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EQUILIBRIUM IMPLICATIONS OF FISCAL POLICY WITH TAX EVASION*

Francesco Busato† Bruno Chiarini‡ Guido M. Rey§

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Abstract

This paper studies equilibrium effects of fiscal policy within a dynamic general equilibrium model where tax evasion and underground activities are explicitly incorporated. There are three main results. **(i)** The underground sector mitigates the distortionary impact of fiscal policies, while lessening the drop (rise) of aggregate production after restrictive (expansionary) tax shifts. In this respect, tax evasion and the informal economy offer a channel for insuring income and consumption from distortions generated by fiscal policy. **(ii)** Tax evasion and underground economy can completely reverse the theoretical predictions of the standard neoclassical growth model and rationalize expansionary responses to contractionary fiscal policies. **(iii)** A dynamic general equilibrium with tax evasion gives a rational justification for a variant of the Laffer curve.

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1 Introduction

This paper studies the equilibrium effects of fiscal policy within a dynamic general equilibrium model in which tax evasion and underground activities are explicitly incorporated.

The macroeconomic literature on the "equilibrium approach to fiscal policy" studies effects of fiscal policy within neoclassical growth models.[1] We are not aware of any contribution, however, that evaluates the macroeconomic effect of fiscal policy explicitly incorporating tax evasion and underground activities.[2]

This might be an important part of the story since underground activities and tax evasion are a fact in many countries; for example, Schneider and Enste (2000) estimates suggest that the average size of the underground sector (as a percentage of total GDP) over 1996-97 account for 39 percent in developing countries, for 23 percent in transition countries, and for about 17 percent in OECD countries. Avoiding tax payments is the very reason for the existence of tax evasion, and this may have an important impact on the effectiveness of fiscal policy to reach the desired objectives.

To investigate relationships between underground economy, taxation and public expenditure, we use a dynamic general equilibrium model in which there are three agents: firms, households, and government.[3] In addition there are two sectors: the regular and the underground sectors. Firms and households are subject to distortionary taxation, but they can use the underground sector to evade taxes, by reallocating labor services into it. Government faces tax evasion originating from the underground sector, and coordinates strategy to address abusive tax evasion schemes. Public expenditure is allocated to the purchase the final consumption goods. Our analysis focuses on the stationary equilibrium of the model.

Here is an overview of our results. Tax evasion and the underground economy mitigate the distortionary impact of fiscal policies, while lessening the drop (rise) of aggregate production after restrictive (expansionary) tax changes. Tax evasion and the informal economy offer a channel for self-insuring income and consumption patterns from distortions generated by fiscal policy.[4] In particular, the elasticities of aggregate GDP to an increase (cut)

[1] Aschauer (1988), and Baxter and King (1993) are seminal contributions sharing an emphasis on the supply-side response of labor and capital to shifts in government demand and tax rates. Recent related contributions are: Braun (1994), McGrattan (1994), Mountford and Uhlig (2002), Burnside Eichenbaum and Fisher (2003), and Fiorito and Kollintzas (2004).

[2] McGrattan, Rogerson and Wright (1994) study fiscal policy effects in a dynamic general equilibrium model for the U.S. economy augmented with a household production sector. The model reveals the significant influence of household production in its affection on official variables. It generates different predictions for the effects of tax changes than similar models without household production. Notice, however, that an underground sector significantly differs from a household production sector.

[3] None of the previous contributions focus on optimal fiscal policy; neither does our model, to allow a consistent comparison with this literature. For quantitative implications of optimal fiscal policy within dynamic general equilibrium models, see, for instance, Chari, Christiano and Kehoe (1995), or Cooley (1993).

[4] We could think, for example, that the government chooses in fact the statutory tax rates, while effective tax rates are endogenously chosen by households and firms relying on the additional dimension represented by tax evasion.

in income and/or corporate tax rates are almost zero under tax evasion, while are negative (positive) without. The negative sign of the elasticities without tax evasion is consistent with the predictions of the neoclassical growth model; on the other hand, the almost zero elasticities under tax evasion are perfectly consistent with consumption and income smoothing done through the underground sector.

In this context, tax evasion can completely reverse the theoretical predictions of the standard neoclassical growth model, under proper conditions, which are formally derived in the sequel. Tax evasion can in fact rationalize expansionary responses of an economy to contractionary fiscal policies (i.e. an increase in tax rates); and notice that these effects would not be possible in a standard dynamic general equilibrium model without tax evasion. If the tax rate goes beyond a certain threshold (which is precisely identified in Section 4.2.1), the additional increase in tax burden motivates households and firms to pursue abusive tax evasion schemes to avoid the excessive tax payments, by reallocating labor services toward the underground sector, because it is not subject to taxation.[5] This additional income (the tax wedge) would then be used for additional consumption and investment, pushing the economy into an expansion.[6]

An additional finding of our paper is that a dynamic general equilibrium model with tax evasion gives a rational justification for a variant of a Laffer curve.[7] It is here shown that a Laffer curve under tax evasion is *almost always* below the one computed for a 100 percent regular economy, as long as tax rates do not exceed the previously mentioned threshold. In this case, however, government revenues would be driven up by the income increase triggered by the underground sector's expansion. A Laffer curve under tax evasion is therefore characterized by an upward sloping tail, for tax rates higher than the critical threshold.

The paper proceeds as follows. Section 2 presents selected stylized facts, and Section 3 details the model. Section 4 analyzes the equilibrium effects of fiscal policies under tax evasion, and Section 5 concludes. Proofs and derivations are sketched in the Appendix.

2 Stylized Facts: Underground Economy and Tax Evasion

We present here data for the Italian economy because it possesses a large underground sector. This analysis, however, is addressed to many European countries and to the United

[5] For example, that tax evasion would negatively affect government revenues, and, by this end, the public expenditure level and its financing.

[6] It is important to underline that we would observe an increase of regular GDP and of government revenues collected from the regular economy; the role of the underground sector is here as the spark that ignites the mechanism.

[7] The Laffer curve is named after Art Laffer who suggested that as taxes are increased from fairly low levels, tax revenue received by the government would also increase. However, there would come a point where people would not regard it as worth working so hard. This lack of incentives would lead to a fall in income and therefore a fall in tax revenue. The logical end point is that with tax rates at 100 percent where no one would work and so tax revenue would become zero. More details below.

States as well.[8] **Figure 1** below presents estimates for size of the underground economy, and for tax evasion. All series are reported as a percentage of aggregate GDP.

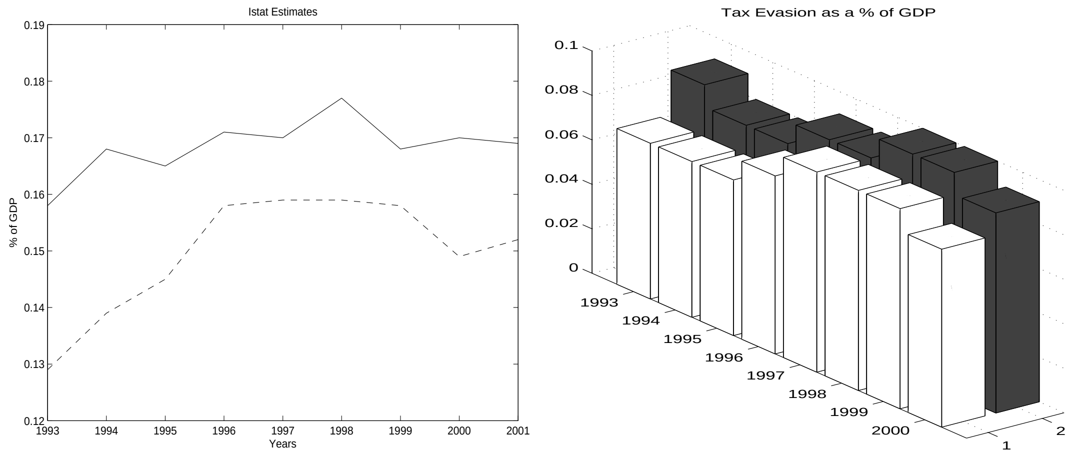

Figure 1: **Underground Economy and Tax Evasion for the Italian Economy**. Left Panel: Underground Economy as a percentage of GDP; the solid (dashed) line represents the highest (lowest) estimate; data are provided by Italy's National Statistical Institute (ISTAT) for the sample 1993:2000. Right Panel: Tax Evasion as a Percentage of GDP; the darker (white) series represents the (lowest) highest estimate; Source: Authors' calculations over the sample 1993:2000.

The size of underground economy ranges between 13 and 18 per cent of the GDP.[9] Given the difficulty to obtain official time series statistics for tax evasion, we attempt a conservative estimate to give an idea of the figures we are talking about. Conservatively assuming that the effective tax rate for the whole economy is the minimum of the effective income (τ_Y) and firm tax rates (τ_F), we compute two approximate measures for tax evasion as $TaxEv_{\min} = (\min(\tau_F, \tau_Y)) \times u_t^{\min} \times GDP_t$ and $TaxEv_{\max} = (\min(\tau_F, \tau_Y)) \times u_t^{\max} \times GDP_t$, where $u_t^{\min}$ and $u_t^{\max}$ denote lowest and highest official estimates for underground economy share as a percentage of the GDP. The right panel of Figure 1 shows that tax evasion accounts for at least 5 percent of the GDP. This is a quite big figure, and an analogous exercise for other European countries and the United States generates qualitatively comparable figures. If governments were effectively able to recollect unpaid taxes, this would generate, on a yearly basis, a significant increase of government revenues.

[8]The average size of underground activities ranges between five percent of the United States GNP (in the Seventies) and 9 percent of the United States GDP (in the Eighties, early Nintes). See Tanzi (1980), Schneider and Enste (2000), Paglin (1994). Even though these figures are below the OECD countries average (17 percent), they still represent a significant amount of resources absconded from tax collection.

[9]See Baldassarini and Pascarella (2003).

3 The Model's Structure

To investigate relationships among underground economy, taxation and public expenditure we use a dynamic general equilibrium model in discrete time. There are three agents in the model: the firms, the households, and the government. In addition there are two sectors: the regular and the underground sectors. Firms and households are subject to distortionary taxation, but they can use the underground sector to evade taxes, by reallocating labor services across sectors.

The firms produce a homogeneous good by combining three production factors: physical capital, regular and irregular labor services. The latter represents the channel through which tax evasion is undertaken.

The households choose consumption, investment, and labor services on each date and in each sector (regular and underground) to maximize the expected discounted value of utility, subject to a sequence of budget constraints, a proportional tax rate on "regular income", and the law of motion for capital stock.[10] The tax system is as parsimonious as possible: the tax base is easily identified and the tax rates (either income or corporate) are constant and proportionally related to the tax base.

Finally, government levies proportional taxes on revenues and incomes, and balances its budget (in expected terms) for each period. In this context, government faces tax evasion originating from the underground sector, and coordinates strategy to address abusive trust schemes. Violations of the Internal Revenue Service Codes may result in civil penalties and/or criminal prosecution, which we model as a surcharge factor over customary tax rates (more details below). Public expenditure is allocated to the purchase of the final consumption good.

3.1 Firms

3.1.1 Production Technologies

Suppose that there exists a continuum of firms, uniformely distributed over the unit interval. Each firm $i \in [0,1]$ produces final output by using two different technologies, one associated with the regular sector $y^i_{M,t}$, and the other with the underground sector $y^i_{U,t}$.

$$y^i_{M,t} = \left(k^i_t\right)^{\alpha} \left(n^i_{M,t}\right)^{1-\alpha} \quad \text{and} \quad y^i_{U,t} = \left(n^i_{U,t}\right)^{1-\sigma}, \quad \alpha, \sigma \in (0,1), \tag{1}$$

where the regular output, $y^i_{M,t}$, is the result of private capital k^i_t , and regular labor, $n^i_{M,t}$ applied to a Cobb-Douglas technology. The underground output, $y^i_{U,t}$, is produced with a production function which uses only underground labor $n^i_{U,t}$ and it displays decreasing

[10]The "regular income" includes income flows generated in the regular sector, including also returns to capital stock. These are declared to the Internal Revenues Services; on the contrary, income flow generated from the underground sector is not included into the tax-base.

returns to scale.[11] The assumption of decreasing returns to scale in the underground labor is based on the existence of un-modeled fixed factors, such as managerial ability to abscond the corresponding tax base from the taxation, or land.

3.1.2 Revenues and Tax Evasion

Denote a price vector for this economy as $\langle \tilde{q}_{M,t}, \tilde{q}_{U,t}, \tilde{w}_{M,t}, \tilde{w}_{U,t}, \tilde{r}_t \rangle$, where $\tilde{q}_{M,t}, \tilde{q}_{U,t}$ represent, respectively, prices for the regularly-produced and the underground-produced commodity, $\tilde{w}_{M,t}, \tilde{w}_{U,t}$ denote labor wages, and $\tilde{r}_t$ is returns to capital. Since we assume that there exists a homogenous consumption good, then the two prices are equal, i.e. $\tilde{q}_{M,t} = \tilde{q}_{U,t} \equiv \tilde{q}_t$. Normalizing the commodity price $\tilde{q}_t$ to unity, the normalized price vector supporting the equilibrium equals $\langle 1, w_{M,t}, w_{U,t}, r_t \rangle$, where $w_{M,t}, w_{U,t}$ and r_t denote equilibrium real wage rates and the real return on capital (see below). Since $\tilde{q}_t = 1$ holds in the equilibrium, we can impose it along the solution. Aggregate output equals therefore the sum of regular and underground produced output: $y_t^i = y_{M,t}^i + y_{U,t}^i$.

Regularly-produced revenues, $\mathcal{R}_{M,t} = (1 - \tau_F) y_{M,t}^i$, are taxed at the rate τ_F, $\tau_F \in (0,1)$. Firms do not pay taxes on underground produced revenues, $\mathcal{R}_{U,t} = y_{U,t}^i$. Firms, however, may be discovered evading, with probability $p \in (0,1)$, and forced to pay the tax rate, τ_F, increased by a surcharge factor, $s > 1$, applied to the standard tax rate.[12] Condition 1 below assumes that the effective tax rate paid when firms are detected is higher than the statutory one ($\tau_F s > \tau_F \Rightarrow s > 1$), but it also suggests that the expected tax payment when evading should be less than the statutory one ($\tau_F sp < \tau_F \Rightarrow sp < 1$); otherwise there would not be tax evasion.

Condition 1 (Penalty and Detection Probability) $s > 1; sp < 1$.

Finally, the chart below summarizes firm revenues' structure in the two states:

$\mathcal{R}_t^i$	$\rightarrow$	**D**etected ($\sim p$)	$\mathcal{R}_{D,t}^i = (1 - \tau_F) y_{M,t}^i + (1 - s\tau_F) y_{U,t}^i$
	$\searrow$		
		Not **D**etected $\sim (1 - p)$	$\mathcal{R}_{ND,t}^i = (1 - \tau_F) y_{M,t}^i + y_{U,t}^i$

Total expected revenues are thus:

$$\mathbb{E}_t \mathcal{R}_t^i = (1 - \tau_F) y_{M,t}^i + (1 - ps\tau_F) y_{U,t}^i. \tag{2}$$

[11] The model can be relabelled by interpreting the regular sector as the manufacturing sector, the underground economy as the service sector, and introducing a relative price for the different commodities. "Manufacturing" uses labor and capital and "services" use just labor with the decresing returns to scale technology. The analyzed fiscal policy shocks can be reinterpreted as changes in these different tax rates.

[12] This quantity is chosen by relying on the Italian Tax Law, because we calibrate the model for this economy. More details are presented below.

Condition 2 (No Bankruptcy) $(1 - ps\tau_F) > 0$ *and* $ps \leq (1-p)$.

Notice that a firm cannot go bankrupt, since $1 - ps\tau_F$ is positive in equilibrium. According to the Italian Tax Law s is calibrated equal to 1.3, and the equilibrium value of τ_F equals 0.4155 (more details to come). This ensures that $1 - s\tau_F > 0$. The second part of the condition states that that the expected surcharge (ps) should be less than $(1-p)$ otherwise the expected returns to a unit of evaded production, $(1-p)\,\tau_F - p\tau_F s$, would be negative, and the firm would have no convenience to operate in the underground sector. The cost structure is presented below.

3.1.3 Costs' Structure and Profit Maximization

Following Prescott and Mehra (1980), we assume that each firm solves a myopic profit maximization problem, on a period-by-period basis, subject to a technological constraint, and to the possibility that it may be discovered producing in the unofficial economy, convicted of tax evasion and subject to a penalty surcharge. We assume optimizing and price taking behavior on the part of all agents, households and firms. Specifically, firms maximize profits on a period-by-period basis.

The cost of renting capital equals its marginal productivity r_t, net of capital depreciation, Ω. The cost of labor is represented by the wage paid for hours worked.[13] At each date t, firm i maximizes period expected profits π_t^i:

$$
\begin{aligned}
\max_{\{n_{M,t}^i, n_{U,t}^i, k_t^i\}} \pi_t^i &= \mathbb{E}_t(\mathcal{R}_t^i - w_{M,t} n_{M,t}^i - w_{U,t} n_{U,t}^i - r_t k_t^i) \qquad (3)\\
\text{subject to} \quad &: \quad y_{M,t}^i = (k_t^i)^{\alpha} (n_{M,t}^i)^{1-\alpha},\ y_{U,t}^i = \left(n_{U,t}^i\right)^{1-\sigma}\\
&: \quad \mathbb{E}_t \mathcal{R}_t^i = (1-\tau_F) y_{M,t}^i + (1 - ps\tau_F) y_{U,t}^i\\
&: \quad n_{M,t}^i > 0, n_{U,t}^i > 0, k_t^i > 0.
\end{aligned}
$$

– *Corporate Efficiency Conditions.*

In a competitive equilibrium factors are marginally priced, as the following first order conditions suggest:

$$
\left\{
\begin{array}{c}
w_{M,t} = (1-\tau_F)\,(1-\alpha)\left(k_t^i\right)^{\alpha} (n_{M,t}^i)^{-\alpha}\\
w_{U,t} = (1 - ps\tau_F)(1-\sigma)(n_{U,t}^i)^{-\sigma}\\
R_t = (1-\tau_F)\,\alpha\left(k_t^i\right)^{\alpha-1} (n_{M,t}^i)^{1-\alpha}.
\end{array}
\right\} \qquad (4)
$$

[13] A more general structure would account for labor costs, too (e.g. social security contributions). This would mean that a worker's cost is augmented by social security contributions only for the regular working time, while there is no tax wedge on the remaining *hidden hours*. This model, however, abstracts from this additional tax rate and leaves its analysis to future investigations.

The firm's equilibrium behavior is characterized by the previous necessary and sufficient conditions. The decreasing returns to scale in the underground sector generates excess profits $\pi_t^i > 0$, which are generated by the underground labor supply. In equilibrium, profits equal:

$$\pi_t^i = (1 - ps\tau_F)\left(n_{U,t}^i\right)^{1-\sigma} \frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma, \tag{5}$$

and are uniformly distributed among all households.

3.2 Households and Preferences

The representative household, indexed with $\gamma \in [0,1]$, has preferences over consumption and labor services. For most of our analysis we specialize momentary utility to have the form:[14]

$$\mathcal{U}_t^\gamma = \log\left(c_t^\gamma + \phi c_{G,t}\right) - B_M \frac{(n_{M,t}^\gamma + n_{U,t}^\gamma)^{1+\xi}}{1+\xi} - B_U \frac{(n_{U,t}^\gamma)^{1+\psi}}{1+\psi}, \quad B_M, B_U \geq 0, \tag{6}$$

where c_t^γ denotes the private consumption profile of household γ, $n_{M,t}^\gamma$ her regular labor services supply, and $n_{U,t}^\gamma$ her underground labor supply; $c_{G,t}$ denotes per-capita government purchases of the homogeneous good produced in the economy, and ϕ is a parameter representing the degree of substitutability between government and private consumption flows.[15] When $\phi = 1$ private and public consumption goods are perfect substitutes. Households would react to a one-unit increase in public consumption by lowering private consumption by one unit. When $\phi = 0$, $c_{G,t}$ does not affect the households' utility.[16] It is assumed that households take $c_{G,t}$ as given, and that there is no congestion effect.[17]

The quantity $B_M \frac{(n_{M,t}^\gamma + n_{U,t}^\gamma)^{1+\xi}}{1+\xi}$ represents the total disutility of working, while the last term, $B_U \frac{(n_{U,t}^\gamma)^{1+\psi}}{1+\psi}$ reflects an idiosyncratic cost of working in the underground sector. Specif-

[14]Our utility function departs from Baxter and King (1993)'s formulation. Their specification is separable among private consumption C_t, leisure L_t, and government expenditure (basic consumption G_t^B and public capital stock K_t^G); using their notation it reads: $u\left(C_t, L_t, G_t^B, K_t^G\right) = u\left(C_t, L_t\right) + \Gamma\left(G_t^B, K_t^G\right)$, where $\Gamma_1, \Gamma_2 > 0$. This preference specification is meant to capture a government spending that does not directly affect private consumption, i.e. military spending.

[15]Also notice that since households are uniformly distributed over the unit interval and there is no population growth, per-capita government spending coincides with the corresponding aggregate quantity.

[16]The role of government expenditure into our utility function is slightly different from the customary one. Precisely, c_G denotes per capital expected government expenditure, because it incorporates expected revenues. The expectation originates from the probability of detecting a firm evading and collecting the absconded tax payments. It should be noted, moreover, that as long as an household allocates labor services to both sectors, she has always incentives to evade taxes. That happens because the value of the additional (tax-free) income is higher than the the value of government consumption added to the individual consumption flow.

[17]We treat government expenditure as a pure public good, and we abstract from congestion typically associated with public goods, in order to sharply identify the tax-evasion impact on fiscal policies. We leave the introductions of different notions of congestions to future investigations.

ically, this cost may be associated with the lack of any social and health insurance in the underground sector. The parameters ξ and ψ represent the inverse labor supply elasticities of regular and unofficial labor supplies, respectively. This utility function is separable between consumption and labor and allows to study how a household allocates its labor supply between the regular and the underground sectors.

Section 3.4.1 below shows that the following condition ensures that the households would find optimal to supply labor in both sectors. It says that the idiosyncratic cost associated with working in the underground sector should not be *too large*:[18]

Condition 3 (Labor Market Equilibrium) $B_U \leq \frac{w_U - w_M(1-\tau_Y)}{C}$,

where C denotes aggregate consumption (i.e. the sum of private and public consumptions, taking into account a possibly different degree of substitutability), w_U and $(1-\tau_Y)w_M$ respectively represent underground and net-of-tax regular salaries (defined before). Notice that the higher the aggregate consumption, the lower is the idiosyncratic cost of working into the underground economy.[19]

In each period the representative household faces a resource constraint saying that the total use of goods for consumption and investment cannot exceed the disposable income, net of income taxes, $\tau_Y \in (0,1)$:

$$c_t^\gamma + i_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_U n_{U,t}^\gamma + \pi_t, \tag{7}$$

where $w_{M,t}$ and $w_{M,t}$ represent regular earnings and earnings from the underground sector, respectively; income generated from the underground sector $w_U n_{U,t}^\gamma$ and per-capital profits $\int_0^1 \pi_t^i di = \pi_t$ are absconded away from income taxation.[20]

Finally, investment increases the capital stock according to a customary state equation:

$$k_{t+1}^\gamma - (1-\Omega)k_t^\gamma = i_t^\gamma,$$

where Ω denotes a quarterly depreciation rate for private capital stock.

The $\gamma - th$ household's sequential problem is therefore the following:

[18] See Section 3.4.1 for a more detailed discussion and the derivation.

[19] In this respect a valuable government expenditure does not rule out the existence of an idiosyncratic cost for the underground labor.

[20] Firms and households are uniformly distributed over the unit interval. Here we implicitly assume that firms' profits are uniformly distributed over the unit mass of households. Therefore distributing the aggregate profits $\int_0^1 \pi_t^i di$ over the unit mass of households yields that per-capita profits equal $\int_0^1 \pi_t^i di = \pi_t$.

$$\max_{\{c_t^\gamma, i_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}_t^\gamma \tag{8}$$

$$\begin{aligned} s.t. \quad &: \; c_t^\gamma + i_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_U n_{U,t}^\gamma + \pi_t \\ &: \; i_t^\gamma = k_{t+1}^\gamma - (1-\Omega) k_t^\gamma \\ &: \; k_{i,0}^\gamma,\ \pi_t \text{ and } c_{G,t} \text{ given, } c_t^\gamma, n_{M,t}^\gamma, n_{U,t}^\gamma > 0 \text{ and } B_U \leq \frac{w_U - w_M (1-\tau_Y)}{C}, \end{aligned}$$

where households take the government purchase of final commodities $c_{G,t}$ and corporate profits as given. Pooling together the feasibility and the capital accumulation constraint, the constraint set is $c_t^\gamma + k_{t+1}^\gamma - (1-\Omega)k_t^\gamma = (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_{U,t} n_{U,t}^\gamma + \pi_t$.

– *Private Efficiency Conditions.*

A representative household chooses how much to consume (c_t^γ), how many labor services to allocate in each sector ($n_{M,t}^\gamma$ and $n_{U,t}^\gamma$), and next period capital stock (k_{t+1}^γ). More formally, her behavior is characterized by the following necessary and sufficient first order conditions:

$$\left\{ \begin{array}{c} B_M (n_t^\gamma)^\xi = \left(c_t^\gamma + \phi c_{G,t}\right)^{-1} (1-\tau_Y)\, w_{M,t} \\ (B_U + B_M)\,(n_{U,t}^\gamma)^\psi = \left(c_t^\gamma + \phi c_{G,t}\right)^{-1} w_{U,t} \\ c_t^\gamma + k_{t+1}^\gamma = (1-\Omega)k_t^\gamma + (1-\tau_Y)\left(w_{M,t} n_{M,t}^\gamma + r_t k_t^\gamma\right) + w_{U,t} n_{U,t}^\gamma + \pi_t. \end{array} \right\} \tag{9}$$

Next, optimal investment choice by the representative agent depends on the following Euler Equation:

$$\left(c_t^\gamma + \phi c_{G,t}\right)^{-1} = \beta \mathbb{E}_t \left(c_{t+1}^\gamma + \phi c_{G,t+1}\right)^{-1} \left((1-\tau_Y)\, r_{t+1} + 1 - \Omega\right). \tag{10}$$

3.3 Government

The government is described as a sequence $\{\varrho_t\}_{t=0}^{\infty}$ of tax rates on households' income, on firms' revenues and of government spending G_t

$$\{\varrho_t\}_{t=0}^{\infty} = \{\tau_Y, \tau_F; G_t\}_{t=0}^{\infty}. \tag{11}$$

Government spending is determined endogenously in equilibrium to balance the public budget constraint.[21] Next, collected tax revenues, denoted by RV_t, read

$$RV_t = \left[(w_{M,t} N_{M,t} + r_t K_t)\,\tau_Y + \tau_F\,(ps Y_{U,t} + Y_{M,t})\right] = c_{G,t}, \tag{12}$$

[21] Notice that this paper does not present an "optimal taxation exercise", in the sense that public consumption or investment choice is not derived from an optimization procedure. In this respect our framework departs from Chari, Christiano and Kehoe (1995), while it follows McGrattan (1994).

where capitalized letters denote aggregate equilibrium quantities, which are defined as $Y_{M,t} = \int_0^1 y_{M,t}^i di$, $Y_{U,t} = \int_0^1 y_{U,t}^i di$, $N_{M,t} = \int_0^1 n_{M,t}^i di$, $K_t = \int_0^1 k_t^i di$. Income tax is collected from households over regular wage and capital revenues $(w_{M,t}N_{M,t} + r_t K_t)$, and from firms over regular $\tau_F(Y_{M,t})$, and underground produced revenues $\tau_F(Y_{U,t}ps)$. The latter quantity denotes expected tax revenues flows, because it takes into account that part of the tax-base is successfully absconded from Internal Revenues Service, with a positive probability p. Government spending is allocated to purchase of final consumption goods $c_{G,t}$ and it equals government revenues.[22]

In this model both households and firms evade taxes. Households evade income taxation producing a tax loss associated to the underground-produced income flow $\tau_Y w_{U,t} n_{U,t}$ and distributed per-capita profits π_t. Firms always try to evade an amount of taxes equal to $\tau_F y_{U,t}^i$; when a firm is detected, however it pays the additional fine $s\tau_F y_{U,t}^i$. That happens with probability p. In the other case, firms do evade, and are not detected; this event happens with probability $1-p$.[23] Hence expected corporate tax evasion is $(1-p)\,\tau_F y_{U,t}^i$.[24] Combining these quantities, total expected tax evasion reads $\mathbb{E}_t TE_t = (1-p)\,\tau_F y_{U,t}^i + \tau_Y w_{U,t} n_{U,t}$.

3.4 Competitive Equilibrium Characterization

A Competitive Allocation is a policy $\{\varrho_t^*\}_{t=0}^{\infty}$, an allocation $\{x_t^*\}_{t=0}^{\infty} = \left\{K_t^*, N_{M,t}^*, N_{U,t}^*, C_t^*\right\}_{t=0}^{\infty}$ and a price system $\left\{1, w_{M,t}^*, w_{U,t}^*, r_t^*\right\}_{t=0}^{\infty}$ such that, given the policy and the price system, the resulting allocation maximizes the representative household utility (conditions (9)) subject to:

(i) the sequence of budget constraints, (condition (7));

(ii) the price system $\left\{1, w_{M,t}^*, w_{U,t}^*, r_t^*\right\}_{t=0}^{\infty}$ (conditions (4));

(iii) the government budget constraint being satisfied on average, (condition (12));

(iv) market clearing conditions holding for each market, and the following aggregate constraint for the economy being satisfied:

$$C_t + I_t + G_t = Y_t,$$

where G_t denotes aggregate government spending, and aggregate consumption, investment, and output are defined as $C_t = \int_0^1 c_t^\gamma d\gamma$, $I_t = \int_0^1 i_t^\gamma d\gamma$ and $Y_t = \int_0^1 y_t^i di$,

[22]The model abstracts from debt accumulation since the government balances its budget on a period by period basis, as equation (12) suggests. Consumption expenditure, furthermore, is fully revenue-financed, and it is implicitly assumed that the government does not issue bonds.

[23]It is also assumed that the proportion of firms evading taxes does not change from period to period.

[24]Firms and households differ along the following dimension: households cannot be discovered, while firms are subject to the probability of being discovered and then fined.

respectively.

The following Proposition 1 shows that the model has a unique stationary state for capital stock, a unique value for equilibrium regular and underground labor services. The stationary state quantities are derived under perfectly elastic labor supply schedules ($\xi = \psi = 0$).[25]

Proposition 1 *For $\xi = \psi = 0$ there exists a unique stationary capital stock $\overline{K} > 0$, and a unique stationary equilibrium for regular labor $\overline{N}_M > 0$, and underground labor $\overline{N}_U$ such that:*

$$
\begin{aligned}
\overline{K} &= \left(\frac{\beta^{-1} - 1 + \Omega}{\alpha (1 - \tau_Y)(1 - \tau_F)} \right)^{\frac{1}{\alpha - 1}} \overline{N}_M \\
\overline{N}_M &= \frac{(1 - \tau_Y)(1 - \tau_F)(1 - \alpha)}{B_M (\mathcal{C}_1 + \phi \mathcal{C}_2)} \left(\frac{\overline{K}}{\overline{N}_M} \right)^{\alpha} \\
\overline{N}_U &= \left(\frac{B_M}{B_U + B_M} \frac{1 - ps\tau_F}{(1 - \tau_Y)(1 - \tau_F)} \right)^{\frac{1}{\sigma}} (1 - \alpha)^{-\frac{1}{\sigma}} \left(\frac{\overline{K}}{\overline{N}_M} \right)^{-\frac{\alpha}{\sigma}},
\end{aligned}
$$

where $\frac{\overline{K}}{\overline{N}_M}$ denotes the stationary capital-labor ratio, and $\mathcal{C}_1$ and $\mathcal{C}_2$ are positive quantities defined in the Appendix.

Proof. See Appendix. ■

Once we have equilibrium values for the stationary capital stock and labor inputs, the remaining equilibrium quantities (consumption, output, investments) are derived from the budget constraint, the production functions and the capital accumulation constraint, all evaluated at the stationary state.

As the quantities in Proposition 1 suggest, the unique stationary state is characterized by a positive equilibrium level for regular and underground labor services. The only economically meaningful scenario in which $\overline{N}_U = 0$ is when $1 = ps\tau_F$ or when $B_U \to \infty$; these case are, however, ruled out by the "no-bankruptcy condition" (see Condition 2) and by the "labor market equilibrium condition" (Condition 3). The next section describes in more details the labor market equilibrium.

3.4.1 Labor Market Equilibrium

Let $\xi = \psi = 0$, as in Proposition 1, and consider, first, the supply side of the underground labor market. A utility maximizing household would allocate labor services to a true underground sector (that is meant to evade taxes) if its additional cost (B_U) is less than (equal at the margin) that produced by distortionary taxation. Therefore a maximizing household would compare the net-of-tax wage ($w_M (1 - \tau_Y) = CB_M$) with the net-of-idiosyncratic

[25] The assumption $\xi = \psi = 0$ is necessary to derive a closed form solution for the stationary state.

cost wage ($w_U - CB_U = CB_M$);[26] at the margin these two quantities should be equal, that is $w_M(1-\tau_Y) = w_M - CB_U$, which implies:

$$w_M(1-\tau_Y) \leq w_U - CB_U \Rightarrow B_U \leq \frac{w_U - w_M(1-\tau_Y)}{C}, \tag{13}$$

where C denotes aggregate consumption, and salaries w_M and w_U are taken as given by the households. This is Condition 3 in Section 3. In other words, the idiosyncratic additional cost associated to underground labor services exploits the opportunity cost related to evading the tax wedge $w_U\tau_Y$. Now, as long as condition (13) is satisfied, a household would supply labor in both sectors. In addition, From the first order conditions for the households we obtain that $B_U = B_M\frac{w_U - w_M(1-\tau_Y)}{w_M(1-\tau_Y)}$, which implies, in turn, that optimal equilibrium labor supplies would be such to equate, at the margin, the two kinds of disutilities (the distortionary taxation and the additional riskiness of the underground labor).[27] When this condition is satisfied (as it happens in equilibrium), it follows that condition (13) holds as well, since B_M is calibrated to a sufficiently small value (see Section 4.2.2 below). It can be concluded that, in equilibrium, households would supply labor services in both sectors.

Consider, next, the demand side of the labor market. Profit maximizing firms equate the gross-of-tax regular wage $\frac{w_M}{1-\tau_F} = (1-\alpha)(K)^{\alpha}(N_M)^{-\alpha}$ with gross-of-expected tax underground salary $\frac{w_U}{1-ps\tau_F} = (1-\sigma)(N_U)^{-\sigma}$. The technology structure ensures, in addition, that a corner solution (that is $n^i_{M,t} = 0$ or $n^i_{U,t} = 0$) would not be an optimal solution for the firms' optimization problem, that is for the demand side of the model.[28] This argument would support a non-negative demand for each labor input.

4 Results

This section discusses how tax evasion and underground activities modify the macroeconomic consequences of permanent shifts in income and corporate tax rates. We focus our attention on the stationary state. Some results are numerically derived in order to convey the main economic message in a more clear way. The parameterization is presented at the end of the section.

[26] From a geometrical perspective in the plane employment-salary, notice that both the idiosyncratic riskiness of underground labor (the parameter B_U) and the distortionary income taxation (the quantity $(1-\tau_Y)$) shift upward households' labor supply schedule.

[27] This implies that the households would appropriate the entire tax wedge. It would also be interesting to consider more sophisticated allocation mechanisms, i.e. a bargaining solution.

[28] The Inada conditions,next, ensure that the productivity (and therefore the salary) of both types of labor goes to infinity when the corresponding labor service goes to zero. Precisely: $\lim_{n^i_{M,t}\to 0} w_{M,t} = \infty$ and $\lim_{n^i_{U,t}\to 0} w_{U,t} = \infty$, where $w_{M,t}$ and $w_{U,t}$ denote the regular and underground labor demand schedule, as conditions (4) state.

4.1 Equilibrium Effect of Fiscal Policy with Tax Evasion

Proposition 2 below derives the long-run elasticities of aggregate production to income and corporate tax rates (permanent) shifts.

Proposition 2 (Long Run Elasticities) *Let $\xi = \psi = 0$; the long-run elasticity of aggregate output to income (ε_{Y,τ_Y}) and corporate tax (ε_{Y,τ_F}) rates are, after ignoring second order quantities:*

$$\varepsilon_{Y,\tau_Y} \simeq -\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{(1-\tau_Y)}\frac{\Delta\tau_Y}{\tau_Y} - \varepsilon_{w_M,\tau_Y} + \sigma\varepsilon_{N_U,\tau_Y} \quad (14)$$

$$\varepsilon_{Y,\tau_F} \simeq -\left(\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{(1-\tau_F)} + \frac{(1-\sigma)ps}{\sigma(1-ps\tau_F)}\right)\frac{\Delta\tau_F}{\tau_F} - \varepsilon_{w_M,\tau_F} + \sigma\varepsilon_{N_U,\tau_F} \quad (15)$$

where $\varepsilon_{N_U,\bullet}$ denotes the elasticity of underground labor supply to changes in tax rates, and is defined below.

$$\varepsilon_{N_U,\tau_Y} = \frac{1}{\sigma}\frac{(1-\alpha)^{-1}}{1-\tau_Y}\frac{\Delta\tau_Y}{\tau_Y}, \qquad \varepsilon_{N_U,\tau_F} = \frac{1}{\sigma}\left(\frac{(1-\alpha)^{-1}}{1-\tau_F} - \frac{ps}{1-ps\tau_F}\right)\frac{\Delta\tau_F}{\tau_F},$$

$$\varepsilon_{w_M,\tau_Y} = \frac{\alpha}{1-\alpha}\frac{1}{1-\tau_Y}\frac{\Delta\tau_Y}{\tau_Y}, \qquad \varepsilon_{w_M,\tau_F} = \frac{\alpha}{1-\alpha}\frac{1}{1-\tau_F}\frac{\Delta\tau_F}{\tau_F}$$

where $\frac{(1-\alpha)^{-1}}{1-\tau_F} - \frac{ps}{1-ps\tau_F} > 0$. When there is no tax evasion and no underground economy, the previous elasticities are denoted with ε^*_{Y,τ_Y} and ε^*_{Y,τ_F} and read:

$$\varepsilon^*_{Y,\tau_j} \simeq -\frac{((2+\alpha)-1)}{(1-\alpha)}\frac{1}{(1-\tau_j)}\frac{\Delta\tau_j}{\tau_j} - \varepsilon_{w_M,\tau_j}, \quad j = Y, F$$

Proof. See Appendix. ■

Consider, first, the equilibrium impact on aggregate output of an income tax rate increase $\left(\frac{\Delta\tau_Y}{\tau_Y} > 0\right)$, the consequence of which is characterized by $\varepsilon_{Y,\tau_Y} = \left(-\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{1-\tau_Y} - \varepsilon_{w_M,\tau_Y}\right)\frac{\Delta\tau_Y}{\tau_Y}$ $+\sigma\varepsilon_{N_U,\tau_Y}$, where $\varepsilon_{w_M,\tau_Y}, \varepsilon_{N_U,\tau_Y} > 0$ and $\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{1-\tau_Y} > 0$ as long as $\sigma > \frac{1}{(2+\alpha)}$. In this case $\left(-\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{1-\tau_Y} - \varepsilon_{w_M,\tau_Y}\right) < 0$; an increase of income tax rate has a negative impact on aggregate production because of the direct effect of the income tax rate $(-\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{1-\tau_Y}\frac{\Delta\tau_Y}{\tau_Y})$, but a positive one driven by the increase in labor input in the underground sector ($\varepsilon_{N_U,\tau_Y} > 0$). Therefore the underground sector mitigates the distortionary impact of fiscal policies, while lessening the fall of aggregate production after restrictive tax changes. A tax cut has a symmetric impact, *mutatis mutandis.* In this sense, tax evasion (underground sector) offers *insurance* to income tax rate shifts. The concept of insurance is present in the sense that underground-produced income flows completely avoid distortionary income taxes. In theory, households can completely offset the distortionary impact of income taxes by reallocating labor supply to the underground sector. That would happen for $\sigma\varepsilon_{N_U,\tau_Y} = \frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{(1-\tau_Y)}\frac{\Delta\tau_Y}{\tau_Y} + \varepsilon_{w_M,\tau_Y}$ and

$$\sigma\varepsilon_{N_U,\tau_F} = \left(\frac{((2+\alpha)\sigma-1)}{(1-\alpha)\sigma}\frac{1}{(1-\tau_F)} + \frac{1-\sigma}{\sigma}\frac{ps}{(1-ps\tau_F)}\right)\frac{\Delta\tau_F}{\tau_F} + \varepsilon_{w_M,\tau_F}.$$

The response of aggregate production to shifts in corporate tax rates τ_F (Eq. 15) differs, because the direct impact of the tax shock is magnified by the quantity $\frac{1-\sigma}{\sigma}\frac{ps}{1-ps\tau_F}$, which incorporates the probability that a firm is detected evading, and eventually fined. It reduces, in expected terms, the insurance opportunity that a firm has, compared with households.[29]

It is also interesting to have a feeling for the relative magnitudes of these elasticities. For our parameterization (see Section 4.2.2 below) tax evasion almost completely offsets the impact of a 1 percent increase in income and corporate tax rates. The elasticities to income tax rate equal to $\varepsilon_{Y,\tau_Y} = 0.006$ in the tax-evasion-case, and to $\varepsilon^*_{Y,\tau_Y} = -0.0167$ in the no-tax-evasion scenario. Similarly, the elasticities to corporate tax rate are $\varepsilon_{Y,\tau_F} = 0.009$ and $\varepsilon^*_{Y,\tau_F} = -0.016$. The elasticities under tax evasion (ε_{Y,τ_Y} and ε_{Y,τ_F}) are almost zero, while those computed without tax evasion (ε^*_{Y,τ_Y} and ε^*_{Y,τ_F}) are negative, consistently with the predictions of the neoclassical growth model. On the other hand, the almost zero elasticity under tax evasion is perfectly consistent with consumption and income smoothing. Tax evasion and the informal economy offer, in other words, a channel for insuring income and consumption patters from distortions generated by fiscal policy. In this sense tax evasion can be interpreted as an income smoothing device available to households and firms. It is like saying that the government chooses in fact the statutory tax rates, while the effective tax rates are endogenously chosen by households and firms by relying on the additional dimension represented by tax evasion.

4.1.1 Expansionary Effects to Recessionary Fiscal Policies

The previous analysis suggests that tax evasion and the underground economy can rationalize expansionary effects of recessionary fiscal policies. This argument may seem related to several contributions that, mainly in the early 90s, studied the so-called "non-keynesian" or non-linear effects of fiscal policies.[30] But here we cannot talk of non-keynesian effects in the spirit of Giavazzi and Pagano (1990) because the government budget constraint of our model holds (in expectations) on a period by period basis. There would be no debt accumulation, and, therefore, no reason for consolidations.

[29] This is a consequence of assuming that households are not subject to the probability of being detected. We argue that this is not a restrictive assumption. Notice, first, that in equilibrium the households own the firms, and therefore they are, in some sense, subject to the probability of being detected, as owners. On the other hand, tax evasion/elusion is a phenomenon that is much more widespread within the corporate sector.

[30] Specifically, a number of authors have investigated "non-keynesian" effects of fiscal contractions or non-linear effects of fiscal policy, stemming from re-establishing credibility. Several studies have found that the effects of fiscal impulses on private consumption are ambiguous (see e.g. Giavazzi and Pagano, 1996; Afonso, 2001). In some cases big fiscal contractions, brought about by government expenditure cuts, have increased private consumption, while in another case a severe fiscal expansion, through a debt financed cut in taxes, has failed to stimulate private consumption. These findings stand in contrast to the keynesian view that output is demand determined so that fiscal expansions stimulate private consumption, while fiscal contractions reduce private consumption.

Notice, however, that our effects are very much keynesian. The government budget constraint suggests, indeed, that an increase (decrease) in taxes would lead (ceteris paribus) to an increase (decrease) in government spending.

The aforementioned expansionary effects to recessionary fiscal policies cannot arise in a standard general equilibrium model without an underground economy and tax evasion. The operating economic mechanism goes as follow. If the underground labor response to a shift in tax rates is sufficiently large, it would offset the classical response of the market sector. Consider the case of an increase in the income tax rate, for example. A standard model with no tax evasion predicts a fall in aggregate output would equal to $\frac{((2+\alpha)-1)}{(1-\alpha)}\frac{1}{(1-\tau_j)}\frac{\Delta\tau_j}{\tau_j}$, $j = Y, F$. A model with tax evasion, however, could generate expansionary effects, if the response of underground labor (and therefore of tax evasion) is sufficiently large, that is if $\varepsilon_{N_U,\tau_Y} > \frac{(2+\alpha)\sigma-1}{(1-\alpha)\sigma}\frac{1}{(1-\tau_Y)}\frac{\Delta\tau_Y}{\tau_Y} + \varepsilon_{w_M,\tau_Y}$. When tax rates are too high ($\tau_Y \to 1$ or $\tau_F \to 1$) an additional increase in tax rates pushes the economy into the underground sector.[31] In this case the reallocation would be so strong that income increases, because the underground-produced income is not subject to taxation. The additional income can be used for purchasing additional consumption and investment (that are normal goods). This pushes the economy into an expansion. In summary, the expansionary effects arise as a consequence of an optimizing behavior (under rational expectations) set up to avoid the excessive burden of taxation.

To derive the threshold values of the tax rates that trigger the expansionary mechanism, it is convenient to rely on a Laffer-curve-based argument.

4.2 Laffer Curve under Tax Evasion

One of the most controversial issues in tax policy analysis is whether a tax cut will boost economic activity to such an extent that the government's budget actually improves (this often referred to as a Laffer's Curve Effect).[32]

Our contribution examines the possibility that tax evasion and underground economy give a rational justification for a variant of a Laffer curve. The Laffer surface under tax evasion is defined as the function $\mathfrak{L}^* (\tau_Y, \tau_F) : [0, 1] \times [0, 1] \mapsto \mathbb{R}_+$ such that

[31] These conditions are derived from the underground labor elasticity, derived in Proposition 2 and are reported below for readers' convenience:

$$\varepsilon_{N_U,\tau_Y} = \frac{1}{\sigma}\frac{(1-\alpha)^{-1}}{1-\tau_Y}\frac{\Delta\tau_Y}{\tau_Y}$$

Notice that for $\tau_Y \to 1$ and for $\sigma \to 0$, then $\varepsilon_{N_U,\tau_Y} \to \infty$.

[32] A Laffer curve can be defined as a curve which supposes that for a given economy there is an optimal income tax level to maximize tax revenues. If the income tax level is set below this level, raising taxes will increase tax revenue. And if the income tax level is set above this level, then lowering taxes will increase tax revenue. Although the theory claims that there is a single maximum and that the further you move in either direction from this point the lower the revenues will be, in reality this is only an approximation.

$$\mathfrak{L}^*(\tau_Y,\tau_F) = ((1-\tau_Y)(1-\tau_F)\tau_Y + \tau_F)(K^*(\tau_Y,\tau_F))^{\alpha}(N_M^*(\tau_Y,\tau_F))^{1-\alpha} + \tau_F ps(N_U^*(\tau_Y,\tau_F))^{1-\sigma},$$

where $\tau_Y, \tau_F \in [0,1]$ and $K^*(\tau_Y,\tau_F)$, $N_M^*(\tau_Y,\tau_F)$, $N_U^*(\tau_Y,\tau_F)$ denote the equilibrium quantities, evaluated at the stationary state (derived in Proposition 1). The first quantity $((1-\tau_Y)(1-\tau_F)\tau_Y + \tau_F)(K^*(\tau_Y,\tau_F))^{\alpha}(N_M^*(\tau_Y,\tau_F))^{1-\alpha}$ represents revenues collected from the regular demand side and the supply sides of the economy, while the second part $\tau_F ps(N_U^*(\tau_Y,\tau_F))^{1-\sigma}$ represents revenues collected, in expected terms, from the underground sector. We parameterize the model for the Italian economy, given the availability of a good data-set for the underground economy (more details in Section 4.2.2 below); then **Figure 2** plots the bi-dimensional Laffer curve (surface) for our economy.

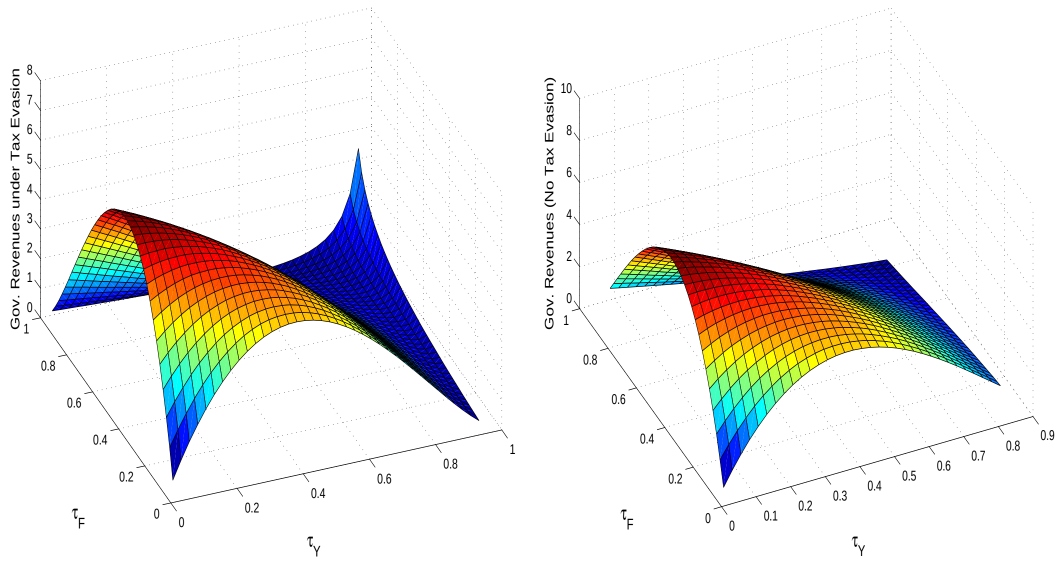

Figure 2: **Laffer Curve with (without) Tax Evasion**. Left Panel: Laffer curve under Tax Evasion; Right Panel: Laffer curve without Tax Evasion. The quantities τ_Y and τ_F denote income and corporate tax rates. The model's parametrization is included in Section 4.2.2

The Laffer curve under tax evasion (left panel) is *almost always* below the corresponding curve computed for a 100 percent regular economy. To better appreciate this result, it is convenient to distinguish between (i) movement along the Laffer's curve, and (ii) shifts of the curve itself. The underground sector, and therefore tax evasion allows for shifts of the curve itself. Indeed, after a tax cut the resource reallocation mechanism moves resources above the ground, and therefore increases the tax base, shifting the Laffer curve upward. In addition, the model with tax evasion also predicts the movement along the curve, which represents the increase of the regular tax base. In addition, notice that for *sufficiently high* tax rates government revenues go to zero in an economy without tax evasion (that is the Laffer curve in the right panel of Figure 2 is flat at zero for (τ_Y, τ_F) large); with tax evasion, on the contrary, government revenues begin rising again for very large tax rates (that is the upward sloping tail of the Laffer curve in the left panel of Figure 2).

The "*upward sloping*" tail of the Laffer curve under tax evasion (left panel of Figure 2) is a consequence of the expansionary effects following "too severe" recessionary fiscal policies (more details to come). Recall that an increase in the tax rate induces the economy to reallocate resources to the underground sector. The higher the tax rate, the stronger would be, *ceteris paribus*, the reallocation toward the informal sector. This reallocation increases households' disposable income, because underground produced revenues are not taxed. Now, if this resource reallocation is sufficiently large, an increase in the tax rate produces an increase in capital accumulation as well (recall that consumption and investment are normal goods, and therefore an increase in income is allocated between both goods). In this case the capital accumulation would be financed though underground-produced revenues; in this respect it can be argued that evaded taxes represent a sort of (still illegal) internal finance for the firms.

4.2.1 Threshold for expansionary effects of contractionary fiscal policies

In this context it is interesting to compute the critical level of household and firm tax rate that triggers the reallocation to underground labor, and, by this end, the appearance of expansionary effects of contractionary fiscal policies. The threshold in this context is defined as the set

$$\mathcal{T} = \left\{ \left(\tau_Y^0, \tau_F^0\right) : RV\left(\tau_Y^0 + \varepsilon, \tau_F^0 + \varepsilon\right) \geq RV\left(\tau_Y^0, \tau_F^0\right), \varepsilon > 0 \right\},$$

where RV denotes government revenues at the steady state. $\mathcal{T}$ defines the set of tax rates beyond which a further increase in tax rate triggers a reallocation toward the underground sector and, eventually, an overall increase in government revenues. In other words if a pair (τ_Y, τ_F) falls in the $\mathcal{T}$ set, it triggers the aforementioned mechanism, which, in turn, pulls the economy toward a tax evasion-driven expansion. It is important to underline, however, that we would observe an increase of regular GDP and of government revenues collected from the regular economy; here the role of the underground sector is being the spark that ignites the mechanism.

Figure 3 below presents this threshold set for the baseline parameterization (the solid line with circles) and for different parameter combinations. The threshold is operationally derived by computing the pairs $\left(\tau_Y^0, \tau_F^0\right)$ at which the numerical gradient of Laffer curve $\mathfrak{L}^*\left(\tau_Y, \tau_F\right)$ is zero, and the numerical Hessian is positive definite.

The figure suggests that expansionary effects of contractionary fiscal policies arise at lower levels of income and corporate tax rates when the regular labor becomes increasingly

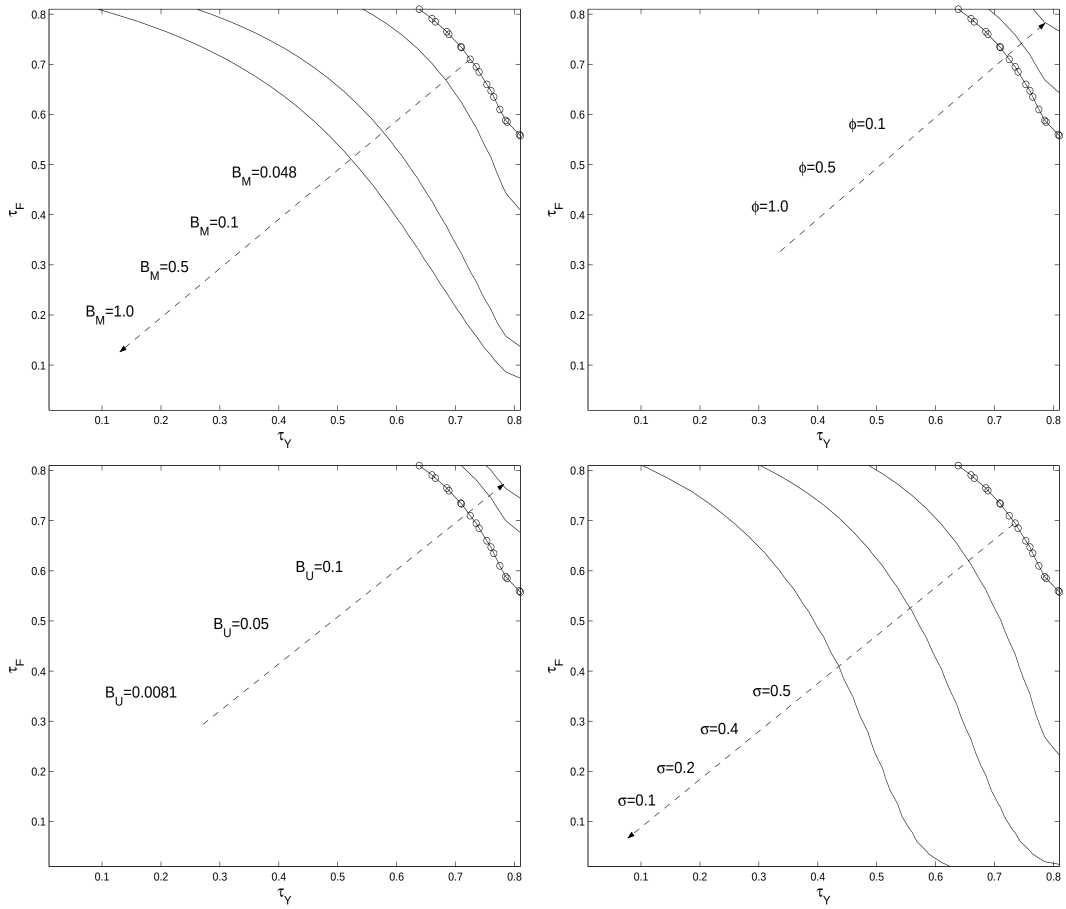

Figure 3: **Threshold for expansionary effects of recessionary fiscal policies**. The panels present the threshold levels of tax rates (τ_Y, and τ_F) beyond which the model displays expansionary effects of recessionary fiscal policies. Solid line with circles: baseline parameterization; solid lines: sensitivity analysis; the dashed arrow denotes the direction in which the threshold set $\mathcal{T}$ moves by perturbing the parameters' space. Upper-left panel: sensitivity with respect to regular labor disutility parameter B_M; upper-right panel: sensitivity with respect to the degree of substitutability between government and private consumptions ϕ; bottom-left panel: sensitivity with respect to the idiosyncratic cost of working in the underground sector B_U; bottom-right panel: sensitivity with respect elasticity of underground labor σ.

costly ($\uparrow B_M$), when underground production becomes more and more flexible ($\downarrow \sigma$). On the contrary, these effects are more unlikely to happen when underground labor is more risky ($\uparrow B_U$) and when the degree of substitutability between private and public consumptions falls ($\downarrow \phi$).

4.2.2 Parameterization

The model is parameterized for the Italian economy. The system of equations we use to compute the dynamic equilibria of the model depends on a set of nine parameters. **Five** pertain to household preferences, $(\psi, \xi, B_M, B_U, \beta)$, **two** to the structural-institutional context (the probability of a firm being detected p, the surcharge factor s), and the remaining **four** parameters to technology (the private capital share α, and the private capital stock quarterly depreciation rate Ω, the underground labor elasticity $1-\sigma$).

Preference and Technology $(\alpha, \beta, \Omega, \sigma)$ are set to commonly used values in this literature (e.g. Fiorito and Kollintzas, 1994). More precisely, we set $\beta^* = 0.984$, $\Omega^* = 0.025$, and $\alpha^* = 0.33$. We choose $\sigma^* = 0.5$. Figure 3 shows the robustness of our results for different values of $\sigma^* = (0.5, 0.4, 0.2, 0.1)$.

Labor supply parameters (B_M, B_U, ψ, ξ) : we assume perfectly elastic labor supply schedules (i.e. $(\psi^* = \xi^* = 0.0)$). The disutility parameters B_M^* and B_U^* are calibrated equal to 0.0482 and 0.0081, respectively, to match the logarithms of the average of the trend component for regular and underground employment; $\log \overline{N}_M$ and $\log \overline{N}_U$ equal to 16.44 and 14.77, respectively. Data are from the Italian Statistical Institute (ISTAT) over the sample 1992-2001.

Degree of substitutability between government and private consumption spending ϕ^* is set to 1, following Christiano and Eichenbaum (1992); this implicitly assumes that public consumption and private consumption are perfect substitutes. For completeness, however, Figure 3 verifies the robustness of our results to $\phi \in [0, 1]$. This sensitivity analysis goes in the spirit of Fiorito and Kollintzas (2004); they investigate the relationship between public and private consumption for twelve European countries.

The probability of being detected $p^* = 0.03$, and the **penalty factor** $s^* = 1.3$ are set to the value calibrated by Busato and Chiarini (2004). Finally, **Effective Tax Rates** are constructed from OECD dataset and calibrated to $\tau_Y = 0.3426$ and $\tau_F = 0.4155$.

Finally, notice that the model we use for assessing consequences of fiscal policy generates time series properties for simulated series that are consistent with corresponding statistics estimated for Italian economy. In this sense it could be used for consistently undertaking fiscal policy experiments.

5 Conclusions

This paper studies equilibrium effects of fiscal policy disturbances within a dynamic general equilibrium model where tax evasion and underground activities are explicitly incorporated. It is here shown that an underground sector mitigates the distortionary impact of fiscal policies, while lessening the drop (and the rise) of aggregate production after restrictive (expansionary) tax changes. Finally, tax evasion and underground economy can be seen as an economic mechanism that rationalizes expansionary effects to recessionary fiscal policies.

In summary, fiscal policy may be significantly affected by tax evasion. Indeed, the GDP elasticities to an increase in tax rates under tax evasion are very close to zero, while those computed without tax evasion are negative, consistently with the predictions of the neoclassical growth model. The almost zero elasticity value under tax evasion is perfectly consistent with consumption and income smoothing. Tax evasion and the informal economy offer, in other words, a channel for insuring income and consumption patters from fluctuations generated by fiscal policy. In this sense tax evasion can be interpreted as a smoothing device available to households and firms. It is like saying that the government chooses in fact statutory tax rates, while the effective tax rates are endogenously chosen by households and firm by relying on the additional dimension represented by tax evasion.

We think that these are crucial observations that have several implications to be exploited from a theoretical perspective and a policy perspective.

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Appendix

Proof of Proposition 1 (Sketch). A competitive equilibrium is characterized by the following conditions, evaluated at the stationary state:

$$\left(\frac{\beta^{-1}-1+\Omega}{\alpha\left(1-\tau_Y\right)\left(1-\tau_F\right)}\right)^{\frac{1}{\alpha-1}}=\frac{\overline{K}}{\overline{N}_M}$$

$$B_M\left(\overline{C}+\overline{C}_G\right)=\left(1-\tau_Y\right)\left(1-\tau_F\right)\left(1-\alpha\right)\left(\overline{K}\right)^{\alpha}\left(\overline{N}_M\right)^{-\alpha} \tag{1}$$

$$\left(B_U+B_M\right)\left(\overline{C}+\phi\overline{C}_G\right)=\left(1-ps\tau_F\right)\overline{N}_U^{-\sigma} \tag{2}$$

$$\overline{C}=-\Omega\overline{K}+\left(1-\tau_Y\right)\left(1-\tau_F\right)\left(\overline{K}\right)^{\alpha}\left(\overline{N}_M\right)^{1-\alpha}+\left(1-ps\tau_F\right)\left(\overline{N}_U\right)^{1-\sigma}\left(1+\frac{\left(1-\sigma\right)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)$$

$$\overline{C}_G=\left(\overline{w}_M\overline{N}_M+r\overline{K}\right)\tau_Y+\tau_F\left(ps\left(\overline{N}_U\right)^{1-\sigma}\left(1+\frac{\left(1-\sigma\right)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)+\left(\overline{K}\right)^{\alpha}\left(\overline{N}_M\right)^{1-\alpha}\right),$$

where $\left(1-ps\tau_F\right)\left(\overline{N}_U\right)^{1-\sigma}+\left(1-ps\tau_F\right)\overline{N}_U^{1-\sigma}\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma=\left(1-ps\tau_F\right)\left(\overline{N}_U\right)^{1-\sigma}\left(1+\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)$, and $ps\left(\overline{N}_U\right)^{1-\sigma}+ps\overline{N}_U^{1-\sigma}\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma=ps\left(\overline{N}_U\right)^{1-\sigma}\left(1+\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)$. Derivation of the stationary state is made of four steps.

Step 1. Stationary equilibrium for $\overline{N}_U$. Combining the (1) and (2) obtain:

$$N_U^{\star}=\left(\frac{B_M}{B_U+B_M}\frac{1-ps\tau_F}{\left(1-\tau_Y\right)\left(1-\tau_F\right)}\right)^{\frac{1}{\sigma}}\left(1-\alpha\right)^{-\frac{1}{\sigma}}\left(\frac{\overline{K}}{\overline{N}_M}\right)^{-\frac{\alpha}{\sigma}}$$

Step 2. Stationary equilibrium for N_M Consider the household's budget constraint, and the equilibrium condition for the government expenditure; factorizing out N_M obtain on the right hand side, the quantity $\overline{C}+\phi\overline{C}_G$ reads $\overline{C}+\phi\overline{C}_G=\overline{N}_M^{-1}\left(\mathcal{C}_1+\phi\mathcal{C}_2\right)$, where $\mathcal{C}_1=-\Omega\frac{K_t}{\overline{N}_M}+\left(1-\tau_Y\right)\left(1-\tau_F\right)\left(\frac{K}{\overline{N}_M}\right)^{\alpha}+\left(1+\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)\left(1-ps\tau_F\right)\left(\frac{\overline{N}_U}{\overline{N}_M}\right)\overline{N}_U^{-\sigma}$ and $\mathcal{C}_2=\left(\left(1-\tau_F\right)\tau_Y+\tau_F\right)\left(\frac{K}{\overline{N}_M}\right)^{\alpha}+\tau_F ps\left(\frac{\overline{N}_U}{\overline{N}_M}\right)\overline{N}_U^{-\sigma}\left(1+\frac{(1-\sigma)^{\frac{1}{\sigma}}}{1-\sigma}\sigma\right)$. Then, combining it with (1) yields

$$N_M^{\star}=\frac{\left(1-\tau_Y\right)\left(1-\tau_F\right)\left(1-\alpha\right)\left(\frac{K}{\overline{N}_M}\right)^{\alpha}}{B_M\left(\mathcal{C}_1+\phi\mathcal{C}_2\right)} \tag{3}$$

Step 3. Stationary value for capital stock. The Euler equation, evaluated at the stationary state, implies that:

$$K^{\star}=N_M^{\star}\left(\frac{\beta^{-1}-1+\Omega}{\alpha\left(1-\tau_Y\right)\left(1-\tau_F\right)}\right)^{\frac{1}{\alpha-1}}, \tag{4}$$

where $N_M^{\star}$ has been derived before.

Step 4. Once we have values for $K_t^{\star}$, $N_M^{\star}$, and $N_U^{\star}$, we can compute all other equilibrium quantities. □

Proof of Proposition 2 (Sketch). **Strategy**: Observe that the model is log-linear. First we derive closed form expressions for $\ln K_t^{\star}$, $\ln N_M^{\star}$, and $\ln N_U^{\star}$; then we compute $\log Y_M^{\star}=\alpha\log K^{\star}+$

$(1-\alpha)\log N_M^\star$ and $\log Y_U^\star = (1-\sigma)\log N_U^\star$; next we approximate $\log Y_M^\star \simeq \log Y_M^\star + \log Y_U^\star$. Finally we compute elasticities with respect income and corporate tax rates, abstracting from second order quantities.$\square$

For more details, see the Technical Appendix available at Francesco Busato's web page, in the "Working Paper"section: `http://www.econ.au.dk/vip_htm/fbusato/index.htm`. It is available upon request.

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