DEPARTMENT OF ECONOMICS

Working Paper

Automatic Stabilizers in an Economy with Multiple Shocks

Morten Spange

Working Paper No. 2004-03

ISSN 1396-2426

UNIVERSITY OF AARHUS • DENMARK

INSTITUT FOR ØKONOMI

AFDELING FOR NATIONALØKONOMI - AARHUS UNIVERSITET - BYGNING 322
8000 AARHUS C - ☎ 89 42 11 33 - TELEFAX 86 13 63 34

WORKING PAPER

Automatic Stabilizers in an Economy with Multiple Shocks

Morten Spange

Working Paper No. 2004-03

DEPARTMENT OF ECONOMICS

SCHOOL OF ECONOMICS AND MANAGEMENT - UNIVERSITY OF AARHUS - BUILDING 322
8000 AARHUS C - DENMARK ☎ +45 89 42 11 33 - TELEFAX +45 86 13 63 34

Automatic Stabilizers in an Economy with Multiple Shocks*

Morten Spange†

Department of Economics, University of Aarhus

May 4, 2004

Abstract

A stochastic general equilibrium model is set up in order to analyse whether automatic stabilizers are a good tool in terms of mitigating risk. It is found that the potential benefits to be derived from automatic stabilizers depend on various factors including the degree of real wage rigidity and the size of the public sector. In countries with large public sectors automatic stabilizers in the sense of procyclical tax revenues can provide significant welfare improvements compared with a passive fiscal policy. For countries with smaller public sectors the predictions of the model are less favorable.

JEL Classification: E32, E62

Keywords: Automatic stabilizers, Optimal fiscal policy, Real wage rigidity

*Comments and suggestions by Torben M. Andersen, Henrik Jensen and participants at the DGPE workshop in Korsør are gratefully acknowledged.

†Mailing address: Department of Economics, Building 322, University of Aarhus, DK-8000 Aarhus C, Denmark. Phone +45 8942 1582; fax: +45 8613 6334; e-mail: mspange@econ.au.dk.

1 Introduction

In recent years inflation targeting has become a predominant strategy for monetary policy. This potentially leaves a greater role for fiscal stabilization policies, especially through the workings of the so-called automatic stabilizers.[1] An important criticism against automatic stabilizers is that they only allow fiscal policy to respond to aggregate economic activity. This is not critical if there is a one–to-one correspondence between output and the variables to be stabilized. If, however, business cycles are generated by a variety of underlying shocks this is not the case, and the appropriate response to output fluctuations depends critically on the kind of shock by which they are driven.[2]

To analyse the potential role for automatic stabilizers we will set up a stochastic general equilibrium model with optimizing agents and wage rigidities. This is a class of models that has been used extensively to study optimal monetary policy, see e.g. Obstfeld and Rogoff (2000) or Sutherland (2000) for open economy models or Woodford (2003) chapter 6 for a closed economy model. Quite recently the framework has also been adopted for the study of optimal fiscal policy, see e.g. Andersen and Spange (2002), Beetsma and Jensen (2002) and Lombardo and Sutherland (2003). Previous papers dealing with fiscal policy has to a large extent taken government spending to be exogenous. Obstfeld and Rogoff (1995), Betts and Devereux (2000), Corsetti and Pesenti (2001) and Ganelli (2003) all study the effects of exogenous government spending shocks in non-stochastic general equilibrium models. Another branch of the literature that considers exogenous fiscal shocks has been occupied with the cyclical implications of a given process for fiscal policy, see e.g. Baxter and King (1993)

[1]In the ECB Monthly Bulletin November 2001, p. 58, it is stated that *By keeping national budgets close to balance or in surplus over the medium term, national governments should be in a position to smoothen the economc effects of cyclical fluctuations through the operation of automatic stabilizers.*

[2]See European Commision (2001), part III for evidence on different types of shocks.

or Linnemann and Schabert (2003) for a model with price stickiness. Papers on optimal fiscal policy have primarily considered the optimal tax profile to finance exogenously given government spending, relevant references here being Lucas and Stokey (1983) and Aiyagari et al. (2002).

Our model will contain both a demand and a supply shock. This allows us to address the above-mentioned criticism concerning the inability of automatic stabilizers to react to the particular type of shock that drives the fluctuations in output. In fact our setup allows us to compare the welfare properties of a fiscal policy based on automatic stabilizers with the theoretically optimal fiscal policy. As stabilization policies are often called for in order to overcome inefficiencies associated with sluggish adjustment of wages and prices, it is assumed in the model that a fraction of the workers have rigid real wages. The fact that it is the real wage and not the nominal wage that is rigid distinguishes our model from the majority of papers in the literature, but especially in a European context our assumption is the appropriate one.[3]

We start out by considering the optimal fiscal policy under the assumption that fiscal policy is contingent on the underlying disturbances. Under this assumption we find that in order to allocate risk efficiently, public spending should respond procyclically to supply shocks. However, in case of demand shocks the results are ambiguous. If the public sector constitutes only a small part of the economy, it is optimal to let public spending be negatively correlated with output. The reason is that in our model a demand shock strengthens the preferences for private consumption. Hence, by contracting public consumption the policy maker can allow the agents to enjoy more private consumption and thereby increase welfare. However, by affecting labour supply, demand shocks also affect output which is important as public spending is assumed to be financed through a proportional tax. A demand shock implies that output is above expectations.

[3]See eg. Berthold et al. (1999) for a quantification of real wage rigidities in Europe and a discussion of the likely consequences thereof.

Hence, assuming that public sector activities are positively related to the size of the demand shock, tax revenues will tend to be high when activity is high, which stabilizes the tax rate. This is especially important when the public sector is large. In fact, if the public sector is sufficiently large, a procyclical fiscal policy may be preferable also under demand shocks due to the tax smoothing argument.

The assumption that the policy maker is able to make policy contingent on the underlying shocks may not be entirely realistic. Hence, we will also consider policies under the constraint that the policy maker cannot observe the individual shocks, but instead bases the policy reaction on aggregate activity. Since automatic stabilizers are in fact characterized by such a mechanical reaction of fiscal policy to variations in activity, we will denote this information constrained policy the policy of automatic stabilizers. The performance of the automatic stabilizers relative to the theoretically optimal policy depends on whether the optimal response to supply and demand shocks are reasonably similar. Since the optimal response to supply and demand shocks are most similar in economies with a large public sector, these economies experience the best performance of automatic stabilizers compared with the optimal shock contingent fiscal policy.

The paper is structured as follows: Section 2 outlines the model. Section 3 develops the optimal policies and provides numerical illustrations to evaluate the quantitative importance of automatic stabilizers in terms of welfare. Section 4 concludes.

2 The model

Consider a stochastic closed economy model with firms, households and a public sector.

Firms

The representative firm produces subject to a decreasing returns to scale production function with labour as the only input

$$Y = Z\frac{1}{\beta}L^{\beta} \quad , \quad \beta < 1 \tag{1}$$

where Y is production, L is input of a labour aggregate, and Z is a measure of productivity which is assumed to be log normally distributed $N(0, \sigma_z^2)$. Firms take the real wage as given, and hence labour demand is given by

$$L = \left(\frac{Z}{W}\right)^{\frac{1}{1-\beta}} \tag{2}$$

where W is the real wage per unit of labour. Firms are owned by households, and profits are assumed to be distributed uniformly across all agents in the economy. To examine how the presence of real rigidities affects the optimal fiscal policy we follow Sutherland (2000) in assuming that labour is defined as an aggregate over two subtypes, type r for which real wages are rigid, and type f for which wages are flexible. The labour aggregate is defined as

$$L = \frac{(L^r)^{\psi}\left(L^f\right)^{1-\psi}}{\psi^{\psi}(1-\psi)^{1-\psi}} \quad , \quad 0 \leq \psi \leq 1$$

Minimizing the cost of obtaining one unit of this aggregate leads to the overall wage index

$$W = (W^r)^{\psi}\left(W^f\right)^{1-\psi}$$

implying that demand for the two labour types is given as

$$\begin{aligned} L^r &= \psi\left(\frac{W^r}{W}\right)^{-1} L \\ L^f &= (1-\psi)\left(\frac{W^f}{W}\right)^{-1} L \end{aligned}$$

The assumption of a unit elasticity of substitution between labour types implies that the fractions of total labour income going to each of the two labour types are constant across states. Since the share of total wages going to the category of labour with rigid wages will be equal to ψ, this parameter is interpreted as the degree of real wage rigidity. To allow for market power in the labour market without having labour suppliers possessing power over the whole economy, each of the two labour types is defined as a composite labour input specified over different subtypes of labour, i.e.

$$L_h = \left[\int_0^1 L_h\left(j\right)^{\frac{\phi-1}{\phi}} dj \right]^{\frac{\phi}{\phi-1}}, \quad , h = r, f$$

Demand for labour of a given type i is given as

$$L_h\left(i\right) = \left(\frac{W\left(i\right)_h}{W_h} \right)^{-\phi} L_h, \quad , h = r, f$$

and the costs of acquiring one unit of the composite labour input can be written

$$W_h = \left[\int_0^1 W(i)_h^{1-\phi} di \right]^{\frac{1}{1-\phi}}, \quad , h = r, f$$

Households

There are two types of households in the economy. A fraction ψ of the households are of type r, each supplying a particular variety of labour type r, whereas the rest of the households are of type f, each supplying a particular variety of type f labour. Since the fraction of workers of a given type equals the fraction of wages paid to each type of labour, all agents have the same income. The households have a utility function depending positively on private and public consumption and negatively on labour supply. Specifically, we assume that utility of household i is given by

$$U\left(i\right) = V \log C\left(i\right) - \lambda L\left(i\right) + \xi \log G \quad (3)$$

where C is private consumption, G is public consumption, L is the amount of labour supplied by the households and V is an exogenous demand shock which is assumed to be log normally distributed $N(0, \sigma_v^2)$. Assuming that V and Z are jointly log normally distributed, all endogenous variables in the model will be log-normally distributed. Demand shocks are assumed to be uncorrelated with supply shock. The households maximize utility subject to a budget constraint

$$C(i) = (1-t)(W(i)L(i) + \Pi) \quad (4)$$

where W is the real wage, Π is real profits, and t is the tax rate. We assume a right to manage structure in which each agent sets the real wage for his particular labour services, and the firms subsequently decide how much of each type of labour to employ at the prevailing wages. The type f households set their wage for the current period after having observed the shocks, whereas the type r households set the wage without knowledge of the shocks. Monopoly power in the labour market implies that the real marginal consumption value of the wage exceeds the marginal disutility labour. Hence, for sufficiently small shocks the agents are willing to supply the amount of labour demanded by the firms.[4] In equilibrium all agents within each of the two groups will set the same wage. This leads to the following first order conditions for wages.

$$W^r = \frac{\lambda\phi}{\phi-1}\frac{E(L^r)}{E(VC^{-1}(1-t)L^r)} \quad (5)$$

$$W^f = \frac{\lambda\phi}{\phi-1}V^{-1}C(1-t)^{-1} \quad (6)$$

The important distinction is that whereas type f wages are a function of the actual values of the endogenous variables, type r wages are set based on expectations.

Public sector

The public sector provides public goods and services to households which are produced by use of the private good as input. For simplicity the production function is assumed linear (output=input=G). Public activities are financed by a proportional income tax, and the government is assumed to run a balanced

[4] It is assumed that this constraint is never violated.

budget.[5] The public budget constraint is given as

$$G = tY$$

As an expression for the steady state level of public consumption, define

$$\mu \equiv \frac{E(G)}{E(Y)} \tag{7}$$

The objective of the government is to maximize consumer welfare. When the shocks equal their expected values, the optimal size of the public sector is[6]

$$\mu = \frac{\xi}{E(V) + \xi}, \qquad \frac{\partial \mu}{\partial \xi} > 0 \tag{8}$$

Obviously, μ is increasing in the weight that the agents attach to public consumption. Since the economy is closed we must impose a resource constraint stating that the sum of private and public consumption equals production, i.e.

$$Y = C + G \tag{9}$$

Economic policy

By considering stabilization as running through automatic stabilizers it is reasonable to assume that fiscal policy responds without delay to aggregate economic activity, and especially that fiscal policy lags are shorter than the contract length for wages. For a generic variable X, define $x \equiv \log\left(\frac{X}{E(X)}\right)$. The fiscal policy rule reflecting automatic stabilizers can now be written

$$r = \kappa_p y \tag{10}$$

where R is tax revenues, and κ_p is the degree of state contingency in public spending or the size of the automatic stabilizer. This relatively simple functional

[5] This assumption is made to allow for an analytical solution of the model.

[6] This expression is derived setting $V = EV = \left(1 + \frac{1}{2}\sigma_v^2\right)$ in the utility function and imposing the resource constraint.

form allows us to compare our κ_p values with the actual sizes of automatic stabilizers in the OECD countries reported by van den Noord (2000). Moreover, if there are stabilization gains using this simple rule, there will also be gains if we allow for more advanced rules. Budget balance implies that

$$g = \kappa_p y \tag{11}$$

For (10) and (11) to be satisfied for any given value of κ_p, the tax rate t is allowed to adjust endogenously. Empirical evidence suggests that fiscal stabilization mainly runs through the revenue side with public expenditures being roughly constant over the cycle, see e.g. van den Noord (2000).[7] By forcing government spending to equal tax revenues we preclude smoothing of public consumption, thereby biasing the gains from an active stabilization policy downwards.

To evaluate automatic stabilizers relative to the optimal "shock-contingent" policy, we consider as a reference point the case in which fiscal policy is contingent on the individual shocks, i.e. we consider the policy rule

$$r = \kappa_z y\left(z\right) + \kappa_v y\left(v\right) \tag{12}$$

$y\left(z\right)$ $\left(y\left(v\right)\right)$ is the log deviation from the output level that would have prevailed under the current realization of the supply (demand) shock, but with the demand (supply) shock set equal to its steady state level.[8] Again budget balance implies $r = g$. Under the policy rule given by (12), private and public consumption can be written as[9]

$$c = \phi_{cz}\left(\kappa_z\right) z + \phi_{cv}\left(\kappa_v\right) v \quad ; \quad g = \phi_{gz}\left(\kappa_z\right) z + \phi_{gv}\left(\kappa_v\right) v \tag{13}$$

[7] See Lane (2003) for an examination of the cyclical properties of a variety of fiscal policy variables.

[8] Since log output is linear in the logs of the shocks, it would make no difference if log revenues were instead written directly as a linear funcion of the logs of the shocks. However, we prefer the current formulation to facilitate comparison with the rule under automatic stabilizers.

[9] These expressions are derived using a log linearized version of the resource constraint. See Appendix B for derivation.

Since the coefficients in (13) are contingent on the parameters of the policy rules, the fiscal authorities are able to affect the variances of private and public consumption. If the shocks are not individually observable, the condition that $\kappa_z = \kappa_v = \kappa_p$ must be imposed.

Welfare

As a measure of welfare we use expected utility of the consumers. Hence, the objective of the government is to maximize

$$EU = E\left[V \log C + \xi \log G - \lambda L\right]$$

We will work with a second order Taylor approximation of EU around the steady state. In appendix B we show that for a given value of μ, maximizing the approximation is equivalent to maximizing the following criterion function

$$\Omega = \phi_{cv}\sigma_v^2 - \frac{1}{2}E\left(V\right) \cdot Var\left(c\right) - \frac{1}{2}\xi Var\left(g\right) \tag{14}$$

This utility measure holds for both types of agents. The optimal shock contingent policy rule is found as the solution to the first order conditions

$$\frac{\partial \Omega}{\partial \kappa_z} = 0 \quad ; \quad \frac{\partial \Omega}{\partial \kappa_v} = 0$$

If stabilization runs through automatic stabilizers, the coefficient of the optimal rule is given as the solution to

$$\frac{\partial \Omega}{\partial \kappa_p} = 0$$

We show in appendix B that the optimal κ_p is a convex combination of κ_z and κ_v, i.e.

$$\kappa_p = \varphi\kappa_z + \left(1 - \varphi\right)\kappa_v, \ \ \varphi \in [0; 1]$$

3 Optimal policies

The government's objective is to choose the κ-values in the policy rule (10) or (12) that maximizes expected utility of the representative agent (14). As a reference point, consider first the optimal parametervalues in the shock contingent reaction function.[10]

$$\kappa_z = \frac{\mu E(V)}{\mu^2 E(V) + \xi(\mu - 1)^2} > 0 \tag{15}$$

$$\kappa_v = \mu \frac{\beta(\psi - 1) E(V) + (1 - \beta\psi)(1 - \mu)}{\beta(\psi - 1)\left(\mu^2 E(V) + \xi(\mu - 1)^2\right)} \lesseqqgtr 0 \tag{16}$$

Assuming that μ is determined by (8) we obtain

$$\kappa_z = 1, \qquad \kappa_v = \frac{(E(V) + \xi)\beta(\psi - 1) + 1 - \beta\psi}{\beta(\psi - 1)(E(V) + \xi)} \lesseqqgtr 0$$

The agents are risk averse with respect to both private and public consumption. In order to stabilize private consumption, the optimal fiscal policy under supply shocks is procyclical. However, as an active fiscal policy induces variability in public consumption, the policy maker is faced with a trade off between private and public sector variability, implying that private consumption is not completely stabilized under the optimal policy. Assuming that the size of the public sector corresponds to its relative weight in the utility function, the optimal policy response to a supply shock ensures that private and public consumption increase by equal proportions. For a given public sector size, stronger preferences for public consumption call for a less aggressive fiscal policy by increasing the welfare loss associated with public sector volatility.

In the case of demand shocks, matters are more complicated as both the level of output as well as the marginal rates of substitution between private consumption, public consumption and leisure are affected. It turns out that the optimal policy response depends qualitatively on the degree of real wage rigidity. The unexpectedly high marginal value of consumption caused by a

[10] Note that we are assuming precommitment to policy rules in the sense that the policy rules are formulated before the marginal utility of consumption (V) is known. However, one can easily show that due to certainty equivalence implied by the linear-quadratic approximation to expected utility, the optimal policy rules under commitment are also optimal ex post after the demand shock is realized. Hence, time inconsisteny is not an issue here.

demand shock implies that it will be optimal for the government to let private consumption increase. If the degree of real rigidity in the economy is high, the expansion in activity caused by the shock will not be large enough to match the desired increase in private consumption, which calls for a contraction of public consumption.

In economies with more real flexibility there is less need to contract public consumption since flex wage agents subject to a demand shock will supply more labour and thus consume more. If the degree of real flexibility is sufficiently high, public consumption should actually increase in periods where preferences for private consumption are particularly strong. The reason is that the increase in labour supply associated with a demand shock expands output. This increase in output lowers the tax rate needed to finance a given level of government spending. Hence, by running a procyclical fiscal policy in response to demand shocks, the government can smooth tax distortions across states.[11] However, because of the decreasing returns to labour in production, more production of private goods makes it more labour demanding to supply a given level of public consumption, which calls for a countercyclical fiscal policy. Which effect dominates depends on the parameters of the model. The relation between the degree of real rigidity and the optimal response to demand shocks can be summarized as follows

$$\lim_{\psi \longrightarrow 1} \kappa_v = -\infty, \quad \lim_{\psi \longrightarrow 0} \kappa_v \lesseqgtr 0, \quad \frac{\partial \kappa_v}{\partial \psi} < 0$$

In addition to the degree of real rigidity, an important determinant for the optimal fiscal response to a demand shock is the relative size of the public sector, μ. We find that

$$\frac{\partial \kappa_v}{\partial \mu} > 0 \qquad \left.\frac{\partial \kappa_v}{\partial \xi}\right|_{\mu = \frac{\xi}{E(V)+\xi}} > 0$$

With large steady state tax revenues, whether they are determined exogenously or endogenously by (8), demand shocks will induce relatively large fluctuations

[11] Lucas and Stokey (1983) analyse how to distribute tax distortions optimally over time.

in the tax rate if tax revenues are held constant across states. Hence, a large public sector is more likely to be associated with a positive κ_v, that is, a policy that increases tax revenues when activity is high. However, since a demand shock makes private consumption relatively more valuable to the agents compared with public consumption, private consumption should increase most. This explains why $\kappa_v < 1$ for all parametervalues, or equivalently, why the tax rate should be countercyclical under demand shocks even though total tax revenues may be procyclical. As with supply shocks, with μ being exogenous, a larger weight on public consumption in the utility function pushes the absolute value of the optimal state contingency parameter towards zero.

Numerical illustrations

To assess the quantitative gains from stabilization we make some numerical illustrations. It is assumed that half of the wages are preset, i.e. $\psi = 0.5$, and to obtain a reasonable capital-labour ratio we set $\beta = \frac{2}{3}$. We choose $\sigma_z^2 = 0.000104$ implying that with 95% probability, productivity will be within a band of $\pm 2\%$ from its expected value. We then set $\sigma_v^2 = 0.00093$ implying that for $\frac{E(G)}{E(Y)} = \frac{1}{3}$ and with κ_p chosen optimally, half of the variation in consumption will be generated by supply and demand shocks respectively. Since the optimal stabilization policy depends on the size of the public sector, the model is calibrated for three different values of ξ, assuming that μ is given by (8). Note that G includes all revenues raised by the public sector, including those passed back to the private sector as transfers. We start out by considering the optimal policy rules under automatic stabilizers and the shock contingent rule, respectively.

Table 1: Optimal policy parameters

	$\frac{E(G)}{E(Y)} = \frac{1}{4}$	$\frac{E(G)}{E(Y)} = \frac{1}{3}$	$\frac{E(G)}{E(Y)} = \frac{1}{2}$
κ_p	0.26	0.34	0.50
κ_z	1.00	1.00	1.00
κ_v	-0.49	-0.32	0.0079

The general pattern is that a larger public sector calls for increased cyclical sensitivity. This is entirely due to the change in the optimal response to demand shocks as with μ determined by (8), $\kappa_z = 1$.

Next we compare the gains from automatic stabilizers with the welfare gains from the optimal shock contingent fiscal policy. The stabilization gains are expressed as the percentage increase in steady state consumption that would bring about an equivalent welfare improvement compared with the case of no stabilization ($\kappa = 0$).

Table 2: Gains from automatic and optimal stabilization

Measure	$\frac{E(G)}{E(Y)} = \frac{1}{4}$	$\frac{E(G)}{E(Y)} = \frac{1}{3}$	$\frac{E(G)}{E(Y)} = \frac{1}{2}$
(i) Optimal policy gain	0.0065	0.0098	0.024
(ii) Automatic stabilizer gain	0.00068	0.0020	0.012
(iii) Ratio (ii)/(i)	0,105	0,207	0,507

In all cases the gains from stabilization seem small.[12] However, the relative performance of automatic stabilizers is better the larger the public sector is. The reason is that with a large public sector it is optimal to let tax revenues move procyclically in response to both supply and demand shocks. This is important since automatic stabilizers are not able to discriminate between the shocks. With a small public sector, supply and demand shocks call for opposite policy

[12]This is a standard result for models with this kind of preferences, see e.g. Lucas 1987. Storesletten et al. (2001) show that the welfare costs of business cycles may increase substantially when heterogeneity is allowed for.

reactions. Hence, when the policy maker cannot make his reaction contingent on the underlying disturbances, the scope for welfare improvements is limited.

4 Conclusions

This paper has analysed under which conditions automatic stabilizers can be used successfully to mitigate risk caused by a mixture of underlying shocks. It has been shown that even with different underlying sources of volatility, the agents benefit from the workings of automatic stabilizers. However, the performance of automatic stabilizers relative to the optimal shock contingent policy depends critically on the exact structure of the economy. The crucial factor is whether fluctuations caused by different kinds of shocks call for reasonably similar policy responses. We find that supply shocks always call for procyclical tax revenues, whereas the result is ambiguous in the case of demand shocks.

Although our model suggests that a policy based on automatic stabilizers may be very inferior to the theoretically optimal policy, this does not necessarily mean that policy makers should aim at conducting a discretionary policy conditioning the policy reaction on the underlying shocks. In the model time lags are absent. However, in actual policy making it is reasonable to assume that both the gathering of information on the shocks as well as the subsequent implementation of the shock contingent policy take a considerable amount of time. Hence, automatic stabilizers may be seen as a valuable second best option when the first best is not feasible due to information and implementation lags.

Our quantitative results suggest that the gains from stabilization are small in any case. This may, however, reflect the fact that the model does not capture all the benefits of stabilization. Comparing the welfare gains from a policy relying on automatic stabilizers with the gains from an optimal shock contingent policy, we find that for reasonable parameter values automatic stabilizers are able to contribute importantly to the stabilization policy in an economy with a large public sector. However, if the public sector is small, automatic stabilizers are very inferior to the theoretically optimal fiscal policy.

In future research it would be interesting to construct an intertemporal model by introducing a capital market and allowing the government to issue debt. Another interesting extension would be to analyse a two country model to focus on international aspects of stabilization policies.

Appendix A: Utility function

In this appendix the approximation to the utility function used to evaluate expected utility is derived. We start by showing that EY, EC, and EG are unaffected by stabilization policy. Letting a lower case letter denote the log-deviation from the mean value of the corresponding upper case letter, the model can be written

$$\begin{aligned}
w^r &= 0 \\
w^f &= -v + y \\
y &= z + \beta l \\
l &= \psi l^r + (1-\psi) l^f \\
l^r &= -w^r + w + l \\
l^f &= -w^f + w + l \\
l &= \frac{1}{1-\beta}(z - w) \\
w &= \psi w^r + (1-\psi) w^f
\end{aligned}$$

Use this system of equations to solve for l^r and y

$$l^r = \frac{1}{1-\beta\psi} z + \frac{\beta(1-\psi)}{1-\beta\psi} v \quad (17)$$

$$y = \frac{1}{1-\beta\psi} z + \frac{\beta(1-\psi)}{1-\beta\psi} v \quad (18)$$

It follows that the choice of policy parameters does not affect the way shocks affect employment of type r workers and output. Hence, all variance and co-variance terms involving either y or l^r are unaffected by policy. Now, using the properties of the log normal distribution, (5) can be written as

$$\log W^r = E\log W^f + RP$$

where

$$RP = -\frac{1}{2}\sigma_v^2 - \frac{1}{2}\sigma_y^2 + \sigma_{vy} - \sigma_{vl^r} + \sigma_{yl^r}$$

In the literature, RP is often referred to as a risk premium demanded by workers with rigid wages to hedge against uncertainty, see e.g. Obstfeld and Rogoff (2000). This is the channel through which risk potentially affects the expectations of the endogenous variables. However, since σ_y^2, σ_{vl^r}, and σ_{yl^r} are unaffected by policy, it follows that RP is unaffected by policy. Hence EY, EC, and EG are unaffected by policy.

We are now ready to work out the welfare measure used in the text, which is based on the following expression for expected utility.

$$EU = E\left[V\log C + \xi\log G - \lambda L\right]$$

First we substitute in for labour. From (5) and (6) we have that

$$\lambda E\left(L^r\right) = \frac{\phi-1}{\phi}E\left(VC^{-1}\left(1-t\right)L^r W^r\right)$$

$$\lambda E\left(L^f\right) = \frac{\phi-1}{\phi}E\left(VC^{-1}\left(1-t\right)L^f W^f\right)$$

But from the following relationships

$$1-t = \frac{C}{Y}$$

$$L^r = \psi\left(\frac{W^r}{W}\right)^{-1}L$$

$$L^f = \left(1-\psi\right)\left(\frac{W^f}{W}\right)^{-1}L$$

$$\beta Y = WL$$

the following can be derived

$$\begin{aligned} E\left(L^r\right) &= \frac{\beta\left(\phi-1\right)\psi}{\lambda\phi}EV \\ E\left(L^f\right) &= \frac{\beta\left(\phi-1\right)\left(1-\psi\right)}{\lambda\phi}EV \end{aligned}$$

implying that

$$\lambda E\left(L\right) = \lambda E\left(L^r + L^f\right) = \frac{\beta\left(\phi-1\right)}{\phi}EV$$

Use this to rewrite expected utility as

$$EU = E\left(V \ln C\right) + \xi E\left(\log G\right) - \frac{\beta\left(\phi-1\right)}{\phi}EV$$

We now approximate the term

$$E\left(V \log C\right) = E\left(\exp\left(v\right)c\right) \tag{19}$$

In appendix B it is shown that

$$\log C = \phi_{cz}z + \phi_{cv}v + \mu_c$$

Substitute this into (19) and get

$$\begin{aligned} & E\left(V \ln C\right) \\ = \; & \phi_{cz}E\left(\exp\left(v\right)z\right) + \phi_{cv}E\left(\exp\left(v\right)v\right) + \mu_c E\left(\exp\left(v\right)\right) \\ = \; & \phi_{cv}E\left(\exp\left(v\right)v\right) + \mu_c E\left(\exp\left(v\right)\right) \end{aligned}$$

where the last equality follows from the independence between the supply and demand shocks. Now consider the expression

$$g\left(v\right) = \exp\left(v\right)v$$

Take a second order Taylor approximation around μ_v and obtain

$$g\left(v\right) \approx \exp\left(\mu_v\right)\mu_v + \left(1+\mu_v\right)\exp\left(\mu_v\right)\left(v-\mu_v\right) + \frac{\left(2+\mu_v\right)\exp\left(\mu_v\right)}{2}\left(v-\mu_v\right)^2$$

Take expectations and remember that

$$\begin{aligned} \mu_v &= 0 \\ \sigma_v^2 &= E\left[\left(v-\mu_v\right)^2\right] \end{aligned}$$

to end up with

$$E[g(v)] = \sigma_v^2$$

By a similar approximation we can show that

$$E(\exp(v)) = 1 + \frac{1}{2}\sigma_v^2$$

Substituting into the expression for expected utility we find that

$$EU = \phi_{cv}\sigma_v^2 + \mu_c\left(\frac{1}{2}\sigma_v^2 + 1\right) + \xi\mu_g + \frac{\beta(\phi - 1)}{\phi}\left(1 + \frac{1}{2}\sigma_v^2\right)$$

Now use that

$$\log E(C) = \mu_c + \frac{1}{2}\sigma_c^2 \quad ; \quad \log E(G) = \mu_g + \frac{1}{2}\sigma_g^2$$

to rewrite expected utility

$$EU = \phi_{cv}\sigma_v^2 + \left(\frac{1}{2}\sigma_v^2 + 1\right)\left(\log EC - \frac{1}{2}Var(c)\right) + \xi\left(\log EG - \frac{1}{2}Var(g)\right) - \frac{\lambda(\phi - 1)}{\phi}\left(1 + \frac{1}{2}\sigma_v^2\right)$$

Since EC and EG are independent of policy, expected utility is maximized with respect to policy when the following criterion function is maximized

$$\begin{aligned}\Omega &= \phi_{cv}\sigma_v^2 - \frac{1}{2}\left(\frac{1}{2}\sigma_v^2 + 1\right)Var(c) - \frac{1}{2}\xi Var(g) \\ &= \phi_{cv}\sigma_v^2 - \frac{1}{2}E(V)\cdot Var(c) - \frac{1}{2}\xi Var(g)\end{aligned}$$

This is the utility measure that we apply in the analysis.

Appendix B: Derivation of optimal policies

Loglinearize (9)

$$y = (1 - \mu)c + \mu g \tag{20}$$

Now apply (20) together with (18) and (12) to derive

$$c = \phi_{cz}(\kappa_z)z + \phi_{cv}(\kappa_v)v \quad ; \quad g = \phi_{gz}(\kappa_z)z + \phi_{gv}(\kappa_v)v$$

where

$$\begin{array}{cc} \phi_{cz}\left(\kappa_z\right) \equiv \frac{1-\kappa_z\mu}{(1-\mu)(1-\beta\psi)} & \phi_{gz}\left(\kappa_z\right) \equiv \frac{\kappa_z}{1-\beta\psi} \\ \phi_{cv}\left(\kappa_v\right) \equiv \frac{(1-\kappa_v\mu)\beta(1-\psi)}{(1-\mu)(1-\beta\psi)} & \phi_{gv}\left(\kappa_v\right) \equiv \frac{\kappa_v\beta(1-\psi)}{1-\beta\psi} \end{array} \tag{21}$$

Hence

$$Var\left(c\right) = \phi_{cz}\left(\kappa_z\right)^2\sigma_z^2 + \phi_{cv}\left(\kappa_v\right)^2\sigma_v^2 \tag{22}$$

$$Var\left(g\right) = \phi_{gz}\left(\kappa_z\right)^2\sigma_z^2 + \phi_{gv}\left(\kappa_v\right)^2\sigma_v^2 \tag{23}$$

So the objective of the government is to maximize (14) subject to (22) and (23). The first order conditions with respect to κ_z and κ_v, respectively, are

$$\left(\frac{1}{2}\sigma_v^2+1\right)\phi_{cz}\frac{\partial\phi_{cz}}{\partial\kappa_z} + \xi\phi_{gz}\frac{\partial\phi_{gz}}{\partial\kappa_z} = 0 \tag{24}$$

$$\left[\left(\frac{1}{2}\sigma_v^2+1\right)\phi_{cv}-1\right]\frac{\partial\phi_{cv}}{\partial\kappa_v} + \xi\phi_{gv}\frac{\partial\phi_{gv}}{\partial\kappa_v} = 0 \tag{25}$$

If we are considering automatic stabilizers we have to solve the minimization problem imposing the condition that $\kappa_z = \kappa_v$. Hence, the solution becomes

$$\left\{\left(\frac{1}{2}\sigma_v^2+1\right)\phi_{cz}\frac{\partial\phi_{cz}}{\partial\kappa_p} + \xi\phi_{gz}\frac{\partial\phi_{gz}}{\partial\kappa_p}\right\} + \frac{\sigma_v^2}{\sigma_z^2}\left\{\left[\left(\frac{1}{2}\sigma_v^2+1\right)\phi_{cv}-1\right]\frac{\partial\phi_{cv}}{\partial\kappa_p} + \xi\phi_{gv}\frac{\partial\phi_{gv}}{\partial\kappa_p}\right\} = 0$$

implying that κ_p will be a weighted average of κ_z and κ_v, i.e.

$$\kappa_p = \varphi\kappa_z + (1-\varphi)\kappa_v,\ \ \varphi \in [0;1]$$

where φ depends on the relative magnitudes of the supply and demand variances. Inserting (21) in (24) and (25) and using that $E\left(V\right) = \left(\frac{1}{2}\sigma_v^2+1\right)$, the optimal parameters are found to be

$$\kappa_z = \frac{\mu E\left(V\right)}{\mu^2 E\left(V\right) + \xi\left(\mu-1\right)^2}$$

$$\kappa_v = \mu\frac{\beta\left(\psi-1\right)E\left(V\right) + \left(1-\beta\psi\right)\left(1-\mu\right)}{\beta\left(\psi-1\right)\left(\mu^2 E\left(V\right) + \xi\left(\mu-1\right)^2\right)}$$

References

Aiyagari, S. R., A. Marcet, T. J. Sargent and J. Seppälä, 2002, Optimal Taxation without State-Contingent Debt, Journal of Political Economy, 110 (6), 1220-1254

Andersen, T. M. and M. Spange, 2002, International Interdependencies in Fiscal Stabilization Policies, CEPR DP 3580

Baxter, M. and R. G. King, 1993, Fiscal Policy in General Equilibrium, The American Economic Review, 83 (3), 315-334

Beetsma, R. M. W. J. and H. Jensen, 2002, Monetary and Fiscal Policy in a Micro-Founded Model of a Monetary Union, European Central Bank, WP no. 166

Berthold, N., R. Fehn and E. Thode, 1999, Real Wage Rigidities, Fiscal Policy, and the Stability of EMU in the Transition Phase, IMF WP/99/83

Betts, C. and M. B. Devereux, 2000, Exchange Rate Dynamics in a Model of Pricing to Market, Journal of International Economics, 50, 215-244

Corsetti, G. and P. Pesenti, 2001, Welfare and Macroeconomic Interdependence, Quarterly Journal of Economics, 116, 421-466

European Central Bank, 2001, The Economic Policy Framework in the EMU, Monthly Bulletin November

European Commission, 2001, Public Fianances in EMU, No 3

Ganelli, G., 2003, Useful Government Spending, Direct Crowding Out and Fiscal Policy Interdependence", Journal of International Money and Finance, 22 (1), 87-103.

Lane, P. R., 2003, The Cyclical Behaviour of Fiscal Policy: Evidence from the OECD, Journal of Public Economics, 87, 2661-2675

Linnemann, L, and A. Schabert, 2003, Fiscal Policy in the New Neoclassical Synthesis, Journal of Money, Credit and Banking, 35 (6), 911-929

Lombardo, G., and A. Sutherland, 2003, Monetary and Fiscal Interactions in Open Economies, Journal of Macroeconomics, forthcomming

Lucas, R. E., 1987, Models of Business Cycles, Basil Blackwell, Oxford

Lucas, R. E. and N. L. Stokey, 1983, Optimal Fiscal and Monetary Policy in an Economy Without Capital, Journal of Monetary Economics, 12, 55-93

Obstfeld, M. and K. Rogoff, 1995, Exchange Rate Dynamics Redux, Journal of Political Economy, 103, 624-660

Obstfeld, M. and K. Rogoff, 2000, New Directions for Stochastic Open Economy Models, Journal of International Economics, 50, 117-153

Storesletten, K., C. I. Telmer and A. Yaron, 2001, The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variations in Idiosyncratic Risk, European Economic Review, 45 (7), 1311-1339

Sutherland, A., 2000, Inflation Targeting in a Small Open Economy, mimeo, University of St Andrews

van den Noord, P., 2000, The size and role of automatic stabilizers in the 1990's and beyond, OECD Working Paper 230

Woodford, M., 2003, Interest and Prices, Princeton University Press

Working Paper

2003-08: Manfred J. Holler and Peter Skott: The Importance of setting the agenda.

2003-09: Niels Haldrup: Empirical analysis of price data in the delineation of the relevant geographical market in competition analysis.

2003-10: Niels Haldrup and Morten Ø. Nielsen: Estimation of Fractional Integration in the Presence of Data Noise.

2003-11: Michael Svarer, Michael Rosholm and Jacob Roland Munch: Rent Control and Unemployment Duration.

2003-12: Morten Spange: International Spill-over Effects of Labour Market Rigidities.

2003-13: Kræn Blume Jensen, Mette Ejrnæs, Helena Skyt Nielsen and Allan Würtz: Self-Employment among Immigrants: A Last Resort?

2003-14: Tue Görgens, Martin Paldam and Allan Würtz: How does Public Regulation affect Growth?

2003-15: Jakob Roland Munch, Michael Rosholm and Michael Svarer: Are Home Owners Really More Unemployed?

2003-16: Gabriel Pons Rotger: Testing for Seasonal Unit Roots with Temporally Aggregated Time Series.

2003-17: Peter Sandholt Jensen and Martin Paldam: Can new aid-growth models be replicated?

2003-17: Sheetal K. Chand and Martin Paldam: The economics of immigration into a welfare state.

2004-1: Gabriel Pons Rotger: Seasonal Unit Root Testing Based on the Temporal Aggregation of Seasonal Cycles.

2004-2: Niels Haldrup and Morten Ø. Nielsen: a Regime Switching Long Memory model for Electricity Prices.

2004-3: Morten Spange: Automatic Stabilizers in an Economy with Multiple Shocks.