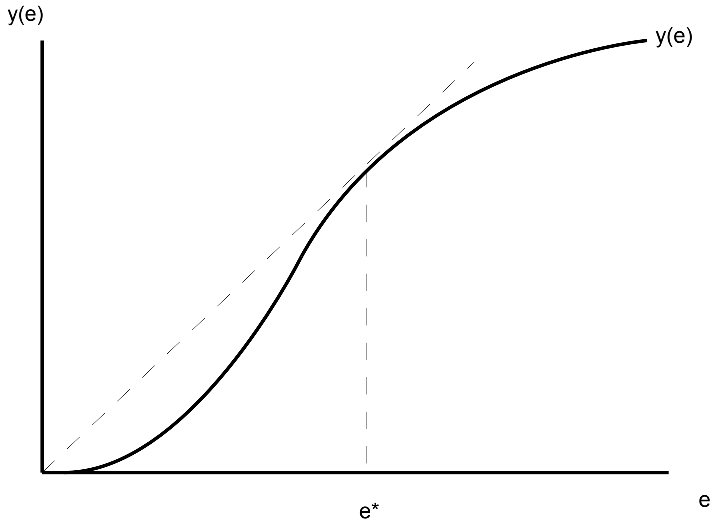
Consider a market with a large exogenous number of n club goods that are distributed across a set of clubs on separate islands. There is an initial distribution of agents across these islands and we let $n(e)$ to denote the number of islands at the start of the period with e incumbents. The total number of agents in this economy is determined by free entry and exit. Therefore, any agent who enters this economy at the start of the period must earn an expected return of W. It also means that any incumbent agent can always choose to leave the island and earn the same expected return as an unmatched agent.[3] A club produces $y(e+z)$ units of transferable utility, where z denotes the number of new members being welcomed unto the island. We assume that large clubs are subject to diminishing returns to scale and that smaller clubs may be subject to increasing returns to scale[4] Therefore, we let e^* denote the maximum output per agent at a club. Figure 2 depicts the function $y(e)$.

The problem of determining club membership and pricing in this economy is modeled as a three stage competing auction game. In the first stage, the clubs choose a selling mechanism (We start by restricting this to a simple auction without a reserve price). In the second stage, there

[3]Alternatively, we could specify a population size and solve for a value of W that is consistent with this parameter.

[4]This club payoff function is consistent with a conventional U shaped average cost curve for the production of output by a group where the costs are measured by the number of members in the group. The properties of this club payoff function are also consistent with the general trade-offs associated with many organizational theories, including Ostrum (2010), Williamson (1971), and Alchian and Demsetz (1972).

Figure 2: Club output as a function of club membership



is entry and exit of agents into this economy. In the third stage, the agents bid competitively for memberships. The model is solved by backwards induction.

3.1 Simple auctions

The bidding function of each agent for membership in a club is given by

$$b(e) = \begin{cases} \triangle_e & if \quad e \geq e^* \\ \frac{y(e)}{e} & if \quad e < e^* \end{cases} \tag{24}$$

where e is the number of bidders and $\Delta_e = y(e) - y(e-1)$ denotes the marginal productivity of the eth agent on an club. In the case of diminishing returns to scale, each bidder extracts their marginal contribution to the output of a club. This is an obvious requirement for efficient decisions within an organization, because the bidders are then rewarded their marginal returns to the value of a match (Refer to Mortensen 1982, Julien, Kennes and King (2008).[5] If the organization is subject to increasing returns, a bid by each agent for their marginal contribution is not feasible given the output of the club. Here we assume that bids are such that each member is rewarded their average contribution. Obviously, for each bidder, it is a best strategy to bid $y(e)/e$, if the others are bidding $y(e)/e$. Bidding higher (i.e. asking for a higher share of club output) than $y(e)/e$ is not feasible, because total output is $y(e)$, and bidding lower than $y(e)/e$

[5]This 'Mortensen rule' can also be derived by cooperative methods. In Wooders (2012) analysis of clubs, she notes that "The Shapley value of a game is the feasible outcome of a game in which all players are assigned their marginal contributions to a coalition when all orders of coalition are equally likely."

does not change the probability of acceptance, which is one if the bidder chooses $y(e)/e$.

Let $q(e)$ denote the expected queue length of entrants to a club with e incumbent members. This queue length is the outcome of free entry of agents subject to the assumption that these agents play common mixed strategies over which seller to visit in this market. Let $\hat{W}(e)$ be the expected utility of an incumbent member at a club with e incumbent agents. If the queue length for any particular club type is $q(e)$, the expected payoff of this incumbent agent is given by

$$\hat{W}(e, q(e)) = \sum_{z=0}^{\infty} b(e+z)\, Pr(z \mid q(e)) \tag{25}$$

where $Pr(z \mid q(e)) = (q(e))^z e^{-q(e)}/z!$ is the number of arriving bidders as a function of the expected queue length. Similarly, if an agents enters this economy, their expected payoff of choosing a club with e incumbent members is given by

$$W(e, q(e)) = \sum_{z=0}^{\infty} b(e+z+1)\, Pr(z \mid q(e)) \tag{26}$$

If the queue length at a club is positive, the free entry of agents gives $W = W(e, q(e))$. An agent in a club with e incumbents can choose to stay, $\sigma(e) = 0$, or exit $\sigma(e) = 1$. Therefore, the incumbent solves

$$\sigma^*(e \mid W) = \arg\max_{\sigma(e)} \left\{ (1 - \sigma(e))\hat{W}(e, W) + \sigma(e) W \right\} \tag{27}$$

If any agent chooses to remain at the club, we must have $\hat{W}(e) \geq W$ since agent can always quit.

3.2 The Mortensen tax and 'club turnover'

The social planner maximizes the value of a club subject to the alternative payoffs of each agent. Consider the solution to this problem for a club that has e incumbents. The social planners can add entrants a cost of W. The planner can also create exits and earn a return of W. The planner takes as given the problem of coordination. Therefore, the club chooses queue lengths q and exits x to maximize the following program

$$Z(e) = \max_{q,x} \left\{ \sum_{z=0}^{\infty} y(e - x + z)\, Pr(z \mid q) - Wq + Wx, 0 \right\} \tag{28}$$

such that $q, x \geq 0$. If the optimal queue length $q^*(e)$ is positive, the solution of the social planning problem is given by the following first-order condition,

$$W = \sum_{i=0}^{\infty} \left[y(e - x + i + 1) - y(e - x + i) \right] \frac{(q^*(e))^i}{i!} e^{-q^*(e)} \tag{29}$$

and the number of exits is equal to zero. If the optimal queue length is zero, the number of exits is determined by the condition that $\hat{W}(e - x, 0) \geq 0$. We can compare this solution to the outcome of the decentralized economy where clubs auction memberships with simple auctions.

Proposition 4. *If incumbent membership is greater than e^*, the social planning solution is equivalent to the decentralized equilibrium with simple auctions. Otherwise the social planner prefers a longer queue of buyers ($q^*(e) \geq q(e)$).*

Proof. Let $t(e)$ denote the difference between the right hand side of the social planner's first-order condition and the right hand side of the free entry condition, $W = W(e, q(e))$. This gives

$$t(e) = \sum_{i=0}^{\infty} \left[y(e - x + i + 1) - y(e - x + i) \right] \frac{(q^*(e))^i}{i!} e^{-(q^*(e))} - \sum b(e + z + 1) Pr(z \mid q^*)$$

$$= \sum_{i=0}^{e^* - e} \left[\triangle_e - \frac{y(e+i)}{e+i} \right] \frac{(q^*(e))^i}{i!} e^{-(q^*(e))} \tag{30}$$

If the club production function $y(e)$ is not subject to increasing returns, then there exists a solution to the simple auction model that is identical to the social planner. However, if there is a region of increasing returns to scale, where $\triangle_e > \frac{y(e)}{e}$, then $t(e)$ is positive if the queue length is equal to $q^*(e)$. The smaller payoffs in this region means that the overall queue length is shorter in the decentralized economy. □

The function $t(e)$ also give the subsidy on buyer entry (i.e. The Mortensen tax on clubs) that can be used to implement the social planner's solution for $q^*(e)$. If the social efficient queue length is positive, then the incumbents should not quit. Therefore, the same subsidy must also be rewarded to incumbents in order to ensure that they do not quit. Consequently, the implementation of the Mortensen tax/subsidy is complicated by the fact that similar transfers must also be rewarded to incumbents.

3.3 Implementation of the Mortensen tax by auctions

Consider a club that advertises an amenity of d. Therefore, the value of club output as a function of club membership is given by $y(e) + d$. Given this amenity, the bid function of an agent is given by

$$b(e,d) = \begin{cases} \triangle_e = b(e) & if \quad e \geq e^* \\ \frac{y(e)+d}{e} & if \quad e < e^* \end{cases} \tag{31}$$

where $\Delta_e = y(e) - y(e-1)$ denotes the marginal productivity of the eth agent on an club. Note that the amenity cause higher bids (greater demands for surplus) only if $e < e^*$. Otherwise, competitive bidding returns the entire cost of the amenity back to the club. Consequently, the amenity has no effect on the club earnings if the club is subject to diminishing returns to scale for all possible memberships. The formula for the queue length as a function of d must satisfy the free entry condition. Thus

$$W = W(e,d,q) = b(e+1,d)e^{-q(e,d)} + b(e+2,d)q\left(e,d\right)e^{-q(e,d)}$$

$$+b(e+3,d)\frac{q\left(e,d\right)^2}{2}e^{-q(e,d)} + b(e+4,d)\frac{q\left(e,d\right)^3}{3!}e^{-q(e,d)} + ...$$

Proposition 5. *The reaction function of buyers to the amenity of sellers is well behaved.*

Proof. We are required to prove that (i.e. $q'(d,e) \equiv \frac{\partial q'(d,e)}{\partial d} > 0$. Totally differentiate the function $W = W(e,d,q)$ as follows.

$$0 = \frac{1}{e+1}e^{-q(e,d)} - b(e+1,d)q'\left(e,d\right)e^{-q(e,d)}$$

$$+\frac{1}{e+2}q\left(e,d\right)e^{-q(e,d)} - b(e+2,d)\left[q'\left(e,d\right)e^{-q(e,d)} + q\left(e,d\right)q'\left(e,d\right)e^{-q(e,d)}\right]$$

$$+\frac{1}{e+3}\frac{q\left(e,d\right)^2}{2}e^{-q(e,d)} - b(e+3,d)\left[q'\left(e,d\right)q\left(e,d\right)e^{-q(e,d)} + \frac{q\left(e,d\right)^2}{2}q'\left(e,d\right)e^{-q(e,d)}\right]$$

Collect terms and write out the functional form of $q'\left(e,d\right)$. The function $q\left(e,d\right)$ is increasing in the amenity d, because the payoff of the buyer is made higher in every realization of membership when the club has increasing returns to scale, $e < e^*$, and this payoff is independent of the realization of membership when these realizations are in the region of decreasing returns to scale, $e > e^*$. $\square$

Given this well behaved reaction function between club queues and club amenities, we can prove the following proposition.

Proposition 6. *The competing auction equilibrium implements efficient entry of club members*

Proof. The seller's problem is given by

$$S(e) = \max_{d} \left\{ \sum_{z=0}^{\infty} \left(y\left(e+z\right)\right) Pr(z \mid q\left(d,e\right)) - Wq\left(d,e\right) \right\} \tag{32}$$

The first order condition of the seller's problem is given by

$$\left[\sum_{i=0}^{\infty} \left[y\left(e+i+1\right) - y\left(e+i\right)\right] \frac{\left(q^{**}\left(e\right)\right)^{i}}{i!} e^{-q^{**}(e)} \right] q'(d,e) = Wq'(d,e).$$

Thus

$$\sum_{i=0}^{\infty} \left[y\left(e-x+i+1\right) - y\left(e-x+i\right)\right] \frac{\left(q^{*}\left(e\right)\right)^{i}}{i!} e^{-q^{*}(e)} = W \tag{33}$$

which is the same condition as the social planner. $\square$

Obviously, an incumbent will also choose not to quit if the queue length is positive in equilibrium. Therefore, this decentralized equilibrium also implements efficient quitting if the queue length is positive. Moreover, if the queue length is zero, the quitting decision is also constrained efficient, because the equilibrium amenity is zero, and thus the private decision to quit is equivalent to that given by the social planner.

4 The dynamic model

The dynamic model is a repeated version of the static model with free entry of agents and a fixed number of clubs.[6] The agents are long lived risk utility maximizers with a common discount factor of β. Each unmatched agent has an opportunity cost of W. A club with e members continues to produce $y\left(e\right)$ units of output, but now each club is subject to a constant and random rate of failure δ such that all of the people using the club are forced to find a new club. The size of the club is bounded by the assumption that there exists a level of membership $\hat{e}$ such that $y\left(\hat{e}\right) \geq y\left(\hat{e}+z\right)$ for all $z \geq 0$.

[6] Alternatively, we can also assume that there is also a fixed number of agents and solve for the expected return to unmatched agent. See Kennes and Knowles (2012), for example.

4.1 Simple Auctions

Let $\Lambda(e)$ denote the joint value of a club good at the time that agents bid for participation in the club. The bidding function of each agent is given by

$$S(e) = \begin{cases} \triangle_e & if \quad e > e^* \\ \frac{\Lambda(e)-\Lambda(0)}{e} & if \quad e \leq e^* \end{cases} \tag{34}$$

where $S(e)$ denotes the bidder's share of joint value. Here we are assuming that the joint surplus of a club is well behaved with increasing returns to scale up to membership level e^* and constant or decreasing returns to scale beyond this point. Note that $e^* \leq \hat{e}$, because additional agents can contribute to match value only if additional workers contribute to output in the club.

The queue lengths at each club will be a function of the number of incumbent agents. In particular, as in the static model, the present value of an unmatched agent choosing to bid on an club membership with e incumbent agents is given by

$$W(e, q(e)) = \sum_{z=0}^{\infty} S(e+z+1)\, Pr(z \mid q(e)) + \beta W \tag{35}$$

where $Pr(z \mid q(e))$ is the same function that we used in in the static model. The expected payoff of an incumbent worker located at an club with e incumbents is given by

$$\hat{W}(e, q(e)) = \sum_{z=0}^{\infty} S(e+z)\, Pr(z \mid q(e)) + \beta W \tag{36}$$

If the queue length at a club is positive, the free entry of agents gives $W = W(e, q(e))$.[7] An agent in a club with e incumbents can choose to stay, $\sigma(e) = 0$, or exit $\sigma(e) = 1$. Therefore, the incumbent solves

$$\sigma^*(e \mid W) = \underset{\sigma(e)}{\arg\max} \left\{ (1-\sigma(e))\hat{W}(e, W) + \sigma(e) W \right\} \tag{37}$$

If any agent chooses to remain at the club, we must have $\hat{W}(e) \geq W$ since agent can always quit.

[7] Note, in the case of an increasing returns to scale club, if there exists a positive equilibrium queue length, there also generally exists another queue length that also solves equation (35). In this case, we can always focus on the longest queue length, which is equivalent to selecting the directed search equilibrium which is most favorable to buyers as in proposition 1.

Prior to the random assignment of agents, the expected joint value of a club is given by

$$J(e) = \sum_{z=0}^{\hat{e}-e} Pr(z \mid q(e)) \Lambda(e+z) \tag{38}$$

The joint value of a club that has $e+z$ members depends on the output of a club and whether the club continues next period. Since a club will admit all members up to the critical value $\hat{e}$, we let $\hat{y}(e+z) = \max\{y(e+z), y(\hat{e})\}$ denote the club output in the current period.

The asset equation for the joint value of a club with a combined number of e members (including entrants and incumbent) within a period (prior to the club activities in the present period) is given by

$$\Lambda(e) = \hat{y}(e) + \begin{cases} \beta\left[\delta(J(0)+eW) + (1-\delta) J(e)\right] & if \quad e \leq \bar{e} \\ \beta\left[\delta(J(0)+eW) + (1-\delta)\left(J(\bar{e}) + (e-\bar{e}) W\right)\right] & if \quad \hat{e} > e > \bar{e} \\ \beta\left[\delta(J(0)+\hat{e}W) + (1-\delta)\left(J(\bar{e}) + (\hat{e}-\bar{e}) W\right)\right] & if \quad e > \hat{e} \end{cases} \tag{39}$$

The joint value of a club is the club output and the continuation value of the club including the opportunities for rematching if the club is subject to dislocation shock. If the club is subject to a dislocation, then the joint value is the startup value of a club $J(0)$ and the expected value of each of its members in the current state, which is W. If the club is not subject to a dislocation shock, the following outcomes are dependent at how many agents are available to be members in the current period. If $e \leq \bar{e}$, then all workers work in the current period and all workers in a viable club are retained in the next period. If $e \in (\bar{e}, \hat{e})$, then all workers work in the current period, but only $\bar{e}$ of the workers are retained in the next period. If $e > \hat{e}$, then only $\hat{e}$ workers work in the current period and only $\bar{e}$ workers are retained in the next period. Workers who are not hired are not included in the calculation of joint value.[8]

The share that a club member gets depends on the current share $S(e)$ and the expected share in the future, which depends on the expected number of z new members new periods. We can let $f(e)$ denote the 'fee' paid by an agent for access to the club. This is related to $S(e)$, W,

[8] If we wish to think of the club good as a 'swimming pool', $\hat{e}$, denotes the maximum number of agents that will be allowed to swim in the period, and $\bar{e}$ denotes the maximum number of agents who will be offered a membership that continues into the next period.

and the function $q(e)$ as follows.

$$S(e) = y(e)/e - f(e) + \begin{cases} \beta\left[\delta W + (1-\delta)\sum_{z=0}^{\infty} Pr\left(z \mid q(e)\right) S(e+z)\right] & if \quad e \leq \bar{e} \\ \beta W & if \quad e > \bar{e} \end{cases} \tag{40}$$

Alternative, as in the static model, we can let the value $b(e) = y(e)/e - f(e)$ denote the agents output share of club output.

Straightforward to work out the distribution of team sizes given the queue lengths $q(e, W)$ and the team destruction rate δ. Let n denote the total number of clubs and let $n(e)$ denote the number of clubs with e incumbent members at the start of the period. Let $N(e+z)$ denote the number of clubs that have e incumbent members and z entrants. The number of clubs without members at the start of the period is given by $n(0) = \delta(1 - N(0)) + N(0)$, which is the unfilled clubs from last period and the number of clubs with members that were not dissolved. Any club that has $N(e)$ current members can survive into next period with probability $(1-\delta)$. Therefore, $n(e) = (1-\delta) N(e)$for all $e \geq 1$. Within the period, the club can increase (or not) in size by recruiting. Thus the number of clubs with e members in the current period is given by

$$N(e) = \sum N(i)\, Pr\left(z = e - i \mid q(e)\right) \tag{41}$$

where $Pr\left(z = e - i \mid q(e)\right)$ is the usual formula, which has the function form of equation (7). Given, the values of $q((e)$, we observe that the equations for $n(e)$ and $N(e)$ form a linear block recursive system of equations that depends on the parameters $Pr\left(z = e - i \mid q(e)\right)$. Block recursivity implies that the equations are linearly independent and thus this system of equations has a unique solution.

4.2 The Mortensen tax and repeated matching rounds

Consider a simple 'chess board' example where each club good is a chess board and two agents are needed to play a game of chess. In this example, the output of the club with a period is given by

$$y(e) = \begin{cases} 1 & if \quad e = 2 \\ 0 & if \quad e \neq 2 \end{cases} \tag{42}$$

The other parameters of the dynamic model are the discount factor β and the separation rate δ. We assume that there is a fixed number of clubs and players, such that the opportunity cost of each player is endogenously determined. (i.e. We simply solve for the value of W that is consistent with our choice of the population of players relative to clubs).

Consider the following experiment where we change the discount factor, β. Here, we assume that the ratio of buyer to sellers is held constant such that the queue lengths at the clubs are always optimal. The following table computes the value of a club with one member net of its value as a club with no members and the value of club with two members net of it value as a club with a single member divided by the number of club members. Since the clubs seek a critical mass of two members, the value of a club with no members is subject to increasing returns to scale since a club with only one member produces no output. Thus

$$(\Lambda(2) - \Lambda(1))/2 \geq \Lambda(1) - \Lambda(0)$$

In the static model, which is analogous to assuming that $\beta = 0$, the value of the club is given by $\Lambda(e) = y(e)$. If the discount factor increases, then $\Lambda(e)$ also increases to reflect the possible returns to future matching rounds. For example, suppose that we choose a separation rate of 10% and a ratio of buyers to sellers such that the constrained efficient model generates an unemployment rate of buyers equal to 10%. The club values as a function of the discount factor are computed in table 1. Table 1 illustrates the key limit result: If the discount factor approaches

Table 1: Changes in asset values as a function of discounting

β	$\Lambda(1) - \Lambda(0)$	$\frac{\Lambda(2)-\Lambda(1)}{2}$	$\Lambda(3) - \Lambda(2)$
0.999	1.4188	1.4436	0
0.998	1.3553	1.4043	0
0.995	1.1793	1.2959	0
0.99	0.9476	1.1561	0
0.98	0.7062	1.0178	0
0.95	0.4867	0.9022	0
0.9	0.3554	0.8292	0
0.8	0.2342	0.7464	0
.7	0.1684	0.6914	0
.6	0.1244	0.6494	0
.5	0.0919	0.6151	0
.4	0.0664	0.586	0
.3	0.0457	0.5608	0
.2	0.0283	0.5384	0
.1	0.0133	0.5184	0
0	0	0.5	0

one, the average asset value of a critical mass club approaches a constant.

$$\lim_{\beta\to 1}\frac{\left(\Lambda\left(2\right)-\Lambda\left(1\right)\right)/2}{\Lambda\left(1\right)-\Lambda\left(0\right)}=1$$

Therefore, opportunities for rematching function to linearize the net payoffs of a club relative to the number of members. The corollary to this result is that the Mortensen tax relative to the value of the club will fall to zero.

A closely related experiment is to increase the frequency of offer rounds holding the discount factor constant (De Fraja and Sakovics (2001)). In this case, we scale the value of output each period by the time available for playing chess given the rate of discounting and we scale the probability of a match shock inside the period to reflect the shorter period length.

Proposition 7. *The Mortensen tax approaches zero if the length of time between offer rounds falls to zero*

Proof. If the time between offer rounds falls to zero, then agents effectively do not discount future matching opportunities and thus $\left(\Lambda\left(2\right)-\Lambda\left(1\right)\right)/2 \cong \Lambda\left(1\right)-\Lambda\left(0\right)$. The asset value of a club is also bounded by $\Lambda\left(2\right) \leq y(1)/\beta$, which is the present value of a club that is filled in each period. Therefore, by proposition 4, the Mortensen tax must fall to zero as the length of time between offer rounds falls, $\square$

The intuition is that a single buyer can effectively purchase the 'chess board' and act as a private goods seller in the next period. This reduces the importance of establishing a critical mass of buyers in any particular period and thus the model functions more closely as a private goods model. Therefore, if we increase the opportunities for repeated matching, the competing auction environment will converge to the simple auction environment.

5 Conclusions

The problem of coordination frictions is a central explanation of an equilibrium DMP 'Diamond-Mortensen-Pissarides' matching technology (Diamond 1982, Mortensen 1982, Pissarides 1984, Rogerson, Shimer, and Wright 2007). In this paper, we demonstrate that sellers of club goods generally fail to implement efficient allocations with simple auctions in this environment. Therefore, in such an environment, policy instruments such as taxes on seller entry are needed to achieve socially efficient outcomes. A failure to implement these Mortensen taxes is also distinct from a failure to implement the Hosios rule. In our 'marriage model', for example, there is no bilateral sharing rule - between a couple and a club - that can implement efficient decisions. This result has broad implications for any analysis that proposes a microfounded model of matching to study organizational behavior. In particular, we cannot appeal to a Hosios rule to cleanly delineate the macroeconomic (extensive margin) questions of determining 'who matches with whom?' from the microeconomic (intensive margin) questions of determining 'who does what and for how much?'.

A failure to implement Mortensen taxes can lead to many undesirable outcomes. The decentralized economy may be subject to too many clubs/too few buyers and thus a problem of coordination concerning the finding of trading partners. A failure to implement these taxes can also lead to outcomes where organizations do not fully internalize the value of the gains to specialization. Similarly, it can lead to outcomes where organizations do not internalize the gains of capacity, and thus the span of control of an organization may be too small in equilibrium (Lucas 1978). Moreover, this failure of implementation can lead to outcomes where organizations do not recruit all sides of the market efficiently, which is the classic failure of two-sided markets (Rochet and Tirole 2003). In a dynamic setting, the failure to implement Mortensen taxes leads to inefficient turnover where agents do not fully internalize the benefits of either search, or the continuation of membership in a club that is presently below economy of scale.

We propose a solution to this coordination problem by extending the competing auction model of McAfee (1993) to allow for the sale of club goods. We believe that this model of organizations offers a foundation for a new theory of social interactions with coordination frictions where 1) agents can vote with their feet; 2) smaller organizational units have distinct advantages over larger units; and 3) local associations compete independently for new members through their choice of local rules of governance. In our competing auction model, local associations choose to compete by advertising positive amenities, which serve the purpose of raising the rents to new members when realized membership numbers are lower than expected. This mechanism is important in a market with coordination frictions, because local bidding gives a reward equal to the members' average product, rather than their marginal product, whenever membership numbers are small. The addition of amenities preserves the desirable properties of the auction format and enables rewards that are aligned with the agents' (and the clubs') marginal contributions.

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