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TESTING FOR SEASONAL UNIT ROOTS WITH TEMPORALLY AGGREGATED TIME SERIES

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Abstract

The temporal aggregation effect on seasonal unit roots and its implications for seasonal unit root testing are discussed. The aggregation effect allows to test with any HEGY-type method for integration at the harmonic frequencies through the Nyquist frequency of properly temporally aggregated series.

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1 Introduction

Temporal aggregation and seasonality are related issues since seasonal variation can only be described with data measured at a particular sampling interval. Available data may contain seasonal cycles with different periods, and in this situation the time unit at which some particular seasonal variation is generated may be different with respect to the time unit the data is used to test for seasonal properties. For example, aggregated consumption is only available at a quarterly basis, while some seasonal variation can be generated inside the quarter. Commonly, the aggregation problem occurs when the practitioner uses data measured at a wider sampling interval than it is generated. However, when we focus on seasonality, we show how an aggregation problem, the presence of complex unit roots, can be present when the practitioner test for integration at a seasonal cycle with data measured at a narrower interval than the cycle is generated.

The purpose of the paper is to study the effect of temporal aggregation on flexible seasonally integrated (SI) processes and discuss the implications for seasonal unit root testing. We focus on monthly and quarterly SI processes, but the theoretical results can be extended to other sampling frequencies. The aggregation theory allows us to propose a simple procedure for HEGY-type testing for unit roots at the harmonic frequencies that avoids complex unit roots.

The outline of the paper is as follows. In section 2 the effect of temporal aggregation on unit root processes is discussed with special attention to SI processes. In section 3 we describe an alternative approach for testing seasonal unit roots at the harmonic frequencies that can be applied to HEGY type tests (Hyllebert et al., 1990). In section 4 a Monte Carlo simulation compares the finite sample properties of the alternative approach for the standard HEGY test. Finally section 5 concludes the paper. The proofs are presented in the appendix.

A word on notation. x_t denotes the disaggregated time series. $X_T \equiv x_{mT}$ denotes a systematically sampled (SS) series where m is a finite positive integer that denotes the order of temporal aggregation. $x_t^a \equiv S_m(L)x_t$ denotes an overlapping summed series where $S_m(L) = 1 + L + \cdots + L^{m-1}$ is the summation filter. $X_T^a \equiv S_m(L)x_{mT} = x_{mT}^a$ denotes an average sampled (AS) series. Systematic sampling is typically applied when y_t is a stock variable, while average sampling is applied to flow and stock variables as well.

2 Temporal Aggregation of Seasonal Unit Root Processes

In this section we discuss the temporal aggregation effect on a flexible SI process. We focus on two particular cases, the monthly and the quarterly flexible SI processes.[1] The empirical evidence has found in many situations that a process with some but not all the seasonal unit roots describes real macroeconomic time series better than the seasonal random walk (Hylleberg et al., 1993). However, the literature on temporal aggregation of seasonal processes has focused on the multiplicative seasonal ARIMA model, that includes the seasonal random walk as a particular case (see Wei, 1978). Thus, there is no study on the effects of temporal aggregation on flexible SI processes, and we try to fill this gap here.[2]

We first discuss the temporal aggregation of AR models that contain single unit roots at different frequencies, and then specialize the aggregated model to the particular cases of monthly and quarterly SI processes. To do so, it is assumed that a univariate time series x_t measured at a sampling interval of length Δt is generated by

$$\phi_p(L)\varphi_d(L)x_t = \varepsilon_t, \tag{1}$$

where the d simple[3] reciprocals of the roots of $\varphi_d(z)$ lie on the unit circle ($z_1 = e^{i\theta_1}$,..., $z_d = e^{i\theta_d}$), the p reciprocals of the roots of $\phi_p(z)$ lie inside the unit circle ($z_{d+1}, ..., z_{d+p}$, with $|z_i| < 1$) and ε_t is a white noise process. The stochastic process (1) is denoted an AR**I**(p, d).[4]

Let us allow the presence of hidden periodicity in model (1). Generally, SI processes are models with hidden periodicity at different frequencies.[5] For

[1] Other situations of temporal aggregation like day-to-week, or week-to-quarter aggregation can be analyzed with our approach.

[2] Franses and Boswijk (1996) analyzed the aggregation effect on the periodic integrated model.

[3] We do not study double or fractionally seasonal integrated processes.

[4] Note that a θ-frequency unit root generates persistent cycles of $2\pi/\theta$ t-periods. The relevant cycles for economic time series are seasonal cycles, $\theta_k = 2k\pi/S$, k =1,...,[$S/2$], business cycles $\theta \leq \pi/S$, and the long-run component $\theta_0 = 0$. Since the coefficients of the lag polynomial $\varphi(L)$ are real, all the complex unit roots ($\theta_i \neq 0, \pi$) come in complex conjugate pairs.

[5] A model is said to be without hidden periodicity of order m if $z_j^m = z_l^m$ iff $z_j = z_l$ $\forall j, l$ (see Stram and Wei, 1986).

example, a quarterly seasonal random walk $\Delta_4 x_t = \varepsilon_t$, has hidden periodicity of order 2 at two frequencies because the roots ± 1 and $\pm i$ are equal when they are raised power 2 ($1^2 = (-1)^2$, and $i^2 = (-i)^2$) and has hidden periodicity of order 4 at one frequency because all the unit roots are equal after being raised power 4 ($1^4 = i^4 = (-1)^4 = (-i)^4$). To include hidden periodicity in the analysis we denote by n the number of subsets of unit roots linked to a particular cycle $m\theta_j$ (j=1, ..., n), h_n is the number of unit roots in any of these subsets, and $h = \Sigma_1^n h_n$ is the total number of hidden unit roots. Similarly, we use the notation n^a, h_{n^a}, and h^a for the summed process with $n^a \leq n$ and $h^a \leq h$. The next lemma presents the representation of the SS process $X_T = x_{mT}$ and the AS process $X_T^a = S_m(L)x_{mT} = x_{mT}^a$ for finite temporal aggregation order m.

Lemma 1 *Let x_t be an ARI(p, d) model with simple reciprocal roots inside or on the unit circle, hidden periodicity of order m for h unit roots and $d - d^a$ common roots with $S_m(L)$:*
*i) The SS process X_T follows the AR***I***MA(p^*, d^*, q^*) :*

$$\Phi_{p^*}(L^m)\Psi_{d^*}(L^m)X_T = \Theta_{q^*}(L^m)E_T,$$

where $\Phi_{p^*}(L^m) = \Pi_1^{p^*}(1 - z_j^m L^m)$, $\Psi_{d^*}(L^m) = \Pi_1^{d^*}(1 - e^{im\theta_j} L^m)$ with $p^* = p$, $d^* = d - h$ and $q^* = [p + d - h - (p + d)/m]$.
*ii) The AS process X_T^a follows the AR***I***MA(p^{**}, d^{**}, q^{**}) :*

$$\Phi_{p^{**}}^a(L^m)\Psi_{d^{**}}^a(L^m)X_T^a = \Theta_{q^{**}}^a(L^m)E_T^a,$$

where $\Phi_{p^{**}}^a(L^m) = \Pi_1^{p^{**}}(1 - z_j^m L^m)$, $\Psi_{d^{**}}^a(L^m) = \Pi_1^{d^{**}}(1 - e^{im\theta_j} L^m)$ with $p^{**} = p$, $d^{**} = d^a - h^a$, and $q^{**} = [p + 1 + d^a - h^a - (p + 1 + d^a)/m]$.

Proof. See in the appendix. ∎

The dynamic properties of aggregated models are determined by the reciprocal roots of the ARI lag polynomial. Thus, a neater description of the aggregation effect is obtained from the aggregated reciprocal roots. From lemma 1 the reciprocal roots of the aggregated models z_j^m are equal to the reciprocal roots of the disaggregated model, z_j, up to m, the length of the sampling interval.

From DeMoivre's theorem z_j^m can be decomposed as follows:

$$z^m = |z|^m e^{im\theta}. \tag{2}$$

Therefore, the effect of temporal aggregation on the reciprocal roots can be decomposed into a effect on the modulus ($|z|^m$) and a effect on the frequency ($e^{im\theta}$). As m increases, the modulus of the roots inside the unit circle decreases, while the modulus of the unit roots is invariant. For large m, the modulus of the stationary roots will be very close to zero and the unit root properties will dominate the dynamics of the process. This will be reflected in a simplification of the stationary part of the model, with smaller orders than those predicted by the theoretical aggregated models p^*, p^{**}, q^*, and q^{**}.

At the same time temporal aggregation changes the frequencies of most of the roots. Concretely, the cycle of the aggregated root does not change if the associated disaggregated root corresponds to a frequency not bigger than the Nyquist frequency ($\theta \leq \pi/m$), while the cycle of the aggregated root changes when the disaggregated root is placed at a frequency bigger that the Nyquist frequency ($\theta > \pi/m$) such that the aggregated cycle has a longer period than the disaggregated cycle. In other words, a cycle smaller than $2m$ t-periods does not really disappear with temporal aggregation but appears at the aggregated process with a lower frequency. This is called the aliasing effect in the spectral analysis literature (see Koopmans, 1974). The aliasing can lead the practitioner to find spurious seasonal integration (see Granger and Siklos, 1995). The frequency effect simplifies the aggregated model through the AR component and for large m reduces the order of the MA polynomial in comparison with the smallest m. The aggregation effect on the reciprocal roots suggests than in practice the temporally aggregated models may be simpler than $AR\mathbf{I}MA(p^*, d^*, q^*)$ (or $AR\mathbf{I}MA(p^{**}, d^{**}, q^{**})$) models.

Let us discuss the aggregation effect on a particular case of (1), a monthly flexible $\mathbf{SI}(d_0, ..., d_6)$ process

$$(1-L)^{d_0}\Pi_{k=1}^{5}\left[(1-e^{i\theta_k}L)(1-e^{-i\theta_k}L)\right]^{d_k}(1+L)^{d_6}x_t = \varepsilon_t$$

with $\theta_k = k\pi/6$, $d_k = 0, 1$ for all k, and ε_t is a stationary and invertible ARMA process.

Proposition 2 *Let x_t be a monthly* $\mathbf{SI}(d_0, ..., d_6)$ *process.*
i) The bimonthly processes are $X_T \tilde{} \mathbf{SI}(d_0^*, d_1^*, d_2^*, d_3)$

$$(1 - L^2)^{d_0^*}(1 - L^2 + L^4)^{d_1^*}(1 + L^2 + L^4)^{d_2^*}(1 + L^2)^{d_3} X_T = E_T,$$

and $X_T^a \tilde{} \mathbf{SI}(d_0, d_1^*, d_2, d_3)$

$$(1 - L^2)^{d_0}(1 - L^2 + L^4)^{d_1^*}(1 + L^2 + L^4)^{d_2^*}(1 + L^2)^{d_3} X_T^a = E_T^a,$$

where $L^2 X_T = X_{T-1}$, $L^2 X_T^a = X_{T-1}^a$, $d_0^* = \max\{d_0, d_6\}$, $d_1^* = \max\{d_1, d_5\}$, *and* $d_2^* = \max\{d_2, d_4\}$.
ii) The quarterly processes are $X_T \tilde{} \mathbf{SI}(d_0^*, d_1^*, d_2^*)$

$$(1 - L^3)^{d_0^*}(1 + L^6)^{d_1^*}(1 + L^3)^{d_2^*} X_T = E_T,$$

and $X_T^a \tilde{} \mathbf{SI}(d_0, d_1^*, d_2^*)$

$$(1 - L^3)^{d_0}(1 + L^6)^{d_1^*}(1 + L^3)^{d_2^*} X_T^a = E_T^a,$$

where $L^3 X_T = X_{T-1}$, $L^3 X_T^a = X_{T-1}^a$, $d_0^* = \max\{d_0, d_4\}$, $d_1^* = \max\{d_1, d_3, d_5\}$, *and* $d_2^* = \max\{d_2, d_6\}$.
iii) The semi-annual processes are $X_T \tilde{} \mathbf{SI}(d_0^*, d_1^*)$

$$(1 - L^6)^{d_0^*}(1 + L^6)^{d_1^*} X_T = E_T,$$

and $X_T^a \tilde{} \mathbf{SI}(d_0, d_1^*)$

$$(1 - L^6)^{d_0}(1 + L^6)^{d_1^*} X_T^a = E_T^a,$$

where $L^6 X_T = X_{T-1}$, $L^6 X_T^a = X_{T-1}^a$, $d_0^* = \max\{d_0, d_2, d_4, d_6\}$, *and* $d_1^* = \max\{d_1, d_3, d_5\}$.

Proof. See in the appendix.[6] ■

As seen in proposition 2, the orders of integration of the seasonal frequencies in the aggregated model $d_1^*, ...d_{S/(2m)}^*$ $(m = 2, 3, 6)$ depend on various orders of seasonal integration of the disaggregated model, and this dependency is the same for both schemes of aggregation.[7,8] Specifically, the order

[6] See the expression of the error terms E_T and E_T^a in the appendix.

[7] The Monte Carlo results of Granger and Siklos (1995) are in contradiction to this common effect. The poor performance of the seasonal unit root tests can explain this situation.

[8] The only exception is the π-frequency unit root at the systematically sampled bimonthly process that depends only on the monthly $\pi/2$-frequency unit for both types of aggregation.

of integration at a particular 'aggregated' frequency depends on the order of integration of the disaggregated component that generates the same cycle and on the orders of integration of the $m-1$ disaggregated components affected by the aliasing. All orders of seasonal integration are at most one because of the hidden periodicity effect. For example, the quarterly $\pi/2$-frequency unit root is linked to the monthly unit roots $\pi/6$, $\pi/2$, and $5\pi/6$. The monthly $\pi/6$ unit root generates the same cycle as the quarterly $\pi/2$ unit root, while the aliasing effect turns the monthly unit roots $\pi/2$ and $5\pi/6$ into a quarterly $\pi/2$ unit root. Then, the quarterly model is affected by the seasonal aliasing problem when the monthly model does not contain the $\pi/6$-frequency unit root but contains a unit root at $\pi/2$ or $5\pi/6$. The seasonal aliasing problem is common for both aggregation schemes because the summation filter does not cancel the roots affected by seasonal aliasing.

The differential effect for the aggregation schemes takes place at the zero-frequency d_0^*. In the case of systematic sampling the zero-frequency component is linked to the monthly zero-frequency and other monthly seasonal unit roots, while for average sampling the aggregated zero-frequency is only linked to the disaggregated zero-frequency d_0, because the summation filter $S_m(L)$ eliminates all unit roots affected by the zero-frequency aliasing. For example, the monthly $2\pi/3$ unit root turns into a zero-frequency unit root after systematic sampling to a quarterly process, and into a covariance stationary component when the process is quarterly averaged.

The aliasing effect obscures the seasonal unit root findings and allows extending the conclusions when a seasonal unit root is rejected with aggregated data. For example, the non-rejection of a quarterly $\pi/2$-frequency unit root does not guarantee that the monthly process has a unit root at the $\pi/6$ frequency. However, the rejection of the quarterly $\pi/2$ unit root implies the rejection of the monthly unit roots $\pi/6$, $\pi/2$ and $5\pi/6$.

Let us consider the temporal aggregation of a quarterly flexible $\mathbf{SI}(d_0, d_1, d_2)$ process $(1-L)^{d_0}(1+L^2)^{d_1}(1+L)^{d_2}x_t = \varepsilon_t$, where ε_t is defined as in the monthly case.

Corollary 3 *Let x_t be a quarterly $\mathbf{SI}(d_0, d_1, d_2)$ process.*
The semi-annual processes are $X_T\tilde{}\,\mathbf{SI}(d_0^*, d_1)$

$$(1-L^2)^{d_0^*}(1+L^2)^{d_1}X_T = E_T,$$

and $X_T^a\tilde{}\,\mathbf{SI}(d_0, d_1)$

$$(1-L^2)^{d_0}(1+L^2)^{d_1}X_T^a = E_T^a,$$

where $L^2 X_T = X_{T-1}$, $L^2 X_T^a = X_{T-1}^a$, *and* $d_0^* = \max\{d_0, d_2\}$.

Note from this corollary that the aggregation of quarterly SI processes is not affected by seasonal aliasing because the semi-annual π-unit root is only linked to the quarterly $\pi/2$-unit root. However, as in the monthly case, zero-frequency aliasing can occur with the systematic sampling of quarterly series. For example, zero-frequency integration of the SS annual process depends on d_0, d_1, d_2. Obviously, zero-frequency aliasing does not occur with average sampling, and therefore the average sampling from a quarterly series is not affected by the aliasing problem.

Table 1 presents the relationship between the seasonal frequencies of monthly, bimonthly, quarterly, semi-annual and yearly models. The single line separates the non aliased frequencies from the aliased frequencies below. As discussed in the preceding paragraphs, seasonal aliasing can occur when monthly or bimonthly series are temporally aggregated. For example, if a monthly series does not contain the $\pi/6$ and $\pi/3$ unit roots and contains the $5\pi/6$ and π unit roots, then the quarterly series will show the same behavior as if the monthly process contained the unit roots at frequencies $\pi/6$ and $\pi/3$.

Table 1: Monthly Seasonal Cycles and Sampling Interval

Month		Bimonth		Quarter		Half-year		Year	
ω	P^1	2ω	P^2	3ω	P^3	6ω	P^6	12ω	P^{12}
0	∞	0	∞	0	∞	0	∞	0	∞
$\pi/6$	12	$\pi/3$	12	$\pi/2$	12	π	12	2π	∞
$\pi/3$	6	$2\pi/3$	6	π	6	2π	∞	4π	∞
$\pi/2$	4	π	4	$3\pi/2$	12	3π	12	6π	∞
$2\pi/3$	3	$4\pi/3$	6	2π	∞	4π	∞	8π	∞
$5\pi/6$	2.4	$5\pi/3$	12	$5\pi/2$	12	5π	12	10π	∞
π	2	2π	∞	3π	6	6π	∞	12π	∞
$7\pi/6$	2.4	$7\pi/3$	12	$7\pi/2$	12	7π	12	14π	∞
$4\pi/3$	3	$8\pi/3$	6	4π	6	8π	∞	16π	∞
$3\pi/2$	4	3π	4	$9\pi/2$	12	9π	12	18π	∞
$5\pi/3$	6	$10\pi/3$	6	5π	∞	10π	∞	20π	∞
$11\pi/6$	12	$11\pi/3$	12	$11\pi/2$	12	11π	12	22π	∞

P^1: Monthly period in months; P^2: Bimonthly period in months; P^3: Quarterly period in months; P^6: Semi-annual period in months; P^{12}: Annual period in months

To summarize, when seasonal unit root tests are applied to systematically sampled series, it is possible to obtain spurious integration at all the frequencies, while if these tests are applied to average sampled series it is possible to obtain a spurious seasonal integration but not spurious zero-frequency integration. The temporal aggregation of monthly time series is potentially affected by both types of aliasing while the aggregation of quarterly series is only affected by zero-frequency aliasing.

3 Testing for all Seasonal Unit Roots With a Standard Unit Root Test

The common practice when testing for seasonal unit roots is testing for integration at the zero-frequency, at the $[S/2]-1$ harmonic frequencies, and at the Nyquist frequency $\pi/\Delta t$ with data measured at one sampling interval. Standard unit root tests apply in the case of the zero and the Nyquist frequency, which are real unit roots, while for the harmonic frequencies, a joint test is necessary because the presence of a pair of conjugate complex roots (Hylleberg et al., 1990).

Recently, Burridge and Taylor (2001a) have shown how the complex unit root test displays a shift in the limiting distribution in the presence of certain types of periodic heteroscedasticity, a common property of some economic time series,[9] while the real unit root tests are unaffected. To solve this problem, Burridge and Taylor (2001a) propose an involved procedure to size-correct the test. A simpler solution is proposed in the following lines. The aggregation theory developed in the preceding section implies that any complex unit root test at a particular sampling interval has its real unit root test counterpart with proper temporally aggregated data. The unit roots at frequencies π/m correspond to the Nyquist frequency with sampled data of order m, so it is possible to interpret these unit roots in terms of cycles with a length defined at the sampling interval m. As seen in table 1, a monthly $\pi-$frequency unit root is a unit root with a period of two months, a monthly $\pi/2$ unit root behaves as a unit root with a period of two bimonths, a monthly $\pi/3$ unit root behaves as a unit root with a period of two quarters, and a monthly $\pi/6$ unit root behaves as a unit root with a period of two half-years. The monthly unit roots $2\pi/3$ and $5\pi/6$ are more difficult to interpret in terms

[9]Many references are given in Burridge and Taylor's paper.

of the sampling interval since they generate periods of 2.4 and 3 months respectively. In a similar way a quarterly π-frequency unit root is a unit root with a period of two quarters, and a quarterly $\pi/2$-unit root is a unit root with a period of two half-years.

On the basis of the relationship between the seasonal unit roots at different sampling intervals, if one tests for a seasonal complex unit root at the longest sampling interval the seasonal cycle is observable without aliasing the test can be performed at the Nyquist frequency. Otherwise, if the seasonal cycle is tested with data measured at some narrower sampling interval then the seasonal root is allocated at an harmonic frequency as a pair of complex conjugate unit roots. In the next subsections we describe how to apply this approach to test for quarterly unit roots and we discuss the problems to extend the procedure to the monthly case.

3.1 Testing for Quarterly Seasonal Unit Roots

The standard quarterly HEGY approach without deterministic terms or transient dynamics is based on the auxiliar regression

$$x_t - x_{t-4} = \pi_0 x_{0,t-1} + \pi_1^\alpha x_{1,t-1}^\alpha + \pi_1^\beta x_{1,t-1}^\beta + \pi_2 x_{2,t-1} + \varepsilon_t, \qquad (3)$$

where $x_{0,t} \equiv x_t + x_{t-1} + x_{t-2} + x_{t-3}$, $x_{1,t}^\alpha \equiv -x_{t-1} + x_{t-3}$, $x_{1,t}^\beta \equiv x_t - x_{t-2}$, and $x_{2,t} \equiv x_t - x_{t-1} + x_{t-2} - x_{t-3}$ are asymptotically uncorrelated (see Hylleberg et al., 1990). The HEGY procedure tests for the roots 1 and -1 with a t-test of $\pi_j = 0$ against $\pi_j < 0$ $(j = 0, 2)$, and tests for the complex conjugate pair $\pm i$ with a F-test for $\pi_1^\alpha = \pi_1^\beta = 0$.[10] All these statistics have non standard distributions related to the Dickey-Fuller distribution.[11]

From the orthogonality of the HEGY regressors, there is no efficiency gain by testing for particular unit roots using all the HEGY regression or only by

[10] This can be done alternatively by a sequential $t-$test first for π_1^β=0 against $\pi_1^\beta \neq 0$ and then, if the null is accepted, for $\pi_1^\alpha = 0$ against $\pi_1^\alpha < 0$. However, the joint test is preferred because its better properties, specially in the presence of periodic heteroscedasticy (see Burridge and Taylor, 2001a), and higher order correlation (see Burridge and Taylor, 2001b).

[11] Another hypothesis of interest are, the null of all seasonal unit roots against some stationary seasonal roots and the null of all unit roots against some stationary roots. These hypothesis can be tested with F-tests (see Ghysels et al., 1994). Finally, the null of all unit roots against all stationary roots, can be tested using the individual tests (see Ghysels and Osborn, 2001).

using the regressors of interest. This property can be used together with our aggregation results to propose an alternative way to test for seasonal complex unit roots through the Nyquist frequency of semi-annual data. Concretely, the quarterly complex unit roots $\pm i$ can be tested with a t-test for $\pi_1 = 0$ vs $\pi_1 < 0$ at the semi-annual regression

$$\Delta_4 X_T = \pi_1 X_{1,T-1} + E_T, \tag{4}$$

where $X_T = x_{2T}$, $X_{1,T} = X_T - X_{T-1}$, $L^2 X_T = X_{T-1}$ and $\pi_1 = \pi_1^{\alpha}$. From (3), $E_T = \pi_1^{\beta} x_{2T-1} + \varepsilon_{2T}$ is serially uncorrelated and asymptotically uncorrelated with $X_{1,T-1}$.[12,13] This procedure can be applied to HEGY-type tests (see Hylleberg et al., 1990; Breitung and Franses, 1998; and Smith and Taylor, 1998, 1999).

The preceding approach has assumed that ε_t is i.i.d. However, in practice it is necessary to deal with the transient dynamics, because the misspecification of the transient dynamics will affect seriously the behavior of the HEGY test. There are different approaches to deal with transient dynamics. The standard approach is to specify a finite order AR component to the short-run dynamics and test for seasonal unit roots in the augmented regression (see Hylleberg et al., 1990; Smith and Taylor, 1998, 1999):

$$\phi(L)\Delta_4 x_t = \pi_0 x_{0,t-1} + \pi_1^{\alpha} x_{1,t-1}^{\alpha} + \pi_1^{\beta} x_{1,t-1}^{\beta} + \pi_2 x_{2,t-1} + \varepsilon_t.$$

This procedure is not suited for our approach since there is an important loss of valuable information for the inference on the semi-annual short-run dynamics due to systematic sampling (see Wei, 1989). A second approach is to treat non-parametrically the short-run dynamics (see Breitung and Franses, 1998). However, the estimation of the long-run variance is affected by the same problem than the standard HEGY approach since the long-run variance is essentially a short-run parameter. A third approach is proposed by Psaradakis (1997) consisting on prewhitening the time series before to apply the HEGY test. The prewhitening approach is the best option for our

[12] We could use alternatively AS semi-annual series $X_T^a = x_{2T}^a$ that is not affected by seasonal aliasing at this frequency. However, the summation filter is not necessary in this case because a π-frequency unit root at semi-annual series is only linked to the $\pi/2$-frequency unit root at the quarterly series. Moreover, from lemma 1, average sampling introduces a MA component higher-order than systematic sampling does, such that the semi-annual auxiliar regression would require a longer augmentation for AS series than for SS series.

[13] Franses and Hobijn (1997) provide critical values for the semi-annual HEGY test.

approach since the transient dynamics are estimated with quarterly time series. Then, including transient dynamics in the model, our approach can be applied in three steps:

1. *Prewhitening.* Filter the quarterly time series x_t from short-run dynamics ($y_t = \phi(L)x_t$). Specify the augmentation under the null by selecting the truncation lag with sequential testing (see Ng and Perron, 1995).[14]

2. *Quarterly Real Unit Root Tests.* Test for unit roots at the zero and π frequency at the filtered quarterly series y_t.

3. *Quarterly π Unit Root Test.* Test for a unit root at the π frequency at the semi-annual systematically sampled filtered series $Y_T = y_{2T}$.

3.2 Testing for Monthly Seasonal Unit Roots

The monthly HEGY test is developed by Beaulieu and Miron (1993) and Taylor (1998). Let us denote by y_t, a monthly time series filtered from short-run dynamics without deterministic terms. Then the auxiliar regression for the monthly HEGY test reads:

$$\Delta_{12} y_t = \pi_0 y_{0,t-1} + \Sigma_1^5 (\pi_j^\alpha y_{j,t-1}^\alpha + \pi_j^\beta y_{j,t-1}^\beta) + \pi_6 y_{6,t-1} + \varepsilon_t,$$

where $y_{0,t}$, $y_{j,t}^\alpha$, $y_{j,t}^\beta$, and $y_{6,t}$ are given in Smith and Taylor (1999).

As in the quarterly case, the seasonal complex unit roots can be tested with different real unit root tests applied to aggregated data. The monthly $\pi/2$ unit root can be tested with a t-test for π_3=0 vs $\pi_3 < 0$ at the bimonthly regression:

$$\Delta_{12} Y_T = \pi_3 Y_{3,T-1} + E_T,$$

where $Y_T = y_{2T}$, $Y_{3,T}$=$Y_T - Y_{T-1} + Y_{T-2} - Y_{T-3} + Y_{T-4} - Y_{T-6}$, and $L^2 Y_T = Y_{T-1}$. The monthly $\pi/3$ unit root can be tested with a t-test for π_2=0 vs $\pi_2 < 0$ at the quarterly regression

$$\Delta_{12} Y_T^f = \pi_2 Y_{2,T-1}^f + E_T,$$

[14]This truncation lag estimator appears to be the best method in Psaradakis' Monte Carlo. However, a deeper analysis of the choice of prewhitening method for the HEGY test should be done.

where Y_T^f=$y_{3T}+y_{3T-1}$, $Y_{2,T}^f$=$Y_T^f-Y_{T-1}^f+Y_{T-2}^f-Y_{T-3}^f$, and $L^3Y_T^f=Y_{T-1}^f$. In this case, it is necessary to remove the π-frequency unit root from the monthly series before systematic sampling to avoid the possibility of aliasing at the quarterly π-frequency.[15] The monthly $\pi/6$ unit root can be tested with a t-test for π_1=0 vs $\pi_1<0$ at the semi-annual regression

$$Y_T^f-Y_{T-2}^f=\pi_1 Y_{1,T-1}^f+E_T,$$

where Y_T^f=$y_{6T}+\sqrt{3}y_{6T-1}+2y_{6T-2}+\sqrt{3}y_{6T-3}+y_{6T-4}$, $Y_{1,T}^f$=$Y_T^f-Y_{T-1}^f$, and $L^6Y_T^f=Y_{T-1}^f$. The remaining harmonic frequencies $2\pi/3$ and $5\pi/6$ can also be tested with real unit root tests. However, in this case there is an important reduction of relevant cycles with temporal aggregation, such that it is likely that the aggregated tests will have worse properties than those of the standard approach.[16]

4 Monte Carlo experiment

In this section, we analyze the size and power of the semi-annual test for the π-frequency unit root (t_1^2) in comparison with the quarterly joint F-test for the $\pi/2$-frequency unit root (F_1^1). We also provide the quarterly t-test for a unit root a the zero (t_0^1) and π (t_2^1) frequencies to compare the properties of the t-test at the different sampling intervals. We consider similar generating mechanisms as those considered in other simulation studies (see Ghysels et al., 1994; Burridge and Taylor, 2001a) to consider some particular features, like near cancellation or periodic heteroscedasticy, that are likely to affect the HEGY test.

The general DGP is given by

$$\varphi(L)x_t=\theta(L)\sigma_s\varepsilon_t,\ t=1,...,T,$$

where $\varphi(L)\in\{1-L^4;1-0.41L^4;1-0.52L^4;1-0.66L^4;1-0.82L^4\}$, $\theta(L)\in\{1;1+0.64L^2\}$, $\boldsymbol{\sigma}$ =$(\sigma_1,\sigma_2,\sigma_3,\sigma_4)$$\in${(1,1,1,1);(5,1,1,1);(5,5,1,1);(5,1,5,1)}, and ε_t˜iidN(0,1). The different AR polynomials $\varphi(L)$ allow us to compare the empirical power, the different MA polynomials $\theta(L)$ allow to analyze the empirical size when there is a near cancellation of the complex unit root, while

[15]Note that the needed filter is different from the summation filter $S_3(L)$.
[16]Critical values for all these tests are provided by Franses and Hobijn (1997).

the different variances of the errors σ_s allow to compare the effects of periodic heteroscedasticy on the finite sample distribution.

The quarterly statistics are computed without deterministic terms. We consider no augmentation for almost all DGPs with the exception of a half-year lag for the near cancellation DGP ($\Delta_4 x_t = 1 + 0.64L^2\varepsilon_t$) for both the quarterly and semi-annual tests. 10,000 repetitions of the experiment are used to estimate the size and power of the different statistics based on the 5% level. The critical values were generated empirically with 50,000 replications. The experiment is run with three different sample sizes $N = \{10, 20, 30\}$.

Table 2 provides the empirical size for different DGPs where we pay attention to the near cancellation of the complex unit root and the presence of periodic heteroscedasticity. First of all, let us consider the near cancellation of the complex unit root ($x_t = x_{t-4} + \varepsilon_t + 0.64\varepsilon_{t-2}$). As seen in the table, all statistics are size biased, particularly those whose test for the complex unit root with quarterly series (F_1^1) or with semi-annual series (t_1^2). The semi-annual test presents a slightly bigger bias than the quarterly test, and therefore the near cancellation problem is not solved by using our approach. A different situation is found when we consider periodic heteroscedasticity. In this case, the limiting distribution of the HEGY statistics associated to the harmonic frequencies may display a shift while the statistics associated to real unit roots are unaffected (see Burridge and Taylor, 2001a). These authors propose a laborious procedure to solve the bad performance of the F test. Our simulations results suggest that our method can be consider as an alternative method to overcome the problem. As seen in the table, while the quarterly F-test is biased for the cases $\boldsymbol{\sigma} = (5,1,1,1)$ and $\boldsymbol{\sigma} = (5,1,5,1)$, the semi-annual statistic is not affected by the different variances of the error term.

Table 2: Empirical size of HEGY statistics

	$x_t = x_{t-4}+\varepsilon_t+0.64\varepsilon_{t-2}$				$x_t = x_{t-4}+\sigma_s\varepsilon_t$, $\sigma_1 = 5$			
N	t_0^1	F_1^1	t_2^1	t_1^2	t_0^1	F_1^1	t_2^1	t_1^2
10	0.108	0.125	0.103	0.166	0.048	0.067	0.045	0.049
20	0.087	0.121	0.084	0.175	0.046	0.060	0.048	0.049
30	0.080	0.118	0.080	0.172	0.048	0.056	0.050	0.047
	$x_t = x_{t-4}+\sigma_s\varepsilon_t$, $\sigma_1 = \sigma_2 = 5$				$x_t = x_{t-4}+\sigma_s\varepsilon_t$, $\sigma_1 = \sigma_3 = 5$			
N	t_0^1	F_1^1	t_2^1	t_1^2	t_0^1	F_1^1	t_2^1	t_1^2
10	0.051	0.054	0.048	0.044	0.053	0.067	0.051	0.047
20	0.047	0.051	0.050	0.053	0.048	0.063	0.049	0.052
30	0.049	0.050	0.053	0.050	0.051	0.067	0.049	0.046

Note: t_0^1: quarterly t-test for a unit root a the zero-frequency; F_1^1: quarterly F-test for a unit root a the $\pi/2$-frequency; t_2^1: quarterly t-test for a unit root a the π-frequency; t_1^2: semi-annual t-test for a unit root a the π-frequency. N: span of the sample measured in years.

If we compare the empirical power of the statistics in table 3, we appreciate how the semi-annual test displays a similar power than the quarterly t-statistics, a power that is slightly smaller than the F-test. However, the power difference decreases with the sample size.

Table 3: Empirical power of HEGY statistics

	$x_t = 0.410x_{t\text{-}4}+\varepsilon_t$				$x_t = 0.522x_{t\text{-}4}+\varepsilon_t$			
N	t_0^1	F_1^1	t_2^1	t_1^2	t_0^1	F_1^1	t_2^1	t_1^2
10	0.634	0.811	0.616	0.590	0.463	0.630	0.454	0.437
20	0.973	0.999	0.979	0.960	0.878	0.984	0.891	0.860
30	0.999	1.000	1.000	0.998	0.992	1.000	0.993	0.988
	$x_t = 0.656x_{t-4}+\varepsilon_t$				$x_t = 0.815x_{t-4}+\varepsilon_t$			
N	t_0^1	F_1^1	t_2^1	t_1^2	t_0^1	F_1^1	t_2^1	t_1^2
10	0.280	0.367	0.271	0.270	0.134	0.154	0.135	0.132
20	0.634	0.849	0.640	0.612	0.259	0.372	0.267	0.257
30	0.890	0.986	0.894	0.875	0.429	0.632	0.434	0.429

See Note of Table 2.

5 Conclusion

We have discussed the relationship between seasonal unit roots at different sampling interval, focusing on monthly and quarterly seasonal unit roots. We have shown how the aliasing affects both the zero-frequency unit root and the seasonal unit roots of temporally aggregated models, such that misleading conclusions on the presence of unit roots can be obtained with temporally aggregated time series. The aggregation theory allows us to design a HEGY-type test for complex unit roots through real unit roots of properly aggregated time series. Obviously, the method implies an important loss of valuable information for the short-run dynamics such that it is necessary to prewhiten the quarterly series, but it does not imply a loss of information for the seasonal cycles of interest since the number of relevant seasonal cycles are invariant with the data transformations applied. A very simple Monte Carlo experiment shows how the usefulness of the alternative approach when the data presents certain types of periodic heteroscedasticity, as long as the sample is big enough. The proposed procedure is better suited for the quarterly case than for the monthly case and can be extend to more powerful HEGY-type tests (see Smith and Taylor, 1999). In addition, our further research will be oriented to Canova and Hansen (1995)'s type tests and seasonal cointegration analysis.

References

Beaulieu, J.J. & J.A. Miron (1993) Seasonal Unit Roots in Aggregate U.S. Data. *Journal of Econometrics* 55, 305-328.

Breitung, H. & P.H. Franses (1998) On the Phillips-Perron type tests for seasonal unit roots. *Econometric Theory* 17, 962-983.

Burridge, P. & A.M.R. Taylor (2001a) On regression-based tests for seasonal unit roots in the presence of periodic heteroscedasticity. *Journal of Econometrics* 104, 91-117.

Burridge, P. & A.M.R. Taylor (2001b) On the Properties of Regression-Based Seasonal Unit Root Tests in the Presence of Higher Order Serial Correlation. *Journal of Business and Economic Statistics* 19, 374-379.

Canova, F. & B. E. Hansen (1995) Are Seasonal Patterns Constant Over Time? A Test For Seasonal Stability. *Journal of Business and Economic Statistics* 13, 237-252.

Franses, P.H. & H.P. Boswijk (1996) Temporal aggregation in a periodically integrated autoregressive process *Statistics & Probability Letters* 30, 235-240.

Franses, P.H. & B. Hobijn (1997) Critical values for unit root tests in seasonal time series. *Journal of Applied Statistics* 24(1), 25-47.

Ghysels, E., H.S. Lee & J. Noh (1994) Testing for unit roots in seasonal time series. *Journal of Econometrics* 62, 415-442.

Ghysels, E. & D. Osborn (2001) *The Econometric Analysis of Seasonal Time Series.* Cambridge University Press.

Granger, C.W.J. & P.L. Siklos (1995) Systematic Sampling, Temporal Aggregation, Seasonal Adjustment, and Cointegration. Theory and Evidence. *Journal of Econometrics* 66, 357-69.

Hylleberg, S., R.F. Engle, C.W.J. Granger & B.S. Yoo (1990) Seasonal Integration and Cointegration. Journal of Econometrics 44, 215-238.

Hylleberg, S., C. Jørgensen & N.K. Sørensen (1993) Seasonality in Macroeconomic Time Series. *Empirical Economics* 18, 321-335.

Koopmans, L.H. (1974) *The Spectral Analysis of Time Series.* Academic Press, New York and London.

Ng, S. & P. Perron (1995) Unit Root Tests in ARMA Models With Data-Dependent Methods for the Selection of the Truncation Lag. *Journal of the American Statistical Association* 90(429), 268-281.

Niemi, H. (1984) The Invertibility of Sampled and Aggregated ARMA Models. *Metrika* 31, 43-50.

Psaradakis, Z. (1997) Testing for Unit Roots in Time Series with Nearly Deterministic Seasonal Variation. *Econometric Reviews* 16, 421-439.

Smith, R.J. & A.M. Taylor (1998) Additional critical values and asymptotic representations for seasonal unit root tests. *Journal of Econometrics* 85, 269-288.

Smith, R.J. & A.M. Taylor (1999) Likelihood Ratio Tests for Seasonal Unit Roots. *Journal of Time Series Analysis* 20, 453-476.

Stram, D.O & W.W.S. Wei (1986) Temporal aggregation in the ARIMA process. *Journal of Time Series Analysis* 7(4), 279-292.

Taylor, A.M.R. (1998) Testing for unit roots in monthly time series. *Journal of Time Series Analysis* 19(3), 349-368.

Wei, W.W.S. (1978) Some consequences of temporal aggregation in seasonal time series models, in *Seasonal Analysis of Economic Time Series*, ed. A. Zellner, 433-444, U.S. Department of Commerce, Bureau of the Census, Washington, DC.

Wei, W.W.S. (1989) *Time Series Analysis. Univariate and Multivariate Methods.* Addison-Wesley Publishing Company, Inc.

Appendix

Proof of Lemma 1. Equation (1) can be written in terms of the reciprocal roots of the AR polynomial:

$$\Pi_1^p(1 - z_j L)\Pi_1^d(1 - e^{i\theta_j} L)x_t = \varepsilon_t, \tag{5}$$

Then, let us multiply both sides of (5) by the lag polynomial $\Pi_1^p(1 - z_j^m L^m)\Pi_1^d(1 - e^{im\theta_j} L^m)/\Pi_1^p(1 - z_j L)\Pi_1^d(1 - e^{i\theta_j} L)$ to obtain the model:

$$\Pi_1^p(1 - z_j^m L^m)\Pi_1^d(1 - e^{im\theta_j} L^m)x_t = \frac{\Pi_1^p(1 - z_j^m L^m)\Pi_1^d(1 - e^{im\theta_j} L^m)}{\Pi_1^p(1 - z_j L)\Pi_1^d(1 - e^{i\theta_j} L)}\varepsilon_t. \tag{6}$$

Model (6) has the convenient property for our purpose that the lags of the AR operator are observable at a longer sampling interval $m\Delta t$. Note that when some of the roots are on the unit circle model (6) is non invertible. For example, the unit root process $x_t = x_{t-1} + \varepsilon_t$ can be written as model $x_t = x_{t-m} + \varepsilon_t + \cdots + \varepsilon_{t-m+1}$.

Let us consider the summed process $x_t^a = S_m(L)x_t$, that from (1) reads:

$$\phi_p(L)\varphi_d(L)x_t^a = S_m(L)\varepsilon_t. \tag{7}$$

The summation polynomial $S_m(L)$ contains $m - 1$ unit roots:

$$S_m(L) = \begin{cases} (1 + L)\Sigma_{j=1}^{m/2-1}(1 - e^{i2j\pi/m}L)(1 - e^{-i2j\pi/m}L), & m \text{ even} \\ \Sigma_{j=1}^{[m/2]}(1 - e^{i2j\pi/m}L)(1 - e^{-i2j\pi/m}L). & m \text{ odd} \end{cases} \tag{8}$$

Some of these roots can be present at the AR unit root component $\varphi_d(L)$. Then, suppose that $\varphi_d(L)$ and $S_m(L)$ have $d - d^a \geq 0$ common unit roots $\alpha(L)$, and let us rewrite the summed process as follows:

$$\phi_q(L)\varphi_{d^a}^a(L)x_t^a = S_m^a(L)\varepsilon_t, \tag{9}$$

an AR**I**(p, d^a) process, where $\varphi_d(L) = \varphi^a_{d^a}(L)\alpha(L)$, and $S_m(L) = S^a_m(L)\alpha(L)$. Then, as in the preceding case, we multiply (9) by $\Pi_1^p(1 - z_j^m L^m)\Pi_1^{d^a}(1 - e^{im\theta_j}L^m)/\Pi_1^p(1 - z_j L)\Pi_1^{d^a}(1 - e^{i\theta_j}L)$ and obtain the model

$$\Pi_1^p(1-z_j^m L^m)\Pi_1^{d^a}(1-e^{im\theta_j}L^m)x_t^a = \frac{\Pi_1^p(1 - z_j^m L^m)\Pi_1^{d^a}(1 - e^{im\theta_j}L^m)}{\Pi_1^p(1 - z_j L)\Pi_1^{d^a}(1 - e^{i\theta_j}L)} S^a_m(L)\varepsilon_t. \tag{10}$$

with all the AR lags multiples of m.

Let us consider a model with hidden periodicity of order m due to h θ_j-frequency unit roots ($2 \leq h \leq m$) such that $e^{i\theta_j} = e^{im\theta_j} = e^{i\theta^*}$ for $j = 1, ..., h$. To simplify the exposition we assume that the model only has unit roots and is given by:

$$\Pi_1^{d-h}(1 - e^{i\theta_j}L)\Pi_{d-h+1}^d(1 - e^{i\theta_j}L)x_t = \varepsilon_t. \tag{11}$$

Let us multiply (11) by $\Pi_1^{d-h}\frac{1-e^{im\theta_j}L^m}{1-e^{i\theta_j}L}\frac{(1-e^{i\theta^*}L^m)^h}{\Pi_{d-h+1}^d(1-e^{i\theta_j}L)}$, such that the disaggregated model reads

$$\Pi_1^{d-h}\left(1 - e^{im\theta_j}L^m\right)(1 - e^{i\theta^*}L^m)^h x_t = \Pi_1^{d-h}\frac{1 - e^{im\theta_j}L^m}{1 - e^{i\theta_j}L}\frac{(1 - e^{i\theta^*}L^m)^h}{\Pi_{d-h+1}^d(1 - e^{i\theta_j}L)}\varepsilon_t.$$

Then, when $h < m$ AR and MA polynomial have the common term $(1 - e^{i\theta^*}L^m)^{h-1}$, such that the model simplifies to:

$$\Pi_1^{d-h}\left(1 - e^{im\theta_j}L^m\right)(1 - e^{i\theta^*}L^m)x_t = \Pi_1^{d-h}\frac{1 - e^{im\theta_j}L^m}{1 - e^{i\theta_j}L}\frac{1 - e^{i\theta^*}L^m}{\Pi_{d-h+1}^d(1 - e^{i\theta_j}L)}\varepsilon_t. \tag{12}$$

When $h = m$, $\Pi_{d-h+1}^d(1 - e^{i\theta_j}L) = 1 - e^{i\theta^*}L^m$, and (12) reduces to:

$$\Pi_1^{d-h}\left(1 - e^{im\theta_j}L^m\right)(1 - e^{i\theta^*}L^m)x_t = \Pi_1^{d-h}\frac{1 - e^{im\theta_j}L^m}{1 - e^{i\theta_j}L}\varepsilon_t. \tag{13}$$

It is straightforward to extend this proof to the case of different groups of hidden roots. ■

Proof of Proposition 2. From lemma 1 and Niemi (1984, Theorem 1), the temporally aggregated models are always invertible because the aggregate MA component does not contain in any case the unit root component (1 −

$e^{im\theta_k}L^m$). The error terms of the biannual processes are

$$
\begin{aligned}
E_T &= \left[\frac{1-L^2}{(1-L)^{d_0}(1+L)^{d_6}}\right]^{d_0^*}\left[\frac{1-L^2+L^4}{(1-\sqrt{3}L+L^2)^{d_1}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*} \\
&\quad \left[\frac{1+L^2+L^4}{(1-L+L^2)^{d_2}(1+L+L^2)^{d_4}}\right]^{d_2^*}\varepsilon_{2T}, \\
E_T^a &= S_2(L)\left[\frac{1-L^2}{(1-L)^{d_0}(1+L)^{d_6}}\right]^{d_0}\left[\frac{1-L^2+L^4}{(1-\sqrt{3}L+L^2)^{d_1}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*} \\
&\quad \left[\frac{1+L^2+L^4}{(1-L+L^2)^{d_2}(1+L+L^2)^{d_4}}\right]^{d_2^*}\varepsilon_{2T},
\end{aligned}
$$

where $d_0^* = \max\{d_0, d_6\}$, $d_1^* = \max\{d_1, d_5\}$, and $d_2^* = \max\{d_2, d_4\}$. The error terms of the quarterly processes are

$$
\begin{aligned}
E_T &= \left[\frac{1-L^3}{(1-L)^{d_0}(1+L+L^2)^{d_4}}\right]^{d_0^*}\left[\frac{1+L^6}{(1-\sqrt{3}L+L^2)^{d_1}(1+L^2)^{d_3}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*} \\
&\quad \left[\frac{1+L^3}{(1-L+L^2)^{d_2}(1+L)^{d_6}}\right]^{d_2^*}\varepsilon_{3T}, \\
E_T^a &= S_3(L)\left[\frac{1-L^3}{(1-L)^{d_0}(1+L+L^2)^{d_4}}\right]^{d_0^*} \\
&\quad \left[\frac{1+L^6}{(1-\sqrt{3}L+L^2)^{d_1}(1+L^2)^{d_3}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*}\left[\frac{1+L^3}{(1-L+L^2)^{d_2}(1+L)^{d_6}}\right]^{d_2^*}\varepsilon_{3T},
\end{aligned}
$$

where $d_0^* = \max\{d_0, d_4\}$, $d_1^* = \max\{d_1, d_3, d_5\}$, and $d_2^* = \max\{d_2, d_6\}$. The error terms of the semi-annual processes are

$$
\begin{aligned}
E_T &= \left[\frac{1-L^6}{(1-L)^{d_0}(1-L+L^2)^{d_2}(1+L+L^2)^{d_4}(1+L)^{d_6}}\right]^{d_0^*} \\
&\quad \left[\frac{1+L^6}{(1-\sqrt{3}L+L^2)^{d_1}(1+L^2)^{d_3}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*}, \\
E_T^a &= S_6(L)\left[\frac{1-L^6}{(1-L)^{d_0}(1-L+L^2)^{d_2}(1+L+L^2)^{d_4}(1+L)^{d_6}}\right]^{d_0} \\
&\quad \left[\frac{1+L^6}{(1-\sqrt{3}L+L^2)^{d_1}(1+L^2)^{d_3}(1+\sqrt{3}L+L^2)^{d_5}}\right]^{d_1^*}
\end{aligned}
$$

$d_0^* = \max\{d_0, d_2, d_4, d_6\}$, and $d_1^* = \max\{d_1, d_3, d_5\}$. ■

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