DEPARTMENT OF ECONOMICS

Working Paper

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Working Paper No. 2006-1

ISSN 1396-2426

UNIVERSITY OF AARHUS • DENMARK

INSTITUT FOR ØKONOMI

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 1322
8000 AARHUS C - ☎ 89 42 11 33 - TELEFAX 86 13 63 34

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8000 AARHUS C - DENMARK ☎ +45 89 42 11 33 - TELEFAX +45 86 13 63 34

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11. January 2006

Abstract

The role of additive outliers in integrated time series has attracted some attention recently and research shows that outlier detection should be an integral part of unit root testing procedures. Recently, Vogelsang (1999) suggested an iterative procedure for the detection of multiple additive outliers in integrated time series. However, the procedure appears to suffer from serious size distortions towards the finding of too many outliers as has been shown by Perron and Rodriguez (2003). In this note we prove the inconsistency of the test in each step of the iterative procedure and hence alternative routes need to be taken to detect outliers in nonstationary time series.

KEYWORDS: Additive outliers, outlier detection, integrated processes.
JEL CLASSIFICATION: C12, C2, C22,

*Corresponding author: Department of Economics, University of Aarhus, Building 1322, DK-8000 Aarhus C, Denmark. E-mail: nhaldrup@econ.au.dk. **Department of Applied Economics University of The Balearic Islands, Ctra. Valldemossa km 7.5, 07122, Spain. E-mail: andreu.sanso@uib.es.

1 Introduction

The detection of outlying observations has attracted much attention in time series econometrics. In the classical autoregressive moving average (ARMA) paradigm it has been suggested to use iterative procedures to locate and identify the types of outliers, see e.g. Box and Tiao (1975) Chen and Liu (1992), and Gómez and Maravall (1996). For integrated data it has been shown by Franses and Haldrup (1994) and Haldrup, Montanés and Sanso (2005a) that unit root testing at both the zero and seasonal frequencies can be much effected by size distortion if no proper account is made to deal with the outliers. In all cases it appears that the detection and location of outliers should be made prior to estimation and testing regarding the essential model parameters, and hence appropriate testing procedures are needed.

Vogelsang (1999) proposes an iterative outlier detection procedure which uses the fact that the null hypothesis of a unit root can be exploited to derive a non-degenerate limiting distribution for the $t-$ratio associated with a relevant one-time dummy variable. Even though this test has the right size under the null of no single outlier, it was shown by Perron and Rodriguez (2003) that when applied in an iterative fashion to select multiple outliers, the test exhibits serious size distortion as an excessive number of outliers will be detected. Consequently, Perron and Rodriguez suggested a modified version of the Vogelsang iterative procedure which had the right size but which nevertheless appeared to suffer from power loss unless the outliers are huge. In the present note we show that even when the Vogelsang test is used to detect a single outlier the test will have asymptotic power equal to the size of the test. Hence the test is generally inconsistent.

2 The Vogelsang test

Consider the univariate process generated by

$$y_t = y_{t-1} + u_t, \qquad t = 1, 2, ..., T \tag{1}$$

where u_t is an I(0) process which for instance can be a linear process $u_t = \varphi(L)e_t$ with

$$\varphi(L) = \sum_{i=0}^{\infty} \varphi_i L^i, \qquad \sum_{i=0}^{\infty} i^2 \varphi_i^2 < \infty \tag{2}$$

and e_t is a mean zero martingale difference sequence with respect to $y_t, y_{t-1}, ...y_1$ and with

$$\sigma_e^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E(\varepsilon_t^2)$$

being finite. Without loss of generality we assume $y_0 = 0$. The sequence u_t satisfies the condition for the application of a functional central limit theorem, whereby

$$T^{-1/2}\sum_{t=1}^{[Tr]} u_t = T^{-1/2}S_t \Rightarrow \sigma^2 W(r)$$

where $W(r)$ is a standard Wiener process and "$\Rightarrow$" denotes weak convergence in distribution and

$$\sigma^2 = \lim_{T\to\infty} T^{-1}E\left(S_T^2\right) < \infty$$

is the long run variance.

The variable being observed is

$$z_t = \mu_t + y_t + \theta\delta_t \tag{3}$$

where μ_t collects the deterministic terms and δ_t is a Bernouilli-type variable independent of u_t, such that $P(\delta_t = 1) = P(\delta_t = -1) = p/2$, $P(\delta_t = 0) = 1 - p$, $0 \leq p < 1$. Accordingly, z_t is an integrated process subject to the presence of additive outliers that occur with a given probability.

The test proposed by Vogelsang (1999) is based on least squares estimation of the sequence of (spurious) regressions

$$z_t = F(t/T)'\widehat{\beta} + \widehat{\theta}D(T_{ao})_t + \widehat{u}_t \tag{4}$$

for any $T_{ao} = 1, 2, ..., T$, where $F(t/T)$ is a vector of deterministic terms such as time trends and seasonal dummy variables. $D(T_{ao})_t$ is a dummy variable that takes value 1 for $t = T_{ao}$ and 0 otherwise. The test statistic is given by

$$\tau = \sup_{T_{ao}} \left|t_{\widehat{\theta}}\left(T_{ao}\right)\right|$$

and the null hypothesis of $\theta = 0$ is rejected if τ is greater than a given critical value. Under the null hypothesis of $\theta = 0$ and the assumption that $\lambda = T_{ao}/T$ remains fixed as T grows, the asymptotic distribution of the test is given by (see Vogelsang, 1999):

$$\tau \Rightarrow \sup_{\lambda}\left|\frac{W^*(\lambda)}{\left(\int_0^1 W^*(r)^2 dr\right)^{1/2}}\right| \tag{5}$$

where $\Rightarrow$ denotes weak convergence of the associated probability measures and $W^*(r)$ are the residuals from the projection of $W(r)$ onto the space spanned by $F(r)$ on (0,1). This asymptotic distribution is free of nuisance parameters and is invariant to the autocorrelation structure of u_t.

Proposition 1 *Under the alternative hypothesis of $\theta \neq 0$, the asymptotic distribution of τ is given by (5).*

Proof. From (3), define $\mu_t = F(t/T)'\beta$. Then the data-generation process under the alternative of $\theta \neq 0$ is given by $z_t = \mu_t + (S_t + \theta\delta_t) = \mu_t + S_{\eta t}$, whereas under the null $S_{\eta t} = S_t$. Let $D^*(T_{ao})_t$ and $S^*_{\eta t}$ denote the residuals from the regression of $D(T_{ao})_t$ and z_t respectively on $F(t/T)$. Following Vogelsang (1999), (Appendix page 251), the t-ratio testing $\theta = 0$ can be written as:

$$t_{\widehat{\theta}}(T_{ao}) = \frac{T^{-1/2} S^*_{\eta T ao}}{\left(T^{-2} \sum S^{*2}_{\eta t} + o_p(1)\right)^{1/2}}$$

Note that the presence of additive outliers does not modify neither the long-run variance of $\Delta y_t : \sigma^2 = \lim_{T\to\infty} E\left(T^{-1} S^2_{\eta T}\right) = \lim_{T\to\infty} E\left(T^{-1}\left(S_T + \theta\delta_T\right)^2\right) = \lim_{T\to\infty} E\left(T^{-1} S^2_T\right)$, nor the asymptotic limits of the numerator and the denominator: $T^{-1/2} S^*_{\eta[rT]} \Rightarrow \sigma W^*(r)$ and $T^{-2} \sum_{[rT]=1}^{T} S^{*2}_{\eta[rT]} \Rightarrow \sigma^2 \int_0^1 W^*(r)^2 dr$ in both cases, for $\theta = 0$ and for $\theta \neq 0$. Hence, the test has the same limit under the null and the alternative hypothesis. ■

The implications of this result is that the power of the test will equal the size even asymptotically; hence the proposed test is inconsistent[1]. Note that the problem is present also when the test is applied in an iterative fashion. One intuition behind this result is that the presence of additive outliers introduces a MA component in I(1) processes, see e.g. Franses and Haldrup (1994). However, the asymptotic distribution of the test, given by (5), is invariant to serial correlation. Another intuition behind the proposition is that an additive outlier will become negligible compared to the I(1) stochastic trend component as the sample size tends to infinity and hence cannot be identified asymptotically.

Some Monte Carlo experiments confirm these findings. Table 1 shows the detection frequencies of the test when there is a fixed outlier in the middle of the sample of a random walk. Four sample sizes, $T = \{50, 100, 200, 400\}$, and values of $\theta = \{0, 5, 10, 15\}$ are considered. When $\theta = 0$, no outliers are present and around 96% of the times the test gets the correct conclusion of absence of outliers using Vogelsangs critical values. For $\theta > 0$, the test only detects the outlier for large values of θ and small sample sizes (say 50). When the sample size grows the performance of the test quickly deteriorates because the influence of the outlier is hidden in the total variation of the variable.

[insert table 1 about here]

Systematic AOs are considered in table 2. Four different probabilities of outliers, $p = \{0.01, 0.025, 0.05, 0.1\}$, two sample sizes, $T = \{100, 400\}$, and values of $\theta = \{5, 15\}$ are considered. The results of the experiments confirm that the test only detects a small amount of the effective number of outliers. Only for large magnitudes of the outlier (say, $\theta = 15$) the total number of detected outliers almost corresponds to the actual number of AOs. As well as for fixed outliers, the sample size deteriorates the ratio between detected outliers and effective outliers.

[1] Of course, the test will be consistent against a sequence of alternatives where the size of the outliers are allowed to increase with the sample size at a given speed. However, in practice we believe this class of models is of little interest.

[insert table 2 about here]

Hence, we conclude that the test proposed by Vogelsang (1999) is generally inadequate for the detection of outliers.

3 Conclusions

We have shown that the testing procedure of Vogelsang (1999) to detect additive outliers in unit root processes is inconsistent. Fortunately, alternative testing procedures are available. In particular, Perron and Rodriguez (2003) have suggested a test for additive outliers adequate for outlier detection in integrated time series. The test uses first differences of the data and has excellent power and size properties. Haldrup, Montañés and Sanso (2005b) generalize the test to data observed with a seasonal frequency and possible seasonal unit roots.

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4 Appendix: Tables

Table 1: Detection frequencies for the Vogelsang τ statistic for a fixed outlier.

θ	T	$n_{ao}=0$	$n_{ao}=1$	$n_{ao}>1$
0	50	0.971	0.012	0.017
	100	0.955	0.027	0.018
	200	0.962	0.018	0.020
	400	0.964	0.011	0.025
5	50	0.781	0.202	0.017
	100	0.878	0.100	0.022
	200	0.948	0.031	0.021
	400	0.961	0.014	0.025
10	50	0.299	0.672	0.029
	100	0.580	0.380	0.040
	200	0.791	0.179	0.030
	400	0.900	0.073	0.027
15	50	0.080	0.890	0.030
	100	0.261	0.696	0.043
	200	0.490	0.478	0.032
	400	0.732	0.240	0.028

Notes: The data-generating process is given by $z_t = y_t + D(0.5T)_t$, $t = 1, 2, ...T$, where $\Delta y_t = \varepsilon_t, \varepsilon_t \sim N(0, 1)$. The auxiliary regression is given by: $y_t = \mu + \hat{\theta} D(T_{ao})_t + \hat{u}_t$. 1000 replications and 10% significance level were used. n_{ao} stands for the number of outliers detected.

Table 2: Detection frequencies for the Vogelsang τ statistic for systematic outliers.

p	θ	T	$\overline{N}_{ao}$	$\overline{n}_{ao}$
	0	100	0	0.024
		400	0	0.235
0.01	5	100	1.023	0.142
		400	4.070	0.273
	15	100	1.023	0.832
		400	4.070	1.238
0.025	5	100	2.509	0.245
		400	9.959	0.285
	15	100	2.509	1.855
		400	9.959	2.519
0.05	5	100	5.074	0.338
		400	20.113	0.341
	15	100	5.074	3.093
		400	20.113	4.265
0.1	5	100	9.974	0.394
		400	40.029	0.352
	15	100	9.974	3.343
		400	40.029	4.585

Notes: The data-generating process is given by $z_t = y_t + \theta\delta_t$, $t = 1, 2, ...T$, where $\Delta y_t = \varepsilon_t, \varepsilon_t \sim N(0,1)$ and δ_t is an independent sequence of Bernouilli variables with $P(\delta_t = 1) = P(\delta_t = -1) = p/2$. The auxiliary regression is given by: $y_t = \mu + \hat{\theta}D(T_{ao})_t + \hat{u}_t$. 1000 replications and a 10% significance level were used. $\bar{N}_{ao}$ stands for the (average) number of outliers in the samples and $\bar{n}_{ao}$ for the (average) number of outliers detected.

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