DEPARTMENT OF ECONOMICS

Working Paper

Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data

Niels Haldrup, Svend Hylleberg, Gabriel Pons,
Jaume Rosselló, and Andreu Sansó

Working Paper No. 2005-03

ISSN 1396-2426

UNIVERSITY OF AARHUS • DENMARK

INSTITUT FOR ØKONOMI

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 322
8000 AARHUS C - ☎ 89 42 11 33 - TELEFAX 86 13 63 34

WORKING PAPER

Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data

Niels Haldrup, Svend Hylleberg, Gabriel Pons,
Jaume Rosselló, and Andreu Sansó

Working Paper No. 2005-03

DEPARTMENT OF ECONOMICS

SCHOOL OF ECONOMICS AND MANAGEMENT - UNIVERSITY OF AARHUS - BUILDING 322
8000 AARHUS C - DENMARK ☎ +45 89 42 11 33 - TELEFAX +45 86 13 63 34

Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data

by

Niels Haldrup*, Svend Hylleberg*, Gabriel Pons**, Jaume Rosselló***, and Andreu Sansó***

22 March 2005

Abstract

This paper presents a new framework for coping with problems often encountered when modeling seasonal high frequency data containing both flow and stock variables. The idea is to apply a multivariate weekly representation of a daily periodic model and to exploit the possible cointegration and common feature properties of the variables in order to obtain a more parsimonious model representation. We introduce the notion of *common periodic correlations*, which are common features that co-vary - possibly with a phase shift - across the different days of the week and possibly also across weeks. The paper also suggests a way of modelling the dynamic interaction of stock and flow variables within a periodic setting that is similar to the concept of multicointegration among integrated variables. The proposed modelling framework is applied to a data set of daily arrivals and departures in the airport of Mallorca.

KEY WORDS: Periodic autoregression, seasonality, high frequency data, cointegration, multicointegration, common features.

JEL CODES: C12, C22, C32.

---

*Department of Economics, University of Aarhus, Building 322, DK-8000 Aarhus C, Denmark. E-mail: nhaldrup@econ.au.dk, shylleberg@econ.au.dk.**Institute of Economics, University of Copenhagen, E-mail: Gabriel.pons@econ.ku.dk ***Departament d'Economia Aplicada. Universitat de les Illes Balears. E-mail: andreu.sanso@uib.es. Gabriel Pons appreciates support from the Danish Research Council. Andreu Sansó acknowledges the financial support of the Direcció General de Recerca de la Conselleria d'Economia, Hisenda i Innovació del Govern de les Illes Balears under grant PRIB-2004-10095.

1 Introduction

A frequent criticism concerning the use of periodic models to describe seasonal phenomena is the fact that such models often require a huge number of parameters to be estimated, a problem which grows with the sampling frequency and periodicity of the observations. The present paper makes two contributions. First, it suggests a method to alleviate the problems associated with the potential overparametrization of periodic models by appropriate imposition of periodic cointegration and common feature restrictions on the short run dynamics. Secondly, within a periodic setting the paper scrutinizes the dynamic interaction that may exist in a system with both flow and stock variables which potentially are non-stationary.

Periodic models are often considered a convenient and flexible framework to model seasonal variation in the data, see e.g. Ghysels and Osborn (2001) and Franses and Paap (2004). In particular, it was demonstrated by Osborn (1988) how the multivariate representation of periodic models due to Gladyshev (1961) and Tiao and Grupe (1980) could be used as a basis for examining the non-stationary properties that frequently characterize economic time series. Because non-periodic models are nested within the periodic model, this is an attractive benchmark for testing various hypotheses. For instance, non-stationarity in the form of periodic integration can be challenged against (non-periodic) integration.

In the present paper, our running example assumes data sampled daily for which a high degree of overparametrization is likely to occur in a periodic context. The sample consists of approximately 8 years of observations of daily arrivals and departures in the airport of Mallorca. The data exhibits a strong form of periodic variation over the days of the week in addition to a strong seasonal variation over the year. The arrivals and departure series can be considered flow variables, and by looking at the difference between arrivals and departures on a given day, the net contribution to the stock of airline passengers visiting Mallorca can be calculated. The stock of visitors naturally follows as the accumulation of the net flow of passengers. Interestingly, the stock variable generated in this fashion tends to co-move with the individual arrivals and departures series, but statistical tests will demonstrate that the stock variable is indeed non-periodically integrated, whereas the arrivals and departure series are periodically integrated. This opens up for a description of how different (periodic or non-periodic) cointegration possibilities may arise in a complicated dynamic system where flows interact with the stock. The phenomenon that a stock variable will cointegrate with the flow variables (from which it is generated) is called *multicointegration* and was initially defined by Granger and Lee (1989, 1991). The notion of multicointegration cannot be directly adopted to a periodic context, but in the paper we describe how periodic models can be formulated to account for similar features.

To obtain more parsimonious representations of periodic models, we suggest applying the concept of serial correlation common features, see Engle and Kozicki (1993), within a periodic framework. The notion of serial correlation common features was initially suggested as a convenient way to restrict the short-run dynamics of multivariate models. However, for a periodic model of a univariate (or possibly multivariate) time series, it means that the periodic serial correlation features for the single days of the week appear to be common across the days and possibly also when linked to other series. By imposing such restrictions on the dynamics, the full model can be greatly simplified. This kind of restrictions will be named *common periodic correlations,* and to our knowledge this way of modelling periodic features has not yet been proposed in the literature. The representation of Hecq *et al.* (2004), which discriminates between strong and weak form features, is adapted to periodic models. Also, we extend the idea of non-synchronous features, Cubbada and Hecq (2001), which allow common features to co-vary - possibly with a phase shift - across the different days of the week and also potentially across weeks. It is shown that the presence of multiple common periodic cycles implies a nested reduced rank structure in a multivariate weekly model, which enables more efficient estimation of the highly parametrized model. When applying the methodology to the airport data, it is found that common periodic correlation features is a distinct property of the data, and by imposing the resulting restrictions the number of estimated parameters can be reduced by 30-35 %.

The characteristic features of the airport data set are described in Section 2, and especially motivation for the stock-flow analysis is discussed. Section 3 contains a presentation and discussion of the periodic autoregressive model for daily data with focus on the representation of such a model for univariate as well as bivariate stock and flow series. Section 4 discusses the common periodic correlation features both within and across weeks. Section 5 contains the empirical application, and the final section concludes.

2 The data set

The data set used in this paper consists of daily arrivals and departures in the Airport of Mallorca. The data spans the period from 1 January, 1994 to 28 February, 2002. This corresponds to 2981 daily observations (425 weeks). The Balearic Islands, and Mallorca in particular, are amongst the most important tourist destinations in the Mediterranean Sea. The annual volume of tourists is around 10 million people of whom over 95% travel by plane. More than 80% of these are tourists visiting Mallorca.

Since Mallorca is a "sun and sand" tourist destination, it is not surprising that passenger data exhibits a high degree of seasonal variation. This is verified by figure 1, which shows the variation of the data over the entire sample period. In addition to the arrivals and departures data, the figure shows the net flow of passengers to Mallorca, (arrivals minus departures), as well as the cumulation of the net flows denoted the stock. Note that the net flow variable indicates the contribution to the number of airline passengers who stay in Mallorca on a given date.[1]

From figure 1 the yearly variation of the transit data is most obvious. In particular, the very close co-movement of arrivals and departures is apparent suggesting a strong common seasonal pattern over the year. However, the day-of-week effect is also very apparent as can be seen from figure 2, where the daily variation for the year 2001 is displayed. Whereas the arrivals and departures exhibit very strong weekly fluctuations, the net flow and the stock variable obviously have much less variation within the week. This seems to indicate that some kind of common seasonal feature exists amongst the arrivals and departures series.

To focs further on the weekly periodicity, figure 3 displays the individual weekday observations for each series for the year 2001. As can be seen, the arrivals and departures have strong day-of-week effects (especially for Saturdays and Sundays), and this feature seems to vary over the year. Moreover, all days seem to co-move, which might indicate that the series potentially can be modelled as periodic seasonal processes. For the net flow series and the stock series, no significant periodic seasonal

[1] The stock variable indicates the *level* of people staying in Mallorca and not the actual figure because the initial value of observations is unknown.

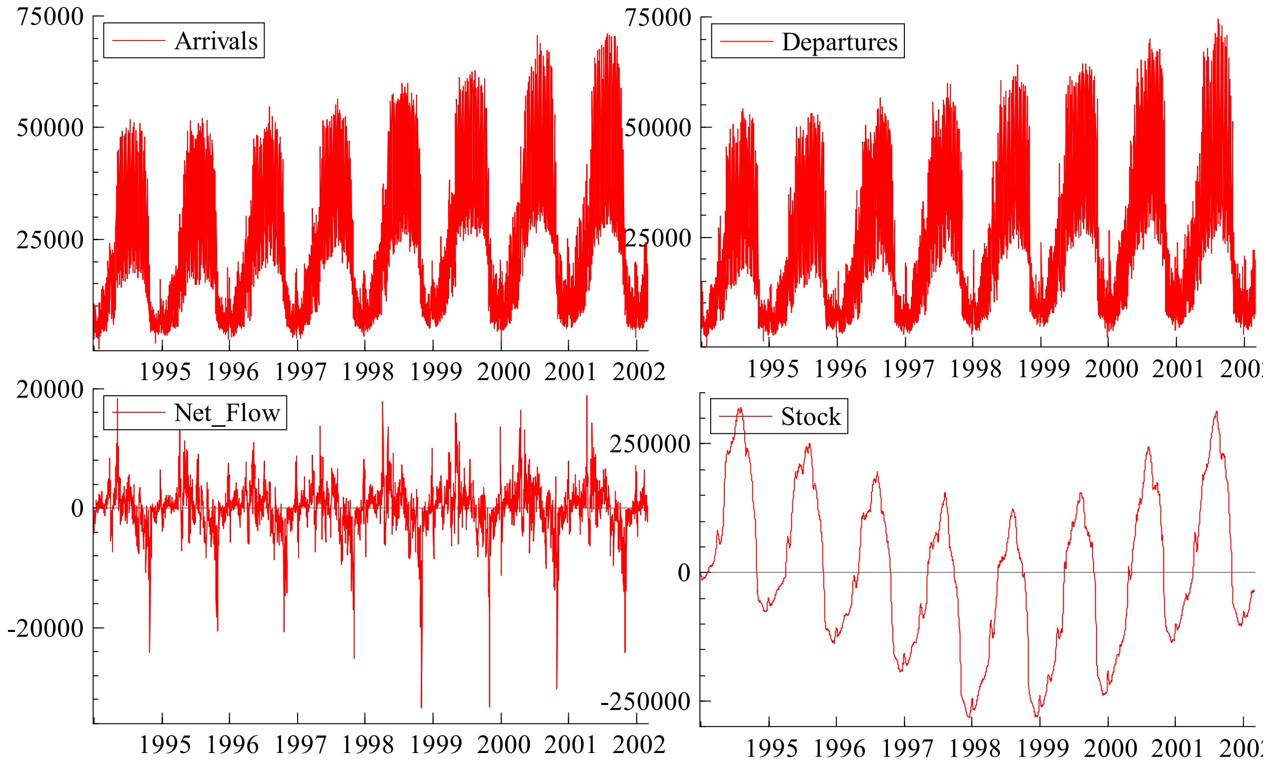

Figure 1: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, 1 January, 1994 - 28 February, 2002.

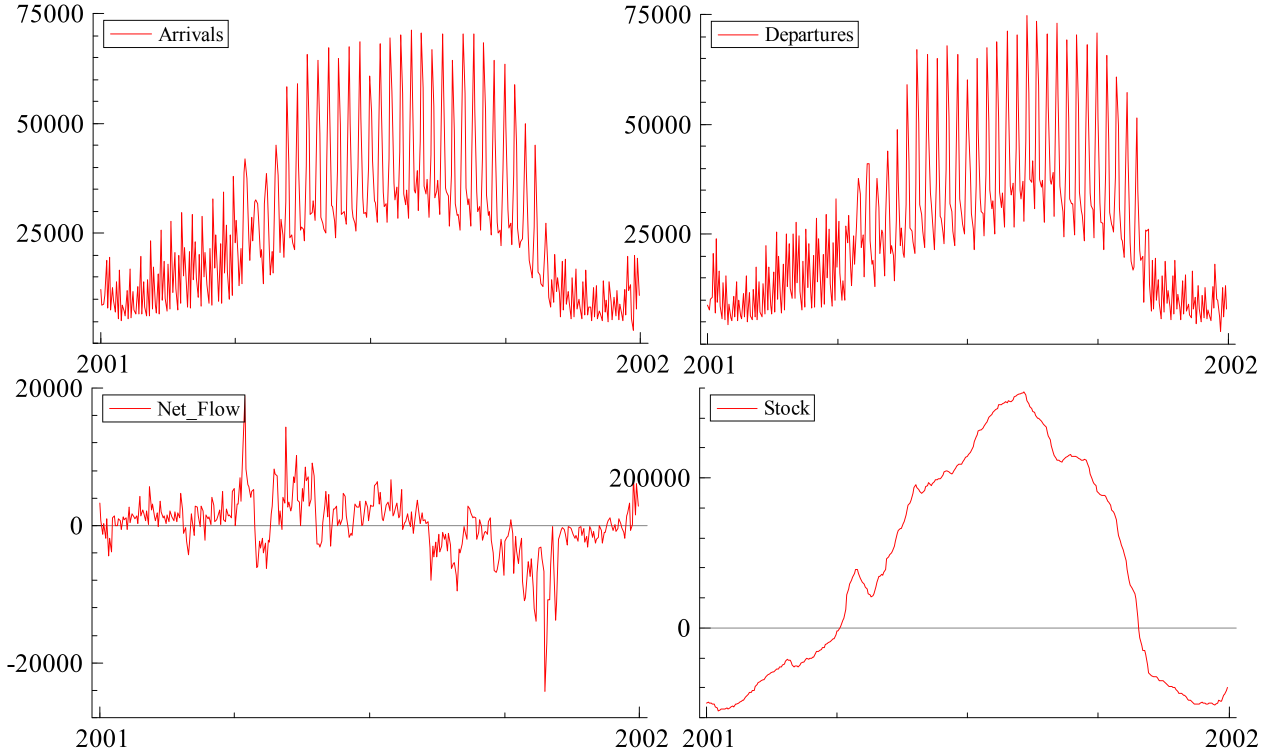

Figure 2: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, 1 January, 2001 - 31 December, 2001.

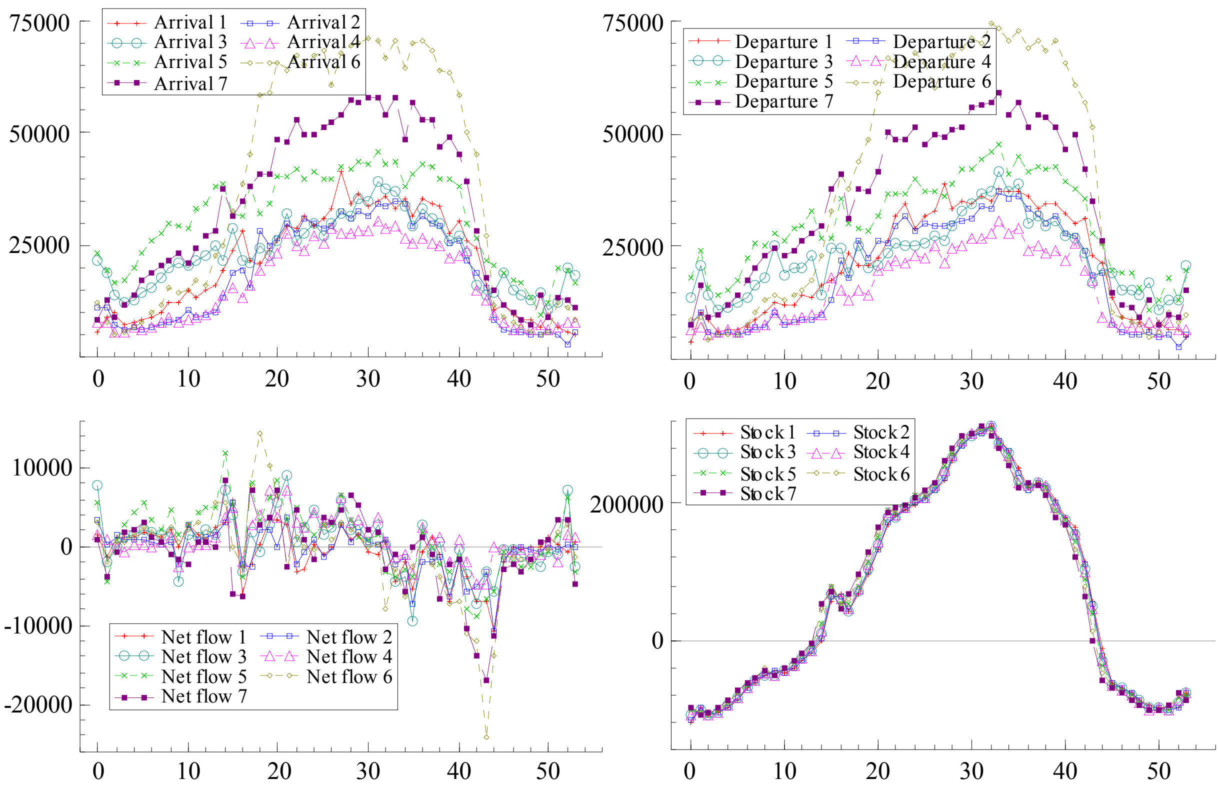

Figure 3: Arrivals, departures, net flow (i.e. arrivals minus departures), and the level of stock (i.e. the cumulated net flow of passengers) in the Airport of Mallorca, for each weekday (Monday 1, Tuesday 2,..., Sunday 7) of 2001.

variation seems to be present.

A further aspect of the present data set concerns the possibility of a multicointegration like feature amongst the series. If we assume that the arrivals and departures series are cointegrated, it is of interest to look at the cumulated net flow series, i.e. the stock variable generated from the arrivals and departures. Interestingly, it appears from figures 1 and 2 that although the stock series has much less (if any) weekly variation, the level around some trend co-varies with both the arrivals and departures series. This is an interesting phenomenon because it allows for the possibility of more than just one cointegrating relationship existing between just two series. The property is often referred to as multicointegration. There are numerous examples of multicointegration (at the zero frequency) in the literature as indicated in the introduction. What is of particular interest is the fact that a similar property is likely to arise in the daily transit data. The challenge of the present paper is to examine simultaneously the interactions between stocks and flows as well as the strong seasonal pattern in the data.

3 A Periodic Autoregressive Model for daily observations

As argued in section 2, it is likely that the arrivals and departures series follow periodic processes. Periodic models have frequently been criticized because such models require a lot of parameters to be estimated. However, in the present case data is not scarce, and a periodic modelling framework seems feasible as well as reasonable. Another implication is that models with fixed parameters and standard seasonal ARIMA processes, including seasonal unit root processes, are encompassed within the periodic model for certain restrictions on the parameters, and hence these restrictions can be tested.

3.1 The representation and properties of the model

Seasonal processes with a periodic correlation structure can be represented by periodic ARMA (henceforth PARMA) models, which allow for different parameters across the seasons. In practice, the estimation of pure PAR models has certain advantages over PARMA models, see Pagano (1978) and the review by McLeod (1995).

Let us describe the most relevant characteristics of the periodic model for a univariate time series where the periodicity is allowed for the day of the week.[2] General comprehensive surveys of periodic models and required inferential tools can be found in e.g. Ghysels and Osborn (2001) and Franses and Paap (2004). In the following we abstract from deterministic components to simplify notation, but extensions to this case are straightforward.

The daily periodic autoregressive process of order p, PAR(p), reads:

$$y_t = \phi_{s,1} y_{t-1} + \phi_{s,2} y_{t-2} + \cdots + \phi_{s,p} y_{t-p} + \varepsilon_t,\ s = 1,..,7 \tag{1}$$

where all the autoregressive parameters $\phi_{s,j}$ $(j = 1, ..., p)$ are allowed to vary with the season s, $(s = 1, ..., 7)$, i.e. the day of the week.[3] It should hold that at least one $\phi_{s,p} \neq 0$. ε_t is a white noise error term with periodic heteroskedasticity, $E\left(\varepsilon_t^2\right) = \sigma_s^2$. Note that, in this model, the parameters are allowed to be different for each day of the week, and therefore the PAR process is non-stationary since the autocorrelation function varies with the season. Another interesting source of nonstationarity frequently observed for economic data is the presence of stochastic trends, which can be examined within a multivariate representation of the PAR process. We denote this the vector of days (VD) representation.

This representation defines the 7-dimensional weekly multivariate process $Y_\tau \equiv (y_{1,\tau}, ..., y_{7,\tau})'$ with $\tau \equiv [(t-1)/7] + 1$ denoting the week. The vector series has the following multivariate (nonperiodic) representation, (see Gladyshev, 1961, or Tiao and Grupe, 1980):

$$\Phi_0 Y_\tau = \Phi_1 Y_{\tau-1} + \cdots + \Phi_P Y_{\tau-P} + E_\tau, \tag{2}$$

where Φ_k $(k = 0, ..., P;\ P = [(p+6)/7])$ are 7×7 matrices of parameters

$$\Phi_0 = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -\phi_{2,1} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ -\phi_{7,6} & \cdots & -\phi_{7,1} & 1 \end{bmatrix},\ \Phi_k = \begin{bmatrix} \phi_{1,7k} & \cdots & \cdots & \phi_{1,7k-6} \\ \vdots & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ \phi_{7,7k+6} & \cdots & \cdots & \phi_{7,7k} \end{bmatrix},$$

for $k = 1, ..., P$, and $E_\tau \equiv (\varepsilon_{1,\tau}, ..., \varepsilon_{7,\tau})' \sim$N(0,$\Sigma$), where $\Sigma =$diag$(\sigma_1^2, \sigma_2^2, ..., \sigma_7^2)$.

[2] Franses and Paap (2004) apply a daily periodic autoregressive model with 5 seasons for financial data.

[3] Note that the season s is related to the observation number t through $s \equiv t - 7\left[(t-1)/7\right]$ where $[\cdot]$ signifies the integer part of its argument.

For instance, the PAR(1) model, $y_t = \phi_s y_{t-1} + \varepsilon_t$, can be written as

$$\begin{pmatrix} 1 & 0 & \cdots & \cdots & 0 \\ -\phi_2 & 1 & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & -\phi_7 & 1 \end{pmatrix} \begin{pmatrix} y_{1,\tau} \\ y_{2,\tau} \\ \vdots \\ \vdots \\ y_{7,\tau} \end{pmatrix} = \begin{pmatrix} 0 & \cdots & 0 & \phi_1 \\ 0 & \cdots & \cdots & 0 \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \begin{pmatrix} y_{1,\tau-1} \\ y_{2,\tau-1} \\ \vdots \\ \vdots \\ y_{7,\tau-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1,\tau} \\ \varepsilon_{2,\tau} \\ \vdots \\ \vdots \\ \varepsilon_{7,\tau} \end{pmatrix}. \quad (3)$$

The unit root properties of the multivariate process Y_τ determine those of the daily process y_t. Define the matrix lag polynomial

$$\Phi\left(L^7\right) = \Phi_0 - \Phi_1 L^7 - \cdots - \Phi_P L^{7P},$$

where $Ly_{s,\tau} = y_{s-1,\tau}$ (with $Ly_{1,\tau} = y_{7,\tau-1}$) and $L^7 y_{s,\tau} = y_{s,\tau-1}$. When all the roots of the characteristic equation $|\Phi(L^7)| = 0$ lie outside the unit circle, the process Y_τ is second order stationary, and y_t is PI(0). Following the PAR(1) example, the necessary and sufficient condition for second order stationarity of this model is $|\phi_1\phi_2 \cdots \phi_7| < 1$. The multivariate process is integrated at the zero frequency if $|\Phi(L^7)| = 0$ has some roots equal to one. Of particular interest are those situations where every weekly process $y_{s,\tau}$ $(s = 1, ..., 7)$ is I(1). This property is known as first order non-stationarity (see Osborn, 2002).

A convenient way to represent the different possibilities of first order non-stationary processes is the error correction representation. Consider the VAR representation of (2) on error correction model form:

$$\Pi(L^7)Y_\tau = U_\tau,$$

where $\Pi(L^7) = I_7 - \Pi_1 L^7 - ... - \Pi_P L^{7P}$, with $\Pi_k \equiv \Phi_0^{-1}\Phi_k$, and $U_\tau = \Phi_0^{-1}E_\tau \sim \mathrm{N}(0, \Phi_0^{-1}\Sigma(\Phi_0^{-1})')$. Decompose the matrix lag polynomial as $\Pi(L^7) = -\Pi L^7 + \Gamma(L^7)(1 - L^7)$ where $\Pi = \Phi_0^{-1}\left(\sum_{j=1}^{P}\Phi_j\right) - I_7$, $\Gamma_0 = I_7$, and $\Gamma_k = \Phi_0^{-1}\Sigma_{j=k+1}^{P}\Phi_j$ $(k = 1, .., P-1)$ such that we obtain the VAR model:

$$\Delta_7 Y_\tau = \Pi Y_{\tau-1} + \sum_{k=1}^{P-1}\Gamma_k \Delta_7 Y_{\tau-k} + U_\tau, \quad (4)$$

with $\Delta_7 = 1 - L^7$.

The different cases of first order non-stationarity are associated with different properties of the impact matrix Π. In particular, when Π has rank 7, the process y_t is periodically integrated of order zero, PI(0), and when Π has rank 6, y_t is periodically integrated of order one, PI(1)[4]. In this case we may distinguish two important cases. When the 6 cointegrating relations are given by $y_{2,\tau}-y_{1,\tau}$, $y_{3,\tau}-y_{2,\tau}$, ..., $y_{7,\tau}-y_{6,\tau}$, then y_t is a non-seasonally and non-periodically integrated process, that is an I(1) process. When the 6 cointegrating relations read $y_{2,\tau}-\phi_2 y_{1,\tau}$, $y_{3,\tau}-\phi_3 y_{2,\tau}$, ..., $y_{7,\tau}-\phi_7 y_{6,\tau}$ with at least one $\phi_s \neq 1$ $(s = 2, ..., 7)$, then y_t is PI(1) where ϕ_s are named the periodic integration coefficients. Hence the I(1) model appears as a special case of the PI(1) model. When y_t ~PI(1), the difference operator Δ does not remove the stochastic trend from y_t. In this case it is necessary to apply a specific difference filter for every season, the quasi-difference filter, $\delta_s(L) \equiv 1 - \phi_s L$ such that $\delta_s(L)y_t$ ~PI(0).

When Π has rank $0 \leq r < 6$, y_t is a seasonally integrated process with $7 - r$ unit roots at seasonal frequencies, see Hylleberg *et al.* (1990), Franses (1994), and Ghysels and Osborn (2001).

Generally, under the reduced rank of Π $(0 < r < 7)$, the impact matrix can be decomposed as $\Pi = \alpha\beta'$, where α and β are $7 \times r$ matrices of full column rank that contain the adjustment vectors and the cointegrating vectors, respectively. Then we can rewrite (4) as

$$\Delta_7 Y_\tau = \alpha\beta' Y_{\tau-1} + \sum_{k=1}^{P-1} \Gamma_k \Delta_7 Y_{\tau-k} + U_\tau. \tag{5}$$

We denote the r-dimensional cointegrating disequilibrium process by $Z_\tau \equiv \beta' Y_\tau$.

For instance, the general PAR(1) process can be represented as

$$\Delta_7 Y_\tau = \begin{pmatrix} -1 & 0 & \cdots & 0 & \phi_1 \\ 0 & \ddots & \ddots & 0 & \phi_1\phi_2 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ \vdots & \cdots & \ddots & -1 & \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 \\ 0 & \cdots & \cdots & 0 & \phi_1\phi_2\phi_3\phi_4\phi_5\phi_6\phi_7 - 1 \end{pmatrix} Y_{\tau-1} + U_\tau.$$

If $\phi_1\phi_2 \cdots \phi_7 = 1$, but not all $\phi_s = 1$, then y_t ~PI(1). In the case of PI(1), the

[4] The PI(0) process was introduced by Gladyshev (1961) denoted a periodically correlated process, and the PI(1) was introduced by Osborn (1988).

cointegrating matrix β will contain six of the coefficients ϕ_s:

$$\beta' = \begin{pmatrix} -\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi_6 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\phi_7 & 1 \end{pmatrix}, \tag{6}$$

and the remaining coefficient is $\phi_1 = (\phi_2\phi_3\phi_4\phi_5\phi_6\phi_7)^{-1}$.

Note that the common stochastic trend can be found as $\beta'_\perp Y_t$ where $\beta_\perp$ is the orthogonal complement of β satisfying $\beta'_\perp\beta = 0$ and thus

$$\beta'_\perp = \begin{pmatrix} \phi_1, & (\phi_3\phi_4\phi_5\phi_6\phi_7)^{-1}, & (\phi_4\phi_5\phi_6\phi_7)^{-1}, & (\phi_5\phi_6\phi_7)^{-1}, & (\phi_6\phi_7)^{-1}, & \phi_7^{-1}, & 1 \end{pmatrix}. \tag{7}$$

The weekly multivariate representation can be used to select among the different first order non-stationary possibilities, by means of multivariate cointegration analysis (see Johansen, 1991). This procedure is proposed by Franses (1994) for quarterly PAR models. The same method can be used to test for periodic integration of the daily flows and stock series. Within the unifying framework of a periodic model we can test for a multitude of different types of first order unit roots, which is somewhat more involved when considering the daily representation of the time series, see Ghysels and Osborn (2001).

3.2 Bivariate Periodic Models

Consider the daily bivariate PAR(p) process $\mathbf{y}_t = (y_t^1, y_t^2)'$ where y_t^1 and y_t^2 can denote, for instance, arrivals and departures:

$$\mathbf{y}_t = \boldsymbol{\phi}_{s,1}\mathbf{y}_{t-1} + \boldsymbol{\phi}_{s,2}\mathbf{y}_{t-2} + \cdots + \boldsymbol{\phi}_{s,p}\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t,\ s = 1, ..., 7,$$

and $\boldsymbol{\phi}_{s,j}$ $(j = 1, .., p)$ are 2-dimensional square matrices of coefficients $\boldsymbol{\phi}_{s,j}^{i,h}$ $(i, h = 1, 2)$, which may vary with the day of the week. Under cointegration we can represent the PAR(p) as

$$D_s\mathbf{y}_t = \boldsymbol{\alpha}_s\boldsymbol{\kappa}'_s\mathbf{y}_{t-1} + \Sigma_{j=1}^{p-1}\gamma_{s,j}D_{s-j}\mathbf{y}_{t-j} + \boldsymbol{\varepsilon}_t,$$

where $D_s = diag(\delta_s^1(L), \delta_s^2(L))$ (with $D_{s-7} = D_s$) is the quasi-difference operator turning the bivariate PI(1) process into a bivariate PI(0) process (see Ghysels and Osborn, 2001).

3.2.1 Cointegration between Flow Variables

In this section we consider the multivariate representation of the daily flow variables y_t^1 and y_t^2. Consider the weekly representation of the daily bivariate process $\mathbf{y}_t = (y_t^1, y_t^2)'$, where now we define the 14−dimensional VD process $Y_\tau \equiv (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$, and assume that it can be represented by a VAR(P) model

$$\mathbf{\Pi}(L^7)\mathbf{Y}_\tau = \mathbf{U}_\tau,$$

where $\mathbf{\Pi}(L^7) = I_{14} - \mathbf{\Pi}_1 L^7 - ... - \mathbf{\Pi}_P L^{7P}$, with $P = [(p+6)/7]$, $\mathbf{U}_\tau$ is a 14-dimensional white noise process with a covariance matrix having 7×7 diagonal square blocks. The error correction representation of the VAR model reads:

$$\Delta_7 \mathbf{Y}_\tau = \mathbf{\Pi Y}_{\tau-1} + \sum_{k=1}^{P-1} \mathbf{\Gamma}_k \Delta_7 \mathbf{Y}_{\tau-k} + \mathbf{U}_\tau. \tag{8}$$

Under the presence of (periodic) cointegration between the daily series, the 14 weekly series have a common stochastic trend, and the impact matrix $\mathbf{\Pi}$ can be written $\mathbf{\Pi} = \boldsymbol{\alpha\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are full column rank 14×13−matrices,

$$\boldsymbol{\beta}' = \begin{bmatrix} I_7 & \mathbf{K} \\ \mathbf{0} & \beta^{2\prime} \end{bmatrix}. \tag{9}$$

$\mathbf{I}_7$ is the 7-dimensional identity matrix, $\mathbf{0}$ is the 6×7-dimensional null matrix, $\mathbf{K}$ is a 7-dimensional matrix containing the cointegrating coefficients on the diagonal, $\mathbf{K}$ =diag$(-k_1, -k_2, ..., -k_7)$, and hence $y_{s,\tau}^1 - k_s y_{s,\tau}^2$ ∼PI(0) (s=1,...,7). β^2 is the 7×6-dimensional matrix containing the periodic integration coefficients associated with y_t^2. We define the general notation

$$\beta^{i\prime} = \begin{bmatrix} -\phi_2^i & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\phi_3^i & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\phi_4^i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\phi_5^i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\phi_6^i & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\phi_7^i & 1 \end{bmatrix}, \tag{10}$$

where ϕ_s^i are the periodic integration coefficients for the series y_t^i.

Osborn (2002) discusses how periodically and non-periodically integrated processes can potentially cointegrate in various cases. When both daily variables y_t^i $(i = 1, 2)$ are I(1), then $k_s = k$ $(s = 1, ..., 7)$, and $\phi_s^2 = 1$ $(s = 1, ..., 6)$, that is, in this case the daily variables are non-periodically cointegrated. When both daily variables y_t^i $(i = 1, 2)$ are PI(1), the cointegrating vectors may be different across the different days of the week, i.e. such that the series are fully periodically cointegrated $k_s \neq k$ (at least for some s), and $k_s \neq 0$ $(s = 1, ..., 7)$. However, the series could also be non-periodically cointegrated such that $k_s = k$ $(s = 1, ..., 7)$. When one daily variable is I(1), and the other one is PI(1), the variables may only be fully periodically cointegrated.

Under the absence of (periodic) cointegration between y_t^1 and y_t^2, the 14 weekly series have two stochastic trends. In particular, the impact matrix reads $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are full column rank $14 \times 12-$matrices, where in particular

$$\boldsymbol{\beta}' = \begin{bmatrix} \beta^{1\prime} & \mathbf{0} \\ \mathbf{0} & \beta^{2\prime} \end{bmatrix},$$

and $\mathbf{0}$ is the 6×7-dimensional null matrix.

The multivariate representation of the flow variables (8) is the basis to testing for periodic cointegration between daily arrivals and daily departures. First, one should test for cointegration using e.g. the ML procedure of Johansen (1991). Then, one may test for non-periodic cointegration through the hypothesis $k_s = k$ $(s = 1, ..., 7)$. Given the relation $\phi_s^1 = \phi_s^2 k_s / k_{s-1}$, the test for non-periodic cointegration can be interpreted as a test of equivalent periodic integration coefficients $\phi_s^1 = \phi_s^2$ $(s = 1, ..., 7)$, such that under non-periodic cointegration, the periodic integration coefficients for the departures are also the periodic integration coefficients of the arrivals.

3.2.2 Cointegration between Stock and Flow Variables

Assume that the net flow $y_t^3 = y_t^1 - y_t^2$ is PI(0). Consequently there exists fully non-periodic cointegration between the daily arrivals y_t^1 and the daily departures y_t^2, and the stock variable $y_t^4 = y_0^4 + \sum_{j=1}^t y_t^3$ is I(1) by definition. It is then feasible that the daily stock y_t^4 may cointegrate with the daily arrivals or with the daily departures. This phenomenon is known as multicointegration (see Granger and Lee, 1989, 1990), and it implies that cointegration may occur not only between the flow variables, but also between the flow and the stock variable, which itself is generated from the flows. The notion of multicointegration does not extend directly to periodic models, but similar features are likely to exist concerning the interaction of stock and flows. The property can be analyzed in the same way as the cointegration between the arrivals and departures by substituting for example the seven weekly arrivals series by the seven stock series in Y_τ.[5]

Potentially, the analysis can be undertaken in a smaller system when the stock variable is *non*-periodically integrated. Hence, if the flow variable is PI(1), and the stock variable is non-periodic I(1), then if these variables are cointegrated, cointegration between the stock and the flow variables can only be periodic (see Osborn, 2002)

$$y^1_{s,\tau} - k_s y^4_{s,\tau} \text{ with } k_s \neq k \text{ for some } k_s.$$

Because the stock variable $y^4_{s,\tau}$ is non-periodically integrated by assumption, it is natural to reduce the model from a 14-dimensional to a 8-dimensional system. Consider the following transformation matrix of dimension 8×14 :

$$\mathbf{W} = \begin{pmatrix} I_7 & \mathbf{0} \\ \mathbf{0} & \frac{1}{7}\beta^{4\prime}_{\perp} \end{pmatrix},$$

where $\beta^4_\perp$ is the orthogonal complement associated with β^4 given in (10). In this particular case, *non*-periodic integration implies that $\phi^4_s = 1$, $(s = 1, ...7)$, and hence the orthogonal matrix according to (7) simplifies to a vector of ones. This means that the reduced system can be defined for the variables $\mathbf{Y}^*_\tau = \mathbf{W}\mathbf{Y}_\tau = (y^1_{1,\tau}, y^1_{2,\tau}, ..., y^1_{7,\tau}, \overline{y}^4_\tau)$ where $\overline{y}^4_\tau = \frac{1}{7}\sum_{s=1}^{7} y^4_{s,\tau}$ is the mean of the stock series over the week. The new system reads

$$\Delta_7 \mathbf{Y}^*_\tau = \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*\prime} \mathbf{Y}^*_{\tau-1} + \sum_{k=1}^{P-1} \mathbf{\Gamma}^*_k \Delta_7 \mathbf{Y}^*_{\tau-k} + \mathbf{U}^*_\tau,$$

where $\boldsymbol{\beta}^*$ in this case takes the form

$$\boldsymbol{\beta}^{*\prime} = \begin{bmatrix} I_7 & \mathbf{k} \end{bmatrix}. \tag{11}$$

[5]Daily arrivals y^1_t could be used in place of daily departures.

where $\mathbf{k} \equiv (k_1, k_2, k_3, k_4, k_5, k_6, k_7)'$ is a 7×1 column vector containing the multicointegrating coefficients such that $y^1_{s,\tau} - k_s \overline{y}^4_\tau$ is PI(0). Note that alternatively the stock variable of the single days represents the common I(1) stock trend.

When y^4_t and y^1_t are not cointegrated, then $\mathbf{\Pi}^* = \boldsymbol{\alpha}^* \boldsymbol{\beta}^{*\prime}$, where $\boldsymbol{\alpha}^*$ and $\boldsymbol{\beta}^*$ are full column rank 8×6 matrices, and the cointegration matrix can be written

$$\boldsymbol{\beta}^{*\prime} = \begin{bmatrix} \beta^{2\prime} & \mathbf{0} \end{bmatrix},$$

where $\mathbf{0}$ is a 6×1 column vector of zeros, and β^2 is given by (10).

4 Common Periodic Correlation Features

When building PAR models it is recommended to introduce restrictions on the periodic components of the model to increase the degrees of freedom, see Ghysels and Osborn (2001). One way to do this is by imposing appropriately tested common feature restrictions on the model like common business cycles, common stationary annual seasonality, or common deterministic annual seasonality[6], Engle and Hylleberg (1996). It is a very plausible assumption that the daily series y_t, in addition to the trend, will share common features across the days of the week. To our knowledge, these kinds of restrictions to describe the common periodicity of the cycles have not yet been proposed in the literature.

4.1 Common Periodic Correlation features within the week

Engle and Kozicki (1993) introduced the notion of serial correlation common features to represent common cycles among different economic time series. For an n-dimensional system like (4) (e.g. with $n = 7, 8, 14$), we get that if there exists an $n \times q$ matrix $\widetilde{\beta}$ that annihilates both the short-run and the long-run dynamics

$$\begin{array}{llll} \text{(i)} & \widetilde{\beta}'\Gamma_k & = & 0 \qquad (k = 1, ..., P-1), \\ \text{(ii)} & \widetilde{\beta}'\Pi & = & -\widetilde{\beta}'\alpha\beta' = 0, \end{array}$$

then Y_τ is said to have serial correlation common features.

[6] In the sequel the notion 'periodic' refers to the daily variation of the data, whereas the remaining seasonality (e.g. within the year) is referred to simply as 'seasonality'.

Under these two conditions, the cofeature matrix $\widetilde{\beta}$ turns the differenced variables into a q-dimensional white noise process $\widetilde{\beta}'\Delta_7 Y_\tau = \widetilde{\beta}' U_\tau$, and the short-run dynamics of the n series is driven by $n-q$ dynamic factors. However, in this case the number of common features $n-q$ is bounded by the cointegration rank r, $r \leq n-q \leq n$. When the daily series is PI(1) or I(1), the number of common features cannot be smaller than 6. Hence, the serial correlation common features allow only little flexibility concerning the imposition of restrictions on the periodicity of the process.

Hecq *et al.* (2004) consider a less restrictive form of common cycles and introduce the idea of a Weak Form (WF) of serial correlation common features, which requires that the cofeature matrix removes the short-run component, but not the long-run. Hence, under i), but not ii), different common factors generate the long-run and the short-run dynamics of the variables. In the present case, under the WF structure there exists an $n \times q$ dimensional cofeature matrix $\widetilde{\beta}$ that turns the differenced variables adjusted for long-run effects into a q-dimensional white noise process $\widetilde{\beta}' (\Delta_7 Y_\tau - \alpha Z_{\tau-1}) = \widetilde{\beta}' U_\tau$. Then the cointegrated system can be expressed as

$$\Delta_7 Y_\tau = \alpha Z_{\tau-1} + \widetilde{\beta}_\perp \mathbf{W}_{\tau-1} + U_\tau,$$

where $\mathbf{W}_\tau = \mathbf{\Upsilon} \mathbf{X}_\tau$ contains the serial correlation common features. $\mathbf{X}_\tau = (\Delta_7 Y_\tau', \ldots, \Delta_7 Y_{\tau-P+2}',)'$, $\mathbf{\Upsilon} \equiv [\Upsilon_1, \ldots, \Upsilon_{P-1}]$ is a $(n-q) \times (n(P-1))$ matrix, and $\widetilde{\beta}_\perp$ is an $n \times (n-q)$ full column rank matrix satisfying $\widetilde{\beta}'\widetilde{\beta}_\perp = 0$. In Hecq *et al.* (2004) inferential procedures for common serial correlation feature models are discussed in detail using canonical correlation techniques.

The notion of WF serial correlation common features is more flexible in our setting than the serial correlation common features, since the number of common features $(n-q)$ is not bounded by the cointegrating rank. Concretely, the number of common features may take any value between 1 and n. For example, in the case of the multivariate representation of one of the daily series, $1 \leq 7-q \leq 7$. When $7-q = 1$, the short-run dynamics of the seven day-of-week series are driven by the same factor. On the other extreme, when $7-q = 7$, the short-run dynamics of the single days is generated by different factors. In our framework, when $0 < q < n$, we name such common features *common periodic correlation (CPC) features*, and $n-q$ denotes the number of CPC features.

The CPC feature in the periodic correlation framework implies that the short-run dynamics of each day of the week (including the stationary annual seasonality as well as the business cycles) is driven by a reduced number of factors. Strictly speaking, the common dynamic factors are asynchronous in terms of the daily model since they relate to different days of the week, but they are synchronous in terms of the weekly representation. Obviously, it is likely that asynchronous common cyclical components occur also in the weekly representation in the sense that the dynamics of consecutive days of different weeks could be as close as the dynamics of consecutive days of the same week.

4.2 Common Periodic Correlation features across the weeks

When considering the presence of common periodic cyclical features between high frequency variables, say the arrivals and the departures, it is likely that such variables will exhibit non-contemporaneous cyclical co-movements in the sense that cycles co-move with a phase shift of a particular number of days *exceeding* a week; a property that is not captured by the CPC features described in the preceding section. The notion of a polynomial serial correlation common feature (Cubadda and Hecq, 2001) can also be considered in its weak form (Hecq *et al.,* 2004), which in this periodic context we name *weak form polynomial CPC features.*

The PI(1) process Y_τ has weak form polynomial CPC of order m ($m < P-1$), denoted CPC(m), if there exists a $7 \times q_m$ polynomial matrix $\widetilde{\beta}_m(L) = \sum_{j=0}^{m} \widetilde{\beta}_{m,j} L^j$ such that $\widetilde{\beta}_{m,0}$ has full column rank, and

$$\widetilde{\beta}'_{m,0}\Gamma_k = \begin{cases} -\widetilde{\beta}'_{m,k} & \text{if } k = 1, ..., \ m, \\ 0 & \text{if } k > m. \end{cases}$$

Under CPC(m), the cofeature matrix reduces the order of the error correction model from $P-1$ to m, $\widetilde{\beta}'_{m,0}\left(\Delta_7 Y_\tau - \alpha Z_{\tau-1}\right) = -\widetilde{\beta}'_{m,1}\Delta_7 X_{\tau-1} - ... - \widetilde{\beta}'_{m,m}\Delta_7 X_{\tau-m} + \widetilde{\beta}'_{m,0}U_\tau$, such that under CPC($m$) the cointegrated system can be written as

$$\Delta_7 Y_\tau = \alpha Z_{\tau-1} + \sum_{k=1}^{m} \Gamma_k \Delta_7 Y_{\tau-k} + \widetilde{\beta}_{m,0\perp} \sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 Y_{\tau-j} + U_\tau,$$

where Υ_j are $(n-q_m) \times 7$ matrices, $\widetilde{\beta}_{m,0\perp}$ is a $7 \times (n-q_m)$ full column rank matrix satisfying $\widetilde{\beta}'_{m,0}\widetilde{\beta}_{m,0\perp} = 0$, and $\sum_{k=m+1}^{P-1} \Upsilon_j \Delta_7 Y_{\tau-j}$ contains the $(n-q_m)$-dimensional common dynamic factor. The presence of the CPC(m) implies restrictions on the periodic coefficients in a similar way as non-polynomical CPC (or CPC(0)), but now involving only more distant lags, and therefore implies complex restrictions among the autocorrelation coefficients of the cyclical component of Y_τ, which is basically the short-run dynamics of y_t.

Notice that CPC(m) of different orders may cohabit in the error correction model and thus imply different restrictions on the autocorrelation structure. To illustrate this consider the 7-dimensional error correction model $\Delta_7 Y_\tau = \alpha\beta' Y_{\tau-1} + \Gamma_1 \Delta_7 Y_{\tau-1} + \Gamma_2 \Delta_7 Y_{\tau-2} + U_\tau$. In the unrestricted case without any CPC features, the short-run matrices Γ_1 and Γ_2 each contain 49 parameters, that is, we have 98 parameters to estimate. Now, consider the case where we have a contemporaneous common feature CPC(0) of $q_0 = 3$. In this case, $\Gamma_1 = \widetilde{\beta}_\perp \Upsilon_1$, and $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$, where $\widetilde{\beta}_\perp$ is $n \times (n - q_0) = 7 \times 4$, and Υ_1 and Υ_2 are both $(n - q_0) \times n = 4 \times 7$ matrices, which implies a total of 84 parameters. Next, consider the case with polynomial common feature CPC(1) of $q_1 = 4$. Under this property there are no synchronous common dynamic factors, and Γ_1 is $n \times n = 7 \times 7$, with 49 parameters. There are three common dynamic factors among the day-of-week series, where at least one of these elements pertains to the preceding week, and $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$ with $\widetilde{\beta}_\perp$ being $n \times (n - q_1) = 7 \times 3$ and Υ_2 being 3×7 matrices. Hence, the number of parameters is $49 + 42 = 91$. Alternatively, consider the case where we have CPC(1) with $q_1 = 6$. Here Γ_1 is $n \times n = 7 \times 7$ as above, while $\Gamma_2 = \widetilde{\beta}_\perp \Upsilon_2$ with $\widetilde{\beta}_\perp$ being 7×1 and Υ_2 being 1×7 matrices, such that the number of short-run parameters is given by $49 + 14 = 63$.

Some limitations of the existing methods to detect the presence of such common features (see Cubadda and Hecq, 2001, and Hecq *et al.*, 2004) are that the statistical tests for q_0 CPC(0) and for q_1 CPC(1) are not independent, the test for non-polynomial common features imposes the same rank for all the short-run matrices, while the test for polynomial common features tells nothing about the first short-run matrices. All in all, we can test for the presence of different CPC(m), but we cannot impose the structure implied by all of them. A solution is thus to select the CPC(m) that is the most parsimonious representation of the short-run dynamics. Returning to our example, we may distinguish two VAR models, one with CPC(0) of $q_0 = 3$ and CPC(1) of $q_1 = 4$, and another one with CPC(0) of $q_0 = 3$ and CPC(1) of $q_1 = 1$. The CPC(0) implies a more parsimonious representation of the short-run dynamics in the first model, while the CPC(1) gets the biggest reduction of parameters in the second model. We suggest selecting the most parsimonious model. Again the estimation procedure of Hecq *et al.* (2004) can be used to estimate the models using canonical correlation analysis.

5 Empirical Application

To perform both the univariate and the bivariate analyses of the airport passenger data described in section 2, we specify the following model

$$\Delta_7 \mathbf{Y}_\tau = \boldsymbol{\mu} + \boldsymbol{\Psi}\mathbf{d}_\tau + \boldsymbol{\Theta}\mathbf{c}_\tau + \boldsymbol{\Pi}\mathbf{Y}_{\tau-1} + \sum_{k=1}^{P-1} \boldsymbol{\Gamma}_k \Delta_7 \mathbf{Y}_{\tau-k} + \mathbf{U}_\tau, \qquad (12)$$

where $\boldsymbol{\mu}$ is a $(n \times 1)$ vector of unrestricted intercepts (and linear trends when $\mathbf{Y}_\tau$ includes stock series), $\boldsymbol{\Psi}$ is a $n \times 12$ matrix of unrestricted parameters associated with $\mathbf{d}_\tau$, which is a matrix of 12 trigonometric variables $\cos(j\pi/26 \times \tau)$ and $\sin(j\pi/26 \times \tau)$ $(j = 1, ..., 6)$ to account for the deterministic annual seasonality in a parsimonious way, $\boldsymbol{\Theta}$ is a $n \times 5$ matrix of unrestricted parameters corresponding to calendar effects.[7] As previously, $\boldsymbol{\Pi}$ and $\boldsymbol{\Gamma}_k$ are $n \times n$ matrices possibly of reduced rank. The $\mathbf{Y}_t$ vector consists of various combinations of the passenger series, i.e. arrivals y_t^1, departures y_t^2, net flow $y_t^3 = y_t^2 - y_t^1$, and the stock of visitors variable $y_t^4 = y_0^4 + \sum_{j=1}^{t} y_t^3$. For the periodic integration and CPC(0) analyses, $n = 7$, and $\mathbf{Y}_\tau = (y_{1,\tau}^i, ..., y_{7,\tau}^i)'$ $(i = 1, 2, 3, 4)$; whereas for the periodic cointegration and multiple CPC(m) analyses $n = 14$, and $\mathbf{Y}_\tau = (y_{1,\tau}^1, ..., y_{7,\tau}^1, y_{1,\tau}^2, ..., y_{7,\tau}^2)'$. Based upon the univariate empirical findings it appears useful for the analysis of stock-flow interactions to consider $n = 8$ and $\mathbf{Y}_\tau = (y_{1,\tau}^2, ..., y_{7,\tau}^2, \overline{y}_\tau^4)'$ where $\overline{y}_\tau^4$ is the weekly average of the stock series.

The daily series are filtered from additive outliers to prevent the potential distortionary effect of such outliers on the cointegration analysis (see Haldrup *et al.*, 2004). All the outliers capture effects not collected by the calendar effect variables.

The order P of the VAR model has been chosen according to the AIC criterion, which performs reasonably well within high dimensional systems (see Gonzalo and Pitarakis, 2002) such that for the univariate analysis $P = 8$ is selected for arrivals

[7] Concretely, we introduce five calendar type dummy variables accounting for Easter, Christmas, end-of-the-year, May-the-first and the All-Saints week of festivals.

and departures (y_t^1 and y_t^2), and $P = 3$ for the net flow and stock variables (y_t^3 and y_t^4). For the bivariate analyses $P = 8$ is selected for both the periodic cointegration analysis of the pairwise flow relations and the stock-flow relations.[8]

5.1 Testing for periodic integration and cointegration amongst stocks and flows

5.1.1 The univariate series

The rank of the matrix $\mathbf{\Pi}$ has been determined according to the Johansen procedure. Table 1 reports the Johansen trace test statistic (LR_r) and the estimated coefficients ϕ_s of the quasi-difference operators $\delta_s(L) \equiv 1 - \phi_s L$ associated with the cointegrating vectors of the four series. The cointegration analysis of the VD series corresponding to the daily arrivals and departures provides strong evidence favouring the PI(1)/I(1) characteristic of such series by detecting 6 cointegrating relations.[9] The fact that 6 cointegrating relations are present means that the 7 daily series exhibit the same stochastic trend which implies that the series cannot be seasonally integrated (Hylleberg *et al.*, 1990, and Franses, 1994).

The cointegration analysis of the net flow series does not detect any cointegrating relationships, while the cointegration analysis of the stock series again detects a cointegration rank of 6 and hence suggests the series to be PI(1) or I(1) depending upon further restrictions. I(1) against PI(1) of the arrivals, departures and the stock series can be tested by restricting the value of the cointegrating vectors. More specifically, when $\phi_s^i = 1$ for all $s = 1, 2, ..., 7$, a (1, -1) cointegrating relation exists across the single days of the VD representation, and hence in this case non-stationarity is non-periodic I(1). As seen from the estimates of the periodic coefficients, both the arrivals and departures series are PI(1), i.e. the periodic coefficients are very different. Hence, the series are potentially non-periodically cointegrated with vector (1,-1), given that the net flow series is stationary. The stock series is seen to have coefficients almost

[8] These lags imply a maximum lag for the daily models of $p = 62$ and $p = 27$ days.

[9] New critical values for the LR_r test have been computed to account for the specific nature of the fitted weekly multivariate model. Specifically, we tabulated critical values for an n-dimensional random walk (independent) process with 425 observations and including the same calendar efects and (trigonometric) deterministic seasonality specified in model (12). n=7,8,14 and an order $P = 3$ and 8.

exactly equal to one, and hence this series is non-periodically integrated I(1), which by and large is a result of the way it is constructed.

Table 1: Periodic integration analysis of the arrivals y_t^1, departures y_t^2, net flow y_t^3 and stock series y_t^4.

	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
y_t^1	168.51***	114.49***	76.70***	50.11***	28.54***	11.41**	1.28
y_t^2	181.64***	115.55***	80.39***	52.99***	32.09***	14.65***	2.31
y_t^3	985.82***	733.18***	545.96***	397.70***	272.08***	168.67***	70.55***
y_t^4	850.82***	634.46***	438.52***	271.06***	167.71***	73.15***	2.91
	Periodic Integration Coefficients						
	$\widehat{\phi}_1^i$	$\widehat{\phi}_2^i$	$\widehat{\phi}_3^i$	$\widehat{\phi}_4^i$	$\widehat{\phi}_5^i$	$\widehat{\phi}_6^i$	$\widehat{\phi}_7^i$
y_t^1	0.910	0.561	3.039	0.683	1.100	1.390	0.617
y_t^2	0.889	0.551	3.087	0.602	1.044	1.583	0.665
y_t^3	-	-	-	-	-	-	-
y_t^4	0.999	0.999	1.000	0.998	0.997	1.001	1.006

Note: LR_r signifies the trace statistic of Johansen. *,**,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

5.1.2 Bivariate analyses of the flow variables and their interaction with the stocks

The univariate properties displayed in the preceding section have several implications for the bivariate analysis. Because the daily arrivals y_t^1 and daily departures y_t^2 are PI(1), and the net flow y_t^3 is I(0), the (flow) arrivals and departures series are potentially non-periodically cointegrated with cointegrating vector $(1, -1)'$, $y_{s,\tau}^1 - y_{s,\tau}^2 \sim I(0)$. However, the analysis may also be considered for a full (14-dimensional) system where the simultaneous analysis of the arrivals and stock series is conducted. This analysis may show whether periodically cointegrating relations amongst the flow variables may exist. Secondly, due to the I(1)-ness of the stock variable y_t^4 and the PI(1)-ness of the flow variables, possible cointegration amongst the stocks and the flows will be periodic, whereby e.g. $y_{s,\tau}^1 - k_s \overline{y}_\tau^4$ is I(0) and $k_s \neq k$ for at least one $s = 1, ..., 7$.

Consider the analysis of the weekly flow series $\mathbf{Y}_\tau \equiv (y^1_{1,\tau}, ..., y^1_{7,\tau}, y^2_{1,\tau}, ..., y^2_{7,\tau})'$. For the extended system we follow the same procedure as for the univariate analysis in section 5.1, that is, we first test for the cointegration rank, and next hypotheses regarding the cointegrating space are tested.

Table 2: Periodic cointegration analysis of the arrivals y^1_t, and departures y^2_t series.

	Trace Test						
	LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6
	769.77***	594.74***	475.16***	376.21***	300.01***	232.61***	175.08***
	LR_7	LR_8	LR_9	LR_{10}	LR_{11}	LR_{12}	LR_{13}
	124.88***	90.72***	61.25***	40.81***	22.20***	9.40	1.35
	Cointegrating Relations						
	$\widehat{k}_1$	$\widehat{k}_2$	$\widehat{k}_3$	$\widehat{k}_4$	$\widehat{k}_5$	$\widehat{k}_6$	$\widehat{k}_7$
	1.003	1.043	1.013	1.105	1.237	1.030	0.977
	$\widehat{\phi}^i_1$	$\widehat{\phi}^i_2$	$\widehat{\phi}^i_3$	$\widehat{\phi}^i_4$	$\widehat{\phi}^i_5$	$\widehat{\phi}^i_6$	$\widehat{\phi}^i_7$
y^1_t	0.895	0.592	3.125	0.646	1.156	1.331	0.609
y^2_t	0.872	0.569	3.219	0.592	1.032	1.598	0.642

Note: $\hat{\kappa}_i$ are the periodic cointegration parameters relating arrivals and departures, whereas $\hat{\phi}^i_j$ are the periodic coefficients of the single series. LR_r signifies the trace statistic of Johansen. *,**,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

The test results are reported in Table 2. The trace test does not reject the null hypothesis for $r = 12, 13$, but the likelihood ratio test ($LR_{12} = 9.40$) is rather close to the 10% critical value 9.67, which leads us to conclude that the rank equals 13. Hence the daily arrivals and departures are cointegrated and thus share the same stochastic trend, which confirms the results of the previous section. The analysis of PI(1)-ness of the flow variables can also be undertaken from the multivariate model of the flows $\mathbf{Y}_\tau$ for $r = 13$ in this highly parametrized model. We test for non PI(1)-ness of the departures variable through the linear hypothesis H$_0$: $\phi^2_1 = ... = \phi^2_6 = 1$, and obtain LR=22.38, which is asymptotically distributed as $\chi^2(6)$ under the null and hence rejects at the 1% level. This reinforces the evidence about PI(1)-ness of the daily departures series.

The next step is to test for nonperiodic cointegration between the arrivals and departures. The estimates of the cointegrating coefficients $\widehat{k}_s$ associated with the arrivals series are also displayed in Table 2. We want to test the hypothesis H$_0$: $k_s = 1$ for all $s = 1, 2, ..., 6$, but in this case, and contrary to the univariate findings, we reject the null at 1% level with a LR=21.21. From the estimated k_ss we recognize that the k_5 coefficient, that is the cointegrating coefficient associated with Friday arrivals and Friday departures, is significatively different from 1. A test for equal cointegrating vectors for all days with the exception of Fridays H$_0$: $k_1 = k_2 = k_3 = k_4 = k_6 = 1$ cannot be rejected (LR=7.64). This slight departure from unity of the k_5 coefficient may be explained by the different periodic integration coefficients of Saturday arrivals and Saturday departures, which are respectively $\widehat{\phi}_6^1 = 1.331$, and $\widehat{\phi}_6^2 = 1.598$. This result was also found in the univariate analysis of the previous section yielding the estimates $\widehat{\phi}_6^1 = 1.390$ and $\widehat{\phi}_6^2 = 1.583$ (see panel B of table 1).

Finally, we test for periodic cointegration amongst the flow and stock series by considering the cointegration analysis of the 8-dimensional process $\mathbf{Y}_\tau^* \equiv (y_{1,\tau}^2, ..., y_{7,\tau}^2, \overline{y}_\tau^4)'$, which includes the departures series (for illustration) and the average stock series (for the reasons previously given). Conducting the Johansen ML-procedure on this system it is found, see Table 3, that the cointegration rank is 7, implying that the stock (derived from arrivals and departures) itself cointegrates with the departure (and arrivals) series. Hence, similarities to the notion of multicointegration seem apparent. Because the flow variables are periodic while the stock variable is non-periodic, the cointegrating relationship is necessarily periodic.

Table 3: Periodic cointegration analysis of the departures, y_t^2, and stock, $\bar{y}_t^4$, series.

LR_0	LR_1	LR_2	LR_3	LR_4	LR_5	LR_6	LR_7
384.81***	288.87***	214.19***	147.03***	92.61***	49.42***	18.60***	0.55

Note: LR_r signifies the trace statistic of Johansen. *,** ,*** indicate rejection of the null hypothesis at 10% , 5%, 1%.

5.2 Testing for common periodic cyclical features

5.2.1 The univariate series

Given the evidence of the cointegration analysis in the previous section, we test for the presence of common periodic correlation features within the week and across the weeks of the individual variables y_t^1, y_t^2, and y_t^4 and the bivariate time series (y_t^1, y_t^2). We use the likelihood ratio test ($\xi_{m=0}(q)$) given by Hecq *et al.* (2004) for the case of contemporaneous cycles and the likelihood ratio test given by Cubadda and Hecq (2001) for the case of common periodic features across different weeks ($\xi_{m>0}(q)$). Because the estimated cointegrating rank $r = 6$ for all the cases, we can safely concentrate out the cointegrating vectors without affecting the limiting distribution.[10]

Table 4: Common periodic correlation feature analysis of the arrivals series y_t^1.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	50.39	33.38	28.34	15.34	10.71	4.37	0.02
2	107.22*	77.47	65.00	35.78	27.82	14.28	0.54
3	196.99***	139.77*	113.24*	62.61	47.59	26.05	1.82
4	306.10***	210.00***	167.96**	107.96	78.75	43.57	10.00
5	432.96***	303.89***	228.06***	154.76*	122.33**	64.56	23.61
6	581.74***	412.46***	307.06***	228.13***	185.20***	108.84**	44.82
7	862.96***	637.36***	503.77***	337.59***	277.53***	168.61***	75.92***

Note: $\xi_m(q)$ signifies the likelihood ratio test. *,** ,*** indicates rejection of the null hypothesis at 10% , 5%, 1%.

Table 5: Common periodic correlation feature analysis of the departures series y_t^2.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	49.12	36.53	24.49	11.65	9.31	2.56	0.01
2	120.19**	81.42	58.56	40.49	30.80	13.84	1.14
3	211.65***	137.26*	107.46	81.31	58.17	31.55	4.67
4	307.36***	212.90***	173.78***	141.49***	102.82***	50.98	15.22
5	441.22***	306.19***	254.94***	209.20***	160.48***	88.58***	29.24
6	581.41***	422.86***	345.65***	292.86***	229.93***	138.92***	58.77***

Note: See table 4.

[10]See the limiting result of Paruolo (2002), and the finite sample results of Hecq *et al.* (2004).

Table 4 shows the results for the arrivals variable. The hypothesis of CPC(0) with $q_0 = 1$ is not rejected at the 10% level. Therefore, we do not reject $n - q_0 = 7 - 1 = 6$ dynamic factors driving the short-run component of the arrivals system. We do not reject at the 10% level, $n - q_1 = 7 - 2 = 5$ CPC(1) factors, $n - q_3 = 7 - 4 = 3$ CPC(3) factors, $n - q_5 = 7 - 5 = 2$ CPC(5) factors, and finally for the last lagged matrix Γ_7 we do not reject $n - q_6 = 7 - 6 = 1$ CPC(6) scalar factor. These results suggest that the more remote the past is, the less influence it has on the present of the short-run dynamics of the arrivals.

The most parsimonious representation of the short-run dynamics is given by CPC(3) of $q_3 = 4$ with 252 parameters (compared to 343 free parameters), while, for example, the CPC(0) of $q_0 = 1$ implies 336 short-run parameters. Similar results are obtained for the departures variable (see table 5). In this case the most parsimonious representation is obtained with the CPC(2) with $q_2 = 3$ with 266 parameters. These two cases illustrate that in our setting there is a relevant efficiency gain by imposing CPC(m) type restrictions.

Table 6 shows the results for the stock of visitors. The test statistics do not reject CPC(0), CPC(1) and CPC(2) with $q_i = 6$ ($i = 0, 1, 2$) which imply that the short-run dynamics of the day-of-week stocks are generated by just one dynamic factor. This reflects the non-periodic behavior of the stock of visitors.

Table 6: Common periodic correlation feature analysis of the stock series y_t^4.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$
1	4.90	2.10
2	17.61	8.54
3	37.09	18.73
4	59.75	32.11
5	92.30	48.72
6	135.96	72.61

Note: See table 4.

Finally, table 7 presents the likelihood ratio tests for the 14-dimensional system including arrivals and departures. As seen in the table, we do not reject $n - q_0 = 14 - 5 = 9$

CPC(0) common factors between arrivals and departures. This suggests that the flow variables do have idiosyncratic and common dynamic factors. In this case, the more parsimonious representation is obtained with CPC(1) of $q_1 = 6$ or CPC(2) with $q_2 = 7$ with a 30% reduction of the number of estimated parameters.

Table 7: Common periodic correlation feature analysis of the arrivals and departures series y_t^1 and y_t^2.

q	$\xi_{m=0}(q)$	$\xi_{m=1}(q)$	$\xi_{m=2}(q)$	$\xi_{m=3}(q)$	$\xi_{m=4}(q)$	$\xi_{m=5}(q)$	$\xi_{m=6}(q)$
1	52.27	42.61	36.66	22.69	17.88	5.83	0.02
2	136.33	102.35	75.61	52.08	40.38	13.55	1.17
3	236.20	173.72	136.88	87.64	67.47	24.49	3.30
4	343.12	257.12	202.66	138.22	99.97	42.25	6.44
5	467.96	347.37	276.21	192.96	140.66	64.49	15.88
6	600.17**	457.92	360.89	260.51	193.04	96.02	26.92
7	772.26***	581.78*	457.75	341.69	253.23	135.95	40.17
8	951.53***	719.11***	566.19**	440.51*	331.96**	181.30	63.61
9	1151.53***	890.87***	705.88***	563.89***	429.98***	228.21	91.21
10	1359.52***	1081.58***	858.38***	706.25***	533.55***	299.35***	124.66**
11	1615.92***	1282.73***	1027.67***	858.45***	658.08***	392.23***	168.34***
12	1924.45***	1493.79***	1207.28***	1018.86***	786.36***	490.94***	222.04***
13	2252.74***	1733.56***	1414.33***	1188.78***	939.01***	607.36***	299.13***

Note: See table 4.

6 Conclusion

Periodic models are often criticized for being too flexible in the sense that they require too many parameters to be estimated. In the present paper, we have suggested restricting the correlation structure of periodic models by identifying common periodic correlation features that can be imposed upon the model. An application to arrivals and departures data for passenger trafic in the airport of Mallorca demonstrated that a significant reduction in the number of estimated parameters can be obtained by such common feature restrictions. We have also suggested a way to model stock and flow data with a daily periodicity of observations, and in so doing we have generalized the notion of multicointegration to a periodic context. It is our belief that the suggested advances are quite promising avenues for future research and in particular for the way of making periodic models parsimonious and operational.

References

Ahn, S.K. and G.C. Reinsel, 1988, Nested Reduced-Rank Autoregressive Models for Multiple Time Series, *Journal of the American Statistical Association* **403**, 849-856.

Cubadda, G. and A. Hecq, 2001, On Non-Contemporaneous Short-Run Co-Movements, *Economics Letters* **73**, 389-397.

Engle, R.F. and S. Kozicki, 1993, Testing for Common Features, *Journal of Business & Economic Statistics* **11**, 369-380.

Engel, R. F. and S. Hylleberg, 1996, Common Seasonal Features. Global Unemployment. *Oxford Bulletin of Economics and Statistics* **58**, 615-630.

Franses, P.H., 1994, A Multivariate Approach to Modeling Univariate Seasonal Time Series, *Journal of Econometrics* **63**, 133-151.

Franses, P.H. and R. Paap, 2004, *Periodic Time Series Models*, Oxford University Press.

Ghysels, E. and D.R. Osborn, 2001, *The Econometric Analysis of Seasonal Time Series*, Cambridge University Press.

Gladyshev, E.G., 1961, Periodically Correlated Random Sequences, *Sov. Math.* **2**, 385-388.

Gonzalo, J. and J-Y. Pitarakis, 2002, Lag Length Estimation in Large Dimensional Systems, *Journal of Time Series Analysis* **23,** 401-423.

Granger, C.W.J. and T.H. Lee, 1989, Investigation of Production, Sales and Inventory Relations using Multicointegration and Non-symmetric Error Correction Models, *Journal of Applied Econometrics* **4**, S145-S159.

Granger, C.W.J. and T.H. Lee, 1990, Multicointegration. In *Long-Run Economic Relationships. Reading in Cointegration, Advanced Texts in Econometrics*, Engle, R.F., C.W.J. Granger (eds). Oxford University Press: Oxford.

Haldrup, N., A. Montanés, and A. Sansó, 2004, Measurement Errors and Outliers in Seasonal Unit Root Testing, forthcoming *Journal of Econometrics*

Hecq A., F.C. Palm and J-P. Urbain, 2004, Common Cyclical Features Analysis in VAR Models with Cointegration, forthcoming in *Journal of Econometrics.*

Hylleberg, S., R.F. Engle, C.W.J. Granger, and S. Yoo, 1990, Seasonal Integration and Cointegration, *Journal of Econometrics* **44**, 215-238.

Johansen, S., 1991, Estimation and Hypothesis Testing of Cointegrating Vectors in Gaussian Vector Autoregressive Models, *Econometrica* **59**, 1551-80.

McLeod, A. I., 1994, Diagnostic Checking Periodic Autoregression Models with Application, *Journal of Time Series Analysis* **15**, 221-233.

Osborn, D., 1988, Seasonality and Habit Persistence in a Life Cycle Model of Consumption, *Journal of Applied Econometrics* **3**, 255-266.

Osborn, D., 2002, Cointegration for Seasonal Time Series Processes. *Mimeo,* University of Manchester.

Pagano, M., 1978, On Periodic and Multiple Autoregressions, *Annals of Statistics* **6**, 1310-1317.

Paruolo, P., 2002, Common Features and Common I(2) Trends in VAR systems, *mimeo* University of Insubria.

Tiao, G.C. and M.R. Grupe, 1980, Hidden Periodic Autoregressive-Moving Average Models in Time Series Data, *Biometrika* **67**, 365-373.

Working Paper

2004-4: Sheetal K. Chand and Martin Paldam: The economics of immigration into a Nordic welfare state - and a comparison to an immigration state and a guest worker state

2004-5: Svend Hylleberg: On the Exploitation of Market Power in the Nordic Electricity Markets. The case of Elsam.

2004-6: Michael Svarer and Michael Rosholm: Estimating the Threat Effect of Active Labour Market Programmes

2004-7: Bo Sandemann Rasmussen: Migration Incentives and Taxation: Do marginal Taxes Matter?

2004-8: Bo Sandemann Rasmussen: On the Possibility and Desirability of Taxing E-Commerce.

2004-9: Bo Sandemann Rasmussen: Preferential Taxation of E-Commerce: Imperfectly Competitive Retail Markets and Trade Costs.

2004-10: Mariarosaria Agostino: Conditionality, Commitment and Investment Response in LDCs.

2004-11: Francesco Busato: Relative Demand Shocks.

2004-12: Francesco Busato, Bruno Chiarini and Enrico Marchetti: Indeterminacy, Underground Activities and Tax Evasion.

2004-13: Michael Rosholm, Michael Svarer and Bo Hammer: A Danish Profiling System.

2004-14: Niels Haldrup, Antonio Montañés and Andreu Sansó: Testing for Additive Outliers in Seasonally Integrated Time Series.

2005-1: Pieter Gautier, Michael Svarer and Coen Teulings: Marriage and the City.

2005-2: Niels Haldrup and Michael Jansson: Improving Size and Power in Unit Root Testing.

2005-3: Niels Haldrup, Svend Hylleberg, Gabriel Pons, Jaume Rosselló, and Andreu Sansó: Common Periodic Correlation Features and the Interaction of Stocks and Flows in Daily Airport Data.