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Relative Demand Shocks

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Relative Demand Shocks.*

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Abstract

This paper introduces the concept of *relative demand shocks* into a multi-sector dynamic general equilibrium model. Relative demand shocks change the *instantaneous* structure of preferences. Under relative demand shocks consumer tastes randomly shift across different commodities, as manifested by unexpected relative increases or decreases in the marginal utility of the various consumption goods. There are no exogenous technology (productivity) shocks in the model. There are three main results. **First**, the model proposes an original theoretical mechanism for generating aggregate fluctuations and sectoral comovement by using inter-sectoral and idiosyncratic shocks. This mechanism is complementary to the standard Real Business Cycle theory. **Second**, the model is effectively able to reproduce the main stylized facts of the U.S. economy, also those that the standard Real Business Cycle model fails to explain. **Third**, the model generates a *false Solow Residual*, even though there is no technological progress in the model. Its size and time series properties are analogous to the actual Solow Residual.

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1 Introduction

Over the last two decades macroeconomists explained aggregate fluctuations as mainly driven by technology shocks. This is the standard Real Business Cycle (RBC) model (e.g. Kydland and Prescott, 1982; Long and Plosser, 1983). Standard RBC models have difficulty explaining several important stylized facts of the U.S. economy, such as the substantial volatility of consumption relative to output, the negative (or null) correlation between marginal productivity of labor and hours worked, the cross correlation (at all leads and lags) of consumption and investment with output, or the high volatility of hours.[1] In addition, a recent body of literature questions the very foundations of RBC theory, by suggesting that positive technological shocks lead to declines in input use, that selected productivity measures are essentially uncorrelated with output, and negatively correlated with input growth (e.g. Basu, Kimball and Fernald, 2002; Basu, 1998; Gali, 1999 and 2003; Francis and Ramey, 2003).[2]

A number of contributions suggest different mechanisms for improving upon the standard model. The explicit introduction of demand shocks in one of them. Indeed several contributions suggest that demand shocks bear significant responsibility for business cycles in the U.S. and in major European countries (France, Germany, and the United Kingdom) (e.g. Blanchard and Quah, 1989; Karras, 1994; Hartley and Whitt, 2003; Gali, 2003 and 1999). Broadly speaking there are three large classes of demand shocks: government spending shocks, monetary shocks, and preference shocks. This paper focuses on the latter one.

It introduces the concept of *relative demand shocks* into a multi-sector model. Relative demand shocks change the *instantaneous* structure of preferences. Under relative demand shocks consumer tastes randomly shift across different commodities, as manifested by unexpected relative increases or decreases in the marginal utility of the various consumption goods. This formulation is complementary to that of Baxter and King (1991), and Bencivenga (1992), whose models rely on aggregate

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[1] Some of these counterfactual prediction are so robust to model specification that are addressed in the literature as *puzzles*. Examples in point are *consumption volatility puzzle*, the *employment variability puzzle* and the *productivity puzzle* (e.g. Stadler, 1994).

[2] The debate whether business cycles are driven by demand and/or supply shocks have been re-opened in the late 80s by the seminal paper of Blanchard and Quah (1989), and it is still open. Indeed, there are other contributions, however, that defend the *technology driven business cycle hypothesis*, and with supporting empirical evidence, and with modified standard RBC models (i.e. Christiano, Eichenbaum and Vigfusson, 2003; Fisher, 2002). Other studies shift their attention on demand shocks.

demand disturbances; i.e. shocks to the marginal utility of the single composite consumption good. The more recent contributions on preference shocks (e.g. Wen, 2003a and 2003b; Benhabib and Wen, 2002) rely either on the Baxter and King (1991) or on the Bencivenga (1992) definition.[3]

We analyze the consequences of these relative demand shocks in the context of a dynamic equilibrium two-sector two-good model with a labor-leisure choice, and where changes in relative demand are driven by autonomous shifts in preferences. While labor services can be reallocated across sectors, consumption and capital goods are sector specific. Aggregate uncertainty here originates from the demand side, and it is modelled by using a state dependent utility function. The benchmark economy is then extended to incorporate inter-sectoral and/or inter-temporal labor adjustment costs, and endogenous capacity utilization. Finally, the model's performance is compared with that of the analogous economy, where fluctuations are driven by relative technology shocks only.

The paper focuses on **five** major issues. **First**, the model proposes an original theoretical mechanism for generating aggregate fluctuations and sectoral comovement by using inter-sectoral and idiosyncratic demand shocks. This mechanism is complementary to the standard Real Business Cycle theory. Relative demand shocks change the desired composition of consumption expenditure on a period by period basis, thereby inducing an inter-sectoral and inter-temporal resource reallocation. A consequence of this variation is that consumers' subjective discount factor changes in tandem with the current compositions of consumption expenditure. **Second**, the model performs quite well in replicating most regularities of the U.S. business cycle. It performs particularly well with respect the aggregate consumption volatility and its cross-correlation with output, the main labor market stylized facts, the consumer price index and the inflation rate volatilities and their correlations with aggregate output. Furthermore the model generates a negative correlation between average productivity of labor and hours worked, which is a stylized fact not explainable by a technology driven model. **Third**, the model generates a *false* Solow residual, whose size and time series properties are consistent with the U.S. Solow residual data. In this model, however, the

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[3] The Baxter and King (1991)'s shock defines a truly *intertemporal* and *aggregate* demand shock. In particular, it urges consumers to substitute aggregate consumption tomorrow (that is saving) with aggregate consumption today. Bencivenga (1992)'s preference shocks directly affect marginal utility of consumption and of leisure. The leisure's shock increases the disutility of labor, generating an inward shift of labor supply schedule. Both the Baxter and King (1991) and the Bencivenga (1992) shocks implicitly assume that all consumers suddenly want to consumer more of all commodities.

false Solow residual measures something completely different from technology or productivity. This suggests that either the Solow Residual is a misspecified measure of productivity *at the business cycle frequencies*, or that relative demand shocks represent a possible explanation for procyclical productivity. **Fourth**, under relative demand shocks the strongest correlations between aggregate consumption and aggregate GDP occur at zero lags(leads), consistent with the data for the U.S. economy (Stock and Watson, 1998). This a significant improvement upon the standard business cycle model where consumption's strongest correlation with output occurs at lag -1; in this sense consumption is said to lag output. That happens because the exogenous increase of income leads to an increase in consumption and investment. Instead, in a demand driven model, the causality order is inverted, since the increase in consumption desire pulls income up, via labor market channel. This also suggests that the model is not subject to the crowding out effect between consumption and investment, as described by Baxter and King (1991) and which is typical of several one-sector models driven by demand shocks.[4] **Fifth**, the stochastic properties of sectoral business cycles are consistent with the U.S. economy. Capital stocks, labor flows, production outputs, investments and consumptions move together in each sector, and, more importantly, all sectoral quantities are procyclical with aggregate GDP. This the most important of the regularities common to all business cycles (Lucas, 1977). In fact the model generates comovements between the sectors of the economy, even under uncorrelated demand shocks. Current economic theory has difficulty accounting for this characteristic (Hornstein, 2000).

Before proceeding, it is important to stress that the goal of this research is **not** to argue that either aggregate shocks of any kind or sectoral technology shocks are irrelevant to the study of macroeconomic fluctuations. It is simply to reduce economists' reliance on them by identifying a role for relative demand shocks in generating sectoral and aggregate co-movements.

The paper is organized as follows. Section 2 details the benchmark economy, and Section 3 introduces the three extensions of the model previously mentioned. Section 4 presents the theoretical mechanism and selected numerical results, and Section 5 concludes. Finally, Section 6 includes all proofs and derivations.

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[4] Baxter and King (1991) notice that when an aggregate demand shock impinges the economy, in a one-sector model, people increase consumption, while reducing investment, and, by this end, capital accumulation. Output, being a a monotone transform of capital stock, subsequently falls, depicting a significant crowding out effect.

2 A Multi-Sector Model with Relative Demand Shifts

This section presents the baseline dynamic equilibrium model with relative demand shocks. Several extensions are discussed in Section 3. Since there are no restrictions to trade, we solve a Planner problem.

2.1 The Benchmark Economy.

The benchmark model is structured as two-sector, two-good economy, with labor-leisure choice, and where changes in relative demand are driven by autonomous changes in preferences. Aggregate uncertainty originates from the demand side, and it is modelled using a state dependent utility function. Consumption and capital goods are sector specific, while labor services can be reallocated across sectors, without bearing any cost of adjustment.[5]

Preferences. Let $C(\mathbf{c}_t)$ be the aggregate consumption/utility index:

$$C(\mathbf{c}_t) = \left\{\lambda_1 \left[u_{(1)}(c_{1,t};\tilde{s}_{1,t})\right]^{\frac{\sigma-1}{\sigma}} + \lambda_2 \left[u_{(2)}(c_{2,t};\tilde{s}_{2,t})\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\mathbf{c}_t = (c_{1,t}, c_{2,t})$, λ_1 and λ_2 ($\lambda_1 + \lambda_2 = 1$) denote the two preference weights, $u_{(1)}(c_1, \tilde{s}_1) = \frac{\tilde{s}_{1,t}(c_{1,t})^{1-q_1}}{1-q_1}$,$u_{(2)}(c_2, \tilde{s}_2) = \frac{\tilde{s}_{2,t}(c_{2,t})^{1-q_2}}{1-q_2}$, where q_1 and q_2 denote the relative risk aversion coefficients over corresponding consumption; the quantities $(\tilde{s}_{1,t}, \tilde{s}_{2,t})$ denote a vector of realizations of sectoral (idiosyncratic) relative preference shocks (defined below), and σ denotes the elasticity of substitution between the two consumption goods. This structure is very general, but there are mainly two cases of interest: the cases of *non-separable preferences* and of *separable preferences.* The former represents the benchmark model, while the latter one is analyzed to check the consistently of the model's mechanism under a different preference specification. When $q_i = 0$, $\sigma \neq 1$ preferences are **non-separable** between the two consumption flows, and index (1) reads:[6]

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[5] Just notice that Section 3 extends the analysis, while investigating the role of inter-temporal and inter-sectoral labor adjustment costs.

[6] Notice that there would be two alternative ways of modelling stochastic changes in the relative desirability between commodities. First it would be to assume that the elasticity of substitution between consumption goods, σ, is a random quantity (i.e. $C\left(\mathbf{c}_t\right) = \left(\lambda_1 \left(c_{1,t}\right)^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}} + \lambda_2 \left(c_{2,t}\right)^{\frac{\tilde{\sigma}-1}{\tilde{\sigma}}}\right)^{\frac{\tilde{\sigma}}{\tilde{\sigma}-1}}$). Alternatively, we could assume that $\lambda = (\lambda_1, \lambda_2)$ is a random quantity, that is $\left(\tilde{\lambda}\left\{\left(c_{1,t}\right)^{\frac{\sigma-1}{\sigma}}\right\} + \left(1-\tilde{\lambda}\right)\left\{\left(c_{2,t}\right)^{\frac{\sigma-1}{\sigma}}\right\}\right)^{\frac{\sigma}{\sigma-1}}$. These formulations are left for future investigation.

$$C\left(\mathbf{c}_t\right)=\left(\lambda_1\left\{\tilde{s}_{1,t}c_{1,t}^{\frac{\sigma-1}{\sigma}}\right\}+\lambda_2\left\{\tilde{s}_{2,t}c_{2,t}^{\frac{\sigma-1}{\sigma}}\right\}\right)^{\frac{\sigma}{\sigma-1}}.$$

Every relative demand shock changes the instantaneous structure of preferences, by making one consumption good more desirable, relative to an other. Preferences over aggregate consumption index $C\left(\mathbf{c}_t\right)$ and leisure ℓ_t are described by a state dependent return function $u(C\left(\mathbf{c}_t\right),\ell_t;\tilde{\mathbf{s}}_t):\mathbb{R}_+^2\cdot\mathcal{S}^2\cdot[0,1]^2\rightarrow\mathbb{R}$:

$$u(\mathbf{c}_t,\ell_t;\mathbf{s}_t)=\frac{\left(C\left(\mathbf{c}_t\right)\right)^{1-\psi}-1}{1-\psi}+\upsilon\left(\ell_t;B\right), \tag{2}$$

where the parameter ψ controls the degree of risk aversion and is inversely proportional to the elasticity of intertemporal substitution.

When $q\neq 0$, $\psi=0$, and $\sigma\rightarrow\infty$, preferences are said to be **separable** between consumption flows. In this case instantaneous preferences (equation (2)) reads: $u(\mathbf{c}_t,\ell_t;\mathbf{s}_t)=\lambda_1\frac{\tilde{s}_{1,t}c_{1,t}^{1-q_1}}{1-q_1}+\lambda_2\frac{\tilde{s}_{2,t}c_{2,t}^{1-q_2}}{1-q_2}+\upsilon\left(\ell_t;B\right)$. Section 4 presents results for both cases.[7]

In both cases assume that $\upsilon\left(\ell_t;B\right)$ is a well behaved (continuous, twice continuously differentiable) function of ℓ_t, representing the utility of leisure ℓ_t, and $B(B>0)$ is a scaling parameter.

Production Technologies. Each good is produced with physical capital and labor, using a sector-specific Cobb-Douglas technology:

$$y_{1,t}=k_{1,t}^{\alpha_1}n_{1,t}^{1-\alpha_1}\text{ and }\ y_{2,t}=k_{2,t}^{\alpha_2}n_{2,t}^{1-\alpha_2}, \tag{3}$$

where $n_{i,t}$ and $k_{i,t}$ denote, respectively, labor demand and capital stock in sector i, for $i=1,2$; notice that there are no random quantities measuring exogenous productivity disturbances.

Feasibility and Capital Accumulation Constraints. Feasibility of the optimal program is ensured by the following two customary constraints where production technologies have been substituted for $y_{1,t}$ and $y_{2,t}$.

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[7] Other cases of interest are the following. Case (i): $(\sigma=1,q_i=1)$ the instantaneous utility function reduces to $\tilde{s}_{1,t}\log c_{1,t}+\tilde{s}_{2,t}\log c_{2,t}$; Case (ii) $(0<\sigma<1,q_i=0)$ it equals $\left\{\tilde{s}_{1,t}\left(c_{1,t}\right)^{\frac{\sigma-1}{\sigma}}+\tilde{s}_{2,t}\left(c_{2,t}\right)^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}$; Case (iii) $(\sigma=0,q_i=0)$ the instantaneous utility function reduces to a Leontiev structure: $\max\left\{\tilde{s}_{1,t}c_{1,t},\tilde{s}_{2,t}c_{2,t}\right\}$.

$$c_{1,t} + i_{1,t} = k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} \quad \text{and} \quad c_{2,t} + i_{2,t} = k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2}, \tag{4}$$

where $i_{i,t}$ denotes investment flows, for $i = 1, 2$. Capital accumulation constraints are defined as follows:

$$k_{1,t+1} = (1 - \Omega_1)k_{1,t} + i_{1,t} \;\; \text{and} \quad k_{2,t+1} = (1 - \Omega_2)k_{2,t} + i_{2,t}, \tag{5}$$

where the $\Omega' s$ denote quarterly depreciation rates. This formulation implicitly assumes that capital is homogenous and not mobile across sector. The idea here is that the capital used in the production of one good cannot easily be used to produce the other one.[8] Consumers first choose how many hours to allocate to labor and to leisure, which is the residual according to the following constraint:

$$\ell_t = 1 - n_t, \tag{6}$$

where available hours are normalized to 1. Then, consumers allocate working hours $n_{1,t}$ and $n_{2,t}$ to each sector. We are expecting, therefore, a rapid movement of labor to where the marginal utility of consumption is higher. Notice that this is an argument distinctive of a demand-driven model. In a model with technology shocks only, labor services shift to the sector where the marginal productivity of labor (wage) is relatively higher.

The aggregate labor index is defined as

$$n_t = \left(n_{1,t}^{-\nu} + n_{2,t}^{-\nu} \right)^{-1/\nu}, \tag{7}$$

where $\nu (\nu \leqq -1)$ denotes the elasticity of substitution between labor services. This specification of the time allocation constraint captures the idea that it is costly to reallocate labor from one sector to the other. The quantity $\left(n_{1,t}^{-\nu} + n_{2,t}^{-\nu} \right)^{-1/\nu}$ may be interpreted as a reverse Constant Elasticity of Substitution technology. A reverse formulation ensures the optimization problem to be concave, since isoquants are concave toward the origin. Now, when $\nu = -1$, the transformation frontier is

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[8] Notice, however, that if instantaneous capital reallocation were allowed, by pooling together equations (5), the model's mechanism would not be affected. It would only generate much less volatility because the system would use that additional dimensional for smoothing income and consumption.

linear, and the transformation rate between hours equals 1. In other words, there are no adjustment cost in reallocating hours worked across sectors: $n_t = n_{1,t} + n_{2,t}$. This is the case of the benchmark economy.

Demand Shocks. The relative (idiosyncratic) demand shocks $\{\tilde{s}_{1,t}, \tilde{s}_{2,t}\}_{t=1}^{\infty}$ have transitory, but persistent effects. Shocks may be (or may be not) positively correlated. Demand shocks follow autoregressive processes in logs $\log \tilde{s}_{i,t+1} = \omega_i \log \bar{s}_i + (1 - \omega_i) \log \tilde{s}_{i,t} + \epsilon_{i,t}$, where $0 \leq \omega_i \leq 1$, and $\epsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_{\epsilon^i}^2\right)$, for $i = 1, 2$.

Model's Solution and Equilibrium Characterization. A Planner maximizes the expected present discounted value of the return function $\mathcal{V}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(\mathbf{c}_t, \ell_t; \mathbf{s}_t)$, subject to the feasibility constraints (4), the capital accumulation constraints (5), and the constraints on total hours (6) and (7). The state of the economy at time t is represented by a vector $\chi_t = \langle k_{1,t}, k_{2,t}, s_{1,t}, s_{2,t} \rangle$. Controls for the problem are consumption flows $\mathbf{c}$, investment flows $\mathbf{i}$, and the labor services ℓ. The function $v(\ell_t; B)$ is then specified as $v(\ell_t; B) = B\frac{(1-n_t)^{1-\gamma}}{1-\gamma}$, where $\gamma \geqq 0$. Introducing dynamic multipliers $\phi_{0,t}$, $\phi_{1,t}$ and $\phi_{2,t}$, forming the Hamiltonian $\mathcal{H}$ yields:

$$\begin{aligned}
\max_{\{c_{i,t}, n_{i,t}, k_{i,t+1}\}_{i=1}^{2}, n_t} \mathcal{H} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(C\left(\mathbf{c}_t\right)\right)^{1-\psi} - 1}{1-\psi} + B\frac{\left(1-n_t\right)^{1-\gamma}}{1-\gamma} + \right. \\
&\quad + \phi_{0,t}\left[n_{1,t} + n_{2,t} - n\right] + \\
&\quad + \phi_{1,t}\left[k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} - c_{1,t} + \left(1-\Omega_1\right) k_{1,t} - k_{1,t+1}\right] + \\
&\quad \left. + \phi_{2,t}\left[k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2} - c_{2,t} + \left(1-\Omega_2\right) k_{2,t} - k_{2,t+1}\right] \right\},
\end{aligned}$$

where $\mathbb{E}_0$ is the expectation operator, conditional on time 0 information, and β $(0 < \beta < 1)$ is a subjective discount factor. First derive the first order condition with respect to i-th consumption flow (FOC($c_{i,t}$) hereafter):

$$c_{i,t} : \left(C\left(\mathbf{c}_t\right)\right)^{-(1+\psi)} \lambda_i \tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}} = \phi_{i,t} \text{ for } i = 1, 2 \tag{8}$$

Then consider FOC(n_t) and FOC($n_{i,t}$) for $i = 1, 2$.

$$n_t \quad : \quad 0 = -B\left(1-n_t\right)^{-\gamma} - \phi_{0,t} \tag{9}$$

$$n_{i,t} \quad : \quad 0 = \phi_{0,t} + \phi_{i,t} APN_{i,t}, \tag{10}$$

where $APN_{i,t} = \left(1-\alpha_i\right) k_{i,t}^{\alpha_i} n_{i,t}^{-\alpha_i} = \left(1-\alpha_i\right)\left(\frac{y_{i,t}}{n_{i,t}}\right)^{\alpha_i}$. Notice that FOC($n_{i,t}$) can be rewritten as $\phi_{0,t} = \phi_{i,t} APN_{i,t}$, since $\phi_{0,t} < 0$ from FOC(n_t). Combining the previous equations, the FOCs for both consumption goods can be rewritten as:

$$\lambda_i \left(C\left(\mathbf{c}_t\right)\right)^{-(1+\psi)} \tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}} APN_{i,t} = B\left(1-n_t\right)^{-\gamma} \text{ for } i = 1, 2 \tag{11}$$

Optimality conditions (11) equate the marginal utility of consumption weighted with the marginal productivity of labor (on the left hand side) with the marginal disutility of leisure (on the right hand side). Then, it is convenient to define the following marginal labor productivity index $\overline{APN}_t = \left(\lambda_1 \tilde{s}_i^{\sigma} \left(APN_{i,t}\right)^{(\sigma-1)} + \lambda_2 \tilde{s}_j^{\sigma} \left(APN_{j,t}\right)^{(\sigma-1)}\right)^{\frac{1}{\sigma-1}}$. After some algebraic manipulations, we derive the equilibrium expression for the individual consumption flows (see Appendix for the details):

$$c_{i,t} = \lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(\frac{APN_{i,t}}{\overline{APN}_t}\right)^{\sigma} C\left(\mathbf{c}_t\right) \text{ for } i = 1, 2 \tag{12}$$

Next, investment dynamics is determined by the following two Euler Equations:

$$\tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}} = \mathbb{E}_t \left\{ \beta \frac{C\left(\mathbf{c}_{t+1}\right)^{-1-\psi}}{C\left(\mathbf{c}_t\right)^{-1-\psi}} \tilde{s}_{i,t+1} c_{i,t+1}^{-\frac{1}{\sigma}} \left(\alpha_i \frac{k_{i,t+1}^{\alpha_i - 1}}{n_{i,t+1}^{\alpha_i - 1}} + 1 - \Omega_i \right) \right\}, \text{ for } i = 1, 2, \tag{13}$$

where $\mathbb{E}_t$ denotes the expectations operator, conditional on information available at time t. Notice how the pricing kernel $\Pi_{i,t} = \frac{C(\mathbf{c}_{t+1})^{-1-\psi}}{C(\mathbf{c}_t)^{-1-\psi}} \frac{\tilde{s}_{i,t+1} c_{i,t+1}^{-\frac{1}{\sigma}}}{\tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}}}$ is affected by the demand shocks. That is a peculiarity of a demand driven model, since this kind of shocks directly affect the marginal utility. Anticipating a result, this generates a significant volatility for "relatively small" shocks. In summary, this Pareto Optimum equilibrium is characterized by the optimality conditions (11), (14) and by the feasibility constraints.

It can be shown that the Planner allocation coincides with that of a recursive competitive economy of the Prescott and Mehra (1980) type.[9] This Planner Equilibrium is equivalent to a competitive equilibrium, in the sense that optimality conditions and constraints will be identical. Since a Pareto Optimal equilibrium exists, so does a Competitive Equilibrium. Since the former is unique, so is the latter. Primitives of the problem satisfy all necessary conditions for existence and uniqueness of the equilibrium.

Before solving the model it is necessary to derive the deterministic steady state around which the dynamic model is log-linearized, and calibrate the main parameters.

Deterministic Steady State. The first order conditions can be used to describe this stationary state where $\tilde{s}_{1,t} = \bar{s}^1$ and $\tilde{s}_{2,t} = \bar{s}^2$ in a recursive manner (Step 1 to Step 5). Equations below describe the deterministic steady state for the benchmark economy; deterministic steady state values for all variables are denoted with a "bar".

Step 1. From the Euler Equations (13) compute the capital/labor ratio $\left(\frac{\bar{k}^i}{\bar{n}_i}\right) = \left(\frac{\alpha_i}{\beta^{-1}-1+\Omega_i}\right)^{\frac{1}{1-\alpha_i}}$.

Step 2. Compute the average productivity of labor services $\overline{APN}^i = (1-\alpha_i)\left(\frac{\alpha_i}{\beta^{-1}-1+\Omega_i}\right)^{\frac{\alpha_i}{1-\alpha_i}}$ for $i = 1,2$ and $\overline{APN} = \left(\lambda_1 \bar{s}_1^\sigma \left(APN^1\right)^{(\sigma-1)} + \lambda_2 \bar{s}_2^\sigma \left(APN^2\right)^{(\sigma-1)}\right)^{\frac{1}{\sigma-1}}$, by using the capital-labor ratio.

Step 3. Then compute the consumption flows in each sector from each first order condition for consumption $\bar{c}_i = \lambda_i^\sigma \bar{s}_i^\sigma \left(\frac{\overline{APN}_i}{\overline{APN}}\right)^\sigma C\left(\bar{\mathbf{c}}\right)$ for $i = 1,2$ and the aggregate consumption index $C\left(\bar{\mathbf{c}}\right)$.

Step 4. Derive steady state labor services from the feasibility constraints $\bar{c}^i + \bar{\imath}^i = \bar{k}_i^{\alpha_i} \bar{n}_i^{1-\alpha_i}$. Since in equilibrium $\bar{\imath}^i = \Omega^i \bar{k}^i$ it is convenient to rewrite it as $\bar{c}^i + \Omega_i \bar{k}^i = \left(\frac{\bar{k}^i}{\bar{n}_i}\right)^{\alpha_i} \bar{n}_i$, and then divide both members by $\bar{n}_i$: $\frac{\bar{c}^i}{\bar{n}_i} + \Omega_i \frac{\bar{k}^i}{\bar{n}_i} = \left(\frac{\bar{k}^i}{\bar{n}_i}\right)^{\alpha_i}$. Since $\frac{\bar{k}^i}{\bar{n}_i}$ is known from Step 1 and $\bar{c}_i$ is known from Step 3, we can solve for $\bar{n}_i = \left(\bar{c}^i\right)^{-1}\left(\left(\frac{\bar{k}^i}{\bar{n}_i}\right)^{\alpha_i} - \Omega_i \frac{\bar{k}^i}{\bar{n}_i}\right)$, $i = 1,2$. Eventually, aggregate labor services are computed using equation $\bar{n} = \left(\bar{n}_1^{-\nu} + \bar{n}_2^{-\nu}\right)^{-1/\nu}$.

Step 5. Once the deterministic steady state value for the labor services is determined, it is possible to solve for $\bar{k}^i = \left(\frac{\alpha_i}{\beta^{-1}-1+\Omega_i}\right)^{\frac{1}{1-\alpha_i}} \bar{n}_i$. Finally investment flows are derived from capital accumulation constraints: $\bar{\imath}^i = \Omega^i \bar{k}^i$.

The deterministic steady state offers additional information for calibrating the model.

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[9] In this context a Recursive Competitive Equilibrium for this economy consists of a set of continuous price functions, **p**, a value function, and optimal policy functions for consumption, investment, such that market clearing conditions hold. Finally, notice that the economy satisfies conditions for the existence and the uniqueness of the Equilibrium as detailed in Prescott and Mehra (1980), to which we refer for details.

2.2 Calibration.

The model is calibrated for the U.S. economy, over the sample 1953:Q1- 1996:Q4. This sample choice allows to compare our results with the benchmark simulations presented in King and Rebelo (1999a), and the data analyzed by Stock and Watson (1998). The parameterization of the model's supply side is standard, as from King and Rebelo (1999). This allows to carry out a meaningful comparison with standard RBC formulation.

We precisely details our calibration below, mainly focusing on the relative demand shocks. Given the model's nature, relative demand shocks could be calibrated using data on consumption of nondurables, of services, and/or on data from wholesale and retail trade. However, it is appropriate to restrict the analysis to different constituent components for nondurables, or using wholesale trade and/or retail trade data. Changes in services' consumption are more associated with technological improvement. In other words, it may be hard to tell a story where consumer preferences shift between "*cheese-burgers*" and "*online banking*". Generalizing the argument, it would be more plausible to argue that services' consumption (e.g. online banking) increases with improvement in (communications) technology (i.e. broad-band internet connections).

Relative demand shocks are thus calibrated using data on expenditures on Food and on Clothing and Shoes, the two largest components of nondurable consumption sales.[10] The non-durable consumption component represents 33.19% of the personal disposable income, over the calibration sample. Food sales and Clothing-Shoes sales account for 53% and for 18% of personal consumption expenditures, respectively. Hodrick-Prescott filtered Food sales (F, hereafter) are less volatile than Clothing and Shoes (CS, hereafter) sales ($\hat{\sigma}_F = 0.87$, while $\hat{\sigma}_{CS} = 1.18$), but are more persistent ($\hat{\rho}_F = 0.83$, while $\hat{\rho}_{CS} = 0.76$). The sales of the two different nondurables components move together ($\hat{\rho}_{F,CS} = 0.48$), and both are positively correlated with aggregate nondurable expenditure ($\hat{\rho}_{F,ND} = 0.89$, and $\hat{\rho}_{CS,ND} = 0.68$).

The system of equations we use to compute the dynamic equilibria of our benchmark model depends on a set of 15 parameters. Five pertain to the supply side $(\alpha_i^*, \Omega_i^*)_{i=1}^2$ and B^*, six belong to demand side (q_1^*, q_2^*, σ^*, ψ^*, β^*, λ^*), and four are associated with the demand shocks (ρ_F^*, ρ_{CS}^*,

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[10] Food and Clothing & Shoes Series. Sales: NIPA Tables 2.2; Price Indexes: NIPA Tables 7.2. Personal Disposable Income: NIPA Tables 2.1. All series are seasonally adjusted.

σ_F^*, σ_{CS}^*).

1. **Supply side parameters**. Both consumption goods we consider belong to the categories of nondurable goods, and therefore we assume that the technology structure is symmetric. We set $\alpha_1^* = \alpha_2^* = 0.33$, the standard value for the U.S. economy (see King and Rebelo, 1999a, and Stock and Watson, 1998), and rates of capital depreciation are chosen to be $\Omega_1^* = \Omega_2^* = .025$ on a quarterly basis, assuming the same capital depreciation rate for both production technologies. A symmetric parameterization allows a direct comparison with alternative formulations, at least along the supply side of the model. Finally, $B^* = 1$.

2. **Demand side parameters**. The subjective discount factor β^* is set to 0.984, a standard value for the U.S. economy. The relative risk aversion coefficients both in the non-separable preference and in the separable preferences case is set to one (that is either $\psi^* = 1$, or $q_1^* = q_2^* = 1$). This implies that instantaneous preferences are logarithmic either over the aggregate consumption index, or over the individual consumption flows. The parameter λ^* ($\lambda_1 = \lambda^*$ and $\lambda_2 = 1 - \lambda^*$) is calibrated to match the ratio of steady state consumption sales. In particular, manipulating the FOCs, it can be showed that: $\frac{\bar{c}^1}{\bar{c}^2} = \frac{\bar{c}^F}{\bar{c}^{CS}} = \left(\frac{\lambda^*}{1-\lambda^*}\right)^{\sigma^*} \left(\frac{APN_{2,t}}{APN_{1,t}}\right)^{\sigma^*}$ where $\frac{\bar{c}^F}{\bar{c}^{CS}}$ denotes the steady state ratio between Food Sales ($\bar{c}^F$) and Clothing and Shoes sales ($\bar{c}^{CS}$). Because of the symmetric production structure, it can be assumed that $\left(\frac{APN_{2,t}}{APN_{1,t}}\right)^{\sigma^*} \approx 1$. Since in equilibrium $\frac{\bar{c}^F}{\bar{c}^{CS}} = 1.70$, $\sigma^* = -0.5$, then λ^* is calibrated to 0.25 by solving $\left\{\lambda^* \text{ solves } 0 = -\frac{\bar{c}^1}{\bar{c}^2} + \left(\frac{\lambda^*}{1-\lambda^*}\right)^{\sigma^*}\right\}$.

3. **Demand Shocks**. Each demand shock process is modelled as AR(1) process in logs in order to facilitate the comparison with standard RBC models. The autocorrelation coefficients and the standard deviation of $\tilde{s}_{1,t}$ and $\tilde{s}_{2,t}$ are chosen to match the autocorrelation and the volatility of Food and Clothing-Shoes sales, respectively. In particular, the autocorrelations are calibrated equal to $\rho_F^* = 0.98$, and $\rho_{CS}^* = .94$. Standard deviation for the innovation process equals $\sigma_F^* = 0.512$ and $\sigma_{CS}^* = 0.912$ (in percentage units).[11] The innovations of the demand shocks are assumed uncorrelated under non-separable preferences (baseline calibration: $corr(\epsilon_{1,t}, \epsilon_{2,t}) = 0$), while the correlation is set to 0.01 for the separable case.This

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[11] It should be acknowledged, however, that the two sectors produce capital goods, as well, while the calibration presented in this paper refers only to consumption flow.

parameter choice is in line with Wen (2003a), Xiao (2003) and Guo and Sturzenegger (1998). They find that estimated persistence parameters range from 0.50 to 0.90 for the U.S. economy.[12]

2.3 Aggregation.

The model generates series for sectoral variables, but it is also interesting to analyze the behavior of the aggregate economy. The aggregate series are computed using the sectoral series and the relative price vector supporting the competitive equilibrium. Precisely, the planner problem is decentralized following Prescott and Mehra (1980), and the relative prices are derived (Theorem 1).

Theorem 1 (Competitive Equilibrium and Relative Prices) *Let the* $p_{1,t}$ *be the numeraire of the system, and let* $\mathbf{p}_t = \left(1, p_t^{c^2}, p_t^{k^1}, p_t^{k^2}, p_t^{n^1}, p_t^{n^2}\right)$ *be the price vector, where* $p_t^{c^2}$ *is the price of* $c_{2,t}$*;* $p_t^{k^1}$ *is the price of* $k_{1,t}$*;* $p_t^{k^2}$ *is the price of* $k_{2,t}$*;* $p_t^{n^1}$ *is the price of* $n_{2,t}$ *and* $p_t^{n^2}$ *is the price of* $n_{2,t}$*. Then, denote the relative price vector as* $\hat{\mathbf{p}}_t = \left(\hat{p}_t, \hat{p}_t^k, \hat{p}_t^n\right)$ *where* $\hat{p}_t$ *denotes relative price for consumption and investment goods,* $\hat{p}_t^k = \frac{\hat{p}_t^{k^1}}{\hat{p}_t^{k^2}}$ *and* $\hat{p}_t^n = \frac{\hat{p}_t^{n^1}}{\hat{p}_t^{n^2}}$ *are relative prices of capital stocks and labor services, respectively.*

$$\hat{p}_t = \frac{MU_{2,t}}{MU_{1,t}} \cdot \frac{MPN_{1,t}}{MPN_{2,t}}$$

$$\hat{p}_t^{k^1} = \alpha_1 \left(k_{1,t}\right)^{\alpha_1 - 1} \left(n_{1,t}\right)^{1-\alpha_1}; \quad \hat{p}_t^{k^2} = \hat{p}_t \alpha_2 \left(k_{2,t}\right)^{\alpha_2 - 1} \left(n_{2,t}\right)^{1-\alpha_2}$$

$$\hat{p}_t^{n^1} = (1-\alpha_1)\left(k_{1,t}\right)^{\alpha_1} \left(n_{1,t}\right)^{-\alpha_1}; \quad \hat{p}_t^{n^2} = \hat{p}_t (1-\alpha_2)\left(k_{2,t}\right)^{\alpha_2} \left(n_{2,t}\right)^{-\alpha_2},$$

where $MU_{i,t}$ *denotes marginal utility from consuming* $c_{i,t}$*, and* $APN_{i,t}$ *represents the marginal productivity of* n_i*, for* $i = 1, 2$*. Investment, consumption and output flows have the same price, in each sector (see Prescott and Mehra, 1980).*

Proof. see Appendix. ■

Notice that the relative price between consumption (and produced) good is a function of the marginal rate of substitution $\frac{MU_{2,t}}{MU_{1,t}}$ and of the marginal rate of transformation between labor

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[12] The correlation between innovations is not comparable since they propose open economy models where the cross-correlation refers to demand shocks in different countries. In addition their calibration focus on aggregate demand shocks. It should be noted, however, that because of sectors' comovement we expect the idiosyncratic demand shocks to be positively correlated with the aggregate one.

flows $\frac{MPN_{1,t}}{MPN_{2,t}}$. Since consumption is sector-specific, it is not possible to directly transform the first consumption good with the second. It is necessary to use labor to do that, which is the only flexible production input. For this reason the marginal rate of transformation between affect the relative price vector.[13]

Since investment goods, consumption goods and outputs have the same price, in each sector, aggregate counterparts are defined as: $C \equiv c_{1,t} + \hat{p}_t c_{2,t}$, $I_t \equiv i_{1,t} + \hat{p}_t i_{2,t}$, and $Y_t \equiv y_{1,t} + \hat{p}_t y_{2,t}$,where $\hat{p}_t$ is defined in Theorem 1. Aggregate labor services and capital stocks are computed by using the corresponding relative prices (previously derived). For the benchmark economy the wage rates are equal in both sectors because of the perfect labor flexibility assumption, since it has been assumed that $n_t = n_{1,t} + n_{2,t}$. Of course this would not hold anymore when labor adjustment costs are introduced. The aggregate capital stock is $K_t \equiv k_{1,t} + \hat{p}_t^k k_{2,t}$,where $\hat{p}_t^k$ is defined Theorem 1. Finally, the consumer price index is defined as $CPI_t \equiv \frac{c_{1,t}}{y_t} + \hat{p}_t \frac{c_{2,t}}{y_t}$, and the inflation rate is $\pi_t = (CPI_t - CPI_{t-1}/CPI_t)$.

Finally, notice there are two main aggregation methodologies: a fixed-weight aggregation method and chain-weighted type procedure. Until 1995 (included) the Bureau of Economics Analysis (BEA) has adopted the traditional fixed-weight approach, while since 1996 BEA has adopted a "*chain-index*" methods, which uses continually updated relative price weights. This paper use the fixed-weight approach since our model is calibrated over the sample 1953:1996, over which national account aggregated were computed with the fixed-index approach.

3 Extensions of the Benchmark Model

The model presented in the previous section is fairly simple, but anticipating some results, it performs quite well in generating fluctuations consistent with actual data. It is, however, natural to ask whether the model would deliver the same qualitative and quantitative results if relative demand shocks were replaced with relative technology shocks, or if inter-temporal labor adjustment costs were added to the model. Moreover, there is one important element that gives to inter-sectoral demand shocks a primary role for explaining business cycles and fluctuations, that is the existence

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[13] If consumption flows were not sector specific, and if the model's structure allowed to substitute consumption flows between them, then the relative price would be equal to the customary ratio between marginal utilities.

of some idle capacity or unused resources available in the economy. These resources will be put in use when demand increases.[14] This suggests that a third natural extension consists in endogenizing the capacity utilization of capital. We consider each of this possible variations in turn.

Relative Technology Shocks. The structure of the economy is symmetric to that presented in the previous page. There are two differences. First, the instantaneous utility function becomes state independent: there are no demand shocks anymore. Second, production technologies are augmented with relative (sector-specific) technology shocks, denoted as $\tilde{\xi}_{i,t}$, $i = 1, 2$. In this context production technologies read:

$$y_{i,t} = \tilde{\xi}_{i,t} k_{i,t}^{\alpha_i} n_{i,t}^{1-\alpha_i}, \;\; \alpha_i \in (0, 1), i = 1, 2,$$

where $\xi_{i,t}$ are assumed to follow customary autoregressive processes in logs $\log \tilde{\xi}_{i,t+1} = \omega \log \bar{\xi}_i + (1 - \omega) \log \tilde{\xi}_{i,t} + \epsilon_{i,t}$, where $\epsilon_{i,t} \sim \mathcal{N}\left(0, \sigma_{\epsilon^i}^2\right)$, for $i = 1, 2$. Steady state values for the sector-specific productivity shocks are set to one: $\bar{\xi}_i = 1$, $i = 1, 2$. All other equations are unchanged.

The Case of Inter-temporal Adjustment Costs. Consider the benchmark model, with relative demand shocks and no exogenous technological improvement, and consider quadratic adjustment costs.[15] In particular, real income is reduced, in each sector, by a positive quantity $\frac{\delta_i}{2}\left(n_{i,t} - b_i n_{i,t-1}\right)^2 k_{i,t}$, where b_i $(0 \leqq b_i \leqq 1)$ is a scaling parameter, and $\delta_i \geqq 0$.[16] Hence feasibility constraints may be rewritten as:

$$c_{i,t} + i_{i,t} = k_{i,t}^{\alpha_i} n_{i,t}^{1-\alpha_i} - \frac{\delta_i}{2}\left(n_{i,t} - b_i n_{t-1}^{i}\right)^2 k_{i,t} \text{ for } i = 1, 2.$$

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[14] Leisure, or variable capacity utilization, or variable effort, or consumption inventories, are examples in cases. Indeed leisure can be reduced when it becomes more convenient to work. Capacity utilization and effort can be increased to satisfy the increased demand; alternatively, consumption inventories can be decreased.

[15] We may imagine that due to the technological and organizational specificity of labor services firms incur hiring costs because they need to inform and instruct newly hired workers before they are as productive as the incumbent workers. The creation and destruction of jobs (turnover) also entails costs for the workers, not only because they may need to learn to perform new tasks, but also in terms of the opportunity cost of unemployment and the costs of moving. The fact that mobility is costly for workers affects the equilibrium dynamics of wages and employment.

[16] Adjustment costs may be strictly convex. In that case, the unit costs of turnover would be an increasing function of the actual variation in the employment level. This would slow down the optimal response to changes in the exogenous variables. There are also good reasons to suppose, however, that adjustment costs are concave. For instance, a single instructor can train more than one recruit, and the administrative costs of a firing procedure may well be at least partially independent of the number of workers involved. A case of linear adjustment costs lies in be-tween these extremes.

Introduction of a labor adjustment cost impacts the first order conditions for the optimal choice of labor services, and the capital accumulation. Concerning the calibration, $\delta_i^* = 1.5$ and $b_i^* = 0.9$.

Variable Capacity Utilization. Consider again the benchmark model, driven by relative demand shocks only, and no exogenous technological improvement. Under variable capacity utilization, production technologies are specified as follows:

$$y_{i,t} = (u_{i,t}k_{i,t})^{\alpha_i} \, n_{i,t}^{1-\alpha_i}, \;\; \alpha_i \in (0,1), i = 1,2,$$

where $u_{i,t}$ denote the capital utilization rates in sector i. To have an interior solution for $u_{i,t}$ it is necessary to assume that the capital stock depreciates faster if it used more intensively. Following Greenwood, Hercowitz and Huffman (1988) the quarterly rate of depreciation is specified as follows: $\Omega_{i,t} = \frac{1}{\theta^i}(u_{i,t})^{\theta^i - 1}$, $\theta^i > 1$, $i = 1,2$. This structure endogenizes capacity utilization, and, at the same time, convexifies capital utilization. In the non-stochastic steady state θ^i is calibrated to 1.625 so that $\Omega_{i,t} = 0.025$, the customary depreciation rate for the U.S. economy (on a quarterly basis). Then, capital accumulation constraints read: $k_{i,t+1} = (1 - \Omega_{i,t})k_{i,t} + i_{i,t}$, for $i = 1,2$, where the $\Omega_{i,t}s$ denote the quarterly *endogenous* depreciation rates.

4 Results

First the theoretical mechanism producing business cycles from inter-sectoral shocks is explained in detail. Next, the empirical performance of the model is presented along several dimensions.

4.1 A Mechanism for Aggregate Fluctuations and Sectors' Comovement.

Combining the first order conditions for consumption flows and labor services, we have the following market clearing condition for the labor market:

$$\underbrace{APN_{i,t}}_{\text{LABOR DEMAND}} = \underbrace{\frac{MU_t^{\ell}}{MU_t^{ci}}}_{\text{LABOR SUPPLY}}, \tag{14}$$

where MU_t^ℓ and $MU_t^{c_i}$ denote the marginal utilities of leisure and consumption respectively, and $APN_{i,t}$ is the average productivity of labor. In the model there are *flexible* inputs, which are the labor services, and *non flexible* production factors, the capital stocks. After every positive idiosyncratic demand shock it is convenient to distinguish between an instantaneous response and an inter-temporal response.

The **instantaneous response** involves the flexible production factors. Every positive idiosyncratic shock increases the marginal utility (and therefore the consumption) of the commodity directly hit by that shock ($\uparrow MU_t^{c_{i,t}} \Rightarrow \uparrow c_i$). Then we have in increase in labor, since the marginal utility of leisure gets smaller, relatively to the marginal utility of consumption ($MU_t^\ell \prec\prec MU_t^{c_i} \Rightarrow \downarrow MU_t^\ell / MU_t^{c_i} \Rightarrow \downarrow \ell_t$).[17] This is the key part of the mechanism. Consumers work more, and therefore become richer; the wealth effect comes into the picture, and people consume more of both commodities, because they are normal goods. Notice that the latter effect reinforces the increase in consumption triggered by the initial shock. This results into an amplification of the propagation mechanism (see Section 4.6 for more details).

The **inter-temporal response** involves the non-flexible production inputs, the capital stocks. Since capital is sector specific and homogenous, investment goods are normal, as well as the final consumption good. Because of the income effect, consumers increase their investment in both commodities. As a result, there is an increase in capital accumulation in both sectors. Finally production output increases in both sectors.

Notice that both sectors of the economy expand, as a result of every inter-sectoral *positive* demand shocks.[18] In terms of intuition, it is like saying that if consumers demand more cars, they will also enjoy more driving vacations. The consumption of one commodity induces the consumption of an other, even if the two sectors are not linked via an input-output structure.[19]

The following sections present the empirical performance of the model. Being highly non-linear, the system has no closed form solution. To study its stochastic properties we apply the well known procedure developed by King Plosser and Rebelo (1988a, b); certainty equivalence is assumed,

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[17] Notice that leisure is not affected directly by any shock. It reacts to a demand shocks, but the causality order of the shock is from the demand side to the supply side.

[18] When the relative demand shock is negative (that is, when it is below its mean), there is a recession in both sectors.

[19] Section 5.5 present a more detailed discussion of sectors' comovement.

the system is linearized around its non stochastic steady state, and is solved by applying linear approximations (e.g. Campbell, 1994; Uhlig, 1999).

A multi-sector model offers several dimensions along which it can be compared to the actual data. We focus first on the aggregate series, presenting the volatility measures and the contemporaneous correlations. Then, we show that the model generates an aggregate and *"false" Solow Residual*, even though there is no exogenous technological improvement. Next, the price side of the model economy (the consumer price index and the inflation rate), and volatility measures and correlations of sectoral variables are analyzed as well. Finally, the propagation mechanism is documented in more details.

4.2 Aggregate Variables: Volatility Measures and Comovements

This section describes how well the model accounts for aggregate fluctuations. **Table 1** reports the relative volatilities with respect to aggregate output for the model's aggregate series, and compares them with their counterparts for the U.S. economy (sample period 1953:Q1-1996:Q4). Also present in Table 1 are the corresponding statistics for standard benchmark Real Business Cycle model of Hansen (1985), for other demand-driven models (e.g. Wen, 2003a and 2003b; Bencivenga, 1992), and for selected multi-sector general equilibrium models (Huffman and Wynne, 1999; Horvath, 2000).[20]

Consumption, Investment and Output Volatility. In all six versions of the model, consumption is less volatile than output, and investment is more volatile than output and than consumption. Both series are highly positively correlated with output. These positive comovements and the relative volatility order among these three variables are two of the most celebrated stylized business cycle facts.

When relative demand shocks are the driving source for the economy, consumption becomes much more volatile than in the standard business cycle models. In this sense our model is not subject to the so called *consumption volatility puzzle*, like all technology driven business cycle models (e.g.

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[20] These schemes are driven by sector specific exogenous productivity shocks. We are not aware of multisector dynamic equilibrium models driven by sector specific demand shocks.

Table 1: Selected Moments, Aggregate Real Series

	σ_X/σ_Y			$\rho(X,Y)$				$\rho(APN,N)$
	N	C	I	N	C	I	APN	
U.S. Economy	0.99	0.76	2.99	0.81	0.83	0.89	0.12	-0.25
(A) **Relative Demand Shocks**	**1.44**	**0.71**	**2.26**	**0.90**	**0.98**	**0.90**	**-0.59**	**-0.85**
(B) Separable Preferences	1.42	0.74	2.05	0.99	0.98	0.97	-0.19	-0.82
(C) Inter-Temp. Labor Adjustment Costs	0.89	0.71	2.07	0.97	0.71	0.97	0.18	-0.48
(D) Inter-Sec. Labor Adjustment Costs	1.36	0.61	2.50	0.99	0.99	0.98	-0.08	-0.83
(E) Endog. Variable Capacity Utilization	1.16	0.89	1.42	0.99	0.99	0.99	-0.02	-0.93
(F) Variable Cap. Util. + Inter-Temp. Adj.	0.89	0.69	2.15	0.97	0.97	0.98	0.09	-0.85
Relative Tech. Shocks	0.78	0.45	3.24	0.99	0.96	0.98	0.77	0.81
Hansen (1985)	0.67	0.61	4.09	0.97	0.94	0.99	0.98	0.87
Wen (2003a)	1.38	0.65	3.68	0.99	0.65	0.90	-	-
Bencivenga (1992)	1.19	1.25	-	0.94	0.98	-	-0.30	-0.60
Huffman-Wynne (1999)	0.80	0.43	2.81	0.87	0.97	0.97	0.74	0.57
Horvath (2000) (a)	0.57	0.58	3.94	0.88	0.86	0.72	0.87	0.49
Horvath (2000) (b)	0.54	0.51	3.18	0.95	0.94	0.85	0.94	0.79

Table 1. (A) Baseline model: relative demand shocks only and non separable preferences; (B) separable preferences; (C), (D) and (E), refer to the introduction of inter-sector and of inter-temporal labor adjustment costs, and of endogenous capacity utilization; (F) model with only sectoral technology shocks, and no demand shocks. The letters N, C, I, and APN denote respectively aggregate employment, consumption, investment, and aggregate labor productivity; σ_X/σ_Y denotes relative volatility between a variable X and aggregate output Y, $\rho(X,Y)$ is the contemporaneous correlation with aggregate output, and $\rho(APN,N)$ represents the contemporaneous correlation between hours worked and the average productivity of labor. All statistics are computed based on 1000 simulations of 176 periods length. Source for U.S. data: Stock and Watson (1998).

the *indivisible labor* version of Hansen, 1985). [21] Notice that also the "relative technology shock" model is subject to this undesirable property.[22] This suggests that the improvement upon this puzzle originates mainly from the source of the uncertainty and in the propagation mechanism (the "relative demand shocks"), and not from the "two sector" structure. Comparing a RBC model with a Relative Demand Shock model, the causality order between exogenous innovations and the model's response is almost symmetric. In a RBC model, first a productivity shock occurs (suppose positive, without loss of generality), and then, as a consequence, consumption and investment increase. In a Relative Demand Shock model, instead, first consumption increases because of the relative demand shock (assumed positive, for consistency), then output increases, and there is a further increase in (both) consumptions triggered by the income effect, as the previous Section underlines. The structure of the Relative Demand Shock formulation turns out to be more efficient in generating a relatively higher consumption volatility.

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[21] The *consumption volatility puzzle* refers to the fact that consumption volatility generated by stochastic growth models is often too small relative to the data.

[22] The "relative technology shock" model has been presented in Section 3. For convenience its main differences with the baseline formulation (driven by demand shocks only) are reported below. First, the instantaneous utility function becomes state independent: there are no demand shocks anymore. Second, production technologies are augmented with relative (sector-specific) technology shocks.

In addition, it should be noted that the model is not subject to the *crowding out* between consumption and investment typical of several one-sector demand-driven formulations.[23] Baxter and King (1991) notice that when an aggregate demand shock impinges on a one-sector model, people increase consumption, while reducing investment, and, by this end, capital accumulation. Output, being a a monotone transform of capital stock, subsequently falls, depicting a significant crowding out effect. That happens because aggregate preference shocks induce the urge to consume, that is to substitute consumption tomorrow (investment) with consumption today. Our model, instead, focuses on inter-sectoral shocks, which induce the consumers to substitute between the two goods, but in relative terms. The argument is interesting and subtle. Section **??** shows that both consumption demands increase after any *positive* relative demand shocks. However, the commodity directly hit by the positive demand shock increases *relatively* more than the other one.[24] There is no crowding out effect between consumption and investment, in each sector, because relative demand shocks perturb the relative desirability between consumption goods.

Labor Market Puzzles. It is also interesting to compare our model's performance along selected labor market dimensions, focusing especially on the so called *productivity puzzle.* The *productivity puzzle* concerns the correlations between average labor productivity and GDP, and between average labor productivity and employment. The puzzle is that average labor productivity (APN) and employment (N) are negatively correlated for most economies ($\rho\left(APN,N\right)<0$), and that the same average labor productivity and GDP presents a weak (or null) correlation ($\rho\left(APN,Y\right)\geqq 0$).[25]

The Stock and Watson (1998)'s estimates for the U.S. economy, in particular, are $\hat{\rho}\left(APN,N\right)=-0.25$ and $\hat{\rho}\left(APN,Y\right)=0.12$ respectively. Under technology shocks, the total productivity (as well as labor and capital productivities) drives the business cycle. Hence, by construction, a RBC model generates a high and positive correlation between GDP and aggregate employment; at the same time it induces a high and positive correlation with the average labor productivity, too. This

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[23] An additional undesirable property originating from the crowding out between consumption and investment, is that consumption and output end up being negatively correlated over the business cycle. That happens because next period capital falls after a reduction in investment. As a consequence, next period production will fall, following the capital decumulation.

[24] For example, suppose that a positive demand shock occurs in the i-th sector. Then both consumption flows increase ($c_{i,t}$ and $c_{j,t}$ increase), but $c_{i,t}$ increases relatively more than $c_{j,t}$ ($\frac{\Delta c_{i,t}}{c_{i,t}}>\frac{\Delta c_{j,t}}{c_{j,t}}$). In this sense the model generates a substitution between $c_{i,t}$ and $c_{j,t}$.

[25] As reported by Stadler (1994) this correlation is negative or zero for almost all the countries.

is, however, in contrast with the facts previously presented. As Table 1 shows the use of relative demand shocks improves upon this difficulty. The discussion that follows explains why.

Consider, first, the correlation between average productivity of labor and employment $\rho\,(APN, N)$. This statistics is negative in all formulations of the relative-demand model, ranging between -0.48 and -0.83. This is consistent with the U.S. economy. On the contrary, technology-driven models induce a large positive correlation. The economic mechanism of our models improves upon this failure, as the first order conditions suggest. In particular, combining the FOCs for consumption and leisure, equation (14), we have that $MU_t^{c_i}/MU_t^{\ell} = APN_{i,t}$, where MU_t^{ℓ} and $MU_t^{c_i}$ denote marginal utilities of leisure and of $i-th$ consumption flow, while $APN_{i,t}$ is the average productivity of $i-th$ sector labor services. The left hand side represents the labor supply, while the right hand side the labor demand. Now, after a positive idiosyncratic demand shock $\uparrow\,\tilde{s}_{i,t}$, $MU_t^{c_i}$ increases, *ceteris paribus.* This shifts out the labor supply schedule, along the labor demand. Labor demand does not shift, inducing a negative correlation between hours worked and wage. In a technology driven model, the mechanism is exactly the opposite. The $APN_{i,t}$ increases after a positive productivity shocks, and labor demand shifts out, along labor supply. This results in a positive correlation between wage and hours, which is, however, absent in the data.[26]

The correlation between wage rate and GDP also deserves mention. It is convenient to analyze this fact in conjunction with volatility of hours worked. The baseline version of the model overpredicts the relative volatility of hours worked $\frac{\sigma_n}{\sigma_y}$,. This seems, however, a feature peculiar of demand driven models (see Bencivenga, 1992; Wen, 2003a). Moreover, this fact has the unfortunate implications of inducing a negative comovement between aggregate GDP and APN. That happens because over the simulated business cycle total employment (N) fluctuates relatively more than aggregate production (Y), inducing a negative correlation between $APN = (1-\alpha)\frac{Y}{N}$ and N. The introduction of intra-sector adjustment costs strengthens comovements between labor flows, thereby increasing the volatility of aggregate hours. Endogenizing the capacity utilization of capital stocks helps to reduce hours' volatility, because the variable capital capacity utilization offers ad-

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[26] Several contributions reformulate the standard technology driven business cycle model for replicating the negative correlation between labor input and technology shocks (e.g. Francis and Ramey, 2003; Campbell, 1998). Others specialize the analysis, suggesting that investment-specific technological change, in the spirit of Greenwood, Hercowitz and Krusell, 1997) account for a large part of business cycle fluctuations (e.g. Fisher, 2002).

ditional flexility to the model.[27] A model with inter-temporal labor adjustment costs *and* variable capacity utilization is capable to generate comovement between both sectors, as well as a procyclical average productivity of labor. The introduction of inter-temporal adjustment costs reduces the ratio $\frac{\sigma_n}{\sigma_y}$ to 0.89. As a consequence the correlation between APN and Y becomes positive. The variable capacity utilization gives back to the system some of the flexibility lost because the cost of adjustment.[28]

Models' Comparison. With a different kind of demand shocks, our model represents an improvement upon the Baxter and King (1991), and Bencivenga (1992) models especially along the consumption volatility dimension, while it performs as well as Wen (2003a and 2003b).[29]

The main advantage relative to a one-sector demand driven model is that our model requires relatively less persistent inter-sectoral demand shocks in order to generate consistent impulse response functions.[30] That happens because the aggregate consumption index helps in propagating the effect of each idiosyncratic demand shock (Section 4.6 offers more details).

Next, compared to multi-sector models driven by technology shocks, our model performs quite well in predicting labor market behavior, and aggregate consumption volatility. Unfortunately, the comparison with alternative formulation remains incomplete, since a detailed set of statistics for all models is usually not available.

4.3 A "*False* Solow Residual"

Prescott (1986) suggests that one way of measuring technological change within the context of real business cycle models is to follow Solow (1957).[31] Prescott's (1986) seminal paper is still today

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[27] This model, however, induces a negative correlation between consumption and investment flows in each sector (statistics are not presented here). That happens because costs of adjustment make it more difficult to increase labor supply after a demand shock, and thus it is more convenient to substitute investment with consumption.

[28] Busato and Argentiero (2004) obtain the same result by introducing into this model a productive government expenditure. There are no labor adjustment cost. Wen (2003b) obtains analogous results in a model with Baxter an King (1991) type of aggregate demand shock.

[29] In particular, Bencivenga (1992)'s model has several undesirable properties, like a a negative correlation between hours and output. Consumption too is very volatile, even more than output (relative variability is 1.25). In summary, the model falls short under several dimensions, and, its results are, in some sense, weakened in the light of Gali (1999)'s contribution. Bencinvenga presents results only for the unconditional moments, still using multiple sources of fluctuations. It would be very interesting to have more information concerning the conditional moments.

[30] Notice that the shocks' persistence mainly refers to the inter-sectoral desirability of the two different commodities, and not to the inter-temporal desirability between consumption and investment.

[31] In this case, Solow growth accounting suggests that the process of the technology parameter is highly persistent. Its volatility, measured with Solow residual's standard deviation, is approximate 0.763 for the U.S. economy.

a source of debate, and the Solow Residual has been directly or indirectly at the center of many discussions, mainly because of a measurement issue. Prescott (1986) stresses there may be errors in measuring the labor and the capital inputs. In calculating it, full and constant utilization of both capital and labor inputs is often assumed.[32] Hall (1988) challenged the assumption that movements in Solow Residual represent exogenous technology shocks. He argues

> "[...] *that under competition and constant returns to scale the Solow residual is uncorrelated with all variables known to be neither cause by productivity shifts, nor the causes of productivity shifts* [...]"

The Solow residual seems, indeed, to be correlated with government expenditure (Hall, 1988), with various monetary aggregates (Evans, 1992), and government consumption (Burnside, Eichenbaum and Rebelo, 1995). Jovanovic (1991) argues that secular changes in organization might explain a large portion of the change in the Solow residual in one country over time. Burnside, Eichembaum and Rebelo (1993) investigate the sensitivity of Solow Residual to the presence of labor hoarding behavior. Quite interestingly, their results are supportive of the view that a large part of fluctuations in Solow residual depends on labor hoarding type behavior. They eventually conclude that the existing real business cycle models substantially overstate the role of technology shocks that accounts for the volatility in the GDP postwar series. Hoover and Salyer (1998) demonstrate that the Solow residual does not carry useful information about technology shocks.

This paper contributes to this debate with the following exercise. First, the aggregate real series are constructed by using the relative prices derived in the decentralization (Theorem 1). Then it is assumed that the aggregate production function for the U.S. economy is Cobb-Douglas, of the kind: $Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$, where K_t is aggregate capital stock, N_t is total employment, A_t is a productivity measure, and α is the share of capital.[33] Finally, three "**false Solow Residuals**" are computed for our economy with relative demand shocks only: one for each sector ($\psi_{i,t}$, $i = 1, 2$) and an aggregate

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[32] Since the utilization of capital is likely to be highly procyclical, it can be argued that this assumption could have important implications for the interpretation of the procyclical behavior and exogeneity of productivity shocks,as well as the degree of increasing returns to scale and market power in the economy.

[33] The definition of the *Solow Residual* (denoted with the quantity $\log A_t$), following the original Solow (1957) contribution, is reported below for convenience: $\log A_t = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(N_t)$. In addition, it should be noted that the model assumes that each sector uses a Cobb-Douglas production function. From a theoretical perspective this does suffices for claiming that the aggregate production function is Cobb-Douglas too. It is, however, a customary assumption for the U.S. economy.

one ψ_t . They are all symmetrically defined, and denoted with Greek letter ψ, since in ancient Greek *ψευδής* (*pseudés*) means *false, untrue.* The aggregate one reads:

$$\log \psi_t = \log(Y_t) - \alpha \log(K_t) - (1-\alpha)\log(N_t). \tag{15}$$

Now, if the Solow Residual is a legitimate measure of productivity improvement, all False Solow Residuals should not be significantly different from zero for each time period, since in the model there is no exogenous productivity improvement. However, this is true for each sector (that is $\{\psi_{i,t} = 0\}_{t=1}^{T}$, $i = 1, 2$), but once the aggregate series are consider, the results are different. The model is capable of generating an aggregate "*false Solow Residual*" whose statistical properties are consistent with the analogous computation using U.S. data (**Table 2**).

Table 2: "Actual" and "False" Solow Residuals: Selected Stochastic Properties

	$\frac{\sigma_{FSR}}{\sigma_Y}$	$\rho_{(FSR,Y)}$
U.S. Economy	0.54	0.78
(A) Relative Demand Shocks	0.13	0.66
(B) Relative Tech. Shocks	0.51	0.99
Benchmark RBC model	0.54	0.78

Table 2. (A) baseline model with non separable preferences, and relative demand shocks only; (B) model with relative technology shock only; $\frac{\sigma_{FSR}}{\sigma_Y}$ and $\rho_{(FSR,Y)}$respectively denote relative volatility and contemporaneous correlation of *False Solow Residual* with the aggregate output Y. All statistics are computed based on 1000 simulations of 176 periods length. Sources: Stock and Watson (1998) for the actual data on the U.S. economy, King and Rebelo (1988a) for the benchmark RBC model.

In particular, that actual and simulated Solow Residual present an analogous contemporaneous correlation with output ($\hat{\rho} = 0.78$, and $\rho^* = 0.66$), while the simulated False Solow residual is less volatile that the actual one, relative to GDP ($\hat{\sigma} = 0.53$ and $\sigma^* = 0.13$).

The False Solow Residual generated by our model does not reflect, by its very construction, any exogenous change in technology and productivity. [34] Therefore, these results suggest that either the Solow residual, as computed following Solow (1957), is a misspecified measure of productivity

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[34] The difference is that this quantity measures something completely different from technology or productivity. It captures pure sectoral demand effects over aggregate factor productivity. Since it arises only for the aggregate economy, it could be interpreted as a measure of inter-sectoral risk hedging against idiosyncratic demand shocks. A representative consumer-shareholder being subject to idiosyncratic demand shocks would indeed benefit from the possibility of allocating resources to both sectors, in order to diversify her risk. The aggregate False Solow Residual here presented could exactly represent this feature.

at the business cycle frequencies, or that relative demand shocks represent a possible explanation for procyclical productivity.[35] The former claim is consistent with Basu (1998) and Basu, Kimball and Fernald (2002), who argue that neoclassical economists have misinterpreted the link between technological changes and business cycle by measuring cyclical technological changes with the Solow Residual. They stress, on the contrary, that Solow Residual was meant to estimate the long run impact of technology over the economy.

4.4 Price Index and Inflation.

Table 3 reports the cross correlation between consumer price index (CPI), inflation rate (π) and aggregate output for the U.S. economy and for the Baseline model (denoted with an "hat" and with a "star", respectively).[36] The upper part of the table presents data on the CPI level, while the bottom part displays corresponding statistics for the inflation rate (that is the CPI growth rate). The data show a negative contemporaneous correlation between CPI and GDP, and positive contemporaneous correlation between inflation rate and GDP. In particular $\hat{\rho}(CPI_t, GDP_t) = -0.51$ and $\hat{\rho}(\pi_t, GDP_t) = 0.35$.[37]

Our competitive model, driven by relative demand shock, accurately reproduced these evidence: $\rho^*(CPI_t, GDP_t) = -0.28$ and $\rho^*(\pi_t, GDP_t) = 0.25$.

Economics literature differently interprets these evidence.[38] One interpretation for this regularity is that supply shocks plays a dominant role in driving the cycle. For example, Barro (1993) interprets these results as evidence in favor of real business cycle models where productivity generates countercyclical price movements, and against new-keynesian models. But, such evidence

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[35] Literature has advanced four main explanations for procyclical productivity. First procyclical productivity may reflect procyclical technology. Second, widespread imperfect competition and increasing returns may lead productivity to rise whenever inputs rise. Third utilization of inputs may vary over the cycle, in a way that is not properly captured by standard input measures. Fourth, reallocation of resources across uses with different marginal products may contribute to procyclicality. For example, if different industries have different degrees of market power, then inputs will generally have different marginal products in different uses. Then aggregate productivity growth is cyclical if sectors with higher markups have input growth that is more cyclical.

[36] For convenience the definition of CPI_t and the inflation rate π_t are here reported: $CPI_t \equiv \frac{c_{1,t}}{y_t} + \hat{p}_t \frac{c_{2,t}}{y_t}$; next $\pi_t = (CPI_t - CPI_{t-1}/CPI_t)$.

[37] Several other studies present evidence of negative correlation between prices and output (e.g. Backus and Kehoe, 1992).

[38] An important reason for examining price-output correlations is to provide evidence about the type of shocks that are important for the business cycle. The logic underlining this approach is that demand shocks cause output and prices to move in the same direction, while supply shocks cause them to move in opposite directions. What follows suggests, however, that it is important to be very careful while making these claims.

Table 3: Consumer Price Index and Inflation Rate: Cross Correlation with Output

lead/lag	-4	-3	-2	-1	0	1	2	3	4
$\rho^*(CPI_t, Y_{t+k})$	0.43	0.39	0.28	0.07	-0.28	-0.33	-0.28	-0.29	-0.33
$\hat{\rho}(CPI_t, Y_{t+k})$	0.12	-0.04	-0.21	-0.38	-0.51	-0.62	-0.68	-0.67	-0.59
$\rho^*(\pi_t, Y_{t+k})$	0.18	0.26	0.35	0.47	0.25	0.19	0.23	0.25	0.25
$\hat{\rho}(\pi_t, Y_{t+k})$	0.58	0.64	0.62	0.52	0.35	0.14	-0.08	-0.27	-0.40

Table 3. $\rho^*(\bullet, Y_{t+k})$ and $\hat{\rho}(\bullet, Y_{t+k})$ denote the simulated and the actual correlations between a time t variable with aggregate output at time $t+k$, respectively. The "star" denotes simulated moments, while the "hat" an estimated one. All statistics are computed based on 1000 simulations of 176 periods length, and refer to the Baseline Model. Source: Stock and Watson (1998) for the U.S. economy.

should be interpreted with caution as a number of studies have shown that standard sticky-price models with only demand shocks can generate negative correlation coefficients between prices and output (e.g. Ball and Mankiw, 1994; Judd and Trehan, 1995).[39]

This paper suggests that it is not necessary to introduce some nominal rigidities for inducing a negative correlation between prices and GDP.[40] The key factor is the existence of some "idle capacity", or some unused resources available for the economy.[41] After any positive idiosyncratic demand shock, these available resources will be used more intensively or put into production, in order to fulfill the additional demand.

We argue this is a consequence of the Permanent Income Hypothesis (PIH). After a relative demand shock, both $c_{1,t}$ and $c_{2,t}$ respond less that aggregate output because of PIH. The relative price $\hat{p}_t$ responds positively to a demand shock on $c_{2,t}$ and negatively to one on $c_{1,t}$. Since the CPI_t is defined as $\frac{c_{1,t}}{y_t} + \hat{p}_t \frac{c_{2,t}}{y_t}$, this implies that during an expansion y_t increases, and $\frac{c_{1,t}}{y_t} + \hat{p}_t \frac{c_{2,t}}{y_t}$ decreases, inducing a negative correlation between CPI and aggregate output.

This mechanism shifts out the aggregate supply, relaxing the pressure of the demand side. The final sign of the correlation depends on the elasticities of the demand and the supply, and on the

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[39] In a classical sticky price model, indeed, a demand shock raises output in the impact period, but it leave price unchanged. In the long run, output returns to its pre-shock level (this is usually defined as *long-run neutrality*) but prices are permanently higher. During the adjustment process, prices are below trend for some periods while output is above trend. This can generate a negative correlation between detrended prices and output.

[40] It also suggests that the price-output correlation does not provide a useful way to evaluate the empirical performance of demand-driven versus supply-driven theoretical models, unless proper restrictions on the relative size and the dynamic properties of demand and technology shocks.

[41] If the economy were operating at 100% capacity, and there were no available production factors, the prices would rise.

size of the adjustment costs, when present. For a standard parameterization and for a plausible size of the adjustment costs, the model generates a negative correlation between CPI and GDP. [42]

4.5 Sectoral Business Cycle and Comovement

Table 4 presents the volatility measures for disaggregated series for production, consumption, investment, and labor services, as generated with the baseline model. The first row reports the volatility of each series relative to corresponding sectoral GDP, while the second one presents the volatility of each series, relative to the corresponding aggregate variable. Consider Food consumption c_F for example: the first row of Table 4 suggests that $\frac{\sigma^*_{c_F}}{\sigma^*_{y_F}} = 0.93$, and the second one that $\frac{\sigma^*_{c_F}}{\sigma^*_{C}} = 0.40$, where $\sigma^*_{y_F}$ and σ^*_C denote the standard deviations of Food production and aggregate consumption, respectively.

Table 4: Volatility Measures for the Disaggregated Series

	n_F	c_F	i_F	r_F	y_F	n_{CS}	c_{CS}	i_{CS}	r_{CS}	y_{CS}
$\sigma^*_{x_i}/\sigma^*_{y_i}$	1.49	0.93	1.69	0.04	1.00	1.48	0.72	2.14	0.04	1.00
$\sigma^*_{x_i}/\sigma^*_{x}$	0.34	0.70	0.26	0.01	0.32	0.80	0.94	0.80	0.01	0.76

Table 4. $\sigma^*_{x_i}/\sigma^*_{y_i}$ denotes the standard deviation of variable x_i relative to corresponding output y_i, while $\sigma^*_{x_i}/\sigma^*_{x}$ denotes the standard deviation of variable x_i relative to corresponding aggregate counterpart x, as generated with the baseline model; The left hand side of the table reports the volatility measures of Food Sales (indexed with the "F"), while the right hand side the corresponding statistics for Clothing and Shoes Sales (indexed with the "CS"); all statistics are computed based on 1000 simulations of 176 periods length.

The Food consumption is more volatile than that of Clothing and Shoes, relative to the total consumption, as generated with the benchmark model. In particular, $\frac{\sigma^*_{c_F}}{\sigma^*_{C}} = 0.70$ and $\frac{\sigma^*_{c_{CS}}}{\sigma^*_{C}} = 0.94$. Unfortunately, the comparison with the data is more complex along these dimensions, since comparable data are available only for the consumption series. The model properly matches the relative volatility of the food sales $\frac{\hat{\sigma}_{c_F}}{\hat{\sigma}_{C}} = 0.89$, while it underpredicts the volatility of Clothing and Shoes sales: $\frac{\hat{\sigma}_{c_{CS}}}{\hat{\sigma}_{C}} = 1.21$ over the sample period 1953:01-1996:01.

Table 5, then, presents the contemporaneous correlations among disaggregated variables in each sector, among aggregate and disaggregate variables. A defining characteristic of the business cycle is the comovement in the pace of economic activities in different sectors of the economy (e.g. Lucas,

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[42] These results are robust to a sensitivity analysis over the critical parameters: labor adjustment costs parameters, and elasticity of substitution between consumption goods.

1977).[43] Given these evidence, two features of the our model deserve more attention. First, all sectors comove, since contemporaneous correlation between capital stocks, labor flows, production output, investments and consumptions are positively correlated within each sector. Second, both disaggregated series comove with the corresponding aggregate series.

Table 5: Comovement across Sectors

	k_F	k_{CS}	K
k_F	1.00		
k_{CS}	0.98	1.00	
K	0.40	0.43	1.00

	n_F	n_{CS}	n
n_F	1.00		
n_{CS}	0.67	1.00	
N	0.89	0.93	1.00

	y_F	y_{CS}	Y
y_F	1.00		
y_{CS}	0.67	1.00	
Y	0.84	0.97	1.00

	c_F	c_{CS}	C
c_F	1.00		
c_{CS}	0.95	1.00	
C	0.79	0.91	1.00

	i_F	i_{CS}	I
i_F	1.00		
i_{CS}	0.97	1.00	
I	0.99	0.99	1.00

Table 5. Statistics denote the contemporaneous correlation between aggregate and sectoral variables, as generated with the baseline model. Capitalized letters denote aggregate variables (e.g. $c_{t,i}$ denote i-th sector's consumption, and C is the aggregate consumption). Subscript "F" refers to Food Sales; subscript "CS" denotes Clothing and Shoes Sales. All statistics are computed based on 1000 simulations of 176 periods length.

Several contributions suggest that multi-sector versions of the neoclassical growth model are consistent with the observed positive comovement across sectors if one accounts for the input-output structure of the economy.[44] This contribution suggests that it is still possible to have comovement without an input-output structure.

In our model there are two key elements that explain the positive comovements across sectors. First, the relative demand shocks change not only the intra-sector desirability between consumption goods, but also modify the inter-temporal preferences of consumers. In addition, under non-separable preferences, the aggregate consumption index enhances the propagation and the transition mechanisms of the model. Next section discusses these issues in more details.

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[43] Lucas (1977) notes that the comovements of economic activities across different sectors of the economy is the most important of the regularities common to all business cycles. This evidence is the prerequisite for a theory of aggregate business cycle. We are aware of only two sectors which employment is almost acyclical or countercyclical: the home production sector as documented by Benhabib, Rogerson and Wright (1991), and the underground sector , as documented by Busato and Chiarini (2004). But investment and employment in various sectors are not perfectly correlated, which suggests that there may be some sector specific driving forces (Huffman and Wynne, 1999; Hornstein, 2000).

[44] See the seminal paper by Long and Plosser, 1983; or Hornstein, 2000; Huffman and Wynne, 1999; Hornstein and Praschnik, 1997; Horvath, 2000. All these contributions assume that technology shocks (aggregate and/or sector specific) are the driving force of the economy.

The positive comovements among sectoral variables and among aggregate and disaggregated variables do not depends on the correlation between demand innovations. Indeed, under non-separable preferences (the baseline model) the relative demand shocks are uncorrelated; when preferences are separable, it has been assumed that they are slightly positively correlated. The model generates consistent impulse-response function for correlation between shocks' innovation higher than 0.025.

4.6 Shocks' Propagation under Relative Demand Shocks

In their well known survey on Real Business Cycle models King and Rebelo (1999) discuss extensively the central role of productivity shocks in driving the business cycle. They also stress how their benchmark model's performance relies on large and highly persistent technology shocks. To generate macroeconomic series consistent with the U.S. and European data, their RBC models require a considerable variability in productivity, and a serial correlation parameter of the stochastic component of productivity near one.

The propagation mechanism of our model differs from the standard one, and it is distinctive of a two-sector model driven by relative demand shocks. Consider the first order condition for one of the two consumption goods ($c_{i,t}$, without loss of generality: $C\left(\mathbf{c}_t\right)^{-(1+\psi)} \lambda_i \tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}} = \phi_{i,t}$) and the aggregate consumption index, from equation (1) $C\left(\mathbf{c}_t\right) = \left(\lambda_1 \left\{\tilde{s}_{1,t} c_{1,t}^{\frac{\sigma-1}{\sigma}}\right\} + \lambda_2 \left\{\tilde{s}_{2,t} c_{2,t}^{\frac{\sigma-1}{\sigma}}\right\}\right)^{\frac{\sigma}{\sigma-1}}$. Below there are the corresponding log-linearized equations:

$$\begin{aligned} \hat{s}_{i,t} &= (1+\psi)\,\widehat{C\left(\mathbf{c}_t\right)} + \frac{1}{\sigma}\hat{c}_{i,t} + \hat{\phi}_{i,t} && (16) \\ \widehat{C\left(\mathbf{c}_t\right)} &= \lambda_i \frac{\sigma}{\sigma-1}\hat{s}_{i,t} + \lambda_i \hat{c}_{i,t} + \lambda_j \frac{\sigma}{\sigma-1}\hat{s}_{j,t} + \lambda_j \hat{c}_{j,t} && (17) \end{aligned}$$

where variables with "*hat*" represent percentage deviations from the steady state values, which are denoted with a "*bar*". To better understand how the propagation mechanism works, it is convenient to decompose the total impact of each demand shock in two parts. Assume that the relative desirability of the i-th commodity increases, that is $\uparrow \hat{s}_{i,t}$.

There is a *direct effect*, which is generated by the increase in marginal utility of the i-th consumption ($\uparrow \hat{s}_{i,t}$) that induces an increase in $\hat{c}_{i,t}$ (see equation (16)). Then, there is an *indirect effect*, which is generated by an increase in the aggregate consumption index ($\uparrow \widehat{C(\mathbf{c}_t)}$) as equation (17) suggests. This induces a further increase in $\hat{c}_{i,t}$, because of the expansion that occurs in the j-th sector.[45]

It is particularly welcome that we obtain these results even if we use a logarithmic utility function for consumption, and very small shocks. We define an improvement in the propagation mechanism of a stochastic growth model in the sense of necessitating a lower autocorrelation coefficient for the process of stochastic disturbances, and a smaller standard deviation of the innovations for replicating business cycle facts.[46]

5 Conclusions

This model proposes an original theoretical mechanism for generating aggregate fluctuations and sectoral comovements by using inter-sectoral and idiosyncratic shocks. This mechanism is complementary to the standard Real Business Cycle theory.

The model performs quite well in reproducing most regularities of the U.S. business cycle. It performs particularly well with respect the aggregate consumption volatility and its cross-correlation with output, the main labor market stylized facts, the price index and the inflation rate volatilities and their correlations with aggregate output. The model furthermore generates a negative correlation between average productivity of labor and hours worked, which is a stylized fact not explainable by a technology driven model.

In this sense, the model can be proposed as a benchmark for the U.S. economy.

Finally, the model generates a *false* Solow residual, whose size and time series properties are consistent with the U.S. Solow residual data. In this model, however, this quantity measures something completely different from technology or productivity.

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[45] The mechanism that generates this comovement between sector has been presented in Section 4.1.

[46] The model still needs some persistence in the process driving the shocks, but this persistence is lower relatively to alternative formulations. In other words, if the relative demand shocks were not persistence, the model's response would not generate impulse response functions consistent with the U.S. economy.

References

[1]Backus D.K., and P.J. Kehoe, 1992, International Evidence of the Historical Properties of Business Cycles, American Economic Review, 82, 4, 864-88.

[2]Ball L., and G. N. Mankiw, 1994, A Sticky-Price Manifesto, National Bureau of Economic Research Working Paper 4677.

[3]Barro R. J., 1993, Economic Growth, Convergence, and Government Policies, Economic policy, Financial Markets, and Economic Growth, pp. 9-28.

[4]Basu S., M. Kimball and J. Fernald, 2002, Harvard Institute of Economic Research Discussion Paper Number: 1986.

[5]Basu S., 1998, Technology and Business Cycles: How Well Do Standard Models Explain the Facts? in Source Beyond Shocks: what Causes Business Cycles? 207-55, Conference Series, 42, Federal Reserve Bank of Boston.

[6]Baxter M. and R. King, 1991, Productive Externalities and Business Cycles, Discussing Paper 53, Federal Reserve Bank of Minneapolis.

[7]Bencivenga, V., 1992, An Econometric Study of Hours and Output Variation with Preference Shocks, International Economic Review, 33, 2, 449-71.

[8]Blanchard O.J. and D. Quah, 1989, The Dynamic Effects of Aggregate Demand and Supply Disturbances, American Economic Review, 79, 4, 655-73

[9]Burnside C., M. Eichenbaum and S. Rebelo, 1995, Capital Utilization and Returns to Scale, NBER Working Papers 5125.

[10]Benhabib J., and Y. Wen, 2002, Indeterminacy, Aggregate Demand, and the Real Business Cycle, manuscript.

[11]Benhabib J., R. Rogerson and R. Wright, 1995, Homework in Macroeconomics: Household Production and Aggregate Fluctuations, Business cycle theory, 473-94.

[12]Busato F. and A. Argentiero, 2004, Fiscal Policy under Relative Demand Shocks, manuscript Columbia University `www.columbia.edu/∼fb117`.

[13]Busato F. and B. Chiarini, 2004, Market and Underground Activities in a Two Sector Dynamic Equilibrium Model, Economic Theory 23, 831 861.

[14]Campbell J.Y., 1994, Inspecting the Mechanism, Journal of Monetary Economics, 33, 463-506.

[15]Campbell J.Y., 1998, Entry, Exit, Embodied Technology, and Business Cycles, Review of Economic Dynamics, 1, 2, 371-408.

[16]Cogley T. and J.M. Nason, 1995, Output Dynamics in Real Business Cycle Models, American Economic Review 85, 492-511.

[17]Christiano L. M. Eichenbaum and R. Vigfusson, 2003, What Happens After a Technology Shock? manuscript, http://www.faculty.econ.nwu.edu/faculty/eichenbaum/research/.

[18]Evans C.L., 1992, Productivity Shocks and Real Business Cycles, Journal of Monetary Economics, 29, 2, 191-208.

[19]Fisher J., 2002, Technology Shocks Matter, Federal Reserve Bank of Chicago, Working Paper Series, 14.

[20]Francis N. and V. Ramey, 2003, Is the Technology-Driven Real Business Cycle Hypothesis Dead? Shocks and Aggregate Fluctuations Revisited, manuscript, http://econ.ucsd.edu/ vramey/.

[21]Gali J., 2003, On the Role of Technology Shocks as a Source of Business Cycles: Some New Evidence, Working Paper.

[22]Gali J., 1999, Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations, American Economic Review, 89, 249-271.

[23]Gali J., 1996, Technology, Employment, and the Business Cycle: Do Technology Shocks Explain Aggregate Fluctuations? Centre for Economic Policy Research, Discussion Paper 1499.

[24]Greenwood J., Z. Hercowitz and P. Krusell, 1997, Long-Run Implications of Investment-Specific Technological Change, American Economic Review, 87, 3, 342-62.

[25]Guo J.Y. and F. Sturzenegger, 1998, Crazy Explanations of International Business Cycles, International Economic Review, 39, 1, 111-33.

[26]Hall R.E., 1988, A Non-Competitive, Equilibrium Model of Fluctuations National Bureau of Economic Research Working Paper: 2576,

[27]Hansen G.D., 1985, Indivisible Labour and Supply and the Business Cycle, Journal of Monetary Economics 16, 309-327.

[28]Hartley P.R. and J.A. Whitt, 2003, Macroeconomic Fluctuations: Demand or Supply, Permanent or Temporary? European Economic Review, 47, 61-94.

[29]Hoover K.D. and D. Salyer, 1998, Technology Shocks or Coloured Noise? Why Real-Business-Cycle Models Cannot Explain Actual Business Cycles, Review of Political Economy, 10, 3, 299-327.

[30]Hodrick. R. J and E. Prescott, 1997, Postwar U.S. Business Cycles: An Empirical Investigation, Journal of Money, Credit, and Banking, 29, 1, 1-16.

[31]Hornstein A., 2000, The Business Cycle and Industry Comovement, Federal Reserve Bank of Richmond Economic Quarterly, 86, 1, 27-48.

[32]Hornstein A. and J. Praschnik, 1997, Intermediate Inputs and Sectoral Comovement in the Business Cycle, Journal of Monetary Economics, 40, 3, 573-95.

[33]Horvath M., 2000, Sectoral Shocks and Aggregate Fluctuations, Journal of Monetary Economics, 45, 1, 69-106.

[34]Huffman, G.W. and M. Wynne, 1999, The Role of Intratemporal Adjustment Costs in a Multisector Economy, Journal of Monetary Economics, 43, 2, 317-50.

[35]Jovanovic B., 1991, Coordination and Spillovers, New York University Economic Research Reports, 91-65

[36]Karras G., 1994, Sources of Business Cycles in Europe: 1960-1988. Evidence from France, Germany, and the United Kingdom, European Economic Review, 38, 9, 1763-78.

[37]Judd J. P., and B. Trehan, 1995, The Cyclical Behavior of Prices: Interpreting the Evidence, Journal of Money, Credit, and Banking, 27, 3, 789-97.

[38]King R.G., c.I. Plosser and S.T. Rebelo, 1988a, Production, Growth and Business Cycles: I. The Basic Neoclassical Model, Journal of Monetary Economics, 21, 195-232.

[39]King R.G., C.I. Plosser and S.T. Rebelo, 1988b, Production, Growth and Business cycles: II New directions, Journal of Monetary Economics 21, 309-342.

[40]King R.G. and S.T. Rebelo, 1999, Resuscitating Real Business Cycles, in Taylor J.B. and Woodford M. (eds.), Handbook of Macroeconomics Vol.1 (Elsevier Science, Amsterdam).

[41]Kydland F. E. and E. C. Prescott, 1982, Time to Build and Aggregate Fluctuations, Econometrica, 50, 6, 1345-70.

[42]Long J. B. Jr and C. Plosser Real Business Cycles, Journal of Political Economy, 91, 1, 39-69.

[43]Lucas R.E. Jr., 1977, Understanding Business Cycles, Journal of Monetary Economics, 5, 0, 7-29.

[44]Prescott E.C., 1986, Theory Ahead of Business Cycle Measurement Federal Reserve Bank of Minneapolis Staff Report 102.

[45]Prescott E.C. and R.Mehra, 1980, Recursive Competitive Equilibrium: the Case of Homogenous Household, Econometrica, 1365-79.

[46]Solow R., 1957, Technical Change and Aggregate Production Function, Review of Economics and Statistics, 39, 312-320.

[47]Stadler G.W., 1994, Real Business Cycle, Journal of Economics Litterature XXXII, 1750-1783.

[48]Stock, J. H., and M. Watson, 1998, Business Cycle Fluctuations in U.S. Macroeconomic Time Series, National Bureau of Economic Research Working Paper, 6528.

[49]Uhlig H., 1999, A Toolkit for Analysing Nonlinear Dynamic Stochastic Models Easily, in R. Marimon and A. Scott (eds) Computational Methods for the Study of Dynamic Economies, (Oxford University Press; Oxford).

[50]Xiao W., 2003, Indeterminacy, Demand Shocks, and International Business Cycle, manuscript.

[51]Wen, Y., 2003a, Fickle Consumer Versus Random Technology, manuscript.

[52]Wen, Y., 2003b, What Does It Take to Explain Procyclical Productivity? manuscript.

6 Appendix

6.1 Derivations

First Order Conditions Derivation The Planner solves the following dynamic problem:

$$\begin{aligned}\max_{c_{1,t},c_{2,t},n_{1,t},n_{2,t},n_t} \mathcal{H} &= \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C(\mathbf{c}_t)^{1-\psi}-1}{1-\psi} + B\frac{(1-n_t)^{1-\gamma}}{1-\gamma} + \right. \\ &\quad +\phi_{0,t}\left[n_{1,t}+n_{2,t}-n\right] + \\ &\quad +\phi_{1,t}\left[k_{1,t}^{\alpha_1} n_{1,t}^{1-\alpha_1} - c_{1,t} + (1-\Omega_1)\, k_{1,t} - k_{1,t+1}\right] + \\ &\quad \left. +\phi_{2,t}\left[k_{2,t}^{\alpha_2} n_{2,t}^{1-\alpha_2} - c_{2,t} + (1-\Omega_2)\, k_{2,t} - k_{2,t+1}\right]\right\},\end{aligned}$$

where $C(\mathbf{c}_t) = \left(\lambda_1 \left\{\tilde{s}_{1,t} c_{1,t}^{\frac{\sigma-1}{\sigma}}\right\} + \lambda_2 \left\{\tilde{s}_{2,t} c_{2,t}^{\frac{\sigma-1}{\sigma}}\right\}\right)^{\frac{\sigma}{\sigma-1}}$, $\mathbb{E}_0$ is the expectation operator, conditional on time 0 information, and $\phi_{i,t}$ are the dynamic multipliers.

Step 1. Derive the first order conditions (FOC hereafter) for $c_{1,t}$ and $c_{2,t}$:

$$\begin{aligned} c_{i,t} &: \quad 0 = C(\mathbf{c}_t)^{-\psi} \left(\lambda_1 \left\{\tilde{s}_{1,t} c_{1,t}^{\frac{\sigma-1}{\sigma}}\right\} + \lambda_2 \left\{\tilde{s}_{2,t} c_{2,t}^{\frac{\sigma-1}{\sigma}}\right\}\right)^{\frac{\sigma}{\sigma-1}-1} \lambda_i \tilde{s}_{i,t} (c_{i,t})^{-\frac{1}{\sigma}} - \phi_{i,t} \\ c_{i,t} &: \quad 0 = C(\mathbf{c}_t)^{-\psi} C(\mathbf{c}_t)^{-1} \lambda_i \tilde{s}_{i,t} (c_{i,t})^{-\frac{1}{\sigma}} - \phi_{i,t} \qquad (1)\end{aligned}$$

Step 2. Derive optimality conditions for n_t and for $n_{i,t}$ for $i = 1, 2$.

$$\begin{aligned} n_t &: \quad 0 = -B(1-n_t)^{-\gamma} - \phi_{0,t} \qquad (2) \\ n_{i,t} &: \quad 0 = \phi_{0,t} + \phi_{i,t} APN_{i,t}, \qquad (3)\end{aligned}$$

where $APN_{i,t} = (1-\alpha_i)(k_{i,t})^{\alpha_i}(n_{i,t})^{-\alpha_i} = (1-\alpha_i)\left(\frac{k_{i,t}}{n_{i,t}}\right)^{\alpha_i}$. Notice that FOC($n_{i,t}$) can be rewritten as $\phi_{0,t} = \phi_{i,t} APN_{i,t}$, since $\phi_{0,t} < 0$ from FOC(n_t).

Step 3. Finally, the optimal investment strategy is derived combining the FOCs for $k_{i,t+1}$ with equations (1). The euler equations of the model read:

$$\tilde{s}_{i,t} c_{i,t}^{-\frac{1}{\sigma}} = \mathbb{E}_t \beta \frac{C(\mathbf{c}_{t+1})^{-1-\psi}}{C(\mathbf{c}_t)^{-1-\psi}} \tilde{s}_{i,t+1} c_{i,t+1}^{-\frac{1}{\sigma}} \left\{\alpha_i \frac{k_{i,t+1}^{\alpha_i-1}}{n_{i,t+1}^{\alpha_i-1}} + 1 - \Omega_i\right\},\ i = 1, 2 \qquad (4)$$

where $\mathbb{E}_t$ denotes the expectations operator, conditional on information available at time t.

Equilibrium Characterization **Step 1**. Isolate $c_{i,t}$ from FOC($c_{i,t}$), $C(\mathbf{c}_t)^{-(1+\psi)} \lambda_i \tilde{s}_{i,t} (\phi_{i,t})^{-1} = c_{i,t}^{\frac{1}{\sigma}}$ and begin constructing the aggregate consumption index. Raising both sides of (**??**) to $(\sigma - 1)$ it reads:

$$C(\mathbf{c}_t)^{-(1+\psi)(\sigma-1)} \lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} (\phi_{i,t})^{-(\sigma-1)} = \lambda_i \tilde{s}_{i,t} c_{i,t}^{\frac{\sigma-1}{\sigma}}. \qquad (5)$$

Combining equations (5) and (3) the FOCs for consumption goods read:

$$C\left(\mathbf{c}_t\right)^{-(1+\psi)(\sigma-1)} \lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(\frac{\phi_{0,t}}{APN_{i,t}}\right)^{-(\sigma-1)} = \lambda_i \tilde{s}_{i,t} \left(c_{i,t}\right)^{\frac{\sigma-1}{\sigma}} \tag{6}$$

$$C\left(\mathbf{c}_t\right)^{-(1+\psi)(\sigma-1)} \lambda_j^{\sigma} \tilde{s}_{j}^{\sigma} \left(\frac{\phi_{0,t}}{APN_{j,t}}\right)^{-(\sigma-1)} = \lambda_j \tilde{s}_{j,t} \left(c_{j,t}\right)^{\frac{\sigma-1}{\sigma}} \tag{7}$$

Step 2. Define the marginal labor productivity index as follow:

$$\overline{APN}_t = \left(\lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(APN_{i,t}\right)^{(\sigma-1)} + \lambda_j^{\sigma} \tilde{s}_{j,t}^{\sigma} \left(APN_{j,t}\right)^{(\sigma-1)}\right)^{\frac{1}{\sigma-1}},$$

Step 3. Then add up equations (6) and (9), and factorize out $C\left(\mathbf{c}_t\right)^{-(1+\psi)(\sigma-1)} \left(\phi_{0,t}\right)^{-(\sigma-1)}$:

$$C\left(\mathbf{c}_t\right)^{-(1+\psi)(\sigma-1)} \left(\phi_{0,t}\right)^{-(\sigma-1)} \left(\lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(APN_{i,t}\right)^{(\sigma-1)} + \lambda_j^{\sigma} \tilde{s}_{j,t}^{\sigma} \left(APN_{j,t}\right)^{(\sigma-1)}\right) = \lambda_j \tilde{s}_{j,t} \left(c_{i,t}\right)^{\frac{\sigma-1}{\sigma}} + \lambda_i \tilde{s}_{i,t} \left(c_{j,t}\right)^{\frac{\sigma-1}{\sigma}}. \tag{8}$$

Next, construct the aggregate labor-productivity index, by raising both members of (8) to $\frac{1}{\sigma-1}$:

$$C\left(\mathbf{c}_t\right)^{-(1+\psi)} \left(\phi_{0,t}\right)^{-1} \left(\lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} APN_{i,t}^{(\sigma-1)} + \lambda_j^{\sigma} \tilde{s}_{j,t}^{\sigma} APN_{j,t}^{(\sigma-1)}\right)^{\frac{1}{\sigma-1}} = \left(\lambda_j \tilde{s}_{j,t} c_{i,t}^{\frac{\sigma-1}{\sigma}} + \lambda_i \tilde{s}_{i,t} c_{j,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}}.$$

Using the fact that $\left(\lambda_j \tilde{s}_{j,t} c_{i,t}^{\frac{\sigma-1}{\sigma}} + \lambda_i \tilde{s}_{i,t} c_{j,t}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} = C\left(\mathbf{c}_t\right)^{\frac{1}{\sigma}}$ the previous equation can be rewritten as: $C\left(\mathbf{c}_t\right)^{-(1+\psi)} \overline{APN}_t = \phi_{0,t} C\left(\mathbf{c}_t\right)^{\frac{1}{\sigma}}$.

Step 4. Solving it for $\phi_{0,t} = C\left(\mathbf{c}_t\right)^{-\frac{\sigma+\psi\sigma+1}{\sigma}} \overline{APN}_t$, and substituting $\phi_{0,t}$ into equation (6) we have:

$$C\left(\mathbf{c}_t\right)^{-(1+\psi)(\sigma-1)} \lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(\frac{C\left(\mathbf{c}_t\right)^{-\frac{\sigma+\psi\sigma+1}{\sigma}} \overline{APN}_t}{APN_{i,t}}\right)^{-(\sigma-1)} = \lambda_i \tilde{s}_{i,t} \left(c_{i,t}\right)^{\frac{\sigma-1}{\sigma}}$$

Raising both members to $\frac{1}{\sigma-1}$ and simplifying we have

$$c_{i,t} = \lambda_i^{\sigma} \tilde{s}_{i,t}^{\sigma} \left(\frac{APN_{i,t}}{\overline{APN}_t}\right)^{\sigma} C\left(\mathbf{c}_t\right) \text{ for } i = 1, 2 \tag{9}$$

6.2 Proofs

Proof of Theorem 1.

Lemma 1 (Firms) *There exist two types of firms: Type I firms produce the first commodity $c_{1,t}$, while Type II firms produce the second one $c_{2,t}$. The choice is without loss of generality. Firms face a sequence of static problems. Each firm maximizes its profits on a period by period basis, given* *market prices p_t. A Type i firms ($i = I, II$) maximized its profits $\pi_{i,t}$:*

$$\begin{aligned} \max \pi_{i,t} &\equiv p_t^{c^i} c_{i,t} + p_t^{i^i} i_{i,t} - p_t^{k^i} k_{i,t} - p_t^{n^i} n_{i,t} \\ s.to &: c_{i,t} + i_{i,t} = k_{i,t}^{\alpha_i} n_{i,t}^{1-\alpha_i}. \end{aligned}$$

Introduce multiplier $\mu_{i,t}$, and form the Lagrangean $\mathcal{L}_{i,t}$

$$\mathcal{L}_{i,t} = p_t^{c^i} c_{i,t} + p_t^{i^i} i_{i,t} - p_t^{k^i} k_{i,t} - p_t^{n^i} n_{i,t} + \mu_{i,t} \left(-c_{i,t} - i_{i,t} + k_{i,t}^{\alpha_i} n_{i,t}^{1-\alpha_i} \right).$$

After algebraic manipulations first order conditions can be written as:

$$\begin{aligned} p_t^{c^i} &= p_t^{i^i} && (10) \\ p_t^{k^i} &= p_t^{c^i} \alpha_i k_{i,t}^{\alpha_i - 1} n_{i,t}^{1-\alpha_i} \equiv p_t^{c^i} MPK_i && (11) \\ p_t^{n^i} &= p_t^{c^i} (1-\alpha_i) k_{i,t}^{\alpha_i} n_{i,t}^{-\alpha_i} \equiv p_t^{c^i} MPN_i, && (12) \end{aligned}$$

for $i = 1, 2$. Just notice that $p_t^{c^i} = p_t^{i^i} = \mu_{i,t} > 0$, since constraint holds with equality.

Lemma 2 (Consumers) *Suppose there exist a continuum of consumers, uniformly distributed over a unit interval, supplying labor to both sectors. Consumer $\gamma \in [0, 1]$ has preference over sequences of consumption flows and labor, and maximizes expected utility as summarized by the lifetime utility function (over consumption flow $(c_{1,t}, c_{2,t})$ and leisure (ℓ_t)) $U_0^\gamma = E_0 \sum_{t=0}^{\infty} \beta^t \{u(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}) + \upsilon(\ell_t)\}$ where E_t is the mathematical expectations operator conditional on information available at time t, $\upsilon(\ell_t)$ is a well behaved (continuous, twice continuously differentiable) function of ℓ_t, representing the disutility of working, and β is a subjective discount factor. Consumer γ solves the following dynamic optimization problem:*

$$\begin{aligned} &\max_{c_{1,t}, n_{1,t}, c_{2,t}, n_{2,t}} \{u(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}) + \upsilon(\ell_t)\}, \\ s.to \ &: p_t^{c^i}(c_{i,t} + i_{i,t}) = p_t^{k^i} k_{i,t} + p_t^{n^i} n_{i,t}\ ,\ i = 1, 2 \\ &: k_{i,t+1} = (1 - \Omega_1) k_{i,t} + i_{i,t},\ i = 1, 2 \\ &: \ell_t = 1 - n_t; \\ &: n_t = n_{1,t} - n_{2,t}, \\ &: \tilde{s}_{i,t+1} = \varrho \tilde{s}_{i,t} + \epsilon_{i,t},\ i = 1, 2 \\ &: k_0^i > 0,\ i = 1, 2 \end{aligned}$$

where 1 denotes total hours available. Introduce multiplier $\theta_{i,t}$, and form the Hamiltonian $\mathcal{H}$

$$\begin{aligned} \max_{c_{i,t}, n_{i,t}, k_{i,t+1}, n_t} \mathcal{H}_0 = \ & \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \{u(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}) + v(1 - n_t) + \\ & + \theta_{0,t}(n_{1,t} + n_{2,t} - n_t) + \\ & + \theta_{1,t} \left(p_t^{k^1} k_{1,t} + p_t^{n^1} n_{1,t} - p_t^{c^1} c_{1,t} - p_t^{c^1} k_{1,t+1} + p_t^{c^1} (1 - \Omega_1) k_{1,t} \right) + \\ & + \theta_{2,t} \left(p_t^{k^2} k_{2,t} + p_t^{n^2} n_{2,t} - p_t^{c^2} c_{2,t} - p_t^{c^2} k_{2,t+1} + p_t^{c^2} (1 - \Omega_2) k_{2,t} \right) \Big\} \end{aligned}$$

First order conditions are, for $i = 1, 2$

$$\begin{aligned} c_{i,t} &: \; u_i\left(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}\right) = \theta_{i,t} p_t^{c^i} && (13) \\ n_t &: \; -v_1\left(1 - n_t\right) = \theta_{0,t} && (14) \\ n_{i,t} &: \; \theta_{0,t} = \theta_{i,t} p_t^{n^i} && (15) \\ k_{i,t+1} &: \; \theta_{i,t} p_t^{c^i} = \beta \mathbb{E}_t \theta_{i,t+1}\left(p_{t+1}^{k^i} + \left(1 - \Omega_i\right) p_{t+1}^{c^i}\right), && (16) \\ TVC &: \; \lim_{t \to \infty} \beta^t \mathbb{E}_t \theta_{i,t} k_{i,t} = 0 \end{aligned}$$

Lemma 3 (Price Vector) *Combining (14) and (15) we have:*

$$\theta_{1,t} p_t^{n^1} = \theta_{2,t} p_t^{n^2}$$

suggesting that, when there are no labor adjustment costs, the marginal productivity of labor is equated between sectors. Substituting then (13) into the previous equation, it reads:

$$\frac{u_1\left(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}\right)}{p_t^{c^1}} p_t^{n^1} = \frac{u_2\left(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}\right)}{p_t^{c^2}} p_t^{n^2}$$

From this equation we have the relative price between consumption commodities, denoted with $\hat{p}_t$*:*

$$\hat{p}_t = \frac{p_t^{c^2}}{p_t^{c^1}} = \frac{u_2\left(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}\right)}{u_1\left(c_{1,t}, c_{2,t}; \tilde{s}_{1,t}, \tilde{s}_{2,t}\right)} \frac{p_t^{n^2}}{p_t^{n^1}}$$

which can be rewritten as

$$\hat{p}_t = \frac{p_t^{c^2}}{p_t^{c^1}} = \frac{MU_{2,t}}{MU_{1,t}} \cdot \frac{MPN_{2,t}}{MPN_{1,t}}$$

where $MU_{i,t}$ *is the marginal utility from consuming the i-th commodity, and* $MPN_{i,t}$ *denotes the marginal productivity of labor services in the i-th sector. The following equality follows from the first order conditions of the two types of firms.*

Let the first commodity be the numeraire of the system, and denote with $\hat{p}_t = \left(1, \hat{p}_t, \hat{p}_t^{k^1}, \hat{p}_t^{k^2}, \hat{p}_t^{n^1}, \hat{p}_t^{n^2}\right)$ *the relative price vector. In particular, the firms' optimality conditions implies:*

$$\begin{aligned} \hat{p}_t &= \frac{MU_{2,t}}{MU_{1,t}} \cdot \frac{MPN_{2,t}}{MPN_{1,t}} && (17) \\ \hat{p}_t^{k^1} &= \alpha_1\left(k_{1,t}\right)^{\alpha_1 - 1}\left(n_{1,t}\right)^{1-\alpha_1} && (18) \\ \hat{p}_t^{k^2} &= \hat{p}_t \alpha_2\left(k_{2,t}\right)^{\alpha_2 - 1}\left(n_{2,t}\right)^{1-\alpha_2} && (19) \\ \hat{p}_t^{n^1} &= \left(1 - \alpha_1\right)\left(k_{1,t}\right)^{\alpha_1}\left(n_{1,t}\right)^{-\alpha_1} && (20) \\ \hat{p}_t^{n^2} &= \hat{p}_t(1 - \alpha_2)\left(k_{2,t}\right)^{\alpha_2}\left(n_{2,t}\right)^{-\alpha_2}, && (21) \end{aligned}$$

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