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An Industry-Equilibrium Analysis of the LeChatelier Principle

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Abstract

By considering firms operating in a perfectly- or monopolistically-competitive industry with free entry, we show that well-established results on the celebrated LeChatelier principle (LCP) do not extend into an endogenous competitive environment. For instance, labour demand may be more elastic in the short run (where capital is fixed) than in the long run even if capital and labour are either complements or substitutes in profits. This may also be true locally at a point of long-run equilibrium. A novel insight is that industry-equilibrium effects introduce an asymmetry such that the LCP may hold for wage increases but not for wage decreases. These results are important for the interpretation of estimated labour-demand elasticities. Finally, we show that the LCP may hold for the total industry labour demand in situations where it does not hold for the labour demand of individual firms.

Keywords: Complements; Substitutes; Monopolistic Competition; Industry Equilibrium; Labour Demand; LeChatelier Principle

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1 Introduction

One cornerstone of comparative statics in economics is the LeChatelier principle (LCP) which arguably has its simplest expression when considering a profit-maximising firm choosing two inputs, say capital and labour. In this context, the LCP states that the elasticity of firms' labour demand with respect to the wage is smaller in magnitude in the short run, where the level of capital is fixed, than in the long run where capital is free to adjust. The requirement for this outcome to hold true is most clearly expressed by Milgrom and Roberts (1996): the LCP requires capital and labour to be complements (substitutes) in the dual sense that a reduction in the wage leads to increased (decreased) demand for capital and that an increase (decrease) in the use of capital leads to increased demand for labour. If this is the case, we say that capital and labour are complements (substitutes) in demand. The LCP holds under these conditions as capital and labour form a positive feedback system where allowing capital to adjust magnifies the adjustment of labour demand (Milgrom, 2006). The question then is to figure out under which conditions the two inputs are indeed complements or substitutes in demand.

Since Samuelson (1947) introduced the LCP into the field of economics, it has been known that the LCP always holds locally (at an initial point of long-run equilibrium) for the labour demand of a firm considered in isolation.[1] However, it quickly became clear that for noninfinitesimal changes in wages, the LCP does not always apply and examples that violate a global LCP now abound; see e.g. Samuelson (1960), de Meza (1981), Milgrom and Roberts (1994), and Milgrom and Roberts (1996). Ample attention has therefore been devoted to formulating conditions that ensure a global version of the LCP.[2] Notably, Milgrom and Roberts (1996) argue that if capital and labour are global complements (substitutes) in the profit function of the firm under scrutiny, then they are global complements (substitutes) in demand as well and consequently, the LCP holds globally for labour demand. This analysis, like most other studies of the LCP, relies on the firm operating in an exogenous competitive environment where the only endogenous variables affecting the firm's profits are its own choices of inputs. While insightful and providing a natural first step, such an approach is not fully satisfying. When considering exogenous shocks that affect all firms in an industry, such as a change

[1] Locally, the two inputs are always either complements or substitutes in demand.

[2] See e.g. Silberberg (1974), Milgrom and Roberts (1994), Milgrom and Roberts (1996), Roberts (1999), Suen et al. (2000), and Quah (2007).

in the wage, precluding the adjustments of competitors from influencing the choices of a given firm is dubious at best.

The present paper takes the next step in the analysis of the LCP by considering the labour demand of firms in industry equilibrium. In particular, we let firms operate in a perfectly- or monopolistically-competitive industry with free entry. Central to our analysis is the fact that exogenous changes in the wage now induce endogenous adjustments in the fierceness of competition. Importantly, the LCP may now fail to hold locally and we describe the conditions under which this is the case. Assuming that capital and labour are either global complements or global substitutes in profits,[3] we show that this assumption is no longer sufficient for these inputs to be global complements or global substitutes in demand. A global LCP is therefore no longer generally valid under these assumptions on the profit function. Further, our setup features cases where the LCP holds globally when the wage increases but not when the wage decreases. The possibilities of breakdowns and, perhaps especially, asymmetries in the LCP call for caution in both obtaining and interpreting estimates of labour-demand elasticities at different horisons. Finally, we note the possibility of a discrepancy between the LCP for firms' labour demand (as considered so far) and an LCP for aggregate labour demand in the industry.

The possible discrepancy between LCPs at the firm and industry levels has also been noted by Koebel and Laisney (2010). These authors focus predominantly on an aggregate LCP for an industry characterised by Cournot competition. As described above, the present paper is primarily concerned with the implications of endogenising the competitive environment for established results regarding the LCP for firms' input demand. While this is done in a setting general enough to encompass both perfect and monopolistic competition, our formulation of the industry equilibrium is admittedly rudimentary. This is intentional as a comprehensive analysis of the LCP in endogenous competitive environments is beyond the scope of this paper. Rather, our goal is to show that, even with our simple notion of industry equilibrium, the implications for the validity of the LCP are profound as evident from the results outlined above.

[3] This is not only a way to make the relation to the existing literature transparent. Milgrom and Roberts (1995) and Topkis (1995) argue that complementarities arise quite naturally between the various dimensions of firms' choices.

2 Setup

The industry under scrutiny is characterised by either perfect or monopolistic competition. Firm profits depend on one endogenous industry-wide variable outside the control of individual firms. We refer to this variable as the demand level, A.[4] Firms use two inputs in production namely capital and labour, K and L, which are in perfectly elastic supply. That is, the industry is small enough not to affect factor prices in the aggregate. Firm profits are given by

$$\pi(K, L; A, \beta) = R(K, L; A) - rK + \beta L - f,$$

where r is the interest rate, $-\beta = w$ is the wage rate, and f is a fixed cost.[5] We use the notation $\beta = -w$ for expositional convenience in the following. Since r and f will be kept constant throughout, we do not write profits as explicitly depending on these parameters. We consider symmetric equilibria and focus on a representative firm. We assume that the revenue function, R, is increasing in $(K, L; A)$ and that a higher value of A makes it more attractive to increase K and L, all else equal. Formally, the latter property means that R has increasing differences in $(K; A)$ and $(L; A)$ which, in turn, implies that R_{LA} and R_{KA} are nonnegative if R is smooth. Subscripts denote partial derivatives. We let K and L be either global complements or global substitutes in the profit function. That is, R, and thus also π, is either super- or submodular in K and L. Supermodularity (submodularity) implies that $R_{KL} \geq 0$ ($R_{KL} \leq 0$) holds globally if R is smooth.

In the following, we focus on the effects of changes in the wage, $-\beta$, on labour demand in the short run, where capital is fixed, and in the long run where capital can adjust. Profit maximisation gives us the optimal levels of K and L,[6]

$$L^*(K; A, \beta) = \arg\max_L \pi(K, L; A, \beta)$$

[4] Under perfect competition, A would simply be the price. Under monopolistic competition, A could e.g. be the demand shifter from the demand function arising from CES preferences; see Section 4.

[5] The fixed cost can e.g. be thought of as a fixed amount of capital, $\tilde{f}$, that must be rented for production to take place, i.e., $f = r\tilde{f}$. The results below will be qualitatively similar if f also comprises a fixed amount of labour.

[6] We implicitly assume that the maximisers exist and treat them as unique. Existence is ensured e.g. if K and L are both chosen from compact choice sets and π is upper semi-continuous in K and L (Milgrom and Roberts, 1996).

and

$$K^*(A,\beta) = \arg\max_K \pi(K, L^*(K;A,\beta);A,\beta).$$

It follows from our assumptions and the monotonicity theorem of Topkis (1978) that L^* is increasing in (A,β). In the case where K and L are complements (substitutes) in profits, K^* is increasing (decreasing) in β and L^* is increasing (decreasing) in K. Our analysis will make extensive use of these properties.

2.1 Industry Equilibrium

To assess the consequences of an endogenous competitive environment, we need to impose an equilibrium condition. We let this be free entry which requires that

$$\pi(K, L^*(K;A,\beta);A,\beta) = 0. \tag{1}$$

Assuming the existence of an equilibrium, this free-entry condition gives us the demand level, $A = A(K;\beta)$.[7] Note that $A(K;\beta)$ is decreasing in β as the left-hand side of (1) is increasing in (A,β). The free-entry condition, (1), is the natural choice for an equilibrium condition in the long run due to our focus on perfect or monopolistic competition. In addition, it is convenient as it only involves the already introduced profit function. We assume that the free-entry condition also holds in the short run. If new firms enter in the short run, they do so with the same (fixed) level of capital as incumbents. Abstracting from the possible complication of different equilibrium conditions at different horisons means that all short-to-long-run effects arising in our setup will solely be the consequence of capital becoming free to adjust. This makes our analysis more directly comparable to the existing literature on the LCP.

3 Comparative Statics

In the following, we restrict attention to cases where short-run labour demand is well-behaved in the sense that it decreases in the wage. That is, cases where

[7] The existence and uniqueness of an equilibrium are guaranteed by the intermediate value theorem if we e.g. let $R(K,L;0) = 0$ and $R(K,L;A)$ be continuous and strictly increasing in A with $R \to \infty$ as $A \to \infty$.

$\hat{L}^*(K;\beta) \equiv L^*(K;A(K,\beta),\beta)$ is increasing in β.[8] For the LCP to hold, the total long-run adjustment in L must exceed the short-run adjustment. As mentioned in the introduction, this requires K and L to be either: (i) complements in the dual sense that increasing β leads to an increase in K in the long run and L^* is increasing in K; or (ii) substitutes in the dual sense that increasing β leads to a reduction in K in the long run and L^* is decreasing in K. Importantly, these properties have to hold when we take the endogeneity of A into account. We refer to case (i) as K and L being complements in demand and to case (ii) as K and L being substitutes in demand. In either of these cases, K and L are said to be part of a positive feedback system where the adjustment in K magnifies the adjustment in L (Milgrom, 2006).

Consider briefly the case of an exogenous demand level, A. Treating A as fixed, it follows immediately from the properties of K^* and L^* that capital and labour being complements (substitutes) in profits implies that they are complements (substitutes) in demand and the LCP holds globally. This is the main result of Milgrom and Roberts (1996); a benchmark result from the related literature in which we take offset. As will become evident below, A being endogenous in our setup implies that the conditions for the LCP to hold (complements or substitutes in demand) do not translate as easily into requirements on primitives (complements or substitutes in profits).

3.1 Local LCP

To show that the endogeneity of A implies that K and L being complements in profits is not sufficient to ensure that they are also complements in demand, let us consider the conditions under which the LCP holds locally at an initial point of long-run equilibrium, assuming R is smooth. Since the results about increases and decreases in β are symmetric in this context, we focus on an increase in β, i.e., a reduction in the wage.

Consider the response in L when K is kept fixed. By the first-order

[8] This does not follow immediately as $A(K;\beta)$ is decreasing in β and L^* is increasing in (A,β). It is however always satisfied in the notable special case where $R(K,L;A) = A\tilde{R}(K,L)$ with $\tilde{R}$ being smooth. Such a revenue function arises under perfect competition where A is the price. Further, under monopolistic competition, it can arise from the combination of a production function and the demand function obtained from consumers having additively separable preferences over varieties in the industry.

condition for L, $R_L = -\beta$, this is given by[9]

$$\frac{\partial \hat{L}^*}{\partial \beta} = \frac{1}{-R_{LL}}(1 - \varepsilon_{R_L,A}\varepsilon_{A,\beta}), \tag{2}$$

where $\varepsilon_{x,y} \equiv \frac{\partial x}{\partial y}\frac{y}{x}$ is used to denote the elasticity of x with respect to y.[10] Let the second-order conditions be satisfied, wherefore R_{LL} is negative. By our focus on well-behaved demand functions, we know that (2) is positive. Note that the short-run increase in L is smaller in magnitude than if A had been exogenous ($\varepsilon_{A,\beta} = 0$).

Next, consider the total response in L when K is allowed to adjust,

$$\frac{\mathrm{d}\hat{L}^*}{\mathrm{d}\beta} = \frac{\partial \hat{L}^*}{\partial \beta} + \frac{\partial \hat{L}^*}{\partial K}\frac{\mathrm{d}K}{\mathrm{d}\beta}.$$

The LCP holds locally if $\frac{\partial \hat{L}^*}{\partial K}\frac{\mathrm{d}K}{\mathrm{d}\beta}$ is positive when evaluated at the initial equilibrium which corresponds to K and L being either (local) complements or (local) substitutes in demand. By the first-order condition for L,[11]

$$\frac{\partial \hat{L}^*}{\partial K} = \frac{R_{KL}}{-R_{LL}}. \tag{3}$$

This derivative is positive if K and L are complements in profits and negative if they are substitutes. Hence, if K and L are complements (substitutes) in profits, then the LCP holds locally only if they are also complements (substitutes) in the sense that an increase in β increases (decreases) K when the endogeneity of A is taken into account.[12] To determine whether this is the case, we consider the derivative,[13]

$$\frac{\mathrm{d}K}{\mathrm{d}\beta} = \frac{R_{KL}(1 - \varepsilon_{R_L,A}\varepsilon_{A,\beta})}{R_{KK}R_{LL} - R_{KL}^2}\left(1 + \frac{\varepsilon_{R_K,A}\varepsilon_{A,\beta}}{\varepsilon_{R_K,L}\varepsilon_{L,\beta}}\right), \tag{4}$$

where the second-order conditions imply that $R_{KK}R_{LL} > R_{KL}^2$. Take first the case of complements in profits ($R_{KL} > 0$). In this case, (4) must be positive

[9]See Appendix A for derivation.

[10]Thus, $\varepsilon_{R_L,A} = \frac{R_{LA}A}{R_L} \geq 0$ and $\varepsilon_{A,\beta} = \frac{\partial A}{\partial \beta}\frac{\beta}{A} \geq 0$. Using the first-order condition for L and (1), the latter can be expressed in terms of primitives as $\varepsilon_{A,\beta} = \frac{R_L L}{R_A A}$.

[11]Here, we have used that $\frac{\partial \pi}{\partial K} = 0$ holds locally, wherefore $\frac{\partial A}{\partial K} = 0$.

[12]In line with the existing literature, the LCP always holds trivially when $\frac{\mathrm{d}K}{\mathrm{d}\beta} = 0$.

[13]For a derivation of this expression, see Appendix A.

for the LCP to hold locally. This is only the case if $\varepsilon_{R_K,A}\varepsilon_{A,\beta} \leq \varepsilon_{R_K,L}|\varepsilon_{L,\beta}|$ where all terms are positive (after taking the absolute value of $\varepsilon_{L,\beta}$). This condition states that the positive effect of β through L on the marginal revenue product of K must be larger in magnitude than the negative effect through the fall in A on the marginal revenue product of K. When this is the case, an increase in β makes firms increase K when allowed to adjust and (4) is positive. The LCP therefore holds locally. On the other hand, if the negative effect of β through A on the marginal revenue product of K dominates the positive effect through L, then K falls when allowed to adjust. Thus, for the local LCP to hold under complements in profits, K and L must be sufficiently complementary such that the indirect effect of a falling demand level is dominated by the direct complementarity effect through L.

Next, consider the case of substitutes in profits ($R_{KL} < 0$). In this case, K always tends to fall when allowed to adjust after a wage decrease.[14] The reason is that both the decrease in A and the increase in L make the firm want to reduce K. Further, as established above, K and L being substitutes in profits implies that this reduction in K makes a firm want to increase L compared to the case where K is fixed. Locally, K and L are therefore always substitutes in demand when they are substitutes in profits. Hence, contrary to the case of complements in profits, the LCP always holds locally when K and L are substitutes in profits. Finally, by (3), it is clear that the LCP always holds locally in a weak form when K and L are independent in profits ($R_{KL} = 0$). The proposition below summarises these findings.[15]

Proposition 1. *The LCP always holds locally when K and L are either substitutes or independent in profits. The LCP holds locally when K and L are complements in profits if and only if $\varepsilon_{R_K,A}\varepsilon_{A,\beta} \leq \varepsilon_{R_K,L}|\varepsilon_{L,\beta}|$.*

3.2 Global Considerations

As we have just seen, the general decline in A following an increase in β may mean that K does not increase in the long run when K and L are complements in profits. Failure in this regard is the only reason for the LCP

[14] When $R_{KL} < 0$, both $\varepsilon_{R_K,A}\varepsilon_{A,\beta}$ and $\varepsilon_{R_K,L}\varepsilon_{L,\beta}$ are positive.

[15] Our focus on well-behaved short-run demand functions is not crucial for obtaining a breakdown of the local LCP. In case $\frac{\partial \hat{L}^*}{\partial \beta}$ is negative, one gets the following results. The local LCP always holds (in an inverted edition) when K and L are complements or independent in profits. When K and L are substitutes in profits, the local LCP holds (in an inverted edition) if and only if $\varepsilon_{R_K,A}\varepsilon_{A,\beta} \leq |\varepsilon_{R_K,L}|\varepsilon_{L,\beta}$.

to break down locally when short-run demand functions are well-behaved. Considering a global version of the LCP, another consideration becomes relevant. Since the purpose of adjusting K is to increase profits, the result is a short-to-long-run decline in A regardless of whether K increases or decreases. Locally, the change in K has no effect on profits (first-order condition) and hence A. However, for noninfinitesimal changes in β, this second-order effect interferes with the LCP.

Let us begin by considering increases in β which mean that the wage isreduced; reductions in β is treated separately in Section 3.3. Let A', A^c, and A'' represent the initial, short-run (constrained), and long-run values of A, respectively. It follows that when $\beta' < \beta''$,

$$A' \geq A^c \geq A''.$$

Further, let K' and K'' denote the initial and long-run equilibrium values of K, respectively. It follows that

$$\begin{aligned} L^*(K''; A'', \beta'') - L^*(K'; A^c, \beta'') = &\underbrace{L^*(K''; A'', \beta'') - L^*(K''; A^c, \beta'')}_{\text{Indirect effect of change in } K} \\ &+ \underbrace{L^*(K''; A^c, \beta'') - L^*(K'; A^c, \beta'')}_{\text{Direct effect of change in } K}. \end{aligned} \tag{5}$$

The previous section already discussed why the direct effect of the change in K could be negative in case of complements in profits. Whenever noninfinitesimal increases in β are considered, the indirect effect through A of changing K must be taken into account as well. Importantly, this effect is always negative since $A^c \geq A''$ and L^* is increasing in A. For the case of complements in profits, this is an additional reason the LCP may not hold globally besides K moving in the wrong direction. For the case of substitutes in profits, this is a reason that the LCP may not hold globally even though it always does so locally.

To illustrate how the indirect effect of the adjustment in K can result in a discrepancy between the local and global validity of the LCP, consider the case where K and L are independent in profits. That is, let π be simultaneously super- and submodular in K and L. When π is smooth, this implies $\pi_{KL} = 0$. In this case, the LCP holds locally; see Proposition 1. However, globally the LCP breaks down. To see why, simply note that the direct effect of the change in K is zero such that (5) is given by the negative indirect effect.

Proposition 2. *For noninfinitesimal increases in β, the LCP does not hold when K and L are independent in profits and the short-to-long-run adjustment in L is nontrivial.*

One could easily imagine that the negative indirect effect of the change in K in (5) dominates whenever K and L are sufficiently weak complements or substitutes in profits. That is, even though the LCP always holds locally for K and L being substitutes, it may not hold globally if they are only weakly so. The example of Section 4 confirms this possibility.

3.3 Decreasing β: Asymmetry in the LCP

In this section, we consider noninfinitesimal reductions in β, i.e., discrete increases in the wage. First note that a reduction in β means that A is larger than initially both in the short and the long run. However, the fact that K can adjust in the long run still implies that A must be lower in the long run relative to the short run. When $\beta' > \beta''$, we therefore get the following ranking of the initial, short-run, and long-run demand levels,

$$A' \leq A'' \leq A^c.$$

Note that a decrease in β reduces L in the short run due to our focus on well-behaved demand functions. Thus, in this case, the LCP holds if L is reduced even further in the long run. But this means that the fact that adjusting K reduces A from the short to the long run works in favour of the LCP holding in contrast to the case where β was increased. In certain cases, the LCP may therefore hold for reductions in β but not for increases. The asymmetry between increases and decreases in β with respect to the LCP can be clearly illustrated if we revisit the case where K and L are independent in profits. In this case, (5) is again given by the negative indirect effect. This means that the short-run reduction in L is magnified in the long run and that the LCP holds.[16]

Proposition 3. *For noninfinitesimal reductions in β, the LCP holds when K and L are independent in profits.*

[16]Note that when K and L are very close to being independent, the reduction in β can increase K in the long run and this adjustment in K can make L fall further than in the short run. That is, K and L can behave as substitutes in demand, regardless of whether they are (very weak) complements or substitutes in profits. Section 4 confirms this possibility.

Propositions 2 and 3 give rise to the following corollary.

Corollary 1. *Whether the LCP holds or not may depend on the direction of change in β.*

Again, one could imagine that, for the same reasons leading to Proposition 3, the LCP holds for reductions in β whenever K and L are sufficiently weak complements or substitutes in profits. That is, asymmetry in the LCP may not be confined to the case of K and L being independent in profits. That there indeed can be asymmetries in the LCP for K and L being weak complements or weak substitutes is confirmed by the example of Section 4.

3.4 LCP at the Industry Level

The last point we want to make before considering an example is that the LCP may hold at the industry level even in cases where it does not hold for individual firms. In order to illustrate this possibility as simply as possible, we let expenditure in the industry be exogenously given by E. Then we get the number of firms as $M = E/R(K, L; A)$ and the total use of labour in the industry is $LM = EL/R(K, L; A)$. It should be clear that endogenous short- and long-run changes in M can cause discrepancies between an LCP for L and one for LM. The following proposition makes this possibility clear.

Proposition 4. *Let $R(K, L; A) = A\tilde{R}(K, L)$. Then the LCP always holds locally at the industry level (for LM) even though it may not do so at the firm level (for L).*

Proof. See Appendix B. □

To see the intuition for Proposition 4, recall the reason that the LCP may not hold locally at the firm level. When K and L are complements in profits, a breakdown happens when K fails to rise following an increase in β. In this case, L rises less when K is allowed to adjust since a decline in K tends to induce a decline in L (complements). However, the (short-to-long-run) declines in K and L also tend to reduce revenue of the individual firm which means that the number of firms must rise (total industry revenues are constant). Thus, whenever there is a force working against the LCP holding locally for individual firms, the same force tends to increase the number of firms. When $R(K, L; A) = A\tilde{R}(K, L)$, which is the case under perfect competition and under monopolistic competition when consumers have additively separable utility across varieties, it turns out that the local LCP at the industry level (for LM) always holds.

4 Example

We conclude our analysis of the LCP by considering an example that illustrates many of the points discussed above. Assume that the revenue function, $R(K, L; A)$, is obtained by combining the isoelastic inverse demand function, $p = Aq^{\rho-1}$, which could originate from consumers with CES preferences across varieties in the industry, with the production function,

$$q = \left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $\sigma > 0$ is the elasticity of substitution in production and $0 < \alpha, \rho < 1$. Profits are thus given by

$$\pi = A\left[\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}} - rK + \beta L - f.$$

Note that these assumptions give rise to a revenue function of the form $R(K, L; A) = A\tilde{R}(K, L)$. Capital and labour are complementary in profits ($\pi_{KL} > 0$) when $\sigma \in (0, \frac{1}{1-\rho})$ and substitutes ($\pi_{KL} < 0$) for $\sigma \in (\frac{1}{1-\rho}, \infty)$.[17] When $\sigma = \frac{1}{1-\rho}$, capital and labour are independent in profits ($\pi_{KL} = 0$). First off, following an increase (reduction) in β, L is larger (smaller) in both the long and the short run when compared to the initial value, regardless of whether K and L are complements, substitutes, or independent in profits.[18] Appendix C shows that the LCP always holds globally if K and L are sufficiently strong complements, $\sigma \in (0, 1)$, regardless of whether β increases or decreases. However, when $\sigma \in (1, \infty)$, the LCP may or may not hold and there may be asymmetries between increases and decreases in β.

The upper half of Figure 1 depicts the total long-run adjustment relative to the short-run adjustment in L for an increase and a reduction in the wage, $w = -\beta$, of 20 percent for different values of σ. In both cases, the LCP holds whenever the relative adjustment is above one. For the increase in

[17] The concavity introduced by ρ implies that K and L are not complements in the profit function for all $\sigma > 0$ even though they are always complements in the production function.

[18] See Appendix C.

β, the LCP holds whenever K and L are sufficiently strong complements or sufficiently strong substitutes in production. When they are independent, the LCP fails to hold (Proposition 2). Further, the LCP fails to hold when K and L are weak complements or weak substitutes. For the reduction in β, the LCP holds both if K and L are strong complements, $\sigma < 1$, and if they are very weak complements, independent (Proposition 3), or substitutes in profits. For the remaining intermediate values of σ, the complementarity is not strong enough to make K fall when β is reduced but still strong enough to make L increase when K increases.[19] This means that L increases from the short to the long run and thus exhibits undershooting. An asymmetry given by the fact that the LCP holds for reductions in β but not for increases (Corollary 1) is clear in the figure for any value of σ close to $\frac{1}{1-\rho}$. That is, when K and L are very weak complements or substitutes in profits.

The bottom half of Figure 1 shows the total long-run adjustment relative to the short-run adjustment in LM. Relating to the local result of Proposition 4, it is clearly seen that the LCP holds at the industry level, both for the increase and for the decrease in β for all values of σ considered, independently of whether the LCP holds at the firm level or not.

5 Concluding Remarks

The present paper has shown that introduction of even very simple industry-equilibrium effects have important implications for the LCP. While research interest in the LCP has somewhat waned in recent years, our contribution emphasises that there is more work to be done in understanding the conditions under which the LCP applies in more realistic environments. More rigorous inquiry into the implications of the specific nature of the competitive environment for the validity of the LCP (both locally and globally) seems like a promising area for future research.

[19] I.e., the complementarity is strong enough to dominate the negative indirect effect of the increase in K; see (5).

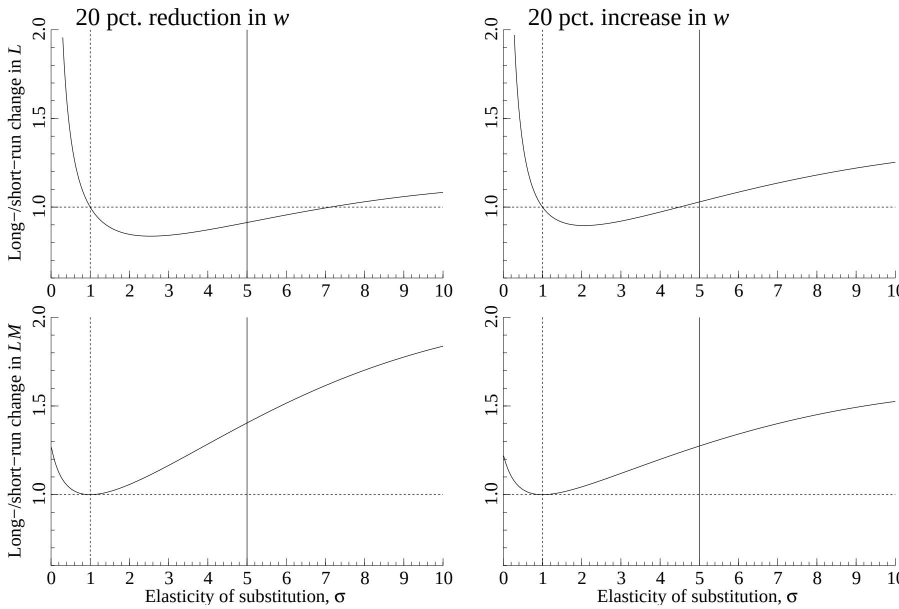

Figure 1: Long-run changes in L (top half) and LM (bottom half) relative to their short-run changes for different values of σ. Whenever the graph is above 1, the LCP holds. The vertical dashed line indicates $\sigma = 1$ and the vertical solid line indicates independence between K and L, $\sigma = \frac{1}{1-\rho}$.

A Derivation of (2) and (4)

To derive (2), we total differentiate the first-order condition $R_L = -\beta$ with respect to β, using that locally $\frac{\partial A}{\partial L} = 0$,

$$R_{AL}\frac{\partial A}{\partial \beta} + R_{LL}\frac{\partial \hat{L}^*}{\partial \beta} = -1.$$

Rearranging, using the first-order condition $R_L = -\beta$, gives (2).

To derive (4), we total differentiate the first-order conditions, $R_K = r$ and $R_L = -\beta$, with respect to β,

$$R_{LL}\frac{\mathrm{d}L}{\mathrm{d}\beta} + R_{KL}\frac{\mathrm{d}K}{\mathrm{d}\beta} = -(1 - \varepsilon_{R_L,A}\varepsilon_{A,\beta}),$$
$$R_{KL}\frac{\mathrm{d}L}{\mathrm{d}\beta} + R_{KK}\frac{\mathrm{d}K}{\mathrm{d}\beta} = -R_{AK}\frac{\partial A}{\partial \beta}.$$

Using Cramer's Rule,

$$\frac{\mathrm{d}K}{\mathrm{d}\beta} = \frac{R_{KL}(1 - \varepsilon_{R_L,A}\varepsilon_{A,\beta}) - R_{AK}R_{LL}\frac{\partial A}{\partial \beta}}{R_{KK}R_{LL} - R_{KL}^2}.$$

Rearranging using the definitions of elasticities and (2) yields (4).

B Local LCP at the Industry Level

This appendix shows that the LCP always holds locally at the industry level, i.e., for the aggregate use of labour, LM, in the industry when $R(K, L; A) = A\tilde{R}(K, L)$. Due to symmetry across the homogeneous firms, we get $A\tilde{R}(K, L)M = E$. From this, we get the aggregate use of labour expressed as

$$LM(K; \beta) = \frac{E\hat{L}^*(K; \beta)}{A(K; \beta)\tilde{R}(K, \hat{L}^*(K; \beta))}.$$

To see that the LCP holds locally for LM, we first note that

$$\begin{aligned} \frac{\partial LM}{\partial \beta} &= \frac{AE\tilde{R}\frac{\partial \hat{L}^*}{\partial \beta} - EL\tilde{R}\frac{\partial A}{\partial \beta} - AEL\tilde{R}_L\frac{\partial \hat{L}^*}{\partial \beta}}{(A\tilde{R})^2} \\ &= \frac{E(A\tilde{R} + \beta L)\frac{\partial \hat{L}^*}{\partial \beta} + EL^2}{(A\tilde{R})^2} > 0, \end{aligned}$$

where we have used $A\tilde{R}_L = -\beta$, $\frac{\partial A}{\partial \beta} = -\frac{L}{\tilde{R}}$, and the fact that $\frac{\partial \hat{L}^*}{\partial \beta} > 0$ in the case we consider here. The LCP will hold locally for LM if

$$\frac{\mathrm{d}LM}{\mathrm{d}\beta} - \frac{\partial LM}{\partial \beta} = \frac{\partial LM}{\partial K}\frac{\mathrm{d}K}{\mathrm{d}\beta} \tag{7}$$

is positive. To see this is the case, note that

$$\begin{aligned} \frac{\partial LM}{\partial K} &= \frac{AE\tilde{R}\frac{\partial \hat{L}^*}{\partial K} - AEL(\tilde{R}_K + \tilde{R}_L\frac{\partial \hat{L}^*}{\partial K})}{(A\tilde{R})^2} \\ &= \frac{E}{A\tilde{R}}\frac{\tilde{R}_{KL}}{-\tilde{R}_{LL}}\left(1 - \frac{\tilde{R}_L L}{\tilde{R}} - \frac{-\tilde{R}_{LL}L\tilde{R}_K}{\tilde{R}_{KL}\tilde{R}}\right). \end{aligned} \tag{8}$$

Now, consider $\frac{\mathrm{d}K}{\mathrm{d}\beta}$, which can be obtained by simplifying (4) using $R = A\tilde{R}$,

$$\frac{\mathrm{d}K}{\mathrm{d}\beta} = \frac{\tilde{R}_{KL}}{A(\tilde{R}_{KK}\tilde{R}_{LL} - \tilde{R}_{KL}^2)} \left(1 - \frac{\tilde{R}_L L}{\tilde{R}} - \frac{-\tilde{R}_{LL} L \tilde{R}_K}{\tilde{R}_{KL}\tilde{R}}\right). \tag{9}$$

It is obvious that (8) and (9) have the same sign such that (7) is positive and the LCP always holds locally at the industry level under the considered functional form of $R(K, L; A)$.

C Comparative Statics in the CES Example

The first-order conditions for K and L are given by

$$\pi_K = A\rho\alpha \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}-1} K^{-\frac{1}{\sigma}} - r = 0, \tag{10}$$

$$\pi_L = A\rho(1-\alpha) \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}-1} L^{-\frac{1}{\sigma}} + \beta = 0. \tag{11}$$

The second-order derivatives are given by

$$\begin{aligned}
\pi_{KK} &= A\rho\alpha^2 \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}-2} K^{-\frac{2}{\sigma}} \left(\rho - 1 - \tfrac{1-\alpha}{\alpha\sigma}\left(\tfrac{L}{K}\right)^{\frac{\sigma-1}{\sigma}}\right), \\
\pi_{LL} &= A\rho(1-\alpha)^2 \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}-2} L^{-\frac{2}{\sigma}} \left(\rho - 1 - \tfrac{\alpha}{(1-\alpha)\sigma}\left(\tfrac{K}{L}\right)^{\frac{\sigma-1}{\sigma}}\right), \\
\pi_{KL} &= A\rho\alpha(1-\alpha) \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\rho\sigma}{\sigma-1}-2} (LK)^{-\frac{1}{\sigma}} \left(\rho - 1 + \tfrac{1}{\sigma}\right).
\end{aligned} \tag{12}$$

It is clear that $\pi_{LL}, \pi_{KK} < 0$ and further, we have

$$\begin{aligned}
\pi_{LL}\pi_{KK} - \pi_{KL}^2 &= A^2\rho^2\alpha^2(1-\alpha)^2 \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{2\frac{\rho\sigma}{\sigma-1}-4} L^{-\frac{2}{\sigma}} K^{-\frac{2}{\sigma}} \\
&\quad \left[\left(\rho - 1 - \tfrac{\alpha}{(1-\alpha)\sigma}\left(\tfrac{K}{L}\right)^{\frac{\sigma-1}{\sigma}}\right)\left(\rho - 1 - \tfrac{1-\alpha}{\alpha\sigma}\left(\tfrac{L}{K}\right)^{\frac{\sigma-1}{\sigma}}\right) - (\rho - 1 + \tfrac{1}{\sigma})^2\right] \\
&= A^2\rho^2\alpha^2(1-\alpha)^2 \left[(1-\alpha)L^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}}\right]^{2\frac{\rho\sigma}{\sigma-1}-4} L^{-\frac{2}{\sigma}} K^{-\frac{2}{\sigma}} \\
&\quad (1-\rho)\left[\tfrac{2}{\sigma} + \tfrac{1-\alpha}{\alpha\sigma}\left(\tfrac{L}{K}\right)^{\frac{\sigma-1}{\sigma}} + \tfrac{\alpha}{(1-\alpha)\sigma}\left(\tfrac{K}{L}\right)^{\frac{\sigma-1}{\sigma}}\right] > 0,
\end{aligned}$$

such that the second-order conditions are satisfied. K and L are complements in profits, $\pi_{KL} > 0$, if $\sigma < \frac{1}{1-\rho}$ and substitutes in profits, $\pi_{KL} < 0$, if $\sigma > \frac{1}{1-\rho}$;

see (12). If $\sigma = \frac{1}{1-\sigma}$, K and L are independent in profits. The second-order conditions hold regardless of whether K and L are substitutes, complements, or independent in profits. The free-entry condition equates optimal profits, π^*, with zero, i.e.,

$$\pi^* = 0. \tag{13}$$

From (10), (11), and (13), we derive the following long-run input demands.

$$K^{LR} = \frac{\left(\frac{\alpha}{r}\right)^{\sigma} f \frac{\rho}{1-\rho}}{(1-\alpha)^{\sigma}(-\beta)^{1-\sigma} + \alpha^{\sigma} r^{1-\sigma}},$$

$$L^{LR} = \frac{\left(\frac{1-\alpha}{-\beta}\right)^{\sigma} f \frac{\rho}{1-\rho}}{(1-\alpha)^{\sigma}(-\beta)^{1-\sigma} + \alpha^{\sigma} r^{1-\sigma}}.$$

It follows that $\frac{\partial L^{LR}}{\partial \beta} > 0$. Further, $\frac{\partial K^{LR}}{\partial \beta} > 0$ if $\sigma < 1$, $\frac{\partial K^{LR}}{\partial \beta} < 0$ if $\sigma > 1$, and $\frac{\partial K^{LR}}{\partial \beta} = 0$ if $\sigma = 1$. Thus, K only rises in the long run following an increase in β if K and L are sufficiently strong complements ($\sigma < 1$). If K and L are only weak complements, independent, or substitutes in profits ($\sigma \in (1, \frac{1}{1-\rho})$, $\sigma = \frac{1}{1-\rho}$, and $\sigma \in (\frac{1}{1-\rho}, \infty)$, respectively), an increase in β ultimately leads to a reduction in K.

Using (11) and (13), the short-run value of L is implicitly determined by

$$\frac{rK}{1-\rho}\left[\frac{-\beta}{r}\frac{\alpha}{1-\alpha}\left(\frac{L^{SR}}{K}\right)^{\frac{1}{\sigma}} - \rho\right] - \beta L^{SR} = \frac{\rho}{1-\rho} f. \tag{14}$$

It follows immediately that $\frac{\partial L^{SR}}{\partial \beta} > 0$. For later use, we note that when $\sigma > 1$, $(-\beta)(L^{SR})^{\frac{1}{\sigma}}$ falls when β increases. Using (14), $\frac{\partial L^{SR}}{\partial K}$ shares sign with

$$1 - \frac{\sigma-1}{\rho\sigma}\frac{-\beta}{r}\frac{\alpha}{1-\alpha}\left(\frac{L^{SR}}{K}\right)^{\frac{1}{\sigma}}. \tag{15}$$

Suppose that $\sigma < 1$. Then it is obvious that $\frac{\partial L^{SR}}{\partial K} > 0$. Thus, in this case, we have that an increase in β implies $L^0 < L^{SR} < L^{LR}$ and a reduction in β implies $L^0 > L^{SR} > L^{LR}$ where L^0 denotes the initial level of labour demand. The LCP therefore holds for both cases. Consider next the case where β increases and $\sigma \in (1, \frac{1}{1-\rho})$ such that K and L are complements in profits, but only weakly so. Then we know that $\frac{\sigma-1}{\rho\sigma} < 1$ and that $(-\beta)(L^{SR})^{\frac{1}{\sigma}}$ falls when β increases. This means that (15) is positive at $K = K^0$. Further, (15) changes continuously as K falls from K^0 to K^{LR} and when it hits $1 - \frac{\sigma-1}{\rho\sigma} > 0$, we are in the new long-run equilibrium. Thus, $\frac{\partial L^{SR}}{\partial K}$ is positive on $K \in (K^{LR}, K^0)$. But then, for $\sigma \in (1, \frac{1}{1-\rho})$, an increase in β implies that $L^0 < L^{LR} < L^{SR}$ and the LCP does not hold in this case.

References

Koebel BM and Laisney F, 2010. The Aggregate LeChatelier Samuelson Principle with Cournot Competition. ZEW Discussion Papers, No. 10-009.

de Meza D, 1981. "Perverse" Long-run and Short-run Factor Demand Curves. *Economica*, 48:299–303.

Milgrom P, 2006. Multipliers and the LeChatelier Principle. In *Szenberg M, Ramrattan L, Gottesman A: Samuelsonian Economics and the Twenty-First Century. Oxford University Press.*

Milgrom P and Roberts J, 1994. Comparing Equilibria. *American Economic Review*, 84(3):441–459.

Milgrom P and Roberts J, 1995. Complementarities and Fit Strategy, Structure, and Organizational Change in Manufacturing. *Journal of Accounting and Economics*, 19:179–208.

Milgrom P and Roberts J, 1996. The LeChatelier Principle. *American Economic Review*, 86(1):173–179.

Quah JKH, 2007. The Comparative Statics of Constrained Optimization Problems. *Econometrica*, 75:401–431.

Roberts K, 1999. Rationality and the LeChatelier Principle. *Journal of Economic Theory*, 87:416–428.

Samuelson PA, 1947. *Foundations of Economic Analysis*. Harvard University Press.

Samuelson PA, 1960. An Extension of the LeChatelier Principle. *Econometrica*, 28:368–379.

Silberberg E, 1974. A Revision of Comparative Statics Methodology in Economics, or, How to do Comparative Statics on the Back on an Envelope. *Journal of Economic Theory*, 7(2):159–172.

Suen W, Silberberg E, and Tseng P, 2000. The LeChatelier Principle: the Long and the Short of It. *Economic Theory*, 16:471–476.

Topkis DM, 1978. Minimizing a Supermodular Function on a Lattice. *Operations Research*, 26:305–321.

Topkis DM, 1995. Comparative Statics of the Firm. *Journal of Economic Theory*, 67:370–401.

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