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Seasonal Adjustment.

Paper prepared for the New Palgrave Dictionary of Economics, 2^{nd} edition

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Abstract

The main objective behind the production of seasonally adjusted time series is to give an easy access to a common time series data set purged of what is considered seasonal noise. Although the application of officially seasonally adjusted data may have the advantage of being cost saving it may also imply a less efficient use of the information available, and one may apply a distorted set of data. Hence, in many cases, there may be a need for treating seasonality as an integrated part of an econometric analysis. In this article we present several different ways to integrate the seasonal adjustment into the econometric analysis in addition to applying data adjusted by the two most popular adjustment methods.

JEL Classification: C10, Keywords: Seasonality

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1 Seasonal Adjustment

Seasonal adjustment of economic time series dates back to the nineteenth century and it is based on an attitude properly expressed by Jevons (1884) *page 4* who wrote:

'Every kind of periodic fluctuation, whether daily, weekly, monthly, quarterly, or yearly, must be detected and exhibited not only as a subject of a study in itself, but because we must ascertain and eliminate such periodic variations before we can correctly exhibit those which are irregular or non-periodic, and probably of more interest and importance'.

The most popular model behind seasonal adjustment in the beginning of the twentieth century was either the so-called additive Unobserved Components model

$$\begin{aligned} X_t &= T_t + C_t + S_t + I_t, \\ t &= 1, 2, ..., n \end{aligned} \tag{1}$$

where the observed series X_t is divided into a trend component, T_t, a business cycle component, C_t, a seasonal component, S_t, and an irregular component, I_t, or the multiplicative UC model

$$\begin{aligned} X_t &= T_t * C_t * S_t * I_t, \\ t &= 1, 2, ..., n \end{aligned} \tag{2}$$

where the latter is applied if the series is positive and the oscillations increases with the level of the series.

The definitions of the individual components could vary, but Mills (1924) page 357 defined the trend component, T_t, as the smoothed, regular, long-term movement of the series X_t, while the seasonal component , S_t, contain fluctuations that are definitely periodic in character with a period of one year i.e. 12 months or 4 quarters. The business cycle component, C_t, is less markedly periodic, but characterized by a considerable degree of regularity with a period of more than one year, while the irregular component, I_t, has no periodicity. A detailed description of the historical development is given in Hylleberg (1986).

The rationale behind seasonal adjustments is that the unobserved components model is useful, that the components are independent, and that the components of interest are the trend and cycle components.

The assumption of independence is a highly questionable assumption, as the actual economic time series is a result of economic agents' reaction to some exogenous seasonally varying explanatory factors such as the climate, the timing of religious festivals and business practices . For a typical economic actors, decisions designed to smooth against seasonal fluctuations will naturally interact with nonseasonal fluctuations, since the costs of such smoothing will necessarily be interrelated, through budget constraints, etc. Therefore, not only is the independence assumption economically unreasonable, seasonal patterns may be expected to change if economic agent's change their behavior rules.

Later Hylleberg (1992, p. 4) defines seasonality as 'A systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy'.

Such a view of seasonality is somewhat different from the views expressed by most statistical data producing agencies. The views of the statistical offices are well represented by the arguments put forward by OECD in OECD (1999) page vii, where the implied definition of seasonality stresses the fixed timing of certain events during the year. Likewise, they indicate that the reasons for changes in the seasonal pattern is "The trading day effect" i.e. the changing number of working days in a month, the changing number of Saturdays, and movable feasts such as Easter, Pentecost, Chinese New Year, Korean Full Moon Day.

Obviously, such factors do influence the seasonal pattern in economic time series, but in the longer run technical progress and economic considerations based on these will imply changes in the seasonal pattern as well.

In addition, the seasonal economic time series may constitute an invaluable plentiful source for testing theories about economic behavior, as the seasonal pattern is a recurrent although changing event, where the pattern despite the changes is somewhat easier to forecast than many other economic phenomena[1].

Seasonal adjustment and treatment of the seasonal components may in practise be undertaken in two ways. Either one simply apply the seasonally adjusted data produced by the statistical agencies, or integrate the modeling and adjustment into the econometric analysis undertaken.

2 The officially applied seasonal adjustment programmes

Several different methods for seasonal adjustment are in actual use, but the most popular programme is the X-12-ARIMA Seasonal Adjustment Programme, see Findley, Monsell, Bell, Otto & Chen (1998a) which is a further development of the popular X-11 seasonal adjustment programme, see Shiskin, Young & Musgrave (1967) and Hylleberg (1986). Another popular seasonal adjustment programme is the TRAMO/SEATS programme developed in Gomez & Maravall (1996).

2.1 X-12-ARIMA Seasonal Adjustment programme.

The main characteristics of the X-11 seasonal adjustment method for the monthly multiplicative model, see (2)

$$X_t = TC_t * S_t * TD_t * H_t * I_t, \tag{3}$$

where TC_t is the combined trend-cycle component, while TD_t is the trading day component, and H_t the holiday component, is the repeated application of selected moving averages such as a 12 month centered moving average to estimate TC_t followed by an actual extraction of the estimated trend-cycle component. The extraction by the moving average filters takes place after a prior adjustment for trading days and certain holidays, and a varying seasonal pattern is taken care of by applying so-called Henderson moving averages with a 9, 13 or 23 number of terms. The Henderson trend filters are used in preference to simpler moving averages because they can reproduce polynomials of up to degree 3, thereby capturing trend turning points.

[1] For a general discussion of seasonality an d the literature, see Hylleberg (1986). For a presentation and discussion of the results since then see Hylleberg (1992), Franses (1996), and Ghysels & Osborn (2001) and Brendstrup, Hylleberg, Nielsen, Skipper & Stentoft (2004) for the latest development.

In addition, treatment for so-called extreme observations was possible, and a refined asymmetric moving averages filter is used at the ends of the series.

In order to robustify the initial seasonally adjusted series against data revisions, the X-11 seasonal adjustment method was improved by extending the series by forecasts and backcasts from an ARIMA model before seasonally adjusting the series, see Dagum (1980).

The X-12-ARIMA Seasonal Adjustment programme described in Findley et al. (1998a), extends the facilities of X-11-ARIMA by adding a modeling module denoted RegARIMA, which not only facilitates modeling the processes in order to forecast and backcast the time series, but also facilitates modeling of trading day and holiday effects, detection of outlier effects, dealing with missing data, detection of sudden level changes, and detection of changes in the seasonal pattern, trading day effects etc. The second major improvement compared to the earlier programmes is the inclusion of a module for diagnostics which contains many helpful "tests". The third improvement is a user-friendly interface.

Although X-12 is a major improvement to X-11 several criticisms are raised such as Wallis (1998), who doubts that the trend estimation procedure taken over from X-11 is still the best available despite the results obtained during the last 30 years, and he emphasis the need for giving the user of the adjusted numbers an indication of their susceptibility to revision.

2.2 TRAMO/SEATS Seasonal Adjustment programme

The main difference between the X-12 programme and the TRAMO/SEATS programme is that the former uses signal-to-noise ratios to choose between the different moving average filters available while SEATS uses signal extraction with filters derived from a time series (ARIMA) model.

The programme also contains a preadjustment programme, TRAMO, which basically performs tasks similar to RegARIMA in X-12.

The signal extraction is based on an additive model such as (1) or

$$Y_t = \mu_t + \gamma_t + \varepsilon_t, \tag{4}$$

where μ_t is the trend-cycle component, γ_t the seasonal component, and ε_t is the irregular component. It is then assumed that the μ_t and γ_t can be modelled as two distinct ARIMA processes

$$\begin{aligned} A_C(L)(1-L)^d \mu_t &= B_C(L)\, v_t \quad \text{and} \\ A_S(L)(1-L^s)^D \gamma_t &= B_S(L)\, w_t \end{aligned} \tag{5}$$

where the processes v_t, w_t and ε_t are independent, serially uncorrelated processes with zero means and variances σ_v^2, σ_w^2 and σ_ε^2, and d and D are integers, while L is the lag operator. This class of model is also called Unobserved Components Autoregressive Integrated Moving-Average Models (UCARIMA) by Engle (1978).

Hence, the TRAMO/SEATS programme requires the estimation of the UCARIMA parameters for each specific series. A non negligible task, which in principle should allow computation of the correct number of degrees of freedom. This is not possible in X-12 due to the adjustments undertaken within the programme based on the characteristics of the individual series.

A discussion of the merits and drawbacks of X-12 and TRAMO/SEATS may be found in Ghysels & Osborn (2001), Hood, Ashley & Findley (2004) and in several working papers from EUROSTAT, see Mazzi & Savio (2005), who finds that X-12 is slightly preferable to TRAMO/SEATS when applied to short time series. An result to be expected as the model based approach requires more data. In fact, the main differences between the two leading competitors is due to the difference between the model based approach of TRAMO/SEATS, which tailor a seasonal filter to each series, and the uniform filter applied by X-12, see below. However, the model based approach relies on a very restrictive set of models, and the uniform filter approach is not really applying the same filter, as individual characteristics like outliers, smoothness etc. have influence on the filter.

3 Seasonal adjustment as an integrated part of the analysis.

The main objective behind the production of seasonally adjusted time series is to give the policy analyst/adviser etc. an easy access to a common time series data set, which has been purged for what is considered noise contaminating the series. Obviously, the application of the seasonally adjusted data may be more or less formal and meticulous, ranking from eyeball analysis to thorough econometric analysis.

However, although the application of officially seasonally adjusted data may have the advantage of being cost saving it also implies that the user runs a severe risk of not making the most effective use of the information available, and maybe more serious, apply a distorted set of data for the specific analysis at hand, - distorted by the applied seasonal filter.

The possible reasons for these shortcomings are

- the seasonal component is a noise component but
  - the wrong seasonal adjustment filter has been applied
  - the data have been seasonally adjusted individually without considering that they are often used as input to a multivariate analysis
- the seasonal components of different time series may be closely connected, and contain valuable information across series.

A filtering of the data before applying them may of course distort the outcome of the analysis if the wrong filter is applied, but even if the "correct" filter is applied, correct seen from the individual series, the filtering may produce biased estimates of the parameters in certain cases where, for instance, a regression model is applied, see Hylleberg (1986) page 3. However, this result is complicated by the application of other transformations to the original series. Which filter to apply may in fact depend on the order of the applied transformations as shown by Ghysels (1997).

Hence, in order to optimally model many economic phenomena, there may be a need for treating seasonality as an integrated part of an econometric analysis based on unadjusted quarterly, monthly, weekly and daily timeseries or panel data observations.

This may be done in many different ways depending on the specific context and the set of reasonable assumptions one can make within that context.

As both X-12 and TRAMO/SEATS seasonal adjustment programmes are available to the individual researcher, they may both be applied as part of an integrated approach and their use somewhat adapted to the specific analysis, but in the following we will discuss some alternative methods. The methods discussed below are in three groups

- Pure noise models,
- Time series models
- Economic models.

3.1 Pure Noise Models

The first group contains the seasonal adjustment methods which are based on the assumption that the seasonal component is noise. Thus, the group also contains the officially applied seasonal adjustment programmes presented earlier. The difference between the seasonal adjustment methods in this group lies in their ability to take care of a changing seasonal component.

3.1.1 Seasonal dummies

The use of seasonal dummy variables to filter quarterly and monthly times series data is a very simple, straightforward and therefore popular method in econometric applications. The dummy variable method is designed to take care of a constant, stable seasonal component. The popularity of the seasonal dummy variable method is partly due to its simplicity and the flexible way it can be used either as a prefiltering device where the series are regressed on a set of seasonal dummy variables and the residuals used in the final regression, or within the regression as an extension of the set of regressors by seasonal dummy variables, see Frisch & Waugh (1933) and Lovell (1963).

3.1.2 Band Spectrum Regression and Band Pass Filters

A natural and quite flexible way to analyze time series with a strong and somewhat varying periodic component is to perform the analysis in the frequency domain, where the time series is represented as a weighted sum of cosine and sine waves. Hence, the time series are Fourier transformed and the seasonal filtering of the time series may take place by removing specific frequency components from the Fourier transformed data series.

Application of such filters dates back a long time, see Hannan (1960), and Band Spectrum Regression was further developed and analyzed by Engle (1974), Hylleberg (1977), and Hylleberg (1986). Later the so-called Real

Business Cycle literature has named it Band Pass Filtering, see Baxter & King (1999). For an overview see Cogley (2006).

Let us assume that we have data series with T observations in a vector y and a matrix X related by $y = X\beta + \varepsilon$, where ε is the disturbance term and β a coefficient vector. Band spectrum regression is then performed as a regression in the transformed model $A\Psi y = A\Psi X\beta + A\Psi\varepsilon$, where the transformation by the matrix Ψ is a finite Fourier transformations of the data. The transformation by the diagonal matrix A with zeros and ones on the diagonal, symmetric around the southwest northeast diagonal, is a filtering, which removes the frequency components corresponding to the elements with the zeros. Hence, by an appropriate choice of zeros in the main diagonal of A the exact seasonal frequencies or in addition a band around them may be filtered from the series.

An obvious advantage of the band spectrum regression representation is that the model $A\Psi y = A\Psi X\beta + A\Psi\varepsilon$ lends itself directly to a test for the appropriate filtering as argued in Engle (1974). In fact the test is just the well known so-called Chow test applied to a stacked model with the null hypothesis that the parameters are the same over the different frequencies.

A drawback of band spectrum regression is that the temporal relations between series may be affected in a complicated way by the two sided filter, see Engle (1980) and Bunzel & Hylleberg (1982).

3.1.3 Seasonal integration and seasonal fractional integration

A simple filter often applied in empirical econometric work is the seasonal difference filter $(1 - L^s)^d$, where s is the number of observations per year, and d the number of times the filter should be applied to render the series stationary at the long run and seasonal frequencies, see Box & Jenkins (1970).

In the unit root literature a time series is said to be integrated of order d if its d'th difference has a stationary and invertible ARMA representation. Hylleberg, Engle, Granger & Yoo (1990) generalized this to seasonal integration and denote for instance a quarterly series $y_t, t = 1, 2, .., T$ represented by the model $\left(1 - L^4\right) y_t = \varepsilon_t$, $\varepsilon_t \sim iid\left(0, \sigma^2\right)$ as integrated of order 1 at frequency θ, since $\left(1 - L^4\right) = (1 - L)(1 + L)(1 + L^2)$ has real roots at the unit circle at the frequencies $\theta = \left\{0, \frac{1}{2}, [\frac{1}{4}, \frac{3}{4}]\right\}$, where θ is given as the share of a total circle of 2π.

Many empirical studies have applied the so-called HEGY test for seasonal unit roots developed by Hylleberg et al. (1990) and Engle, Granger, Hylleberg & Lee (1993) for quarterly data, extended to monthly data by Beaulieu & Miron (1993), and to daily data integrated at a period of one week by Kunst (1997). These tests are extensions of the well known Dickey-Fuller test for a unit root at the long-run zero frequency Dickey & Fuller (1979) and at the seasonal frequencies Dickey, Hasza & Fuller (1984)

The HEGY test is the simplest and most easily applied test for seasonal unit roots. In the quarterly case the test is based on an autoregressive model $\phi(L)y_t = \varepsilon_t$, $\varepsilon_t \sim iid\left(0, \sigma^2\right)$ where $\phi(L)$ is a lagpolynomial with possible unit roots at frequencies $\theta = \left\{0, \frac{1}{2}, [\frac{1}{4}, \frac{3}{4}]\right\}$. A rewritten linear regression model where the possible unit roots are isolated in specific terms is

$$\begin{aligned} \phi^*(L)y_{4t} &= \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t \qquad (6) \\ y_{1t} &= \left(1 + L + L^2 + L^3\right) y_t \\ y_{2t} &= -\left(1 - L + L^2 - L^3\right) y_t \\ y_{3t} &= -\left(1 - L^2\right) y_t \\ y_{4t} &= \left(1 - L^4\right) y_t. \end{aligned}$$

The lag polynomial $\phi^*(L)$ is a stationary and finite polynomial by assumption. Denoting integration of order d at frequency θ by $I_\theta(d)$ we thus have $y_{1t} \sim I_0(1)$, $y_{2t} \sim I_{\frac{1}{2}}(1)$,and $y_{3t} \sim I_{[1/4,3/4]}(1)$ while $y_{1t} \sim I_{\frac{1}{2},[1/4,3/4]}(0)$, $y_{2t} \sim I_{0,[1/4,3/4]}(0)$, $y_{3t} \sim I_{0,\frac{1}{2}}(0)$and $y_{4t} \sim I_{0,\frac{1}{2},[1/4,3/4]}(0)$ provided $y_t \sim I_{0,\frac{1}{2},[1/4,3/4]}(1)$.

The HEGY tests of the null hypothesis of a unit root are conducted by 't-value' tests on π_1 for the long-run unit root, π_2 for the semiannual unit root, and 'F-value' tests on π_3 and π_4 for the annual unit roots. In fact the 't-value' tests on π_1 is just the unit root test of Dickey and Fuller with a special augmentation applied. As in the Dickey-Fuller cases the statistics are not t or F distributed, but have non-standard distributions, which for the "t"are tabulated in Fuller (1976) while critical values for the "F" test are tabulated in Hylleberg et al. (1990).

As in the Dickey-Fuller case the correct lag-augmentation in the auxiliary regression (6) is crucial. The errors need to be rendered white noise in order for the size to be close to the stipulated significance level, but the use of too many lag coefficients reduces the power of the tests.

Obviously, if the DGP contains a moving average component, the augmentation of the autoregressive part may require long lags, see Hylleberg (1995) and the HEGY test may be seriously affected by autorcorrelation in the errors, moving average terms with roots close to the unit circle, so-called structural breaks, and noisy data with outliers.

The existence of seasonal unit roots in the DGP implies a varying seasonal pattern where "summer may become winter". In most cases such an extreme situation is not logically possible and the findings of seasonal unit roots should be taken as an indication of a varying seasonal pattern and the unit root model as a parsimonious approximation to the DGP.

Another test where the null is no unit root at the zero frequency is suggested by Kwiatkowski, Phillips, Schmidt & Shin (1992) and extended to the seasonal frequencies by Canova & Hansen (1995), and further developed by Busetti & Harvey (2003). See Hylleberg (1995) for a comparison of the Canova-Hansen test and the HEGY test. See also Taylor (2002) for a variance ratio test.

Recently, Arteche (2000) and Arteche & Robinson (2000) have extended the analysis to include non-integer values of d in the definition of a seasonally integrated process. In case d is a number between 0 and 1 the process is called fractionally seasonally integrated. The fractionally integrated seasonal process is said to have strong dependence or long memory at a frequency ω since the autocorrelations at that frequency die out at a hyperbolic rate in contrast to the much faster exponential rate in the weak dependence case where $d = 0$. In the integrated case where $d = 1$ the autocorrelations never die out.

The difficulty with the fractional model is estimation of the parameter d, and even in the quarterly case there are three possible d parameters, and the testing procedure may become very elaborate, requiring for instance a sequence of clustered tests as in Gil-Alana & Robinson (1997).

3.2 The Time Series Models

The time series models may be univariate models such as the Box-Jenkins model, unobserved components model, time varying parameter models or evolving seasonal models, or multivariate models with seasonal cointegration, periodic cointegration or models with seasonal common features.

3.2.1 Univariate seasonal models

The Box-Jenkins model. In the traditional analysis of Box and Jenkins, see Box & Jenkins (1970), the time series where s is the number of quarters, months etc. in the year, were made stationary by application of the filters $(1-L)$ and/or $(1-L^s) = (1-L)S(L)$, where $S(L) = (1 + L + L^2 + L^3 + ...L^{s-1})$, as many times as was deemed necessary from the form of the resulting autocorrelation function. After having obtained stationarity the filtered series were modelled as an Autoregressive Moving Average model or ARMA model. Both the AR and the MA part could be modelled as consisting of a non seasonal and seasonal lag polynomial. Hence, the so-called Seasonal ARIMA model has the form

$$\phi(L)\phi_s(L^s)(1-L^s)^D(1-L)^d y_t = \theta(L)\theta_s(L^s)\varepsilon_t \tag{7}$$

where $\phi(L)$ and $\theta(L)$ are invertible lag polynomials in L, while $\phi_s(L^s)$ and $\theta_s(L^s)$ are invertible lag polynomials in L^s, and D and d integers.

In light of the results mentioned in the section on seasonal unit roots the modeling strategy of Box and Jenkins may easily be refined to allow for situations were the nonstationarity exists only at some of the seasonal frequencies.

The "Structural" or Unobserved Components Model. When modeling processes with seasonal characteristics, intractable, complicated and high ordered polynomials must be applied in the ARMA representation. As an alternative to this the Unobserved Components model (UC) discussed earlier was proposed.

It is easily seen that the UCARIMA model is a general ARIMA model with restrictions on the parameters. Alternatively, the UC model may be specified as a so-called structural model following Harvey (1993).

The structural model is based on a very simple and quite restrictive modeling of the components of interest such as trends, seasonals and cycles. The model is often specified as (4). The trend μ_t is normally assumed only to be stationary in first or second differences whereas the seasonal component γ_t is stationary when multiplied by the seasonal summation operator $S(L)$. In the Basic Structural Model (BSM) the trend is specified as

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \qquad (8) \\ \beta_t &= \beta_{t-1} + \zeta_t \end{aligned}$$

where each of the error terms is independently distributed[2]. The seasonal component is specified as

$$S(L)\gamma_t = \sum_{j=0}^{n-1} \gamma_{t-j} = w_t \tag{9}$$

where s is the number of periods per year and where $w_t \sim N\left(0, \sigma_w^2\right)$.[3,4] The BSM model can also be written as

$$y_t = \frac{\xi_t}{\Delta^2} + \frac{w_t}{S(L)} + \varepsilon_t, \tag{10}$$

where $\xi_t = \eta_t - \eta_{t-1} + \zeta_{t-1}$ is equivalent to an MA(1) process. Expressing the model in the form (10) makes the connection to the UCARIMA model in (4) clear.

Estimation of the general UC model is treated in Hylleberg (1986) and estimation of the structural model is treated in Harvey, Koopman & Shephard (2004).

In the structural approach the problems of specifying the ARMA models for the components is thus avoided by a priori restrictions. Harvey & Scott (1994) argue that the type of model above which has a seasonal component evolving relatively slowly over time can fit most economic time series, irrespective of the apparently strong assumptions of a trend component with a unit root and a seasonal component with all possible seasonal unit roots present.

Periodic Models and other Time Varying Parameter Models. The periodic model extends the nonperiodic time series models by allowing the parameters to vary with the seasons. The so-called periodic autoregressive (PAR) model assumes that the observations in each of the seasons can be described using different autoregressive models, see Franses (1996).

Consider a quarterly times series y_t which is observed for N years. The

[2] If $\sigma_\xi^2 = 0$ this collapses to a random walk plus drift. If $\sigma_\eta^2 = 0$ as well it corresponds to a model with a linear trend.

[3] This specification is known as the dummy variable form, since it reduces to a standard deterministic seasonal component if $\sigma_w^2 = 0$.

[4] Specifying the seasonal component this way makes it slowly changing by a mechanism that ensures that the sum of the seasonal components over any s consecutive time periods has an expected value of zero and a variance that reamins constant over time.

stationary PAR(h) quarterly model can be written as

$$y_t = \sum_{s=1}^{4} \mu_s D_{s,t} + \sum_{s=1}^{4} \phi_{1s} D_{s,t} y_{t-1} + ... + \sum_{s=1}^{4} \phi_{hs} D_{s,t} y_{t-h} + \varepsilon_t \tag{11}$$

with $s = 1, 2, 3, 4$, $t = 1, 2, ..., T = 4N$, and where $D_{s,t}$ are seasonal dummies, or as $y_t = \mu_s + \phi_{1s} y_{t-1} + ... + \phi_{ps} y_{t-h} + \varepsilon_t$.

It has been shown that any PAR model can be described by a non-periodic ARMA model Osborn (1991). In general, however, the orders will be higher than in the PAR model. For example, a PAR(1) corresponds to a non-periodic ARMA(4,3) model. Furthermore, it has been shown that estimating a non-periodic model when the true DGP is a PAR can result in a lack of ability to reject the false non-periodic model,Franses (1996). Fitting a PAR model does not prevent the finding of a non-periodic AR process, if the latter is in fact the DGP. It practice it is thus recommended that one starts by selecting a PAR(h) model and then tests whether the autoregressive parameters are periodically varying using the method described above.

A major weakness of the periodic model is that the available sample for estimation $N = n/s$ often is too small. Furthermore, the identification of a periodic time series model is not as easy as it is for non periodic models.

Now, let us rewrite the series $y_t, t = 1, 2, 3, ...T$, as $y_{s,\tau}$, where $s = 1, 2, 3, 4$ indicating the quarter, and $\tau = 1, 2, ..., n$ indicating the year. The PAR(1) process can then be written as

$$y_{s,\tau} = \phi_s y_{s-1,\tau} + \varepsilon_{s,\tau}, s = 1, 2, 3, 4; \tau = 1, 2, ..., n \tag{12}$$

or in vector notation

$$\Phi(L) Y_\tau = U_\tau \tag{13}$$

where

$$
\begin{aligned}
\Phi(L) &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 \\ 0 & 0 & -\phi_4 & 1 \end{bmatrix} \\
& - \begin{bmatrix} 0 & 0 & 0 & \phi_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} L \\
Y_\tau' &= [y_{1,\tau}, y_{21,\tau}, y_{2,\tau}, y_{2,\tau}] \\
U_\tau' &= [\varepsilon_{1,\tau}, \varepsilon_{21,\tau}, \varepsilon_{2,\tau}, \varepsilon_{2,\tau}]
\end{aligned}
$$

with L operating on the seasons. The PAR(1) process in (13) is stationary provided $|\Phi(z)| = 0$ has all its roots outside the unit circle, which is the case if and only if $|\phi_1\phi_2\phi_3\phi_4| < 1$.

The model may be estimated by Maximum Likelihood or OLS. Testing for periodicity in (11) amounts to testing the hypothesis $H_0 : \phi_{is} = \phi_i$ for $s = 1, 2, 3, 4$ and $i = 1, 2, ..., p$, and this can be done with a likelihood ratio test, which is asymptotically χ^2_{3p} under the null, irrespective of any unit roots in y_t, see Boswijk & Franses (1995).

The vector representation of the PAR model forms an effective vehicle for generating estimation and testing procedures directly from the general result for stationary VAR models, but it also create an effective way to handle the non stationary case and compare the periodic models to the models with seasonal integration.

In the non stationary case, a periodically integrated process of order 1, $PI(1)$ is defined as a process, where there exists a quasi-difference

$$
\begin{aligned}
D_s y_{s,\tau} &= 1 - \alpha_s y_{s-1,\tau} \\
\alpha_1\alpha_2\alpha_3\alpha_4 &= 1 \\
\text{not all } \alpha_s &= 1, s = 1, 2, 3, 4.
\end{aligned} \tag{14}
$$

such that $D_s y_{s,\tau}$ has a stationary and invertible representation. Notice, that the $PI(1)$ process is neither an integrated $I_0(1)$ process nor a seasonally integrated $I_{0,\frac{1}{2},[1/4,3/4]}(1)$ process as shown in Ghysels & Osborn (2001).

The periodic models can be considered special cases of what is referred to as the Time-Varying Parameter models, see Hylleberg (1986). These are regression models of the form

$$\begin{aligned} Y_t &= X_t'\beta_t + u_t \\ B(L)\left(\beta_t - \bar{\beta}\right) &= A\gamma_t + \xi_t \end{aligned} \tag{15}$$

which can be written in state-space form and estimated using the Kalman filter. However, the number of parameters is often large compared to the number of observations, and in practice one may be forced to restrict the parameter space. A sensible assumption is that the parameters vary smoothly over the seasons, an assumption used by Gersovitz & MacKinnon (1978) applying Bayesian techniques.

The Evolving Seasonals Model The evolving seasonals model was promulgated by Hannan, Terrell & Tuckwell (1970). The model was revitalized by Hylleberg & Pagan (1997), who showed that the evolving seasonals model produces an excellent vehicle for analyzing different commonly applied seasonal models as it nests many of them. Recently, the model has been used by Koop & Dijk (2000) to analyze seasonal models from a Bayesian perspective.

The evolving seasonals model for a quarterly time series is based on a representation like

$$\begin{aligned} y_t &= \alpha_{1t}\cos(\lambda_1 t) + \alpha_{2t}\cos(\lambda_2 t) + 2\alpha_{3t}\cos(\lambda_3 t) + 2\alpha_{4t}\sin(\lambda_3 t), \\ &= \alpha_{1t} + \alpha_{2t}\cos(\pi t) + 2\alpha_{3t}\cos(\pi t/2) + 2\alpha_{4t}\sin(\pi t/2), \\ &= \alpha_{1t}(1)^t + \alpha_{2t}(-1)^t + \alpha_{3t}[i^t + (-i)^t] + \alpha_{4t}[i^{t-1} + (-i)^{t-1}], \end{aligned} \tag{16}$$

where $\lambda_1 = 0$, $\lambda_2 = \pi$, $\lambda_3 = \pi/2$, $\cos(\pi t) = (-1)^t, 2\cos(\pi t/2) = [i^t + (-i)^t]$, $2\sin(\pi t/2) = [i^{t-1} + (-i)^{t-1}]$, $i^2 = -1$, while $\alpha_{jt}, j = 1,2,3,4$, is a linear function of its own past and a stochastic term $e_{jt}, j = 1,2,3,4$. For instance,

$$\begin{aligned} \alpha_{1t} &= \rho_1\alpha_{1,t-1} + e_{1t}, \\ \alpha_{2t} &= \rho_1\alpha_{2,t-1} + e_{2t}, \\ \alpha_{3t} &= \rho_3\alpha_{3,t-2} + e_{3t}, \\ \alpha_{4t} &= \rho_4\alpha_{4,t-3} + e_{4t}. \end{aligned} \tag{17}$$

In such a model, $\alpha_{1t}(1)^t = \alpha_{1t}$ represents the trend component with the unit root at the zero frequency, $\alpha_{2t}(-1)^t$ represents the semiannual component with the root -1, while $\alpha_{3t}[i^t + (-i)^t] + \alpha_{4t}[i^{t-1} + (-i)^{t-1}]$ represents the annual component with the complex conjugate roots $\pm i$. In Hylleberg &

Pagan (1997) it is shown that the HEGY auxiliary regression in (6) has an evolving seasonals model representation, and also the Canova-Hansen test and the PAR(h) model may be presented in the framework of the evolving seasonals model.

3.2.2 Multivariate seasonal time series models

The idea that the seasonal components of a set of economic time series are driven by a smaller set of common seasonal features seems a natural extension of the idea that the trend components of a set of economic time series are driven by common trends.

If the seasonal components are seasonally integrated, the idea immediately leads to the concept of seasonal cointegration, introduced in Engle, Granger & Hallman (1989), Hylleberg et al. (1990), and Engle et al. (1993). In case the seasonal components are stationary, the idea leads to the concept of seasonal common features, see Engle & Hylleberg (1996), while so-called periodic cointegration considers cointegration season by season, see Birchenhal, Bladen-Howell, Chui, Osborn & Smith (1989), and Ghysels & Osborn (2001).

Seasonal Cointegration Seasonal cointegration exists at a particular seasonal frequency if at least one linear combination of series, which are seasonally integrated at the particular frequency, is integrated of a lower order.

Consider the quarterly case where y_t and x_t are both integrated of order 1 at the zero and at the seasonal frequencies, i.e. the transformations corresponding to 6 are $\{y_{1t}, x_{1t}\} \sim I_0(1)$, $\{y_{2t}, x_{2t}\} \sim I_{\frac{1}{2}}(1)$,and $\{y_{3t}, x_{3t}\} \sim I_{[1/4,3/4]}(1)$. Cointegration at the frequency $\theta = 0$ then exists if $y_{1t} - k_1 x_{1t} \sim I_0(0)$ for some nonzero k_1, cointegration at the frequency $\theta = \frac{1}{2}$ exists if $y_{2t} - k_2 x_{2t} \sim I_{\frac{1}{2}}(0)$ for some nonzero k_2, while cointegration at the frequency $\theta = [1/4, 3/4]$ exists if $y_{2t} - k_3 x_{2t} - k_4 x_{2,t-1} \sim I_{[1/4,3/4]}(0)$ for some nonzero pair $\{k_3, k_4\}$. The complex unit roots at the annual frequency $[1/4, 3/4]$ lead to the concept of polynomial cointegration, where cointegration exists if one can find at least one linear combination including a lag of the seasonally integrated series which is stationary.

In Hylleberg et al. (1990) and Engle et al. (1993) seasonal cointegration was analyzed along the path set up in Engle & Granger (1987).

The well known drawbacks of this method, especially when the number of variables included exceeds two, is partly overcome by Lee (1992) who extended the maximum likelihood based methods of Johansen (1995) for cointegration at the long run frequency, to cointegration at the semiannual frequency $\theta = \frac{1}{2}$.

To adopt the ML based cointegration analysis at the annual frequency $\theta = [1/4, 3/4]$ with the complex pair of unit roots $\pm i$, is somewhat more complicated, however. The general results may be found in Johansen & Schaumburg (1999), and Cubadda (2001) applies the results of Brillinger (1981) on the canonical correlation analysis of complex variables to obtain tests for cointegration at all the frequencies of interest, i.e. at the frequencies 0 and π with the real unit roots ± 1 and at the frequency $\theta = [1/4, 3/4]$ with the complex roots $\pm i$.

Periodic cointegration The periodic cointegration extends the notion of seasonal cointegration by allowing the coefficients in the cointegration relations to be periodic, see Ghysels & Osborn (2001).

Consider the example given above with two quarterly time series y_t and x_t, $t = 1, 2, ..., T$ which are integrated of order 1 at the zero and seasonal frequencies implying that a transformation by the fourth difference $1 - L^4$ will make the two series stationary. Such series are called seasonally integrated series. Let us rewrite the series as $y_{s,\tau}$ and $x_{s,\tau}$ with $s = 1, 2, 3, 4$ indicating the quarter, and $\tau = 1, 2, ..., n$ indicating the year. Hence, the eight yearly series $y_{s,\tau}$, $x_{s,\tau}$, $s = 1, 2, 3, 4$ are all integrated of order 1 at the zero frequency.

Hence, full periodic cointegration exists, see Boswijk & Franses (1995), if $y_{\tau t} - k_s x_{\tau t} \sim I_0(0)$ for some nonzero $k_s, s = 1, 2, 3, 4, , \tau = 1, 2, 3$. In case stationarity is only obtained for some $s = 1, 2, 3, 4$, partially periodic cointegration exists.

Several interesting and useful results reviewed in Ghysels & Osborn (2001) follow:

- Two seasonally integrated series may fully or partially periodically cointegrate

- Two $I_0(1)$ processes cannot be periodically cointegrated. They are either non-periodically cointegrated or not cointegrated at all.

- If two $PI(1)$ processes cointegrate in one quarter they cointegrate in all four quarters.

Periodic cointegration is a promising, but currently not fully exploited area of research, which has the inherent problem that it requires a large sample. It is therefore not surprising, that the recent advances in this area happen when data are plentiful (daily) and it is possible to restrict the model appropriately, see Haldrup, Hylleberg, Pons & Sansó (2006).

Common Seasonal Features Although economic time series often exhibit non-stationary behavior, stationary economic variables exist as well, especially when conditioned on some deterministic pattern such as linear trends, seasonal dummies, breaks etc. However, a set of stationary economic times series may also exhibit common behavior, and for instance share a common seasonal pattern. The technique for finding such patterns, known as Common Seasonal Features, see Engle & Hylleberg (1996), and Cubadda (1999), is based on earlier contributions defining common features by Engle & Kozicki (1993) and Vahid & Engle (1993).

Consider a multivariate autoregression written in error correction form as

$$\Delta Y_t = \sum_{j=1}^{p} B_j \Delta Y_{t-j} + \Pi v_{t-1} + \Gamma z_t + \varepsilon_t, t = 1, 2, ..., T, \tag{18}$$

where Y_t is $k \times 1$ vector of observations on the series of interest in period t and the error correction term is Πv_{t-1}. The vector v_t contains the cointegrating relations at the zero frequency, and the number of cointegrating relations is equal to the rank of Π. If Π has full rank equal to k the series are stationary. In the quarterly case the vector z_t is a vector of trigonometric seasonal dummies, such as $\{\cos(2\pi ht/4 + 2\pi j/T), h = 1, 2;\ j \in (-\delta T \leq j \leq \delta T),$ $\sin(2\pi h4 + 2\pi j/T), h = 1, 2;\ j \in (-\delta T \leq j \leq \delta T), j \neq 0,$ when $h = 2\}$. The use of trigonometric dummy variables facilitates the "modeling" of a varying seasonal pattern, since a proper choice of δ takes care of the neighboring frequencies to the exact seasonal frequencies. If δ is 0, the filter is equivalent to the usual seasonal dummy filter.

The implication of a full rank of the $k \times m$ matrix Γ, equal to $\min[k, m]$, is that different linear combinations of the seasonal dummies in z_t are needed in order to explain the seasonal behavior of the variables in Y_t. However, if there are common seasonal features in these variables we do not need all the different linear combinations, and the rank of Π is not full. Thus, a test of the number of common seasonal features can be based on the rank of Π, see Engle & Kozicki (1993).

The test is based on a reduced rank regression similar to the test for cointegration by Johansen (1995). Hence, the hypotheses are tested using a canonical correlation analysis between of z_t and ΔY_t, where both sets of variables are purged of the effect from the other variables in (18).

This kind of analysis has proved useful in some situations, but it is difficult to apply in cases where the number of variables is large, and the results are sensitive to the lag-augmentation as in the case of cointegration. In addition, the somewhat arbitrary nature of the choice of z_t poses difficulties.

3.3 Economic Models of Seasonality

Many economic time series have a strong seasonal component, and obviously economic agents must react to that. Hence, the seasonal variation in economic time series must be an integrated part of the optimizing behavior of economic agents, and the seasonal variation in economic time series must be a result of the optimizing behavior of economic agents, reacting to exogenous factors such as the weather, the timing of holidays etc.

The fact that economic agents react and adjust to seasonal movements on one hand and influence them on the other, implies that the application of seasonal data in economic analysis may widen the possibilities for testing theories about economic behavior. The relative ease at which the agents may forecast at least some of the causes of the seasonality may be quite helpful in setting up testable models for production smoothing, for instance.

Apart from what is caused by the easiness of forecasting exogenous factors, the type of optimizing behavior and the agents' reactions to a seasonal phenomenon may be expected not to differ fundamentally from what is happening in a non-seasonal context. However, the recurrent characteristic of seasonality may be exploited.

The economic treatment of seasonal fluctuation has been discussed in the Real Business Cycle literature, e.g. Chatterjee & Ravikumar (1992), or Braun & Evans (1995), working with a utility optimizing consumer faced with some feasibility constraint. However, in most of this RBC branch, seasonality arises from deterministic shifts in tastes and technology. A few other papers incorporate seasonality through stochastic productivity shocks, see e.g. Wells (1997) and Cubadda, Savio & Zelli (2002).

Another area is the production smoothing literature as for instance Ghysels (1988), Miron & Zeldes (1988), and Miron (1996), and habit persistence as in Osborn (1988), who present a model for seasonality and habit persistence in a life cycle consumption model.

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