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Overconfidence and Moral Hazard

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Abstract

In this paper, I study the effects of overconfidence on incentive contracts in a moral-hazard framework in which principal and agent knowingly hold asymmetric beliefs regarding the probability of success of their enterprise. Agent overconfidence can have conflicting effects on the equilibrium contract. On the one hand, an overconfident agent disproportionately values success-contingent payments, and thus prefers higher-powered incentives. On the other hand, if the agent is overconfident in particular about the extent to which his actions affect the likelihood of success, lower-powered incentives are sufficient to induce any given effort level. If the agent is overall moderately overconfident, the latter effect dominates; because the agent bears less risk in this case, he actually benefits from his overconfidence. If the agent is significantly overconfident, the former effect dominates; the agent is then exposed to an excessive amount of risk, which is harmful to him. An increase in overconfidence—either about the base probability of success or the extent to which effort affects it—makes it more likely that high levels of effort are implemented in equilibrium.

Keywords: overconfidence, heterogeneous beliefs, moral hazard.

JEL Classification: A12, D81, D82.

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1 Introduction

It is not uncommon to observe incentive contracts that appear puzzling in the light of a standard principal-agent model. For instance, in 1995 Continental Airlines offered \$65 to every hourly employee in every month that Continental's on-time performance ranked amongst the top five in the industry. This seemingly small incentive had, rather surprisingly, notable results.[1] More generally, low- and middle-rank employees receive performance bonuses that are relatively small but seem to "get the job done": they induce the employees to exert effort at work and act with the company's best interest in mind. At the same time, millions of dollars in many top executives' compensation packages acutely depend on their company's performance. I argue that allowing for heterogeneous beliefs—agent overconfidence in particular—will aid our understanding of incentives in contracts.

Consider the problem facing a business owner when hiring a manager as her agent. If the owner (the principal) cannot monitor the actions undertaken by the manager (the agent) and these actions affect profits, she will offer an incentive contract (e.g. consisting of a salary and a performance bonus). In the standard treatment of this moral-hazard problem, it is usually assumed either that the parties hold identical beliefs regarding the distribution of profits conditional on the manager's actions, or that asymmetries in beliefs arise solely from private information. Motivated by extensive psychological evidence that people are overconfident about their ability and future prospects[2], this paper introduces heterogeneous beliefs of which principal and agent are aware: they "agree to disagree."

Section 2 introduces the main assumptions of the model, devoting special attention to the assumption that principal and agent knowingly hold asymmetric beliefs, which is crucial in the model and implies there can be no further updating of beliefs upon observing each other's actions. There are two dimensions on which the asymmetry of beliefs is important in the model: an (overall) overconfident agent can be overconfident about the *base probability of success* of the project—over all possible choices of effort available to him—and he can be overconfident about the *value of his*

[1] Knez and Simester (2001) study Continental's case. The authors argue that mutual monitoring among employees was the main reason behind the success of the incentive scheme. We will see that such effectiveness of low-powered incentives can also be explained within a setting of agent overconfidence.

[2] In the psychology literature, a technical use of the term "overconfidence" refers to overestimating the precision of one's forecast. In this paper, we will refer to overestimating the probability of favorable outcomes following the agent's actions as "agent overconfidence." Some literature refers to this type of self-serving bias as unrealistic optimism (or simply optimism). Others share the use of the term with this paper. When discussing ability and the repercussions of one's actions, I believe that overconfidence is a more appropriate term than optimism, which suggests a passive role in relation to outcomes.

effort—the marginal contribution of his effort to the probability of success.[3]

Section 3 develops the main results of the model, exploring the effects of overconfidence in a setting in which several principals compete to contract with the agent. Competition between principals will drive their expected profits to zero in equilibrium, which allows for an intuitive exposition of the effects of overconfidence.[4]

Because of the parties' awareness about the asymmetry in beliefs, there are no signaling or screening concerns in my model, so the effects of overconfidence on optimal contract design are isolated from its consequences in terms of adverse selection.[5] Agent overconfidence about the probability of success of the enterprise can have conflicting effects on the equilibrium contract. On the one hand, when the agent is overconfident in particular about the marginal contribution of his effort to the project's probability of success, lower-powered incentives are sufficient to induce any given effort level. This is the ***incentive effect*** of overconfidence, and it pushes the equilibrium contract to exhibit lower-powered incentives. On the other hand, because an overconfident agent disproportionately values success-contingent payments, he finds high-powered incentive contracts more attractive than a "realistic" agent. Because the principal believes that she will pay the bonus infrequently, she finds such a contract—with a higher performance bonus and a lower base salary—an inexpensive way of hiring the agent. This consequence of the divergence in evaluating payments is the ***wager effect*** of overconfidence, and it pushes the equilibrium contract to exhibit higher-powered incentives.

[3] The agent could even be *underconfident* about the value of effort, while still being overconfident overall. Imagine, for example, an agent who believes he has the "Midas touch": just because he's involved, the enterprise must succeed. This agent is overall very overconfident in the sense that he always overestimates the probability of success of the project, but at the same time underestimates the contribution of his effort to increasing the probability of success.

[4] Section 5 studies the case of a principal making a take-it-or-leave-it offer to the agent, and Section 6 extends the model to allow for a continuum of effort levels that the agent can choose from. Section 4 is a discussion about the welfare effects of agent overconfidence, which is particularly relevant in the competing-principals framework.

[5] Recent studies allow for asymmetric beliefs in principal-agent models, but the main focus has been on the effects of asymmetric beliefs in an adverse-selection framework. Fang and Moscarini (2005) allow for overconfident agents in an adverse selection model, and find that a principal might prefer not to differentiate wages to avoid the negative effects that revealing her private information about the agents' true ability may have on workers' morale. Koufopoulos (2002) suggests that bias in the perception of risk might explain some empirical observations related to asymmetric information in competitive insurance markets. Maskin and Tirole (1990) and (1992) introduce private information held by the principal regarding the extent to which she values the agency relationship in an adverse-selection model. Villeneuve (2000) considers the possibility that the principal is better informed than the agent in an insurance-market setting, which he refers to as "reverse adverse selection." Van den Steen (2005) considers asymmetric beliefs in the absence of private information (as this paper does) when there is disagreement about the best course of action. I consider disagreement about outcome distribution conditional on actions, but in my model the parties agree about which one generates a better distribution.

The degree of overall overconfidence determines which of these effects dominates in equilibrium. The incentive effect dominates when the agent is only slightly overconfident overall. When the agent is significantly overconfident, however, incentive provision becomes secondary to the fact that principal and agent value outcome-contingent payments differently. As a consequence, the wager effect dominates, and greater agent overconfidence about either the base probability of success or the effect of the agent's effort on the probability of success results in higher-powered incentives in equilibrium. Because of the potentially conflicting effects of overconfidence, the power of incentives of the equilibrium contract depends both on the degree and the kind of agent overconfidence. In contrast, the level of effort implemented by the equilibrium contract unambiguously increases with overconfidence.[6]

Section 4 discusses the welfare effects of overconfidence in the competing-principals framework. As it turns out, moderate overconfidence can be welfare-enhancing in this setting. Because an agent who is only slightly overconfident about the value of effort receives more insurance than an agent who holds realistic beliefs, he actually benefits from the effects of his overconfidence on the equilibrium contract. An agent who is underconfident about the value of effort or significantly overconfident overall, in contrast, bears an excessive amount of risk, so his overconfidence is harmful to him. Subsection 4.2 is a brief discussion on how the results carry over when we allow the competing principals to hold asymmetric beliefs amongst themselves.

Section 5 studies the implications of the model when one principal can make a take-it-or-leave-it offer to an agent—the standard setting in the moral-hazard literature. The main difference is that an exogenous agent-participation constraint replaces the endogenous one generated by competing contract offers, so the principal faces a remarkably similar optimization problem to the one faced by a principal who competes with others. The qualitative effects of overconfidence on the equilibrium contract in the one-principal case thus mirror the results discussed in the competing-principals framework. Subsection 5.1 discusses the welfare effects of agent overconfidence in the one-principal setting; because the principal extracts all the surplus from the agency relationship, these effects are quite different than in the case in which principals compete. Subsection 5.2 discusses the implications of the model in a situation in which the principal has a choice regarding which agent to hire from a pool of agents with different levels of ability and overconfidence. Subsection 5.3 studies the possibility that it is the agent who designs the contract and makes a take-it-or-leave-it offer to the principal.

[6] There is a caveat to this statement. Formally, the implemented level of effort (or the probability that high effort is implemented in equilibrium in the two-action case) unambiguously increases, *ceteris paribus*, with overconfidence of each kind. Given the two kinds of overconfidence, this does not mean that agents who are more overconfident *overall* always exert more effort in equilibrium.

Section 6 extends the one-principal, one-agent framework to allow for a continuum of effort levels that the agent can choose from. The incentive and wager effects of overconfidence carry over to this setting. A consequence of both effects is that, if the problem has an interior solution, the implemented level of effort is (continuously) increasing in each kind of overconfidence. For this reason, in contrast to the discrete-choice case, the power of incentives of the optimal contract might increase with overconfidence about the value of effort even when the agent is only slightly overconfident overall.

Section 7 concludes. Interesting applications of the model could include entrepreneurship and executive compensation, and the results seem to be consistent with recent empirical observations. I also discuss some potentially interesting avenues for further research.

2 Framework

The main assumption of the model that differs from those in conventional moral-hazard models is that principal and agent hold heterogeneous beliefs regarding the distribution of outcomes, and both are aware of this asymmetry. Therefore, principal and agent do not update their beliefs upon play of the game (principal and agent simply "agree to disagree"). Because this assumption is crucial to the results of this paper, I will discuss its validity before moving on to setting up the model.

There are both empirical and methodological reasons for assuming that parties do not fully update their beliefs upon learning the beliefs held by others. This assumption may be very appropriate in a moral-hazard framework; in relation to the agent's ability, arguments like "I know myself better than anybody else" for the agent and "everyone thinks they're better than average" for the principal would allow them both to rationalize not revising their beliefs. Consider, for example, the extreme situation in which the principal judges the agent's ability according to the population mean, knowing that agents tend to be overconfident. If she believes that agents' beliefs are independent of their underlying ability, she will disregard those beliefs as uninformative. In this scenario, the principal's beliefs are independent of the individual agent's true ability, so the agent can also disregard them as uninformative. Principal and agent have nothing to teach each other in terms of the agent's true ability in a one-shot game. Furthermore, the assumption allows me to study the effects of overconfidence on the equilibrium incentive contract, isolating them from any signaling or screening concerns.

Heterogeneous posterior beliefs can also result from differing prior beliefs. Morris (1995) discusses the assumption of heterogeneous priors in the context of economic models, and makes a case for allowing this possibility. An alternative explanation involves errors in processing information. If players update their beliefs in a non-Bayesian way following the observation of a given signal (for example, each participant in a private-information game overestimates the informative value of their own private signal), their posterior beliefs will differ even if all private information is revealed in equilibrium. Eyster and Rabin (2005) explore another channel through which participants can maintain asymmetric posterior beliefs: if players in a private-information game fail to interpret other players' actions as conveyors of private information, asymmetric posterior beliefs will survive even in fully-separating equilibria of the game.

In the presentation of the results, I focus on the case of *agent overconfidence*: the agent holds overly optimistic beliefs, relative to the principal, regarding the probability of success of the project. The propositions, however, accommodate the possibility of a relatively pessimistic agent. Research in the field of psychology suggests that individuals tend to overestimate the probability of favorable events, and that such bias is more pronounced when they have some control over the likelihood of those events. Weinstein (1980) found that students were overly optimistic about the likelihood of good or bad events happening to them relative to same-gender students in their school—such as enjoying their post-graduation job or attempting suicide. He also found that the degree of such "unrealistic optimism" depended, among other things, on a notion of control over the likelihood of a given event. Taylor and Brown (1988) present a review of psychology literature that supports the view that, in general, individuals' assessment of their own abilities, talents, and social skills are overly optimistic. Fiske and Taylor (1991), and Kunda (1999) also discuss the tendency of individuals to be overconfident, referencing both theoretical and empirical studies in psychology.

Researchers in business and economics have also taken notice of the propensity of individuals to be overconfident. Larwood and Whittaker (1977) found company managers to be unrealistically optimistic about the future performance of their firms relative to the competition. Cooper, Woo, and Dunkelberg (1988), in a survey of nearly three thousand entrepreneurs, report that entrepreneurs are notably optimistic about their chances of success when setting up a business. Evidence from experimental economics supports the case for overconfidence as well: Camerer and Lovallo (1999), for example, find that there is excess entry into a hypothetical capacity-constrained market when participants' payoffs after entering depend on skill, but not when they depend on chance. This suggests that agents not only hold overconfident beliefs, but also act on them.[7]

[7] There are theoretical approaches to overconfidence in the economics literature as well. Gervais and Odean (2001) explain overconfidence in a dynamic framework in which agents overweight success and underweight failure when updating their beliefs about their own ability. Bénabou and Tirole (2002) model a self-deception game in which multiple equilibria regarding the level of overconfidence may arise. Goel and Thakor (2002) explore the costs and benefits of overconfidence in a tournament setting.

The other assumptions of the model are in line with the standard treatment of moral hazard. Assume there is a project that can be undertaken by a principal and an agent if they decide to enter a contractual relationship. There are two possible outcomes: the project can succeed or fail. The project yields revenue x_0 if it fails, and revenue $x_1 > x_0$ if it succeeds. The probability of success of the project depends on a non-contractible action e chosen by the agent, which can be interpreted as his choice among effort levels.

The principal's utility is expected revenue from the project net any payments made to the agent (the principal is risk neutral). The agent's utility is separable in money and effort, so that his utility after receiving payment s from the principal and exerting effort level e is

$$u(s) - c(e),$$

where $c(e)$ denotes the disutility to the agent from exerting effort. I assume that $u : \mathbb{R} \to \mathbb{R}$ has full range, and that it is continuous and twice continuously differentiable, with $u' > 0$ and $u'' < 0$ (the agent is risk averse).

As previously noted, principal and agent knowingly hold asymmetric beliefs regarding the probability of success of the project. The principal believes that, conditional on the agent choosing effort level $e \in [0, 1]$, the project will succeed with probability $\Pr(x_1 \mid e) = q + ve$. Let a tilde denote the agent's beliefs: he believes that the conditional probability of success is $\widetilde{\Pr}(x_1 \mid e) = \tilde{q} + \tilde{v}e$. This particular parameterization will prove to be subsequently useful for the analysis, because it highlights the two dimensions (levels and differences) on which the asymmetry in beliefs is relevant in the model. The parameters q, $\tilde{q}$, v, and $\tilde{v}$ are assumed to be positive; the probability of success of the project is perceived by both parties to be increasing in effort. Beliefs are also restricted to $q + v < 1$ and $\tilde{q} + \tilde{v} < 1$.[8]

There are two ways in which the beliefs held by principal and agent can differ. The agent is said to be overconfident about the *base probability of success* if $\tilde{q} > q$. The agent is said to be overconfident about the *value of effort* if $\tilde{v} > v$; he believes that the marginal contribution of his effort to the probability of success is greater than what the principal believes. We will refer to these as different kinds of overconfidence, and say that the agent is overconfident *overall* if $\tilde{q} > q$ and $\tilde{q} + \tilde{v} > q + v$. The possibility of agent underconfidence about the value of effort ($\tilde{v} < v$) is consistent with overall overconfidence and may be relevant according to some views regarding self-enhancing biases. Hoorens (1993) notes that most self-enhancing biases seem to be motivated by a desire to

[8]The assumption that $\tilde{q} + \tilde{v} < 1$ avoids the possibility of a trivial forcing contract—one that infinitely punishes the agent in case of project failure and thus trivially implements effort at first-best cost. Assuming $q + v < 1$ (so that principal and agent agree on the subset of outcomes that occur with probability zero) avoids the possibility that the principal can unboundedly increase the agent's perceived expected utility at no cost to herself.

see oneself as particularly "good" and consequently a perception of superiority (pp. 131–2). A sense of superiority might lead an agent to believe that the probability of success of a project in which he engages is very high, independent of effort level, (a very high $\tilde{q}$) and underestimate the value of his effort ($\tilde{v} < v$). The agent's beliefs about the value of effort affect his perception of the rewards to effort of a given incentive contract. Even though I am partial to interpret the evidence regarding overconfidence as pointing to overconfidence about the value of effort, it is useful to remain open to the possibility of overall overconfidence coupled with underconfidence of this kind.

I will consider both the case of many principals who compete to contract with an agent, and the case of one principal making a take-it-or-leave-it contract offer to an agent. The timing of the game is as follows: the principal(s) first make contract offer(s) to the agent. The agent then decides whether to accept one or reject all offers. If he accepts an offer, he then chooses how much effort to exert. The outcome of the project is then realized, payoffs are distributed according to the contract's terms, and the agency relationship ends. The solution concept used is subgame-perfect Nash equilibrium: at every decision node of the game, the relevant player chooses an optimal response, even if she had expected not to reach that node in equilibrium. I focus on pure-strategy equilibria of the game (in particular, each principal offers a given contract with probability one in equilibrium); this is a substantive assumption in terms of the equilibrium strategy, but does not affect the main message of the model.[9] Without loss of generality, I restrict attention to contract offers of the form $\langle s_1, s_0 \rangle$—a schedule of outcome-contingent payments to the agent—given that project outcome is the only mutually-observable signal in the model.[10]

[9] In the model with one principal, she always offers the optimal contract to the agent. In the case that principals compete to contract with the agent, it will be shown that in equilibrium principals receive zero expected profits. The principals will therefore be indifferent between offering what is characterized as the equilibrium contract, or any other contract that yields zero expected profits (e.g. one that is rejected by the agent). Equilibrium requires, however, that the agent accepts the characterized equilibrium contract with probability 1; if the agent accepted a different contract with positive probability, there would be a profitable deviation for some principal.

[10] See Holmstrom (1979) for a discussion about observability and contracting under moral hazard. Because of the agent's risk aversion, it is in general not optimal to introduce unnecessary "noise" to the payment structure. If signals besides project outcome are observable by both parties, the terms of the equilibrium contract may be contingent on those as well. Under asymmetric beliefs, if there is some signal that the agent believes to be correlated with his effort, the principal can reduce the cost of implementing effort by offering payments that are also contingent on this signal, even if she believes it to be completely uninformative. I assume that outcome is the only signal that principal and agent can contract on.

3 Competing Principals

Consider the case of multiple principals who compete to contract with one agent. This setup is appropriate if agents are scarce in the sense that there are more principals who wish to hire an agent than there are qualified agents. If we are concerned with particularly talented or specialized agents (superstar occupations for example), this model will be more suitable than the standard one-principal, one-agent framework. This model is also useful when considering a situation in which the agent has proprietary rights over the project, rather than the principal. Imagine, for example, a risk-averse entrepreneur deciding whether or not to set up a business. There are potential principals (banks or venture capital funds) willing to bear some of the risk inherent to the enterprise. Establishing an agency relationship in which the principal absorbs some of this risk would be mutually beneficial.

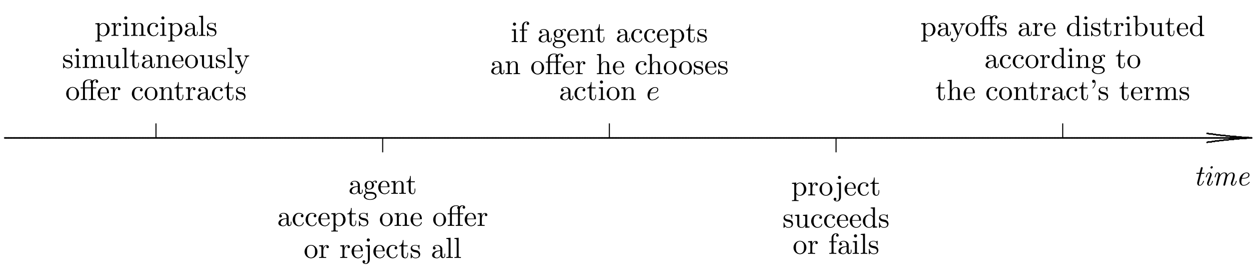

Figure 1: Timing of the model when principals compete

The timing of the model is as follows. First, principals make simultaneous contract offers to the agent. The agent then chooses which offer (if any) to accept. If the agent chooses to accept a contract offer, he chooses some action that affects the outcome distribution of the project. The outcome of the project is realized and observed by both parties. Payoffs are then distributed according to the provisions in the contract, and the agency relationship ends. If the agent chooses not to accept any contract, the project will not be undertaken, and the players receive payoffs according to some outside option. The participants' outside option is their opportunity cost of entering the contractual relationship. The outside option for each principal is not contracting with the agent, which yields zero profits. I assume that the outside option for the agent is low enough so that he always accepts an offer in equilibrium, and thus the equilibrium contract is independent of his outside option.

Assume the agent has two actions to choose from; $e \in \{0, 1\}$.[11] A straightforward way to interpret this two-action space is that the agent can simply choose whether or not to exert effort.

[11] Section 6 presents an extension of the model in which the agent's action choice set is the continuous unit interval.

I normalize the cost of not exerting effort to zero so that $c(0) = 0$ and $c(1) = c$.

Principals and agent evaluate any given contract according to their own beliefs. Assume that competing principals share the same beliefs about the probability of success of the project; I discuss the possibility of different principals holding different beliefs in Subsection 4.2 below. Each principal wishes to maximize her expected profits. Expected profits for the principal whose contract offer $\langle s_1, s_0 \rangle$ is accepted by the agent in equilibrium, conditional on each of the agent's possible effort levels, are:

$$\begin{aligned}\mathbb{E}[\pi \mid e = 1] &= (q + v)(x_1 - s_1) + [1 - (q + v)](x_0 - s_0) \\ \mathbb{E}[\pi \mid e = 0] &= q(x_1 - s_1) + [1 - q](x_0 - s_0).\end{aligned}$$

The agent's objective is to maximize his expected utility when choosing which contract offer to accept and how much effort to exert once he engages the project. After accepting a given contract offer $\langle s_1, s_0 \rangle$, the agent's expected utility conditional on his choice of effort is:

$$\begin{aligned}\tilde{\mathbb{E}}[u(s_x) \mid e = 1] - c &= (\tilde{q} + \tilde{v})\, u(s_1) + [1 - (\tilde{q} + \tilde{v})]\, u(s_0) - c \\ \tilde{\mathbb{E}}[u(s_x) \mid e = 0] &= \tilde{q} u(s_1) + [1 - \tilde{q}]\, u(s_0).\end{aligned}$$

We can now turn to characterizing the equilibrium contract. After accepting the contract offer that the agent finds most attractive, he chooses whichever action he believes will yield him higher expected utility given the terms of the contract. The competing principals take this into account when designing their offers. In particular, if a principal wishes to induce effort, the contract must be "incentive compatible"—the contract terms must be such that, if the agent accepts it, he finds it in his best interest to exert effort:

$$(\tilde{q} + \tilde{v})\, u(s_1) + [1 - (\tilde{q} + \tilde{v})]\, u(s_0) - c \geq \tilde{q} u(s_1) + [1 - \tilde{q}]\, u(s_0).$$

We can rewrite the incentive-compatibility constraint above as

$$\tilde{v}\,(u(s_1) - u(s_0)) \geq c. \tag{IC}$$

Intuitively, the perceived expected utility gain for the agent from exerting effort (receiving excess utility $(u(s_1) - u(s_0))$ with additional probability $\tilde{v}$), must be no less than his disutility from exerting effort. I will refer to the differential $u(s_1) - u(s_0)$ as the contract's *power of incentives*. Note that the power of incentives necessary to induce effort is decreasing in $\tilde{v}$. It is, however, independent of $\tilde{q}$: the agent's beliefs regarding the base probability of success does not affect his perception of the rewards to effort of a given incentive scheme.

An equilibrium contract is such that no other contract can *(i)* attract the agent by offering him terms that he strictly prefers and *(ii)* yield higher expected profits for the offering principal (i.e. there is no profitable deviation for any principal from an equilibrium contract). Because principals compete in offering contracts to the agent and they all evaluate profits based on the same beliefs, expected profits for the principal whose offer is accepted by the agent in equilibrium must be zero (equal to the principals' outside option).

Lemma 1 *If principals share the same beliefs regarding outcome distribution conditional on the agent's actions, in equilibrium expected profits will be zero for all principals according to their beliefs.*

All formal proofs are relegated to the appendix. Intuitively, if a principal made positive expected profits, another principal could outbid that contract offer—provide a slightly higher expected payment to the agent—without affecting incentives, thus attracting the agent and earning positive expected profits. When principals do not share the same beliefs, this zero-expected-profits condition will no longer hold, but the intuition behind the results carries over to such a setting. We relax the assumption that principals share the same beliefs in Subsection 4.2.

The equilibrium contract depends crucially on the effort level that is implemented in equilibrium. I will, in turn, characterize the equilibrium contract assuming that effort is not implemented and assuming that effort is implemented, and subsequently analyze the effect of overconfidence on the level of effort actually implemented in equilibrium.

Assume first that effort is not implemented in equilibrium. If principals and agent held identical beliefs, the risk-neutral principal would absorb all the risk from the project, and offer a fixed payment to the agent (i.e. independent of project outcome). If the agent is overconfident about the base probability of success, however, he will be exposed to risk in equilibrium.

Proposition 1 *Assuming effort is not implemented in equilibrium, the only equilibrium contract* $\langle s_{1*}, s_{0*} \rangle$ *is characterized by the conditions*

$$\frac{\tilde{q}}{1-\tilde{q}} \frac{u'(s_{1*})}{u'(s_{0*})} = \frac{q}{1-q}$$

and $q(x_1 - s_{1*}) + [1-q](x_0 - s_{0*}) = 0$. *The agent bears risk in equilibrium if* $\tilde{q} \neq q$.

Consider the case of agent overconfidence about the base probability of success, in which $\tilde{q} > q$. The intuition of Proposition 1 is that an overconfident agent is willing to wager on success against the (relatively pessimistic) principal. Starting from a riskless contract (one that specifies $s_1 = s_0$), because the marginal cost for the agent from bearing additional risk is zero at that point, principal and agent evaluate marginal changes in payments based on their effect only in terms of expected payment. Consider, then, an increase in the success-contingent payment, coupled with a decrease in the failure-contingent payment, that leaves expected payment unchanged according to the principals' beliefs. The agent is relatively optimistic about receiving the success-contingent payment, so *according to his beliefs* such deviation yields a higher expected payment. When $\tilde{q} > q$, there is a first-order gain perceived by the agent from such higher expected payment, and only a second-order loss from higher risk exposure, compared to a riskless contract. Therefore, an agent who is overconfident about the base probability of success bears risk in equilibrium.

Because of the disagreement between principal and agent regarding the probability of success of the project, the agent is willing to be exposed to more risk in equilibrium than a "realistic" agent. This ***wager effect*** pushes the equilibrium contract towards higher-powered incentives. Note that, absent moral-hazard concerns, the equilibrium contract allows for Pareto-optimal risk sharing. In the identical-beliefs case, this implies that the risk-neutral principal will absorb all of the risk. In the heterogeneous-beliefs case, it implies that the agent bears risk in proportion to the disagreement in beliefs.[12]

If the contract characterized in Proposition 1 satisfies the incentive-compatibility constraint (IC), it must be the case that the agent exerts effort in equilibrium. Whenever $\frac{\tilde{q}}{1-\tilde{q}}\frac{u'(s_1)}{u'(s_0)} > \frac{q}{1-q}$ and $\langle s_1, s_0 \rangle$ does not implement effort, $\langle s_1, s_0 \rangle$ cannot be an equilibrium contract. A principal could deviate and offer a higher-powered incentive contract that will both attract the agent and yield positive expected profits. Not implementing effort might thus be infeasible under heterogeneous beliefs.

Assume now that effort is implemented in equilibrium. If principals and agent held identical beliefs, the equilibrium contract offer would be characterized by zero expected profits for the offering principal and the binding incentive-compatibility constraint (IC). There is a tradeoff between incentives and insurance: if not for the incentive-provision problem, efficiency gains would result from providing more insurance to the agent (i.e. reducing the power of incentives). Implementing effort requires that the agent be exposed to a discrete amount of risk. The efficiency loss that arises,

[12] Another way to frame the wager effect is from the viewpoint of asset trading under uncertainty. Principal and agent have the opportunity to trade payments in the "success" and "failure" states of the world. The principal, being risk neutral, is willing to trade infinitely at what she believes to be the actuarially fair price of these securities. Principal and agent evaluate each trade differently. In particular, a trade of higher payment to the agent in the "success" state coupled with a lower payment in the "failure" state that the principal judges to be actuarially fair is regarded as better than actuarially fair by the agent. Because of his risk aversion, the agent is not willing to trade infinitely at that price—only as long as the gains from a higher perceived expected payment compensate him for the additional risk he bears. Adrian and Westerfield (2005) uncover the wager effect in a continuous-time agency model.

given the agent's risk aversion, is referred to as the *cost of agency*; if the agency relationship was not necessary, this cost would be avoided (for instance, if the risk-neutral principal could undertake the project on her own and carry out the agent's task).

Given this incentive-insurance tradeoff, the contract analogous to the identical-beliefs equilibrium contract is a natural candidate for a potential equilibrium contract when we allow for heterogeneous beliefs.

Definition 1 *Let $\langle \bar{s}_1, \bar{s}_0 \rangle$ denote the contract that satisfies (IC) with equality and yields zero expected profits according to the principals' beliefs:*

$$\tilde{v}\left(u\left(\bar{s}_1\right) - u\left(\bar{s}_0\right)\right) = c$$
$$\left(q+v\right)\left(x_1 - \bar{s}_1\right) + \left[1 - \left(q+v\right)\right]\left(x_0 - \bar{s}_0\right) = 0.$$

This contract will in fact be the equilibrium contract when the beliefs held by the agent differ only slightly from the principals' beliefs. In other words, if the agent is only slightly overconfident overall, the contract with incentives just powerful enough to implement effort will be the equilibrium contract. The intuition from the identical-beliefs setting carries over to this case: a principal cannot provide more insurance to the agent without destroying the incentives for the agent to exert effort.

Proposition 2 *Assuming effort is implemented in equilibrium, $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the only equilibrium contract if $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})}\frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} \leq \frac{q+v}{1-(q+v)}$.*

Given that $\bar{s}_1 > \bar{s}_0$, the condition $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})}\frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} \leq \frac{q+v}{1-(q+v)}$ holds for some values $\tilde{q}+\tilde{v} > q+v$. If this is the case, we will say that the agent is only "slightly overconfident overall," since $\tilde{q}+\tilde{v} \ngtr\!\!\!\ngtr q+v$. There is no profitable deviation from $\langle \bar{s}_1, \bar{s}_0 \rangle$. To see why, note that providing more insurance to the agent (by decreasing s_1 and increasing s_0) would destroy the incentives for the agent to exert effort, increasing both payments would decrease expected profits, and a contract with lower payments would not attract the agent away from $\langle \bar{s}_1, \bar{s}_0 \rangle$. Consider then an increase in s_1 coupled with a decrease in s_0, so that effort is still implemented and expected profits do not decrease. Providing less insurance to the agent would never be profitable in the identical-beliefs framework, but such deviation could seem attractive to an overconfident agent because of the wager effect. The condition in Proposition 2 above illustrates how each party evaluates marginal changes in payments at $\langle \bar{s}_1, \bar{s}_0 \rangle$. The agent perceives that a marginal increase in s_1 increases his utility by $u'\left(\bar{s}_1\right)$ with probability $\left(\tilde{q}+\tilde{v}\right)$, and that a marginal decrease in s_0 decreases his utility by $u'\left(\bar{s}_0\right)$ with probability $\left[1-\left(\tilde{q}+\tilde{v}\right)\right]$. The principal believes, on the other hand, that she will pay the marginal increase in s_1 with probability $\left(q+v\right)$ and save the marginal decrease in s_0 with probability

$[1-(q+v)]$. If the inequality holds, a principal cannot draw the agent away from $\langle \bar{s}_1, \bar{s}_0 \rangle$ and increase expected profits by providing less insurance. Intuitively, it is too costly for the principal to compensate the agent for bearing more risk. Because marginal utility is assumed to be decreasing, the fact that there is no profitable marginal deviation from $\langle \bar{s}_1, \bar{s}_0 \rangle$ implies that there is no profitable discrete deviation from $\langle \bar{s}_1, \bar{s}_0 \rangle$ either.

When the condition in Proposition 2 holds, overconfidence about the value of effort ($\tilde{v} > v$) allows a principal to provide more insurance to an overconfident agent without destroying incentives compared to a "realistic" agent. This is the ***incentive effect*** of overconfidence. Any contract that implements effort exposes the agent to a discrete amount of risk, so as to give him sufficient incentives to exert effort. Even though the wager effect implies that an overconfident agent is willing to bear some risk when contracting with a principal, this amount of risk is continuous in the degree of disagreement in beliefs. If the agent is only slightly overconfident overall, the amount of risk required by incentive provision is greater than the amount of risk he would willingly bear as a consequence of the wager effect. The incentive effect therefore dominates the wager effect in this case, and the power of incentives of the equilibrium contract depends solely on the agent's beliefs about the value of effort.

When the agent is only slightly overconfident overall, the incentive-insurance tradeoff present in the case of identical beliefs remains. If the agent is overconfident about the value of effort, lower-powered incentives are sufficient to implement effort than if he held "realistic" beliefs; a principal can then offer a contract that provides more insurance (thus reducing the cost of agency) without destroying incentives. If, however, the agent is underconfident about the value of effort, the incentive effect implies that higher-powered incentives will be necessary to implement effort. Applying Lemma 1, we can say that $\frac{d\bar{s}_1}{d\tilde{v}} < 0$ and $\frac{d\bar{s}_0}{d\tilde{v}} > 0$ when the agent is only slightly overconfident overall. Therefore, **the power of incentives of the equilibrium contract decreases in overconfidence about the value of effort when the agent is only slightly overconfident overall**. The agent's beliefs regarding the base probability of success ($\tilde{q}$) do not affect the equilibrium contract in this case. The principals' beliefs do not affect the power of incentives of the equilibrium contract either, but they do affect the agent's expected payment. If the principals are optimistic and judge the probability of success to be high, they believe the project's expected revenue is high, and can offer a higher expected payment to the agent accordingly.

The degree of asymmetry in beliefs held by principals and agent define whether or not the agent is only slightly overconfident overall. As the next proposition shows, if the agent is instead significantly overconfident overall, the equilibrium contract exhibits excessively powerful incentives. Because of the wager effect, a very overconfident agent substantially overestimates the probability of success, so he prefers a contract that rewards him handsomely for success and punishes him harshly for failure over the $\langle \bar{s}_1, \bar{s}_0 \rangle$ contract (which provides as much insurance as possible while implementing effort). He judges the higher expected payment from an excessively risky contract as sufficient to compensate him for the cost of bearing more risk.

Proposition 3 *Assuming effort is implemented in equilibrium, if* $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} > \frac{q+v}{1-(q+v)}$ *then* $\langle \bar{s}_1, \bar{s}_0 \rangle$ *is not an equilibrium contract. The only equilibrium contract* $\langle s_1^*, s_0^* \rangle$ *is characterized by*

$$\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(s_1^*)}{u'(s_0^*)} = \frac{q+v}{1-(q+v)}$$

and $(q+v)(x_1 - s_1^*) + [1-(q+v)](x_0 - s_0^*) = 0$. *The equilibrium contract has higher-powered incentives than necessary to implement effort.*

If $\tilde{q}+\tilde{v} \gg q+v$, so that $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} > \frac{q+v}{1-(q+v)}$, we will say that the agent is "significantly overconfident overall." If this is the case, he is actually content to bear more risk in equilibrium than he would under contract $\langle \bar{s}_1, \bar{s}_0 \rangle$. Parallel to the analysis in the case of slight overconfidence, consider a deviation from $\langle \bar{s}_1, \bar{s}_0 \rangle$ towards less insurance: a marginal increase in s_1 together with a marginal decrease in s_0. Recall that the agent perceives such a change as an increase in utility of $u'(\bar{s}_1)$ with probability $(\tilde{q}+\tilde{v})$ and a decrease in utility of $u'(\bar{s}_0)$ with probability $[1-(\tilde{q}+\tilde{v})]$. Because the agent considerably overestimates the likelihood of receiving the success-contingent payment, a principal can offer less insurance to the agent and attract the agent away from $\langle \bar{s}_1, \bar{s}_0 \rangle$, while increasing her expected profits. Since $\langle s_1^*, s_0^* \rangle$ maximizes the agent's perceived expected utility subject to the zero-expected-profits condition, there is no profitable deviation from this contract. Because of the wager effect, if the agent is significantly overconfident overall, he judges the $\langle s_1^*, s_0^* \rangle$ contract to yield a significantly higher expected payment than the $\langle \bar{s}_1, \bar{s}_0 \rangle$ contract—so much higher that it more than compensates him for the excessive amount of risk he bears.

When the agent is significantly overconfident overall, the agent's bias in evaluating payments overshadows the incentive-insurance tradeoff present in the identical-beliefs case. In that setting, the principal is prevented from providing more insurance to the agent because of the need to provide appropriate incentives. When the agent is significantly overconfident overall, there is no such tradeoff in equilibrium. The agent's relative bias then overrides his risk aversion when evaluating payoffs, and thus incentive provision becomes secondary to the difference in how principal and agent evaluate outcome-contingent payments. As in the case in which no effort is implemented, the equilibrium contract allows for Pareto-optimal risk sharing; the amount of risk the agent bears as a consequence of the heterogeneity in beliefs is more than enough to provide incentives.

In contrast to the effect of overconfidence about the value of effort on the power of incentives when the agent is slightly overconfident, now $\frac{ds_1^*}{d\tilde{v}} > 0$ and $\frac{ds_0^*}{d\tilde{v}} < 0$. Likewise, $\frac{ds_1^*}{d\tilde{q}} > 0$ and

$\frac{ds_0^*}{d\tilde{q}} < 0$. Because it is the wager effect of overconfidence that dominates in this case, **the power of incentives of the equilibrium contract increases in overconfidence of either kind when the agent is significantly overconfident overall**. The equilibrium contract exhibits excessively powerful incentives—more powerful than necessary to induce effort—independent of the composition of agent overconfidence. As before, we would expect more-optimistic principals to offer a higher expected payment to the agent. Given that in this case it is the degree of divergence in beliefs that drives contract design, however, the power of incentives is decreasing in both q and v: the divergence is smaller when principals are more optimistic.

Whether or not effort is implemented in equilibrium depends on which of the potential equilibrium contracts identified above gives the agent higher perceived expected utility (recall that principals always receive zero expected profits in equilibrium). The incentive effect reduces the cost of implementing effort, and a higher degree of overconfidence of either kind makes the potential equilibrium contract that implements effort relatively more attractive to the agent. Therefore, higher levels of overconfidence of either kind increase the likelihood that effort is implemented in equilibrium.

Proposition 4 *Ceteris paribus, higher levels of overconfidence of either kind increase the likelihood that effort is implemented in equilibrium: if effort is implemented given agent beliefs* $(\tilde{q}, \tilde{v})$*, then effort will be implemented when his beliefs are* $(\tilde{q}, \tilde{v}^*)$ *for any* $\tilde{v}^* \geq \tilde{v}$ *or* $(\tilde{q}^*, \tilde{v})$ *for any* $\tilde{q}^* \geq \tilde{q}$*.*

For different reasons, the potential equilibrium contract that implements effort becomes relatively more attractive to the agent as overconfidence of either kind increases, whether it is the incentive or the wager effect of overconfidence that dominates. Recall that when the incentive effect dominates, higher overconfidence about the value of effort reduces the cost of implementing effort. The agent reaps the benefits of this efficiency gain when principals compete. Higher overconfidence about the base probability of success increases the agent's perceived expected utility under the contract that *does not* implement effort; however, because he receives a higher success-contingent payment under the contract that *does* implement effort, there is a greater increase in his perceived expected utility under this contract. For this same reason, higher overconfidence of either kind makes the contract that implements effort comparatively more attractive to the agent when the wager effect dominates.[13]

[13] Note that Proposition 4 does not imply that effort is more likely to be implemented, in general, when dealing with an overall more overconfident agent. Imagine, for example, a case in which effort would be implemented if the agent held "realistic" beliefs. Suppose that the agent is overall slightly overconfident, but underconfident about the value of effort. Because higher-powered incentives are then necessary to implement effort, the increase in the cost of agency reduces the likelihood that effort is implemented in equilibrium.

4 Welfare Analysis

This section studies the welfare effects of agent overconfidence. Principals and agent hold heterogeneous beliefs, so at least one of them must be incorrect. The effects of this deviation from a strong interpretation of rationality on the participants' well being are not immediately clear. For expositional purposes, Subsection 4.1 analyzes the welfare effects of overconfidence in which all principals share the same beliefs, as was assumed in the previous section. Subsection 4.2 extends the analysis to allow for principals holding differing beliefs.

4.1 Principals Sharing the Same Beliefs

An overconfident agent will have a biased outlook on his expected utility; in what follows, I evaluate the agent's *actual expected utility* based on the true probabilities of success and failure. I believe that this is a good initial measure of the agent's *ex-ante* well being. If the agent cares solely about the actual payments that he receives (in addition to his cost of effort), this is the appropriate measure—it is the actual expected value of his future utility. Further considerations about factors that influence individuals' well being have conflicting implications in terms of welfare analysis. If the agent derives utility from anticipating how richly he will be rewarded once the project succeeds, along the lines of Kőszegi (2005), an overconfident agent will enjoy higher utility than what I calculate as his actual expected utility. On the other hand, once payoffs are realized, the agent will be disappointed whenever he does not receive the success-contingent payment he so confidently anticipates. An agent could evaluate receiving the low failure-contingent payment not only for its own worth, but also as a loss relative to his "reference point" as introduced by Kőszegi and Rabin (2005). In this case, the agent would be worse off than what I calculate. One could also imagine a concept of welfare consistent with individual sovereignty: we could calculate each participant's expected utility based on their subjective beliefs. Such analysis, however, ignores the fact that individuals do care about utility derived from actual consumption made possible by income, and not just their expectations about future utility.

Assume, for now, that the principals hold accurate beliefs. This assumption is convenient because the zero-expected-profits condition implies that social welfare depends solely on the agent's well being and that the agent's beliefs only affect the power of incentives of the equilibrium contract—not the actual expected payment the agent receives. We will relax this assumption in short. When principals compete, the agent receives expected payment equal to the project's expected revenue. Because, given some implemented level of effort, the agent's expected payment is independent of the terms of the equilibrium contract, his actual expected utility depends exclusively on the implemented level of effort and the amount of risk he bears in equilibrium.

Keeping with the structure of the previous section, I first identify the welfare effects of overconfidence assuming that effort is not implemented in equilibrium, and that changes in overconfidence do not affect the implemented level of effort. In that case, zero expected profits and the assumption that principals hold accurate beliefs imply that actual expected payment to the agent is the project's expected revenue:

$$qx_1 + (1 - q)\, x_0.$$

As shown in Proposition 1, as a consequence of the wager effect, an agent who is overconfident about the base probability of success always bears risk in equilibrium, so any level of overconfidence of this kind harms the risk-averse agent.

Assume now that effort is implemented in equilibrium, and that changes in overconfidence do not affect the implemented level of effort. Actual expected payment to the agent is then

$$(q + v)\, x_1 + [1 - (q + v)]\, x_0.$$

When the agent is only slightly overconfident overall, the equilibrium contract is characterized by Proposition 2. If this is the case, the power of incentives depends solely on the agent's beliefs regarding the value of effort. Due to the incentive effect, an agent who is overconfident about the value of effort is exposed to less risk in equilibrium than he would be if he held realistic beliefs because lower-powered incentives are sufficient to induce effort. Thus, the agent's well being *increases* with overconfidence regarding the value of effort. Slight overconfidence about the value of effort is therefore beneficial to the agent: he benefits from the efficiency gain of a lower cost of agency. The agent's beliefs about the base probability of success do not affect the equilibrium contract if he remains only slightly overconfident overall.

When the agent is significantly overconfident overall, the equilibrium contract is characterized by Proposition 3. As a consequence of the wager effect, the amount of risk that the agent is exposed to increases in overconfidence of either kind. Thus, when the agent is significantly overconfident, his well being *decreases* with overall overconfidence. Higher overconfidence increases the agent's exposure to what is already an excessive amount of risk.

As shown by Proposition 4, another effect of higher overconfidence of either kind is that it makes effort more likely to be implemented. If higher agent overconfidence drives effort to be implemented in equilibrium, the actual expected payment to the agent increases (by as much as expected revenue does) because effort exertion increases the probability of success of the project. A marginal increase in overconfidence that drives effort to be implemented might, therefore, benefit the agent. This will only be the case, however, if the increase in actual expected payment compensates the agent for the disutility of exerting effort (effort must therefore be the first-best action, so that $v(x_1 - x_0) > c$) and the additional risk he bears.

When principals compete to contract with the agent, some overconfidence about the value of effort benefits the agent because lower-powered incentives are then sufficient to induce effort. A risk-neutral principal can provide more insurance without destroying incentives. In contrast, if the agent is underconfident about the value of effort or significantly overconfident overall, he bears an excessive amount of risk (which is costly to him).

Allowing for principals to hold inaccurate beliefs, the effects of overconfidence outlined above are reinforced if principals are overly optimistic about the probability of success of the project. When this is the case, higher-powered incentive contacts that yield zero expected profits according to the principals' beliefs yield lower actual expected payment to the agent. As the agent holds higher stakes in the project, his actual expected payment decreases from the optimistic principals' estimate of expected revenue towards actual expected revenue.[14]. The agent's welfare therefore decreases as the power of incentives of the equilibrium contract increases, even more sharply than in the case in which principals hold accurate beliefs. The effects of agent overconfidence on his well being are therefore reinforced when principals are overly optimistic: the agent benefits from moderate overconfidence about the value of effort because he bears less risk *and* he receives a higher actual expected payment in equilibrium. Underconfidence about the value of effort or significant overall overconfidence harm the agent both because he is exposed to a higher amount of risk and because he receives a lower actual expected payment in equilibrium.

If the principals are overly pessimistic relative to the true probability of success, the effects of agent overconfidence on his well being are ambiguous. In this case, when expected profits are zero according to the principals' beliefs, higher-powered incentive contracts yield higher actual expected payment to the agent. As the agent holds higher stakes in the project, his actual expected payment now increases from the overly pessimistic principals' estimate of expected revenue towards actual expected revenue. Changes in overconfidence that result in lower power of incentives still benefit the agent by shielding him from risk, but provide him with a lower actual expected payment. Conversely, changes in overconfidence that result in higher power of incentives expose the agent to more risk but also provide higher actual expected payment to him. For instance, a significantly overconfident agent may actually benefit from his overconfidence when contracting with an overly pessimistic principal, particularly if his beliefs are close to the true outcome distribution and he

[14] This is true as long as $s_1 < x_1$ and $s_0 > x_0$. I assume that this is the case for the remainder of the paper, but briefly discuss the alternative in Subsection 4.2 below. Common sense, if not legal provisions, should prevent the agent from signing a contract in which he takes on more risk than the intrinsic risk of the project.

has high risk tolerance.

4.2 Principals With Differing Beliefs

I derived the results of the model assuming that all competing principals hold identical beliefs. This simplified the analysis because competition then resembles Bertrand competition, so that expected profits are driven to zero when as few as two principals compete. Given that I am allowing for principal and agent to hold heterogeneous beliefs, this assumption seems particularly strong. If we allow for principals to knowingly hold disagreeing beliefs regarding the probability distribution of outcomes, two main complications arise.[15]

First, so long as the number of competing principals is finite, expected profits according to the beliefs of the principal whose offer is accepted in equilibrium will generically be positive. The most optimistic principal needs only to ensure that her contract offer yields zero expected profits according to the beliefs of the second-most-optimistic principal. The second-most-optimistic principal is not willing to outbid some offers that yield positive expected profits to the most optimistic principal if they hold heterogeneous beliefs.[16] When allowing for principals to hold disagreeing beliefs, it is generally in each principal's best interest to hold accurate beliefs, which will allow her to design her contract offer optimally. If the second-most-optimistic principal overestimates the true probability of success, the most optimistic principal (whose contract is accepted in equilibrium) will suffer losses in expectation. Optimistic bias will therefore tend to harm the principal. An overly pessimistic principal, on the other hand, will tend to be outbid by more optimistic principals. If all principals are pessimistic relative to the actual probability of success, any principal whose offer is rejected in equilibrium would benefit from correctly updating her beliefs; she could then attract the agent and earn positive expected profits.

The second complication is pinpointing the offer which is accepted by the agent in equilibrium. If we assume monotonic bidding strategies by the principals, then the offer made by the most

[15] When the principals hold heterogeneous beliefs, it would be optimal for them to set up a secondary side-betting market on project outcome (if it is publicly observable). Because of risk neutrality, this side-betting would be unbounded—and so would expected profits from each principal's point of view. We assume that such a side-betting market is infeasible, so that principals behave optimally in the contract-offer-design stage.

[16] Note that the result of generically-positive perceived expected profits for the principal whose offer is accepted in equilibrium does not depend on the assumption that principals are aware of each others' beliefs. Dropping this assumption (if principals know only the agent's and their own beliefs), competition would then resemble a first-price sealed-bid auction. If there are finitely many principals, each of them knows that some contract offers which yield strictly positive expected profits will be accepted by the agent with positive probability. Each principal will therefore offer a contract to the agent that, conditional on being accepted, gives the principal strictly positive perceived expected profits according to her own beliefs.

optimistic principal will tend to be the one accepted in equilibrium. There is, however, an extreme case in which the offer made by the most *pessimistic* principal could be the one accepted by the agent: if the agent is fairly risk neutral and very overconfident relative to the most pessimistic principal, the $\langle s_1^*, s_0^* \rangle$ contract as characterized in Proposition 3 could be such that $s_1^* > x_1$ and $s_0^* < x_0$—the principal bets on project failure.[17] If this is the case, the principal earns higher profits when the project fails than when it succeeds. The principal would then have incentives to sabotage the project if possible, and the agent should be wary of accepting such a contract offer. This extreme seems unrealistic, for the same reason that contracts with payments that are non-monotonic in the principal's objective variable (e.g. output) seem unrealistic: one of the participants would then have incentives to destroy output.

5 One Principal and One Agent

Consider now the setting that tends to be discussed more often in agency literature, in which one principal can make a take-it-or-leave-it contract offer to one agent. Owing to her bargaining power, the principal extracts all the surplus from the agency relationship. This framework is more appropriate than the competing-principals setting when the pool of potential agents is large relative to the number of principals. The case of salespeople in retail could be an example of such a situation.

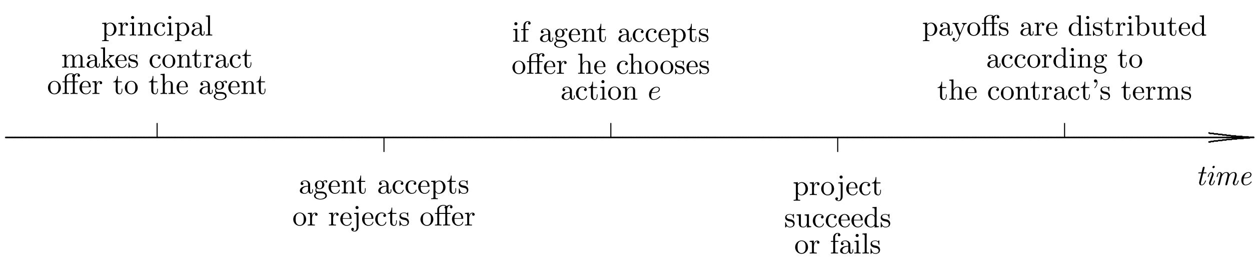

Figure 2: Timing of the model with one principal and one agent

The timing of the model is as follows. First, the principal makes a take-it-or-leave-it contract offer to the agent. The agent can accept or reject the offer. If he accepts it, he chooses whichever action maximizes his perceived expected utility given the terms of the contract. The outcome of the project is then realized, payoffs are distributed according to the terms of the contract, and the agency relationship ends. If the agent rejects the principal's offer, both will receive utility according

[17] This case may be so extreme that holding excessively pessimistic beliefs could be better than holding accurate beliefs for a principal: she then "wins" and earns the chance to bilk the agent for profit, whereas the offer made by a principal holding accurate beliefs would be rejected.

to their outside option. I assume that the agent's outside option is exogenous and independent of his overconfidence.[18] The principal's outside option does not affect the equilibrium contract as long as the agency relationship yields sufficient surplus for her to engage in it; I assume this to be the case.

Let $\underline{u}$ denote the agent's perceived expected utility from his outside option. The principal's contract offer, if it is to be accepted by the agent, must provide him perceived expected utility no lower than $\underline{u}$. This participation, or "individual rationality"—IR—constraint restricts the possible optimal contract offers to those that satisfy

$$(\tilde{q} + \tilde{v}e)\,u\,(s_1) + [1 - (\tilde{q} + \tilde{v}e)]\,u\,(s_0) - c\,(e) \geq \underline{u}, \tag{IR}$$

where e is the action (freely chosen by the agent) that the principal wishes to implement. The incentive compatibility constraint that the contract must satisfy if the principal wishes to implement effort remains unchanged:

$$\tilde{v}\,(u\,(s_1) - u\,(s_0)) \geq c. \tag{IC}$$

The principal's objective when designing the optimal contract offer is to maximize expected profits, taking into account the relevant constraints:

$$\max_{e,s_1,s_0}\,(q + ve)\,(x_1 - s_1) + [1 - (q + ve)]\,(x_0 - s_0)$$

subject to

$$\begin{aligned} \tilde{v}\,(u\,(s_1) - u\,(s_0)) &\geq c \text{ if } e = 1, \\ \tilde{v}\,(u\,(s_1) - u\,(s_0)) &\leq c \text{ if } e = 0, \text{ and} \\ (\tilde{q} + \tilde{v}e)\,u\,(s_1) + [1 - (\tilde{q} + \tilde{v}e)]\,u\,(s_0) - c\,(e) &\geq \underline{u}. \end{aligned}$$

Note that the problem of a principal who competes with others to contract with the agent, studied in Section 3, can be reinterpreted as follows: she wishes to maximize expected profits, taking into account that the agent chooses e optimally and that he will accept the offer only if it is better than the next-best contract offer. This maximization problem can be written as:

$$\max_{e,s_1,s_0}\,(q + ve)\,(x_1 - s_1) + [1 - (q + ve)]\,(x_0 - s_0)$$

[18] This assumption allows me to isolate the effect that overconfidence, only regarding the probability of success of the project, has on the equilibrium contract. If the agent was also overconfident about his outside option, he would demand a higher perceived expected utility in order to accept any given offer by the principal.

subject to

$$\tilde{v}\,(u\,(s_1) - u\,(s_0)) \geq c \text{ if } e = 1$$

$$\tilde{v}\,(u\,(s_1) - u\,(s_0)) \leq c \text{ if } e = 0$$

$$(\tilde{q} + \tilde{v}e)\,u\,(s_1) + [1 - (\tilde{q} + \tilde{v}e)]\,u\,(s_0) - c\,(e) \geq U(\text{next-best offer}),$$

where U(next-best offer) stands for the perceived expected utility that the agent would receive by accepting the best alternative offer (and choosing his optimal effort level accordingly).

The only difference between these two problems is that the agent's participation constraint is exogenous when one principal makes a take-it-or-leave-it offer, and endogenously generated by competing offers in the case of competing principals. The results regarding the equilibrium contract (Propositions 1–4) remain, except that the expected payment to the agent is determined by his exogenous participation constraint (IR), rather than the zero-expected-profits condition. Note in particular that, given a set of parameters of the model, the effort level implemented in equilibrium is independent of market structure. Intuitively, there is no profitable deviation from the equilibrium contract in the competing-principals setting—there is no other contract that attracts the agent (gives him higher perceived expected utility) and yields higher expected profits for the offering principal. The optimal contract when one principal makes a take-it-or-leave-it offer is such that no other contract retains the agent (gives him at least as much perceived expected utility as his outside option) and yields higher expected profits for the principal.

Because of the duality of this problem, the effects of overconfidence on the equilibrium contract carry over from the competing-principals model accordingly. If the agent is only slightly overconfident overall, the incentive effect dominates: if the agent is overconfident about the value of effort, then lower-powered incentives are sufficient to induce effort, while if he is underconfident on this dimension, higher-powered incentives are necessary to implement effort. If the agent is significantly overconfident overall, the wager effect dominates: the optimal contract exhibits excessively powerful incentives that increase with agent overconfidence on either dimension. Finally, *ceteris paribus*, increases in overconfidence of either kind make it more likely that effort is implemented under the optimal contract. The optimal contract is derived explicitly in Subsection A.2 of the Appendix.

5.1 Welfare Analysis

Because the principal is able to extract all the surplus from the agency relationship, the welfare effects of overconfidence differ from those in the competing-principals setting.[19] In equilibrium,

[19] In an independent study, Santos-Pinto (2006) analyzes the effects of agent overconfidence on firm profits in a tournament setting. Our results in terms of welfare are consistent.

she offers the agent perceived expected utility only as high as the utility he would derive from his outside option. As before, first assume that the principal's beliefs are accurate.

Recall that the cost of agency is reduced when the agent is overconfident about the value of effort. The principal benefits from this efficiency gain because she captures all the surplus from the relationship; it reduces the cost of satisfying the incentive-compatibility constraint. Furthermore, because an overconfident agent is overly optimistic about the probability of receiving the success-contingent payment, overall agent overconfidence reduces the principal's cost of satisfying his participation constraint. For this reason, even slight overconfidence about the value of effort hurts the agent.[20] Agent underconfidence about the value of effort increases the cost of agency. Besides hurting the agent, who is exposed to more risk, it might reduce the principal's expected profits if the increase in the cost of agency is greater than the savings in terms of an easier-to-satisfy participation constraint.

When the agent is significantly overconfident overall, he bears an excessive amount of risk in equilibrium. This, coupled with a lower actual expected payment (consequence of overestimating the probability of receiving the success-contingent payment), implies that his actual expected utility quickly decreases with overconfidence in this range. The principal's expected profits increase with agent overconfidence of either kind in this range, because it is cheaper for her in expectation to provide the agent a perceived expected utility comparable to his outside option—whether or not effort is implemented in equilibrium.

5.2 Choosing From a Pool of Agents

Given that agent beliefs affect the principal's expected profit—even if we hold actual agent productivity constant—the question regarding whether the principal prefers to contract with a more- or less-overconfident agent follows naturally. Consider the problem that the principal faces when choosing an agent from a pool of applicants with different levels of overconfidence and true ability. This question is particularly relevant when the pool of agents is large relative to the number of principals willing to hire an agent, our main motivation for discussing the one-principal setting.

All else equal, the principal will prefer a more overconfident agent. Imagine a situation in which many potential agents share the same underlying characteristics, but differ in their self-confidence—e.g. a group of individuals who pass several aptitude tests. The principal will choose the most overconfident agent, because it is cheapest for her to satisfy his participation constraint.[21]

[20] The assumption that the agent's valuation of his outside option is independent of his level of overconfidence is crucial for this result. In some instances, one might expect an overconfident agent to be overconfident about the opportunities that he passes by, just as he is about the project he actually engages.

[21] As discussed before, if an agent is overconfident overall but underconfident about the value of effort, the cost

Clearly, the assumption that the outside option is evaluated equally by agents with different beliefs is crucial for this result: less overconfident agents could become attractive if overconfidence also affects their perception regarding their outside option, since less overconfident agents would accept contracts yielding lower perceived expected utility.

When facing a pool of agents who share the same beliefs about their ability, on the other hand, the principal will hire the agent that she judges to be most able. An applicant who responds to a job announcement, for example, probably believes he is well-suited for the position. Because choosing a higher-ability agent (i.e. an agent who generates a better outcome distribution) yields higher expected revenue and the cost of inducing any of these agents to exert effort is the same, the principal naturally prefers the most able agent.

In short: When the principal faces a pool of same-ability agents who differ only in their level of overconfidence, she will tend to hire the *most* overconfident agent. In contrast, when she faces a pool of applicants who share the same beliefs but differ in underlying ability, the principal will choose the *least* overconfident—most able—agent.

5.3 Comment on the Agent Designing the Contract

With a reinterpretation, the competing-principals framework of Section 3 can be used to study the setting in which it is the agent who designs the contract and makes a take-it-or-leave-it offer to a principal. The agent extracts all the surplus from the relationship, just as when principals compete. Recall that competition drives expected profits to zero, so the equilibrium contract in that case maximizes the agent's perceived expected utility among those that yield non-negative expected profits for the principal. This is precisely the agent's objective when designing a take-it-or-leave-it contract offer. If the agent is aware of the principal's beliefs, the equilibrium contract is identical to the equilibrium contract that would follow competition between several principals sharing the same beliefs. If the overconfident agent was instead oblivious about the fact that the principal holds different beliefs, he would offer a contract in line with the equilibrium of a standard identical-beliefs model. The principal would reject such an offer if she was relatively pessimistic about the probability of success, judging it to yield negative expected profits.

of agency increases. The principal might choose a less (overall) overconfident agent if lower-powered incentives are sufficient to induce this agent to exert effort. More precisely, fixing actual agent ability and one kind of overconfidence across agents, the principal will prefer the agent with highest overall overconfidence.

6 Extension: The Continuous-Action Case

In this section, I generalize the one-principal, one-agent model to allow the agent to choose from a continuum of possible effort levels. As we will see, most of the results of the two-action model generalize to a continuous-action setting. The timing and other assumptions in this framework are identical to those in the one-principal, one-agent setting studied in the previous section, except that the agent has a continuum of possible effort levels to choose from: he will choose some $e \in [0, 1]$ if he accepts the principal's contract offer. The disutility cost of effort is a function $c(e)$; assume that $c'(\cdot) > 0$ and $c''(\cdot) > 0$. This assumption implies that the agent's effort level choice will be proportionately related to the contract's power of incentives as long as his choice is an interior solution to his perceived expected utility maximization problem. Assume that $c'(0) = 0$ and $\lim_{e \to 1} c'(e) = \infty$ so that it is, in fact, an interior solution whenever $s_1 > s_0$.[22]

As discussed before, because the principal who can make a take-it-or-leave-it offer to an agent faces a very similar problem to the one faced by a principal who competes with others, the results in a competing-principals setting and a one-principal setting differ only in the level of expected payment to the agent. I focus on a continuous-action-space extension of the model in a one-principal, one-agent setting. Because the agent receives perceived expected utility equal to that of his (exogenous) outside option in equilibrium, his two possible outcome-contingent utility levels have a fairly simple closed-form solution in this setting.

The principal's problem when designing the optimal contract offer is to maximize her expected profits, taking into account that the agent will choose his effort level optimally and that he will accept the offer only if he judges it to yield expected utility no lower than his outside option.

If the agent accepts a given contract offer $\langle s_1, s_0 \rangle$, he will subsequently choose his effort level so as to maximize his perceived expected utility:

$$\max_{e \in [0,1]} (\tilde{q} + \tilde{v}e)\, u(s_1) + [1 - (\tilde{q} + \tilde{v}e)]\, u(s_0) - c(e).$$

The first-order condition for the agent's problem is

$$\tilde{v}\,[u(s_1) - u(s_0)] = c'(e),$$

which defines the agent's choice of effort after accepting contract offer $\langle s_1, s_0 \rangle$. The incentive effect is apparent from this condition: a lower-powered incentive contract is sufficient to implement any given effort level e when the agent is overconfident about the value of effort. The agent's

[22] Note that if the agent chose a corner solution ($e = 0$ or $e = 1$), as long as his choice of effort remains at a given corner or shifts discretely to the other, the analysis reduces to the two-action model studied before.

participation constraint can be written as:

$$(\tilde{q}+\tilde{v}e)\,u(s_1)+[1-(\tilde{q}+\tilde{v}e)]\,u(s_0)-c(e)\geq \underline{u},$$

where e is chosen optimally by the agent. Note that it must be binding in equilibrium. If not, the principal could marginally reduce both payments s_1 and s_0 while keeping the power of incentives $[u(s_1)-u(s_0)]$ constant (so as to implement the same effort level). The principal would then increase expected profits, which contradicts equilibrium.

In order to characterize the relationship between overconfidence, the power of incentives, and the implemented level of effort under the optimal contract, it is useful to reinterpret the principal's problem.

Definition 2 *Given the agent's effort-choice problem, and that the principal will optimally set the participation constraint to bind, the best contract that implements effort level e, $\langle s_1(e), s_0(e)\rangle$, is implicitly defined by*

$$\begin{aligned} u(s_1(e)) &= \underline{u}+c(e)+[1-(\tilde{q}+\tilde{v}e)]\,\frac{c'(e)}{\tilde{v}} \text{ and} \\ u(s_0(e)) &= \underline{u}+c(e)-(\tilde{q}+\tilde{v}e)\,\frac{c'(e)}{\tilde{v}}. \end{aligned}$$

Taking this into account, we can reduce the principal's problem to

$$\max_{e\in[0,1]}(q+ve)\,(x_1-s_1(e))+[1-(q+ve)]\,(x_0-s_0(e)).$$

Clearly, the power of incentives of the optimal contract depends not only on the agent's beliefs but also on the effort level that the principal chooses to implement, which in turn depends on all the parameters in the model (including the particular functional form of the agent's utility with respect to payments and disutility cost of effort). While explicitly solving for the optimal implemented level of effort seems fruitless, it is possible to study the qualitative effects of changes in each kind of overconfidence.

Assume that the principal's profit-maximization problem when choosing which effort level to implement is *well behaved*: it has a unique, interior, local and global maximum. Let e^* denote the effort level that solves the principal's profit maximization problem, at which the marginal revenue from increasing the implemented level of effort equals its marginal cost:

$$MR_{e^*}=MC_{e^*}.$$

Note that the marginal revenue of effort is constant:

$$MR_e=v\,(x_1-x_0).$$

By marginally increasing the implemented level of effort, the additional revenue in the event of project success $(x_1 - x_0)$ will come about with marginally higher probability (how much higher depends on v, but not on the agent's beliefs).

The marginal cost of implementing effort, on the other hand, is

$$MC_e = v\left(s_1(e) - s_0(e)\right) + (q + ve)\frac{ds_1(e)}{de} + \left[1 - (q + ve)\right]\frac{ds_0(e)}{de}$$

where

$$\begin{aligned} \frac{ds_1(e)}{de} &= \frac{1}{u'(s_1(e))}\left[1 - (\tilde{q} + \tilde{v}e)\right]\frac{c''(e)}{\tilde{v}}, \text{ and} \\ \frac{ds_0(e)}{de} &= -\frac{1}{u'(s_0(e))}(\tilde{q} + \tilde{v}e)\frac{c''(e)}{\tilde{v}}. \end{aligned}$$

Note, in particular, that the marginal cost of implementing effort depends crucially on the agent's beliefs. It is clear that for the principal's profit-maximization problem to be well behaved, it is sufficient that the marginal cost of implementing effort be an increasing function of effort level. Further discussion about the conditions under which the principal's profit-maximization problem is well behaved is relegated to Subsection A.3.1 of the Appendix.

Consider the effect of marginally higher agent overconfidence about the base probability of success on the marginal cost of implementing effort, evaluated at the optimal e^*:

$$\frac{\partial MC_{e^*}}{\partial \tilde{q}} = -\frac{c''(e^*)}{\tilde{v}}\left[(q + ve^*)\frac{1}{u'(s_1(e^*))} + \left[1 - (q + ve^*)\right]\frac{1}{u'(s_0(e^*))}\right] < 0.$$

Given that the marginal revenue of implementing any effort level is constant, and that the marginal cost of implementing effort increases with effort level, it follows that the principal will choose to implement higher effort if dealing with an agent who is more overconfident about the base probability of success: $\frac{de^*}{d\tilde{q}} > 0$. This result is analogous to its counterpart in the two-action case, summarized in Proposition 4. As a consequence of the wager effect of overconfidence, because a more-overconfident agent prefers higher-powered incentive contracts, it is cheaper for the principal to implement a higher level of effort in the margin.

Consider now the comparable effect of marginally higher agent overconfidence about the value of effort:

$$\begin{aligned} \frac{\partial MC_{e^*}}{\partial \tilde{v}} = -e^*\frac{c''(e^*)}{\tilde{v}}&\left[(q + ve^*)\frac{1}{u'(s_1(e^*))} + \left[1 - (q + ve^*)\right]\frac{1}{u'(s_0(e^*))}\right] \\ &- \frac{1}{\tilde{v}}v\left[(x_1 - s_1(e^*)) - (x_0 - s_0(e^*))\right] < 0. \end{aligned}$$

The first term of the equation above reflects the wager effect of overconfidence. Just as in the case of overconfidence about the base probability of success, it is less costly for the principal to implement higher effort levels. The second term of the equation reflects the incentive effect of overconfidence. The contract $\langle s_1(e^*), s_0(e^*)\rangle$ will implement some effort level greater than e^* following an increase in the agent's overconfidence about the value of effort. This benefits the principal as long as $(x_1 - s_1(e^*)) \geq (x_0 - s_0(e^*))$.[23] Implementing a higher level of effort increases the expected revenue of the project. As a consequence of both the wager and the incentive effects of overconfidence, higher overconfidence about the value of effort results in a higher implemented effort level: $\frac{de^*}{d\tilde{v}} > 0$. This is analogous to the corresponding result in the two-action case, exposed in Proposition 4.

The effect of overconfidence about the base probability of success on the power of incentives of the optimal contract is straightforward. Given that a higher effort level is implemented, and that this kind of overconfidence does not directly affect the incentive structure of the contract holding effort constant, it follows that overconfidence about the base probability of success always implies higher-powered incentives. The wager effect of overconfidence drives the optimal contract towards higher-powered incentives in the continuous-action case, even if the agent is only slightly overconfident overall, because the implemented effort level continuously increases with overconfidence.

The effect of overconfidence about the value of effort on the power of incentives is, on the other hand, ambiguous. This is because, as in the two-action case, the incentive effect of overconfidence on this dimension pushes toward lower-powered incentives, while the wager effect pushes toward higher-powered incentives. Furthermore, both effects of overconfidence imply that the optimal contract implements a higher effort level, which also pushes toward higher-powered incentives. The intuition can also be explained in terms of the two-action setting. In that case, an increase in overconfidence about the value of effort that drives effort to be implemented in equilibrium is likely to result in a higher-powered incentive contract.

The actual change in the power of incentives of the optimal contract as a consequence of changes in agent overconfidence regarding the value of effort is formally derived in Subsection A.3.2 of the Appendix. Whether there is some range of overall slight overconfidence over which the power of incentives decreases in overconfidence about the value of effort depends on all the parameters of the model. Such a range exists if the agent is very risk averse and the marginal cost of exerting effort does not increase sharply. Intuitively, because a small increase in the amount of risk born by

[23] As argued in footnote 14 and the discussion that follows it in the text, it seems very reasonable to assume that in equilibrium $x_1 - x_0 \geq s_1(e^*) - s_0(e^*)$, so that the agent is not exposed to more risk than inherent in the project. If this assumption is violated, the principal would enjoy greater profits in the case of project failure than in the case of success, so may prefer to implement lower rather than higher effort levels. Such a contract seems unrealistic, since the principal would then have incentives to sabotage the project. Common sense, if not legal restrictions, should prevent the agent from accepting such a contract.

the agent is sufficient to induce higher effort exertion, and lower-powered incentives are sufficient to implement any given effort level, the power of incentives of the optimal contract decreases with overconfidence about the value of effort. If, on the other hand, the agent is not very risk averse and his marginal cost of exerting effort increases sharply, the additional power of incentives necessary to implement higher effort is large, so there is no such range and the power of incentives is everywhere increasing in overconfidence of either kind.

7 Conclusion and Discussion

This paper attempts to provide insight into the effects of overconfidence in equilibrium within a moral-hazard framework, which include potential benefits and pitfalls. It gives one possible explanation to why we may observe incentive contracts that seem excessively powerful in some situations, and others that seem surprisingly flat in other situations. It also helps explain why overconfidence can be valuable, not only for the agent, but also for the principal—even though she is mainly concerned with the agent's underlying ability.

The results of the paper suggest that incentive contracts are sensitive to the *kind* of overconfidence, not only to the presence of overconfidence *per se*. Because of this, experimental and field studies exploring how (besides whether or not) individuals are overconfident would help our understanding about how incentive contracts respond to changes in beliefs. For instance, if agents tend to be significantly overconfident overall and agent overconfidence is procyclical (as suggested by Gervais and Odean [2001]), our model predicts that fast-paced growth should be followed by more powerful incentive contracts being implemented. In contrast, if agents tend to be only slightly overconfident about the value of effort, less powerful incentives would follow.

Some recent empirical observations are consistent with the model. For example, in a survey of almost three thousand entrepreneurs, Cooper, Woo, and Dunkelberg (1988) find that entrepreneurs tend to overestimate the probability of success of their enterprise, and invest many hours in it (more than 60 hours a week according to many of the respondents). In Weinstein's (1980) terms, when entrepreneurs form expectations they may be comparing themselves to a hypothetical entrepreneur who chooses to enter an industry with merely good prospects and puts little effort into making the enterprise succeed. Given that the average success rate of businesses is readily available information, entrepreneurs may use this average as their benchmark when forming expectations. When doing so, they fail to internalize the fact that most other entrepreneurs choose to enter an industry which they deem particularly profitable and work hard towards success, just like they do. The results of my model imply that entrepreneurs' choice of long hours could arise simply from their overconfidence rather than, for example, a particularly low cost of effort for entrepreneurs because they enjoy their work more than others.

Cooper, Woo, and Dunkelberg (1988) also find that the entrepreneurs' degree of overconfidence seems to be independent of factors that affect their actual probability of success (like experience in the industry and education level). This suggests an empirical test for the relevance of overconfidence in entrepreneurs' decisions: holding the beliefs of the entrepreneur constant, if the most overconfident agents are *significantly overconfident* (as implicitly defined by Proposition 3) then my model can have implications opposite to those of a standard moral-hazard model which does not allow for overconfidence. A standard identical-beliefs model would predict a positive correlation between the proportion of funds invested in the project by the entrepreneur and the probability of success of the enterprise. Entrepreneurs who invest a higher proportion of their own funds face more powerful incentives to exert effort. In that case, there is less of an agency problem, so the probability of success should always be *positively* correlated to the proportion of funds invested by the entrepreneur. When we allow for overconfidence, and given the observation that beliefs tend to be fairly homogeneous, in this setting the most overconfident entrepreneurs (those with relatively low underlying ability) will tend to "underinsure" and invest a higher proportion of their own funds in the project. They will tend to have a lower success rate than more able, less overconfident entrepreneurs who do not underinsure. Less overconfident entrepreneurs find the terms of the principals' offers (e.g. the terms of a bank loan) to be more in line with their own beliefs; because of a smaller wager effect of overconfidence, they face less powerful incentives in equilibrium (they invest a smaller proportion of their own funds). Thus, the probability of success of an enterprise may be *negatively* correlated to the proportion of the entrepreneur's own funds invested in it. Of course, even allowing for overconfidence, the effect of incentives on the effort that the entrepreneur exerts pushes towards a positive correlation. Observing a negative correlation would strongly suggest that overconfidence is relevant in entrepreneurs' decision-making processes.

The model can serve to reinterpret some previous empirical results. Smith and Watts (1992) present an empirical study which discusses executive compensation. One of their observations is that firms with larger "investment opportunity sets" pay their CEOs more, and are more likely to use stock options and other forms of performance-contingent pay. They interpret this result in light of a standard moral-hazard framework: they argue that a manager's actions are less readily observable if the firm has more investment opportunities. In a world where overconfidence is relevant, it seems intuitive that a larger opportunity set will go hand-in-hand with higher CEO overconfidence about the chosen course of action, since he will tend to choose whichever action he is most optimistic about. In this sense, my model is consistent with a positive correlation between larger investment opportunity sets and higher-powered compensation packages, and the connection more straightforward than monitoring problems that increase with the amount of possible investment projects. Moral hazard, however, does not seem to be the main driving force behind optimal contract design in the context of executive compensation. A model in which the agent holds private information regarding the best course of action seems more appropriate to study this topic. Gervais, Heaton and Odean (2003) explore one such model and argue that the power of incentives should be lower if a manager is overconfident than if he is realistic. Their insight is that an overconfident manager will act in a less risk-averse manner, so lower-powered incentives are sufficient to align the agent's objectives to those of the principal. The wager effect of overconfidence identified in this paper, however, carries over to this setting. If the principal is aware that the agent tends to overestimate the probability of success after choosing a course of action, the wager effect pushes towards higher-powered incentives. I study the effects of overconfidence in an investment-decision setting in de la Rosa (2006), and Van den Steen (2005) studies a similar problem (from the viewpoint of intrinsic and extrinsic motivation).

Another direction for further research that could yield interesting results is overconfidence in a self-selection (or sorting) setting. According to adverse-selection models that allow for overconfidence, the most overconfident agents are naturally attracted to riskier endeavors. This is consistent with the fact that some agents in dangerous jobs do underestimate the probability of a bad outcome, as noted by Akerlof and Dickens (1982). My model implies, however, that different kinds of overconfidence can have conflicting effects in terms of the amount of risk born by the agent in equilibrium. If this is the case, agents with similar degrees of overall overconfidence might sort themselves into very different positions.

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A Appendix

A.1 Competing Principals

Lemma 1 *If principals share the same beliefs regarding outcome distribution conditional on the agent's actions, in equilibrium expected profits will be zero for all principals according to their beliefs.*

Proof. Suppose not. Suppose that a principal offers, and the agent accepts, a contract $\langle s_1, s_0 \rangle$ in equilibrium that yields positive expected profits for the offering principal:

$$\Pr(x_1 \mid e)(x_1 - s_1) + [1 - \Pr(x_1 \mid e)](x_0 - s_0) > 0,$$

where e is chosen optimally by the agent. Other principals, whose contracts are not accepted by the agent, receive zero profits in equilibrium. Consider the following deviation by one of these principals: offer contract $\langle s_1 + \epsilon, s_0 \rangle$ if $\langle s_1, s_0 \rangle$ implements effort, and $\langle s_1, s_0 + \epsilon \rangle$ if $\langle s_1, s_0 \rangle$ does not implement effort. In either case, the new contract implements the same effort level as $\langle s_1, s_0 \rangle$ does, makes the agent strictly better off, and will thus be accepted by the agent. For ϵ close enough to zero, the principal making the new contract offer enjoys positive expected profits. Given that there is a profitable deviation, $\langle s_1, s_0 \rangle$ cannot be the equilibrium contract. ■

Proposition 1 *Assuming effort is not implemented in equilibrium, the only equilibrium contract* $\langle s_{1*}, s_{0*} \rangle$ *is characterized by the conditions*

$$\frac{\tilde{q}}{1-\tilde{q}} \frac{u'(s_{1*})}{u'(s_{0*})} = \frac{q}{1-q}$$

and $q(x_1 - s_{1*}) + [1-q](x_0 - s_{0*}) = 0$. *The agent bears risk in equilibrium if* $\tilde{q} \neq q$.

Proof. Assuming that effort is not implemented in equilibrium, $\langle s_{1*}, s_{0*} \rangle$ characterized by $\frac{q}{1-q} = \frac{\tilde{q}}{1-\tilde{q}} \frac{u'(s_{1*})}{u'(s_{0*})}$ and zero expected profits for the principal maximizes the agent's perceived expected utility subject to non-negative profits for the offering principal. It can be shown that, because of the agent's risk aversion, this contract is the unique global maximum. Any other contract that does not implement effort and gives non-negative expected profits to the offering principal therefore yields strictly lower perceived expected utility to the agent. Conversely, any other contract that does not implement effort and yields the same perceived expected utility to the agent yields strictly negative expected profits for the offering principal. Thus, there is no profitable deviation from $\langle s_{1*}, s_{0*} \rangle$. ■

Proposition 2 *Assuming effort is implemented in equilibrium,* $\langle \bar{s}_1, \bar{s}_0 \rangle$ *is the only equilibrium contract if* $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} \leq \frac{q+v}{1-(q+v)}$.

Proof. We need to show that there is no profitable deviation from this contract; i.e. that no other effort-implementing contract exists such that the agent receives higher expected utility, and the offering principal enjoys positive expected profits. This rules out contracts with higher payment to the agent in both success and failure, and those with lower payment to the agent in both events. The perceived expected utility for the agent under $\langle \bar{s}_1, \bar{s}_0 \rangle$ is

$$\tilde{\mathbb{E}}\left[u\left(\bar{s}_x\right) \mid e=1\right] = \left(\tilde{q}+\tilde{v}\right) u\left(\bar{s}_1\right) + \left[1-\left(\tilde{q}+\tilde{v}\right)\right] u\left(\bar{s}_0\right) - c.$$

A marginal change in payments that leaves the agent indifferent satisfies

$$\left(\tilde{q}+\tilde{v}\right) u'\left(\bar{s}_1\right) ds_1 + \left[1-\left(\tilde{q}+\tilde{v}\right)\right] u'\left(\bar{s}_0\right) ds_0 = 0$$

or

$$ds_0 = -\frac{\tilde{q}+\tilde{v}}{1-\left(\tilde{q}+\tilde{v}\right)} \frac{u'\left(\bar{s}_1\right)}{u'\left(\bar{s}_0\right)} ds_1$$

where necessarily $ds_1 > 0$ and $ds_0 < 0$, since the incentive compatibility constraint would be otherwise violated. Such a change implies that the agent would bear more risk under the new contract. Since the agent is risk-averse, he must be compensated with a higher expected payment, which is why the increase in s_1 must be more than actuarially fair *according to his beliefs*.

The change in expected profits for the offering principal from a marginal change in the payment structure that leaves the agent indifferent is

$$\begin{gathered} -\left(q+v\right) ds_1 - \left[1-\left(q+v\right)\right] ds_0 = \\ \left(-\left(q+v\right) + \left[1-\left(q+v\right)\right] \frac{\tilde{q}+\tilde{v}}{1-\left(\tilde{q}+\tilde{v}\right)} \frac{u'\left(\bar{s}_1\right)}{u'\left(\bar{s}_0\right)}\right) ds_1 \leq 0. \end{gathered}$$

If $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} \leq \frac{q+v}{1-(q+v)}$, expected profits decrease for the principal offering the new contract that exposes the agent to more risk than $\langle \bar{s}_1, \bar{s}_0 \rangle$ does. It follows that there is no profitable marginal deviation from the $\langle \bar{s}_1, \bar{s}_0 \rangle$ contract. Given that $\frac{u'(s_1)}{u'(s_0)}$ falls as s_1 increases and s_0 decreases, discrete deviations from $\langle \bar{s}_1, \bar{s}_0 \rangle$ that implement effort and leave the agent indifferent also imply expected losses for the offering principal. Therefore, $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the only equilibrium contract. ■

Proposition 3 *Assuming effort is implemented in equilibrium, if* $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} > \frac{q+v}{1-(q+v)}$ *then* $\langle \bar{s}_1, \bar{s}_0 \rangle$ *is not an equilibrium contract. The only equilibrium contract* $\langle s_1^*, s_0^* \rangle$ *is characterized by*

$$\frac{\tilde{q}+\tilde{v}}{1-\left(\tilde{q}+\tilde{v}\right)} \frac{u'\left(s_1^*\right)}{u'\left(s_0^*\right)} = \frac{q+v}{1-\left(q+v\right)}$$

and $\left(q+v\right)\left(x_1 - s_1^*\right) + \left[1-\left(q+v\right)\right]\left(x_0 - s_0^*\right) = 0$. *The equilibrium contract has higher-powered incentives than necessary to implement effort.*

Proof. The proof is analogous to the proof of Proposition 1. Assuming that effort is implemented in equilibrium, $\langle s_1^*, s_0^* \rangle$ (uniquely) maximizes the agent's perceived utility subject to non-negative profits for the offering principal. It follows that any other contract that implements effort and yields the same perceived expected utility to the agent yields strictly negative expected profits for the offering principal. There is no profitable deviation from $\langle s_1^*, s_0^* \rangle$.

Given that by assumption $\frac{q+v}{1-(q+v)} < \frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)}$, since $u''(\cdot) < 0$, and expected profits for the principal are zero under both $\langle \bar{s}_1, \bar{s}_0 \rangle$ and $\langle s_1^*, s_0^* \rangle$, it follows that $s_1^* > \bar{s}_1$ and $s_0^* < \bar{s}_0$. By construction, $\tilde{v}\left(u\left(\bar{s}_1\right) - u\left(\bar{s}_0\right)\right) = c$, and thus

$$\tilde{v}\left(u\left(s_1^*\right) - u\left(s_0^*\right)\right) > c;$$

$\langle s_1^*, s_0^* \rangle$ has higher-powered incentives than necessary to implement effort. ■

Proposition 4 *Ceteris paribus, higher levels of overconfidence of either kind increase the likelihood that effort is implemented in equilibrium: if effort is implemented given agent beliefs $(\tilde{q}, \tilde{v})$, then effort will be implemented when his beliefs are $(\tilde{q}, \tilde{v}^*)$ for any $\tilde{v}^* \geq \tilde{v}$ or $(\tilde{q}^*, \tilde{v})$ for any $\tilde{q}^* \geq \tilde{q}$.*

Proof. Overconfidence affects the implemented level of effort differently, depending on whether the incentive or the wager effect of overconfidence dominates.

First, consider changes in agent overconfidence regarding the value of effort.

If $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the potential equilibrium contract that implements effort, the power of incentives decreases with overconfidence regarding the value of effort. Expected profits are zero in equilibrium, so a marginal increase in $\tilde{v}$ implies:

$$-\left(q+v\right)\frac{d\bar{s}_1}{d\tilde{v}} - \left[1-\left(q+v\right)\right]\frac{d\bar{s}_0}{d\tilde{v}} = 0,$$

where $\frac{d\bar{s}_1}{d\tilde{v}} < 0$ and $\frac{d\bar{s}_0}{d\tilde{v}} > 0$. The effect of such a change in the agent's perceived utility when $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the potential equilibrium contract that implements effort is

$$u\left(\bar{s}_1\right) - u\left(\bar{s}_0\right) + \left(\tilde{q}+\tilde{v}\right)u'\left(\bar{s}_1\right)\frac{d\bar{s}_1}{d\tilde{v}} + \left[1-\left(\tilde{q}+\tilde{v}\right)\right]u'\left(\bar{s}_0\right)\frac{d\bar{s}_0}{d\tilde{v}} \geq c > 0,$$

taking into account that $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the potential equilibrium contract only if $\frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})} \frac{u'(\bar{s}_1)}{u'(\bar{s}_0)} \leq \frac{q+v}{1-(q+v)}$.

If $\langle s_1^*, s_0^* \rangle$ is the potential equilibrium contract that implements effort, the envelope theorem implies that the change in the agent's perceived utility from a marginal increase in $\tilde{v}$ is

$$u\left(s_1^*\right) - u\left(s_0^*\right) > c > 0.$$

There is no change in the agent's perceived expected utility under $\langle s_{1*}, s_{0*} \rangle$ from a marginal increase in $\tilde{v}$. Therefore, if effort is implemented given agent beliefs $(\tilde{q}, \tilde{v})$, then effort will be implemented when his beliefs are $(\tilde{q}, \tilde{v}^*)$ for any $\tilde{v}^* \geq \tilde{v}$.

Consider now changes in agent overconfidence regarding the base probability of success.

If $\langle \bar{s}_1, \bar{s}_0 \rangle$ is the potential equilibrium contract that implements effort, the power of incentives is independent from overconfidence regarding the base probability of success. A marginal increase in $\tilde{q}$ implies a change in the agent's perceived expected utility of

$$u(\bar{s}_1) - u(\bar{s}_0) = c.$$

If $\langle s_1^*, s_0^* \rangle$ is the potential equilibrium contract that implements effort, the envelope theorem again implies that the change in the agent's perceived utility from a marginal increase in $\tilde{q}$ is

$$u(s_1^*) - u(s_0^*) > c.$$

The change in the agent's perceived expected utility following a marginal increase in $\tilde{q}$ under the potential equilibrium contract that does not implement effort is

$$u(s_{1*}) - u(s_{0*}) \leq c.$$

Therefore, if effort is implemented given agent beliefs $(\tilde{q}, \tilde{v})$, then effort will be implemented when his beliefs are $(\tilde{q}^*, \tilde{v})$ for any $\tilde{q}^* \geq \tilde{q}$. ■

A.2 One Principal and One Agent

Solving the principal's profit-maximization problem explicitly

Assume that effort is implemented in equilibrium. The principal's problem is

$$\max_{s_1, s_0} (q+v)(x_1 - s_1) + [1 - (q+v)](x_0 - s_0)$$

subject to

$$\begin{aligned} \tilde{v}(u(s_1) - u(s_0)) &\geq c \\ (\tilde{q} + \tilde{v})u(s_1) + [1 - (\tilde{q} + \tilde{v})]u(s_0) - c &= \underline{u} \end{aligned}$$

We can solve this problem by setting up a Lagrangian. Let λ denote the Lagrange multiplier associated with the IC constraint, and μ the multiplier associated with the IR constraint. Since the IR constraint binds in equilibrium, it follows that $\mu > 0$.

If the IC constraint binds in equilibrium as well, so that $\lambda > 0$, then the IR and IC constraints holding with equality define the equilibrium contract. This contract is analogous to the contract discussed in the previous section when the difference in beliefs held by principal and agent was small (which we denoted by $\langle \bar{s}_1, \bar{s}_0 \rangle$). As before, when the beliefs held by principal and agent do not diverge significantly, the IC constraint will in fact bind, and the contract with incentives just powerful enough to induce effort exertion is the equilibrium contract. This result is analogous to Proposition 2. The optimal contract is then characterized by

$$u(s_0) = \underline{u} - \frac{\tilde{q}}{\tilde{v}}c$$
$$u(s_1) = \underline{u} + \frac{1-\tilde{q}}{\tilde{v}}c.$$

If the IC constraint binds, the incentive effect of overconfidence dominates. The power of incentives is, as in the case of competing principals, decreasing in the agent's estimation of the value of his effort ($\tilde{v}$). The power of incentives is independent of the agent's belief about the base probability of success ($\tilde{q}$), and of the principal's beliefs. An agent who is overconfident about the base probability of success (he overestimates $\tilde{q}$) will, however, be paid less both in the event of success and failure. Overconfidence of this kind makes it cheaper for the principal to satisfy the agent's participation constraint, since the agent deems it more likely that he will receive the "high" payment s_1. The incentive effect of overconfidence will in fact dominate—and the incentive compatibility constraint will bind in equilibrium—if principal and agent hold beliefs that are not significantly divergent.

If the IC constraint is slack in equilibrium instead, so that $\lambda = 0$, the first-order conditions of the principal's maximization problem yield

$$(\tilde{q}+\tilde{v})\,\mu u'(s_1) = (q+v)$$
$$[1-(\tilde{q}+\tilde{v})]\,\mu u'(s_0) = [1-(\tilde{q}+\tilde{v})]$$

which substituting for μ yields

$$\frac{q+v}{1-(q+v)} = \frac{\tilde{q}+\tilde{v}}{1-(\tilde{q}+\tilde{v})}\frac{u'(s_1^{**})}{u'(s_0^{**})}.$$

This condition characterizes the power of incentives of the equilibrium contract when the beliefs held by principal and agent differ significantly. This result is analogous to Proposition 3 in the case of competing principals. The expected payment in this case is grounded by the IR constraint instead of the zero-expected-profits condition.

When the agent is significantly overconfident overall, the power of incentives of the optimal contract offered by the principal is increasing in overconfidence of either kind.

Assume now that effort is not implemented in equilibrium. The wager effect of overconfidence implies that the agent will bear some risk in equilibrium. The equilibrium contract's power of incentives are characterized by the expression $\frac{q}{1-q} = \frac{\tilde{q}}{1-\tilde{q}}\frac{u'(s_1)}{u'(s_0)}$. This result is analogous to Proposition 1. The expected payment to the agent is determined by the IR constraint.

Finally, the likelihood that the principal will implement effort is increasing in either kind of overconfidence. If the agent is overconfident about the value of effort, the cost of agency when implementing effort is reduced, so the profits for the principal when implementing effort increase. Furthermore, given that an overconfident agent overestimates the probability of receiving the success-contingent payment, it is increasingly cheaper for the principal to satisfy the agent's participation constraint the higher this payment is; a contract that implements effort is increasingly cheaper (as overconfidence increases) to implement for the principal relative to a contract that does not implement effort. The likelihood that effort is implemented under the optimal contract is therefore increasing in each kind of overconfidence. This result is analogous to Proposition 4.

A.3 The Continuous-Action Case

A.3.1 Conditions for the principal's profit-maximization problem to be well behaved

Recall that we can write the principal's profit-maximization problem as:

$$\max_{e\in[0,1]} (q+ve)(x_1 - s_1(e)) + [1-(q+ve)](x_0 - s_0(e)).$$

subject to

$$\begin{aligned} u(s_1(e)) &= \underline{u} + c(e) + [1-(\tilde{q}+\tilde{v}e)]\frac{c'(e)}{\tilde{v}} \\ u(s_0(e)) &= \underline{u} + c(e) - (\tilde{q}+\tilde{v}e)\frac{c'(e)}{\tilde{v}}. \end{aligned}$$

Let e^* denote the solution to the first-order condition of this problem, the level of effort at which the principal's marginal revenue equals her marginal cost of implementing effort:

$$MR_{e^*} = MC_{e^*}.$$

Recall that

$$MR_e = v(x_1 - x_0),$$

and

$$MC_e = v(s_1(e) - s_0(e)) + (q+ve)\frac{ds_1(e)}{de} + [1-(q+ve)]\frac{ds_0(e)}{de},$$

where

$$\begin{aligned} \frac{ds_1(e)}{de} &= \frac{1}{u'(s_1(e))}[1-(\tilde{q}+\tilde{v}e)]\frac{c''(e)}{\tilde{v}} \\ \frac{ds_0(e)}{de} &= -\frac{1}{u'(s_0(e))}(\tilde{q}+\tilde{v}e)\frac{c''(e)}{\tilde{v}}. \end{aligned}$$

This problem will have a unique, interior, local and global maximum if the marginal cost of implementing effort is strictly increasing in implemented level of effort.

The change in the marginal cost of increasing effort is:

$$\frac{dMC_e}{de} = v\left(\frac{ds_1(e)}{de} - \frac{ds_0(e)}{de}\right) + (q+ve)\frac{d^2 s_1(e)}{de^2} + [1-(q+ve)]\frac{d^2 s_0(e)}{de^2}.$$

The first component of this expression is positive, since $\frac{ds_1(e)}{de} > 0$ and $\frac{ds_0(e)}{de} < 0$. Assuming for simplicity that $c''(e) = k$, a constant, the second and third components of the expression above are:

$$\frac{d^2 s_1(e)}{de^2} = \frac{k}{u'(s_1(e))}\left[-\frac{u''(s_1(e))}{u'(s_1(e))}\frac{[1-(\tilde{q}+\tilde{v}e)]^2}{\tilde{v}^2}\frac{k}{u'(s_1(e))} - 1\right],$$
$$\frac{d^2 s_0(e)}{de^2} = \frac{k}{u'(s_0(e))}\left[-\frac{u''(s_0(e))}{u'(s_0(e))}\frac{(\tilde{q}+\tilde{v}e)^2}{\tilde{v}^2}\frac{k}{u'(s_0(e))} - 1\right].$$

If both of these components are positive, it follows that the marginal cost of implementing effort will be strictly increasing in effort level. This will be the case if:

- $c''(e)$ is large enough

  If the cost to the agent of choosing higher levels of effort is convex enough, then the cost to the principal of implementing higher levels of effort will be convex as well.

- the agent is sufficiently risk averse

  A large coefficient of absolute risk aversion $-\frac{u''(s_x)}{u'(s_x)}$ also makes it increasingly costly to implement higher effort, since the principal must compensate the agent for the higher risk he must bear as higher levels of effort are implemented.

- the agent is wealthy

  It is increasingly costly to power-up incentives and implement higher levels of effort when changes in the payments have little effect on the agent's utility level. When the agent is wealthy, his marginal utility $u'(s_x)$ is relatively low.

A.3.2 The power of incentives and agent overconfidence regarding the value of effort

Recall that the solution to the agent's problem yields

$$[u(s_1(e^*)) - u(s_0(e^*))] = \frac{c'(e^*)}{\tilde{v}}.$$

The change in the power of incentives of the equilibrium contract is thus

$$\frac{d\left[u\left(s_1\left(e^*\right)\right)-u\left(s_0\left(e^*\right)\right)\right]}{d\tilde{v}}=\frac{c''\left(e^*\right)}{\tilde{v}}\frac{de^*}{d\tilde{v}}-\frac{c'\left(e^*\right)}{\tilde{v}^2}.$$

Recall, as well, that the solution to the principal's problem e^* is such that

$$MR_{e^*}=MC_{e^*},$$

or

$$v\left(x_1-x_0\right)=v\left(s_1-s_0\right)+\left(q+ve^*\right)\frac{ds_1\left(e^*\right)}{de}+\left[1-\left(q+ve^*\right)\right]\frac{ds_0\left(e^*\right)}{de}.$$

Assume that $c''(e)=k$, a constant. Taking the total derivative of the equation above with respect to $\tilde{v}$ yields

$$\begin{aligned}
0=&\frac{de^*}{d\tilde{v}}\left(\left\{\frac{\left[1-\left(\tilde{q}+\tilde{v}e^*\right)\right]}{u'\left(s_1\left(e^*\right)\right)}+\frac{\left(\tilde{q}+\tilde{v}e^*\right)}{u'\left(s_0\left(e^*\right)\right)}\right\}v+\left\{\frac{\left[1-\left(\tilde{q}+\tilde{v}e^*\right)\right]}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}+\frac{\left(\tilde{q}+\tilde{v}e^*\right)}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\right\}v\right.\\
&-\left\{\frac{\left(q+ve^*\right)}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}+\frac{\left[1-\left(q+ve^*\right)\right]}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\right\}\tilde{v}\\
&+\frac{k}{\tilde{v}}\left\{-\frac{u''\left(u\left(s_1\left(e^*\right)\right)\right)}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}\frac{\left(q+ve^*\right)\left[1-\left(\tilde{q}+\tilde{v}e^*\right)\right]^2}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}\right.\\
&\left.\left.-\frac{u''\left(u\left(s_0\left(e^*\right)\right)\right)}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\frac{\left[1-\left(q+ve^*\right)\right]\left(\tilde{q}+\tilde{v}e^*\right)^2}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\right\}\right)\\
&-\frac{1}{\tilde{v}}\left\{\frac{\left(q+ve^*\right)\left[1-\left(\tilde{q}+\tilde{v}e^*\right)\right]}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}+\frac{\left[1-\left(q+ve^*\right)\right]\left(\tilde{q}+\tilde{v}e^*\right)}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\right\}\\
&-e^*\left\{\frac{\left(q+ve^*\right)}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}+\frac{\left[1-\left(q+ve^*\right)\right]}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\right\}.
\end{aligned}$$

Given that we are interested in the effect of overconfidence regarding the value of effort on the power of incentives when the agent is slightly overconfident, I evaluate the change in the power of incentives of the equilibrium contract at the point that principal and agent agree in their beliefs:

$$\left.\frac{d\left[u\left(s_1\left(e^*\right)\right)-u\left(s_0\left(e^*\right)\right)\right]}{d\tilde{v}}\right|_{\tilde{v}=v,\tilde{q}=q}=\frac{k}{v}\frac{de^*}{d\tilde{v}}-\frac{c'\left(e^*\right)}{v^2}$$

$$\begin{aligned}
=&\left[\left(\left\{\frac{\left[1-\left(q+ve^*\right)\right]}{u'\left(s_1\left(e^*\right)\right)}-\frac{\left(q+ve^*\right)}{u'\left(s_0\left(e^*\right)\right)}\right\}+\left\{\frac{1}{\beta_1}-\frac{1}{\beta_0}\right\}\right)v\left(\frac{v}{k}\right)\right.\\
&\left.+\left\{-\frac{u''\left(u\left(s_1\left(e^*\right)\right)\right)}{u'\left(u\left(s_1\left(e^*\right)\right)\right)}\frac{\left[1-\left(q+ve^*\right)\right]}{\beta_1}-\frac{u''\left(u\left(s_0\left(e^*\right)\right)\right)}{u'\left(u\left(s_0\left(e^*\right)\right)\right)}\frac{\left(q+ve^*\right)}{\beta_0}\right\}\alpha\right]^{-1}\\
&\cdot\left[\left\{\frac{1}{\beta_1}-\frac{1}{\beta_0}\right\}\frac{\alpha}{v}+\left\{\frac{\left(q+ve^*\right)}{\beta_1}-\frac{\left[1-\left(q+ve^*\right)\right]}{\beta_0}\right\}e^*\right]\\
&-\frac{c'\left(e^*\right)}{v^2}
\end{aligned}$$

where

$$\alpha = (q + ve^*)\left[1 - (q + ve^*)\right], \beta_1 = u'\left(u\left(s_1\left(e^*\right)\right)\right), \beta_0 = u'\left(u\left(s_0\left(e^*\right)\right)\right);$$

note $\frac{1}{\beta_1} - \frac{1}{\beta_0} > 0$.

The expression above shows that the power of incentives will be decreasing in overconfidence about the value of effort when the agent is only slightly overconfident overall if the agent's action is very responsive to the power of incentives. This will be the case if the agent is very risk averse (as measured by $-\frac{u''(u(s_x))}{u'(u(s_x))}$, which resembles the coefficient of absolute risk aversion), or if the increase in the marginal cost of effort is sufficiently low (as measured by $k = c''\left(e^*\right)$). The power of incentives of the optimal contract will be everywhere increasing in both kinds of overconfidence if the agent is sufficiently risk neutral, or if the disutility cost of effort rapidly increases.

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