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## Choosing to keep up with the Joneses

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#### Abstract

Does a rise in income inequality induce people to work harder to stay in the rat race ("keep up with the Joneses") or to simply drop out? We investigate this issue in a simple new framework in which heterogeneous ability agents get extra utility if their consumption keeps up with the economy's average. The novelty is that agents are allowed to choose whether they want to stay in or drop out of the rat race. We show that sufficiently high ability agents choose to keep up with the Joneses and they enjoy higher consumption but lower leisure than those who don't. When income inequality rises in a mean-preserving manner, average leisure in the economy may fall. Our analysis touches on the question, why are Americans working so much compared to the Europeans? We posit that higher income inequality in the US, by inducing more people to join the rat race there, may be partly responsible for the transatlantic leisure divide.

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### 1 Introduction

Human beings have always had an ambivalent attitude towards the commonly observed practice of "keeping up with the Joneses". In the context of the Bible, for example, Gordon (1963) argues that it must "strike the modern man as strange that the Ten Commandments end with a prohibition against coveting<sup>1</sup>, something which is neither criminal nor punishable in any society... A man who does not want to raise his standard of living to that enjoyed by the other fellow is considered ambitionless, and as such reprehensible. To be sure, if coveting results only in unconstructive greed, ostentation, or pushiness (e.g., "keeping up with the Joneses"), and therefore entails an unseemly response to the potentially desirable stimulus of coveting, it may be frowned upon.<sup>2</sup> But when coveting impels us to greater effort, so that we may rise constructively toward the level of our more affluent neighbors, we are on the path that universally leads to approbation.<sup>3</sup> Why, then, does the Bible make a great issue of coveting, grouping it with such evil offenses as murder?".

The aforediscussed equivocal belief manifests itself in modern times in one of two reactions; either people can and are willing to run on the "hedonic treadmill" of keeping up with some reference group (i.e., they participate in the "rat race"), or they stay out of it.<sup>4</sup> In this paper, we propose a simple description of preferences to capture this idea – that some people, acting in their own self interest, *choose* to participate in the rat race, while others choose to drop out.

Our framework is in line with recent work on models with "keeping up with the Joneses" preferences – preferences summarized by utility functions which depend not just on one's own level of consumption, but also on the economy's average.<sup>5</sup> Such models have become immensely popular in many areas of economics and finance. Classic references in this literature include the seminal

<sup>&</sup>lt;sup>1</sup>Exodus 20:2-20:14, also known as the Ten Commandments, reads as follows "You shall not covet your neighbor's house; you shall not covet your neighbor's wife, or male or female slave, or ox, or donkey, or anything that belongs to your neighbor."

<sup>&</sup>lt;sup>2</sup>Johnson (1988) reporting on the dark side of keeping up during Victorian times finds "that some of the poorest families in York in 1899 were starving, and hastening their own death, in order to maintain the insurance payments that would let them have a really good funeral. And every Monday morning throughout Britain, thousands of bundles of Sunday clothes would be deposited with the pawnbroker, and then redeemed the following Saturday at enormous long-term cost, just to maintain appearances on the Sabbath."

<sup>&</sup>lt;sup>3</sup>Perkin (1968) argues that it is keeping up that explains "the massively expansive consumer demand" necessary to sustain the Industrial Revolution. "[...] the key to the Industrial Revolution was the infinitely elastic home demand for mass consumer goods. And the key to that demand was social emulation, 'keeping up with the Joneses', the compulsive urge for imitating the spending habits of one's betters, which sprang from an open aristocracy in which every member from top to bottom of society trod closely on the heels of the next above."

 $<sup>^{4}</sup>$ We use the terminologies "rat race" and "keeping up with the Joneses" interchangeably. Also note that our notion of a rat race is not the same as that in the literature on adverse selection; see for example, Landers, Rebitzer, and Taylor (1996). There, agents may try to signal their high type by agreeing to work longer than most, and firms may use this willingness to work long hours as an indicator of high type.

 $<sup>{}^{5}</sup>$ In a seminal piece, Hopkins and Kornienko (2004), following up on pioneering work by Frank (1985, 1999), present an entirely different angle. In their setup, people care both about their own consumption and also about status – their ordinal rank in the distribution of consumption. As society gets richer, those whose incomes do not keep up with the rest of society spend more on "conspicous consumption" so as to maintain their rank on the consumption distribution.

pieces by Abel (1990) and Gali (1994), followed by more recent work by Ljungqvist and Uhlig (2000) and Alonso-Carrera, Caballé, and Raurich (2005), among numerous others. Underlying the preference formulations used by all these models is the presumption that everyone *wants* to keep up with the Joneses; hence, the issue of "what if they don't?" has never been addressed. The novelty of our paper is that we model the decision to participate in the rat race as a choice.

In our setup, people who stay in the rat race derive utility not just from consumption but also from their success in keeping up with the Joneses. That is, when their level of consumption beats the average, they get a special boost in utility from knowing they have "made it" in life. We posit, for example, Mr. X receives utility not just from owning a 2000 square feet house but *also* from knowing that his house is bigger than the average house (1500 sq. feet) in his neighborhood. In contrast, Ms. Y, who owns a 1200 square feet house in that same neighborhood, derives utility solely from the housing services her house provides, nothing extra. We wish to know why Mr. X and Ms. Y choose to own their respective sized houses, and whether Ms. Y, if she could work harder and earn enough to afford it, would ever buy a 2000 sq. feet house?

Our preference formulation shares many similarities with those studied in Dupor and Liu (2003).<sup>6</sup> In their paper, an increase in per capita consumption causes a) marginal utility from own consumption to rise relative to leisure (what they term "Keeping up with the Joneses" or KUJ) and/or b) ceteris paribus, it lowers the agent's own utility ("jealousy"). In our setup, those who consume more than per capita consumption behave exactly as people with KUJ and jealousy preferences in Dupor and Liu (2003). The critical difference is that Dupor and Liu (2003) do not study the possibility that some people may choose not to keep up. In our framework, those who choose to drop out, do so, because they do not wish to work as hard as those who participate. On the one hand, they avoid exposure to the jealousy associated with keeping up; on the other hand, they relinquish any utility 'kick' from successfully competing in the rat race.

The model economy is inhabited by heterogenous ability agents who share the same set of preferences – they all like consumption and leisure and receive an extra boost in utility only if their consumption exceeds the (endogenously determined) average of the economy. We show that there is a unique cut-off level of ability above which people choose to keep up and below which they choose not to. Additionally, members of the former group enjoy higher consumption, albeit lower leisure, than members of the latter. When income inequality rises (modeled here as a

<sup>&</sup>lt;sup>6</sup>Falk and Knell (2004) argue that models incorporating some measure of relative comparison typically rely on three assumptions. First, utility is defined not only in terms of an individual's own consumption c, but also in terms of consumption relative to that of a reference group r, i.e., U = U(c, r). Second,  $U_1 > 0$  and  $U_2 < 0$ . Third, the reference standard r is assumed to be exogenously given. Our formulation shares the aforementioned features, except a) we allow agents to choose whether to have their utility valued according to U(c, r) or simply U(c), b) agents who successfully keep up get an extra boost in utility, and c) even though agents do not choose their own r, there is a common reference standard for all and it is endogenously determined.

mean-preserving increase in the spread of the ability distribution), it is possible that more people participate in the rat race, and on average, people work more.

This ties in with the larger question recently popularized by Prescott (2004) and others: in recent times, why are Americans working so much more than their European counterparts?<sup>7</sup> Prescott (2004) has argued that higher marginal income tax rates in Europe relative to the U.S. are closely correlated with their shorter workweeks and expanding vacations. Others such as Glaeser, Sacerdote, and Scheinkman (2003) and Alesina, Glaeser, and Sacerdote (2005) have suggested that stronger unionism in Europe is to blame: unions bargain for shorter workweeks (especially when it is hard to secure wage increases) raising the marginal benefit from taking vacations with friends whose workweeks are also shorter. Our analysis, albeit qualitative, has the potential to add to this line of inquiry. In the model economy, when income inequality rises, it is possible that more people choose to participate in the rat race and consequently work harder, thereby reducing average leisure. The logic of the argument is complete once it is noted that income inequality is (and has been) much higher in the U.S. than in Europe. More generally, our analysis suggests a new channel by which inequality may influence work hours via its effect on the composition of keeper-uppers and drop-outs in a society.

The empirical connection between inequality and work hours is a strong one. Bell and Freeman (2001), for example, provide empirical evidence from U.S. and German data that suggests wage inequality within detailed occupation/industry cells is positively associated with work hours for those working 35 hours a week or longer. They argue that inequality induces longer hours, especially if that latter acts as a proxy for unobservable quality. Using panel data from 10 OECD countries on average annual work hours and income inequality, Bowles and Park (2005) find that the latter "is a predictor of work hours in both OLS and fixed-effects estimates; its effects are large, and estimates are robust across a variety of specifications."<sup>8</sup>

<sup>&</sup>lt;sup>7</sup>Prescott (2004) cites OECD labor market statistics that suggest "Americans aged 15-64, on a per-person basis, work 50% more than the French. Comparisons between Americans and Germans or Italians are similar."

Landsburg (2006) summarizes the differences between Americans and Europeans as follows: "The average American works 25 hours a week; the average Frenchman 18; the average Italian a bit more than 16 and a half. Even the hardest-working Europeans – the British, who put in an average of 21 and half hours – are far more laid-back than their American cousins.

Compared with Europeans, Americans are more likely to be employed and more likely to work longer hours – employed Americans put in about three hours more per week than employed Frenchmen. Most important, Americans take fewer (and shorter) vacations. The average American takes off less than six weeks a year; the average Frenchman almost 12. The world champion vacationers are the Swedes, at 16 and a half weeks per year."

<sup>&</sup>lt;sup>8</sup>Bowles and Park (2005) also study a simple model of the choice of work hours. Theirs is a formalization of Veblen's persuasive argument that everyone in society tries to emulate the richest – the leisure class – who in Veblen's time did not do much work. Their model predicts that work hours are increasing in the degree of income inequality.

While there are several superficial similarities, the differences between our papers stand out prominently. In Bowles and Park (2005), the need to emulate the leisure class is modeled as a consumption externality in the utility function of the poor; the rich are in no way impacted by the actions of the poor. This consumption externality becomes stronger when the rich earn exogenously higher wages – their interpretation of an increase in inequality. More importantly, they do not allow for the possibility for agents to choose to enter or drop out of the rat race.

The plan for the rest of the paper is as follows. Section 2 outlines the model and characterizes the set of agents that keep up and those that drop out. Section 3 studies the impact of a change in the spread of the ability distribution; it contains our main results, concluding with the one showing that a marginal increase in inequality increases the number of agents keeping out, thereby raising average hours worked. Concluding remarks are contained in Section 4. Appendix A studies several dimensions of robustness with respect to some of our simplifying assumptions. Proofs of major results are relegated to appendices B-G.

### 2 The Model

#### 2.1 Preliminaries

Consider a simple static model in which people (indexed by *i*) work for a fixed wage *w*. Agents differ in their effective units of time endowment (or innate ability),  $1 + h^i$ . Agent *i* draws  $h^i$ from a continuous distribution with cdf  $\mathcal{F}(h^i)$  and density  $f(h^i)$  with support  $[h_l, h_u]$  where  $0 \leq h_l < h_u < \infty$ . Let  $\bar{h}$  denote the mean value of *h*, and let  $\sigma$  denote the spread of the distribution of *h*.

Agents' preferences over consumption (c) and leisure (l) are summarized by the utility function U(c, l) as follows:

$$U(c,l) \equiv \begin{cases} u(c+\theta(c-\kappa)) + \phi v(l) & \text{if } c > \kappa \\ u(c) + \phi v(l) & \text{if } c \le \kappa \end{cases}$$

Here  $\theta \ge 0$  is a parameter capturing the relative importance of keeping-up (see below) and  $\kappa > 0$ is the to-be-specified level of consumption the agent aspires to keep up. Also,  $\phi$  is a parameter capturing the relative importance of leisure. We assume the functions u and v are strictly increasing and strictly concave. In what follows, we label people enjoying consumption greater than  $\kappa$  as *keeper-uppers* – those participating in the *rat race* of keeping up with the Joneses; all others will be described as having dropped out – the *drop outs*.

The interpretation of the "consumption portion" of these preferences is as follows. When consumption is at or below  $\kappa$ , the agent derives utility according to u(c); if it exceeds  $\kappa$ , she derives utility according to  $u(c + \theta (c - \kappa))$ . This means that when consumption exceeds  $\kappa$ , her "effective consumption" rises to  $c + \theta (c - \kappa)$ . Intuitively, this captures the idea that a 1200 square feet house may appear to its owner to be "bigger and better" in a neighborhood with mainly 800 square feet houses as opposed to one with primarily 2000 square feet houses.<sup>9</sup>

<sup>&</sup>lt;sup>9</sup>Another motivating example might be the following. A grocery store manager may get a extra boost in utility from wearing a \$100 suit in the company of check-out personnel; and yet, that same suit "does nothing" for him if he has to wear it to a fancy wedding.

In utility terms, if  $\theta = 0$ , there is no utility benefit to keeping up with  $\kappa$ , and as such,  $\kappa$  is irrelevant. When  $\theta > 0$  holds, and an agent's consumption crosses  $\kappa$ , she receives utility not just from consumption but also from the *very act* of successfully keeping up with the Joneses.<sup>10</sup> To carry on with the house example, she gets utility not just from owning a house but also because hers is a bigger/better/fancier house than that of her neighbors.<sup>11</sup>



Figure 1: Utility from dropping-out and keeping-up

It deserves mention here that the marginal utility of consumption is higher to the right of  $\kappa$  than to the left. Consequently, a kink appears at  $\kappa$ .<sup>12</sup> The implication is that keeping up bestows higher total utility and higher marginal utility: an additional Alphonso mango tastes sweeter and each marginal bite tastes sweeter to those who keep up relative to those who don't. This effect is strongest near  $c = \kappa$ : the marginal utility of consumption just to the right of  $\kappa$  in Figure 1 is much higher than to the left of it. But for those consuming far above  $\kappa$ , the effect of  $\kappa$  on marginal utility is quite muted.<sup>13</sup>

<sup>&</sup>lt;sup>10</sup>Boss (2006) argues that keeping-up behavior may be a social norm: "We're trained to gaze up one level from where we are and to aspire to get what those people have. Once we accomplish that much, we're looking up again. By cultural design, there is no end to it." For more on this line of thinking, see Akerlof (2007).

<sup>&</sup>lt;sup>11</sup>Note that the utility she gets from "housing services" is not separable from the utility she gets because her house is superior to those of the Joneses. Also, bear in mind she faces no penalty for *not* keeping up.

There is also a sense in which these are 'meta' preferences; she is effectively choosing how she would like her consumption to be valued and this "choice" is influenced by the consumption of a specified reference group.

<sup>&</sup>lt;sup>12</sup>Parenthetically, note that  $\theta$  introduces non-homotheticity in preferences. When  $\theta$  is positive, homotheticity obtains separately for  $c \leq \delta$  and for  $c > \delta$ , but not for the entire range of c.

<sup>&</sup>lt;sup>13</sup>It also deserves mention that for  $\theta > 0$ , the "utility from keeping up",  $\theta(c - \kappa)$ , is linearly increasing in c implying that the *rate* at which utility from keeping up increases with c is constant. Arguably, it is more reasonable

In other respects, this formulation of preferences is fairly standard. Utility from consumption and leisure is assumed to be continuous and concave in each segment. As is common in other specifications of keeping-up preferences, higher  $\kappa$  ("jealousy") hurts the agent since it makes keeping-up harder.<sup>14</sup>

The rest of the model is deliberately kept extremely simple. Agent i allocates her effective time endowment between work (x) and leisure (l), and so

$$x^i + l^i = 1 + h^i$$

holds. Then agent i's budget constraint is given by

$$c^i = wx^i.$$

As is evident, some agents may not have a effective time endowment high enough to finance a level of consumption higher than  $\kappa$ , *even* when they enjoy no leisure. This implies that agents with h such that

$$h^{i} < h^{F} \equiv \max\left(\left(\frac{\kappa}{w} - 1\right), h_{l}\right)$$

holds are those who are forced (by their low time endowment) to stay out of the rat race. Of course, if  $h^F = h_l$ , no agent is forced out of the rat race.

We now consider the possibility that some agents may choose to stay out. In other words, restrict attention to those *i* for whom  $h^i > h^F$ . When such an agent is *contemplating* dropping out, her leisure and consumption are computed by solving

$$\max_{c,l} \quad u(c) + \phi v(l)$$

subject to  $c_d^i = w \left(1 + h^i - l_d^i\right)$ , taking  $\kappa$  as given; the subscript *d* refers to "dropping out". This yields the following first order condition:

$$w u' \left( w \left( 1 + h^i - l_d^{i*} \right) \right) = \phi v' \left( l_d^{i*} \right).$$

$$\tag{1}$$

A valid interior optimum  $l_d^{i*}$  additionally satisfies  $w(1 + h^i - l_d^{i*}) \equiv c_d^{i*} \leq \kappa$ . When that same agent contemplates keeping up, her leisure and consumption are computed by solving

$$\max_{c,l} \quad u\left(c + \theta\left(c - \kappa\right)\right) + \phi v\left(l\right)$$

to think that the aforementioned rate falls with c (those who beat  $\kappa$  by a wide margin get lower incremental utility from a small increase in their consumption than those who just "make it"). This angle could be introduced, for example, by assuming  $u (c + \theta (c - \kappa)^{\gamma})$  where  $\gamma \in [0, 1]$ . However, as will be clear below, setting  $\gamma < 1$  would render the analysis analytically intractable.

<sup>&</sup>lt;sup>14</sup>Luttmer (2005) finds convincing evidence that "an increase in neighbors' earnings and a similarly sized decrease in own income each lead to a reduction in happiness of about the same order of magnitude."

subject to  $c_k^i = w \left(1 + h^i - l_k^i\right)$ , taking  $\kappa$  as given; the subscript k refers to "keeping up". This yields

$$(1+\theta) w u' \left( (1+\theta) w \left( 1+h^i - l_k^{i*} \right) - \theta \kappa \right) = \phi v' \left( l_k^{i*} \right).$$

$$\tag{2}$$

Again, a valid interior optimum  $l_k^{i*}$  also satisfies  $w\left(1+h^i-l_k^{i*}\right)\equiv c_k^{i*}>\kappa$ .

It follows that if the agent chooses to keep up, her consumption will be higher and leisure lower than if she chooses to drop out.<sup>15</sup> It is also clear from (1) and (2) that for agent *i*, while the marginal benefit from an additional unit of leisure is the same if she keeps up or drops out, the accompanying marginal cost of lost consumption is higher (because  $\theta > 0$ ) if she keeps up relative to if she drops out.

This is exactly where  $\kappa$  (jealousy) matters to the keeper uppers. Ceteris paribus, a rise in  $\kappa$  reduces the effective consumption of a keeper-upper and raises her marginal valuation of effective consumption over leisure. Keeping up entails allowing oneself to be negatively influenced by the consumption of a reference group. When the latter rises, a keeper upper's effective consumption falls and she counteracts by substituting out of leisure (into work), earning more income, and raising her own consumption. Needless to say, a drop out is protected from all this.

It is relatively straightforward to verify that for any i, there is a unique  $l_d^*$  that solves (1) and a unique  $l_k^*$  that solves (2). Also,  $l_d^*$  and  $l_k^*$  are each non-decreasing in h suggesting that leisure is a normal good for both keeper uppers and drop outs.<sup>16</sup> Note that it is quite possible that a sufficiently rich agent who chooses to keep up has higher leisure than a sufficiently poor agent who chooses to drop out.<sup>17</sup> Of course, at valid optima, it is not possible for a keeper upper to receive less consumption than a drop out.

#### 2.2 Who keeps up, who doesn't

It is useful to know more about the identities of who chooses to keep up and who doesn't. To that end, and for reasons of analytical tractability, we henceforth assume a quasi-linear form:

$$U(c,l) \equiv \begin{cases} (c+\theta(c-\kappa)) + \phi \ln l & \text{if } c > \kappa \\ c+\phi \ln l & \text{if } c \le \kappa \end{cases}$$

<sup>16</sup>From (1), it follows that  $w^2 u''(\cdot) \left(1 - \frac{\partial l_d^i}{\partial h^i}\right) = \phi v''(\cdot) \frac{\partial l_d^i}{\partial h^i} \iff \frac{\partial l_d^i}{\partial h^i} = \frac{w^2 u''(\cdot)}{\phi v''(\cdot) + w^2 u''(\cdot)}$ . Since  $u'' \leq 0$  and  $v'' \leq 0$ , it follows that  $\frac{\partial l_d^i}{\partial h^i} \geq 0$ .

<sup>&</sup>lt;sup>15</sup>This is reminiscent of the idea of downshifting, defined as "changing voluntarily to a less demanding work schedule in order to enjoy life more". Downshifters, arguably, are not dropouts; they are in pursuit of a more balanced lifestyle.

<sup>&</sup>lt;sup>17</sup>See footnote 8 for Veblen's characterization of a similar idea.

For this specification, the marginal utility of consumption of anyone who drops out is 1, and for anyone who keeps up, is constant at  $1 + \theta$ . Let  $\phi_u \equiv w (1 + h_l)$  and

$$\phi \le \phi_u. \tag{A.1}$$

It is easy to check that the first order conditions (1) and (2) imply that

$$l_d^i = \frac{\phi}{w}; \quad c_d^i = w \left( 1 + h^i - \frac{\phi}{w} \right) \tag{3}$$

$$l_k^i = \frac{\phi}{w\left(1+\theta\right)}; \quad c_k^i = w\left(1+h^i - \frac{\phi}{w\left(1+\theta\right)}\right) \tag{4}$$

hold. Then (A.1) implies that  $c_d^i, c_k^i \ge 0$  and  $l_d^i, l_k^i \le 1 + h^i$  for each *i*. For these candidate solutions to be valid optima, we must verify that  $c_d^i \le \kappa$  and  $c_k^i > \kappa$  hold. From (3)-(4), it follows that  $l_k^i < l_d^j$  for any *i*, *j* implying that any agent who chooses to keep up would enjoy less leisure than any other agent who chooses to drop out.<sup>18</sup>

An agent will choose to stay out of the rat race if the utility from staying out exceeds that of participating. Define  $b(h^i; \kappa)$  to be the difference between the indirect utility from keeping up and dropping out – the net benefit from keeping up. Then it is easily verified that

$$b(h^{i};\kappa) = \frac{\theta\phi}{1+\theta} + \theta\left(w\left(1+h^{i}-\frac{\phi}{w(1+\theta)}\right)-\kappa\right) - \phi\ln\left(1+\theta\right).$$
(5)

The first term represents the additional utility from higher consumption received when keeping up; the second term is the utility 'kick' the agent gets from the very act of keeping up, and the third term is the net loss in utility incurred from needing to work harder so as to be able to keep up. Formally, agents for whom the following inequality holds choose to keep up:  $b(h^i; \kappa) \ge 0$  for any  $h^i \ge \hat{h}$  where  $\hat{h}$  is defined by  $b(\hat{h}; \kappa) \equiv 0$ . Straightforward algebra shows

$$h^{i} > \hat{h} \equiv \left(\frac{\kappa}{w} - 1\right) + \frac{\phi \ln\left(1 + \theta\right)}{\theta w} \ge h^{F}.$$
(6)

Note that when  $\phi = 0$ ,  $\hat{h} = h^F$ ; i.e., if no one derives any utility from leisure, then a person keeps up if and only if they *can*. When  $\phi$  is positive, people care about leisure and only then does dropping-out become a choice for some. Notice that (5) implies that

 $b(h^{i};\kappa) = \theta w(1+h^{i}-\kappa) - \phi \ln(1+\theta).$ 

It follows that as  $\phi$  rises, the net loss in utility incurred from needing to work harder (so as to be able to keep up) rises. This raises the cut-off,  $\hat{h}$ ; only those who get a sufficiently high kick

<sup>&</sup>lt;sup>18</sup>The principal benefit of our choice of a quasi-linear form for utility is now apparent. Neither  $l_d^i$  nor  $l_k^i$  depends on  $h_i$  and hence the lesiure of anyone who keeps up  $\left(l_k = \frac{\phi}{w(1+\theta)}\right)$  is always less than that of someone who drops out  $\left(l_d = \frac{\phi}{w}\right)$ . This makes the subsequent analysis analytically tractable. However, as discussed earlier, this is not true in general. Robustness issues are discussed in Appendix A.2 below.

in utility from keeping up can rationally forego the accompanying loss in utility from requiring to work harder.

We now verify that  $c_d^i \leq \kappa$  for anyone who chooses to drop out and  $c_k^i > \kappa$  for any agent who chooses to keep up, and hence any agent who chooses to keep up would enjoy more consumption than *any other* agent who chooses to drop out.

**Lemma 1** If  $h^i \leq \hat{h}$ , agent *i* drops out and chooses consumption  $c_d^i \leq \kappa$ ; if  $h^j > \hat{h}$ , agent *i* keeps up and chooses consumption  $c_k^j > \kappa$ .

The previous analysis suggests that, for a given reference point  $\kappa$ , the entire population gets divided into two categories. Those with effective time endowment  $h^i < \hat{h}$  drop out of the rat race (either by choice or by force) and they are the "drop-outs"; those with  $h^i \in (\hat{h}, h_h]$  stay in, and they are the keeper-uppers. Formally,

$$\mathcal{D} \equiv \left\{ i : h^i < \hat{h} \right\}$$

is the fraction of agents in the population who drop out and

$$\mathcal{K} \equiv \left\{ i : h^i \ge \hat{h} \right\}$$

is the set of keeper uppers.<sup>19</sup> To foreshadow, a change in the effective time endowment distribution will matter because it will influence the size of the sets  $\mathcal{D}$  and  $\mathcal{K}$ , which in turn will affect the aggregate consumption and leisure.

#### 2.3 The reference point

It remains to discuss the determination of the cut-off  $\kappa$ . While there is obviously no uniquely acceptable way to choose  $\kappa$ , we posit without loss of much generality, that  $\kappa = \lambda \delta$ , where  $\delta$  is the cross-sectional mean consumption in the economy, and  $\lambda$  is a positive scalar.<sup>20</sup> In that case,

$$\delta = \int_{h_l}^{\hat{h}} c_d^i \, d\mathcal{F}\left(h^i\right) + \int_{\hat{h}}^{h_u} c_k^i \, d\mathcal{F}\left(h^i\right)$$

Using the expressions for  $c_d^i$  and  $c_k^i$  derived earlier, it is straightforward to check that

$$\delta = w \left( 1 + \bar{h} \right) - \frac{\phi \left( 1 + \mathcal{F} \left( \hat{h}; \sigma \right) \theta \right)}{(1 + \theta)},\tag{7}$$

<sup>&</sup>lt;sup>19</sup>A November 2004 poll conducted by the US News and World Report found that 48 per cent of Americans have done at least one of the following in the past five years: "cut back their hours at work, declined or did not seek a promotion, lowered their expectations for what they need out of life, reduced their work commitments, or moved to a community with a less hectic way of life."

 $<sup>^{20}</sup>$ Falk and Knell (2004) develop a theory of the determination of reference groups. In their setup, individuals choose "which Joneses to keep up with" by optimizing against two opposing criteria: "self-enhancement" – people compare with others to make themselves feel better – and "self improvement" – people compete with others to improve their own performance.

where  $\mathcal{F}(\hat{h};.)$  is the measure of agents (for given spread,  $\sigma$ ) with effective time endowments less than or equal to the cut-off  $\hat{h}$ , or more succinctly, the probability of an agent being in  $\mathcal{D}^{21}$  It is clear from (7) that average consumption in the economy falls when  $\mathcal{F}(\hat{h};.)$  rises. This is intuitive because those in the set  $\mathcal{D}$  consume less than those in  $\mathcal{K}$  and hence an increase in the probability mass of the former group lowers average consumption.

A few additional words about eq. (7) are in order. Notice when  $\phi = 0$ , people do not value leisure and hence average consumption in the economy is just  $w(\bar{h}+1)$ , the income of the agent with average h. When  $\phi > 0$ , but  $\theta = 0$ , people care about leisure but not about keeping up. In that case, preferences become homothetic for the entire range of consumption, and hence the distribution of h does not influence the determination of average consumption, i.e.,  $w(\bar{h}+1) - \phi < w(\bar{h}+1)$  holds.<sup>22</sup> Notice when  $\theta > 0$  holds, then  $\mathcal{F}(\hat{h};\sigma) \leq 1 \Rightarrow \frac{1+\theta \mathcal{F}(\hat{h};\sigma)}{(\theta+1)} \leq 1$ , and hence  $\phi\left(\frac{1+\theta \mathcal{F}(\hat{h};\sigma)}{(\theta+1)}\right) \leq \phi$  obtains, implying (see eq. (7)) that average consumption is higher when people care about keeping up. In short,

$$\delta_{\phi=0} > \delta_{\phi>0,\theta>0} > \delta_{\phi>0,\theta=0}$$

obtains.

Using  $\kappa = \lambda \delta$  in (6) along with (7), we get a single equation in the cut-off, h:

$$\left(\hat{h} - \bar{h}\right) + \frac{1}{w} \left(\frac{\lambda \phi \theta}{1 + \theta}\right) \mathcal{F}\left(\hat{h}; \sigma\right) = \frac{1}{w} \left(\frac{\phi \ln\left(1 + \theta\right)}{\theta} - \frac{\lambda \phi}{(1 + \theta)}\right) + (\lambda - 1)\left(1 + \bar{h}\right).$$
(8)

Let the left hand side of (8) be denoted by

$$l(h) \equiv \left(h - \bar{h}\right) + \frac{1}{w} \left(\frac{\phi \lambda \theta}{1 + \theta}\right) \mathcal{F}(h; \sigma)$$
(9)

and the right hand side by

$$g \equiv \frac{1}{w} \left( \frac{\phi \ln \left( 1 + \theta \right)}{\theta} - \frac{\lambda \phi}{\left( 1 + \theta \right)} \right) + \left( \lambda - 1 \right) \left( 1 + \bar{h} \right).$$
<sup>(10)</sup>

Also, denote

$$\underline{\lambda} \equiv \frac{1+h_l - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta}}{1+\bar{h} - \frac{\phi}{w} \frac{1}{(1+\theta)}}; \quad \overline{\lambda} \equiv \frac{1+h_u - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta}}{1+\bar{h} - \frac{\phi}{w}}.$$

The next lemma outlines a necessary and sufficient condition for the existence of a unique cut-off.

**Lemma 2** There exists a unique  $\hat{h} \in [h_l, h_u]$  iff

$$\underline{\lambda} < \lambda < \overline{\lambda}. \tag{A.2}$$

<sup>21</sup>Since  $\frac{\phi(1+\mathcal{F}(\hat{h};\sigma)\theta)}{(1+\theta)} \leq \phi$ , it follows from (A.1) that  $\delta > 0$ .

<sup>&</sup>lt;sup>22</sup>Average consumption is lower when  $\phi > 0$ . The opportunity cost of leisure in terms of goods is  $\frac{\phi}{1+\phi}$  and this is subtracted from average consumption.

Henceforth, we maintain Assumption (A.2). Notice that  $\underline{\lambda} < 1 < \overline{\lambda}$  holds.

An implication of Lemma 2 is that the population gets split *exactly* into two groups,  $\mathcal{D}$  and  $\mathcal{K}$ . There is a single cut-off level of h below which people drop out of the rat race and above which they stay in. This renders impossible a situation, for example, in which an agent with a very high h chooses to drop out and enjoy leisure; the loss in utility from the resultant decline in her consumption would simply hurt her too much.

## 3 Changes in the "income distribution"

We are now in a position to ask the question, how do changes in the income distribution affect average consumption and leisure in the economy? We will consider the effective time endowment distribution as a proxy for the income distribution.<sup>23</sup> Our focus is restricted to a mean preserving increase in spread (MPIS) of the effective time endowment distribution.

Such an increase in the spread will redistribute mass away from the center and towards the tails. For a fixed cutoff, the mass redistribution would have a first order effect on the number of drop-outs – the people with time endowment below the cutoff. This, in turn, would influence average consumption (and leisure). In our setup, since the cutoff *itself* depends on average consumption, there would be an additional second order effect; that is, the first order change in average consumption would change the cutoff and hence the mass of keeper-uppers, and ultimately average consumption.

#### 3.1 Position of the cut-off

It is well known that if a new unimodal distribution,  $\mathcal{F}(h, \sigma_2)$ , is obtained from an old one,  $\mathcal{F}(h, \sigma_1)$ , via a mean-preserving increase in spread from  $\sigma_1$  to  $\sigma_2$ , it exhibits the single crossing property. That is, there exists an  $h^*$  such that  $\mathcal{F}(h, \sigma_2) \geq \mathcal{F}(h, \sigma_1)$  whenever  $h \leq h^*$ . Also, when the mean is preserved, and  $\mathcal{F}(.)$  is a symmetric and unimodal distribution, the single crossing takes place at the mean, i.e.,  $h^* = \bar{h}.^{24}$  See Figures 2a and 2b for an illustration. Intuitively, for a symmetric and unimodal distribution, a mean-preserving increase in spread redistributes the mass evenly away from the mean towards the tails, implying draws near the mean under the new

<sup>&</sup>lt;sup>23</sup>In the economy studied, agent *i*'s income is endogenous and is given by  $wx^i = w(1 + h^i - l^i)$ . The distribution of the effective time endowment  $(h^i)$  is exogenous and is a worthy proxy for the income distribution. More so here, since  $l^i$  does not directly depend on  $h^i$ .

<sup>&</sup>lt;sup>24</sup>Recall, that for a density function f symmetric about its mean  $\bar{h}$ , the skewness given by  $\mu_3 \equiv E \left(h^i - \bar{h}\right)^3$  is zero, and  $\mathcal{F}(\bar{h}) = 1/2$ . If  $\mu_3 > 0$ , the density is often called skewed to the right, or having long tails to the right.

distribution are less likely and draws near the tails are correspondingly more likely.



The analytical results below are derived under the assumption that  $\mathcal{F}(.)$  is a symmetric and unimodal distribution.<sup>25</sup> In that case, the position of the cut-off relative to the mean will be critical in what follows.<sup>26</sup> This is because for a symmetric distribution, when  $\sigma$  increases in a mean preserving manner,  $\mathcal{F}(\hat{h};\sigma)$  rises (falls) for all  $\hat{h}$  below (above) the single-crossing point (which, in this case, is the mean  $\bar{h}$ ). In short, for a symmetric distribution,  $\mathcal{F}_2(\hat{h},\sigma) \gtrless 0$  as  $\hat{h} \lessapprox \bar{h}$ .

**Lemma 3** Let  $\mathcal{F}(;\sigma)$  be a symmetric and unimodal distribution. Define

$$\lambda^{\dagger} \equiv \frac{1 + \bar{h} - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta}}{1 + \bar{h} - \frac{\phi}{w} \frac{1}{(1+\theta)} \left(1 + \frac{\theta}{2}\right)}$$

Then, a)  $\hat{h} \gtrless \bar{h}$  iff  $\lambda \gtrless \lambda^{\dagger}$ , and b)  $\underline{\lambda} < \lambda^{\dagger} < \overline{\lambda}$ .

This means for any  $\lambda$  in the range  $(\lambda^{\dagger}, \overline{\lambda}]$ ,  $\hat{h} > \overline{h} \Leftrightarrow \mathcal{F}_2(\hat{h}, \sigma) < 0$  will obtain; similarly, for any  $\lambda$  in the range  $[\underline{\lambda}, \lambda^{\dagger}]$ ,  $\hat{h} < \overline{h} \Leftrightarrow \mathcal{F}_2(\hat{h}, \sigma) > 0$  will obtain.<sup>27</sup> We are now ready to state a very important result.

**Proposition 1** a) The sign of  $\frac{d\hat{h}}{d\sigma}$  is opposite that of  $\mathcal{F}_2(\hat{h}, \sigma)$ . b) Suppose  $\lambda > \lambda^{\dagger}$ . Then, for a symmetric and unimodal distribution,  $\frac{d\hat{h}}{d\sigma} \ge 0$  holds, implying a mean-preserving increase in the spread raises the cut-off.

To summarize: from Lemma 3, we know that for a symmetric and unimodal distribution of effective time endowments, the cut-off,  $\hat{h}$ , will exceed the mean,  $\bar{h}$ , for high-enough  $\lambda$ . Proposition

<sup>&</sup>lt;sup>25</sup>The robustness of our results extends to skewed distributions and is examined via numerical methods in Appendix A below.

<sup>&</sup>lt;sup>26</sup>More generally, what is crucial is the position of the cut-off  $\hat{h}$  relative to the single crossing point  $h^*$  (which, for asymmetric distributions, may not coincide with the mean).

<sup>&</sup>lt;sup>27</sup>By inspection,  $\lambda^{\dagger} > 1$ . This is an artifact of our simplifying assumptions of symmetric distributions and quasilinear utility. We have verified that for a more general CRRA form utility,  $\lambda^{\dagger} \leq 1$  is possible. In any case, it is not unreasonable to imagine that people who *are* in the rat race aspire to reach consumption levels that are "above average".

1 takes it further by claiming that for symmetric distributions, and for high enough  $\lambda$ , a meanpreserving increase in the spread *raises* the cut-off  $\hat{h}$ .

As discussed above, for sufficiently high  $\lambda$ ,  $\hat{h} > \bar{h}$  holds and this implies  $\mathcal{F}(\hat{h}; \sigma)$  falls when  $\sigma$  increases. In other words, a mean-preserving increase in spread reduces the mass on those not keeping up. Since the consumption of the latter is lower than that of those who do keep up, average consumption rises, and hence the cut-off rises.

#### 3.2 Effect on leisure

Lastly, define average leisure  $(\overline{l})$  in the economy as follows:

$$\bar{l} = \int_{h_l}^{\hat{h}} l_d^i f(h^i) \, dh^i + \int_{\hat{h}}^{h_u} l_k^i f(h^i) \, dh^i.$$
(11)

This is the weighted average of the leisure enjoyed by those who drop out and those who keep up; in our case, these weights are endogenous since  $\hat{h}$  is endogenous. Using the expressions for  $l_d^i$  and  $l_k^i$  derived earlier, it follows that

$$\bar{l} = \frac{\phi}{(1+\theta)w} \left[ 1 + \theta \mathcal{F}\left(\hat{h};\sigma\right) \right].$$
(12)

Alternatively, notice that  $c_i = wx_i = w(1 + h^i - l_i) \forall i$  implying

$$\int_{i} c_{i} = \int_{i} w \left( 1 + h^{i} - l_{i} \right) \Rightarrow \delta = w \left( 1 + \bar{h} \right) - w \bar{l}.$$

$$\tag{13}$$

From (13), it is clear that average consumption and average leisure are inversely related.

**Proposition 2** a)  $\frac{\partial \bar{l}}{\partial \sigma}$  has the same sign as  $\mathcal{F}_2(\hat{h}, \sigma)$ . b) Suppose  $\lambda > \lambda^{\dagger}$ . Then, for a symmetric and unimodal distribution,  $\frac{\partial \bar{l}}{\partial \sigma} \leq 0$ .

In other words, for sufficiently high  $\lambda$ , a mean preserving increase in the spread of the effective time endowment distribution reduces average leisure. Loosely speaking, Proposition 2 says that when people desire to keep up with more than average consumption, an increase in income inequality pushes them to work harder on average.



Figure 3: A mean-preserving increase in spread

To get a sense of the intuition for Proposition 2, focus attention on Figure 3. In this figure, the heavier purple curve represents a mean-preserving increase in spread of the original distribution (shown as the thin black line). For ease of presentation, we start by considering a fixed  $\hat{h}$  (the bold red vertical line drawn here to the left of the point m; arguments similar to the ones presented below hold when  $\hat{h}$  is to the right of m). Under the original distribution, the mass of people dropping out is given by the area B + C + E + F; under the new distribution, the mass becomes A + E + F. Since both distributions are symmetric, B + E = A + E = 0.5 implying that the mass of agents below  $\bar{h}$  – all drop-outs – is the same. Above the mean, the mass of drop-outs in the original distribution is C + F while it is F under the new distribution; this implies that the total mass of drop-outs is now *reduced* by the area  $C.^{28}$  Since those that keep up consume less leisure than those who drop out, the net effect is that average leisure goes down.

More generally though, we know from Lemma 3 that a mean-preserving increase in the spread raises the cutoff  $\hat{h}$  when  $\hat{h}$  lies above  $\bar{h}$  (i.e., it moves the red line to the right). Why? Above we established that the net effect of an increase in spread is that average leisure goes down for a given  $\hat{h}$ ; this means average consumption rises and this causes the cutoff  $\hat{h}$  to rise as well. The increase in  $\hat{h}$  further serves to increase average leisure; evidently, this last effect is of second-order magnitude.

<sup>&</sup>lt;sup>28</sup>Under the old distribution, the mass keeping up is D+H and under the new, it is H+G. Since C+D+F+H = F+H+G = 0.5, it follows that C+D=G, i.e., the net increase in the mass of keeper-uppers is C=G-D.

The analysis above ties in well with the larger question raised by Prescott (2004) and others: in recent times, why are Americans working so much harder than the Europeans? To the list of explanations that have already been provided (see the introduction), we add our own. Over the last three decades or so, income inequality has been much higher in the United States than in Europe while average incomes have been roughly similar. Our analysis suggests that all else being similar across the two shores of the Atlantic, the higher income inequality in the United States may induce more Americans (relative to Europeans) to join the rat race and work harder on average than their European counterparts.

### 4 Concluding remarks

From biblical times, human beings have struggled with an innate desire to covet their neighbors' possessions. For many, this translates into a seemingly endless pursuit of keeping up to the consumption standards set by their neighbors. Others simply drop out of this rat race. In this paper, we take the stance that people who successfully keep up with their neighbors receive an extra boost in utility relative to those who drop out. We embed this new preference formulation in a simple new framework in which heterogenous ability agents get utility not just from consumption but also from the very act of keeping up with the average level of consumption. We find that only the high ability agents choose to keep up and they enjoy higher consumption but lower leisure than those who stay out of the rat race. When income inequality rises in a mean preserving manner, average leisure in the economy may fall. The analysis sheds some light on the transatlantic leisure differential issue recently popularized by Prescott (2004) and others. It suggests that higher income inequality in the US may be responsible for inducing many more people to join the rat race in the US (relative to Europe) and work harder, thereby reducing average leisure. In sum, our modelling approach is novel and seems to fit squarely with the basic notions of what drives individuals to work harder. Perhaps more importantly, the connection between inequality, work, and excess consumption we espouse does not emerge from a standard keeping-up-with-the-Joneses framework.

Our preference formulation has other potential applications, such as in the area of asset pricing, which we haven't explored here. Similarly, the model could be extended to allow different income classes to keep up with consumption levels closer to their own. A dynamic version of this economy that includes a consumption-saving choice may shed light on the connection between income inequality and aggregate savings. We leave these, and other issues, for future research.

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## Appendix

### A Robustness

This section establishes that all our results go through even when we relax some of the modelling assumptions made earlier for reasons of analytical tractability. This also allows us to identify the roles played by these different assumptions more clearly. Two issues are discussed below. First, do our results extend to skewed distributions? Second, is it possible to relax the assumption of quasi-linear utility? We take these up in turn.

#### A.1 Skewed distributions

In the main text, we focussed our attention on symmetric unimodal distributions of the effective time endowment. This was analytically convenient for it allowed us to use a well-known result that if a new distribution is obtained from an old one via a mean preserving increase in spread, it exhibits the single crossing property, and when the original distribution is symmetric, the single crossing takes place at the mean. To the best of our knowledge, no such clean result is known in general for skewed distributions. We now take up this issue using numerical techniques. Without loss of generality, we stick to a Beta distribution  $B(\alpha, \beta)$  which is flexible enough to allow for all directions of skewness and, as we demonstrate below, also permits mean-preserving increases in spread.

Consider a beta distribution  $B(\alpha, \beta)$ , and suppose h is distributed  $B(\alpha, \beta)$ .<sup>29</sup> Then mean of h is given by

$$E\left(h\right) = \frac{\alpha}{\alpha + \beta}$$

and variance (spread) of h by

$$V(h) = \frac{\alpha\beta}{\left(\alpha + \beta\right)^2 \left(\alpha + \beta + 1\right)}$$

In order to achieve a mean-preserving increase in spread, we treat V(h) as an exogenous variable and change it but leave E(h) fixed (hence preserving the mean). This means we have to figure out two things: a)  $\alpha$  and  $\beta$  as functions of V(h), and b) how  $\alpha$  and  $\beta$  should change so as to leave E(h) fixed at  $\frac{\alpha}{\alpha+\beta}$ .

What value of  $\alpha$  will keep the mean fixed at its original level of  $\frac{\alpha}{\alpha+\beta}$ ? Since  $E(h) = \frac{\alpha}{\alpha+\beta}$ , we solve for  $\alpha$  to get

$$E(h) = \frac{\alpha}{\alpha + \beta} \Rightarrow \hat{\alpha} = \frac{E(h)\beta}{1 - E(h)}$$

so henceforth, we set  $\alpha$  to this value,  $\hat{\alpha}$ . How are  $\beta$  and V(h) connected? For  $\alpha = \hat{\alpha}$ , what is the variance?

$$V(h) = \frac{\hat{\alpha}\beta}{(\hat{\alpha} + \beta)^2 (\hat{\alpha} + \beta + 1)} = \frac{E(h)}{(1 - E(h))^2 (1 + \beta - E(h))}$$

<sup>29</sup>The Beta distribution is described by

$$B(x;\alpha,\beta) = \frac{x^{\alpha-1} (1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1} (1-u)^{\beta-1} du}; \ x \in [0,1]$$

Then  $\beta$  as a function of V(h) (obtained by solving the previous equation for  $\beta$ ) is given by

$$\hat{\beta} = \frac{E(h)}{V(h)(1-E(h))^2} - (1-E(h)).$$

Plug this value of  $\beta$  into the expression for  $\hat{\alpha}$  to get

$$\hat{\alpha} = \frac{E(h)\beta}{1 - E(h)} = \frac{[E(h)]^2}{V(h)(1 - E(h))^3} - E(h).$$

This gives  $\hat{\alpha}$  as a function of V(h). It is easy to verify that

$$\frac{\hat{\alpha}}{\hat{\alpha} + \hat{\beta}} = \frac{\alpha}{\alpha + \beta} = E(h)$$

and so we know that when we change V(h), the mean stays fixed at  $\frac{\alpha}{\alpha+\beta}$ . Therefore as functions of the spread (after substituting  $E(h) = \frac{\alpha}{\alpha+\beta}$ ), we get

$$\hat{\alpha} = \frac{\left(\frac{\alpha}{\alpha+\beta}\right)^2}{V(h)\left(1-\frac{\alpha}{\alpha+\beta}\right)^3} - \left(\frac{\alpha}{\alpha+\beta}\right)$$
$$\hat{\beta} = \frac{\frac{\alpha}{\alpha+\beta}}{V(h)\left(1-\frac{\alpha}{\alpha+\beta}\right)^2} - \left(1-\frac{\alpha}{\alpha+\beta}\right)$$

Hence, starting from a distribution  $h \sim B(\alpha, \beta)$ , a mean-preserving increase in spread is achieved by raising V(h) and changing  $\alpha$  and  $\beta$  to  $\hat{\alpha}$  and  $\hat{\beta}$  respectively.

**Example 1** Suppose  $w = 1, \phi = \theta = 0.5, \lambda = 1.2$ , and let  $E(h) = \frac{\alpha}{\alpha+\beta} = 0.4$ . Figure 4a shows a mean-preserving increase in spread from 0.2 to 0.4. Figures 4b-4f plot the various parameters and variables of interest against changes in the spread parameter  $\sigma$  from 0.2 to 1. Figures 4b and 4c show how  $\alpha$  and  $\beta$  have to be modified as the spread rises so as to keep the mean fixed at 0.4. The cut-off level of ability (Figure 4d) starts off above the mean (0.4) and rises. The mass on people who are not keeping up falls (Figure 4e). As Figure 4f makes clear, a mean-preserving increase in the spread reduces average leisure.





Figure 4f: The change in average leisure

#### A.2Additive log preferences

To check if the assumption of quasi-linear preferences can be relaxed, let us briefly sketch the model in the case where

$$U(c,l) \equiv \begin{cases} \ln(c+\theta(c-\kappa)) + \phi \ln l & \text{if } c > \kappa \\ \ln c + \phi \ln l & \text{if } c \le \kappa \end{cases}$$

In this case, it is easy to verify that (3)-(4) is replaced by

$$l_d^i = \frac{\phi(1+h^i)}{1+\phi}; \quad l_k^i = l_d^i - \frac{\phi\theta\kappa}{w(\phi+1)(\theta+1)}$$
(14)

$$c_d^i = w \frac{(1+h^i)}{1+\phi}; \quad c_k^i = c_d^i + \frac{\phi \theta \kappa}{(\phi+1)(\theta+1)}.$$
 (15)

Below, we will prove that  $l_k^i > 0$  for all keeper uppers. Notice that the gap between  $l_k^i$  and  $l_d^i$ (parenthetically, the gap between  $c_k^i$  and  $c_d^i$ ) depends on  $\kappa$ : as  $\kappa$  rises, these gaps rise. Also note that it is no longer the case that  $l_k^i < l_d^j$  for any i, j but  $c_k^i > c_d^j$  obviously continues to hold.

An agent will choose to stay out of the rat race if the utility from participating exceeds that of staying out. Formally, those  $h^i$  for whom the following inequality hold will participate:

$$\ln\left(c_k^i + \theta\left(c_k^i - \kappa\right)\right) + \phi \ln l_k^i > \ln c_d^i + \phi \ln l_d^i.$$

Straightforward algebra shows that when  $\kappa = \lambda \delta$ , the previous inequality reduces to

$$h^{i} > \hat{h} \equiv \frac{\theta}{\left(\theta + 1\right) - \left(\theta + 1\right)^{\frac{\phi}{1 + \phi}}} \left(\frac{\delta}{\lambda w}\right) - 1,\tag{16}$$

where  $(\theta + 1) - (\theta + 1)^{\frac{\phi}{1+\phi}} > 0$  and  $\theta/[(\theta + 1) - (\theta + 1)^{\frac{\phi}{1+\phi}}] > 1.^{30}$ Using the expressions for  $c_d^i$  and  $c_k^i$  derived earlier, it is straightforward to check that

$$\delta = \frac{w\left(\theta+1\right)\left(\bar{h}+1\right)}{1+\phi+\theta+\theta\lambda\phi\mathcal{F}\left(\hat{h};\sigma\right)}.$$
(17)

Using (16) and (17), we derive the analog of eq. (8):

$$\frac{w\left(\theta+1\right)\left(\bar{h}+1\right)}{1+\phi+\theta+\theta\lambda\phi\mathcal{F}\left(\hat{h};\sigma\right)} = w\left(\frac{\left(\theta+1\right)-\left(\theta+1\right)^{\frac{\phi}{1+\phi}}}{\theta\lambda}\right)\left(1+\hat{h}\right).$$

**Example 2** Suppose  $w = \phi = \theta = 1$ ,  $\lambda = 1.3$ , and let  $E(h) = \frac{\alpha}{\alpha + \beta} = 0.5$ . An increase in the spread reduces average leisure, as illustrated in Figure 5.

$$l_{k}^{i} > 0 \Leftrightarrow 1 + h_{i} - \frac{\theta \lambda \delta}{w \left(\theta + 1\right)} > 0$$

It is straightforward to verify that the previous inequality holds for all  $h_i \ge \hat{h}$ . This means every keeper-upper enjoys a strictly interior level of leisure.

<sup>&</sup>lt;sup>30</sup>Note that



Figure 5: Average leisure versus spread in Example 2

## B Proof of Lemma 1

Using the expression for  $c_d^i$ , we get

$$c_d^i = w\left(1 + h^i - \frac{\phi}{w}\right) < \kappa \Longleftrightarrow 1 + h^i < \frac{\kappa + \phi}{w}$$

Since

$$1 + \hat{h} = \frac{\kappa}{w} + \frac{\phi \ln (1 + \theta)}{\theta w} = \frac{\kappa}{w} + \frac{\phi}{w} \frac{\ln (1 + \theta)}{\theta}$$

and drop-outs are those with  $h^i < \hat{h}$ , we have

$$1 + h^{i} < 1 + \hat{h} = \frac{\kappa}{w} + \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta} < \frac{\kappa+\phi}{w}$$

implying

$$h^i < \hat{h} \Rightarrow c^i_d < \kappa$$

Similarly,

$$c_{k}^{i} = w\left(1 + h^{i} - \frac{\phi}{w\left(1 + \theta\right)}\right) > \kappa \Longleftrightarrow 1 + h^{i} > \frac{\kappa}{w} + \frac{\phi}{w\left(1 + \theta\right)}$$

and since, keeper-uppers are those with  $h^i > 1 + \hat{h}$ , we have

$$1+h^i > 1+\hat{h} = \frac{\kappa}{w} + \frac{\phi}{w} \frac{\ln\left(1+\theta\right)}{\theta} = \frac{\kappa}{w} + \frac{\phi}{w\left(1+\theta\right)} \frac{\left(1+\theta\right)\ln\left(1+\theta\right)}{\theta} > \frac{\kappa}{w} + \frac{\phi}{w\left(1+\theta\right)}$$

because  $\frac{(1+\theta)\ln(1+\theta)}{\theta} > 1$ .

## C Proof of Lemma 2

First note that  $\underline{\lambda} < 1 < \overline{\lambda}$  such that the relevant interval for  $\lambda$  is non-empty. Further, it follows from (9) that l'(h) > 0, implying there is a unique equilibrium if  $l(h_l) < g$  and  $l(h_u) > g$  where g is defined in (10). First consider  $l(h_l) < g$ . Since  $\mathcal{F}(h_l) = 0$ ,

$$l(h_l) < g \Leftrightarrow h_l + 1 - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta} < \left(1 + \bar{h} - \frac{\phi}{w} \frac{1}{(1+\theta)}\right) \lambda$$
(18)

By (A.1),  $\phi < \phi_u \equiv w (1 + h_l)$ . Then, it follows that

$$h_l + 1 - \frac{\phi}{w} \frac{\ln\left(1+\theta\right)}{\theta} \ge h_l + 1 - \frac{\phi_u}{w} \frac{\ln\left(1+\theta\right)}{\theta} = (h_l + 1)\left(1 - \frac{\ln\left(1+\theta\right)}{\theta}\right) > 0$$

and

$$1 + \bar{h} - \frac{\phi}{w} \frac{1}{(1+\theta)} \ge 1 + \bar{h} - \frac{\phi_u}{w} \frac{1}{1+\theta} = (1+h_l) \left(\frac{1+\bar{h}}{1+h_l} - \frac{1}{1+\theta}\right) > 0$$

implying both sides of the above inequality in (18) are positive. Hence, it follows that

$$l(h_l) < g \Leftrightarrow \underline{\lambda} \leq \lambda.$$

Now consider  $l(h_u) > g$ . Since  $\mathcal{F}(h_u) = 1$ ,

$$g < l(h_u) \Leftrightarrow \lambda \left(1 + \bar{h} - \frac{\phi}{w}\right) < 1 + h_u - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta} \Leftrightarrow \lambda < \overline{\lambda}$$

### D Proof of Lemma 3

Under (A.2), the condition provided in Lemma 2, there exists a unique  $\hat{h}$ . Since l(h) is increasing and initially smaller than g, we know that  $\hat{h} > \bar{h}$  is equivalent to the knowledge that the crossing of l and g has not yet occurred at  $\bar{h}$ . If this is the case,  $l(\bar{h})$  must be smaller than g. The argument for  $\hat{h} < \bar{h}$  is exactly equivalent. Therefore, it follows that  $\hat{h} \gtrless \bar{h}$  iff  $l(\bar{h}) \gtrless g$ . Noting that for a symmetric distribution,  $\mathcal{F}(\bar{h}) = 1/2$ , we get:

$$l\left(\bar{h}\right) < g \Leftrightarrow \frac{1}{w} \left(\frac{\phi\lambda\theta}{1+\theta}\right) \mathcal{F}\left(\bar{h}\right) < \frac{1}{w} \left(\frac{\phi\ln\left(1+\theta\right)}{\theta} - \frac{\lambda\phi}{(1+\theta)}\right) + (\lambda-1)\left(1+\bar{h}\right)$$

Further simplification yields

$$1 + \bar{h} - \frac{1}{w} \frac{\phi \ln (1+\theta)}{\theta} < \lambda \left( 1 + \bar{h} - \frac{\phi}{2w} \frac{2+\theta}{1+\theta} \right) \Leftrightarrow \lambda^{\dagger} \equiv \frac{1 + \bar{h} - \frac{\phi}{w} \frac{\ln(1+\theta)}{\theta}}{1 + \bar{h} - \frac{\phi}{w} \frac{1}{(1+\theta)} \left( 1 + \frac{\theta}{2} \right)} < \lambda.$$

The remaining part of the lemma is trivial to verify.

### **E Proof of Proposition 1**

Differentiating (8) yields

$$\frac{\partial \hat{h}}{\partial \sigma} + \frac{1}{w} \left( \frac{\lambda \phi \theta}{1 + \theta} \right) \left( f\left( \hat{h}, \sigma \right) \frac{\partial \hat{h}}{\partial \sigma} + \mathcal{F}_2\left( \hat{h}, \sigma \right) \right) = 0$$

which upon rearrangement produces

$$\frac{\partial \hat{h}}{\partial \sigma} = -\frac{\frac{1}{w} \left(\frac{\lambda \phi \theta}{1+\theta}\right) \mathcal{F}_2\left(\hat{h}, \sigma\right)}{1 + \frac{1}{w} \left(\frac{\lambda \phi \theta}{1+\theta}\right) f\left(\hat{h}, \sigma\right)}.$$
(19)

Hence, the sign of  $\frac{\partial \hat{h}}{\partial \sigma}$  is the opposite of the sign of  $\mathcal{F}_2\left(\hat{h},\sigma\right)$ . For symmetric distributions, when  $\lambda > \lambda^{\dagger}$ ,  $\hat{h}$  lies to the right of the single crossing point  $(\bar{h})$  and so  $\mathcal{F}_2\left(\hat{h},\sigma\right) < 0$ ; hence  $\frac{\partial \hat{h}}{\partial \sigma} < 0$ .

# F Proof of Proposition 2

Straightforward differentiation of (12) yields

$$\frac{\partial \bar{l}}{\partial \sigma} = \frac{\phi \theta}{\left(1 + \theta\right) w} \left( f\left(\hat{h}, \sigma\right) \frac{\partial \hat{h}}{\partial \sigma} + \mathcal{F}_2\left(\hat{h}, \sigma\right) \right)$$

Substituting the expression for  $\frac{\partial \hat{h}}{\partial \sigma}$  from (19) and rearranging, we get

$$\frac{\partial \bar{l}}{\partial \sigma} = \frac{\phi \theta}{\left(1 + \theta\right) w} \left( \frac{\mathcal{F}_2\left(\hat{h}, \sigma\right)}{1 + \frac{1}{w} \left(\frac{\lambda \phi \theta}{1 + \theta}\right) f\left(\hat{h}, \sigma\right)} \right)$$

which has the same sign as  $\mathcal{F}_{2}\left(\hat{h},\sigma\right)$ .

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