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Welfare Effects of Tax and Price Changes and the CES-UT Utility Function

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Abstract

Dixit's 1975 paper "Welfare Effects of Tax and Price Changes" constitutes a seminal contribution to the theory of tax reform within a second-best general equilibrium framework. The present paper clarifies ambiguities with respect to normalisation which has led to misinterpretation of some of Dixit's analytical results. It proves that a marginal tax reform starting from a proportional tax system will improve social welfare if it increases the supply of labour, whatever the rule of normalisation adopted. In models which impose additive separability between consumption and leisure in household preferences this insight cannot be articulated. This paper proposes as an alternative a parameterised utility function with explicit representation of the use of time, the CES-UT, which allows a flexible representation of the relationship between consumption and leisure. It also demonstrates how standard compensated price elasticities can be derived from the parameters of the CES-UT and how it may be used for applied tax reform analysis.

Keywords:
Public economics, optimal taxation, tax reform, tax simulation, CGE models

JEL classification codes:
H2

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1. Introduction

Evaluating welfare effects of alternative tax regimes is of central importance for public policy. Nevertheless, despite Dixit's seminal and widely cited (1975) paper on "Welfare Effects of Tax and Price Changes" and numerous subsequent contributions, the policy implications of analytical results have remained elusive. One possible reason is that Dixit's analysis did not properly account for the rules of normalisation in such models and for that reason failed to provide an intuitive insight into what constitutes desirable direction of tax reform and thus what determines optimal systems of taxation. Simulation studies based on parameterised functional forms have from the outset been used in attempts to shed light on this question. However, these efforts received an early set-back with the realisation of the straitjacket imposed by the assumption of additive separability, generally adopted in applied work. A proposed solution to this impasse was the use of so-called flexible forms (see for example Wales and Woodland 1979). Yet, since these functional forms in general do not globally satisfy the conditions on utility functions of monotonicity and quasi-convexity, flexible forms turned out to be of limited use for tax simulations studies. Irrespective of this early insight, the functional forms used in Computable General Equilibrium, (CGE), models have typically imposed separability between leisure and consumption. In many cases this has compromised the policy relevance of the results obtained.

The limitations for tax simulations of additive separable functional forms, as well of flexible forms, motivated Atkinson and Stern (1980, 1981) to try a different tack based on the realisation of two related aspects of labour supply and commodity demand: First, that goods are usually purchased for use in particular activities, and second, that these activities involve the use of time. In conjunction with Gomulka they estimated, based on British survey data for 1973, a linear demand system augmented with the representation of the use of time according to the Becker theory of household production (Becker 1965). They used the estimated system to evaluate the implication of a switch from direct to indirect taxation. Importantly, they demonstrated that the standard theory of demand can still be applied to the resulting demand system when incorporating household production into the utility function[1]. However, although they justified their approach with the importance for the optimal tax system of the interaction of consumption with leisure, they did not take the analysis to its logical conclusion. They neither investigated whether a tax reform involving higher taxes on goods with high time requirements would improve welfare, nor did they derive optimal tax rates.

The Leontief specification of the relationship between the use of time and consumption inherent in the Becker approach is very restrictive. A much less restrictive alternative is to represent this relationship by a constant elasticity of substitution (CES) function. This specification under the name Constant Elasticities of Substitution with explicit representation of the Use of Time, CES-UT for short, has been employed to analyse optimal support to low income households (Munk 1998) and the welfare implications of green tax reforms (Munk 1999), and more recently to

[1] In this finding they drew on results established by Pollak and Wachter (1975).

illustrate the importance the size of the informal sector for the desirability of using border taxes in developing countries (Munk 2006).

In this paper we first highlight the ambiguities in Dixit's original contribution with respect to normalisation which may have created a barrier for the intuitive understanding of what determines desirable directions of tax reforms, and thus what determines optimal tax systems. Having clarified the issue of normalisation we attempt to provide such intuition. We then define the concept of a utility function with explicit representation of the use of time and analyses the properties of the corresponding demand system to provide a tool to illustrate this insight. We point out the limitations for tax simulation studies of using the Leontief specification, and demonstrate that in contrast the CES-UT is a flexible tool for tax reform analysis as it allows differences between various commodities with respect to their complementarity with leisure to be represented, while remaining relatively easy to interpret and to implement in applied work. The paper thus supplements the work of Atkinson and Stern (1980, 1981) by generalising the parameterised functional form they consider.

The paper is structured as follows. In Section 2 we derive conditions for social welfare improving moves away from a proportional tax system. In Section 3 we define a utility function with direct representation of the use of time and the CES-UT parameterisation of such a utility function. We then analyse the properties of the corresponding demand system and illustrate how it may be used in applied work to represent the insight obtained in the theoretical analysis. A final section summarises and concludes the paper.

2. Tax reform analysis

The commodity tax model and normalisation

Following Dixit (1975), we consider a competitive economy with one representative household and a government where there is one primary factor labelled 0 and produced commodities labelled 1,..,N. We denote the set of commodities *FC* and the set of produced commodities *C*. The household's vector of endowments is $\boldsymbol{\omega} \equiv (\omega_0, 0,..,0)$, its vector of consumptions $\mathbf{c} \equiv (c_0, c_1,.., c_N)$, and its net trade vector thus $\mathbf{x} \equiv (x_0, x_1,.., x_N) \equiv \mathbf{c} - \boldsymbol{\omega}$. The primary factor can best be thought of as time, making x_0 labour measured negatively, and c_0 the household's consumption of its time endowment, traditionally referred to as "leisure", but better called "untaxed use of time". Consumer prices are $\mathbf{q} \equiv (q_0, q_1,.., q_N)$ and producer prices are $\mathbf{p} \equiv (p_0, p_1,.., p_N)$. The government's resource requirements, $\mathbf{x}^G \equiv (x_0^G, x_1^G,.., x_N^G)$, are financed by commodity taxes, $\mathbf{t} \equiv \mathbf{q} - \mathbf{p}$. The household's preferences are represented by a strictly quasi-concave utility function, $u(\mathbf{c})$. Production takes place according to a linear production structure where, only the primary factor is used as input in the production of each product. Producer prices, and by implication the government's expenditures, $G = \mathbf{p}'\mathbf{x}^G$, may therefore be considered as fixed as a matter of normalisation.

We consider two alternative ways of representing the household's preferences. The household's preferences may, using the dual approach, be represented either in terms of

$$E(\mathbf{q},u) = \min_{\mathbf{x}} \mathbf{q}'\mathbf{x} \text{ subject to } u = u(\boldsymbol{\omega} - \mathbf{x}) \tag{1}$$

or by

$$M(\mathbf{q},u) = \min_{\mathbf{c}} \mathbf{q}'\mathbf{c} \text{ subject to } u = u(\mathbf{c}) \tag{2}$$

Using the subscript notation, we write net demand functions, as $\mathbf{E_q}(\mathbf{q},u) \equiv \{x_i(\mathbf{q},u), i \in FC\}$ and gross demand functions, as $\mathbf{M_q}(\mathbf{q},u) \equiv \{c_i(\mathbf{q},u), i \in FC\}$, and the corresponding partial demand derivatives as $\mathbf{E_{qq}}(\mathbf{q},u) \equiv \left\{\frac{\partial \tilde{x}_k}{\partial q_i}, i, j \in FC\right\}$ and $\mathbf{M_{qq}}(\mathbf{q},u) \equiv \left\{\frac{\partial c_k}{\partial q_i}, i, j \in FC\right\}$.

We formulate conditions for equilibrium under two alternative assumptions about the tax base. Assuming that taxation is *based on net trade*, the conditions for a tax vector, $\mathbf{t} \equiv \mathbf{q} - \mathbf{p}$, to correspond to an equilibrium situation may be formulated as

$$E(\mathbf{q},u) \leq 0 \tag{3}$$

$$\mathbf{t}'\mathbf{E_q}(\mathbf{q},u) \geq G \tag{4}$$

Equation (3) requires the level of utility for the household to be consistent with the level of unearned income, which, since the household receives neither profit income nor net transfers from the government, is equal to zero. Equation (4) is the government's budget constraint. These two constraints also represent the general equilibrium conditions for profit maximisation, utility maximisation and material balance (see Dixit and Munk 1977).

Under the alternative assumption that taxation is *based on consumption*, the conditions for a tax vector, $\mathbf{t} \equiv \mathbf{q} - \mathbf{p}$, to correspond to an equilibrium situation are

$$M(\mathbf{q},u) \leq p_0\omega_0 \tag{5}$$

$$\mathbf{t}'\mathbf{M_q}(\mathbf{q},u) \geq G \tag{6}$$

Multiplying equilibrium prices $\mathbf{q}$ and $\mathbf{p}$ by the same constant, (3), (4), (5) and (6) remain satisfied, but remain satisfied. Multiplying only $\mathbf{q}$ by a constant, (3), (4) or (6) also remain satisfied, but (5) does not. This leads to following extension of *Proposition 1* in Munk (1978):

Proposition 1: *In tax models at least one price (either a producer price or a consumer price) must be fixed in order for there to be a unique solution to the maximisation problem. However, if taxation is based on net trade (rather than consumption), and if*

a) 100% profit tax is imposed on profit, or

b) there are constant returns to scale,

then one consumer price and at least one producer price must be fixed.

It follows from *Proposition 1* that in the case of taxation based on net trade, and no profit income, one commodity *may* be assumed untaxed as a matter of normalisation, as is customary in optimal tax models. However, notice that it is not the case that one commodity *must* be assumed untaxed to have a unique solution. A totally adequate normalisation rule is to assume that the tax on one commodity is fixed at some other value than 0.

Dixit (1975, op. cit. p, 106) considers (3) and (5) as alternative interpretations of the household's budget constraint[2]. However, as he adopts the primary factor as numeraire in the sense that it cannot be taxed, rather than in the sense that the producer price is equal to 1, he obscures the distinction between, the behavioural assumption *that the consumption of the primary factor cannot be taxed* and the assumption *that the supply of the primary factor to the market is untaxed*. The former assumption constrains the set of feasible solutions, while the latter does not.

The transformation from the first best allocation (where the consumption of all commodities are taxed at the same rate) to the second best allocation (where only the consumption of the produced commodities are taxed at the same rate), clearly results in a decrease in the price of leisure compared to those of all other commodities. Such a price change will therefore - compared with the first best allocation - increase the household's consumption of leisure and the consumption of all commodities complementary to leisure and reduce the consumption of those which are substitutes for leisure. Starting from a proportional tax system, this suggests that increasing tax rates on those commodities which are complementary with leisure, and decreasing them for those which are substitutes for leisure, will increase social welfare.

To analyse this conjecture we consider, as Dixit (1975, op. cit.), the effect of a tax reform starting from a proportional tax system, $T_i = T$, $i \in C$. Such a reform changes

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[2] Dixit write the constraint that the value of the expenditure function must be equal to the households full income as $E(1,q,u) = Z - T + P$, where P is untaxed profit and T is a lump sum tax, and where, providing the equivalent expression in our notation, in the case of (3), $E(1,q,u) \equiv E(\mathbf{q},u)$ and $Z \equiv 0$, whereas in the case of (5), $E(1,q,u) \equiv M(\mathbf{q},u)$ and $Z \equiv \omega_0$. Dixit thus assumes that $p_0 = q_0 = 1$; however, in the presence of untaxed profit and when the household's consumption of its endowment can be taxed, one commodity cannot be assumed untaxed without loss of generality. Dixit analysis has been reinterpreted taking into account the first point in Dixit and Munk (1977); here we deal with the second point. For simplicity we assume a linear production structure such that producer prices are fixed, and that lump sum taxation is not available to the government. One commodity may also be assumed untaxed without loss of generality with variable producer price and in the presence of profit, if the profit is taxed at 100%; the optimal tax rules are in that case the same as in the case of a linear production structure, but the analysis is then complicated by producer prices being endogenous (see Munk 1978).

the equilibrium consumer price vector to $\left(\mathbf{q}+\mathrm{d}\mathbf{q}\right)$, and the utility to $\left(u+\mathrm{d}u\right)$, leaving the government's revenue unchanged.

Taking total differentials of (3) and (4), we obtain

$$\mathbf{E}_{\mathbf{q}}\mathrm{d}\mathbf{q}+\mathbf{E}_{u}\mathrm{d}u=0 \tag{7}$$

$$\mathbf{E}_{\mathbf{q}}\mathrm{d}\mathbf{q}+\mathbf{t}'\mathbf{E}_{\mathbf{qq}}\mathrm{d}\mathbf{q}+\mathbf{t}'\mathbf{E}_{\mathbf{q}u}\mathrm{d}u=0 \tag{8}$$

Solving for $\mathrm{d}u$, using that $\mathbf{t}=\mathbf{q}-\mathbf{p}$ and that by homogeneity that $\mathbf{q}'\mathbf{E}_{\mathbf{q}u}=\mathbf{E}_{u}$, we have

$$\mathrm{d}u=\Phi\mathbf{t}'\mathbf{E}_{\mathbf{qq}}\mathrm{d}\mathbf{q} \tag{9}$$

Where $\Phi=1/\left(\mathbf{E}_{u}+\mathbf{p}'\mathbf{E}_{\mathbf{q}u}\right)$. We assuming that $\left(\mathbf{E}_{u}+\mathbf{p}'\mathbf{E}_{\mathbf{q}u}\right)>0$ (see Dixit op. cit, p107 for justification).

We now define $\mathbf{t}_{-0}$ and $\mathrm{d}\mathbf{q}_{-0}$ as equal to the corresponding vectors where the 0th element has been removed, $\mathbf{E}_{\mathbf{q}_{-0}\mathbf{q}_{-0}}$ as equal to $\mathbf{E}_{\mathbf{qq}}$, where the 0^{th} row and the 0th column has been removed, and $\mathbf{E}_{\mathbf{q}_{-0}0}$ as equal to the first row of $\mathbf{E}_{\mathbf{qq}}$. Assuming that $\mathrm{d}q_0=0$ we have

$$\mathbf{t}'\mathbf{E}_{\mathbf{qq}}=t_0\mathbf{E}_{\mathbf{q}_{-0}0}\mathrm{d}\mathbf{q}_{-0}+\mathbf{t}_{-0}'\mathbf{E}_{\mathbf{q}_{-0}\mathbf{q}_{-0}}\mathrm{d}\mathbf{q}_{-0} \tag{10}$$

If we assume that the initial tax system is proportional, i.e. that the supply to the market of the primary factor is taxed at a fixed rate, t_0 and $t_i=\frac{T-1}{T}q_i>0$ for $i\in$C, then $\frac{p_i+t_i}{p_i}$ $=T>1$. With reference to *Proposition 1* we can fix $t_0<0$ as a matter of normalisation. We now consider a tax reform which changes consumer prices by $\mathrm{d}\mathbf{q}_{-0}$. Substituting for $\mathbf{t}'\mathbf{E}_{\mathbf{qq}}$ in (9) using (10) we have

$$\mathrm{d}u=\Phi\left(t_0\mathbf{E}_{\mathbf{q}_{-0}0}\mathrm{d}\mathbf{q}_{-0}+\frac{T-1}{T}\mathbf{q}_{-0}'\mathbf{E}_{\mathbf{q}_{-0}\mathbf{q}_{-0}}\mathrm{d}\mathbf{q}_{-0}\right) \tag{11}$$

Since by the homogeneity of degree zero of $\mathbf{E}_{\mathbf{q}_{-0}}$ in $\mathbf{q}$, $q_0\mathbf{E}_{\mathbf{q}_{-0}0}+\mathbf{q}_{-0}'\mathbf{E}_{\mathbf{q}_{-0}\mathbf{q}_{-0}}=0$ and by the symmetry of demand derivatives, $\mathbf{E}_{\mathbf{q}_{-0}0}=\mathbf{E}_{0\mathbf{q}_{-0}}'$

$$\mathrm{d}u=-\Phi\left(t_0-\frac{T-1}{T}q_0\right)\mathbf{E}_{0\mathbf{q}_{-0}}'\mathrm{d}\mathbf{q}_{-0} \tag{12}$$

Substituting by $\mathrm{d}x_0$, the change in the supply to the market of the primary factor (measured negatively), in (12) for $\mathbf{E}_{\mathbf{q}_{-0}0}\mathrm{d}\mathbf{q}_{-0}$, and exploiting the symmetry of the derivatives of compensated demand, we have

$$\mathrm{d}u = \Phi\left(t_0 - \frac{T-1}{T}q_0\right)\mathrm{d}x_0 > 0 \tag{13}$$

This leads to the following proposition.

Proposition 2: *In a competitive economy with constant producer prices and one primary factor, in an equilibrium with a proportional tax system in terms of the produced commodities, a small change in tax rates holding commodity tax revenue constant will increase welfare if the change in tax rates result in an increase in the compensated supply of the primary factor.*

A increase in the tax on commodity *j* balanced by a decrease in the tax on a commodity *i* changes the compensated supply of the primary factor by (see Dixit op. cit. p116)

$$\mathrm{d}x_0 = E_{0i}dt_i + E_{0j}dt_j \tag{14}$$

$$\mathrm{d}x_0 = \alpha_0\sigma_{i0}x_i dt_i + \alpha_0\sigma_{j0}x_j dt_j \tag{15}$$

As tax reform to be welfare improving must have $x_i dt_i + x_j dt_j < 0$ (see (7)) and thus

$$\mathrm{d}x_0 \le \left(\sigma_{j0} - \sigma_{i0}\right)\alpha_0 x_j dt_j \tag{16}$$

A welfare increasing tax reform which balance an increase in the tax on *j* by a reduction in the tax on i will thus if $\sigma_{i0} > \sigma_{j0}$ increase the supply of the primary factor $\left(\mathrm{d}x_0 < 0\right)$. Combining this result with *Proposition 5* we have (cf. Corlett and Hague 1953) we obtain the following proposition which confirms the conjecture formulated above.

Proposition 3: *In a competitive economy with constant producer prices and one primary factor, in an equilibrium with a proportional tax system a small decrease in the tax on one commodity balanced by a increase of the tax of another commodity less complementary with the untaxed use of the primary factor than the first commodity, will increase welfare.*

Dixit's

Theorem 6*: In a competitive economy with constant producer prices and an initial equilibrium with equal proportional distortions, a small change in tax rates holding commodity tax revenue constant will increase welfare if all commodities whose prices and lowered are better substitutes for the numeraire that all those whose prices are raised.*

follows from *Proposition 3* when “the numeraire” is replaced by “the untaxed use of the primary factor”. Dixit’s *Theorem 6* is not valid if any other commodity than the primary factor is chosen as untaxed numeraire. To establish the point consider the case where in the initial equilibrium the tax system is proportional with $T_0 < 1,\ T = 1$.

In this case any produced commodity may be considered an untaxed numeraire, but *Theorem 6* does not apply. The importance, suggested by Dixit's *Theorem 6*, of the degree of complementarity with the *untaxed numeraire* for what constitutes welfare improving directions of tax reform is thus coincidental, attributable to the fact that Dixit in his derivation has chosen as numeraire the commodity of which the household has an initial endowment, i.e. the primary factor.

Dixit's

Theorem 7: *Lowering the price of any one commodity towards its marginal cost will increase welfare if the commodity is complementary to all those with a greater proportional distortion and substitute for all other including the numeraire.*

may similarly be reinterpreting by replacing "the numeraire" by "the untaxed use of the primary factor"

Thus reinterpreted Dixit's theorems 6 and 7 give rise to the following more general proposition:

Proposition 4: *The government's problem of choosing desirable tax reforms and optimal tax rates to finance a given resource requirement by taxes on market transactions rather than by lump-sum taxes, may be seen as a problem of finding a compromise between achieving*

1) *the objective of not distorting the first-best pattern of consumption of the produced commodities,* x_i *,* $i \in C$*, (Objective 1), and*
2) *the objective of stimulating the supply of labour (Objective 2).*

In the following section we define and analyse a parameterised utility function with the explicit representation of the use of time, the CES-UT, which can be used in CGE models to represent this insight.

3. Representation of household preferences

3.1. Justification for an explicit representation of the use of time

Much applied work, and in particular applied work based on the use of general equilibrium models, adds structure to the general representation of household preferences by employing additively separable utility functions. However, this assumption severely limits the flexibility of the estimating equations from the point of view of optimal tax theory, always making a proportional tax system the optimal solution. Deaton (1981, p 1) have formulated it as follows: "*It is likely that empirically calculated tax rates, based on econometric estimates of parameters, will be determined in structure, not by the measurement actually made, but by arbitrary, untested (and even unconscious) hypotheses chosen by the econometrician for practical convenience*" (quoted from Atkinson and Stern 1980). As our initial analysis has suggested, the interaction between consumption of leisure and the consumption of the various commodities is important for whether a not a tax reform is desirable. It is therefore essential in adding structure to the representation of household preferences not to assume this interaction away, as is the case when assuming separability between leisure and consumption in the household's preferences.

3.2. Utility function with an explicit representation of the use of time

In order to avoid these pitfalls of using parameterised utility functions, we have in applied work adopted a utility function, which allows the interaction between the consumption of produced commodities and leisure to differ between produced commodities (see Munk 1998, 1999 and 2006). We define a utility function with an explicit representation of the use of time, as

$$U\left(c_0^0, C\left(C_1\left(x_1, c_0^1\right), C_2\left(x_2, c_0^2\right), .., C_N\left(x_N, c_0^N\right)\right)\right) \tag{9}$$

where $c_0^0 = \omega_0 - \sum_{i \in C} c_0^i + x_0$ is "pure leisure"[3]. For each composite good, C_i , the preference for the amount purchased of the commodity, x_i , and the time used for its consumption, c_0^i, is expressed by a concave function $C_i = C_i(x_i, c_0^i)$. These functions may be interpreted as representing either household production or consumption activities[4]. "Leisure" or "non-market use of time" is therefore $c_0 = \omega_0 + x_0 = \sum_{i \in C} c_0^i + c_0^0$. Aggregate consumption, C, is a concave function of the composite goods, $C = C\left(C_1, C_2, .., C_N\right)$, and $U\left(c_0^0, C\right)$ is a utility function with standard properties.

The assumption that the household maximises utility subject to its budget constraint may, using the expenditure function approach, be expressed as

[3] "Pure leisure" is thus defined as the amount of time spent on activities, which are not associated with the consumption of purchased commodities or the supply of labour to the market. Pure leisure may for empirical purposes be interpreted as non-market use of time which cannot be related to the consumption of any specific commodity, as for example the use of time for relaxation in ones home which typically involves the use of many durable commodities at the same time. This definition of pure leisure and the assumption that it is separable from consumption of produced commodities is useful in applied work, and we are not here going to be drawn on whether or not from an ontological point of view all time is spent on either consumption or labour. Any scientific theory has to be developed in such a way that it can be applied either to data that are available or to data that can potentially be made available, and not by trying to represent the world as it "really" is.

[4] The aggregation functions may be interpreted as home production functions, or as just constraining household preferences. In the first case, the composite commodity, C_i, is a physical entity resulting from the combinations of a purchased commodity and time as in the case of food prepared in the home; in the second case, the composite commodity is just a theoretical concept helping to structure the household's preferences, as in the case of childcare where alternative combinations of non-marketed time and the purchase of a marketed commodity can satisfy the same well-defined need. The one interpretation may be used for one application and the other for another, but it does not matter for the formal analysis.

$$E(q_0, q_1, .., q_N, u) \equiv \underset{c_0^i,\ i \in FC;\ x_i,\ i \in FC}{\text{Min}}\ q_0 x_0 + \sum_{i \in C} q_i x_i \ \ s.t.$$
$$u = U\left(\omega_0 + x_0 - \sum_{i \in C} c_0^i, C_1\left(x_1, c_0^1\right), C_2\left(x_2, c_0^2\right), .., C_N\left(x_N, c_0^N\right)\right) \tag{10}$$

which has all the standard properties of an expenditure function.

Assuming that $C_i = C_i(x_i, c_0^i)$, $i \in C$, and $C = C\left(C_1, C_2, .., C_N\right)$ are homogenous of degree 1, we have

$$\tilde{E}\left(q_0, Q, u\right) \equiv \underset{c_0^i, C}{\text{Min}}\ q_0 c_0^0 + QC \text{ s.t. } U\left(c_0^0, C\right) = u \text{ , where} \tag{11}$$

$$Q = Q\left(Q_1, Q_2, .., Q_N\right) \equiv \left(\underset{C_1, C_2, .., C_N}{\text{Min}}\ \sum_{i \in C} Q_i C_i \text{ s.t. } C\left(C_1, C_2, .., C_N\right)\right) / C \text{ , where} \tag{12}$$

$$Q_i = Q_i\left(q_0, q_i\right) \equiv \left(\underset{c_0^i, x_i}{\text{Min}}\ q_0 c_0^i + q_i x_i \text{ s.t. } C_i\left(c_0^i, x_i\right)\right) / C_i \qquad i \in C \tag{13}$$

Since $\tilde{E}\left(q_0, Q, u\right)$, $Q\left(Q_1, Q_2, .., Q_N\right)C$, and $Q_i\left(q_0, q_i\right)C_i$, $i \in C$ are expenditure functions

$$E(q_0, q_1, .., q_N, u) \equiv \tilde{E}\left(q_0, Q\left(Q_1\left(q_0, q_1\right), Q_2\left(q_0, q_2\right), .., Q_N\left(q_0, q_N\right)\right), u\right) - q_0\ \omega_0 \tag{14}$$

will also have the standard properties of an expenditure function[5]. This is an essential point; it implies that the corresponding demand system can be analysed using standard demand theory and that the insight of tax reform theory can be applied directly. That we can apply standard results is not only of considerable analytical convenience, but also facilitates the interpretation of results, which exploit the explicit representation of the use of time, as has been pointed out by Atkinson and Stern (1980)[6]. But in order to exploit those advantages it is naturally important to make a correct mapping of the variables based on a general utility function and a utility function with an explicit representation of the use of time.

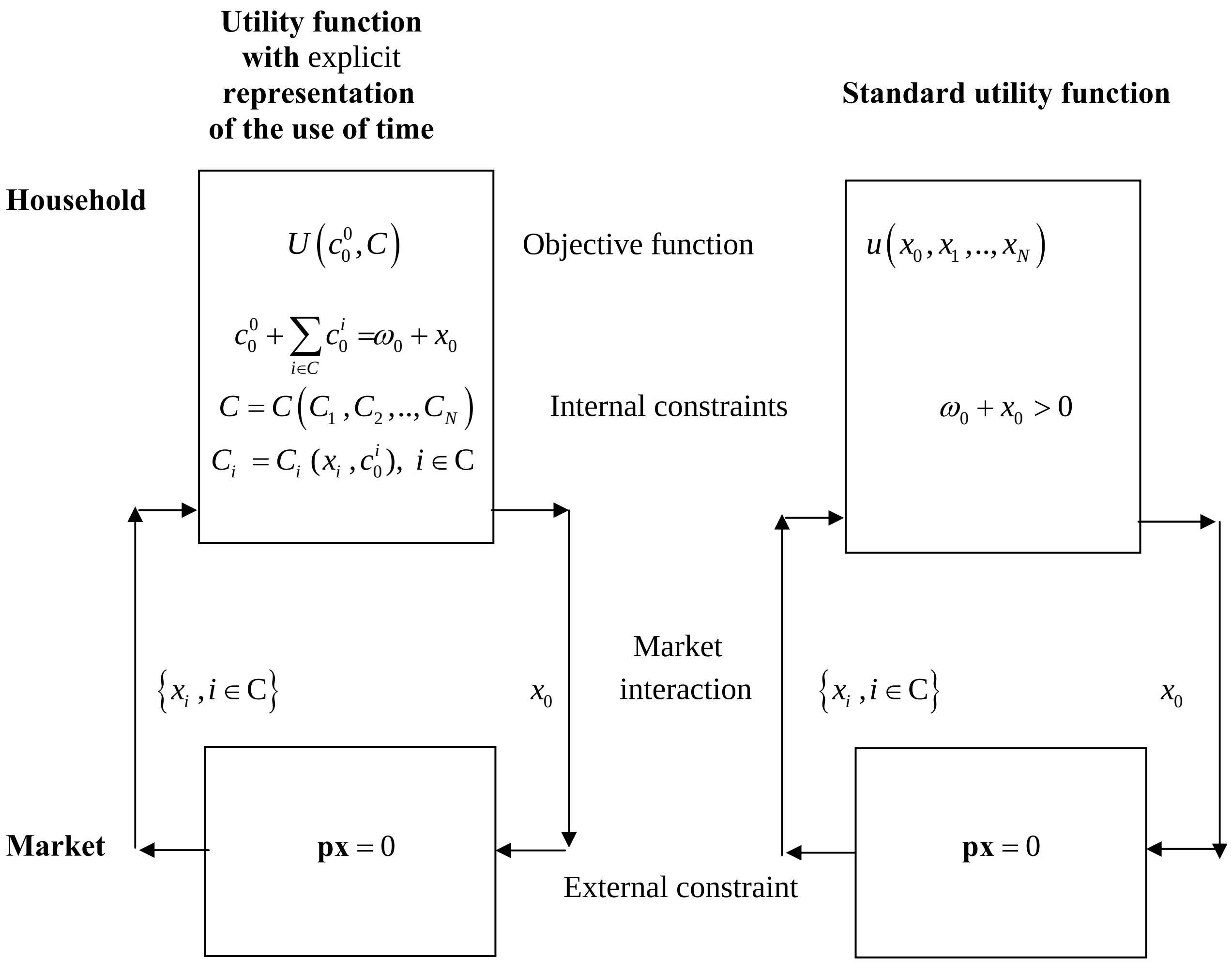
The mapping between the standard formulation of the household's maximisation problem and the formulation with the explicit representation of the use of time is illustrated in *Figure 1*. In the case of a standard utility function the household's preferences are defined directly on net trade, $x_0, x_1, .., x_N$. In the case of a utility function with the explicit representation of the use of time, the household's preferences are defined directly only on pure leisure, c_0^0, and aggregate consumption, C, where aggregate consumption is a function of the consumption of composite commodities, $C\left(C_1, C_2, .., C_N\right)$, and where the consumption of each composite commodity, C_i, $i \in C$, is a function, $C_i(x_i, c_0^i)$, of the purchase of the corresponding

[5] See also see Pollak and Wachter (1975)

[6] Atkinson and Stern (1980) write: *In the formal sense [a model incorporating home production] is no different, and we can apply the standard theory of demand, a fact which is worth emphasising in view of the claims sometimes made to the contrary. That we can apply standard results is a considerable analytical convenience, and allows us to see more clearly how the interpretation of the results differs [when incorporating home production]*

commodity , x_i , and of the time used for its consumption, c_0^i. What we have shown is that the behaviour implied by the utility function with explicit representation of the use of time, $U\left(c_0^0, C\left(C_1\left(x_1, c_0^1\right), C_2\left(x_2, c_0^2\right), .., C_{\mathrm{N}}\left(x_{\mathrm{N}}, c_0^{\mathrm{N}}\right)\right)\right)$, is the same as that of an appropriate utility function defined on net trade, $u\left(x_0, x_1, .., x_{\mathrm{N}}\right)$.

Figure 1: Alternative representation of household behaviour



The net trade vector $\mathbf{x} \equiv \left(x_0, x_1, .., x_{\mathrm{N}}\right) \equiv \boldsymbol{\omega} - \mathbf{c}$ indicates the household's interaction with the rest of the world, which in optimal tax theory is the only aspect of household behaviour, which is assumed to be observable by the government. The use of utility functions with the explicit representation of the use of time provides more structure to the explanation of changes in $\mathbf{x}$ in response to price changes than the general formulation, but remains, as we have seen, a special case of the general formulation. It is thus inconceivable that optimal tax results obtained for the general case should not be applicable to cases where household preferences are represented by utility functions with the explicit representation of the use of time.

3.3. *Flexibility with respect to the complementarity of leisure*

We now define a parameterised utility function, CES-UT (Constant Elasticities of Substitution utility function with explicit representation of the Use of Time, (see Munk 1998, Annex 1) as

$$U\left(c_0^0, C\left(C_1\left(x_1, c_0^1; \sigma^{11}\right), C_2\left(x_2, c_0^2; \sigma^{12}\right), .., C_{\mathrm{N}}\left(x_{\mathrm{N}}, c_0^{\mathrm{N}}; \sigma^{1\mathrm{N}}\right); \sigma^2\right)\sigma^3\right) \tag{15}$$

where $C_i\ (x_i\ , c_0^i; \sigma^{1i})$, $i \in$C, $C\left(C_1, C_2, .., C_{\mathrm{N}}; \sigma^2\right)$ and $U\left(C, c_0^0; \sigma^3\right)$ are CES functions characterised by elasticities of substitution σ^{1i}, $i \in C$, σ^2 and σ^3, respectively.

We want to demonstrate that the CES-UT is quite flexible with respect to representation of household preferences in particular, allowing different commodities to have different degrees of complementarity with leisure.

Differentiating (14) we get the corresponding demand system

$$x_i\left(\mathbf{q}, u\right) = \frac{\partial \tilde{E}}{\partial Q}\frac{\partial Q}{\partial Q_i}\frac{\partial Q_i}{\partial q_i} \qquad i \in \mathrm{C}\ (16)$$

$$x_0\left(\mathbf{q}, u\right) = \frac{\partial \tilde{E}}{\partial q_0} + \frac{\partial \tilde{E}}{\partial Q}\sum_{j \in \mathrm{C}}\frac{\partial Q}{\partial Q_j}\frac{\partial Q_j}{\partial q_0} - \omega_0 \tag{17}$$

We define

$$\varepsilon_{ii}^1 \equiv \frac{\partial^2 Q_i}{\partial q_i\, \partial q_i}(q_i\ , q_0)C_i \Big/ \frac{x_i}{q_i}\ ; \qquad -\varepsilon_{i0}^1 \equiv \frac{\partial^2 Q_i}{\partial q_i\, \partial q_0}(q_i\ , q_0)C_i \Big/ \frac{x_0}{q_i} \qquad i \in \mathrm{C}$$

$$\varepsilon_{ij}^2 \equiv \frac{\partial^2 Q}{\partial Q_i\, \partial Q_j}(Q_1, Q_2, ., Q_{\mathrm{N}})C \Big/ \frac{C_i}{Q_j} \qquad i,j \in \mathrm{C}$$

$$\varepsilon_{CC}^3 \equiv \frac{\partial^2 E^2}{\partial Q \partial C}(q_0, Q, u) \Big/ \frac{C}{Q}\ ; \qquad -\varepsilon_{C0}^3 \equiv \frac{\partial^2 E^2}{\partial Q \partial \mathrm{q}_0}(q_0, Q, u) \Big/ \frac{x_0}{Q}$$

$$\alpha_i \equiv \frac{q_i x_i}{q_0 x_0}$$

Differentiating (16) with respect to q_j and q_0, respectively, and defining $a_j \equiv \frac{q_j x_j}{Q_j C_j}$ as the share of the costs of the consumption of commodity *j* in the total costs of composite *j* (which include the cost of the consumption of time) and $b_j \equiv \frac{Q_j C_j}{QC}$ the share of the composite *j* in the total cost of consumption (including the consumption of time except pure leisure), we get

$$\varepsilon_{ii} = \varepsilon_{ii}^1 + a_i\, \varepsilon_{ii}^2 + a_i\, b_i\ \ \varepsilon_{CC}^3 \qquad i \in \mathrm{C}\ \ (18)$$

$$\varepsilon_{ij} = a_j \varepsilon_{ij}^2 + a_j b_j\ \ \varepsilon_{CC}^3 \qquad j \neq i \in \mathrm{C}\ \ (19)$$

$$\varepsilon_{i0} = \varepsilon^1_{i0} + \sum_{j\in C}(1-a_j)\varepsilon^2_{ij} + \sum_{j\in C} a_j b_j \varepsilon^3_{C0} \qquad i \in C \quad (20)$$ [7]

$$\varepsilon_{0i} = \alpha_i \left(\varepsilon^1_{i0} + \sum_{j\in C}(1-a_j)\varepsilon^2_{ij} + \sum_{j\in C} a_j b_j \varepsilon^3_{C0} \right) \qquad i \in C \quad (21)$$

In the case of the CES-UT, $\varepsilon^1_{ii} = \varepsilon^1_{i0} = -\left(1-a_i\right)\sigma^{1i}$, $\varepsilon^2_{ij} = b_j \sigma^2$ for $i, j \neq i \in C$ $\varepsilon^2_{ii} = -\left(1-b_i\right)\sigma^2$ for $i \in C$ and $\varepsilon^3_{CC} = -\varepsilon^3_{C0} = -(1-c)\sigma^3$, where $c \equiv \dfrac{QC}{q_0 c^0_0 + QC}$, we have

$$\varepsilon_{ii} = -\left(1-a_i\right)\sigma^{1i} - a_i\left(1-b_i\right)\sigma^2 - a_i b_i (1-c)\sigma^3 \qquad i \in C \quad (22)$$

$$\varepsilon_{ij} = a_j b_j \sigma^2 - a_j b_j (1-c)\sigma^3 \qquad i, j \in C \quad (23)$$

$$\varepsilon_{i0} = \left(1-a_i\right)\sigma^{1i} + \left(a_i - \overline{a}\right)\sigma^2 - \overline{a}(1-c)\sigma^3 \qquad i \in 1,2 \quad (24)$$

$$\varepsilon_{0i} = \alpha_i \left(\left(1-a_i\right)\sigma^{1i} + \left(a_i - \overline{a}\right)\sigma^2 - \overline{a}(1-c)\sigma^3 \right) \qquad i \in 1,2 \quad (25)$$

where $\overline{a} = \sum_{i\in C} a_i b_i$.

The elasticities of substitution between the commodities and leisure, σ_{io}, $i \in C$, are related to the compensated elasticities by $\varepsilon_{i0} = s_0 \sigma_{io}$ where s_0 is the share of labour income in full income. Differences in ε_{i0} therefore reflect differences in the complementarity with leisure of the different commodities.

The compensated elasticity of commodity *i* with respect to the price of labour, ε_{i0}, depends on three elements (see 24), : The *within element*, the *between element* and the *pure leisure element*.

The *within element* is represented by the first term, is given by $\varepsilon^1_{i0} = \left(1-a_i\right)\sigma^{1i}$. This element is always positive with respect to the value of ε_{i0}, and is larger, the larger the amount of time used for the consumption of commodity *i*, $\left(1-a_i\right)$, and the larger the elasticity of substitution between time and the commodity within the composite commodity *i*, σ^{1i}.

The *between element* is represented by the second term: $\sum_{j\in C}(1-a_j)\varepsilon^2_{ij} = \left(a_i - \overline{a}\right)\sigma^2$. This element may be positive or negative with respect to the value of ε_{i0} depending on whether commodity *i* requires a relatively large amount of time for its

[7] This formula may alternatively be derived from (18) and (19) using that $\varepsilon_{i0} = -\sum_{j\in C}\varepsilon_{ij}$ and $\varepsilon^1_{i0} = -\varepsilon^1_{ii}$ and $\varepsilon^3_{C0} = -\varepsilon^3_{CC}$.

consumption, i.e. $a_i < \bar{a}$, or relatively small amount, i.e. $a_i > \bar{a}$. In the first case, an increase in the price of the commodity and hence in the corresponding composite commodity results in a shift to composite commodities which involve the use of relatively less time, drawing in the direction of a small ε_{i0}; in the second case the opposite will be the case, drawing in the direction of a relatively large ε_{i0}. The elasticity between composite commodities, σ^2, amplifies the effect whatever its direction.

The pure leisure element is represented by the third term: $\bar{a}(1-c)\sigma^3$ is always positive with respect to the value of ε_{i0}. Furthermore the larger the share of the household's time endowment used for pure leisure $(1-c)$ and the larger the elasticity of substitution between leisure and consumption, σ^3, the larger is this element.

In particular we see that

1) for $a_i > \bar{a}$, i.e. when a relatively small amount of time is used for the consumption of commodity *i*, then relatively large substitution elasticities σ^{1i}, σ^2 imply relatively large ε_{i0}, and

2) for $\sigma^2 > \sigma^{1i}$, i.e. when the *between element* dominates, then a relatively large amount of time, $(1-a_i)$, used for the consumption of commodity *i* imply a relatively small ε_{i0}, but

3) for $\sigma^2 < \sigma^{1i}$, i.e. when the *within element* dominates, then a relatively large amount of time, $(1-a_i)$, used for the consumption of commodity <u>*i*</u> implies a relatively large ε_{i0}.

3.4. Comparison with the Atkinson and Stern (1980, 1981)[8]

In this section we show that the utility function suggested by Atkinson and Stern (1980) is a special case of the CES-UT utility function and its use for tax reform analysis.

The utility function considered by Atkinson and Stern (1980, 1981) is a special case of a utility function with explicit representation of time (9) where $U\left(c_0^0, C\left(C_1, C_2, .., C_N\right)\right)$ is a Stone-Geary utility function and the $C_i\left(x_i, c_0^1\right), i \in \mathrm{C}$ are Leontief functions. The corresponding expenditure function is

$$E(\mathbf{q},u) \equiv \tilde{E}\left(q_0, Q_1\left(q_0, q_1; \alpha_1\right), Q_2\left(q_0, q_2; \alpha_2\right), ..Q_N\left(q_0, q_2; \alpha_N\right); \gamma_i, i \in FC, \beta_i, i \in FC, u\right) \quad (29)$$

[8] This section was written prior to Kleven (2004)

$$E\left(\mathbf{q},u\right) \equiv \sum_{i\in C}\left(q_i + \left(1-\alpha_i\right)q_0\right)\gamma_i + \prod_{i\in C}\left(q_i + \left(1-\alpha_i\right)q_0\right)^{\beta_i} \quad (30)$$

where γ_i and β_i are parameters of the Stone Geary utility function. Estimating this functional form on British survey data for 1973 they discovered significant difference in the time requirement of different goods (high for tobacco, low for services).

The advantage of this specification compared with the CES-UT is that it easy to estimate, but it is as we shall see not suitable for tax reform analysis.

A proportional tax system, $\frac{p_i + t_i}{p_i} = T > 1$, $i \in$FC, based on the consumption of all commodities, including leisure, is a first-best solution whatever the structure of the household's preferences. Such a tax system involves higher consumer prices, $\mathbf{q}'' \equiv \left(Tp_0, Tp_1,..,Tp_N\right)$, and thus higher prices for composite commodities, $Q_i'' = Q_i\left(Tp_0, Tp_i\right)$, $i \in$C than the first-best solution based on lump sum taxation where the prices for composite commodities are $Q_i' = Q_i\left(p_0, p_i\right)$, $i \in$C.

If we impose the constraint that leisure cannot be taxed, i.e. that $q_0 = p_0$, it is, as we have seen, in general not possible to achieve the first-best solution. Although it is possible to choose tax rates for produced commodities to generate prices for composite commodities which create no distortion between the composite commodities, within the aggregation function $C\left(C_1, C_2,.., C_N\right)$, such tax rates will involve higher prices for produced commodities relative to the price of non-market use of time, thus in general distorting the allocation between the consumption of produced commodities and time within the aggregation functions for the composite commodities, $C_i\,(x_i, c_0^i)$, $i \in$C.

However, in the case where all the aggregation functions for the composite commodities, $C_i\,(x_i, c_0^i)$, $i \in C$, are Leontief, and where the household's consumption of pure leisure, c_0^0, is either a function only of the level of utility or nil[9], distorting the price ratio between produced commodities and time does not distort the allocation. A tax vector where leisure is untaxed, i.e. $\mathbf{t} = \left(0, q_1 - p_1,.., q_N - p_N\right)$, will therefore establish a first-best solution if the tax rates on produced commodities are chosen so that the relative prices for the composite commodities are the same as if the government's resource requirement had been financed by a proportional tax system based on the consumption on all commodities, i.e. if

$$Q_i\left(p_0, p_i + t_i\right) = Q_i\left(\left(1/(1-\tau)\right)p_0, \left(1/(1-\tau)\right)p_i\right) \quad i \in C \ (26)$$

where $\tau < 1$ is the rate of tax on the household's endowment of time net of the fixed amount used for pure leisure required to finance the government's resource requirement. Since for this utility function the aggregation functions are Leontief

[9] This is the case if $U\left(c_0^0, C\right)$ is Leontief or if c=1, i.e. if the household consumes no pure leisure.

$$Q_i' = 1/(1-\tau)\left(q_0\left(1-a_i\right)+p_i a_i\right) = \left(q_0\left(1-a_i\right)+\left(q_i + t_i\right)a_i\right) \qquad i\in\text{C} \; (27)$$

Therefore assuming $t_0 = 0$ as a matter of normalisation, we see that

$$\frac{t_i}{q_i} = \frac{\tau}{1-\tau}\frac{1}{a_i} \qquad i\in\text{C} \; (28)$$

is a first-best solution. Commodities that require relative much time for their consumptions, i.e. where a_i is relatively small, is thus taxed at a relatively high rates.[10]

The functional form used by Atkinson and Stern (1980) for tax reform analysis therefore implies that the optimal solution is first best, in other words that the first best solution need not be associated with distortionary costs.

Numerical examples illustrating the points made above are provided in *Table 2 and 3* in terms of different sets of parameter values of the CES-UT and the corresponding optimal tax systems for the bench mark data set provided *Table 1*. The corresponding compensated demand elasticities are provided in Appendix. Notice in particular the results for *Set 1* which corresponds to the situation where a first best solution can be achieved by taxes on net trade without lump sum taxation.

Table 1. Benchmark data where the government requirement is financed by a lump sum tax

Government requirement	x_0^G	50
Consumption of commodity 1	x_1	10
Consumption of commodity 2	x_2	90
Supply of labour	$-x_0$	150
Real income	$R\left(p_0, p_1, p_2, -L; p_0, p_1, p_2\right)$	100

[10] This case was first identified (but not provided the same interpretation) by Kleven (2000). He derives (expressed in our notation) the conditions $\frac{t_k}{q_k}\varepsilon_{kk}^1 + \sum_{i\in\text{C}}\frac{t_i}{q_i}a_i\varepsilon_{ki}^2 = -\theta,\ k\in C$ to be satisfied by an optimal tax structure from the government's tax optimisation problem constrained by a Becker type representation of home production assuming no pure leisure. He then derive the tax formula $\frac{t_k/q_k}{t_j/q_j} = \frac{a_i}{a_k}$, $k, j\in\text{C}$ from these conditions in the case where $\varepsilon_{kk}^1 = 0, k\in C$. This is not strictly correct since at the first best optimum $\theta = 0$.

Table 2: Parameters and net trade vectors

Parameters	**Set 1**	**Set 2**	**Set 3**	**Set 4**
Elasticity of substitution between commodity 1 and the time used for its consumption, σ^{11}	0,00	0,20	0,00	0,20
Elasticity of substitution between commodity 2 and the time used for its consumption, σ^{12}	0,00	0,20	0,00	0,20
Elasticity of substitution between the composite commodities, σ^{2}	0,50	0,00	0,50	0,20
Elasticity of substitution between consumption and pure leisure σ^{3}	0,50	0,50	0,50	0,50
Share of time in composite commodity 1, a_1	1/7	1/7	1/7	1/7
Share of time in composite commodity 2, a_2	1/2	1/2	1/2	1/2
Leisure coefficient, $1-c$	0,00	2,00	2,00	2,00

Table 3: Optimal tax solutions ant the corresponding net trade vectors and real income

Results		**Set 1**	**Set 2**	**Set 3**	**Set 4**
Tax on labour	t_0	0,00	0,00	0,00	0,00
Tax on consumption of commodity 1	t_1	1,40	0,22	1,45	0,55
Tax on consumption of commodity 2	t_2	0,40	0,58	0,42	0,55
Labour supply	x_0	150,00	142,30	146,42	141,52
Consumption commodity 1	x_1	10,00	9,29	9,64	9,15
Consumption commodity 2	x_2	90,00	83,00	86,78	82,37
Real income		100,00	98,21	99,17	98,02

It is thus relatively easy represent different degrees of complementarity with leisure for the different commodities by specifying different values of the parameters of the CES-UT, a_i , σ^{1i}, σ^{2}. In contrast to the additive separable utility functions, as the CES, the CES-UT therefore allows the complementarity with leisure to differ between commodities. Furthermore each of the parameters of the CES-UT has a clear economic interpretation, facilitating the intuitive understanding of the relation between the parameters and the elasticities.

3. Summary and concluding remarks

In this paper we have drawn attention the importance of distinction between the behavioural assumption *that the consumption of the primary factor cannot be taxed* and the assumption *that the supply of the primary factor to the market is untaxed*. The former assumption constrains the set of feasible solutions, while the latter does not. We have explained why ambiguity with respect to this distinction has created a barrier for identification of what constitute desirable directions of tax reform, and thus for what determines the optimal tax system. Reviewing Dixit's original analysis, we have provided an intuitive explanation of what determines the optimal tax system as a trade-off between two objectives: 1) the objective of not distorting the consumption of produced commodities, and 2) the objective of encouraging the supply of labour emphasised that this insight does not depends on the choice of numeraire as has been suggested.

Furthermore, we have thus in the spirit of the endeavour by Atkinson and Stern (1980, 1981) provided a parameterisation of a utility function with the explicit representation of the use of time, the CES-UT, which may be used to illustrate this trade-off. This may in applied work be used as an alternative to functional forms, which as they impose separability between consumption and leisure, may result in misleading conclusions from tax simulations studies.

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Appendix

Compensated elasticities corresponding to parameters of the CES-UT utility function in Table 2 for the benchmark dataset provided in Table 1.

Set 1: ε_{ij}	Price *j*		
Quantity *i*	1	2	0
1	-0,100	0,210	-0,110
2	0,040	-0,080	0,040
0	0,020	-0,030	0,020
Set 2: ε_{ij}	Price *j*		
Quantity *i*	1	2	0
1	-0,170	-0,080	0,250
2	-0,010	-0,160	0,170
0	-0,020	-0,150	0,170
Set 3: ε_{ij}	Price *j*		
Quantity *i*	1	2	0
1	-0,120	0,140	-0,020
2	0,030	-0,160	0,130
0	0,000	-0,110	0,110
Set 4: ε_{ij}	Price *j*		
Quantity *i*	1	2	0
1	-0,200	0,010	0,190
2	0,000	-0,190	0,190
0	-0,020	-0,170	0,190

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