SCHOOL OF ECONOMICS AND MANAGEMENT

ECONOMICS WORKING PAPER 2007-19

Optimal research effort and product differentiation in network industries

Christian Dahl Winther

UNIVERSITY OF AARHUS

BUILDING 1322 - 8000 AARHUS C - DENMARK ☎ +45 8942 1133

Optimal research effort and product differentiation in network industries

Christian Dahl Winther*

Abstract: This paper studies the introduction of a new and incompatible technology in a spatial market with network externalities. In competition with an established network the paper investigates how long an entrant optimally should do research before entering the market and what level of product differentiation should be chosen in order to maximize its present value profits. Research time is important as it not only determines the quality of the technology that is introduced; it also has consequences for how successfully the two competing firms build their network of users.

First, the paper derives a function characterizing the intertemporal evolution in market shares resulting from the newcomer's choices. Second, it is shown that each level of technological quality is associated with both a minimum and a maximum level of product differentiation that should be chosen in equilibrium. Third, the entrant's problem is solved by numerical methods.

JEL classification: D85, L1, O33

Keywords: Network externalities, product differentiation, product introduction, technological change.

---

*School of Economics and Management, University of Aarhus, Universitetsparken, Building 1322, DK - 8000 Aarhus C, Denmark. E-mail: cwinther@econ.au.dk.

I thank Ott-Siim Toomet for his interest in and comments on this project. All remaining errors are mine.

1 Introduction

This paper studies the introduction of a new and incompatible technology in a spatial market with network externalities. In competition with an established network the paper investigates how long an entrant optimally should do research before entering the market and what level of product differentiation should be chosen in order to maximize its present value profits. Research effort is important as it not only determines the quality of the technology that is introduced; it also has consequences for how successfully the two competing firms build their network of users.

By introducing heterogeneity in buyers' preferences, the model is able to appreciate how differences in consumers' tastes on the demand side influence the choices made on the supply side. Since network externalities make past sales an asset in present competition, the location choice of a firm influences not only the scope of potential markups but also the market shares now and in the future. Together the choices of R&D strategy and location in the industry, and their interaction, are important factors to understand for a future entrant. Not only do they determine the speed and direction of changes in market shares by influencing buyers' adoption decisions, they also affect the prices that can be charged. This relationship has not yet received much attention in existing literature. Consistent with empirical evidence the model produces a smooth transition in demands such that the market gradually tips over time; when the CD was introduced in 1982 it did not immediately stunt all sales of music on existing audio medias such as audio cassettes and vinyl recordings, even though it offered a strong improvement in sound quality and storage capacity.[1] This should be seen in contrast to the abrupt changes in demand predicted by models of homogeneous demand,[2] in which a firm may sell to every buyer in one period, only to sell nothing in the next if a better technology enters the market.

The contributions of this model are the following: The paper describes the intertemporal evolution in market shares whether they grow or decline, as a function of the entrant's R&D strategy and chosen level of product differentiation, and how it relates to the characteristics of the market (such as speed of technological progress, strength of network externalities, and size of heterogeneity in consumers' tastes). The

[1] See the distribution of record sales on different formats in "The recording industry in numbers (2005)". This is an annual publication made by the International Federation of the Phonographic Industry (IFPI). Park (2004) shows that competition between the Betamax and VHS standards displayed a similar adoption pattern.

[2] For example Katz and Shapiro (1992) and Regibeau and Rockett (1996), and Kristiansen (1998).

relationship between minimum and maximum levels of product differentiation and the timing of product introduction is derived. Generally speaking, stronger differences in consumer tastes lead to faster introduction of new products that will be of lower quality for this reason. Even though the exact connection is complex, the higher quality a new product can offer consumers the less differentiation should be chosen by the firm.

The dynamic nature of network economies in combination with demand heterogeneity is not without costs, as analytical solutions to the problem in some cases do not exist; only when firms' market shares are constant over time does the problem reduce in complexity to allow for analytical results. To solve the problem in general, numerical methods are used to describe how the equilibrium of the model depends on the parameterization of the economy.

The literature most closely related to the present analysis is Katz and Shapiro (1992) and Regibeau and Rockett (1996) both studying firms' endogenous choices of introduction dates when buyers are homogeneous. Katz and Shapiro focus on the timing of product introduction of a second-generation technology when consumers hold perfect foresight on future network sizes and introduction times. In Regibeau and Rockett the buyers are myopic, as is the case my model, and they solve for the optimal entry dates of introduction for both generations of technology. The present analysis lends the basic structure of timing decisions, but adds a spatial dimension to capture the connection between timing and location choices.

The present model also borrows from location theory, starting with Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979) in their consideration how firms choose locations in a spatial market. Mitchell and Skrzypacz (2006) study a dynamic duopoly with network externalities in a spatial market where firms are exogenously located at each their extremity of the city and endowed with products of fixed quality. Firms choose prices strategically knowing how sales increase the value of their networks. However, firms have an incentive to 'harvest' high market shares by setting higher prices, thus becoming less attractive to later adopters. Mitchell and Skrzypacz aim at explaining whether market shares stabilize or diverge over time and the efficiency of the private market. The authors show that strong network effects and patient firms lead to a single standard being adopted by all in the long run. This paper is concerned with how pricing behavior affects market shares when each firm only can set a single price per period which applies to all consumers. I assume that firms can divide consumers into segments and charge different prices to different segments. In this environment I study the evolution of market shares as a function of the newcomer's choices of product quality and positioning in the market. My paper reaches the same conclusion as Mitchell and Skrzypacz by predicting that only one standard prevails in the long run. Yet if network effects are small then the two competing technologies may still coexist for an extended period of time. Doganoglu (2003) derives the conditions for the existence of a stable Markov Perfect Equilibrium in a model quite similar to Mitchell and Skrzypacz', and concludes that network feedback must be sufficiently weak for this to hold.

The model is outlined as follows: The model is laid out in section 2 and firms' equilibrium pricing strategies are derived. Section 3 shows how intertemporal demands are related to the entering firm's choice of timing and location. The set of potential strategies is reduced in section 4. Section 5 solves the entrant's problem numerically, and section 6 concludes.

2 Setup

Two firms, A and B, sponsor technologies that are mutually incompatible. Firm A enters exogenously at time $s = 0$ with a product of quality α. The focus of the paper is on firm B, which is faced with the decisions of choosing how long to do research before entering the industry and how much to differentiate its product in comparison to firm A. Firm B can be thought of as incorporating the state-of-the-art technology in its product; the longer it delays entry, the better technologies become available. The technological improvement is assumed to be a linear function of R&D time; $\beta + \theta s$, where β represents the initial quality of the product, and the coefficient θ is the rate at which the technological quality improves as a function of research time.

By definition firm B introduces its product at date $s = \Delta$, where $\Delta \in [0; \infty)$. Technological quality is a vertical attribute that is shared by all customers. To focus on the interaction between R&D effort and network externalities, it is assumed that it is not possible to upgrade technologies once they have been introduced. Changes in network size are therefore the only channel through which product values can change. This assumption is best met in industries where performance of a new technology would suffer greatly from carrying such historical technological baggage or when dealing with a 'hard-coded' technology.[3]

[3]Katz and Shapiro (1992) note that this assumption is likely to hold in markets where it is

Let the demand side of the market be given as the unit line $x \in [0;1]$ with a uniform distribution of consumers with density 1. To make the analysis of the entrant's choice of product differentiation as clean as possible it is assumed that the incumbent is located at $x = 0$. One could argue that $x = 0.5$ is a more appropriate location as this minimizes the average transportation cost incurred by the incumbent's customers during its monopoly period. As shown by Tyagi (2000), however, an incumbent optimally cedes the best location (that is right in the middle of the market) in favor of a less vulnerable position when the entrant is expected to have an advantage in production.[4] Tyagi shows that as this differential increases, the incumbent will locate further and further away from the centre perhaps even outside the market as a buffer to profits. It is therefore reasonable to assume that the incumbent is located at one of the extremities of the unit line. Let Ω denote the distance between the established network and the newcomer, and a such represent the degree of product differentiation of the two technologies; the higher Ω the more different are products. Once a location is chosen it remains fixed throughout the game. There are several reasons why this may be so; firms can have difficulties changing the reputation of their product, lock-in to a certain marketing strategy, organizational inflexibility, contractual ties etc.

Similar to the Hotelling model, the distance between a buyer and a firm represents the fit between the (horizontal) characteristics of the product and the buyer's personal taste. It is assumed that the reduction in customers' willingness to pay is quadratic in the distance traveled. Let the parameter d measure the degree of heterogeneity in consumers' preferences, where $d > 0$. If $d = 0$ then buyers would have identical tastes, making a firm's location choice irrelevant since buyers do not incur any travel costs. To simplify calculations it is assumed that $\alpha \geq d$ to ensure that the incumbent can profitably attract the buyer who is least inclined to buy its product even if the firm has no network to offer. It seems reasonable to discuss this assumption; if $\alpha < d$, buyers would generally be less inclined to adopt technology A especially during early periods thus hampering the network accumulation of firm A. Faced with a competitor of lesser value the entrant more quickly reaches the relative value it would otherwise want leading to faster entry. One can therefore view the conclusions drawn by the

important to remain compatible with the existing user base. Furthermore, Regibeau and Rockett (1996) suggest that the analysis of the problem remains essentially unchanged if introduction only decreases the rate of technological enhancements as long as it applies to both players.

[4]In Tyagi's model firms have different production costs for products of identical qualities, whereas my treatment involves identical production costs for products of different qualities. These two approaches to value creation for the firm are technically equivalent.

model in its present formulation as an upper bound to research effort.

The market size increases continuously because a new and uncommitted group of buyers arrives at each moment in time. Myopia makes consumers unable to foresee future events making their adoption decision hinge entirely on the current state of the market. This assumption is discussed further in the next section. Moreover, consumers do not act strategically, an assumption consistent with each individual being small. Once a consumer has arrived in the market he will join a network or disappear from the economy. Joining a network makes the consumer a permanent part of the associated network adding to its appeal to future consumers. The value of a network is a linear function of the size of the installed base of users. Let γ measure the importance of network effects, where $\gamma \geq 0$. To ensure that the newcomer picks positive levels of research, let $\theta \geq \gamma$ such that the technological value created during a period of research is no smaller than the value increase experienced by the incumbent via its network accumulation. This restricts the applicability of the model to industries where innovation rates are sufficiently high.

Buyers have inelastic demands for one unit of technology, they are infinitely lived, and products are infinitely durable. A firm ceasing to make positive sales remains in the market as an option to buyers. Time is discounted by a factor δ, assumed to satisfy the inequality $\delta > \frac{\gamma}{d\Omega}$ in order for profits to be finite. By appropriately redefining the discount factor one can interpret technologies as becoming obsolete with a constant hazard rate rather than lasting indefinitely.

The population can be divided into segments allowing firms to practice third-degree price discrimination. Think of the continuum that is the Hotelling market as an abstraction of finely segmented markets. Competition is in prices and firms produce at zero marginal costs,[5] and it is assumed that firms do not engage in below cost pricing. When a firm is active in the market it specifies the price charged to each segment in the market at each point in time. For each segment along the market competition drives the price of the less valuable technology down to cost, while the firm sponsoring the more valuable network sets a price that just induces the buyer to adopt. The markup earned by the winning firm therefore equals the excess value it offers over the segment's best alternative. This is what Scherer (1980, p. 315) calls get-the-most-from-each-region, offering this example:

[5] An alternative interpretation is to view product qualities as the net value of technologies in excess of any marginal production costs incurred.

> "Prices are persistently held higher in regions where competition is weak than where it is strong. Thus, for many years European high-fidelity phonograph records were sold for less in the United States than in the countries where they were produced."

Since the more valuable network at each location is adopted in equilibrium, the pivotal consumer who is indifferent between the two networks can be used as a tool for determining network sizes. This property is useful in the next section.

Assuming away prices below marginal costs is not an entirely innocent assumption, but several factors can limit firms' incentives for setting prices below costs. First, the very reason for setting low prices is to attract users to the network to gain a strategic advantage in future competition, but if network effects are weak, the return to the 'strategic investment' is small. Second, heavy discounting makes future gains from owning a network less significant. Third, strong heterogeneity in consumer tastes makes it expensive for a firm to persuade a buyer to join his least preferred network, while the strategic value of winning this buyer remains unchanged. Fourth, since marginal costs are zero then prices would have to be negative.[6] As a final justification of the assumption recall that marginal costs are zero, which indicates that someone should receive, rather than pay, money for using a given product, which is rarely observed. From a theorist's point of view this assumption is not ideal, but from a practical perspective it does seem to be quite reasonable. In any case it simplifies the model considerably and is therefore maintained.

The value of a network is assumed to be additively separable in its components. The entrant's choices of timing of product introduction and the level of differentiation are key to consumers' valuations by influencing the relative quality of technologies and play very important roles in network formation. Moreover, a consumer's valuation for a network is dependent on how well the product is aligned with the buyer's tastes. Equations 1 and 2 show the valuation of consumer segment x at time t for networks A and B respectively, where t denotes time elapsed *since* the introduction of product B:

$$V^A(\Delta, \Omega, x, t) = \alpha + \gamma\Delta + \gamma \int_0^t \tilde{x}(\Delta, \Omega, \tau) d\tau - d(x)^2 \tag{1}$$

[6] Legal issues may also prevent firms from engaging in such penetration pricing. While low costs do not in their own right qualify as a bad act, in fact this is usually the goal of having competition laws in the first place; charging low prices with intent to force competitors out of the industry is generally frowned upon. See Bolton *et al.* for a comprehensive study showing how economic theory connects to practical cases.

$$V^B(\Delta,\Omega,x,t) = \beta + \theta\Delta + \gamma \int_0^t (1 - \tilde{x}(\Delta,\Omega,\tau))d\tau - d(\Omega - x)^2 \tag{2}$$

As firm A captures every consumer during its monopoly period its network has value $\gamma\Delta$ after the first Δ periods, at which point firm B enters the market, and firms will share the market according to the attributes chosen by the newcomer. The terms $\gamma \int_0^t \tilde{x}(\Delta,\Omega,\tau)d\tau$ and $\gamma \int_0^t (1 - \tilde{x}(\Delta,\Omega,\tau))d\tau$ show the value of the firms' accumulated networks during the first t periods (of duopoly competition) which are a summation of the market shares during these periods.

At $s = 0$ firm B decides on an introduction date, Δ, and a level of product differentiation, Ω, for its new technology to maximize present value of profits. The solution concept is Markov perfect equilibrium. The model does not consider mixed strategies.

3 Intertemporal changes in market shares

Since the most valuable network is adopted by each segment across the market, the location of the pivotal agent can be utilized to separate firms' market shares intertemporally as a function of the newcomer's choice of research effort and product differentiation. Let $\tilde{x}(\Delta,\Omega,t)$ denote this transition function, which can be derived from equations 1 and 2 as shown in Appendix A.

$$\tilde{x}(\Delta,\Omega,t) = \left[\frac{\alpha - \beta - (\theta - \gamma)\Delta - d(1-\Omega)\Omega}{2d\Omega}\right] e^{\frac{\gamma}{d\Omega}t} + \frac{1}{2} \tag{3}$$

Equation 3 is a differential equation that separates those consumers joining network A from those joining network B, such that for each t all consumers located at $x < \tilde{x}(\Delta,\Omega,t)$ join A and all consumers at $x > \tilde{x}(\Delta,\Omega,t)$ join B. The consumer at $x = \tilde{x}(\Delta,\Omega,t)$ adopts either network with equal probability. Since buyers are distributed on $[0;1]$, the demand for network B at time t equals $1 - \tilde{x}(\Delta,\Omega,t)$.[7]

The transition function shows that the diffusion path is exponential implying that the market will tip at an ever increasing rate. This resembles the self-perpetuating process of technological change in epidemic models. See Stoneman (2002) for a deeper look into this literature. Unlike those models assuming homogeneous demand, this model can support coexistence of more than one network within a time period. Em-

[7] In the pathological case in which B enters with a product of the exact same value, and at the exact same spot as firm A, there would exist a continuum of indifferent consumers. In this case, however, firm B would make zero profits, while every other location would result in strictly positive profits. Thus it seems reasonable to disregard this situation.

pirically this is consistent with network competition in many industries.

Consider some properties of the transition function. Research effort increases the technological quality of the newcomer's network, and the higher is demand. The location choice, Ω, influences the entrant's market shares in two different ways. First, a lower degree of product differentiation will increase the entrant's demand, shifting the position of the pivotal agent towards a smaller x,[8] since the network becomes more appealing to consumers that otherwise would have joined the incumbent. Second, less differentiated networks increase the speed of tipping, because the more similar networks become, the smaller is the value difference needed for a customer buying one product rather than the other. Market shares will therefore shift more easily over time. This effect influences the firm differently dependent on whether it experiences increasing or decreasing market shares. This drives a wedge in optimal choice of product differentiation for two otherwise closely related industries. This is explored further in the section on numerical simulation of the entrant's optimal choices.

The more heterogeneous the population is the more rigid is the transition in market shares. Greater network sizes are then needed to compensate for the increased transportation costs and slow tipping follows. Stronger network effects influence the entrant in two ways. It increases the value of the installed base the incumbent builds during its monopoly period, which makes it more difficult for the entrant to introduce a dominant product. Also it makes markets tip faster.

Given the chosen strategy pair (Δ, Ω) the industry can evolve in one of five general trajectories. Formal conditions characterizing each case are identified using equation 3. Figure 1 illustrates possible transition paths of intertemporal demands.

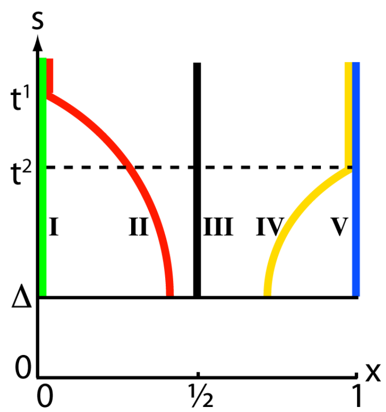

Figure 1

[8]This holds only as long as the new network is more valuable to the segment most in favor of it in comparison to the established network.

■ A new technology that is considered superior by every consumer can capture the entire market immediately upon its introduction. By the same token the incumbent's network becomes obsolete over night. In this case equation 3 must satisfy $\tilde{x}(\Delta, \Omega, 0) \leq 0$. Denote strategy pairs for which this condition holds as belonging to area I. Strong technological progress and/or low heterogeneity in consumers' preferences makes this situation most likely to occur. One example is industry for computer processors; as a faster processor is introduced the market for the old kind will quickly vanish.

■ Let area II represent situations where the new network is unable to capture every buyer right away, but is sufficiently valuable to attract more than half the market, such that $\tilde{x}(\Delta, \Omega, 0) \in \left(0; \frac{1}{2}\right)$ holds. Since network size is the only factor that changes relative valuations of products in the duopoly, this market will tip network B becoming the industry-wide standard. Calculating $\tilde{x}(\Delta, \Omega, t) = 0$ and solving for t shows the time period at which network A is preempted by network B; $t^1 = \ln\left(\frac{-d\Omega}{[\alpha-\beta-(\theta-\gamma)\Delta-d(1-\Omega)\Omega]}\right)\frac{d\Omega}{\gamma}$. Intuitively it is appealing that a new technology does not capture all demand on arrival, and only with time grows strong enough to take command over the full market. This is consistent with competition in many real world network markets, for instance the VCR standards war between VHS and Betamax formats. See Cusumano *et al.* (1992) for a thorough look into one of the most famous battles of business history. Collectively areas I and II represent 'winning' new technologies.

■ Suppose that (Δ, Ω) is such that the market is evenly split at the outset of duopoly. In this situation both firms grow at the exact same pace and networks remain equally valuable through future periods. Let area III characterize this stalemate, with demands described by $\tilde{x}(\Delta, \Omega, t) = \frac{1}{2}$ for all t.

■ Empirically, far from every new technology ends up dominant in their industry. Area IV consists of strategy pairs satisfying $\tilde{x}(\Delta, \Omega, 0) \in \left(\frac{1}{2}; 1\right]$. As network B captures less than half the consumers at $t = 0$ it grows more slowly than the incumbent's network. Formally, the newcomer is preempted at $t^2 = \ln\left(\frac{d\Omega}{\alpha-\beta-(\theta-\gamma)\Delta-d(1-\Omega)\Omega}\right)\frac{d\Omega}{\gamma}$ found by isolating t in $\tilde{x}(\Delta, \Omega, t) = 1$. Such networks can be thought of as 'losing' in the sense that the technological improvement they offer is only enough to be appealing to a niche in the market and will therefore never become the market-wide standard.

Even though the CD was introduced in 1982 it has been able to fight off competing standards such as the MiniDisc, which arguably has ten years of additional R&D

activity built into it (introduced 1992). The benefits of the MiniDisc are its smaller size, practical when running, the capability to create a 'mixed tape' on the rewritable discs, and that music can be transferred from CDs to MiniDiscs. These improvements are nice but not sufficient for consumers to abandon the CD. Another example is the standard competition in keyboard layouts fought by the Dvorak and QWERTY systems. I return to this example in a later section.

■ The last possible situation arises if the newcomer is unable to develop a product that exceeds the value of the established network, in which case the firm will never be adopted by anyone. Area V in figure 2 denotes those strategy pairs for which it holds that $\tilde{x}(\Delta, \Omega, t) > 1$ for all t.

Table 1 shows the conditions characterizing areas I-V. Figure 2 shows the combinations of (Δ, Ω) that will result in outcomes belonging to each of the areas.

Area I	$\alpha-\beta-(\theta-\gamma)\Delta+d\Omega^2 \leq 0$
Area II	$\alpha-\beta-(\theta-\gamma)\Delta+d\Omega^2 > 0$ $\alpha-\beta-(\theta-\gamma)\Delta-d(1-\Omega)\Omega < 0$
Area III	$\alpha-\beta-(\theta-\gamma)\Delta-d(1-\Omega)\Omega = 0$
Area IV	$\alpha-\beta-(\theta-\gamma)\Delta-d(1-\Omega)\Omega > 0$ $\alpha-\beta-(\theta-\gamma)\Delta-d(2-\Omega) < 0$
Area V	$\alpha-\beta-(\theta-\gamma)\Delta-d(2-\Omega) \geq 0$

Table 1

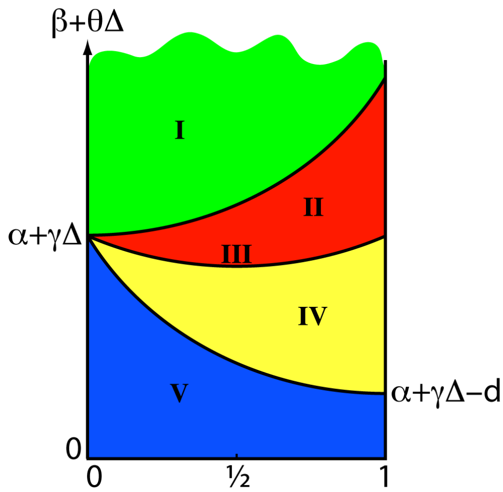

Figure 2

This paper assumes that buyers hold myopic expectations. Naturally, this is an extreme assumption and therefore deserves some discussion. When a consumer only uses past and present information as basis for his adoption decision, the direction of movement of a company's market share is not fully appreciated. As such the buyer may join a network today, but regret this decision tomorrow if he could do it over again. With perfect foresight on the other hand the future evolution of the industry is taken into account, and he may be reluctant to join a loosing network anticipating that it will have relatively less to offer in terms of additional network benefits. A consumer with perfect foresight will therefore be more inclined to jump on the bandwagon of networks on the rise which may be small today but dominant in the future. I would therefore expect the market to tip faster under perfect foresight in comparison to myopic expectations.

Perfect foresight has been criticized for being too demanding on consumers' capabilities as they would have to anticipate all relevant future events, such as firms' choices of product introduction strategies, as well as make the correct inference on the adoption decision of his fellow adopters. So while myopia is surely too simplifying an assumption, perfect foresight is too sophisticated, and the true speed of tipping probably lies somewhere in between these two extreme benchmarks.

4 Reducing the set of potential entry strategies

The natural strategy at this point would be to set up firm B's profit maximization problem and find the optimal solutions of timing and level of product differentiation. Due to the complexity of the problem, however, it is only possible to get analytical solutions for the three situations in which market shares do not evolve over time (areas I, III, and V). This is unfortunate because the problem cannot be solved for the more interesting cases where market shares shift intertemporally (areas II and IV). To overcome this problem the globally optimal choices of the entrant are solved numerically, which I will return to in section 5. However, it is possible to draw some conclusions on the behavior of firm B without solving its profit maximization problem; while firm B is free to choose any pair (Δ, Ω) it desires, some combinations will never be chosen in equilibrium. Given the relative values of the competing network I derive both a minimum and a maximum boundary to the level of product differentiation that an entrant should want to play, which has implications for the technological designs that will emerge in network industries.

To give a sense of the boundaries derived below, figure 3 illustrates the set of potential equilibria remaining after elimination. The thick lines represent the outer boundaries to the surviving strategy pairs.

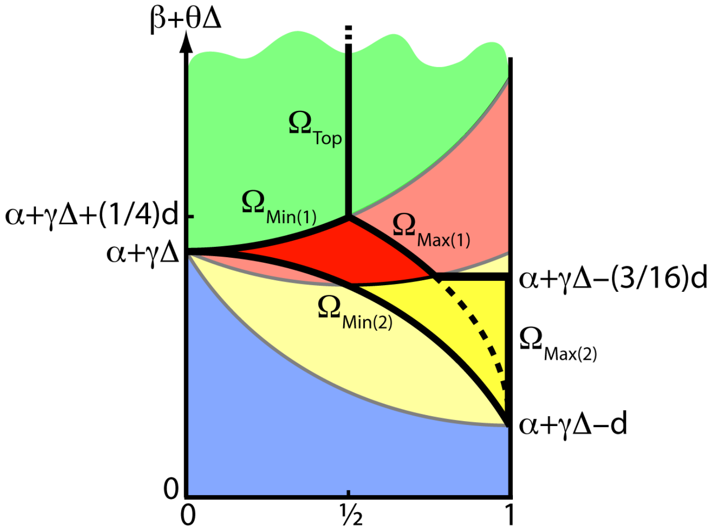

Figure 3

The profitability of a strategy (Δ, Ω) can be boiled down to a consideration of how it will influence market shares and per period profits now and in future periods. As market shares are determinant for the speed at which network sizes change, and network sizes are the only element that changes the relative value of firms in equilibrium, the size of the initial market share determines future market shares in a deterministic way in accordance with the transition function in equation 3. Thus, a strategy that involves a higher initial market share than some other strategy also yields higher market shares in future periods. This holds whether market shares increase or decrease over time.

The choice of product differentiation affects the firm's markups in every period. As the market can be segmented it is possible for the winning network to soak up the excess value it offers through the price charged. The firm is therefore interested in choosing a location in the market that does a good job of minimizing the average loss to transportation for those consumers joining its network, in order to command higher prices and maximize profits. The formal values of networks are given by equations 1 and 2. In the analysis below I do not use these equations directly but rather derive the conclusions from the properties that they possess such as being convex and downward sloping around the location of the firm.

The following analysis exploits that for some fixed level of research some locations yield both lower initial market shares *and* lower revenues than other locations that could have been chosen. Such pairs are inferior in terms of the level of present value profits and should therefore never be chosen by the entrant.

■ There exists a negative relationship between the maximum level of product differentiation and research time for strategy pairs belonging to areas II and III. Figure 4 illustrates competition in the first period of the duopoly.

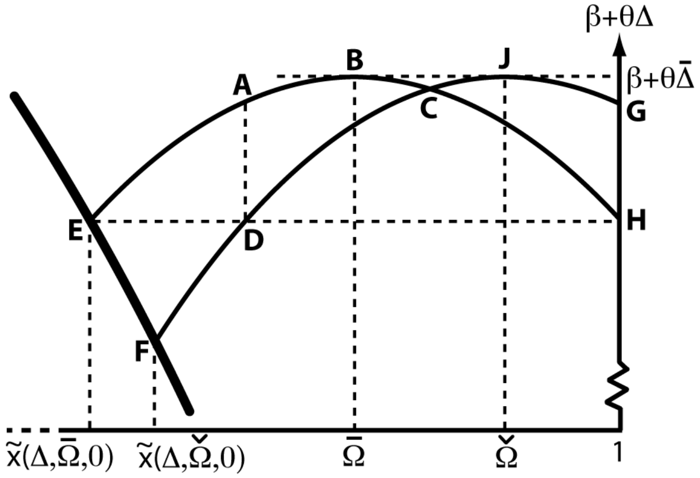

Figure 4

Compare two potential strategy pairs $(\bar{\Delta}, \bar{\Omega})$ and $(\bar{\Delta}, \check{\Omega})$ where $\bar{\Omega} < \check{\Omega}$. Doing research for $\bar{\Delta}$ periods leads to a product with a final quality of $\beta + \theta\bar{\Delta}$. By choosing $(\bar{\Delta}, \bar{\Omega})$ the entrant's value function during the first period of duopoly competition has its apex at point B in figure 4. Likewise, the value function from choosing $(\bar{\Delta}, \check{\Omega})$ attains its apex at point J. The incumbent's value function is represented by the thick line running through points E and F. Furthermore, $(\bar{\Delta}, \bar{\Omega})$ has the special property that $\bar{\Omega} = \tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right) + \frac{1-\tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right)}{2}$. This makes firm B locate at the position that maximizes revenue for its users during the first period of competition, since this is the location that minimizes the average loss to transportation. I will now argue that $\left(\bar{\Delta}, \bar{\Omega}\right)$ leads to present value profits that dominate those of $(\bar{\Delta}, \check{\Omega})$.

First, since value functions are downward sloping there is a business stealing effect of moving closer to the competitor, as this increases the newcomer's appeal to consumers otherwise inclined to adopt the incumbent's network. Since $\bar{\Omega} < \check{\Omega}$ then $\tilde{x}(\bar{\Delta}, \bar{\Omega}, 0) < \tilde{x}(\bar{\Delta}, \check{\Omega}, 0)$ showing that $\left(\bar{\Delta}, \bar{\Omega}\right)$ yields higher market shares than $\left(\bar{\Delta}, \check{\Omega}\right)$. Second, higher market shares today lead to higher market shares in future periods.

The markups earned under $(\bar{\Delta}, \bar{\Omega})$ exceed those of $(\bar{\Delta}, \check{\Omega})$ by an area corresponding to BCDFEAB. On the other hand, $(\bar{\Delta}, \check{\Omega})$ has an excess value corresponding to the area JGHCJ. Since value functions are symmetric around firms' locations, area BCDAB mirrors to area JGHCJ through an imaginary vertical axis through point C. Choosing $(\bar{\Delta}, \bar{\Omega})$ therefore leaves firm B with an additional profit equal to area

ADFEA, establishing that $(\bar{\Delta}, \bar{\Omega})$ yields higher profits at least during the first period. What happens in the ensuing periods? By construction, all strategy pairs belonging to area II lead to increasing market shares for network B, and since $\bar{\Omega} < \check{\Omega}$ then the former does a better job of maximizing revenues in all future periods also. In conclusion, $(\bar{\Delta}, \bar{\Omega})$ leads to strictly higher markups than $(\bar{\Delta}, \check{\Omega})$ in all current and future periods.

The equation describing the formal relationship between $\bar{\Delta}$ and $\bar{\Omega}$ making up this boundary is derived in Appendix B. The result is reported in equation 4:

$$\Omega_{Max(1)} = \frac{1}{3} + \left(\frac{1}{9} + \frac{\alpha - \beta - (\theta - \gamma)\,\Delta}{3d} \right)^{\frac{1}{2}} \tag{4}$$

The greater the research effort the higher is the quality of the network. Higher quality makes the network appeal to a wider range of segments in the market, and to accommodate this new demand, the firm has an incentive to move closer to the incumbent to reduce the average loss due to transportation costs incurred by its buyers. Given some Δ satisfying $\alpha + \gamma\Delta - \left(\frac{3}{16}\right) d \leq \beta + \theta\Delta \leq \alpha + \gamma\Delta + \frac{d}{4}$ then all $\Omega \in \left(\Omega_{Max(1)}; 1\right]$ are excluded from the set of potential profit maximizing strategy pairs.

■ Similar arguments do not, however, apply in area IV where network B experiences declining market shares. While it remains true that $\bar{\Omega}$ yields higher market shares today and in the future, it is no longer certain that markups are higher in *all* future periods; as market shares decline the firm *may* be able to earn higher markups at some point in time by choosing a greater level of product differentiation than $\bar{\Omega}$. Without the aid of more sophisticated analysis, one cannot rule out the possibility that the entrant chooses the maximal level of differentiation. Therefore define the boundary $\Omega_{Max(2)} = 1$ applicable when relative qualities satisfy $\alpha + \gamma\Delta - d \leq \beta + \theta\Delta < \alpha + \gamma\Delta - \left(\frac{3}{16}\right) d$. Even though equation 4 does not hold with certainty in area IV, it is still likely to remain valid in industries with strong network effects and/or high discounting.

■ Strategy pairs belonging to area I allow the newcomer to capture the entire market immediately upon arrival. In this case it is possible to derive the analytical solution to the problem since market shares remain constant over time. See derivation in Appendix B.

$$\Omega^*_{Top} = \frac{1}{2} \text{ and } \Delta^*_{Top} = \frac{\alpha - \beta - \frac{d}{4}}{\theta - \gamma} + \frac{(\theta - 2\gamma)}{\delta(\theta - \gamma)} \tag{5}$$

It is not surprising to find that the profit-maximizing location is right in the middle of the market as this is the location that minimizes average transportation cost. Given this location choice, research must satisfy $\beta+\theta\Delta > \alpha+\gamma\Delta+\frac{d}{4}$ for the strategy pair to belong to area I. The research effort reflects the balance between the loss of remaining an outsider to the market for another period, $\beta-\alpha+\frac{d}{4}+(\theta-\gamma)\Delta$, versus the present value of the net gain for an additional period of research, $\frac{\theta-2\gamma}{\delta}$. The term $\frac{d}{4}$ is the average travel expense saved[9] for consumers across the market for joining network B at $x=\frac{1}{2}$ rather than joining network A at $x=0$. This can be interpreted as the newcomer's gain from its ability to product differentiate. The stronger heterogeneity in consumers' preferences get, the higher d is, the faster does the new network enter the market, since this increases the potential loss for remaining an outsider to the market but leaves the value of further research unaffected. Even though heterogeneity does not affect competition in area I directly via market shares, it does add a new channel of profits that is not captured by models with identical consumers, which has implication for research time.

■ In area V the quality of network B is so low that even under maximal differentiation the firm is not adopted by anyone in equilibrium. For this to hold formally, research must satisfy the condition $\beta+\theta\Delta < \alpha+\gamma\Delta-d$. In this case optimal location is a matter of definition.

Proposition 1 summarizes the findings on the reduced set of potential entry strategies so far, which are illustrated as the collection of the boundaries $\Omega_{Top,}$ $\Omega_{Max(1)}$, and $\Omega_{Max(2)}$ in figure 3.

> **Proposition 1.** Research time has a nonpositive relationship with the maximal feasible level of product differentiation for $\alpha+\gamma\Delta-d \leq \beta+\theta\Delta$.

Having derived the boundaries to maximum product differentiation, the next part of this section considers the minimum boundaries to product differentiation.

■ The minimum level of product differentiation is decreasing in technological quality for all strategy pairs belonging to areas IV, III, and II. As before, consider the two candidate strategy pairs $(\bar{\Delta},\bar{\Omega})$ and $(\bar{\Delta},\hat{\Omega})$, where $\bar{\Omega}>\hat{\Omega}$, and compare market shares and profits they give rise to. Figure 5 illustrates the first period of duopoly competition under the two potential strategy pairs. The value functions resulting from $(\bar{\Delta},\bar{\Omega})$

[9] Average travel costs when A covers the market are $\int_0^1 d(x)^2\partial x=\frac{d}{3}$. Average travel costs when B covers the market *and* locates at $\Omega=\frac{1}{2}$ are $\int_0^1 d\left(\frac{1}{2}-x\right)^2\partial x=\frac{d}{12}$. The difference equals $\frac{d}{4}$.

and $(\bar{\Delta}, \hat{\Omega})$ have apices at A and G respectively. The incumbent's value function is represented by the thick line along AED.

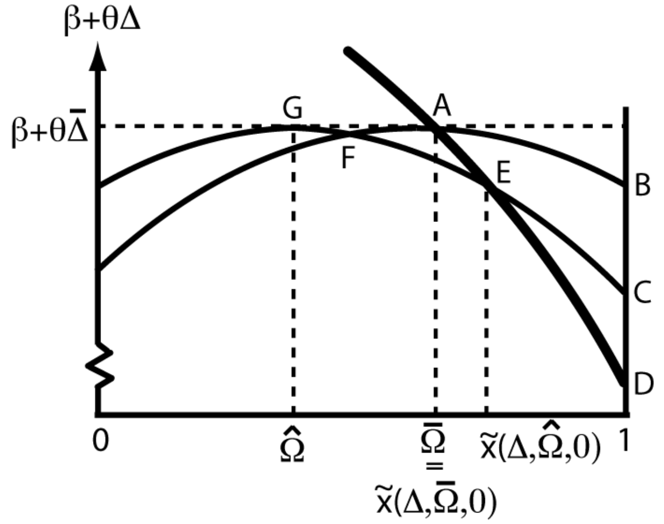

Figure 5

Let $(\bar{\Delta}, \bar{\Omega})$ satisfy the equality $\tilde{x}(\bar{\Delta}, \bar{\Omega}, 0) = \bar{\Omega}$ such that network B initially is located at the same address as the pivotal agent; it wins no one to the left but everybody to the right. Since $(\bar{\Delta}, \hat{\Omega})$ involves a less differentiated product but has the same intrinsic quality as $(\bar{\Delta}, \bar{\Omega})$, the newcomer cannot offer the die-hard fans of network A that lives at $\hat{\Omega}$ enough value for them to adopt. Yet, by the convexity of value functions it is possible that B will win some segments that dislike network A a lot. Since $\tilde{x}(\bar{\Delta}, \bar{\Omega}, 0) < \tilde{x}(\bar{\Delta}, \hat{\Omega}, 0)$ the entrant receives a higher market share today by locating at $\bar{\Omega}$ rather than at $\hat{\Omega}$, and by the same arguments as made above, all future market shares are higher as well.

Consider the effect of location on markups. During the initial period the profit made under $(\bar{\Delta}, \bar{\Omega})$ equals ABCDEA, which is greater than the profit resulting from $(\bar{\Delta}, \hat{\Omega})$, namely ECDE, since value functions are downward sloping. Furthermore, profits in all *future* periods under $(\bar{\Delta}, \bar{\Omega})$ are also higher than they would have been under $(\bar{\Delta}, \hat{\Omega})$ no matter whether the entrant's market shares increase or decrease intertemporally: If market shares are increasing under $(\bar{\Delta}, \bar{\Omega})$ then it must be the case that $\bar{\Omega} < \frac{1}{2}$ (by the assumption that $\tilde{x}(\bar{\Delta}, \bar{\Omega}, 0) = \bar{\Omega}$). Observing that the location that minimizes the average loss to transportation can never be smaller than $x = \frac{1}{2}$, then choosing $\hat{\Omega}$ puts the firm in a less favorable position to earn revenues than at $\bar{\Omega}$ since $\hat{\Omega} < \bar{\Omega} < \frac{1}{2}$. If market shares are decreasing then the location that should be chosen to maximize markups at each point in time will be higher and higher, but as $\hat{\Omega} < \bar{\Omega}$

then the firm is strictly better off choosing $\bar{\Omega}$ rather than $\hat{\Omega}$ in each period.

The relationship between timing and location that satisfies the definition of strategy pairs $\left(\bar{\Delta}, \bar{\Omega}\right)$, that is $\tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right) = \bar{\Omega}$, can be derived from the transition function in equation 3. See Appendix B. The result is as follows:

$$\Omega_{Min(2)} = \left[\frac{\alpha - \beta - (\theta - \gamma)\Delta}{d}\right]^{\frac{1}{2}} \tag{6}$$

Equation 6 shows the relationship between research time and the minimal, feasible level of product differentiation. For research levels satisfying $\alpha + \gamma\Delta - d \leq \beta + \theta\Delta \leq \alpha + \gamma\Delta$ then all $\Omega \in \left[0; \Omega_{Min(2)}\right)$ should not be chosen in equilibrium since they are dominated by another strategy pair that yields higher market shares as well as higher markups, which could have been chosen for the same research effort. The intuition behind this result is that an entrant with a relatively poor product should not aim for a strategy of low product differentiation; doing so would make B offer the most value to buyers it cannot win because they have a strong preference for A. At the same time the firm would fail to capitalize on those buyers not much in favor of A as they would incur high travel costs to reach B. Instead it serves the entrant better to focus on minimizing travel costs for the segment of the market that has tastes very different from A's product.

■ Consider the situation in which B possesses a technology that is of a sufficiently high quality to take over the entire market provided that the firm is willing to introduce a product sufficiently close to the incumbent's technology. The problem of this business-stealing strategy is naturally that low differentiation is synonymous with low markups. So while the entrant wants to have a high market share, there is no reason to locate closer to the incumbent than needed to just cover the entire market. We are therefore interested in identifying the connection between Δ and Ω which just makes every consumer segment across the market adopt network B in equilibrium immediately upon introduction. Rather than deriving the relationship in the usual way, we can exploit the condition in table 1 that separates areas I and II, namely $\alpha - \beta - (\theta - \gamma)\Delta + d\Omega^2 = 0$. Rewrite to get

$$\Omega_{Min(1)} = \left(-\frac{\alpha - \beta - (\theta - \gamma)\Delta}{d}\right)^{\frac{1}{2}} \tag{7}$$

For all Δ satisfying $\alpha + \gamma\Delta < \beta + \theta\Delta \leq \alpha + \gamma\Delta + \frac{d}{4}$ then all $\Omega \in \left[0; \Omega_{Min(1)}\right)$ are wasteful because these locations only lead to lower markups, but no gain through higher market shares.

Proposition 2 summarizes our knowledge on the relationship between research time and the minimum level of product differentiation as illustrated by boundaries $\Omega_{Min(2)}$, $\Omega_{Min(1)}$, and Ω_{Top} in figure 3.

> **Proposition 2.** Research is negatively related to the minimum level of product differentiation when $\alpha+\gamma\Delta-d \leq \beta+\theta\Delta \leq \alpha+\gamma\Delta$. Research has a nonnegative relationship with product differentiation for $\alpha+\gamma\Delta < \beta+\theta\Delta$.

Together the minimum and maximum boundaries show the levels of product differentiation that will, or at least should, be chosen in equilibrium as a function of the quality of the newcomer's network.

5 Numerical results

Having derived the set of potential equilibria, this section analyzes the tensions that go into optimizing behavior of the entrant in more detail. Numerical computations can extend the analysis from section 4 to determine the precise global solution (Δ, Ω) to the entrant's problem in a particular market. By means of numerical integration the entrant's present value profits across a two-dimensional space spanned by the endogenous choice variables are calculated for an industry j described by the set of parameters, $S_j \equiv \{\theta, \gamma, \alpha, \beta, d, \delta\}$. Naturally, this sort of analysis can never represent the model in a general way, but it does give a taste of the entrant's problem and the mechanisms behind its choices. Table 2 reports the globally optimal strategy pairs for a specific industry characterized by the parameter set $S = \{-, 0.25, 4, 0, 4, 0.2\}$ allowing for a range of different technological improvement rates in order to illustrate how optimal choices change with the market. Figure 6 illustrates such a sequence of equilibria which I will refer to as the 'equilibrium path'.

θ	Δ	Ω
0.9	7.8	0.50
0.8	8.5	0.46
0.7	10.0	0.43
0.6	11.8	0.40
0.5	15.4	0.39
0.4	23.3	0.42
0.3872	24.8	0.44
0.3872	16.2	0.83
0.35	15.5	0.90
0.3	15.1	0.96
0.2	----	----
0.1	----	----

Table 2

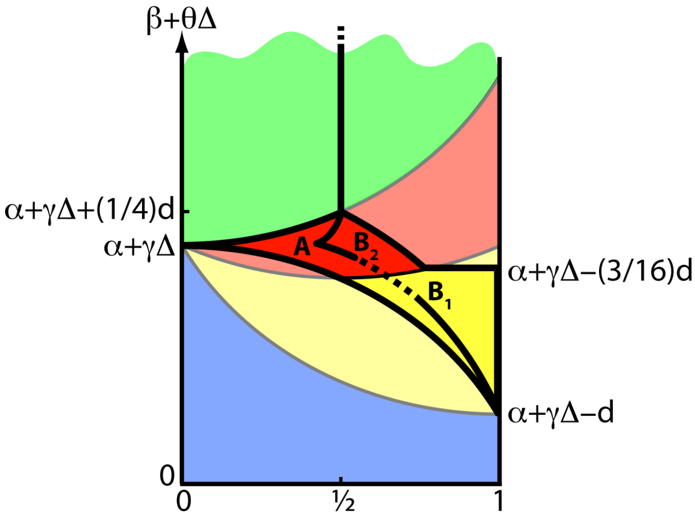

Figure 6

The entrant's choices under incompatible standards display two interesting features resulting from network effects. A) Location choices for winning technologies are backward-bent. B) The entrant's optimal choices display a discontinuity.

The first feature to observe is that the newcomer's equilibrium choices of research and location are backward-bent for winning technologies as the parameterization of the industry becomes increasingly favorable. See point A in figure 6. Consider first a situation where the return to research effort is relatively small, the entrant's incentive to do research is somewhat limited and the resulting level of product differentiation is high. If, for example, the technological improvement rate increases then research becomes more attractive and the newcomer will want to enter the industry with a more valuable product. To ease the burden of preempting the established firm, a smaller level of product differentiation is chosen to cut into current and future network sizes of firm A as well as increase the speed of tipping. Yet there is an upper limit to this business stealing effect, as increasingly higher quality levels make for increasingly faster preemption of the established network. And as preemption moves closer, the newcomer becomes increasingly interested in choosing a level of product differentiation that puts it in a good spot with respect to revenues in future periods. As a matter of course the newcomer's profit maximizing location moves towards $x = \frac{1}{2}$ from the left. Taken together, network effects make the optimal strategy pairs a backward-bent shape for a range of different industries, but only as long as there are network effects present; absent network effects the newcomer would not have the incentive to stray from the position that maximizes markups in order to steal market shares, and there would be no kink in the optimal in optimal location choices.

The second feature shows that for a 'continuous' change in the parameterization of the industry, the newcomer's equilibrium path exhibits a discontinuity.[10] Thus, there exists a pivotal parameterization that produces two separate strategy pairs yielding the same level of present value profits. The discontinuity can be seen in table 2 at $\theta = 0.3872$ where the optimal research time jumps from 16.2 to 24.8, and the optimal location shifts from 0.83 to 0.44. In figure 6 B_1 and B_2 illustrate such two equilibria. Everywhere else a small change in a parameter leads to a correspondingly small change in the equilibrium. As discussed in relation to the transition function in equation 3, lower product differentiation increases both initial demand *and* speed of tipping. The discontinuity emerges because the newcomer experiences the effect of more speedily tipping differently depending on whether it sponsors a winning or loosing technology. The optimal level of product differentiation is therefore lower for winning technologies, which as a result have higher market shares. If the firm can sell its product to more people, there is a higher return to the research investment, which naturally propels the research effort. For loosing products, on the other hand, increased speed of tipping is undesirable since it leads to faster preemption. Thus, the optimal level of product differentiation is higher for loosing technologies. And a low market share reduces the entrant's incentive to do research, because smaller demand reduces the scope of returns to R&D. Without network effects the market does not tip and there would be no such discontinuity. The result is that around this pivotal parameterization the newcomer will choose between a strategy of a small research effort and high product differentiation or a strategy of greater research effort and a lesser degree of product differentiation. One should therefore expect to observe one of two network types. The first type is networks of high quality with broad appeal to the market. These networks will, at least with time, become adopted by all consumers. The other type of network is more highly differentiated with less build-in. By targeting a niche the sponsoring firm can secure reasonable profits in the short run but must at the same recognize that this decision seals its own faith.

Examples of better quality technologies having dethroned established goods are plentiful in the literature and a few have already been mentioned in this paper as well. However, I find it more interesting to consider some of those cases where a

[10]In the example given in table 2 it is the rate of technological progress that changes, but the feature can be derived from changes to the other parameters as well.

new technology has not been able to reach the critical mass needed in order to become dominant. One example is the standards war in keyboard layouts between the QWERTY (designed in the 1860s[11]) and Dvorak systems (patented in 1936) competing along such dimensions as speed of typing and ergonomics. The Dvorak system is designed to minimize finger movement, while QWERTY originates from minimization of the number of jams by the hammers on a typewriter. There has been much discussion about the efficiency of Dvorak in comparison to QWERTY. David (1985) argues that Dvorak is in fact superior, while Liebowitz and Margolis (1990) present evidence to the contrary. Liebowitz and Margolis also dispute the alleged ergonomic benefits of the Dvorak system referring to studies in the ergonomics literature. Whether or not Dvorak is a superior system it is safe to say that the difference in technological quality is rather limited. In this case the present model implies that the entrant (in this case the Dvorak system) should pursue a strategy of high differentiation. Since only the A and M keys are placed at identical spots on the two layouts[12] this seems to be the case.

If one believes that Dvorak *is* a major improvement then this example illustrates a failure to recognize that a lower degree of product differentiation would be needed to achieve the critical mass of users to get the bandwagon rolling in its favor. For instance the level of differentiation could be reduced by maintaining the position of the keys in the lower left hand section of the board that accesses many macros, such as ‘copy’, ‘paste’, ‘select all’ familiar to many users of the QWERTY system.

6 Conclusions

This paper has studied the connection between a newcomer’s choice of technological quality and product differentiation in competition against an established network. The inclusion of heterogeneity on the demand side allows the model to gain insight into the diffusion of networks, and how the fight for market shares influences product design.

Comparing revenues and demands of different strategies I have been able to eliminate some strategies that cannot be part of equilibrium. This analysis is therefore useful in making predictions on what should be expected from future technologies.

[11] During the first decades of the QWERTY standard’s lifetime, several minor changes were made to the individual key’s placements. It is therefore difficult to pinpoint the exact introduction time. The current layout is accredited by Liebowitz and Margolis (1990) to Christopher Sholes.

[12] See Shy (2001, p. 43) for an illustration of both layouts.

Moreover, numerical computations reveal that new networks will in general be of either high quality with broad appeal in the market or low quality niche products.

Returning to the issue of consumer expectations, forward looking buyers will be more hesitant to adopt a loosing network than their myopic counterparts. This will limit the opportunities to enter with a fringe product. If one were to assume a fixed cost of development this would further limit the scope of introduction of such networks. On the other hand, the marginal value of additional users may decline as the network grows. A Consumer may therefore want to adopt a product that is closely aligned with her taste even though its network is small and grows more slowly than other networks. This alludes to the survival, even in the long run, by a small network as long as it achieves some critical mass of users. The Mac speaks for the case that niche products can survive and thrive in the face of a greater opponent.

Appendix A

Equate utilities $V^A(\Delta, \Omega, x, t)$ to $V^B(\Delta, \Omega, x, t)$ in equations 1 and 2 to find the location of the indifferent buyer in the duopoly as a function of time

$$\tilde{x}(\Delta, \Omega, t) = \frac{\alpha - \beta - (\theta - \gamma)\Delta + d\Omega^2 - \gamma t + 2\gamma \int_0^t \tilde{x}(\Delta, \Omega, \tau)\, d\tau}{2d\Omega}$$

Define $D \equiv \alpha - \beta - (\theta - \gamma)\Delta + d\Omega^2$ and $C \equiv 2d\Omega$ to get

$$\tilde{x}(\Delta, \Omega, t) = \frac{D}{C} - \frac{\gamma}{C}t + \frac{2\gamma \int_0^t \tilde{x}(\Delta, \Omega, \tau)\, d\tau}{C} \tag{8}$$

Apply Leibniz formula for differentiation of integrals to calculate $\frac{\partial \tilde{x}(\Delta,\Omega,t)}{\partial t}$.

$$\frac{\partial \tilde{x}(\Delta, \Omega, t)}{\partial t} = -\frac{\gamma}{C} + \frac{2\gamma}{C}\tilde{x}(\Delta, \Omega, t)$$

The general solution to this differential equation is

$$\tilde{x}(\Delta, \Omega, t) = We^{\left(\frac{2\gamma}{C}\right)t} + \frac{1}{2} \tag{9}$$

where W is an unknown constant. The particular solution is obtained using the initial condition that firm B has accumulated no network immediately upon entry. Thus, at $t = 0$, equation 8 becomes

$$\tilde{x}(\Delta, \Omega, 0) = \frac{D}{C} - \frac{\gamma}{C}(0) + \frac{2\gamma \int_0^0 \tilde{x}(\Delta, \Omega, \tau)\, d\tau}{C}$$

where $\int_0^0 \tilde{x}(\Delta, \Omega, \tau)\, d\tau = 0$ by construction of the integral. Thus $\tilde{x}(\Delta, \Omega, 0) = \frac{D}{C}$. Equating this to equation 9 evaluated at $t = 0$ gives $W = [\frac{D}{C} - \frac{1}{2}]$. Inserting W into equation 9 gives $\tilde{x}(\Delta, \Omega, t) = [\frac{D}{C} - \frac{1}{2}]e^{\frac{2\gamma}{C}t} + \frac{1}{2}$. Plugging back D and C and rewrite to get the transition function in equation 3.

$$\tilde{x}(\Delta, \Omega, t) = \left[\frac{\alpha - \beta - (\theta - \gamma)\Delta - d(1 - \Omega)\Omega}{2d\Omega}\right] e^{\frac{\gamma}{d\Omega}t} + \frac{1}{2}$$

Appendix B

■ Solving the problem in area I

Area I represents the situation where the entrant wins every consumer on $x \in [0;1]$ in the duopoly for all $t \geq 0$. As discussed in the setup the winning firm sets a price for each segment equal to the excess value it can offer over the best alternative. The price is derived from the difference between equation 2 and 1 using the property that $\tilde{x}(\Delta, \Omega, t) = 0$ for all $t \geq 0$. The resulting price is $p^B = \beta - \alpha + (\theta - \gamma)\Delta - d(\Omega)^2 + 2d\Omega x + \gamma t$. Integrate over all locations in the market $x \in [0, 1]$ and all $t \geq 0$. Discount back Δ periods to get $\pi^B(\Delta, \Omega) = e^{-\delta\Delta} \int_0^\infty \int_0^1 \left(p^B\right) e^{-\delta t} dx dt$. With some calculus the present value profits can be rewritten as $\pi^B = \frac{1}{\delta}e^{-\delta\Delta}\left(\frac{\gamma}{\delta} - (\alpha - \beta - (\theta - \gamma)\Delta - d(1 - \Omega)\Omega)\right)$. Constrained maximization subject to conditions $\alpha - \beta - (\theta - \gamma)\Delta + d\Omega^2 < 0$ (this condition restricts attention to strategy pairs in area I) and $\Omega \in [0; 1]$, one finds that the optimal solutions to the Kuhn-Tucker problem is

$$\Omega^*_{Top} = \frac{1}{2} \text{ and } \Delta^*_{Top} = \frac{\alpha - \beta - \frac{d}{4}}{\theta - \gamma} + \frac{(\theta - 2\gamma)}{\delta(\theta - \gamma)}$$

■ **Boundary** $\Omega_{Max(1)}$

The strategy pair $\left(\bar{\Delta}, \bar{\Omega}\right)$ satisfies the equality $\bar{\Omega} = \tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right) + \frac{1-\tilde{x}\left(\bar{\Delta},\bar{\Omega},0\right)}{2}$. Rewrite as $2\bar{\Omega} - 1 = \tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right)$. Insert equation 3 evaluated at $t = 0$ to get the functional form for the relationship between $\bar{\Delta}$ and $\bar{\Omega}$:

$$\begin{gathered} 2\bar{\Omega} - 1 = \left[\tfrac{\alpha-\beta-(\theta-\gamma)\bar{\Delta}-d(1-\bar{\Omega})\bar{\Omega}}{2d\bar{\Omega}}\right] + \tfrac{1}{2} \\ \Leftrightarrow 3d\bar{\Omega}^2 - 2d\bar{\Omega} - \left[\alpha - \beta - (\theta - \gamma)\bar{\Delta}\right] = 0 \end{gathered}$$

Solve the second-degree polynomial for $\bar{\Omega}$ to get the relevant solution

$$\bar{\Omega} = \frac{1}{3} + \left(\frac{1}{9} + \frac{\alpha - \beta - (\theta - \gamma)\,\bar{\Delta}}{3d}\right)^{\frac{1}{2}} \equiv \Omega_{Max(1)}$$

This solution is relevant in areas II and III only. Thus, given the above relationship Δ must satisfy $\alpha + \gamma\Delta + \frac{d}{4} > \beta + \theta\Delta \geq \alpha + \gamma\Delta - \left(\frac{3}{16}\right)$.

■ **Boundary** $\Omega_{Min(2)}$

The strategy pair $\left(\bar{\Delta}, \bar{\Omega}\right)$ satisfies the equality $\bar{\Omega} = \tilde{x}\left(\bar{\Delta}, \bar{\Omega}, 0\right)$. Use equation 3 evaluated at $t = 0$ to find the functional form for the relationship between $\bar{\Delta}$ and $\bar{\Omega}$.

$$\bar{\Omega} = \left[\frac{\alpha - \beta - (\theta - \gamma)\bar{\Delta} - d(1 - \bar{\Omega})\bar{\Omega}}{2d\bar{\Omega}}\right] + \frac{1}{2}$$

Isolate $\bar{\Omega}$ to get

$$\bar{\Omega} = \left[\frac{\alpha - \beta - (\theta - \gamma)\bar{\Delta}}{d}\right]^{\frac{1}{2}} \equiv \Omega_{Min(2)}$$

This solution holds for $\alpha + \gamma\Delta \geq \beta + \theta\Delta \geq \alpha + \gamma\Delta - d$ for $\bar{\Omega} \in [0; 1]$.

References

- Bolton, P., J.F. Brodley, and M.H. Riordan, 2000, "Predatory pricing: Strategic theory and legal policy", *Georgetown Law Journal*, 88, 2239-2330.

- Cusumano, M.A., Y. Mylonadis, and R.S. Rosenbloom, 1992, "Strategic maneuvering and mass-market dynamics: The triumph of VHS over Beta", *Business History Review*, 66, 51-94.

- d'Aspremont, C., J.J. Gabszewicz, and J.-F. Thisse, 1979, "On Hotelling's stability in competition", *Econometrica*, 47, 1145-1150.

- David, P.A., 1985, "CLIO and the economics of QWERTY", *American Economic Review*, 75, 332-337.

- Doganoglu, T., 2003, "Dynamic price competition with consumption externalities", *Netnomics*, 5, 43-69.
- Hotelling, H., 1929, "Stability in competition", *Economic Journal*, 39, 41-57.
- Katz, M.L. and C. Shapiro, 1992, "Product introduction with network externalities", *Journal of Industrial Economics*, 40, 55-83.
- Kristiansen, E.G., 1998, "R&D in the presence of network externalities: Timing and compatibility", *RAND Journal of Economics*, 29, 531-547.
- Liebowitz, S.J. and S.E. Margolis, 1990, "The fable of the keys", *Journal of Law and Economics*, 33, 1-25.
- Mitchell, M. F. and A. Skrzypacz, 2006, "Network externalities and long-run market shares", *Economic Theory*, 29, 621-648.
- Park, S., 2004, "Quantitative analysis of network externalities in competing technologies: The VCR case", *Review of Economics and Statistics*, 86, 937-945.
- Regibeau, P. and K.E. Rockett, 1996, "The timing of product introduction and the credibility of compatibility decisions", *International Journal of Industrial Organization*, 14, 801-823.
- Scherer, F.M., 1980, Industrial market structure and economic performance, 2nd ed., Chicago: Rand McNally & Co.
- Shy, O., 2001, The economics of network industries, Cambridge: Cambridge University Press.
- Stoneman, P., 2002, The economics of technological diffusion, Oxford: Blackwell Publishers.
- Tyagi, R.K., 2000, "Sequential product positioning under differential costs", *Management Science*, 46, 928-940.

Other resources

- The recording industry in numbers (2005) - Published by IFPI London.

Legend

Economics Working Paper

2007-05 William Addessi and Francesco Busato: Labor Unions and Asset Prices

2007-06 Marias Halldor Gestsson: Foreign Prices Shocks in a Small Open Economy

2007-07 Helena Skyt Nielsen, Nina Smith and Aycan Celikaksoy: The Effect of Marriage on Education of Immigrants: Evidence from a Policy Reform Restricting Spouse Import

2007-08 Leonidas Enrique de la Rosa: Overconfidence and Moral Hazard

2007-09 Torben M. Andersen and Michael Svarer: Flexicurity – labour market performance in Denmark

2007-10 Michael Svarer: The Effect of Sanctions on the Job Finding Rate: Evidence from Denmark

2007-11 Richard C. Barnett and Joydeep Bhattacharya: Rejuveniles and Growth

2007-12 Richard C. Barnett, Joydeep Bhattacharya and Helle Bunzel: Resurrecting Equilibria Through Cycles

2007-13 Hristos Doucouliagos and Martin Paldam: Explaining development aid allocation by growth: A meta study

2007-14 Maria Knoth Humlum, Kristin J. Kleinjans and Helena Skyt Nielsen: An Economic Analysis of Identity and Career Choice

2007-15 Hristos Doucouliagos and Martin Paldam: A meta-analysis of development aid allocation: The effects of income level and population size

2007-16 Michael Rosholm, Jonas Staghøj and Michael Svarer: A Statistical Programme Assignment Model

2007-17 Nabanita Datta Gupta and Marianne Simonsen: Non-cognitive Child Outcomes and Universal High Quality Child Care

2007-18 Agne Lauzadyte: Unemployment, Employment and Inactivity in Denmark: An Analysis of Event History Data

2007-19: Christian Dahl Winther: Optimal research effort and product differentiation in network industries