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Directional Congestion and Regime Switching in a Long Memory Model for Electricity Prices*

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Abstract

The functioning of electricity markets has experienced increasing complexitiy as a result of deregulation in recent years. Consequently this affects the multilateral price behaviour across regions with physical exchange of power. It has been documented elsewhere that features such as long memory and regime switching reflecting congestion and non-congestion periods are empirically relevant and hence are features that need to be taken into account when modeling price behavior. In the present paper we further elaborate on the co-existence of long memory and regime switches by focusing on the effect that the *direction* of possible congestion episodes has on the price dynamics. Under non-congestion prices are identical. The direction of possible congestion is identified by the region with excess demand of power through the sign of price differences and hence three different states can be considered: Non-congestion and congestion periods with excess demand in the one or the other region. Using data from the Nordic power exchange, Nord Pool, we find that the price dynamics and long memory features of the price series generally are rather different across the different states. Also, there is evidence of fractional cointegration at some grid points when conditioning on the states.

Keywords: Cointegration, electricity prices, forecasting, fractional integration and cointegration, long memory, Markov switching.

JEL Classification: C2, C22, C32.

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1 Introduction

The design of many electricity markets is characterized by some interesting features which affect the price dynamics in such markets. In particular, regions can be physically interconnected bilaterally in the exchange of electricity whereby the price formation is dependent upon whether the market is subject to congestion or non-congestion. For instance, the Nordic power exchange, Nord Pool, is organized such that when no bottlenecks or congestions exist bilaterally at exchange points the prices across regions will be identical, whereas the market mechanism makes prices depart in situations with capacity constraints. It is thus natural to consider price processes which accommodate regime switching subject to the presence or absence of congestion. Furthermore, the direction of the congestion reflecting the region of excess demand for power may be of importance. Yet another aspect of electricity prices is that the price series seem to indicate a high degree of long memory measured in terms of their fractional order of integration, see e.g. Escribano, Pena, and Villaplana (2002) and Haldrup and Nielsen (2005).

In some studies, for instance Granger and Ding (1996), Diebold and Inoue (2001), and Granger and Hyung (2004), it has been argued that under certain conditions time series variables can spuriously have long memory when measured in terms of their fractional order of integration (see Granger and Joyeux (1980), Beran (1994), and Baillie (1996)), when in fact the series exhibit non-linear features such as regime switching. Haldrup and Nielsen (2005) suggest a model framework which allows for separate long memory price dynamics depending upon whether the bilateral market is subject to congestion or non-congestion. The model is of the Markov switching type originally defined by Hamilton (1989). However, because the defining property of e.g. a non-congestion state is that prices are identical, the state variable is observable as opposed to being a latent variable. An important feature of the model is that the price processes in the different regimes can have different degrees of long memory which gives rise to a number of interesting possibilities. For instance, consider the state with non-congestion and assume that the associated bivariate prices are fractionally integrated of a given order. It follows that prices are fractionally cointegrated in this case, i.e. extending the notion of Granger (1981) and Engle and Granger (1987), in the sense that individual prices are fractionally integrated but price differences are identically zero. Thus, an extreme form of cointegration occurs in this situation because the prices are identical and hence are governed by exactly the same price shocks. The price behavior in the congestion state can (and typically will) be very different. That is, the bivariate prices can be fractionally cointegrated in a more conventional way or the prices can appear not to cointegrate. Hence the model can potentially exhibit state dependent fractional cointegration.[1] By not appropriately conditioning on the congestion state, i.e. when having a model with no regime switching, the full

[1] In the literature, Markov switching (integer-valued) cointegration models have been suggested by a number of authors, see *inter alia* Krolzig (1997), Krolzig, Marcellino, and Mizon (2002), and Hansen and Seo (2002).

sample estimates are likely to be a convex combination of the behavior in the single states and hence misleading inference is likely to result. Indeed, this is one of our major empirical findings in Haldrup and Nielsen (2005).

In the present paper we further explore this class of models by allowing for an increased number of states. In particular, the Nord Pool electricity price data at hand allows us to identify the direction of a possible congestion. Consider electricity prices in two regions A and B, p_A and p_B. If $p_A = p_B$ prices are identical and there is a free flow of power across the regions, i.e. the non-congestion state. The congestion state considered by Haldrup and Nielsen (2005) concerns the case $p_A \neq p_B$. However, when $p_A > p_B$ there is excess demand in region A whereas when $p_A < p_B$ there is excess demand in region B. It is of interest to further explore the price dynamics in these separate states. First of all, the spurious results argued to potentially exist when disregarding the congestion/non-congestion states are likely to extend to the case where the price dynamics depends upon the direction of the congestion but the direction is left unmodeled. In other words, if (fractional) cointegration cannot be found to exist in the congestion state (as defined in Haldrup and Nielsen (2005)) it is still possible, for instance, that cointegration may be present when there is excess demand in region A ($p_A > p_B$) but not when there is excess demand in region B ($p_A < p_B$).

Identifying separate price dynamics is important for several reasons. Since the operation of electricity markets is similar to the operation of financial markets with electricity power derivatives being priced and traded in highly competitive markets, the dynamics of the price process is important, both in means and variances, even though our focus in the present paper only concerns the modeling of the mean-process. Furthermore, the price dynamics is of interest with respect to competition analysis of electricity markets where market delineation is a central issue, see e.g. Sherman (1989) and Motta (2004). Even though most power markets are highly liberalized there is still a scope for regulating authorities to closely follow the market behavior, see also Fabra and Toro (2005). Under non-congestion there is obviously a single price existing in the market and the relevant geographical market consists of the regions with identical prices. However, when there is directional congestion in one way or the other it is of interest to follow the price dynamics closely because suppliers can have a dominating position. The geographical market delineation thus becomes less straightforward in this case. If the price dynamics appears to be very different there is scope for further examination of the market conditions by regulatory authorities.

The plan of the paper is as follows. In the next section we give a very brief description of the way the Nordic power market functions. Section 3 presents the data, and preliminary analyses are conducted which support the idea of separating between the three distinct regimes previously defined. The subsequent section presents the 3-state long memory model with regime switching, and section 5 presents the empirical results. The analysis shows that there is strong support for very different time series properties of the price data in the different regimes. In some states the prices cointegrate fractionally and in other states they do not. When estimation is made without appropriate conditioning on the relevant states the analysis shows that there is a high risk of making spurious inference regarding the price dynamics. In the final section we conclude.

2 The power market in the Nordic countries

Often, electricity markets, like the Nordic Power market, have capacity barriers which tend to affect the relevant market delineation, depending upon the existence or absence of bottlenecks across neighboring regions.[2] The Nordic power market consisting of Norway, Sweden, Finland, and Denmark has undergone a remarkable development towards liberalization over the past decade or so and today all Nordic power markets have adapted to the new competitive environment and serves as a model for the restructuring of other power markets[3].

The supply of electricity power in Norway is almost 100% hydropower whereas Sweden and Finland use nuclear plants, fossil-fuel powered plants, and hydropower. Approximately 90% of the Danish electricity is produced from conventional thermal plants and combined heating and power facilities; a minor proportion (10-12%) of Danish supply is from wind power turbines[4]. The hydropower production is mainly found in the northern parts of the Nordic power web whereas thermal power plants are located in the south. In general, the relatively cheap hydropower generation is transmitted to the heavily populated southern regions which of course requires a well established power grid transmission capacity to facilitate the flow. When the reservoir levels are adequate, the less costly hydropower production causes the market to prefer this energy source and thus causes low spot prices. In these cases national and cross-border transmission systems will be used to their capacity in order to level out price discrepancies across regions. On the other hand, when reservoir levels are low there will be a net flow from south to north, and the market will see relatively high prices for thermally generated electricity.

Figure 1 about here

With respect to the connection points in the transmission of power, Norway is divided into three regions (North Norway (NNO), Mid Norway (MNO), and South Norway (SNO)), Denmark is divided into two regions (East Denmark (EDK) and West Denmark (WDK)), and Sweden (SWE) and Finland (FIN) each constitute their own region. This division reflects the physical linkages of

[2] In microeconomics a related subject concerns the development of peak-load pricing models for non-storable public goods. Often, these models are studied in the context of regulation of, e.g., electricity supply. Classical treatments of this subject are Boiteux (1949) and Steiner (1957) and a recent review is Crew, Fernando, and Kleindorfer (1995). Some extensions that are relevant for electricity markets in particular include the consideration of multiple technologies and time periods in Crew and Kleindorfer (1976) and the introduction of supply-side uncertainty in Kleindorfer and Fernando (1993).

[3] See Nord Pool (2003a) which provides a detailed description of the Nordic power market.

[4] The total power supply for the Nordic area is 55% hydro, 24% nuclear, 20% thermal and combined heating, and 1% renewable.

power exchanges and the bidding areas with respect to the pricing of electricity as we shall explain shortly. Figure 1 displays the actual electricity exchange points.

The power spot market[5] - operated by Nord Pool Spot A/S - is an exchange where market participants trade power contracts for physical delivery the next day and is thus referred to as a day-ahead market. The spot market is based on an auction with bids for purchase and sale of power contracts of one-hour duration covering the 24 hours of the following day. At the deadline for the collection of all buy and sell orders the information is gathered into aggregate supply and demand curves for each power-delivery hour. From these supply and demand curves the equilibrium spot prices - referred to as the system prices - are calculated.

In a situation where no grid congestions (or grid bottlenecks) exist across neighboring interconnectors there will be a single identical price across the areas with no congestions. However, when there is insufficient transmission capacity in a sector of the grid, a grid congestion will arise and the market system will establish different price areas. This is because the Nordic market is partitioned into separate bidding areas which become separate price areas when the contractual flow between bidding areas exceeds the capacity allocated by the transmission system operators for spot contracts. The direction of the flow congestion can be easily identified from observed prices since the bidding area with the largest price is the area with excess demand. When no capacity constraints exist in a given hour, the spot system price is also the spot price for the entire Nordic power exchange area, i.e. the system price. The situation where different price areas arise due to bilateral congestions is relevant within the Norwegian power system and the border interconnectors between the Nordic countries. Because separate prices may coexist depending upon regional supply and demand causes the relevant market definition to vary with time. Many different price area combinations will occur in practice. In some hours there will only be a single price area, in other hours there will be two or more price areas.

Our main interest in the present paper is to analyze separate prices *bilaterally* across grid points and in particular we will focus on the direction of the flow congestion. Hence we will not address the price dynamics for multiple price areas simultaneously even though such an analysis might be of interest in future work.

3 The data

For the Nord Pool area hourly spot electricity prices are available for the period 3 January 2000 - 25 October 2003, thus yielding more than 33000 observations. For East Denmark the sample period starts slightly later and only about 27000 observations are available. We decided not to include North Norway in the

[5] Since only the spot market will be relevant for the present study, only this market will be described here, see also Nord Pool (2003b). In Nord Pool (2003c) a description is given of the futures and forward markets of the Nordic power exchange which are used for price hedging and risk management.

study since most of the year this market is merged with Mid Norway.

Figure 2 about here

Figure 2 displays the electricity log price series. Most price series are characterized by huge fluctuations and outliers, however, the general level of these series tends to be highly persistent possibly with mean reversion. These features are well-known to exist for electricity prices, see e.g. Escribano, Pena, and Villaplana (2002), Carnero, Koopman, and Ooms (2003), and Atkins and Chen (2005). Since weather conditions are dominant factors influencing equilibrium prices through changes in both supply and demand, it seems reasonable that prices will exhibit mean reversion, see e.g. Lucia and Schwartz (2002) and Knittel and Roberts (2005). Also, the year-to-year variation in water reservoirs is rather significant and the fact that more than 50% of total electricity supply is from hydropower plants explains an important part of the within year seasonal variation.

In Haldrup and Nielsen (2005) both the unit root I(1) and I(0) hypotheses were tested using Phillips-Perron and KPSS tests (see Phillips and Perron (1988) and Kwiatkowski, Phillips, Schmidt, and Shin (1992)). Both these hypotheses were strongly rejected and hence suggest that neither an I(1) nor an I(0) description of the price series is appropriate. An alternative way of measuring long memory and mean reversion is by estimation of fractionally integrated processes for the price series (see Granger and Joyeux (1980), Beran (1994), and Baillie (1996)). In fact, as shown by Lee and Shie (2004) and Lee and Schmidt, both the Phillips-Perron and the KPSS tests are consistent against fractional alternatives if the fractional order is less than unity. Estimates of the fractional order of integration of the series were obtained by specifying a multiplicative seasonal ARFIMA (SARFIMA) model

$$A(L)(1-aL^{24})(1-L)^d(y_t-\mu)=\varepsilon_t, \qquad \varepsilon_t \sim nid(0,\sigma_\varepsilon^2), \tag{1}$$

where $A(L)$ is a lag polynomial of order 8 capturing the within-the-day effects, the polynomial $(1-aL^{24})$ corresponds to a daily quasi-difference filter, and μ is a mean. The mean component captures deterministic seasonality which was obtained by regression on seasonal dummy variables (hour-of-day, day-of-week, and month-of-year) prior to the estimation of (1). We tried other specifications including longer $A(L)$ polynomials and weekly stochastic seasonality instead of daily, but (1) was found to be the superior model in terms of in-sample fit and whiteness of the residuals. The estimates were obtained by conditional maximum likelihood estimation[6] of (1) and typically lie in the interval $0.41 < d < 0.52$ with West Denmark being an exception with a slightly lower estimate, 0.31.[7] Carnero *et al.* (2003) also find long memory in Norwegian electricity price

[6] Note that the estimation method and asymptotic normality of the estimates do not require Gaussianity of the errors, but only that they are *i.i.d.* or martingale differences, and furthermore the data can be both stationary or non-stationary, for details see Tanaka (1999) and Nielsen (2004).

[7] These estimation results are presented in Haldrup and Nielsen (2005).

data but less so in electricity markets of The Netherlands, Germany, and France. Whether this feature reflects the fact that a significant amount of power within the Nord Pool area is generated from hydropower plants is an open question, however, it is a well-known empirical finding that e.g. river flows and water reservoir levels exhibit long memory, see Hurst (1951, 1956), and this might have an effect on power supply and prices.

Figure 3 about here

The characteristics of prices given above are entirely univariate and do not reflect cross-region linkages. In particular, we are interested in the regime switching features of the data, that is, the feature that in certain hours capacity constraints prevent electricity from flowing freely across grid points. Under congestion the market prices across neighboring grid point regions will differ whereas prices will be identical under non-congestion. In Figure 3 scatter plots for each of the seven grid points within the Nord Pool area are displayed. As seen, there is a clear tendency for a significant number of observations to lie on a 45 degree line. Obviously, these observations reflect non-congestion hours. On the other hand, observation pairs off the 45 degree line reflect congestion hours.

The analysis in Haldrup and Nielsen (2005) focused on the possibility of two regimes: The congestion state and the non-congestion state. However, the data provides further insights regarding the *direction* of congestion in the congestion state. In particular, if referring to A as the region on the vertical axis and B as the region on the horizontal axis, observations above the 45 degree line indicate that $p_A > p_B$, and hence there is excess demand for electricity in region A. On the other hand, observation pairs below the 45 degree line indicate that $p_A < p_B$ and hence indicating excess demand in region B. In the following we define the 3 states as follows:

Regime 0:	No congestion, $p_A = p_B$	
Regime 1:	Congestion with excess demand in region A,	$p_A > p_B$
Regime 2:	Congestion with excess demand in region B,	$p_A < p_B$

Table 1 about here

It follows from Figure 3 that the direction of a given congestion is not evenly distributed for a particular grid point. In Table 1 the number of observations in each regime is indicated for each of the seven grid points. As seen, some grid points are more subject to congestion than others. Conditional upon congestion, there are clear differences regarding which area cause a possible bottleneck. There can be many reasons behind insufficient capacity and congestion originating from both the demand and the supply side, but obviously there is also the possibility that congestion (of either types) is caused by exploitation of market power, and hence is a scope for further analysis by regulating authorities.

Table 2 about here

We assume that the transition between states follows a Markov process, c.f. Hamilton (1989), although the states are observable as mentioned above. Since the separate states are observable it is fairly easy to calculate the transition dynamics across the different states and the mean duration of each of the states. The estimated transition matrices for each grid point is defined as

$$P = \begin{bmatrix} p_{00} & p_{01} & p_{02} \\ p_{10} & p_{11} & p_{12} \\ p_{20} & p_{21} & p_{22} \end{bmatrix} \tag{2}$$

with

$$\sum_j p_{ij} = 1 \text{ for } i, j = 0, 1, 2.$$

Estimates of the transition probabilities are reported in Table 2 using the formula

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}, \text{ for } i, j = 0, 1, 2, \tag{3}$$

where n_{ij} is the number of observed transitions from state i to state j.

Table 3 about here

The mean durations of the particular states are reported in Table 3. For all gridpoints the separate states appear to be fairly persistent even though the non-congestion state generally is most persistent (the links including West Denmark being an exception). Also, the transition back to the non-congestion state appears to be very similar regardless of initially being in states 1 or 2, i.e. $\widehat{p}_{10} \approx \widehat{p}_{20}$. On the other hand, the probabilities of moving into states 1 and 2 when initially being in the non-congestion state, state 0, are relatively dissimilar. The transitions directly from states 1 to 2 (and opposite) are generally few with the Mid-South Norway connection being an exception.

4 A 3-state regime switching model with long memory

In some studies it has been argued that long memory in the form of fractional integration can easily be interchanged with non-linear models. For instance, Diebold and Inoue (2001) demonstrate that mixture or regime switching models with suitably adapted time varying transition probabilities can generate an autocovariance structure similar to fractionally integrated processes, see also Granger and Ding (1996). In addition, Bos, Franses, and Ooms (1999), Haldrup and Nielsen (2003), and Granger and Hyung (2004) argue that level shifts that are not appropriately dealt with can result in spurious indication of long memory and one may conjecture that in fact many types of hidden non-linearity can be expected to generate long memory as a result of model misspecification.

Here we extend the model of Haldrup and Nielsen (2005) which accommodates *both* fractional integration *and* regime switching simultaneously. The model in Haldrup and Nielsen (2005) allows for 2 regimes: congestion versus non-congestion. The extension made here concerns the number of states in accordance with the discussion in the previous section. A number of different scenarios can be considered. Assume that in the non-congestion state the bilateral price series across a particular grid point are fractionally integrated. In this case an extreme form of (fractional) cointegration occurs because the single prices are identical and thus the difference in log prices is identically zero. In the congestion state the price behavior can be very different. Separating the congestion state into the regime 1 and regime 2 states where the location of the excess demand region is further detailed, very dissimilar price dynamics can exist. Comparing prices without considering the different regime possibilities it is hard to say what to expect from the data, however, the mixing of the processes across regimes is likely to produce series which have a behavior being a convex combination of separate state processes.

Extending the model (1) to be state dependent we can define a 3-state regime switching multiplicative SARFIMA or RS-SARFIMA[8] as follows,

$$A_{s_t}(L)\left(1 - a_{s_t}L^{24}\right)(1 - L)^{d_{s_t}}\left(y_t - \mu_{s_t}\right) = \varepsilon_{s_t,t}, \qquad \varepsilon_{s_t,t} \sim nid\left(0, \sigma^2_{s_t}\right), \quad (4)$$

where $A_{s_t}(L)$ is a 8th order lag polynomial and $s_t = 0, 1, 2$ denotes the regime, determined by a Markov chain with transition probability matrix (2). All states are observable and hence differ from Hamilton's (1989) class of regime switching models, where the Markov process generating the states is unobserved.

For each bilateral price pair, the (univariate) series y_t may denote one of the two individual log price series or the log relative price series. In each case, y_t has been corrected for deterministic seasonality prior to the estimation of (4), and to reflect the regime switching nature of the model and using the fact that the regimes are observable, the coefficients on the dummy variables are allowed to differ across states. When y_t denotes a log relative price, all parameters are zero when $s_t = 0$ including σ^2_0, i.e. a deterministic state. We experienced with several alternative specifications of the regime switching model (4), i.e. with a longer $A_{s_t}(L)$ polynomial and weekly instead of daily stochastic seasonality, but (1) was found to be the superior model in terms of in-sample fit.

We have previously described how the transition probabilities can be calculated. Estimation of the remaining parameters is by conditional maximum likelihood following the results of Tanaka (1999) and Nielsen (2004) for the non-switching model (1). By normality of the errors in (4), the likelihood function reads

$$L = -\frac{T}{2}\ln\left(\frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}^2_{s_t,t}\right) - \frac{T}{2}\left(1 + \ln(2\pi)\right), \quad (5)$$

where

$$\hat{\varepsilon}_{s_t,t} = \hat{A}_{s_t}(L)\left(1 - \hat{a}_{s_t}L^{24}\right)(1 - L)^{\hat{d}_{s_t}}\left(y_t - \hat{\mu}_{s_t}\right), \qquad s_t = 0, 1, 2,$$

[8]Note that the model (4) is a regime switching version of the non-switching model (1).

and we use the convention that $\hat{\varepsilon}_{j,t} = 0$ if $s_t \neq j$ for $j = 0, 1, 2$. That is, we take advantage of the fact that the regimes are observable which allows us to extract the residual series to maximize the likelihood function (5). Equivalently, the estimation procedure can be described as the maximization of (5) with $\hat{\varepsilon}_t = \sum_{j=0}^{2} \hat{\varepsilon}_{j,t} 1 (s_t = j)$ instead of $\hat{\varepsilon}_{s_t,t}$ and $1(\cdot)$ being the indicator function. Because regimes are observable, the maximization problem can be solved by minimizing the residual sum-of-squares $\sum_{t=1}^{T} \hat{\varepsilon}_t^2$. The maximum likelihood estimate of the variance is

$$\hat{\sigma}_{s_t}^2 = \frac{1}{T} \sum_{t=1}^{T} \hat{\varepsilon}_{s_t,t}^2, \qquad s_t = 0, 1, 2,$$

where $\hat{\varepsilon}_{j,t} = 0$ if $s_t \neq j$ for $j = 0, 1, 2$. The starting values for the numerical maximization of (5) were the estimates from the two-state multiplicative SARFIMA model of Haldrup and Nielsen (2005), i.e. both regimes 1 and 2 used the estimates from the congestion state of Haldrup and Nielsen (2005) as starting values.[9]

According to the results of Tanaka (1999) and Nielsen (2004) for the non-switching model, errors do not in fact have to be Gaussian, but may be martingale differences. This is important in our case where the errors are likely to be heavy tailed. Another important aspect discussed in these references is that the estimation results do not depend on the stationarity or non-stationarity of the data; standard asymptotics apply for all values of the fractional integration parameters.

Disregarding the matrix P, which is not needed for the estimation of the remaining parameters, the RS-SARFIMA model (4) has exactly three times as many parameters as the non-switching SARFIMA model (1). Since the estimation is by conditional maximum likelihood, the significance of the RS-SARFIMA model relative to the simpler and more parsimonious SARFIMA model can be tested by means of e.g. a likelihood ratio (LR) test. Such an LR test would thus be asymptotically χ^2 distributed with degrees of freedom equal to twice the number of parameters in each state. This test can be calculated because the states are observable as opposed to the Hamilton (1989) class of models with latent regimes.

In addition, another LR test can be calculated to test the significance of our 3-state regime switching model against the two-state model of Haldrup and Nielsen (2005), i.e. a test of the significance of the directional congestion effect. Such a test would be χ^2 distributed with degrees of freedom equal to the number of parameters in each state. In the subsequent empirical analysis we will apply both these two LR tests (null of no switching and null of no directional effect) to examine the significance of our regime switching model in the Nord Pool data.

[9]Throughout we used the Ox programming language, see Doornik (2001), for all the calculations.

5 Empirical findings

In this section we adopt the model and estimation framework described above to the Nord Pool electricity data set. Each data set considered includes a pair of log prices for two regions connected physically in a grid point plus the associated log relative price. Before estimation, each log price series y_t (which for each data series can be either of the two individual price series or the log relative price) had deterministic seasonality removed by regression on dummy variables corresponding to hour-of-day, day-of-week, and month-of-year. The coefficients on the dummy variables are allowed to differ across the different states. In case y_t is a log relative price, all dummy variable coefficients in regime 0, i.e. the non-congestion state, are estimated to be zero. Hence the pre-filtering removes deterministic seasonality while allowing for seasonal effects to be different across states.

Table 4 about here

Table 4 presents the empirical results. The first three columns present the estimates when no regime switching is allowed for and correspond to estimation of the model (1). The estimate $\widehat{d}_1$ corresponds to the fractional d estimate of the first region, whereas $\widehat{d}_2$ corresponds to the second region, and $\widehat{d}_3$ signifies the estimate for the log relative price. Note that these estimates correspond to the imposition of the restriction that all parameters, and in particular the estimated d values, should be identical across states. These estimates are very similar to those reported in Haldrup and Nielsen (2005). Differences occur because the detrending was done prior to estimation of (1) and hence slightly different detrending is applied due to the differently defined states. Similarly, the estimates of d for a particular region are subject to the connecting region comparison is made with since different subsamples are used due to the different definition of states.

The next 9 columns contain the estimates of d for the same series, but conditional upon the state, i.e. these estimates correspond to estimation of (4) where the superscript signifies the regime. All estimates of parameters in the non-congestion regime 0 are identically zero when y_t is a relative price, that is, $\widehat{d}_3^0 \equiv 0$. In Table 4 all standard errors of estimates are provided in parentheses.

Table 5 about here

Table 5 reports a number of likelihood ratio tests for the adequacy of the regimes. The first three columns test the null of no regime switching against the model with three regimes whilst the last three columns report the LR tests where the null of no directional effect is tested against the model with a directional effect. In other words, the first tests test the non-switching model against the 3 regime model, whereas the remaining tests test the 2 regime model against the 3 regime model. Note that the 2 regime model only separates between congestion and non-congestion. All the tests reject at any reasonable level of significance, and hence there is strong support *for* the presence of 3 regimes.

Next, we examine some of the empirical results concerning the long memory properties of the price series at the individual grid points. Consider first the West Denmark - Sweden (WDK-SWE) connection. The non-switching model provides no evidence of fractional cointegration amongst the series in this case. The single series appear to be integrated of different orders (0.30 respectively 0.39) and the log relative price has $\widehat{d}_3 = 0.26$. Addressing the regime-switching model, the non-congestion state tells a different story. In this case estimates for the single prices are 0.40 and 0.44, respectively, and the log relative has $\widehat{d}_3^0 = 0$. Hence, fractional cointegration is particularly strong in this case. In Haldrup and Nielsen (2005) it was further found that the congestion state did *not* exhibit cointegration for this connection. The present analysis tells a somewhat different story because a further detailed analysis of the congestion state can be made. The state 1 regime in this case reflects the state where there is excess demand in West Denmark. The integration orders in this case are estimated to be 0.20 and 0.21 for West Denmark and Sweden, respectively, and hence are very similar. The log relative prices are close to being $I(0)$ in this case where d_3^1 is estimated to be 0.02 (with a 0.06 standard error). Hence there is also strong indication of fractional cointegration to occur in state 1, although the type of fractional cointegration is of a more conventional form.[10] In state 2, where congestion originates from excess demand in Sweden, there is no indication of cointegration, i.e. $\widehat{d}_1^2 = 0.23$, $\widehat{d}_2^2 = 0.25$ and the log relative price has $\widehat{d}_3^2 = 0.32$. When considering the full sample there is thus a strong evidence that the state 2 price behavior dominates the overall process behavior because no cointegration was found in the model with no regime switching.[11]

In general, the qualitative results for the Mid-South Norway (MNO-SNO) connection are very similar to those just discussed for the WDK-SWE link except that the non-switching model indicated fractional cointegration. Hence, the "fractional cointegration" part of the sample observations seems to dominate the series based on the full sample in this case.

For the West Denmark-South Norway (WDK-SNO) link, there is no indication of cointegration in the non-switching model, and in the switching model fractional cointegration only takes place in the non-congestion state, state 0. However, the likelihood ratio test still indicates that the price behavior is different in the two congestion states, but as seen parameter estimates do not indicate fractional cointegration in either of the two congestion states.

With respect to the South Norway-Sweden (SNO-SWE) and Mid Norway-Sweden (MNO-SWE) connections, the non-regime switching model suggests fractional cointegration, and equally so for the non-congestion regime in the regime switching model. However, no cointegration seems to exist in either of congestion states. These findings support the findings of the 2-regime model of Haldrup and Nielsen (2005) although the likelihood ratio tests find strong support for the 3 regime model.

[10] The cointegration feature occuring in state 1 may originate from prices being indirectly connected through the South Norway connection, even though there is a physical bottleneck between West Denmark and Sweden.

[11] State 2 covers about 22% of all observations for this connection.

Finally, we comment briefly on the East Denmark-Sweden (EDK-SWE) and Sweden-Finland (SWE-FIN) connections. In both these cases, the numerical optimization of the likelihood function for the 3-state regime switching model failed to converge for the log relative prices (although it did converge for the individual prices and for the non-switching and two-state regime switching models for the log relative prices). We attribute this failure to converge to the fact that, exactly for these two connections, very few observations fall into regimes 1 and 2 relative to regime 0. In particular, c.f. Table 1, for EDK-SWE $n_1 + n_2 = 2,429$ relative to $n_0 = 25,276$ and for SWE-FIN $n_1 + n_2 = 3,911$ relative to $n_0 = 29,373$ (indeed, for EDK-SWE only 99 observations appear in regime 2). For all other connections, $n_1 + n_2 \geq 9,253$. Some general comments can be made about these two connections, though, based on the estimates that are available in Table 4. In both cases, there are clear signs of fractional cointegration based on the non-switching model and also in the non-congestion state of the switching model. However, even with the estimates available it appears that the inference from the non-switching model is spurious in the sense that there is clearly no cointegration when there is excess demand in SWE (regime 2 for EDK-SWE and regime 1 for SWE-FIN) since in that case the individual price series appear to be I(0).

6 Conclusion

We have presented a 3-state regime switching model for electricity prices which allows for different (long memory) price dynamics in the separate states. The analysis refines the model set up of Haldrup and Nielsen (2005) to allow for more than two states depending on the direction of the congestion between grid points. The analysis shows that for Nord Pool data the model extension is empirically relevant and hence suggesting in many cases a different long memory price behavior depending upon the nature of the market conditions at a particular point in time. Further analysis of the price behaviour by e.g. the regulatory authorities may be called for in these cases. In particular, deeper analysis of the reasons underlying different congestion states may be conducted. Our analysis can be considered a first test to identify grid points with very separate price behavior in different congestion states.

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Table 1: Observations in each regime

Bivariate series	n_0	n_1	n_2
EDK-SWE	25276	1430	99
WDK-SWE	19590	6049	7645
WDK-SNO	15865	9961	7458
MNO-SWE	24031	3256	5997
SNO-SWE	22775	2324	8185
MNO-SNO	21017	7975	4292
SWE-FIN	29373	2255	1656

Note: n_i is the number of hours the link has been in regime i, e.g. n_1 means the price in the first exchange is higher.

Table 2: Estimated matrices of transition probabilities

EDK-SWE	0	1	2
0	0.987	0.012	0.001
1	0.212	0.787	0.001
2	0.263	0.020	0.717

WDK-SWE	0	1	2
0	0.874	0.057	0.069
1	0.199	0.787	0.014
2	0.166	0.022	0.812

WDK-SNO	0	1	2
0	0.877	0.055	0.068
1	0.098	0.892	0.010
2	0.131	0.027	0.842

MNO-SWE	0	1	2
0	0.955	0.018	0.027
1	0.131	0.869	0.000
2	0.110	0.000	0.890

SNO-SWE	0	1	2
0	0.953	0.015	0.032
1	0.143	0.857	0.000
2	0.091	0.000	0.909

MNO-SNO	0	1	2
0	0.943	0.036	0.021
1	0.094	0.878	0.028
2	0.106	0.051	0.843

SWE-FIN	0	1	2
0	0.980	0.013	0.007
1	0.165	0.835	0.000
2	0.138	0.001	0.861

Note: The transition probability matrices are estimated as in (2) and (3) for each of the seven physical connections.

Table 3: Mean duration of states, λ (hours)

Bivariate series	$\widehat{\lambda}_0$	$\widehat{\lambda}_1$	$\widehat{\lambda}_2$
EDK-SWE	76.92	4.69	3.54
WDK-SWE	7.93	4.70	5.32
WDK-SNO	8.15	9.27	6.37
MNO-SWE	22.22	7.63	9.12
SNO-SWE	21.23	7.00	11.05
MNO-SNO	17.48	8.19	6.37
SWE-FIN	48.78	6.05	7.20

Note: $\widehat{\lambda}_i$ is the estimate of the mean duration of state i in hours.

Table 4: Switching model estimates of d for log prices and log relative prices

	SARFIMA (1)			RS-SARFIMA (4)								
Bivariate series	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_3$	$\hat{d}_1^0$	$\hat{d}_1^1$	$\hat{d}_1^2$	$\hat{d}_2^0$	$\hat{d}_2^1$	$\hat{d}_2^2$	$\hat{d}_3^0$	$\hat{d}_3^1$	$\hat{d}_3^2$
EDK-SWE	0.43 (0.012)	0.43 (0.012)	0.06 (0.019)	0.46 (0.011)	0.24 (0.023)	−0.02 (0.016)	0.46 (0.011)	0.10 (0.015)	−0.01 (0.013)	0	–	–
WDK-SWE	0.30 (0.016)	0.39 (0.012)	0.26 (0.018)	0.40 (0.021)	0.20 (0.033)	0.23 (0.021)	0.44 (0.012)	0.21 (0.014)	0.25 (0.006)	0	0.02 (0.060)	0.32 (0.016)
WDK-SNO	0.28 (0.017)	0.38 (0.012)	0.25 (0.018)	0.35 (0.021)	0.22 (0.033)	0.22 (0.022)	0.39 (0.010)	0.17 (0.025)	0.26 (0.006)	0	0.17 (0.061)	0.32 (0.016)
MNO-SWE	0.42 (0.011)	0.41 (0.012)	0.21 (0.015)	0.42 (0.008)	0.04 (0.006)	0.22 (0.015)	0.44 (0.010)	−0.01 (0.007)	0.18 (0.015)	0	0.34 (0.046)	0.28 (0.028)
SNO-SWE	0.44 (0.012)	0.42 (0.012)	0.18 (0.019)	0.42 (0.007)	0.09 (0.009)	0.24 (0.010)	0.43 (0.009)	0.02 (0.010)	0.19 (0.012)	0	0.41 (0.033)	0.27 (0.029)
MNO-SNO	0.43 (0.011)	0.44 (0.011)	0.15 (0.018)	0.34 (0.006)	0.36 (0.013)	0.33 (0.012)	0.34 (0.006)	0.35 (0.013)	0.31 (0.013)	0	0.09 (0.030)	0.28 (0.063)
SWE-FIN	0.39 (0.012)	0.38 (0.012)	0.24 (0.017)	0.43 (0.010)	0.03 (0.006)	0.19 (0.012)	0.43 (0.011)	−0.00 (0.008)	0.13 (0.011)	0	–	–

Notes: Standard errors are in parentheses. The subscripts denote the price region (3 is the log relative price) and the superscripts denote the state.

Table 5: LR tests of switching models for log prices and log relative prices

	Null of no switching			Null of no directional effect		
Bivariate series	LR_1	LR_2	LR_3	LR_1	LR_2	LR_3
EDK-SWE	2228^{**}	2650^{**}	5168^{**}	848^{**}	1088^{**}	–
WDK-SWE	448^{**}	3738^{**}	2787^{**}	236^{**}	3530^{**}	319^{**}
WDK-SNO	518^{**}	3760^{**}	2150^{**}	316^{**}	3288^{**}	330^{**}
MNO-SWE	3188^{**}	2512^{**}	6882^{**}	1576^{**}	800^{**}	968^{**}
SNO-SWE	3724^{**}	2542^{**}	5850^{**}	1912^{**}	1000^{**}	512^{**}
MNO-SNO	3670^{**}	3506^{**}	4068^{**}	550^{**}	220^{**}	1276^{**}
SWE-FIN	7670^{**}	6218^{**}	–	652^{**}	1560^{**}	–

Notes: LR_i is the likelihood ratio test for price region i ($i = 3$ is the relative price). The LR test of the null of no switching, i.e. a test of equal coefficients in all three states, is χ^2 distributed with 24 degrees of freedom (1% critical value is 43.0), and the LR test of the null of no directional effect is a test of equal coefficients in states 1 and 2 which is χ^2 distributed with 12 degrees of freedom (1% critical value is 26.2). One and two asterisks denote significance at the 5% and 1% level, respectively.

Figure 1: Map of the Nord Pool Power Grid

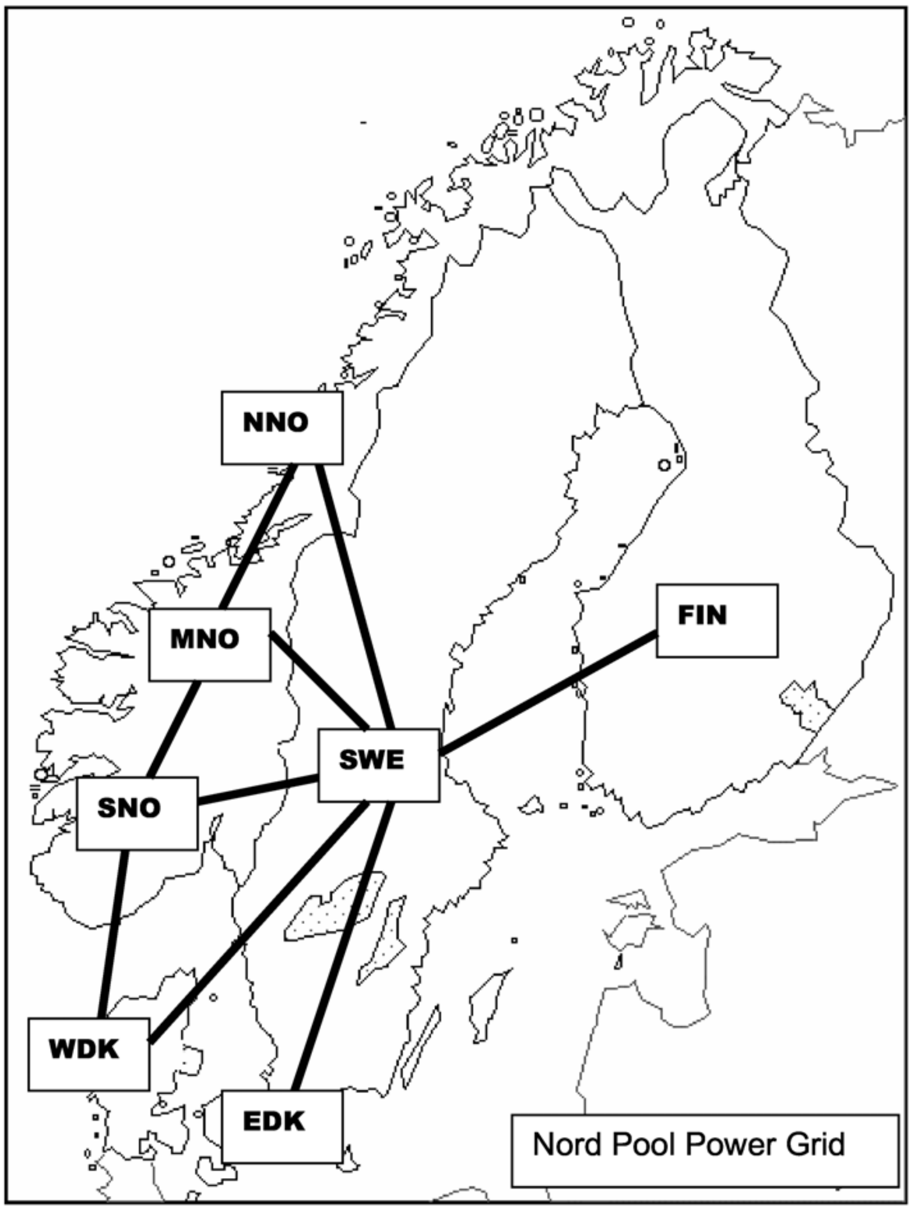
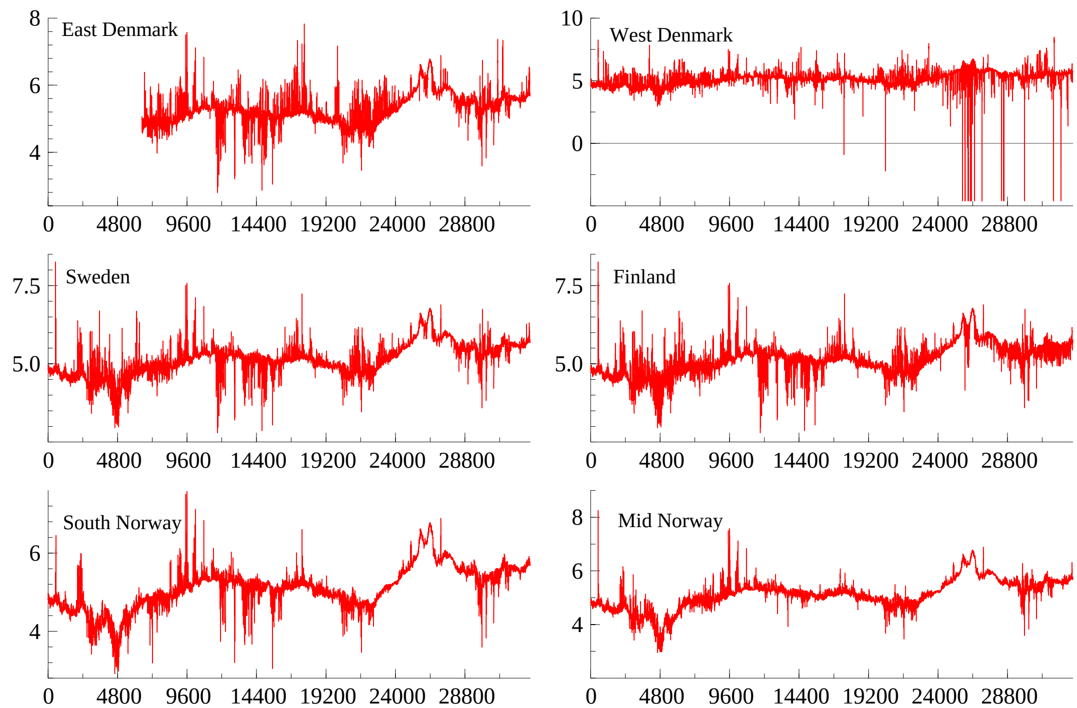
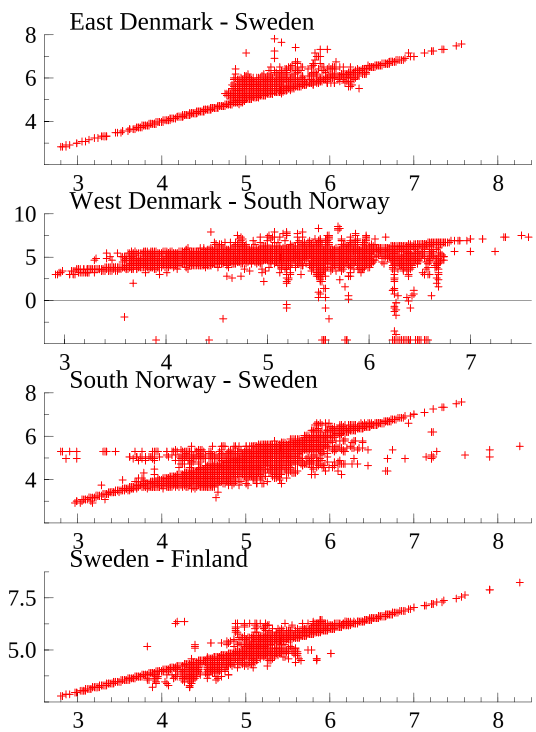


Figure 2: Hourly log spot electricity prices for the Nord Pool area



Note: Observations are from 3 January 2000 to 25 October 2003.

Figure 3: Scatter plots of hourly log prices across Nord Pool regions



Note: Observations are from 3 January 2000 to 25 October 2003.

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