Economics Working Paper

2011-4

Non-existence of Steady State Equilibrium in the Neoclassical Growth Model with a Longevity Trend

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NON-EXISTENCE OF STEADY STATE EQUILIBRIUM IN THE NEOCLASSICAL GROWTH MODEL WITH A LONGEVITY TREND

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April 15, 2011

ABSTRACT. Longevity has been increasing in the developed countries for almost two centuries and further increases are expected in the future. In the neoclassical growth models the case of population growth driven by fertility is well-known, whereas the properties of population growth caused by persistently declining mortality rates have received little attention. Furthermore, the economic literature on the consequences of changing longevity has relied almost entirely on analysis applying a once and for all change in the survival probability. This paper raises concern about such an approach of comparison of steady state equilibrium when considering the empirically observed trend in longevity. We extend a standard continuous time overlapping generations model by a longevity trend and are thereby able to study the properties of mortality-driven population growth. This turns out to be exceedingly complicated to handle, and it is shown that in general no steady state equilibrium exists. Consequently analytical results and long run implications cannot be obtained in a setting with a realistic demographic setup.

Keywords: Longevity, Population growth, Overlapping generations models, Steady state equilibrium, Existence

JEL Classification: J11, C62, O41, E13

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The author gratefully acknowledge comments from Torben M. Andersen and Valdemar Smith.

1. Introduction

Demographics are changing significantly in most developed countries, and the large number of elderly people relative to the labor force challenges the sustainability of public pension schemes and tax financed welfare services. The development is neither new nor transitory; longevity (life expectancy at birth) has been rising for a long period and is expected to continue to do so (cf. Section 2). At the same time fertility rates have generally fallen and thereby mitigated an increase in the population growth rate (which by definition is the fertility rate minus the death rate). In the neoclassical growth model fertility is usually the source of population growth which is easy to handle and the results are well-known.[1] The alternative case of persistently increasing longevity as the driver of population growth has so far received little attention. This paper contributes to the scarce literature by studying the properties of a neoclassical growth model with longevity driven population growth. We show that compared to the fertility case this is much more complicated to handle, and in general no steady state equilibrium (defined by constant growth rates for per capita variables) exists.

Many parts of the economy have been and will continue to be influenced by the demographic changes, and a large economic literature on the consequences now exists.[2] It spans from the impact on public finances, in particular social security, to international capital flows, human capital accumulation and growth in general. However, it is rather striking that most of the literature (cf. Section 3) disregards the trend in longevity and model longevity as stationary over time. To address the consequences of increasing longevity in such a set-up, one has to rely on steady state equilibrium comparisons, i.e., study the effects of once and for all positive shocks to longevity. This is a convenient model simplification, but it is in sharp contrast to what we observe empirically. Old age mortality rates have been declining for many years, and life expectancy at birth has shown an almost linear trend for more than a century (cf. Section 2). The magnitude of future lifetime improvements is of course uncertain, but in the past the projections have been revised upwards several times. Thus, there should be little doubt that expected lifetime is going to be higher for future generations. Since the time horizon is important for many economic decisions, we would expect people to incorporate future longevity improvements in their information set. Therefore, it is problematic to draw conclusions from models with constant longevity across time and cohorts. This is already pointed out by Hamermesh

[1] See e.g. the treatment in standard textbooks such as Barro and Sala-i-Martin (2004) or Romer (2006).

[2] See e.g. Ehrlich and Lui (1991), Adema, Meijdam and Verbon (2008), Andersen (2008), and Heijdra and Romp (2009).

(1985). In a questionnaire survey he finds that people are well aware of changes in life expectancy. They extrapolate past improvements in longevity and are able to predict their own lifetime fairly well.[3]

Besides looking into the case of population growth via longevity, we accommodate this critique by specifying a realistic demographic setup in the sense that life expectancy at birth exhibits a positive time trend. We do this by setting up an overlapping generations model in the spirit of Blanchard (1985) but extended with an exogenous demographic process such that newborn cohorts face mortality rates equal to or lower than those applying for previous cohorts. Since nothing indicates that the longevity trend is lowering off (cf. Section 2), and to address the extreme case of population growth caused by persistently decreasing mortality rates, we allow the longevity trend to continue forever. Thus, no maximum attainable age is assumed. Apart from the demographic setup the model is very simple; agents face a standard maximization problem of choosing consumption over their life cycle, and the supply side is given by a standard neoclassical production function. The paper thus addresses the fundamental question of whether population growth caused by declining mortality rates can be a source of balanced growth. The answer turns out to be negative, and a non-existence result of steady state equilibrium is provided. Intuitively, existence fails because individual consumption becomes cohort dependent and the age distribution changes over time. Consequently, the dynamics of aggregate consumption becomes too complicated and uncontrollable to ensure balanced growth. The result is discouraging and it questions the reliability of results derived from the models with simpler demographic frameworks implying that steady state equilibrium exists.

The paper is organized as follows. In Section 2 we give some empirical facts to illustrate the upward trend in longevity. A brief review of the literature on longevity and growth is given in Section 3. Section 4 presents the model and outlines the demographic structure of both age and cohort specific mortality rates. The key non-existence result of steady state equilibrium is provided in Section 5. Section 6 discusses possible extensions and Section 7 concludes.

2. The upward trend in longevity

Longevity has been increasing for more than 150 years in most developed countries. It can be described by a positive and almost linear trend (at least from around 1960). The case for the USA from 1933 and with projections until 2050 is plotted in Figure 1. The case for Sweden from 1751 until 2050 is plotted in Figure 2. Clearly, there is an upward

[3]Later studies have confirmed the results, see e.g. Smith, Taylor and Sloan (2001).

trend in longevity, and nothing indicates that it is leveling off.[4] The idea that there is some upper limit on human longevity has not yet been proven (Oeppen and Vaupel, 2002), and further increases should be expected in the future (United Nations, 2008) as seen from the figures.

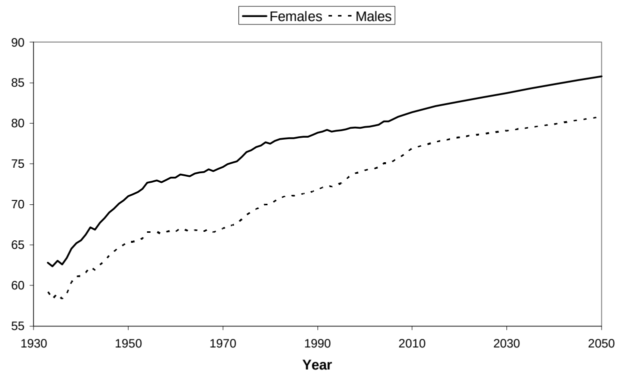

Figure 1: Life expectancy at birth, USA, 1933–2050.

Source: Human Mortality Database (2011) and United Nations (2008)

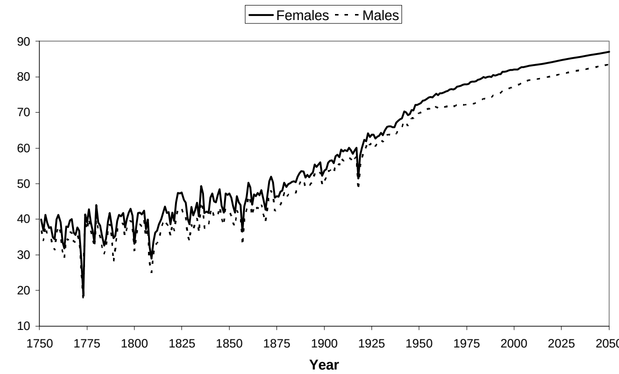

Figure 2: Life expectancy at birth, Sweden, 1751–2050.

Source: Human Mortality Database (2011) and United Nations (2008)

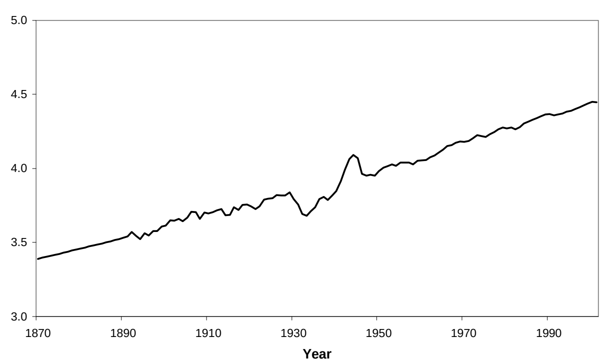

Figure 3: log GDP per capita, USA, 1870–2001.

Source: Maddison (2003)

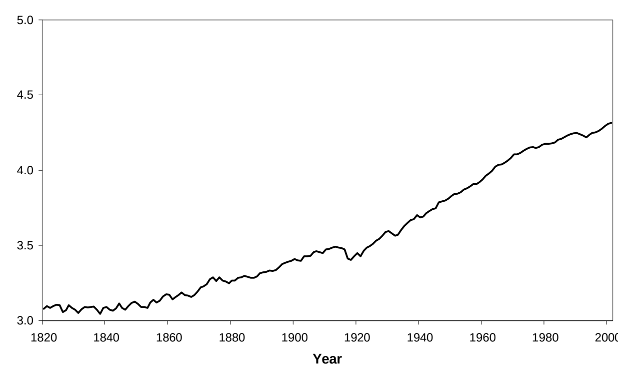

Figure 4: log GDP per capita, Sweden, 1820-2001.

Source: Maddison (2003)

Simultaneously with the improvements in longevity the developed countries have experienced sustained economic growth. This can be seen in the development of log GDP per capita, which is plotted for the USA and Sweden in Figure 3 and 4. Both series fit a straight line quite well, implying that the growth rate of GDP per capita seems to be fairly constant in the long run.

The drivers behind the mortality improvements in Figure 1 and 2 have changed a lot over time. Initially, reductions in childhood mortality were the main reason for higher life expectancy, whereas the improvements since 1970 are almost entirely due to declining

[4]Similar patterns hold for other developed countries, cf. the Human Mortality Database (2011).

mortality at old ages (cf. table 2 in Christensen, Doblhammer, Rau and Vaupel (2009)). Therefore, the lower life expectancy growth relative to the early years in the USA (Figure 1) is not due to a slowing down of mortality improvements at old ages. In fact, the decline of mortality at old ages has accelerated since around 1970 (Wilmoth, 2000; Vaupel, 1998), and current population growth is almost entirely due to mortality decline at old ages (Johnson, 2000). This can also be seen from the period survival curves plotted in Figure 5 and 6 – changes from 1960 to 1980 and 2007 take place almost entirely at old ages.

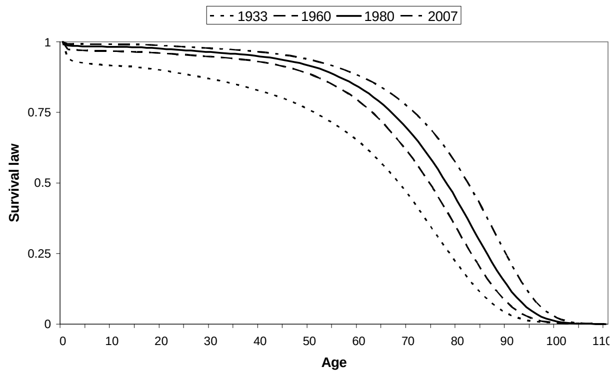

Figure 5: Period survival curves, USA.

Source: Human Mortality Database (2011)

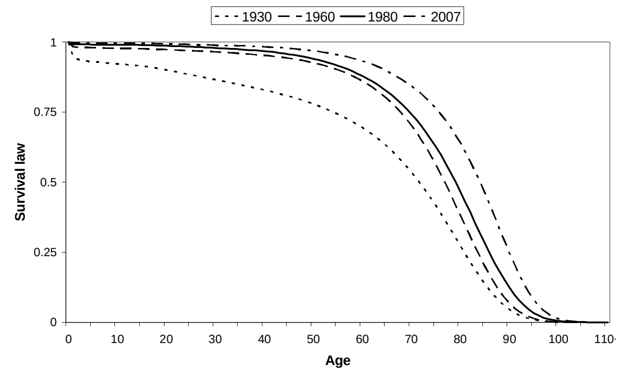

Figure 6: Period survival curves, Sweden.

Source: Human Mortality Database (2011)

When studying the consequences for the labor force of rising longevity and the derived influence on growth, it is crucial to know whether people are mostly gaining healthy years of life. Christensen et al. (2009) survey various studies of this and find that people below the age of 85 now live longer and generally with better health than previous generations. Above the age of 85 the conditions are less clear, and there is some evidence that exceptionally old people have on average worse health than previous cohorts. This could be a result of medical improvements that make it possible to help frail and ill people into advanced old age. Overall, Christensen et al. (2009) conclude that people are living longer and with less disability and fewer functional limitations than previous generations. Hence, we can in principle expect the demographic changes to continuously extend the labor force.[5]

The stylized economic facts of longevity and GDP per capita are thus that they increase together in the long run, the latter at a fairly constant rate and the former by an almost

[5]This has so far not been the case since retirement ages have been fairly constant or even declining, cf. Andersen and Hermansen (2010).

linear trend in life expectancy at birth (at least the recent development due to lower old age mortality rates).[6]

3. Literature review

The economic consequences of demographic changes have been analyzed mainly by applying the Blanchard model (Blanchard, 1985) in continuous time and by applying the Diamond model (Diamond, 1965) extended by an exogenous probability of survival into old-age in discrete time. In both models extended longevity is analyzed using comparative statics by varying the survival parameter. Although the models are analytically attractive, the simplified demographic setup limits the generality and level of detail of the results. One faces the classical trade-off of analytical tractability versus descriptive accuracy. Several attempts to build models with a more realistic demographic specification have been made. We briefly review this literature to outline the current state of replicating the longevity trend in theoretical work.[7,8]

The main drawback (and advantage) of the Blanchard model is the age invariant survival rate. Relaxing this complicates the expressions for aggregate variables considerably. Boucekkine, de la Croix and Licandro (2002) improve the realism of the demographic structure considerably by assuming that the survival rate is a decreasing function of age. By applying a specific functional form for the survival rate, assuming a maximum attainable age and linear utility, their model of human capital can be solved analytically. Different demographic shocks can then be analyzed by varying the two parameters in the survival rate function. Azomahou, Boucekkine and Diene (2009) apply the same survival rate function in a model combining Blanchard (1985) with a learning-by-doing externality (Romer, 1986). They investigate the impact of higher life expectancy on economic growth and find with age-dependent survival probabilities a positive convex (concave) relationship for low (high) values of life expectancy.

[6] For empirical studies on the relationship between longevity and growth see e.g. Kelley and Schmidt (1995), Boucekkine, de la Croix and Licandro (2003), and Acemoglu and Johnson (2007).

[7] Here we restrict attention to exogenous longevity. There is also a small but growing literature on endogenous longevity (see e.g. Aísa and Pueyo (2004), Chakraborty (2004), Bhattacharya and Qiao (2007), and Schneider and Winkler (2010)). However, we are not aware of any models that generate an upward trend in equilibrium. Instead the steady state equilibrium is typically characterized by constant longevity, and changes in mortality rates can only be explained as a transition towards a new steady state. Recalling the century-long trend in longevity seen in Section 2, this is not a very satisfying answer.

[8] In discrete time a more realistic demographic specification amounts to dividing length of life into more time periods. However, analytical tractability is typically already lost by three or four periods. Therefore, the focus in this paper is on continuous time models.

Faruqee (2003) takes a similar approach. First he generalizes the mortality rate of the Blanchard model to be time and cohort dependent. He then makes a functional form assumption (age-dependent mortality rate) to achieve analytical results. The model can, however, only be solved analytically for exogenous factor prices and no results for transitional dynamics can be obtained.

d'Albis (2007) applies a general hazard rate of death (the framework introduced by Yaari (1965)), and uses Lotka's (1998) stable population assumption[9] to be able to aggregate and show existence of steady state equilibrium. The focus of the paper is the relationship between the population growth rate and capital per capita, and not explicitly increasing longevity. Although the general mortality rate is very attractive in the attempt to replicate demographic observations, the stable population assumption is obviously not empirically supported (cf. Section 2).

Another way to simplify aggregation is to consider a small open economy in which factor prices are exogenously given. Making this assumption enables Heijdra and Romp (2008) to solve the Blanchard model with a general age-dependent mortality rate (in a numerical analysis they use a Gompertz-Makeham process). Contrary to Boucekkine et al. (2002) and d'Albis (2007) no maximum attainable age is assumed in this paper, instead the probability of survival tends toward zero when the age tends toward infinity.

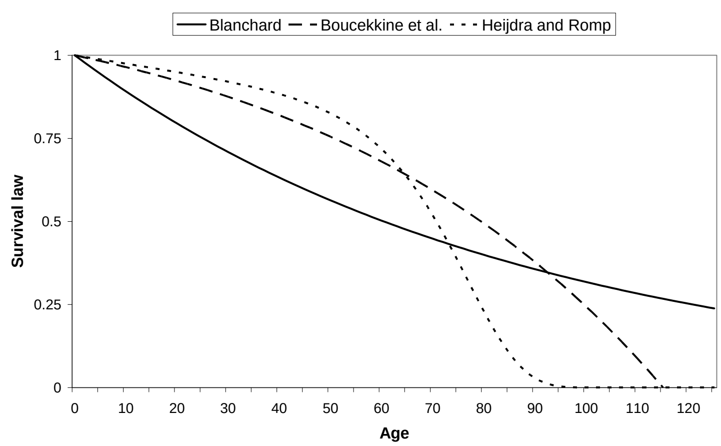

Figure 7: Survival law functions.

The survival rate functions applied in Blanchard (1985), Boucekkine et al. (2002), and Heijdra and Romp (2008) are plotted in Figure 7. Comparing this to the empirical counterpart in Figure 5 and 6, we see that the latter two make large progress relative to

[9]Defined by a constant population growth rate and a fixed age distribution over time. See also Section 5.

the Blanchard case and are able to track the empirical survival curve in a given year quite well. It is tempting to compare two steady states for two given survival curves that fit the empirical curves well and argue that such an analysis displays a realistic demographic setup. But this is not the case since a steady state equilibrium obviously requires that all adjustments have taken place, which first of all happens very slowly when we deal with demographic changes and secondly requires that the change is a one time shock. Figure 5 and 6 highlight that a realistic demographic setup requires the survival curve to move outwards persistently.

Attempts to make the survival probabilities time or cohort dependent are seen in Boucekkine et al. (2003), Faruqee (2003), and Azomahou et al. (2009). However, to show existence of steady state equilibrium, they all consider time-invariant parameters and end up analyzing extended longevity by comparative statics. Andersen and Gestsson (2010) assume that survival rates are cohort specific and are thus able to capture the empirically observed longevity trend. However, they use a partial equilibrium model (or small open economy framework) since the focus of the paper is intergenerational equity and determination of optimal policy. Hence, existence of steady state equilibrium is not addressed.

4. Overlapping generations model with cohort and age dependent mortality

Even though attempts have been made to make mortality rates time or cohort specific, no steady state equilibrium with a longevity trend has been shown to exist. To clarify the reasons for this and study the case of population growth via longevity, we take a standard neoclassical growth model in continuous time and extend it with a very general demographic structure.[10] The key requirement is that life expectancy at birth should be increasing over time, or alternatively that decreasing mortality rates should be the source for population growth. A simple way to achieve this is by letting mortality rates be cohort specific, which implies that individuals become heterogeneous. Therefore, it is most attractive to work in continuous time to ease aggregation.

Since this is a first step in handling the longevity trend, we keep the model simple and use standard assumptions in order to maximize the possibilities for existence of steady state equilibrium. Thus, labor supply is exogenous, firms are perfectly competitive, and markets for life insurance and annuities exist. There is no endogenous growth in the

[10]The model in this paper is closest to the one in Azomahou et al. (2009) (Section 5). However, in this paper we generalize demography even further by working with a general survival function.

model – the only source of growth is exogenous population growth due to decreasing mortality rates. More specifically, the links between longevity and growth in this model are an exogenous positive effect on the size of the labor force and a potential reaction in the (endogenous) savings rate. If a steady state equilibrium can be shown *not* to exist under these admittedly strong assumptions, we see no way to achieve a balanced growth path with longevity driven population growth and thus no purpose in relaxing the model assumptions. After specifying the structure of the demography, we set up the household's maximization problem before aggregating and studying the conditions for existence of steady state equilibrium.

4.1. **Demography: Cohort and age dependent mortality.** The aim of the demographic specification is to make it as general and realistic as possible while keeping it analytically tractable.[11,12] The instantaneous probability of death (the hazard rate) depends on time of birth (s) and age (a)

$$\mu(s,a) \geq 0, \quad \mu_s \leq 0, \ \mu_a > 0. \tag{4.1}$$

The probability for an agent born at time s to survive until age a is thus

$$m(s,a) = m_0 e^{-\int_0^a \mu(s,x)dx}.$$

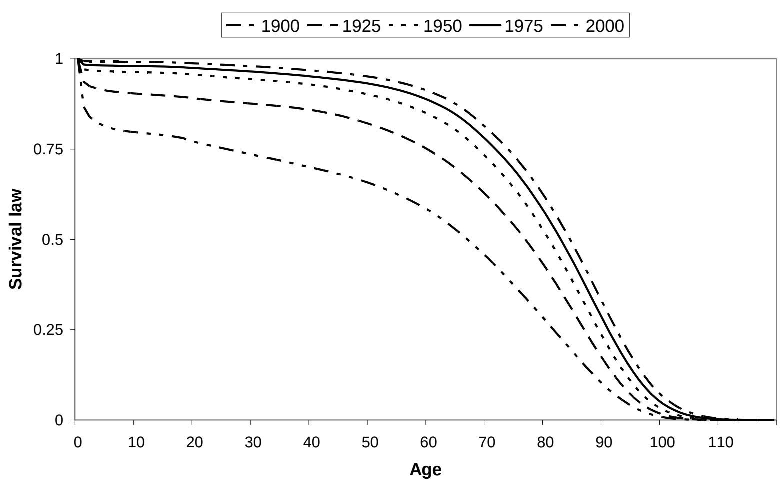

Figure 8: Projected cohort survival curves, USA.

Source: Berkeley Mortality Database (2011)

[11]Instead of cohort specific mortality rates, one could also consider time specific rates. If longevity improvements are mainly driven by new discoveries in medicine and health care, this would be a better approximation. Unfortunately, the hazard rate will turn up in the steady state condition, and if the rate is time dependent, no steady state equilibrium can exist in such a setting.

[12]The demographic setup builds on Faruqee (2003).

The empirical counterpart to this is shown in Figure 8 for five different US cohorts.[13] To track this we impose the boundary conditions of a fixed probability of death at birth and that no one lives forever[14]

$$\begin{aligned} m(s,0) &= m_0 \in (0,1] \; \forall s \\ \lim_{a\to\infty} m(s,a) &= 0 \; \forall s. \end{aligned}$$

We can calculate the life expectancy at birth as[15]

$$LE(s) = \int_s^{\infty} (t-s)\, m(s,t-s)\, dt = \int_s^{\infty} (t-s)\, e^{-\int_0^{t-s}\mu(s,x)dx} dt. \tag{4.2}$$

Due to the assumptions (cohort specific mortality, $\mu_s \leq 0$), life expectancy increases over cohorts and thereby tracks the longevity trend (cf. Figure 1 and 2) as requested[16]

$$\frac{dLE(s)}{ds} = -\int_s^{\infty} (t-s) \left(\int_0^{t-s} \mu_s(s,x)\, dx \right) e^{-\int_0^{t-s}\mu(s,x)dx} dt \geq 0.$$

As is standard in this type of model, we assume that lifetime is uncertain at the individual level, but that the cohorts are so large that at the aggregate level everything is deterministic (the law of large numbers). We denote the total population size at time t by $N(t)$ and the size of the cohort born at time s at time t by $n(s,t)$. The crude fertility rate is denoted $b(s)$ and allowed to change over time. The size of the newborn generation at time s is then

$$n(s,s) = b(s)\, N(s), \tag{4.3}$$

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[13] Whereas the period survival curves in Figure 5 and 6 show the probability of survival at a given point in time as a function of age, Figure 8 shows the survival curve for the cohorts born in $s \in \{1900, 1925, 1950, 1975, 2000\}$.

[14] The assumptions made for $\mu(.,.)$ ensure that the function $m(.,.)$ is decreasing in age and increasing in the cohort index. The second derivative w.r.t. age is

$$\frac{\partial^2 m(s,a)}{\partial a^2} = \left[\mu(s,a)^2 - \mu_a(s,a)\right] m_0 e^{-\int_0^a \mu(s,x)dx} \lesseqgtr 0.$$

To replicate the empirical survival function we can assume

$$\mu(s,a)^2 - \mu_a(s,a) \lesseqgtr 0 \text{ for } a \lesseqgtr a^*$$

where a^* is the inflection point, where the function shifts from being concave to being convex.

[15] As is standard in this type of model, childhood is neglected and individuals are assumed to enter at the age of 20. For notational convenience we let $m_0 = 1$ in the rest of the paper.

[16] This can be shown by noting that the assumption $\lim_{t\to\infty} m(s,t-s) = 0$ implies $\lim_{t\to\infty} e^{\int_0^{t-s}\mu(s,x)dx} = \infty$, so L'Hôpital's rule can be applied.

and the following relations apply

$$\begin{aligned} n(s,t) &= n(s,s)\, m(s,t-s) = b(s)\, N(s)\, e^{-\int_0^{t-s}\mu(s,x)dx} && (4.4)\\ \dot{n}(s,t) &= -\mu(s,t-s)\, n(s,t) && (4.5)\\ N(t) &= \int_{-\infty}^{t} n(s,t)\, ds. && (4.6) \end{aligned}$$

We can work out the population growth rate $(v(t))$ as the difference between the fertility rate and the instantaneous probability of death averaged over the present cohorts (the crude death rate)[17]

$$v(t) = \frac{\dot{N}(t)}{N(t)} = \frac{n(t,t)}{N(t)} - \int_{-\infty}^{t} \mu(s,t-s)\, \frac{n(s,t)}{N(t)} ds = b(t) - \overline{\mu}(t). \tag{4.7}$$

Furthermore, using (4.4) and (4.7) in (4.6) yields the relation

$$\int_{-\infty}^{t} b(s)\, e^{-\int_s^t [v(x)+\mu(s,x-s)]dx} ds = 1, \tag{4.8}$$

between the three rates. (4.8) implicitly defines $v(.)$ for given choices of $b(.)$ and $\mu(.,.)$.

4.2. **Households.** The household's maximization problem is standard. Agents inelastically supply one unit of labor at each instant of time, and we assume that individuals work for their entire lifetime. Any disutility of work is therefore normalized to zero to simplify. Expected utility at birth time for an agent belonging to cohort s is

$$E[U(s)] = \int_s^{\infty} e^{-\rho(\tau-s)} m(s,\tau-s)\, u(c(s,\tau))\, d\tau = \int_s^{\infty} e^{-\int_0^{\tau-s}[\rho+\mu(s,x)]dx} u(c(s,\tau))\, d\tau,$$

where ρ is the subjective discount rate, $c(s,\tau)$ is consumption at time τ for a cohort s agent, and $u()$ is a standard increasing and concave utility function.[18] Complete markets for annuities and life insurance are assumed to exist, and it will thus be optimal for the agent to fully annuitize to insure against longevity risk (Yaari, 1965). For simplicity agents are assumed to be equally productive across age and cohorts such that the wage rate is the same for all and it only depends on time. Hence, the flow budget constraint is given by

$$\dot{z}(s,\tau) = [r(\tau) + \mu(s,\tau-s)]\, z(s,\tau) + w(\tau) - c(s,\tau) \;\; \forall \tau \geq s, \tag{4.9}$$

[17] The crude death rate is likely to increase over time even though mortality rates at old age fall. The reason is that old individuals compose a proportionately larger fraction of the population. Therefore, it is not in itself a very informative measure.

[18] Note that the utility function is assumed to be invariant across time and cohorts. Since this is standard in the literature we will not attempt to relax it here.

where $z(s,\tau)$ is the wealth for cohort s at time τ, $r(\tau)$ the interest rate, and $w(\tau)$ the wage rate. By investing wealth in life annuities individuals receive a premium equal to the instantaneous probability of death, $\mu(s,\tau-s)$. In return wealth is transferred to an insurance company if the individual dies. We assume that individuals are born without wealth, $z(s,s)=0\ \forall s$, and a no-Ponzi-game condition applies. Maximization yields the following Euler equation

$$\frac{\dot{c}(s,\tau)}{c(s,\tau)} = -\frac{u'(c(s,\tau))}{u''(c(s,\tau))\,c(s,\tau)}\left[r(\tau)-\rho\right] \equiv \theta(s,\tau)\left[r(\tau)-\rho\right], \tag{4.10}$$

where $\theta(s,\tau)$ is the inverse measure of relative risk aversion. By applying (4.9), (4.10), and the no-Ponzi-game condition consumption can be expressed as a function of human and non-human wealth

$$c(s,t) = \phi(s,t)\left[z(s,t)+h(s,t)\right], \tag{4.11}$$

where human wealth or the present value of future labor income is defined by

$$h(s,t) \equiv \int_t^\infty e^{-\int_t^\tau [r(x)+\mu(s,x-s)]dx} w(\tau)\,d\tau, \tag{4.12}$$

and it evolves according to

$$\dot{h}(s,t) = \left[r(t)+\mu(s,t-s)\right]h(s,t) - w(t). \tag{4.13}$$

The marginal propensity to consume is defined by

$$\phi(s,t) \equiv \left[\int_t^\infty e^{-\int_t^\tau [[1-\theta(s,x)]r(x)+\mu(s,x-s)+\theta(s,x)\rho]dx} d\tau\right]^{-1}.$$

Note that an increase in the instantaneous probability of death, $\mu(.,.)$, increases $\phi(.,.)$ since the expected lifespan to consume falls.

4.3. **Firms.** We follow a standard formulation of the production side to keep focus on the effects of introducing a longevity trend. Firms produce according to a standard neoclassical production function with capital and labor as the only input factors. Firms are perfectly competitive and the production factors are thus paid their marginal product

$$r(t) = f'(k(t)) - \delta \tag{4.14}$$

$$w(t) = f(k(t)) - k(t)\,f'(k(t)), \tag{4.15}$$

where $k(t)$ is the capital-labor ratio and δ is the depreciation rate.

4.4. **Aggregation.** As mentioned above the size of cohort s at time t is given by $n(s,t)$. Therefore, aggregate consumption, human wealth, and non-human wealth are given by

$$Q(t) = \int_{-\infty}^{t} n(s,t)\, q(s,t)\, ds, \tag{4.16}$$

where $(Q,q) \in \{(C,c), (H,h), (Z,z)\}$.

The dynamic evolvement for consumption follows from differentiation of (4.16) and by use of (4.10)

$$\begin{aligned} \dot{C}(t) &= c(t,t)\, n(t,t) - \int_{-\infty}^{t} \mu(s,t-s)\, n(s,t)\, c(s,t)\, ds \\ &\quad + [r(t) - \rho] \int_{-\infty}^{t} \theta(s,t)\, n(s,t)\, c(s,t)\, ds. \end{aligned} \tag{4.17}$$

The first term is consumption by the newborn cohort at time t, the second is the fall in aggregate consumption caused by individuals dying at time t, and the last term is the change due to intertemporal substitution (the Keynes-Ramsey rule).

For aggregate human wealth we can apply (4.13) and obtain

$$\dot{H}(t) = h(t,t)\, n(t,t) - w(t)\, N(t) + r(t)\, H(t). \tag{4.18}$$

The first term is the increase in human wealth due to the newborn cohort, the second is the fall caused by wage being actually paid out, and the last term is the return on human wealth (the difference between rate of return and the mortality rate).

Finally, aggregate non-human wealth evolves according to

$$\dot{Z}(t) = r(t)\, Z(t) + w(t)\, N(t) - C(t). \tag{4.19}$$

The term $\mu(s,t-s)\, Z(t)$ does not enter since this reflects transfers (between insurance companies and survivors), and does not affect aggregate wealth. Note that the simplicity of (4.18) and (4.19) is due to the assumption of complete markets for annuities and life insurance. Relaxing this would complicate the model considerably.

5. Steady state equilibrium

To look for a steady state equilibrium, we consider per capita variables[19] denoted by $q(t) = \frac{Q(t)}{N(t)}$, with (Q,q) defined as above. Recall that the population growth rate is

[19] We have also tried to scale by other measures than total population size. However, it turns out that the scaling variable has to be proportional to $N(t)$ for a steady state to exist. To see this let $X(t)$ be some unspecified scaling variable. Then the dynamic law for scaled non-human wealth $\left(\widetilde{z}(t) = \frac{Z(t)}{X(t)}\right)$ becomes

$$\dot{\widetilde{z}}(t) = w(t) \frac{N(t)}{X(t)} + [r(t) - x(t)]\, \widetilde{z}(t) - \widetilde{c}(t)$$

denoted by $v(t)$. By applying (4.17) and (4.7) the dynamic law for per capita consumption is obtained

$$\begin{aligned}\dot{c}(t) &= \frac{\dot{C}(t)}{N(t)} - \frac{C(t)\dot{N}(t)}{N(t)^2} \\ &= [r(t)-\rho]\int_{-\infty}^{t} \theta(s,t)\, c(s,t)\,\frac{n(s,t)}{N(t)}ds \\ &\quad + c(t,t)\,b(t) - \int_{-\infty}^{t} \mu(s,t-s)\,c(s,t)\,\frac{n(s,t)}{N(t)}ds - v(t)\,c(t). \qquad (5.1)\end{aligned}$$

The first term is the effect of all agents adjusting consumption optimally according to the Euler equation (4.10). If we (as is often done) assume a utility function of the constant relative risk aversion (CRRA) form, the first integral will reduce to a constant times consumption per capita. The second term is the increase due to addition of newborn agents who start to consume out of human wealth. The third term is the decrease due to consumption given up by dying agents from all existing cohorts. Finally, since we are working with consumption in per capita terms, a correction for population growth is needed. Clearly, the presence of cohort specific mortality complicates matters, and without further restrictions no simple expressions for the change in per capita consumption due to generational turnover (the second and third term) can be obtained.[20]

Per capita human and non-human wealth evolve according to

$$\dot{h}(t) = h(t,t)\,b(t) - w(t) + [r(t)-v(t)]\,h(t) \qquad (5.2)$$

$$\dot{z}(t) = [r(t)-v(t)]\,z(t) + w(t) - c(t). \qquad (5.3)$$

We consider a closed economy, therefore the only form of non-human wealth is capital, implying $Z(t) = K(t)$ and $z(t) = k(t)$. Using this and the factor prices (4.14) and (4.15), (5.3) can be written as

$$\dot{k}(t) = f(k(t)) - c(t) - [\delta + v(t)]\,k(t). \qquad (5.4)$$

where $x(t) = \frac{\dot{X}(t)}{X(t)}$ is the growth rate of the scaling variable. From the term $w(t)\frac{N(t)}{X(t)}$ it is clear that a necessary condition for existence of a steady state (with a constant wage) is that the scaling variable is proportional to the population size.

[20]Note that in the Blanchard case ($\mu(s,t-s) = \mu$, $b(t) = b$ and $\theta = 1$), the dynamic law reduces to

$$\dot{c}(t) = [r(t)-\rho]\,c(t) + bc(t,t) - [\mu + v]\,c(t),$$

and after some simplification, one obtains the familiar expression

$$\dot{c}(t) = [r(t)-\rho]\,c(t) - b(\mu+\rho)\,z(t).$$

A steady state equilibrium is characterized by constant per capita variables c and k (i.e. zero growth rates).[21] Therefore, necessary and sufficient conditions for existence of steady state equilibrium are that the two equations $\dot{k}(t) = 0$ and $\dot{c}(t) = 0$ have a solution c^* and k^* for all t. Unfortunately, this is generally not the case as outlined in Proposition 1.

Proposition 1. *In general, the model has no steady state equilibrium.*

Proof. The proof requires that the inequality $\mu_s \leq 0$ in (4.1) is strict, i.e., we assume that a newborn cohort can expect to live strictly longer than existing cohorts.

From (5.4) the condition $\dot{k}(t) = 0$ yields

$$0 = f(k) - c - [\delta + v(t)]\,k.$$

Clearly constant population growth, $v(t) = v\ \forall t$, is a necessary condition for existence of a solution. Thus, for a given mortality schedule $\mu(s, t-s)$ we need to put strong assumptions on the birth rate for this to be satisfied.

From (5.1) the condition $\dot{c}(t) = 0$ yields

$$\begin{aligned} 0 \;=\; & [f'(k) - \delta - \rho] \int_{-\infty}^{t} \theta(s,t)\, \frac{n(s,t)}{N(t)} c(s,t)\, ds \\ & + c(t,t)\, b(t) - \int_{-\infty}^{t} \mu(s, t-s)\, c(s,t)\, \frac{n(s,t)}{N(t)} ds - v(t)\, c. \end{aligned}$$

For a given choice of $\mu(s, t-s)$ and $b(t)$ nothing ensures that we can find a c and k which satisfy this for all t. Hence, we conclude that, in general, no steady state equilibrium exists. $\square$

Proposition 1 is driven by the heterogeneity arising across generations due to cohort specific mortality rates. We can obtain expressions for aggregate variables, but we are unable to simplify or control aggregate consumption in a way that ensures existence of steady state equilibrium. As Blanchard (1985) notes, simple aggregation is, in general, impossible when agents are finitely lived. The reason can be seen from the consumption function (4.11); individuals of different age and cohort have different levels of wealth and different propensities to consume. Simple expressions for aggregate variables are not a necessary condition for existence of steady state equilibrium (cf. Proposition 2 below),

[21] In principle we should also look for steady state equilibrium with constant positive growth rates. However, from standard growth theory (see e.g. Barro and Sala-i-Martin (2004)) we know that with a neoclassical production function such balanced growth paths can be ruled out by the transversality condition. Endogenous growth requires some externality which is not present in this model.

but in combination with a changing age distribution existence generally fails in our case. Next, we turn to a very special case where a steady state equilibrium can be shown to exist.

Proposition 2. *Under the following restrictions the model has a steady state equilibrium.*

I. *CRRA utility* $(\theta(s,t) = \theta \ \forall t, s)$
II. *For a given mortality schedule,* $\mu(s, t-s)$*, the birth rate* (b) *is determined endogenously by* (4.8) *such that the population growth rate is constant* $(v(t) = v \ \forall t)$
III. *The mortality rate* $\mu(s, t-s)$ *evolve such that the generational turnover term is a function of per capita consumption and capital only, i.e.,*

$$\int_{-\infty}^{t} \mu(s, t-s)\, c(s,t)\, \frac{n(s,t)}{N(t)} ds - c(t,t)\, b(t) \equiv \Gamma(k,c) \ \ \forall t.$$

Proof. Applying restriction (I) and (II) the equilibrium conditions reduce to

$$\dot{k}(t) = 0 : c = f(k) - [\delta + v]\, k \tag{5.5}$$

$$\dot{c}(t) = 0 : c = \frac{\int_{-\infty}^{t} \mu(s,t-s)\, c(s,t)\, \frac{n(s,t)}{N(t)} ds - c(t,t)\, b(t)}{\theta [f'(k) - \delta - \rho] - v}. \tag{5.6}$$

It remains to be show that an appropriate choice of $\mu(s, t-s)$ can make Γ a function of k and c only. Using (4.4), Γ can be written as

$$\Gamma(k,c) = \int_{-\infty}^{t} \mu(s,t-s)\, c(s,t)\, b(s)\, e^{-\int_0^{t-s}[v+\mu(s,x)]dx} ds - c(t,t)\, b(t). \tag{5.7}$$

Recall that

$$\begin{aligned} c(t,t) &= \phi(t,t)\, h(t,t) \\ &= \frac{\int_t^{\infty} [f(k(\tau)) - k(\tau) f'(k(\tau))]\, e^{-\int_t^{\tau}[f'(k(x)) - \delta + \mu(t,x-t)]dx} d\tau}{\int_t^{\infty} e^{-\int_t^{\tau}[(1-\theta)[f'(k(x))-\delta] + \mu(t,x-t) + \theta\rho]dx} d\tau}, \end{aligned}$$

and

$$c(s,t) = \phi(s,s)\, h(s,s)\, e^{\int_s^t \theta[f'(k(x)) - \delta - \rho]dx}.$$

Hence, $b(t)$ and $\mu(s, t-s)$ are implicitly defined by (4.8) and (5.7).

The restrictions (I)-(III) ensure that the equilibrium conditions reduce to two equations in k and c. To be complete we should also show that a solution (k^*, c^*) exists. This follows from the properties of the neoclassical production function $f(k)$. In a (k,c) diagram (5.5) gives the usual hump-shaped graph, and provided that the numerator in (5.6) is positive and bounded, the $\dot{c}(t) = 0$ locus is increasing and has a vertical asymptote at the k satisfying $\theta[f'(k) - \delta - \rho] - v = 0$. That the numerator is positive follows from the fact that individual consumption is increasing when $f'(k) - \delta - \rho > 0$ (cf. 4.10) which is the case to the left of the asymptote. Therefore, per capita consumption will tend to increase. So, to counteract this and obtain $\dot{c}(t) = 0$, consumption of the newly deceased must be greater than consumption of newborn cohorts, which is exactly what we have in the numerator. Hence, the two lines $\dot{k}(t) = 0$ and $\dot{c}(t) = 0$ must cross at least once and define a steady state equilibrium. $\square$

The idea behind Proposition 2 is to consider each term in the equilibrium conditions and impose a restriction such that the term becomes constant or a function of k and c only. CRRA utility is often assumed and not that critical. The demographic structure imposed in point (II) links the fertility rate to people's lifetime. Such a causality is hard to defend although the assumption of a constant population growth rate might be reasonable.[22] Point (III) is obviously the strongest assumption. Not only does it again link the behavior of the newborns and the newly deceased, it also strongly restricts the choice of the functional form for the hazard rate $\mu(s, t-s)$. Thus, the restrictions in Proposition 2 are so strong and the complexity of the equilibrium so high that this special case is of little interest.

Returning to the case in Proposition 1, the main problem is the dynamics of aggregate consumption. Blanchard (1985) simplified this by assuming constant probability of death across age and cohorts. A less restrictive solution is to assume a stable population, i.e., a constant population growth rate and a fixed age distribution over time (Lotka, 1998). Existence can then be shown by rewriting the consumption function into a function of age only. Then given the stable population, consumption per capita will be constant under mild conditions. This is formally shown in Proposition 3.

Proposition 3. *In the special case of age-specific mortality only, a constant birth rate, and CRRA utility ($\mu(s, t-s) = \mu(t-s)$, $b(t) = b$, and $\theta(s,t) = \theta$) a steady state equilibrium exists.*

Proof. Without the cohort specific mortality rate, and given a constant birth rate, the population growth rate becomes constant. This can be seen by rewriting the population

[22] For the USA the population growth rate has been in the range of 0.5 to 2.5% per annum in the period 1870–2003 (Maddison, 2003).

growth rate (4.7)

$$\begin{aligned} v &= b - \int_{-\infty}^{t} \mu(t-s) \frac{bN(s) e^{-\int_0^{t-s} \mu(x)dx}}{N(t)} ds \\ &= b - \int_{-\infty}^{t} \mu(t-s) b e^{-\int_0^{t-s}[v(x)+\mu(x)]dx} ds \\ &= b - \int_{0}^{\infty} \mu(a) b e^{-\int_0^{a}[v(x)+\mu(x)]dx} da. \end{aligned}$$

The equilibrium conditions are then

$$\dot{k}(t) = 0 : c = f(k) - [\delta + v] k \tag{5.8}$$

$$\dot{c}(t) = 0 : c = \frac{\int_{-\infty}^{t} \mu(t-s) c(s,t) \frac{n(s,t)}{N(t)} ds - c(t,t) b}{\theta [f'(k) - \delta - \rho] - v}. \tag{5.9}$$

To show existence we rewrite the consumption function such that it is a function of age, the interest rate, and the wage rate only.

The marginal propensity to consume

$$\phi(s,t) = \left[\int_t^{\infty} e^{-\int_t^{\tau}[r(x)+\mu(x-s)-\theta[r(x)-\rho]]dx} d\tau\right]^{-1} = \left[\int_a^{\infty} e^{-\int_a^{\tau}[r(x+s)+\mu(x)-\theta[r(x+s)-\rho]]dx} d\tau\right]^{-1}.$$

Human wealth

$$h(s,t) = \int_t^{\infty} e^{-\int_t^{\tau}[r(x)+\mu(x-s)]dx} w(\tau) d\tau = \int_a^{\infty} e^{-\int_a^{\tau}[r(x+s)+\mu(x)]dx} w(\tau+s) d\tau,$$

and finally consumption

$$c(s,t) = \phi(s,s) h(s,s) e^{\theta \int_s^t [r(x)-\rho]dx} = \phi(s,s) h(s,s) e^{\theta \int_0^a [r(x+s)-\rho]dx}.$$

Recall that the interest rate and the wage rate are functions of k. Thus, we have reduced individual consumption to depend on time only through $k()$.

The first equation (5.8) is a function of k and c only. In the second equation (5.9) the term, $c(t,t)b$, is as just shown a function of $k()$ only. The integral can be rewritten as follows

$$\int_{-\infty}^{t} \mu(t-s) c(s,t) \frac{n(s,t)}{N(t)} ds = \int_{-\infty}^{t} \mu(t-s) c(s,t) b e^{-\int_0^{t-s}[v+\mu(x)]dx} ds.$$

Except for $c(s,t)$ time enters through age $a = t-s$ only. Hence, since the population is assumed to be stable (constant growth rate and age distribution), and the equilibrium conditions (5.8) and (5.9) have been shown to depend on c and k only, a steady state equilibrium exists if the two equations have a solution (k^*, c^*). That this is the case follows from an argument analogous to the one in the proof of Proposition 2. □

The assumption of a stable population (or a demographic steady state) has been explored in a number of recent papers, see e.g. d'Albis (2007) and Heijdra and Romp (2008). The advantage is a realistic description of differences in mortality rates across age. Yet, as outlined in Section 2, a fully realistic demographic description also includes differences across cohorts. This paper shows that generally this is incompatible with steady state equilibrium. Intuitively, existence of steady state equilibrium generally fails because the age distribution cannot be made constant over time when a longevity trend is present. The model becomes non-stationary. Technically, the impossibility of a longevity trend and a fixed age distribution at the same time creates the non-existence result.

6. Extensions

In the current setting individuals work for as long as they live. This is not very realistic, and it might be more relevant to consider growth in a setting with retirement. A first step would be to introduce a mandatory retirement age which is exogenous to the individual and allowed to vary across cohorts, i.e., the retirement age is a function of birth time, $R(s)$. The relevant question is then: can we specify a retirement age function to ensure that the economy has a balanced growth path? The answer is no (see Appendix A). We get a policy instrument (the retirement age) which can be used to control the growth rate of the labor force or to let the labor force be proportional to the population. But the change in aggregate consumption due to exit and entry of new cohorts is still as complex as above. So besides controlling the labor force we need more restrictions on individual consumption to generate steady state equilibrium. For now consumption depends on age and cohort. The dependence generates a complicated expression for the dynamics of aggregate consumption. Recall that the model, except for the longevity trend, already contains strong simplifying assumptions (wage rate independent of age, perfect annuity market etc.). Therefore, further simplifying assumptions would spoil the ability to track what we observe in reality, and left would be a model that can fit the demographic development (by assumption!) and not much more.

Why should we care whether a steady state equilibrium exists or not? The purpose of studying steady state equilibrium derives from the idea that there exist forces ensuring relatively stable economic growth. Moreover, if the economy moves towards the steady state equilibrium, analytical results and long run implications for the economy can be derived. In this model the economy does *not* converge to a steady state equilibrium, and unfortunately it is not possible analytically to determine what direction the economy moves in then. This will, among other things, depend on initial values and the functional form for $\mu(s, t-s)$.

7. Concluding remarks

It is an empirical fact that longevity follows an upward trend with no indication of an upper limit approaching. We show that extending a standard continuous time overlapping generations model to include a longevity trend by assuming cohort specific mortality rates implies that no steady state equilibrium exists. Intuitively, existence fails since individual consumption becomes heterogeneous and the age distribution changes over time, which makes aggregate consumption too complex to maintain on a balanced growth path. We have made a number of strong simplifying assumptions with the purpose of creating the best conditions for a steady state equilibrium to exist. Since this could be rejected, we see no way to achieve a balanced growth path with longevity driven population growth and thus no purpose in relaxing the model assumptions. Therefore, we must conclude that population growth caused by declining mortality rates cannot be consistent with balanced growth. Consequently, we cannot analytically make long run predictions for an economy with a longevity trend, or at least we cannot do that in the type of model studied here.

Furthermore, this paper raises questions about the standard method of steady state equilibrium comparison applied in the literature on the consequences of longevity improvements. The underlying assumption in these models seems to be that longevity improves but eventually approaches some constant level after which the economy moves to steady state equilibrium again. However, there is no indication of longevity approaching an upper limit, and the magnitude of today's longevity improvements (further stressed by the systematic underestimation of future gains) suggests that the quantitative effects of exit and entry of new cohorts with different mortality rates (as seen in (4.17)) are non-trivial.

The disappointing result of this paper raises the question of how to proceed. Is it impossible to obtain useful analytical results on the consequences of raising longevity? More work on how to handle the longevity trend is needed. An active area of research is endogenous longevity models, but so far researchers have only been able to generate stationary equilibrium (i.e. constant longevity in equilibrium). Extending this to the non-stationary case (i.e. an endogenous longevity trend) is an important future area of research.

Appendix A. Extended model with retirement

In this appendix we introduce a mandatory retirement age which is exogenous to the individual and allowed to vary across cohorts, i.e., the retirement age is a function of birth time, $R(s)$. We start by defining an indicator variable for being part of the labor force as a function of cohort (s) and time (t)

$$I_R(s,t) = \begin{cases} 1 & \text{if} \quad t < s + R(s) \\ 0 & \text{if} \quad t \geq s + R(s). \end{cases}$$

The labor force is given by

$$L(t) = \int_{-\infty}^{t} I_R(s,t)\, n(s,t)\, ds,$$

and since the population and the labor force now differ, we have to modify the factor prices found above. Total production is now

$$Y(t) = F(K(t), L(t)) = L(t)\, F\left(\frac{K(t)}{N(t)}\frac{N(t)}{L(t)}, 1\right) = L(t)\, f\left(k(t)\frac{N(t)}{L(t)}\right),$$

and the factor prices

$$\begin{aligned} r(t) &= f'\left(k(t)\frac{N(t)}{L(t)}\right) - \delta \\ w(t) &= f\left(k(t)\frac{N(t)}{L(t)}\right) - f'\left(k(t)\frac{N(t)}{L(t)}\right) k(t)\frac{N(t)}{L(t)}. \end{aligned}$$

The household's maximization problem with the exogenous retirement age, $R(s)$, now become

$$\begin{aligned} &\max_{\{c(s,\tau)\}_{\tau=s}^{\infty}} \int_{s}^{\infty} e^{-\int_0^{\tau-s}[\rho+\mu(s,x)]dx} u(c(s,\tau))\, d\tau \\ s.t. \quad : \quad & \dot{z}(s,\tau) = [r(\tau) + \mu(s,\tau-s)]\, z(s,\tau) + I_R(s,\tau)\, w(\tau) - c(s,\tau) \\ & \lim_{\tau\to\infty} z(s,\tau)\, e^{-\int_t^{\tau}[r(x)+\mu(s,x-s)]dx} = 0 \ \forall t \geq s. \end{aligned}$$

From here we can derive an Euler equation identical to the one in Section 4.2. Human wealth now becomes a function of the retirement age

$$h_R(s,t) = \int_t^{s+R(s)} w(\tau)\, e^{-\int_t^{\tau}[r(x)+\mu(s,x-s)]dx} d\tau,$$

and optimal consumption is similar to (4.11) except for the new human wealth function

$$c(s,t) = \phi(s,t)\, [z(s,t) + h_R(s,t)]. \tag{A.1}$$

The expression for marginal propensity to consume ($\phi(s,t)$) is unchanged.[23] The reason is that the retirement age is exogenous to the agent and only influences the flow of wage income. The level of consumption and non-human wealth is of course different from the case without retirement age.

The dynamics of aggregate non-human wealth is now

$$\dot{Z}(t) = r(t) Z(t) + w(t) L(t) - C(t).$$

Since the population and the labor force now differ, we can consider both per capita and per worker variables in the search for steady state equilibrium. Normalizing by the total population as in Section 5, the steady state equilibrium conditions in terms of per capita variables are

$$\dot{k}(t) = 0 : f\left(k(t)\frac{N(t)}{L(t)}\right)\frac{L(t)}{N(t)} - [\delta + v(t)] k(t) - c(t) = 0 \tag{A.2}$$

$$\begin{aligned} \dot{c}(t) = 0 : [f'(k(t)) - \delta - \rho] \int_{-\infty}^{t} \theta(s,t) \frac{n(s,t)}{N(t)} c(s,t)\, ds + c(t,t)\, b(t) \\ - \int_{-\infty}^{t} \mu(s,t-s)\, c(s,t) \frac{n(s,t)}{N(t)} ds - v(t)\, c(t) = 0. \end{aligned} \tag{A.3}$$

The ratio between the labor force and the population is now present, and since the retirement age for each cohort is adjustable, it is possible to make this ratio constant. However, the population growth rate ($v(t)$) is still present in (A.2), and as shown in Section 5, strong assumptions on the demography are needed to make this constant. Furthermore, the equilibrium condition for per capita consumption possesses the same problems as above, and therefore we reach the same conclusion.

Moving on to per worker variables, we define $q_R(t) = \frac{Q(t)}{L(t)}$ and $g(t) = \frac{\dot{L}(t)}{L(t)}$, then the equilibrium conditions are

$$\dot{k}_R(t) = 0 : f(k_R(t)) - [\delta + g(t)] k_R(t) - c_R(t) = 0 \tag{A.4}$$

$$\begin{aligned} \dot{c}_R(t) = 0 : [f'(k(t)) - \delta - \rho] \int_{-\infty}^{t} \theta(s,t) \frac{n(s,t)}{L(t)} c(s,t)\, ds + c(t,t) \frac{n(t,t)}{L(t)} \\ - \int_{-\infty}^{t} \mu(s,t-s)\, c(s,t) \frac{n(s,t)}{L(t)} ds - g(t)\, c_R(t) = 0. \end{aligned} \tag{A.5}$$

The dynamic equation for capital per worker (A.4) now features the growth rate of the labor force ($g(t)$). Since the retirement age for each cohort is fully adjustable, it is possible to make this growth rate constant, which would imply that the equilibrium condition (A.4)

[23] $\phi(s,t)$ depends on the equilibrium interest rate, and since this is likely to differ from the case without retirement, the value of the marginal propensity to consume will differ in the two cases.

would be a function of k_R and c_R only as desired. The dynamic equation for consumption per worker (A.5) can, however, not be made a function of k_R and c_R only, and therefore we do not find a steady state equilibrium in terms of per worker variables either.

In conclusion, introducing a mandatory and cohort specific retirement age is not sufficient to ensure the existence of steady state equilibrium when a longevity trend is present.

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