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Empirical analysis of price data in the
delineation of the relevant geographical market
in competition analysis

Niels Haldrup

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3. July 2003

Abstract

This paper reviews a number of modern as well as classical econometric techniques suitable for empirically determining whether commodities in physically separated markets belong to the *same* geographical market. Even though the tools presented generalize to the delineation of the relevant *product market* our main focus is on the *geographical delineation* of markets. The analyses rely entirely on the use of price data of different types in an attempt to operationalize the so-called SSNIP methodology for price comparisons. In particular, the stationarity versus non-stationarity of price data appears important because otherwise spurious results can potentially occur. Both bivariate and multivariate price comparisons will be discussed. We also consider situations with data observations covering a relatively long period and where it is likely that structural changes have occurred in the sample period whereby the degree of market integration is likely to have changed. New techniques to deal with such recursive features will be suggested. For the methods presented a discussion of the practical problems and concerns facing the model builder are addressed.

JEL-CODES: C22, C32, D4

KEYWORDS: Market delineation, SSNIP methodology, price test, competition analysis, price data

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1 Introduction

The purpose of this document is to present a number of modern as well as classical econometric techniques that are suitable for empirically determining whether commodities in physically separated markets belong to the *same* geographical market. Although the techniques presented in many cases generalize to the delineation of the relevant *product market* our main focus is on the *geographical delineation* of markets. Thus, *delineation* will generally refer to geographical markets although in some situations, where appropriate, the joint delineation of the relevant product and geographical markets will be considered.

The analyses rely entirely on the use of price data of different types in an attempt to operationalize the so-called SSNIP methodology for price comparisons. The statistical and econometric analysis of real data can be rather advanced when data is of a quality for which such methods can be used. But the complexity of the techniques is not an argument for not adopting such methods. However, we also acknowledge the fact that in some cases data availability will be scarce and in these cases somewhat more pragmatic approaches need to be taken. In such cases the cost is likely to be that no robust inferences can be drawn which is adequate for documentation in competition cases.

This document argues for the importance of appropriately analyzing price data with the focus of delineating the relevant geographical market. In particular, the stationarity versus non-stationarity of price data appears important because otherwise spurious results can occur in the sense that the conclusions drawn from the statistical analysis can be misleading. Also, we consider the situation that when data observations cover a relatively long period, then it is likely that structural changes have occurred in the sample period whereby the degree of market integration has changed. New techniques to deal with such recursive features will be discussed.

In presenting the various methods it will of course be impossible to give a complete characterization of the techniques, but appropriate references to the relevant literature will be given where appropriate. Instead, priority is given to presenting the techniques in a relatively non-technical and intuitive fashion where the practical applicability of the methods will be stressed. For each method presented a section is dedicated to a discussion of the practical problems and concerns the model builder should be aware of when analyzing the price data.

1.1 Making the SSNIP methodology operational as a device for market delineation

1.1.1 The SSNIP methodology and price tests

Our approach to market delineation builds on three fundamental pillars:

- A well-structured guiding framework,
- a careful quantitative documentation and

- a consistent and intuitive story to tell.

We find that quantitative documentation is extremely important. If an analyst has postulated a relevant market and cannot deduce a behaviour that can be observed in the market and eventually verified in the available data, we would be very reluctant to give much credit to the conclusions about the relevant market. *Quantitative documentation is the trademark of a good anti-trust analyst.*

In this report we document a large number of econometric tests based on price behaviour. We will refer to these tests as price tests. We are well aware of the criticism of price tests, but we have nevertheless the pragmatic position that price tests are the most feasible and relevant tools for market delineation. However, we don't believe that a price test is the test for market delineation and we don't believe that the result of a price test is credible on its own unless it fits into a consistent and intuitive story or can be confirmed by other studies or observations.

The guiding framework we apply is the SNIPP-methodology. We find that the SSNIP-methodology is a convenient and attractive framework for thinking about market definition, although the methodology is not immediately operational. We are – like others - not aware of any serious alternative to applying the SSNIP-methodology and in our view, the only alternative to the SSNIP-methodology is inconsistency and lack of transparency.

The SSNIP-methodology is a systematic method to identify substitutability between products and can be used to define a number of dimensions of the relevant market. SSNIP is an acronym for **S**mall, **S**ignificant, **N**on-transitory **I**ncrease in **P**rices. The point of departure for the SSNIP-methodology is a thought experiment along the following lines: We want to test whether two products belong to the same market (that being a product or a geographical market). In our mind we speculate whether it is possible to let one of the producers increase profits by raising prices (Increase in Prices) by 5-10 percent (small, but significant) for a period not shorter than twelve months (non-transitory). If the two products are substitutable, we will expect the second product to capture market shares and reduce the profitability of price increases of the first product. If the second product raises market share sufficiently to render the initial price increase unprofitable, we will say that the two products belong to the same relevant market. If the two products are not substitutable, the second product will not be able to increase market shares and in this constrain the behaviour of the first producer. In this case, we will say that the two products belong to different markets.

In particular, when the focus of attention is on products traded in different countries, the SSNIP methodology amounts to asking

whether sufficient arbitrage and substitutability exist across the different regions for these to belong to the same market and hence delineating what is considered *the relevant geographical market*.

We are aware that the concept of an anti-trust market as defined by the SSNIP-methodology is different from the concept of an economic market as defined, for example, by Stigler and Sherwin (1985): “*A market for a good is the area within which the price of a good tends to uniformity, allowance being made for transportation costs*”. Both Werden and Froeb (1993) and Sleuwagen et al (1999, 2001) have good discussions of the differences, but we tend to disagree with the rather bombastic conclusion of Sleuwagen et al (2001), that “*... the delineation of an economic market is completely different from the delineation of an anti-trust market*”. Even though we realize that the market concepts are different, we will maintain that much useful information can be extracted from analyses based on the economic market concept, that is relevant for the anti-trust market and that cannot be extracted in any other way.

We are also aware of a large number of concrete criticisms raised against price tests. We agree with some of them, we disagree with others and the rest are irrelevant in the sense that they are so general that they are valid for any kind of empirical analysis, be it residual demand analysis or price tests. While they may be important to keep in mind when interpreting results, they are hardly damaging for price correlation tests as such.

Werden and Froeb (1993) argue that price correlation test cannot be applied when prices are non-stationary. This is correct, but it does not invalidate the idea of comparing price trends, but just requires you to use other econometric techniques, namely co-integration techniques and stationarity analysis.

They also argue that normally pair-wise comparison of prices is applied, leaving out the possibility that a set of goods may be substitutes with the candidate good even though each single good is not sufficiently substitutable. It is correct that pair-wise comparisons are widely used, but as shown in this report there are several techniques available for simultaneous comparison.

Finally, they argue that price tests often have to rely on data extrapolated outside their original range and that one relies on historical data that in some cases may not adequately reflect the markets under scrutiny. While these criticisms are correct, they are hardly relevant only for price tests. We prefer to interpret these comments as an urge to be cautious and modest when interpreting the results from any kind of empirical analysis.

It is also true that price correlation tests can reach erroneous conclusions if there is spurious correlation in the data originating from common factors unrelated to competitive forces. However, as we shall argue, statistical and econometric techniques exist which can isolate such common factors and hence avoid this criticism.

Sleuwagen et al also (2001) argue that co-integration methods cannot be used if price series are stationary and are without unit roots. This is correct, but it is no problem. Co-integration methods are the appropriate tools whenever

series are non-stationary, correlation methods the appropriate tools whenever series are stationary. Notwithstanding, price adjustment schemes in the form of error correction models are equally valid for stationary and non-stationary price processes, and hence such models are equally useful in describing price dependencies and feed-back mechanisms in price adjustment across products and regions.

Bishop and Walker (1996) argue that price correlation methods are less applicable for comparing prices between countries whenever exchange rates are volatile. This is probably correct, but seems hardly relevant in a Europe with little exchange rate volatility for most of the nineties (for most of the countries) and is certainly of no relevance in Euro-land. Again, it is also plausible that the same reservations may apply for any analysis that involves prices from different countries, be it residual demand analysis or price correlation tests.

Furthermore, Werden and Froeb (1993) argue that there is nothing else but arbitrary guidelines for determining whether a high price correlation is sufficient to declare that two markets are integrated or not. This is correct, but we believe – and show in this report - that in some cases it is possible to obtain some non-arbitrary guidance by benchmarking correlations between candidate markets on correlations between markets for which we are convinced that they are either not integrated or very well integrated.

Finally, Werden and Froeb (1993) as well as Sleuwaegen (1999) argue that a high price correlation between two market areas is neither a necessary, nor a sufficient condition for the two market areas being integrated in the sense of an anti-trust market. Werden and Froeb (1993) develop a small theoretical model and prove that under specific circumstances it is possible to have high correlations between prices only under circumstances where the two markets are not integrated in the SSNIP-sense and vice versa. We will not question this line of argument and acknowledges that the use of price correlation tests implies a risk of making type I-errors (rejecting market delineation when it truly exists) and type II-errors (accepting market delineation when in fact it is absent). However, any statistical test is due to these fallacies but naturally we want to use tests that make these error types small. In this case we need two ask the following two questions: 1) Are there any other analytical method that is superior and equally feasible? and 2) Does price correlation work in practice even though we realize that it may fail in theory?.

The answer to the first question is probably no. There exist superior analytical methods, as residual demand analysis, that in most cases remain hopelessly infeasible because of their huge data requirements. And there exist more feasible methods, as price level comparisons and trade flows that are very feasible, but certainly not superior to price correlation tests.

The answer to the other question is unknown, even though it is highly relevant. It may be that price correlation tests in theory are not a reliable predictor of the delineation of the relevant market, but it may be that for the majority of parameter outcomes that can be observed in real life, this observation is just a theoretical oddity that can be dismissed for any practical application. We don't know the answer.

1.1.2 Price correlations, absolute, and relative price convergence

The SSNIP methodology is indeed a thought experiment and hence there is a need to make the notion operational. Although, potentially there exist several ways of operationalizing the concept, the present set up focuses on the price-behaviour of given products across a range of countries. The idea is thus to define products to belong to the same geographical markets when arbitrage and substitutability will ensure that prices tend to move together in a particular fashion.

In price correlation analysis a maintained assumption is that prices either have or have not converged within a given sample period and hence indicating that the commodities belong or do not belong to the same geographical market. More precisely we say that two price series for separate geographical regions *have converged*, and hence belong to the same *geographical market*, if the difference between them is *stable* in a way we are going to define. If the initial observations are unimportant, the stability requires that the difference between the series is stationary for the entire sample period. When the mean of the price difference is zero the prices are in a state of *absolute convergence* whereas a non-zero mean indicates *relative convergence* in the sense that convergence exists but the gap is not completely filled. The latter possibility may occur if e.g. increasing costs of convergence or possible barriers to absolute price convergence exist. Testing stability of price differences can thus be considered an efficient tool for the delineation of the relevant geographical market and we shall make intensive use of this approach.

It is of importance to note here that absolute convergence is not considered necessary for commodities to belong to the same geographical market. Rather it is of importance for market delineation to see whether the price behaviour of one commodity transmits to other commodities across borders; this is satisfied even for the weaker notion of relative price convergence. This notion is closer to the SSNIP methodology than requiring absolute price convergence in the sense of (almost) identical prices across regions. It is the co-movement of prices (in the sense of relative convergence) rather than the law of one price that is of importance in market delineation. However, this does not preclude that absolute convergence is an equally interesting notion to analyze. This is a stronger notion, but it is not strictly required for market delineation.

In the analysis of price correlations we stress the importance of discriminating between price processes being either stationary or non-stationary. Making this distinction is essential in properly and validly analyzing the price data. This is discussed in section 2. Subsequently, in section 3 price correlation analysis for the pairwise comparison of price data is provided for stationary and non-stationary data, respectively. For stationary data the (classical) methods of correlation analysis are presented whilst for non-stationary data the notion of co-integration of price data is presented. Looking at data pairwise is always extremely useful, but the analysis is limited by the fact that it can only be judged whether particular goods considered in pairs can be considered belonging to the same geographical market. Extensions to the more general case with

multiple price processes are given in section 4 where the particular hypotheses to be tested in the market delineation discussion is attached a particular weight.

1.1.3 Price convergence as an evolving process

The operationalization of the SSNIP notion is complicated by the fact that for real data and for data covering a sufficiently long span of time periods, several years for instance, it is likely that gradual changes of the degree of market integration have taken place. For instance, it can occur that markets happen *not* to be integrated initially in the sample period whereas later changes have taken place due to reforms in institutional settings, transaction costs may have been lowered, changes may have occurred in the exchange rate regime, and so on. These are factors which potentially could have caused an increased degree of market integration and possibly a common geographical market for the products could be established.

Price correlation analysis will be inadequate in this case because the convergence can be considered a process *evolving over time*. Econometric techniques to deal with the convergence problem will be discussed in section 5.

1.2 Data considerations

This section serves to clarify the types of data we will be dealing with. Some notation and appropriate data transformations will be presented.

1.2.1 Types of data

In general three types of data will be available: Time series, cross section, and panel data. For the analysis at hand panel data is most relevant since prices across a range of countries exist for a number of time periods. Typically we have p_{it} indicating the price of a given commodity i (or a commodity of region i) measured at time t , where the indices are $i = 1, 2, \dots, q$ and $t = 1, 2, \dots, T$. That is, we have for instance q regions and T time periods.

The single price variables p_{it} can be considered a time series whereas the stacked process

$$\mathbf{p}_t = \begin{pmatrix} p_{1t} \\ p_{2t} \\ \vdots \\ p_{qt} \end{pmatrix}$$

can be considered a panel. In the analysis of the delineation of the relevant geographical market having observations entirely on the cross section dimension will not be useful for empirical analysis.

Pairwise *price differences* between the single countries or regions, country i and country j , say, are defined

$$d_{ijt} = p_{it} - p_{jt}.$$

These price series are time series.

It appears important for the analysis of the data to consider the frequency s of the data observed. Typically the frequency will be annual, quarterly, monthly, daily, or even hourly. Each type of frequency generates problems that need to be appropriately dealt with in practice. In particular, for price comparisons and the calculation of price correlations it becomes of importance to appropriately remove common co-movements of the data which is the result of a common cyclical pattern rather than an intrinsic common element in the price movements. How to deal with these difficulties will be discussed accordingly.

It needs to be stressed that for competition analysis it is preferable to deal with as high a frequency of observations as possible. Not only will this provide more sample points, but it is also in (relative) high frequency data that price correlation patterns are most easily identified.

1.2.2 Data transformations

Sometimes it will be useful to transform the data before analysis. For instance, rather than looking at the price levels, we may want to consider the *price changes* of the single time series (for each country):

$$\Delta p_{it} = p_{it} - p_{it-1}$$

where $\Delta = 1 - L$ is the difference operator and L is the lag operator defined as $L^k p_t = p_{t-k}$.

It is also frequently of value to consider the log transformed data $\ln p_{it}$ whereby the differenced log series

$$\Delta \ln p_{it} = \ln p_{it} - \ln p_{it-1} \approx \frac{p_{it} - p_{it-1}}{p_{it-1}}$$

approximately measures the *growth rate* of the series, i.e. price inflation.

Note that price differences in log transformed data amounts to considering the log transformed ratio of the price series:

$$\ln p_{it} - \ln p_{jt} = \ln \frac{p_{it}}{p_{jt}}$$

Whether one should use log or non-log transformed data is frequently a matter of taste. One advantage of using the log transformed data is firstly, that the changes in the series become scale invariant and will measure the growth rate of the series, secondly, it is frequently found that the statistical specification of log-transformed data appears to be better compared to the non-transformed data. Finally, using log transformed data becomes most attractive when *price indices* rather than the original prices series are available. The reason is that the scaling factor associated with the indexation becomes a constant term which will have no influence on the price correlations.

After section 2 in this document the notation p_{it} can mean either the log-transformed data *or* the original series. The methods equally apply to both types of data.

1.2.3 Exchange rates

When comparing prices across regions it is necessary to account for changes in the exchange rates and to make the price comparisons denominated in the same currency. Hence p_{it} should be compared with prices $p_{jt}e_{ijt}$ where e_{ijt} is the exchange rate of currency i to currency j measured at time t . In logs, the comparison¹ is made of $\ln p_{it}$ and the variable $\ln p_{jt}^* = (\ln p_{jt} - \ln e_{ijt})$.

1.2.4 Absolute and relative convergence in price comparisons

Sometimes it is of interest to distinguish between the notions of absolute and relative convergence of price series. If we continue addressing log-transformed data we will say that p_{it} and p_{jt}^* have *converged in absolute terms* if

$$\lim_{T \rightarrow \infty} E(\ln p_{it} - \ln p_{jt}^*) = 0$$

In other words, for $T \rightarrow \infty$ the law of one price will apply. On the other hand, if

$$\lim_{T \rightarrow \infty} E(\ln p_{it} - \ln p_{jt}^*) = \alpha$$

for $\alpha \neq 0$ we would say that p_{it} has *converged relative* to p_{jt}^* .

Observe that if p_{it} and p_{jt} are price indices then the price levels are affected in the comparisons. Hence it does not make sense to consider a potential absolute convergence amongst the prices in the above sense. The two price series will deviate by a quantity α reflecting the indexation and hence only relative convergence can be analyzed from index data.

Asking whether prices have converged relatively can also be undertaken by addressing the co-variation of price-changes $\Delta \ln p_{it}$, $\Delta \ln p_{jt}^*$, say. This comparison can be made both when the series are price indices and raw price series.

2 Univariate time series analysis of price levels, growth rates, and price differences

2.1 Why are the univariate price characteristics important in price analysis? - A requirement for market delineation

By the univariate characteristics of a price series we primarily focus on the property of the price level being either *stationary* or *non-stationary*. When prices are stationary it means that the time process will fluctuate around some well defined mean although the swings around the mean can tend to be rather persistent in time. As a contrast, a non-stationary time series is a time series

¹For notational simplicity, we assume from section 2 and on-wards that prices p_{it} and p_{jt} are denominated in the same currency.

which does not have a well-defined mean to which it will eventually return. One example of a nonstationary time series is a random walk with drift which reads

$$p_t = p_{t-1} + \mu + \varepsilon_t$$

The parameter μ measures the drift of the process whereas ε_t are the shocks to the price series, that is, $\mu + \varepsilon_t$ measures the change in the prices from time period $t - 1$ to time period t . The price level can be written as

$$p_t = p_0 + \mu t + \sum_{j=1}^t \varepsilon_j \quad (1)$$

whereby it can be seen that prices will have a linear trend plus an accumulative term reflecting all historical shocks to the process. The latter component is frequently being referred to as a stochastic trend. In contrast, a *stationary* price process will not have a stochastic trend in its levels representation. A typical way of writing a so-called trend stationary process is by

$$p_t = \alpha^t p_0 + \mu t + \sum_{j=1}^t \alpha^j \varepsilon_{t-j}, \quad \alpha < 1. \quad (2)$$

As can be seen the non-stationary process results when $\alpha = 1$ in (2). In this case we say that prices p_t have a *unit root*.

In describing the univariate characteristics of price data most price series can be described either in terms of the process (1) or the process (2) and hence for empirical analysis it is of interest to test whether the one or the other description is to be preferred. The reason for this is that when the focus of attention is on price behaviour amongst many price series, then a necessary condition for two price series to co-vary is that the underlying series both need to be either stationary or non-stationary. When price levels are non stationary in the sense described above, we say the series is integrated of order one², I(1), that is, when $\alpha = 1$ in (2). An I(1) process therefore has a unit root. When the price level is stationary, $\alpha < 1$, we say the series is integrated of order zero, I(0). A first (necessary, though not sufficient) test of whether prices comove in a systematic way (i.e. such that the underlying markets can be considered belonging to the *same* geographical market), is that both series are integrated of the same order. That is, the series are both stationary *or* both nonstationary. The property generalizes to situations with multiple price series:

A necessary requirement for market delineation

A necessary (but not sufficient) requirement for commodities belonging to the same geographical market is that the price level of each series is integrated of the same order.

²Observe that statistical notion of *integration* should not be confused with the economic notion of *market integration*.

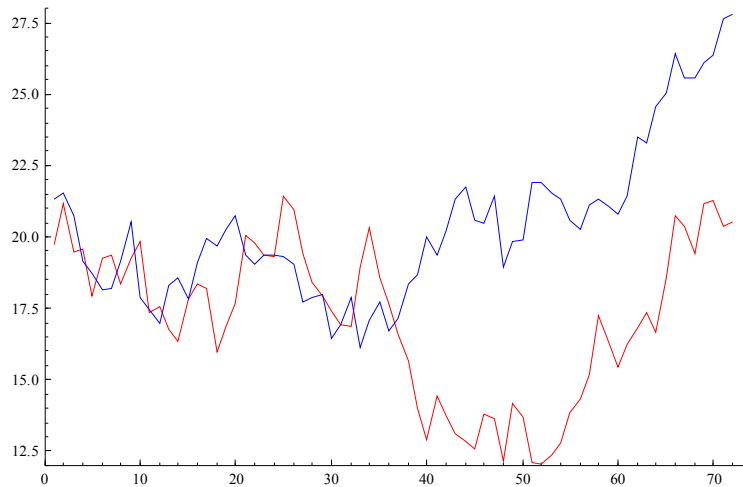


Figure 1: Examples of non-stationary, I(1) price processes.

In figures 1 and 2 examples of non-stationary, I(1), and stationary, I(0), price processes are displayed. In both cases it is assumed that $\mu = 0$ such that there is no deterministic trend in the series. As can be seen the two price series behave fundamentally differently. The intuition is thus, that a first test of whether commodities belong to the same geographical market is that prices behave "in a similar fashion". This question can be accomplished empirically by testing the order of integration of the single price series.

2.2 Test for the order of integration

As argued in the previous section, it is of crucial importance to test whether the single price level series are integrated of the same order. One particular test, the Dickey-Fuller test³, DF, and its generalization, the Augmented Dickey Fuller test, ADF, seem to be most popular in applied work.

2.2.1 The ADF test

The DF test is based on a test of whether $\alpha = 1$ in the model (2), against the alternative $\alpha < 1$. Under the null hypothesis the price level is thus integrated of order one, whereas under the alternative prices are integrated of order zero. In its most simple form the test can be conducted by estimating by OLS the model:

$$\Delta p_t = (\hat{\alpha} - 1)p_{t-1} + \hat{v}_t \quad (3)$$

³Due to Dickey and Fuller (1979). See also Fuller (1976) and the text books Hamilton (1994), and Maddala and Kim (1998).

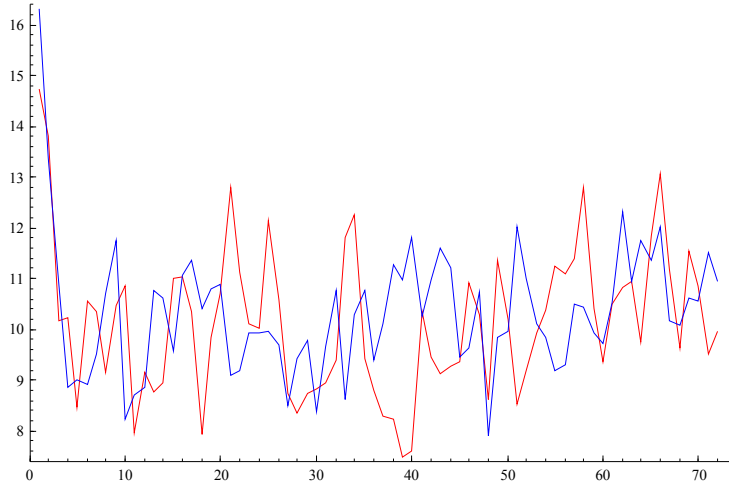


Figure 2: Examples of stationary, $I(0)$, price processes

where $\hat{\alpha} - 1$ is the estimated parameter. Under the null hypothesis the t -ratio, $t_{\hat{\alpha}-1}$, follows a distribution known as the Dickey-Fuller distribution. The distribution is tabulated in Fuller (1976), but in practical applications the finite sample critical values in McKinnon (1991) are recommended. It should be noted in particular, that the t -statistic does *not* follow a t -distribution.

Uncritical use of the simple regression model (3) should be warned against because in most cases the assumptions underlying the simple model, c.f. later, will not be satisfied. Instead, the augmented Dickey-Fuller regression with deterministics should be conducted. The auxiliary regressions read:

$$\Delta p_t = (\tilde{\alpha} - 1)p_{t-1} + \tilde{m}_0 + \sum_{j=1}^k \tilde{\gamma}_j \Delta p_{t-j} + \tilde{v}_t \quad (4)$$

$$\Delta p_t = (\hat{\alpha} - 1)p_{t-1} + \hat{m}_0 + \hat{m}_1 t + \sum_{j=1}^k \hat{\gamma}_j \Delta p_{t-j} + \hat{v}_t \quad (5)$$

which also include an intercept and an intercept plus a time trend in addition to an appropriate number of lags of the differenced series. The t -statistic, $t_{\hat{\alpha}-1}$, from the regression (4) or the t -statistic, $t_{\hat{\alpha}-1}$, from the regression (5) is again used for testing whether $\alpha = 1$. The critical values of the test can also be found in McKinnon (1991). However, note that the *relevant* critical values should reflect whether only an intercept or intercept and trend have been included in the auxiliary regression.

2.2.2 Practical problems and concerns

There are a large number of pitfalls in unit root testing. Below a list of the problems and concerns is given, and suggestions of how to deal with the difficulties are presented.

Should a trend be included in the ADF-regressions ? It is always recommended to include at least an intercept in DF regressions. The reason for including a time trend is to obtain a test that is powerful against the possibility of stationarity around a time trend.

- For the analysis of the price level (or log price level) it is *always* recommended to include a time trend, that is, the regression (5) should be used.
- For the analysis of the price changes or log differences (growth rates) of prices, it is recommended at least to include a constant term, that is an analogue of regression (4) should be used.

Observe that when the focus is on the price differences, the ADF-test can be conducted for the series $p_t^* = p_t - p_{t-1}$ in place of p_t in the regressions (4) and (5). In this case the hypothesis being tested for the price level is that of I(2) against I(1). When the series have been log-transformed it therefore means that the growth rates (price inflation) is tested to be nonstationary I(1) against the alternative that the price inflation is stationary, I(0).

It should also be mentioned in the choice between a model with or without trend it is *not* legitimate to use the t -test of the trend regressor and test via a t -distribution whether the coefficient is zero or non-zero. The choice of which deterministic to include is given a priori.

Lagged differences. It is of crucial importance that the ADF regressions (with or without trend) have residuals with no remaining autocorrelation. In other words, it is important that the "right" value of k is selected. Autocorrelation in the residuals will invalidate the tests. On the other hand, having too many lags in the auxiliary regressions will give a test with very bad properties. The recommendation is thus to include sufficiently many lags of Δp_t in the auxiliary model in order to remove all residual autocorrelation. Following the general-to-specific testing principle, insignificant lags are next removed but only if removal of a particular lag does not result in residual autocorrelation. It is fully legitimate to remove insignificant intermediate parameters. Note that a standard t -distribution with an appropriate number of degrees of freedom can be used to select which lagged differences to include in the model, - but only the lagged differences.

In checking for residual autocorrelation it is *not* valid to use a Durbin Watson test due to the presence of lagged differences and levels. Instead an LM-type test, a Box-Pierce test, or a Ljung Box test should be used.

Whereas residual autocorrelation will invalidate the ADF tests, it should be noted that heteroscedasticity does not invalidate the test, and hence no action needs to be taken in this respect.

Seasonality and frequency of observations. One type of residual autocorrelation that may occur is that of seasonality. Depending upon the frequency of observations hourly, daily, weekly, monthly, or bi-annual seasonal dummy variables can be used in the auxiliary regressions. In fact, the use of seasonal dummy variables will frequently reduce the number of lagged differences needed in the regressions. For instance, if the frequency of observations is equal to q ($q = 4$:quarterly data, $q = 12$: monthly data), the auxiliary regression with a time trend will take the form

$$\Delta p_t = (\hat{\alpha} - 1)p_{t-1} + \sum_{j=1}^{q-1} \hat{\delta}_j D_{jt} + \hat{m}_0 + \hat{m}_1 t + \sum_{j=1}^k \hat{\gamma}_j \Delta p_{t-j} + \hat{v}_t$$

where D_{jt} takes the value 1 in season j and is zero otherwise.

Outliers, missing values, and irregularly spaced observations. Outlying, aberrant, and abrupt observations can sometimes invalidate the DF tests. If the analyst suspects certain observations to be due to such anomalies it is recommended to include dummy variables in the auxiliary models in order to exclude the effect of their influence.

It is also frequently seen that data observations are missing or irregularly spaced. There are a number of ways to deal with this problem. In Ryan and Giles (1998), drawing on work by Shin and Shakar (1994), it is recommended simply to ignore the missing observations and to abstract from "holes" in the sampling data.

Small samples. It is always preferable to have long data series compared to short data series. However, in competition cases it is frequently of interest to compare price movements at short horizons for a relatively high frequency of data, e.g. monthly, and often data will not be available except for a few years. In general, increasing the frequency of observations can only partially compensate for a short span of data. The reason is that new problems, e.g. seasonality problems and outliers, are more likely to arise when the frequency of observations increases. After all, it is extremely important when higher frequencies of data are analyzed that the seasonality issue is carefully dealt with.

The guideline for the minimum number of observations to be used in the present kind of data analysis lies around 30 observations. When dealing with samples of this size it is recommended to use appropriate small sample critical values for hypothesis testing, see McKinnon (1991).

2.3 Test of stationarity: The KPSS test

The augmented Dickey-Fuller unit root tests are designed to test the null of non-stationarity against the null of stationarity. Sometimes it may appear adequate to reverse the hypotheses by testing the null of stationarity against the alternative of non-stationarity. For instance, in delineating the relevant geographical market it may sometimes seem more relevant to test the null of stationarity of price differences or relative prices, and thus delineating a common geographical market, against non-stationarity of price differences, i.e. the hypothesis of no common market. The so-called KPSS-test suggested by Kwiatkowski, Phillips, Schmidt, and Shin (1992) has been derived for this purpose. In Forni (2002) it has been advocated for the use of stationarity tests in market delineation. After all, our suggestion is to consider both types of tests in order to obtain some common evidence for the relevant hypotheses of interest. Hence the tests should be considered complementary rather than substitutes.

The KPSS test is constructed along the following lines. Assume the price series can be represented as

$$p_t = \delta t + \zeta_t + \varepsilon_t$$

where ε_t is stationary whilst ζ_t is a random walk component given by

$$\zeta_t = \zeta_{t-1} + u_t, \quad u_t \sim i.i.d(0, \sigma_u^2)$$

The null hypothesis of stationarity can be formulated as a test on the variance of the random walk component, that is under the hypothesis

$$H_0 : \sigma_u^2 = 0$$

the random walk part of the price process is absent. The KPSS test statistic reads:

$$KPSS = T^{-2} \sum_{t=1}^T S_t^2 / s^2(l) \quad (6)$$

where

$$\begin{aligned} s^2(l) &= T^{-1} \sum_{t=1}^T e_t^2 + 2T^{-1} \sum_{\tau=1}^l w_{\tau l} \sum_{t=\tau+1}^T e_t e_{t-\tau} \\ S_t &= \sum_{i=1}^t e_i \quad t = 1, 2, \dots, T \\ w_{\tau l} &= 1 - \frac{\tau}{l+1} \end{aligned}$$

and e_t are the residuals from a regression of p_t onto a constant (and possibly a time trend). The function $w_{\tau l}$ is a weight function. This can take different forms but the one reported above, known as a Bartlett window, is most frequently used in empirical applications. The expression $s^2(l)$ is a variance estimator accounting for the fact that serial correlation will typically exist in the series of interest.

2.3.1 Practical problems and concerns

The asymptotic distribution of the KPSS test statistic given in (6) is reported in Kwiatkowski *et al* (1992) for two cases: with demeaned data, i.e. when e_t are residuals from a regression of p_t on a constant, and for detrended data, that is, when e_t are residuals from a regression on both a constant and a time trend. The time trend case is considered when the price series is judged to have a deterministic trend.

No finite sample critical values are available which can be problematic if only a limited number of observations are available as the test can be shown to be somewhat size distorted in finite samples. Hence the test should be used with caution in small samples.

In practice, the truncation number l is chosen by the analyst. There are no clear directions concerning the choice of this value except that l should grow with the number of data points available. A practical advice is however to try different values of l . When the frequency of the data is s , e.g. $s = 12$ for monthly data, it is recommended at least to calculate the test for $l = s$.

3 Pairwise comparison of price data

In order for price comparisons to make sense it is essential that the price series to be compared are of the same order of integration, c.f. the first requirement for market integration. If the price series under scrutiny are of different orders of integration there is no way that the underlying markets can belong to the same geographical market. On the other hand, if the price series are in fact integrated of the same order this does not imply market integration, but it is necessary for market integration.

In this section pairwise price comparisons are described for two situations:

- When the price levels are both stationary, $I(0)$.
- When the price levels are both non-stationary, $I(1)$.

Initially, only contemporaneous correlations are addressed, that is the dynamic aspect of price movements is abstracted from. However, the dynamic aspect is rather important in practical situations and hence a subsequent section is dedicated to tools designed for the dynamical considerations in price movements.

3.1 Contemporaneous correlation analysis of stationary price series

3.1.1 Simple correlation vs partial correlation

A frequent tool for analyzing the co-movement of two *stationary* price series is to consider **the simple correlation coefficient**.

Consider two price series p_{1t} , and p_{2t} for which a total of T observations are available. The simple correlation is calculated as

$$\rho_{12} = \frac{\sum_{t=1}^T (p_{1t} - \bar{p}_1)(p_{2t} - \bar{p}_2)}{\sqrt{\sum_{t=1}^T (p_{1t} - \bar{p}_1)^2 \sum_{t=1}^T (p_{2t} - \bar{p}_2)^2}}, \quad \bar{p}_1 = \frac{1}{T} \sum_{t=1}^T p_{1t}, \quad \bar{p}_2 = \frac{1}{T} \sum_{t=1}^T p_{2t}$$

The correlation coefficient is a scale invariant measure lying in the interval $-1 \leq \rho_{12} \leq 1$. The closer ρ_{12} is to 1, the higher (contemporaneous) association exists between the two price series. There is no generally agreed level or threshold which defines whether series are moving sufficiently together for the single commodity markets to belong to the same geographical market. After all, the simple correlation of two price series should be treated with much care due to the potential risk of measuring a spurious relationship between the series. A high degree of correlation between two price series can occur because intrinsically the two price series do co-move. However, a high degree of correlation can also arise as a result of a both series correlating with one or several other factors. For instance, a high (simple) correlation can occur as a result of a common seasonal pattern in the series without the single series being directly related to each other. Also, the *general price level* of the implied markets can tend to co-move or the commodity price is much sensitive to a production input being externally priced. In practice we would like to correct for the influence of such factors.

Assume that the factors we would like to correct for are the variables $x_{1t}, x_{2t}, \dots, x_{kt}$. These variables could for instance be seasonal dummy variables if we want to correct for seasonal effects. Correcting p_{1t}, p_{2t} for the influence of these factors is made by least squares regression:

$$p_{1t} = \hat{\beta}_{10} + \sum_{j=1}^k \hat{\beta}_{1j} x_{jt} + p_{1t}^* \quad (7)$$

$$p_{2t} = \hat{\beta}_{20} + \sum_{j=1}^k \hat{\beta}_{2j} x_{jt} + p_{2t}^* \quad (8)$$

The series p_{1t}^* and p_{2t}^* are the regression residuals and can be interpreted as p_{1t} and p_{2t} *corrected* for the influence of $x_{1t}, x_{2t}, \dots, x_{kt}$. **The partial correlation coefficient** is defined as the correlation between the adjusted series and (because the mean of the corrected series is zero) reads:

$$\rho_{12|x_1, x_2, \dots, x_k} = \frac{\sum_{t=1}^T p_{1t}^* p_{2t}^*}{\sqrt{\sum_{t=1}^T p_{1t}^{*2} \sum_{t=1}^T p_{2t}^{*2}}}$$

If the correlation between p_{1t} and p_{2t} is spurious it is typically seen as the simple correlation ρ_{12} being relatively large whereas the partial correlation $\rho_{12|x_1, x_2, \dots, x_k}$ is relatively small. In general, however, the partial correlation coefficient can be either smaller or bigger than the simple correlation coefficient

and the coefficients may not even have the same sign even though one would expect this in most cases analyzing price data.

3.1.2 Practical problems and concerns

When is correlation high ? - the use of benchmark correlations. In the analysis of delineation of the relevant geographical market of a given commodity it is *always* recommended to base calculations on the partial correlation coefficients for the reasons just given. It remains, however, to determine when a partial correlation is sufficiently high for single markets or products to belong to the same geographical market. Generally, we will be reluctant to put forth a threshold for correlations being "high". However, in some cases we suggest using *threshold correlations* by comparing estimated correlations with those occurring between markets where we are convinced that either no integration or very well integration exists. Although such a procedure is not a test we believe that such an approach is preferable to the use of ad-hoc correlation thresholds in product and geographical market delineation.

The order of Integration and the *law of one price* in absolute and relative terms. As previously mentioned, calculation of the above correlation quantities only makes sense when the underlying price level series are stationary, $I(0)$. When the series are $I(1)$, calculation of the quantities will result in spuriously high correlation coefficients even though the series appear not to be significantly correlated and hence inference can be very misleading. Notwithstanding, when price levels are indeed $I(1)$ or $I(0)$ it still makes sense (and it is perfectly valid statistically) to calculate the partial correlation coefficients of the price changes, e.g. correlations of the transformed series $\Delta p_{1t} = p_{1t} - p_{1t-1}$ and $\Delta p_{2t} = p_{2t} - p_{2t-1}$. If p_{1t}, p_{2t} are log-transformed the correlation of the growth rates (or price inflation) of the series will thus be calculated. In this situation the economic interpretation of the correlations differ because it is the price changes rather than the price levels that potentially will correlate. In this situation it is a *relative* rather than an *absolute* version of the *law of one price* being tested. A "high" correlation of price changes across commodities for different geographical regions still indicates that these comove and hence suggests products to belong to the same geographical market.

Only contemporaneous effects. Partial correlation analysis should be considered one way of extracting information from price data and only information associated with the contemporaneous correlations can be calculated. The dynamical aspects of standard simple and partial price correlations are thus being abstracted from. Price dynamics of price levels and price changes will subsequently be discussed.

3.2 Dynamical correlation analysis of stationary price series

3.2.1 Granger causality tests

Price levels as well as price changes can be correlated over time in the sense that there need not be any immediate effect from one price to another. One way of formalizing whether there is a time lag effect in price dynamics is by virtue of a Granger causality test, see Granger (1969) and Granger and Newbold (1986). Granger causality or noncausality is concerned with the question of whether lagged values of p_{2t} do or do not improve on the explanation of p_{1t} obtainable from only lagged values of p_{1t} itself and vice versa. In other words, the Granger causality test helps in identifying the channels through which the price series interact dynamically.

The idea behind the Granger test is the following. Consider two price series p_{1t}, p_{2t} both being $I(0)$. For instance, the price levels can themselves be $I(0)$. Alternatively we can focus on the price changes of $I(1)$ price levels series, that is Δp_{1t} and Δp_{2t} , which are then $I(0)$ by their construction. Assuming the price levels are addressed, the model underlying the Granger test reads:

$$p_{1t} = \alpha_{10} + \sum_{j=1}^k \alpha_{1j} p_{1t-j} + \sum_{j=1}^k \beta_{1j} p_{2t-j} + \varepsilon_{1t} \quad (9)$$

$$p_{2t} = \alpha_{20} + \sum_{j=1}^k \alpha_{2j} p_{2t-j} + \sum_{j=1}^k \beta_{2j} p_{1t-j} + \varepsilon_{2t} \quad (10)$$

The following possibilities exist:

- a) When $\beta_{1j} = 0$ for all values $j = 1, \dots, k$ then p_2 does *not* Granger cause p_1
- b) When $\beta_{1j} \neq 0$ for some $j = 1, \dots, k$ then p_2 *does* Granger cause p_1

Similarly,

- c) When $\beta_{2j} = 0$ for all values $j = 1, \dots, k$ then p_1 does *not* Granger cause p_2
- d) When $\beta_{2j} \neq 0$ for some $j = 1, \dots, k$ then p_1 *does* Granger cause p_2 .

Obviously, nothing excludes that causality or non-causality can be in both directions. An easy way of testing whether one price series does not Granger cause the other price series is by testing via a standard F -test the hypotheses given in *a* and *c* above. The test can be easily conducted from any econometrics soft-ware package doing linear regression analysis. When the null hypothesis underlying a particular model is rejected then there is Granger causality from one variable to the other.

3.2.2 Practical problems and concerns

Selecting the truncation parameter "k". A practical problem concerns the choice of "k", the truncation parameter, in the auxiliary regressions (9) and (10) when conducting the Granger test. In practice, it is often seen that results are reported for different values of k . It is important though, that the results reported for any particular choice of k is based on a regression with no residual autocorrelation. Again, the Durbin-Watson test adopted for testing autocorrelation cannot be used due to the presence of a lagged left hand side variable, and instead LM type of tests are recommended.

Correcting for external effects. The above procedure suffers from the same potential problem as underlying standard correlation analysis discussed in section 3.1.1. That is, a dynamic correlation can result as a consequence of "third" factors such as the general price level effects, seasonality, dependency upon prices of product inputs etc. In order to avoid this difficulty the price series need to be adjusted for such external factor influences. This can be accomplished by prior correction of the series as in (7) and (8) (possibly by allowing some x variables to be lagged values) and conducting the auxiliary regressions (9) and (10) with the series p_{1t}^* and p_{2t}^* in place of p_{1t} and p_{2t} . In fact, such a procedure is *always* recommended. In many case it is also desirable to account for other price series than just p_{1t} and p_{2t} . Further elaboration on this multivariable case is discussed in section 4.1.3.

A crude measure of dynamic correlation. The significance of the F -test of Granger noncausality indicates the strength of the dynamic influence (or causality) amongst the series, if any. In practice, however, it can be difficult from the F -test to judge the degree of causality amongst the price processes. For instance, there can be causality from one price to another, but the association amongst the series need not be sufficiently strong for considering the respective markets to belong to a common geographical market. A crude measure of the dynamic association can be obtained according to the following lines.

Assume for instance that the F -test for Granger non-causality in the model (9) rejects. Hence there is an indication that some correlation exists. We would like to know how much of the variation in prices p_{1t} can be explained solely in terms of the lagged values of p_{2t} , that is, the lagged prices $p_{2t-1}, p_{2t-2}, \dots, p_{2t-k}$. In so doing we regress each of the variables $p_{1t}, p_{2t-1}, p_{2t-2}, \dots$, and, p_{2t-k} on a constant, as well as $p_{1t-1}, p_{1t-2}, \dots, p_{1t-k}$. For each of these series we save the residuals which subsequently constitute new 'corrected' series. Denote these adjusted series $\tilde{p}_{1t}, \tilde{p}_{2t-1}, \tilde{p}_{2t-2}, \dots$, and, \tilde{p}_{2t-k} . Next, \tilde{p}_{1t} is regressed on a constant and the other adjusted series $\tilde{p}_{2t-1}, \tilde{p}_{2t-2}, \dots$, and, \tilde{p}_{2t-k} . The coefficient of determination R^2 from this regression measures how large a fraction of the variation in prices p_{1t} can be explained by the lagged prices $p_{2t-1}, p_{2t-2}, \dots, p_{2t-k}$ after correcting for the influence inherited in the dynamics of the price series p_{1t} itself. The larger the value of R^2 the stronger dynamic adjustment exists across markets.

Of course, when external effects, (see above), are suspected, it is recommended to use the series p_{1t}^* and p_{2t}^* in place of p_{1t} and p_{2t} in the above analysis.

Granger non-causality *and* absence of contemporaneous correlation.

Frequently it will be of interest to examine whether jointly there is no contemporaneous or dynamic dependency across two price series. If a statistical test concludes the absence of such a dependency the two markets are separate and cannot be part of a common geographical market. The test considers jointly the possibility of Granger non-causality *and* absence of contemporaneous correlation. The test can be constructed as a variant of the Granger non-causality test by augmenting equation (9), say, with the regressor p_{2t} and testing by an F -test the jointly nullity of all parameters associated with contemporaneous and lagged values of p_{2t} . A difficulty with this test is, however, that it assumes no joint determination of prices, i.e. one of the price series needs to be exogenously determined. This can be a practical problem because we would frequently assume endogeneity of both prices. In these cases the "crude" version of the Granger causality test, with no contemporaneous variables, is recommendable.

3.2.3 Error Correction Models for stationary variables

Error correction models are typically used in describing the dynamics of cointegrated time series and we will address this topic later in this report. However, for stationary processes it is equally valid to model the dynamic adjustment processes by error correcting behaviour. Essentially the estimated model corresponds to reformulated Granger-causality test type auxiliary regressions, but the advantage is that an attractive interpretation of the estimation results can be given.

The idea is to explain price changes in terms of lagged values of the price changes as well as the lagged value of the price gap between prices p_{1t} and p_{2t} , that is $(p_{1t-1} - p_{2t-1})$. That is, regressions of the form:

$$\begin{aligned}\Delta p_{1t} &= m_1 + \alpha_1 (p_{1t-1} - p_{2t-1}) + \sum_{j=1}^k \gamma_{11j} \Delta p_{1t-j} + \sum_{j=1}^k \gamma_{12j} \Delta p_{2t-j} + \varepsilon_{1t} \\ \Delta p_{2t} &= m_2 + \alpha_2 (p_{1t-1} - p_{2t-1}) + \sum_{j=1}^k \gamma_{21j} \Delta p_{1t-j} + \sum_{j=1}^k \gamma_{22j} \Delta p_{2t-j} + \varepsilon_{2t}\end{aligned}$$

are conducted. These equations can be estimated by ordinary least squares, making sure that the value of k is selected to make the residuals behave nicely. For error correction to take place we expect α_1 to be negative and α_2 to be positive. Hence, if a positive (or negative) price gap is observed in the previous period, then p_{1t} and p_{2t} adjust in order to partially fill the gap. For the two commodities to be long to the same market we thus expect at least one of α_1 and α_2 to be non-zero (with the expected sign). Testing the nullity of one of the α -coefficients can be undertaken by a standard t -test.

3.2.4 Impulse response analysis

In the previous subsections we have defined Granger-causality from p_{2t} to p_{1t} and vice versa. In principle, the definition involves all the variables of the system (in this case two) which can be considered a concern when a larger system is suspect to generate the interaction amongst the price series. In applied work it is often of interest to know the response of *one* variable to an impulse in another variable. Of course, if there is a reaction of one variable to an impulse in another variable we may call the latter causal for the former. This kind of impulse response analysis is often called *multiplier analysis*, see e.g. Luthkepohl (1990). For instance, in a system consisting of two stationary price series the dynamic effect of an increase in one price on the behaviour of the other price series seems highly relevant.

3.2.5 Practical problems and concerns

It can be tempting to conduct impulse response analysis in the implementation of the SSNIP test as sketched above. However, the practical experience with impulse response analysis is that these are sensitive to a number of factors which are critical for the evaluation of such models. There are several problems with the interpretation of estimated impulse response functions. Firstly, the estimated impulse responses appear to be very sensitive to the variables that are included in the model. Focusing on just two variables will generally *not* be sufficient to trace out the adjustment dynamics and even when extending the information set to several variables (as discussed in the next sections) the estimated impulse responses can vary dramatically and often there is no clear guide as to which variables to select. Secondly, even when a "correct" system is specified the estimated impulse responses will depend upon the ordering of the variables in the model. This is naturally a serious critique because there is nothing guiding us towards which variables to include - and in which order - of a model. And finally, the identification of the shocks to be analyzed is itself a critical problem where no guidance seems available.

The overall conclusion is thus, despite its theoretical appeal, that impulse response analysis of price data is not appropriate for the delineation of the relevant geographical market. The critique applies both to stationary and non-stationary variables in "small" as well as in "big" models.

3.3 Correlation analysis of non-stationary price series: Cointegration

So far we have examined data analysis of stationary, or $I(0)$, price series. In particular, we have noted that when time series are $I(1)$, standard methods are inadequate (unless their differences are being analyzed) since it is likely that spurious relationships amongst the processes will result by use of such methods. It occurs that in order to meaningfully identify relationships between price series that are integrated of order one, $I(1)$, the series need to share a

common stochastic trend. When integrated time series have the same stochastic trend we say the series are *co-integrated*.

In the following the properties of co-integrated processes will be discussed for the situation where just two price series are analyzed. In section 4.2 we extend the analysis to the situation with multiple (>2) price processes.

3.3.1 Cointegration analysis of pairwise (bivariate) price data

Assume we have two price series p_{1t} and p_{2t} which are both integrated of order one. Univariate representations of the series (abstracting from deterministic components) can thus be stated as follows:

$$\begin{aligned} p_{1t} &= p_{10} + \sum_{j=1}^t \varepsilon_{1j} \\ p_{2t} &= p_{20} + \sum_{j=1}^t \varepsilon_{2j} \end{aligned}$$

The single price series exhibit a stochastically trending behaviour, each with the (stochastic) trends $\sum_{j=1}^t \varepsilon_{1j}$ and $\sum_{j=1}^t \varepsilon_{2j}$, respectively. For the underlying commodities to belong to the same geographical market, we require that the prices *co-move* in the sense that the stochastic trends vanish when a particular linear combination of the data series is considered. It is natural in the present context to consider the linear combination defining the price gap $p_{1t} - p_{2t}$. If it occurs that the price differential does not contain a stochastic trend it means that there is a common stochastic trend element in p_{1t} and p_{2t} and hence the series tend to move together. Technically, the series are said to be *co-integrated*. The single price series have a stochastic trend when viewed in isolation but when looked at together the trend appears to be the same. This means that the price gap can be given the representation:

$$p_{1t} - p_{2t} = \alpha^t (p_{10} - p_{20}) + \sum_{j=1}^t \alpha^j v_{t-j}, \quad \alpha < 1.$$

which is thus a stationary, $I(0)$, process. In figure 3 two co-integrated price series are displayed together with the price differential. As can be seen the single series exhibit a non-stationary pattern whilst the difference between the two is stationary.

This suggests a natural testing procedure: First, the single series are tested to have a unit root, i.e. the series are tested to be integrated of order one. If both series cannot be rejected to be $I(1)$, the price differential is subsequently tested to be $I(1)$. If this latter hypothesis is rejected it means that p_{1t} and p_{2t} are co-integrated. These tests can be conducted by the ADF tests presented in section 2.2. We say that the series are co-integrated with co-integration vector $(1, -1)$ because the linear combination:

$$(p_{1t}, p_{2t}) \begin{pmatrix} 1 \\ -1 \end{pmatrix} = p_{1t} - p_{2t}$$

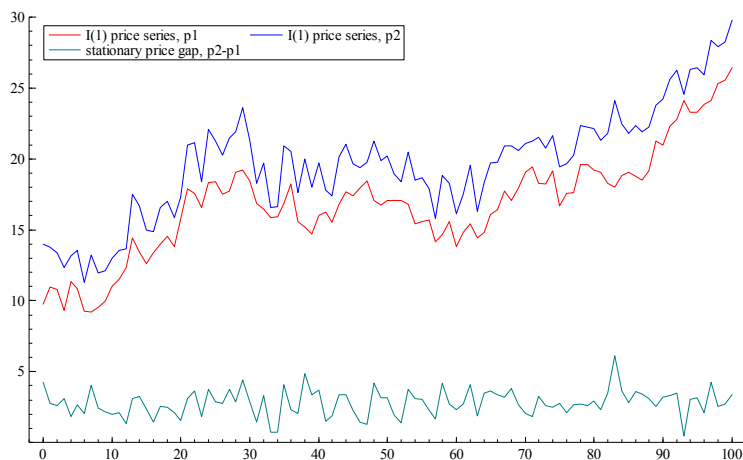


Figure 3: Example of two cointegrated I(1) price series together with their difference, the price gap.

is stationary, I(0).

When the focus of the analysis is on market delineation, testing cointegration vectors to have the above form appears the most natural because it indicates that there is a one-to-one correspondence between the price movements. More generally, however, price series can still co-move without the cointegration vector taking the form $(1, -1)$. In general, the quasi-difference $p_{1t} - bp_{2t}$ can be I(0) in which case the series cointegrate with cointegration vector $(1, -b)$.

In this situation there is still a lot of information in terms of the co-movement amongst the series, but there is no one-to-one association between the series in this case. After all, if the series *do* cointegrate, it is always of interest to examine whether the cointegration vector takes the particular form $(1, -1)$, because this relationship will have a nice interpretation in terms of the price gap.

Testing for cointegration when the cointegration vector is *not* given as a one minus one relationship needs to be undertaken by using an alternative procedure. A frequently used method is the Engle-Granger test procedure, Engle and Granger (1987).

3.3.2 The Engle-Granger test for cointegration

Assume again, that p_{1t} and p_{2t} are both I(1) which has been concluded by prior testing. When testing whether the two series cointegrate with a *general* cointegration vector a commonly used procedure is to conduct the auxiliary regression

$$p_{1t} = a + bp_{2t} + v_t$$

and test whether the residual series v_t is integrated of order zero. When this occurs the series cointegrate with cointegration vector $(1, -b)$.

The testing of the order of integration can be accomplished by use of the ADF test discussed in section 2.2 and accounting for the practical problems and concerns associated with that discussion. However, the critical values of the Dickey-Fuller t -test statistic cannot be used in the present situation because the (potential) cointegration vector has been *estimated*. Instead the critical values associated with cointegration regressions reported in McKinnon (1991) need to be used. The procedure is frequently referred to as the Engle-Granger two-step procedure for obvious reasons.

3.3.3 The Johansen procedure for the analysis of cointegration

The Engle-Granger procedure can be considered a single equation procedure for the testing of cointegration. In this sense it has certain limitations if in fact the system of price series is generated simultaneously. An improved inference and estimation procedure can be used by taking a systems approach to cointegration analysis. This appears to be relatively simple for the two variable case, but when extending this to multiple price processes it occurs that the systems approach will gain much additional insight into the complicated dynamic interaction existing in simultaneous multivariable price systems. Demonstrating the nature of the cointegration model in the bivariate case will be very instructive and highly adequate for much analysis of the pairwise movement of price series.

In this section the intuition behind the systems approach to cointegration analysis (frequently referred to as the Johansen procedure, see Johansen (1988, 1991, 1995)) is briefly described. In section 4.2.4 a more elaborate and detail discussion will be given for multiple price processes.

A common way of representing multiple time series is by virtue of a vector autoregressive (VAR) model. For two $I(1)$ price series a $VAR(1)$, for instance, can be written:

$$\begin{aligned} p_{1t} &= m_1 + \delta_{11}p_{1t-1} + \delta_{12}p_{2t-1} + \varepsilon_{1t} \\ p_{2t} &= m_2 + \delta_{21}p_{1t-1} + \delta_{22}p_{2t-1} + \varepsilon_{2t} \end{aligned}$$

where

$$\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{pmatrix} \right)$$

In this case p_{1t} and p_{2t} are determined simultaneously (the covariance between the error terms is non-zero) and both price series depend upon the lagged value of each of the two series. A compact way of writing this is:

$$\begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} \begin{pmatrix} p_{1t-1} \\ p_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

or, using vector notation,

$$\mathbf{p}_t = \mathbf{m} + \boldsymbol{\delta}\mathbf{p}_{t-1} + \boldsymbol{\varepsilon}_t.$$

An equivalent way of writing the VAR(1) model is

$$\Delta \mathbf{p}_t = \mathbf{m} + \mathbf{\Gamma} \mathbf{p}_{t-1} + \boldsymbol{\varepsilon}_t$$

It occurs that if p_{1t} and p_{2t} are co-integrated, then the matrix $\mathbf{\Gamma}$ can be written as the product of two matrices, i.e.

$$\mathbf{\Gamma} = \boldsymbol{\alpha} \boldsymbol{\beta}' = \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \end{pmatrix} \begin{pmatrix} 1 & -\beta_{21} \end{pmatrix}$$

and the VAR model can thus be reparametrized as

$$\begin{pmatrix} p_{1t} - p_{1t-1} \\ p_{2t} - p_{2t-1} \end{pmatrix} = \Delta \begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} \alpha_{11} \\ \alpha_{21} \end{pmatrix} \begin{pmatrix} 1 & -\beta_{21} \end{pmatrix} \begin{pmatrix} p_{1t-1} \\ p_{2t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \quad (11)$$

or

$$\Delta \mathbf{p}_t = \mathbf{m} + \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{p}_{t-1} + \boldsymbol{\varepsilon}_t \quad (12)$$

This representation is frequently referred to as an *error correction model*. The representation has the following characteristics.

Although $\mathbf{p}_t = (p_{1t}, p_{2t})'$ is I(1), the single terms in (11) and (12) are stationary I(0). $\Delta \mathbf{p}_t$ is so by definition, and $\boldsymbol{\beta}' \mathbf{p}_{t-1}$ is so because it represents the cointegrating relation, that is, $\boldsymbol{\beta}' \mathbf{p}_{t-1} = p_{1t-1} - \beta_{21} p_{2t-1}$ is stationary.

Observe, that the cointegrating relation can be interpreted as an attractor relation towards which the two series tend to be attracted. The attraction between the prices exists because the products belong to, and hence delineate, a common geographical market. Hence p_{1t} and p_{2t} adjust in such a way that in equilibrium $p_{1t} = \beta_{21} p_{2t}$. The variable $p_{1t} - \beta_{21} p_{2t}$ can thus be interpreted as the deviation from equilibrium in period t . The dynamics of the model is such that by virtue of the adjustment parameters in $\boldsymbol{\alpha}$, ($-1 < \alpha_{11} < 0, 0 < \alpha_{21} < 1$), a positive (negative) disequilibrium error causes Δp_{1t} to be negative (positive) and Δp_{2t} to be positive (negative). In other words, the adjustment is such that the price series move back on track over time in response to being out of equilibrium. The dynamics of the model is illustrated in figure 4.

The actual adjustment and dynamic reaction of the price series in response to shocks will naturally depend upon the parameters of the model. In particular, zero values of α_{11} or α_{21} contain valuable information about the dynamic adjustment process. Assume for instance that a statistical test cannot reject that $\alpha_{21} = 0$ and that $\beta_{21} = 1$. In this case the price of commodity 1, p_{1t} , is governed by that of p_{2t} :

$$\Delta \begin{pmatrix} p_{1t} \\ p_{2t} \end{pmatrix} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} + \begin{pmatrix} \alpha_{11} \\ 0 \end{pmatrix} (p_{1t-1} - p_{2t-1}) + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$

In other words, prices in market 1 are determined by the prices in market 2. If the price gap was positive in the past period (and hence p_{1t} was too

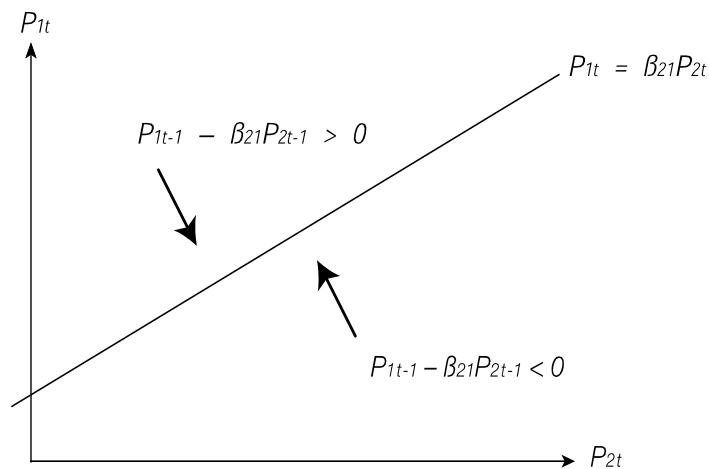


Figure 4: Equilibrium correction towards the attractor in a bivariate cointegrated system.

high compared to its equilibrium level p_{2t}) then market forces in a "common" integrated market will tend to reduce the price p_{1t} in the subsequent period. If $\alpha_{21} > 0$ we would see both prices move in opposite directions towards each other in order to eliminate the disequilibrium.

The statistical analysis of problems associated with the above model will be thoroughly discussed in section 4.2.4. However, as can be seen from the presentation the empirical problem concerns the question of *whether* $\mathbf{\Gamma}$ factorizes as $\mathbf{\Gamma} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, and hence the price series appear to be cointegrated. It can be shown that this problem corresponds to a kind of multivariate Dickey-Fuller test. Next, it is of relevance to test hypotheses about the α and β parameters. In the present context of questions in relation to market delineation, zero restrictions on α coefficients and unit restrictions on the β coefficients appear especially relevant.

It is of interest to note that when the price series cointegrate there will be Granger-causality in at least one direction. This means that in (11) either α_{11} or α_{21} (or both) will be non-zero.

3.3.4 Practical problems and concerns

Many of the problems associated with univariate testing of the number of unit roots equally apply in the context of pairwise comparison of $I(1)$ series. In particular, the considerations of which deterministics to employ when using ADF tests on price differentials are of equal relevance. Also, the problems associated with lag truncation, seasonality, and outlying observations equally apply.

Historically, the Engle-Granger test has been much used in empirical appli-

cations. However, despite its simplicity, theoretical studies indicate that the procedure suffers from a number of caveats, especially due to simultaneity problems, and for practical applications nowadays the Johansen procedure for VAR models is preferable. A comprehensive treatment of this procedure, its pros and cons, will be given in section 4.2.4.

An important property of cointegrated systems concerns the fact that when price series are found to cointegrate, then this property is invariant to the information set used in the study. This limits the need for extending the information set when particular price series are compared even though the inclusion of seasonal dummy variables and the appropriate number of lags in the truncation of the VAR model is of relevance with respect to the statistical properties of the cointegration techniques.

4 Analysis of multiple (>2) price series

So far it has been assumed that only two price series are compared. In many cases such pairwise comparisons will be necessary due to limited data availability but pairwise comparisons is also of separate interest even when there is access to many more price variables. If sufficient data observations are available for multiple price series a careful analysis of the series is naturally called for. In the following we will again distinguish between comparison of price series being integrated of order zero or one. Price series being integrated of different orders cannot be associated with commodities belonging to the same geographical and product market, as we have previously argued.

4.1 Contemporaneous correlation analysis of stationary price series

4.1.1 Simple and Partial Correlations

Simple and partial correlation, by their definition, are measures associated with bivariate relations amongst series. However, the appropriate conditioning on "third" variables relates the measures of association in a multivariate context. These issues are already discussed in section 3.1.1. However, the previous discussion is altered when multiple price series are available by the fact that in addition to correcting the price series p_{1t} and p_{2t} , for instance, for some exogenous factors $x_{1t}, x_{2t}, \dots, x_{kt}$, further correction can be made by extending the set of "third" variables to include $p_{3t}, p_{4t}, \dots, p_{qt}$, i.e. some other price series. In so doing, a high simple coefficient of correlation and a low partial coefficient of correlation will indicate that the price series may co-move due to their co-movement with other price series or some exogenous variables. In fact, the partial correlations can be calculated for a range of conditioning variables which will thus help in identifying the possible groups of variables through which the correlation structure will follow.

4.1.2 Principal components and factor analysis

One of the difficulties concerning the estimation of simple and partial correlation coefficients when multiple price series exist is that it can be hard to judge which price series co-move considered as *a group*, and hence can be interpreted to belong to the same underlying geographical market. Principal components and factor analysis partially accounts for this problem because such an analysis will provide an idea of the number of separate factors that generate the price data. If there is just a single factor it may indicate the existence of a single geographical market. If there are several factors one may consider the price series to be governed in separate markets. The case of a single geographical market is clearly of most practical interest in the conduction of principal components and factor analysis because it is only in this case the common market can really be identified unproblematically. When several markets are indicated to exist, it can be a hard empirical problem to identify which commodities belong to the separate markets and hence will be of little practical relevance. One concern regarding the use of principal components and factor analysis is that there is no real testing of the number of markets (factors) can be given. The problem is much like the problem associated with the interpretation of correlation coefficients: When can we judge that correlation is sufficiently high for market delineation to exist?

Note that principal components and factor analysis are only valid when all of the single price series are integrated of zero and hence are stationary. For I(1) series the analysis can be conducted for the price changes, e.g. the growth rates when the price series are log transformed.

Principal components analysis. Principal components analysis, see e.g. Johnson and Wichern (2002) concerns the problem of explaining the variance-covariance structure in a set of multiple (price) series through a few *linear* combinations of the original variables. The general objective is to 1) reduce the variation in the data to a limited number of separate components and 2) to interpret these. For the market delineation problem, the components can be considered a partitioning of the price variability into a limited number of separate origins with the interpretation of these belong to separate geographical markets.

Although q price series are required to reproduce the total system variability, often much of this variability can be accounted for by a small number, k , of the principal components. If so, there is (almost) as much information in the k components as there are in the original q price series. In a sense, the k principal components can then replace the initial q price variables, and the original data set, consisting of T observation points on q variables, is reduced to one consisting of T observations of k principal components. However, only when a single component appears to represent the data is an interpretation in terms of market delineation most useful.

Technically, the principal components can be calculated in the following way. The principal components are (particular) linear combinations of the q price series $p_{1t}, p_{2t}, \dots, p_{qt}$ and it occurs that these solely depend on the covariance

(correlation) matrix of the price series. The calculation of the principal components does not require that the data is normally distributed but it is needed (approximately) when inference concerning the components need to be made.

Consider the following linear combinations of the price series:

$$\begin{aligned} F_1 &= l'_1 \mathbf{P} = l_{11}p_1 + l_{21}p_2 + \dots + l_{q1}p_q \\ F_2 &= l'_2 \mathbf{P} = l_{12}p_1 + l_{22}p_2 + \dots + l_{q2}p_q \\ &\vdots \\ F_q &= l'_q \mathbf{P} = l_{1q}p_1 + l_{2q}p_2 + \dots + l_{qq}p_q \end{aligned}$$

The variance of F_i is given as

$$Var(F_i) = l'_i \boldsymbol{\Sigma} l_i \quad (13)$$

where $\boldsymbol{\Sigma}$ is the variance-covariance matrix of $p_{1t}, p_{2t}, \dots, p_{qt}$. The principal components are those *uncorrelated* linear combinations F_1, F_2, \dots, F_q whose variances in (13) is as large as possible. The first principal component is the linear combination with the maximum variance, i.e. the linear combinations l_1 maximizing $Var(F_1) = l'_1 \boldsymbol{\Sigma} l_1$ subject to the normalization that l_1 has unit length in the following sense: $l'_1 l_1 = 1$.

The second principal component is given by the linear combinations $F_2 = l'_2 \mathbf{P} = l_{12}p_1 + l_{22}p_2 + \dots + l_{q2}p_q$ that maximizes $Var(F_2) = l'_2 \boldsymbol{\Sigma} l_2$ subject to $l'_2 l_2 = 1$ and $Cov(F_1, F_2) = 0$. In other words, the second principal component is that linear combination which has the highest explanatory power but is uncorrelated with the first principal component. The above procedure proceeds recursively for the remaining principal components.

It occurs that the principal components can be found by calculating the eigenvalue of the covariance matrix $\boldsymbol{\Sigma}$ associated with the price series. If the q (ranked) eigenvalues are denoted

$$\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_q \geq 0$$

then the linear combinations l_i defining the i 'th principal component is given by the associated eigenvector, which we can denote $\mathbf{e}'_i = (e_{1i}, e_{2i}, \dots, e_{qi})$. Note that if some elements in \mathbf{e}_i are equal to zero, then this suggests that the price series associated with these entries are uncorrelated with the other components. In practice, however, it can be difficult to identify such structures.

An important result is that the total variation of the price series can be written as

$$\sum_{i=1}^q Var(p_{it}) = \lambda_1 + \lambda_2 + \dots + \lambda_q$$

and consequently, the proportion of the total variance explained by the k th principal component is

$$\frac{\lambda_k}{\lambda_1 + \lambda_2 + \dots + \lambda_q}, \quad k = 1, 2, \dots, q. \quad (14)$$

For instance, if most (80 or 90 %, say) of the total population variance can be attributed to the first principal component, then this can be taken as indicative of a high degree of co-movement amongst the price series. On the other hand, if the first principal component is relatively small, it may indicate a lesser degree of market integration. The practical problem lies in the determination of the threshold for which market delineation can be assumed, unfortunately there is little guidance in this respect from the statistical analysis but a choice of 80 or 90 % of the quantity (14) seems to be a pragmatic choice.

The above presentation has been in terms of population quantities, but sample equivalents can be straightforwardly defined.

Practical problems and concerns. The limitations of the principal components analysis are several. First, as indicated there is no clear answer to the question of when a component is sufficiently strong to justify market integration and hence the tool should be used as a pragmatic device, just like standard correlation analysis. It is only when most of the statistical variation in the data can be described by a single component that the application of the procedure to market delineation is most useful. In practice, we expect some degree of co-variation of prices, even when products do not belong to the same geographical market. Requiring the separate components to be orthogonal (and hence uncorrelated) seems to be a too strong requirement.

Also one should be aware that the scaling of the variables may matter (because the variances will change) and hence the principal components based on the covariance matrix rather than on the correlation matrix need not be the same. Such dependency of scaling is naturally critical. Hypothesis testing regarding the eigenvectors and hence the form of the principal components can be made in principle, but normally rather many observations will be needed for validity of the technique.

A final comment concerns the correction of the influence from exogenous factors such as seasonality. A dominating first principal component can result from the influence of a significant external factor and hence, in practical situations we would like to correct for this. Prior correction of the price series for exogenous factors x_{1t}, x_{2t}, \dots etc. can be made using the procedure described in section 3.1.1.

Factor analysis. In principle, factor analysis can be considered a next step, following extraction of principle components. The essential purpose of factor analysis is to describe, if possible, the relationships amongst many variables in terms of a few underlying, but unobservable, random quantities called *factors*. Basically, the factor model is motivated by the following argument. Suppose variables can be grouped by their correlations. That is, all variables within a particular group are highly correlated among themselves but have relatively small correlations with variables in a different group. It is conceivable that each group of variables represents a single underlying construct, or factor, that is responsible for the observed correlations. For example, price correlations in

separate markets can constitute such separate factors and if just a single factor is given, it can be interpreted as a justification for all the price series delineating a single market for a particular good.

There are several ways to specify the factors. However, it seems natural to build upon the outcome from the principal components analysis and making the estimated m primary orthogonalized components represent the relevant factors. The factor model can now be written as

$$\begin{aligned}
 p_1 - \mu_1 &= l_{11}F_1 + l_{12}F_2 + \dots + l_{1m}F_m + \varepsilon_1 \\
 p_2 - \mu_2 &= l_{21}F_1 + l_{22}F_2 + \dots + l_{2m}F_m + \varepsilon_2 \\
 &\vdots \\
 p_q - \mu_q &= l_{q1}F_1 + l_{q2}F_2 + \dots + l_{qm}F_m + \varepsilon_q
 \end{aligned} \tag{15}$$

where μ_i are the means of each price series, and F_j , $j = 1, 2, \dots, m$ is the j th price factor representing market j . ε_i , $i = 1, 2, \dots, m$ are price specific error terms. Hence price variation is due to a factor specific component and a residual component. Note, that in (15) m indicates the number of separate markets but nothing is said concerning which price series belong to the separate markets. This needs further testing. For instance, if $m = 2$ it may be of interest to impose zero restrictions on the parameters l_{ij} (frequently referred to as the factor loadings) in order to see whether particular price series are affected by particular price factors. After all, the estimated values of the factor loadings l_{ij} are indicative of the relative importance of the separate price factors in the determination of prices of the goods of different origin. If it has been found that $m = 1$, e.g. by principal components extraction, the interest in estimation a model such as (15) lies in the determination of whether some l_{i1} coefficients can be tested to zero and thus indicating that the p_i th price is unaffected by common price factors of the other commodities.

Practical problems and concerns. The literature on the estimation of factor models and the choice of factor representing combinations of the data is rather huge and it would take us too far to go into a further debate on this issue. Here we just note that such models *can* be estimated, typically by maximum likelihood methods. Much soft-ware implements these methods. The real problem is to determine *the number of factors* in a multivariable price process, and as it has been argued the testing device in this respect is not well developed. Principal component analysis can be indicative of the number of factors, and factor analysis can be useful in understanding the adjustment mechanisms applying in such a model.

A strategy that can be considered in principal component and factor analysis is the following: Perform a principal component analysis which is particularly appropriate for a first pass through the data. Next, a maximum likelihood estimation is made of a factor model with the primary principal components given as the factors. The factor analysis solutions are next compared and it is checked whether the factor loadings group in a systematic way across the price

series. It can be useful to consider a different number of factors in the factor analysis to check the robustness of the results.

A serious limitation of principal component and factor analysis concerns the fact that no dynamics are allowed for in the specification. In many cases we would expect dynamics to play an important role in the transmission of price changes across and within separate geographical markets. To this problem we next turn although the approach appears somewhat different.

4.1.3 Granger causality

Granger causality test. Tests for Granger-causality in the presence of multiple time series largely follows the test in the bivariate case except that now the single price series are properly adjusted for the influence of lagged values of "third" variables, c.f. the discussion above. That is, either the single price series p_{1t} and p_{2t} are corrected for the influence of lagged values of $p_{3t}, p_{4t}, \dots, p_{qt}$ (and any other *exogenous* factors $x_{1t}, x_{2t}, \dots, x_{kt}$) and then using the residual series in a standard Granger causality test regression. Alternatively, all the series upon which the conditioning is made are included in the auxiliary regression. When multiple price series are considered jointly the Granger test captures the pairwise dynamic interaction of the variables after appropriate conditioning upon other price series. However, a joint test where lagged values of multiple price series are tested via a standard F -test of linear restrictions is perfectly valid. But in this case it cannot be identified directly whether one or the other price series causes the price variable at hand. Only the joint causality can potentially be identified.

Error correction models for stationary variables. In section 3.2.3 we advocated for the use of error correction models for stationary processes as a convenient framework for interpreting Granger causality in an attractive way. This method can be easily extended to multiple series, i.e. by the allowance for several price gaps in the estimation equations.

4.2 Cointegration analysis of multiple non-stationary price series

In section 3.3.3 an introduction to the notion of cointegration in systems of equations was given for the situation where two price series are compared. In this section we assume that multiple (>2) price series are available and hence we want to model the joint association amongst these series. We will also relate the statistical properties of such systems to the economic interpretation of the underlying commodities potentially belonging to a common geographical market. The maintained assumption in the sequel is that the single price series are integrated of order one which can be tested, of course, given the procedures presented in section 2.

4.2.1 Intuition behind cointegration analysis in systems

Assume we have q price series $p_{1t}, p_{2t}, \dots, p_{qt}$ which are all integrated of order one, $I(1)$. When viewed in isolation each of these series can be characterized as having a stochastic trend. However, like in the $q = 2$ case, it can occur that the trends, or at least some of the trends, move very closely such that common stochastic trends are shared amongst the series. When this happens some of the series co-integrate whereby linear combinations (price gaps, for instance) of the price series appear to be stationary. The number of stationary relations existing between the q variables is denoted the *cointegration rank* and we denote this number r . Each of the stationary relations can be interpreted as *attractor* relations that are tied together via economic forces, for instance by the fact that commodities belong to the same geographical market whereby price differences are eliminated through arbitrage trading. It can be shown that for q variables and r cointegration relations the number of common stochastic trends is given by $q - r$. Assume for instance, that we have $q = 3$ price series, and it can be found that $r = 2$. This means that $q - r = 3 - 2 = 1$ common stochastic trend drives all of the three price series. In other words, the price trend in each market is the same because the underlying commodities belong to the same geographical market. Another example is when $q = 4$ and $r = 2$. In this case there are two attractor relations and two common stochastic trends driving the four price levels. A natural way of interpreting this possibility is that the four prices delineate into two separate markets which are each driven by their own price trends. If the price trends in the two markets were in fact co-moving in a stationary sense it would mean that the markets were really a unified market, and hence r would equal 3 rather than 2. In figures 4 and 5 the above two situations are illustrated. Each figure displays the single series as well as the stationary relations which are designed to be the relevant price differentials.

The above intuition suggests a natural outline for the empirical analysis of attractor relations and market delineation for multiple price processes.

- Identify that the single series are integrated of order one
- By use of a statistical test, estimate the number r indicating the number of attractor relations, and hence the number of separate markets. When r is identified it means that $q - r$ price trends drive the markets.
- Check whether the price differentials can represent the attractor relations, i.e. is it such that $p_{it} - p_{jt}$ for $i \neq j$ for instance is one of the stationary relations even though the single price processes are non-stationary.

In the sequel a thorough description of the empirical analysis of these problems will be given.

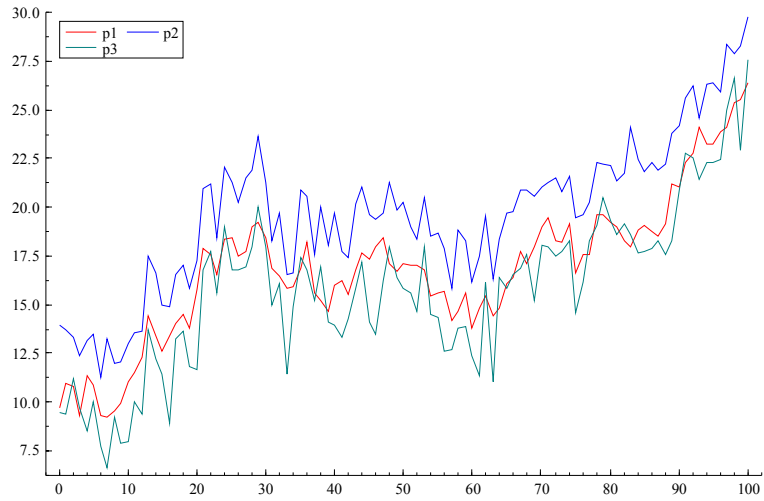


Figure 5: Cointegrated series with $q = 3$, $r = 2$, and hence $q - r = 1$ common stochastic trend.

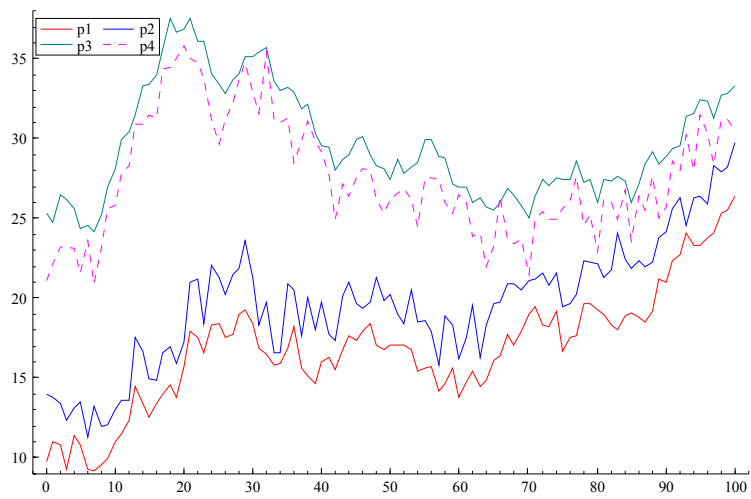


Figure 6: Cointegrated series with $q = 4$, $r = 2$, and hence $q - r = 2$ common stochastic trend.

4.2.2 A formal definition of cointegration and error correction in simultaneous price systems

To make the notation clear we define formally what is meant by cointegration in a multivariable context. The original definition by Engle and Granger, (1987) was made for general orders of integration. Here we assume the price series are integrated of order one.

Definition of cointegration. We say that the $q \times 1$ vector process $\mathbf{P}_t = (p_{1t}, p_{2t}, \dots, p_{qt})'$ is cointegrated of order 1,1 ($\mathbf{P}_t \sim CI(1, 1)$) if the single price series in $\mathbf{P}_t \sim I(1)$ whilst there exists a $q \times r$ matrix β such that $\beta' \mathbf{P}_t \sim I(0)$. We denote β the matrix of cointegration parameters/vectors and the total number of stationary cointegrating relations equals r .

Cointegration dictates that although the single series are fluctuating wildly according to unit root (integrated) processes, there exist some linear combinations of the data where the stochastic variation is tied together due to e.g the presence of some economic attractors (for instance, the market forces which through arbitrage delineate a common geographical market).

The relations

$$\beta' \mathbf{P}_t$$

are the cointegrating relations defining the attractor relations. The actual values of $\beta' \mathbf{P}_t$ can be interpreted as the *deviations from the "equilibrium relations"*. Non-zero values therefore suggest that some kind of adjustment in the prices can be expected to eliminate price differences for instance.

Assume that $q = 3$, and $r = 2$. In this case $\mathbf{P}_t = (p_{1t}, p_{2t}, p_{3t})' \sim I(1)$ and the cointegrating relations are given by:

$$\begin{aligned} \beta' \mathbf{P}_t &= \begin{pmatrix} 1 & \beta_{12} & \beta_{13} \\ \beta_{21} & 1 & \beta_{23} \end{pmatrix} \begin{pmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{pmatrix} \\ &= \begin{pmatrix} p_{1t} + \beta_{12}p_{2t} + \beta_{13}p_{3t} \\ \beta_{21}p_{1t} + p_{2t} + \beta_{23}p_{3t} \end{pmatrix} = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} \end{aligned}$$

Suppose the parameter values are such that the system can be rewritten as

$$\begin{aligned} \beta' \mathbf{P}_t &= \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{pmatrix} \\ &= \begin{pmatrix} p_{1t} - p_{2t} \\ p_{2t} - p_{3t} \end{pmatrix} = \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} \end{aligned}$$

whereby the stationary relations are given as the price differences. Observe that two ($r = 2$) cointegrating relations exist: z_{1t} and z_{2t} , i.e. the economically interesting attractor relations are the price gaps.

Error Correction. An interpretation can be given as follows: If a positive (negative) equilibrium error (price difference) is observed in the previous period for the first relation, $z_{1t-1} > 0$, ($z_{1t-1} < 0$) then we expect that p_{1t} decreases (increases) in the subsequent period and that p_{2t} increases (decreases) in the subsequent period, i.e. the two series adjust in such a way that the gap $p_{1t} - p_{2t}$ tends to be eliminated in the following period. When such adjustment mechanisms exist we say the model is *error correcting*.

In fact, when cointegration exists, it means that the dynamics of the model can be represented as an *error correction model*. In the above example this can be written as:

$$\begin{aligned} \begin{pmatrix} \Delta p_{1t} \\ \Delta p_{2t} \\ \Delta p_{3t} \end{pmatrix} &= \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} p_{1t-1} \\ p_{2t-1} \\ p_{3t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \\ &= \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{pmatrix} \begin{pmatrix} p_{1t} - p_{2t} \\ p_{2t} - p_{3t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{pmatrix} \end{aligned} \quad (16)$$

where the α coefficients determine *how much* the single price series react in response to price differences across the markets.

Observe, that in the above case there are $q = 3$ commodities and price series. p_{1t} and p_{2t} are tied together and p_{2t} and p_{3t} are tied together. However, considering the linear combination $z_{1t} + z_{2t} = (p_{1t} - p_{2t}) + (p_{2t} - p_{3t}) = p_{1t} - p_{3t}$ it is seen that p_{1t} and p_{3t} are also tied together in the sense that $p_{1t} - p_{3t}$ is stationary (because z_{1t} and z_{2t} are stationary). The result also follows naturally because the three series are driven by $q - r = 1$ common stochastic trend. As indicated in the example, each of the commodities can thus be considered to belong to the same geographical market.

A general representation. In general, the *Error Correction Model* can be written in the compact form:

$$\Delta \mathbf{P}_t = \boldsymbol{\alpha} \boldsymbol{\beta}' \mathbf{P}_{t-1} + \sum_{j=1}^k \boldsymbol{\Gamma}_j \Delta \mathbf{P}_{t-j} + \boldsymbol{\varepsilon}_t \quad (17)$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $q \times r$ matrices and $\boldsymbol{\Gamma}_j$ is $q \times q$. The columns in $\boldsymbol{\beta}$ are denoted the *cointegration vectors*. Note that in practice adjustments in terms of lagged price changes are needed.

The empirical problem is

- Determine r , the number of cointegrating relations
- Put identifying restrictions onto $\boldsymbol{\beta}$ to interpret the cointegrating relations economically, typically in terms of price differences
- Test restrictions on $\boldsymbol{\beta}$ and possibly test restrictions on $\boldsymbol{\alpha}$ to see which variables respond in reaction to disequilibrium

4.2.3 Engle Granger test for cointegration - single equation approach

In section 3.3.2 the Engle-Granger two-step procedure was presented for the case with $q = 2$. The procedure generalizes naturally to the case with $q > 2$. However, because the procedure is only valid for a cointegration rank of $r = 1$ it is suggested in the present context to apply the previously described method for $q = 2$ for each pair of prices in turn. Similarly, the price differences $p_{it} - p_{jt}$, $i \neq j$, can be tested for all combinations of the price series to have a unit root according to the procedure outlined in section 3.3.1.

If all of the pairwise comparisons result in each of them being cointegrated, then the number of cointegration relations is given as $q - 1$ and hence all price series are governed by one common stochastic price trend and each commodity will belong to the same market.

The Engle-Granger procedure is not an optimal procedure because all the price series are not addressed simultaneously. The so-called Johansen procedure is a joint procedure which addresses the joint interaction of the series. This is a clear advantage although for practical purposes one should be aware that increased data requirements in terms of sample observations are needed.

4.2.4 The Johansen procedure for systems of variables

The VAR model for I(1) variables. The starting point for the systems analysis of cointegration is a VAR(k) model for the *levels* of the price series

$$\mathbf{P}_t = \mathbf{m} + \mathbf{A}_1\mathbf{P}_{t-1} + \mathbf{A}_2\mathbf{P}_{t-2} + \dots + \mathbf{A}_k\mathbf{P}_{t-k} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{P}_t = (p_{1t}, p_{2t}, \dots, p_{qt})$ and \mathbf{A}_i are $q \times q$ matrices with the parameters associated with each lag of the price series, and \mathbf{m} is a vector with the intercept terms of each equation. It is assumed that the error term in the VAR model is normally distributed with a covariance matrix $\boldsymbol{\Sigma}$:

$$\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \boldsymbol{\Sigma})$$

If data is seasonally varying it can also be considered to include seasonal dummy variables. Sometimes also a trend is included; we return to this complexity later. A practical concern is to make sure that k is chosen such that the errors in each equation have no autocorrelation and preferably are also normally distributed.

After k has been determined the model is put on so-called error correction model form (most computer packages will do this automatically):

$$\Delta\mathbf{P}_t = \Pi\mathbf{P}_{t-1} + \sum_{j=1}^{k-1} \boldsymbol{\Gamma}_j\Delta\mathbf{P}_{t-j} + \boldsymbol{\varepsilon}_t \quad (18)$$

Balancing requires that each side of the equality are integrated of the same order even though both differences and levels appear in the equation. This gives rise to a number of possibilities which determine the number of cointegrating relations and thus the number of common stochastic price trends in the data.

The Cointegrated VAR model for I(1) variables. It occurs that the cointegration properties of the data are given by conditions concerning the matrix $\mathbf{\Pi}$. Technically we have to address (and estimate) the rank of the matrix. The following possibilities arise:

- a) Rank $\mathbf{\Pi} = q$ (full rank) $\Rightarrow \mathbf{P}_t$ is stationary, I(0).
- b) Rank $\mathbf{\Pi} = 0$ (zero rank) $\Rightarrow \Delta\mathbf{P}_t$ is stationary, I(0), (or \mathbf{P}_t is I(1)).
- c) Rank $\mathbf{\Pi} = r < q$ (reduced rank) $\Rightarrow \mathbf{\Pi P}_{t-1}$ is stationary I(0). In this case there are r cointegrating relations and thus $q - r$ separate price trends.

The case "c" is obviously most interesting because this is the situation where attractor relations exist between the price levels and hence determine market delineation.

The practical problem is thus to determine r . Technically this means that an eigenvalue problem has to be solved because the rank of a matrix ($\mathbf{\Pi}$ in this case) is determined by the number of non-zero eigenvalues of the matrix. The associated empirical problem is to determine by some test the number of eigenvalues equal to zero. This number corresponds to the number of common stochastic trends amongst the price variables, that is $q - r$.

Assume now, that the rank of $\mathbf{\Pi}$ equals r ($0 < r < q$). In this case

$$\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$$

where $\boldsymbol{\alpha}, \boldsymbol{\beta}$ are both $q \times r$.

This yields the Error Correction Model formulation of the model in the case of cointegration (17) which is rewritten here as

$$\Delta\mathbf{P}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{P}_{t-1} + \sum_{j=1}^{k-1} \boldsymbol{\Gamma}_j \Delta\mathbf{P}_{t-j} + \boldsymbol{\varepsilon}_t.$$

The error correction model given in (16) is a special case of this general error correction model formulation.

Test of Cointegration Rank. Testing the cointegration rank can be undertaken by use of the so-called Johansen procedure. The procedure is implemented in most modern econometric software such as PcGive, MicroFit, Eviews, and TSP. We will not give a detailed description of the technicalities in the present context but rather provide some practical advice and intuition in the implementation of the procedure. Stepwise, the procedure reads:

- Estimate the VAR model (18) for a given value of k , making sure that the errors follow approximately a normal distribution. It is advisable to start with a relatively big model (depending upon the number of observations) and testing down the model until the design criteria are no longer satisfied. It is recommendable to have as simple a model as possible (in terms of the choice of k), however, it is *extremely important that no residual autocorrelation is left in the residuals*.

- The model parameters are estimated by the Maximum Likelihood method. The software implementing the Johansen cointegration analysis will next estimate the eigenvalues of the Π matrix in (18) by so-called reduced rank regression. The solution yields q *estimated eigenvalues* which can be ranked such that:

$$1 > \hat{\lambda}_1 > \hat{\lambda}_2 > \dots > \hat{\lambda}_r > \dots > \hat{\lambda}_q \geq 0$$

- Associated with the eigenvalues are the q *estimated eigenvectors*

$$\hat{v} = (\hat{v}_1, \hat{v}_2, \dots, \hat{v}_r, \dots, \hat{v}_q)$$

Now, because the rank of the Π -matrix is given by the number of non-zero eigenvalues, the empirical problem is given by that of testing the $q - r$ smallest eigenvalues to equal zero:

$$1 > \underbrace{\lambda_1 > \lambda_2 > \dots > \lambda_r}_{r \text{ eigenvalues } \neq 0} > \underbrace{0 = 0 \dots = 0}_{q-r \text{ eigenvalues } = 0}$$

- The test of the hypothesis that the $q - r$ smallest eigenvalues equal zero, that is: $H_0 : \lambda_{r+1} = \lambda_{r+2} = \dots = \lambda_q = 0$ can be conducted using the Likelihood Ratio principle. The null hypothesis can also be written as:

$$\begin{aligned} H_0 & : \text{rank}\Pi = r && (r \text{ cointegration vectors and hence } q - r \text{ common stochastic trends}) \\ H_0 & : \text{rank}\Pi > r && (\text{more than } r \text{ cointegration vectors and hence less than } q - r \\ & && \text{common stochastic trends}) \end{aligned}$$

The LR-test statistic (the trace-test) reads:

$$-2LnLR = -T \sum_{i=r+1}^q \ln(1 - \hat{\lambda}_i)$$

The distribution of the test statistic does not follow a χ^2 distribution as LR tests normally do. Instead the distribution follows a multivariate version of the Dickey-Fuller distribution. The critical values are automatically reported in most software implementing the Johansen procedure. The testing procedure is sequential: First $r = 0$ is tested. If this is rejected one proceeds to testing $r = 1$, and so forth, until the value of r for which the test does not reject. This determines r , and hence the number of cointegrating relations, (as well as the the number of price trends generating the entire system of price series).

- Assume that r is now determined. Corresponding to the r largest eigenvalues the associated eigenvectors can be found. These correspond to estimates of the cointegration parameters:

$$\hat{\beta} = (\hat{v}_1, \hat{v}_2, \dots, \hat{v}_r).$$

- Estimates of α follow directly from the procedure.

Testing hypothesis on α and β . After the number of cointegrating relations r has been determined this is *fixed* for all subsequent analysis. Likelihood ratio tests on α and β conditional on a fixed value of the cointegration rank are all χ^2 distributed which of course eases hypothesis testing. When testing restrictions on α and β it should be noted that *only overidentifying restrictions of the parameters can be tested*.

Assume the restrictions to be tested look like:

$$\begin{aligned} H_0 & : \alpha = \alpha_0 \\ H_A & : \alpha \neq \alpha_0 \end{aligned}$$

or

$$\begin{aligned} H_0 & : \beta = \beta_0 \\ H_A & : \beta \neq \beta_0 \end{aligned}$$

The procedure for calculating the tests reads:

- For a fixed value of r , estimate the model *without restrictions*, and calculate the eigenvalues: $\{\widehat{\lambda}_1, \widehat{\lambda}_2, \dots, \widehat{\lambda}_r\}$, and the unrestricted likelihood value $\ln L$.
- For the same value of r , estimate the model *with restrictions*, and calculate the restricted eigenvalues $\{\widehat{\lambda}_1^*, \widehat{\lambda}_2^*, \dots, \widehat{\lambda}_r^*\}$, and the restricted likelihood value, $\ln L^*$.
- The LR-test statistic now reads:

$$\begin{aligned} -2 \ln LR & = -2(\ln L^* - \ln L) = T \sum_{i=1}^r \ln \left(\frac{(1 - \widehat{\lambda}_i^*)}{(1 - \widehat{\lambda}_i)} \right) \\ & \sim \chi^2(\text{df}=\#(\text{overidentifying}) \text{ restrictions}) \end{aligned}$$

- Most soft-ware automatically calculates the degrees of freedom.

The relevant hypotheses in the testing of market delineation. The cointegration vectors are the columns in the β vector, and these span the so-called cointegration space. This space is identified. However, the single cointegration vectors are unidentified and further identifying restrictions are needed in order to interpret and test the single cointegrating relations. In fact, this is the usual identification problem in simultaneous equation models. Consider the estimated β matrix for $q = 5$ and $r = 3$:

$$\beta = \begin{pmatrix} 1 & \beta_{12} & \beta_{13} \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \\ \beta_{41} & \beta_{42} & \beta_{43} \\ \beta_{51} & \beta_{52} & \beta_{53} \end{pmatrix} \quad (19)$$

In this (unidentified) model there are 3 cointegrating relations and 2 common stochastic trends. However, we cannot interpret the single columns in (19) because "they all look like the same". We need to impose some *identifying* restrictions on the parameters. Restrictions important for the market delineation problem read as follows.

First note, that because $q - r = 2$ there are two separate price trends, and hence two separate markets can potentially be identified. One set of restrictions that will do so reads:

$$\boldsymbol{\beta} = \begin{pmatrix} 1 & \beta_{12} & \beta_{13} \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \\ \beta_{41} & \beta_{42} & \beta_{43} \\ \beta_{51} & \beta_{52} & \beta_{53} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{pmatrix} \quad (20)$$

In this case there are $r(q - r) = 6$ overidentifying restrictions which for the 5 price series $\mathbf{P}_t = (p_{1t}, p_{2t}, \dots, p_{5t})'$ determine the relations $\boldsymbol{\beta}'\mathbf{P}_t$:

$$\begin{aligned} p_{1t} - p_{2t} \\ p_{2t} - p_{3t} \\ p_{4t} - p_{5t} \end{aligned} \quad (21)$$

Hence, p_{1t}, p_{2t} , and p_{3t} all move together and can be considered to delineate one geographical market. The price series p_{4t} and p_{5t} also comove, but only together, not along with the other price series p_{1t}, p_{2t} , and p_{3t} . The given example with segmented markets is a particular case of what Granger and Haldrup (1997) coined *separate cointegration*.

A situation where all commodities belong to the same geographical market would require 1) that $r = 4$, (such that there is $q - r = 5 - 4 = 1$ common price trend) *and* the restricted $\boldsymbol{\beta}$ vector would read:

$$\boldsymbol{\beta} = \begin{pmatrix} 1 & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & 1 & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & 1 & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & 1 \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & & -1 \end{pmatrix}. \quad (22)$$

Now there are $r(q - r) = 4$ overidentifying restrictions which for the 5 price series $\mathbf{P}_t = (p_{1t}, p_{2t}, \dots, p_{5t})'$ determine the relations $\boldsymbol{\beta}'\mathbf{P}_t$

$$\begin{aligned} p_{1t} - p_{2t} \\ p_{2t} - p_{3t} \\ p_{3t} - p_{4t} \\ p_{4t} - p_{5t} \end{aligned}$$

Hence, in this example all the series co-move in pairs because there is just a single common price trend. Observe, that one can take arbitrary linear combinations

of the above price differentials, which will demonstrate that all pairs tend to co-move. For instance $(p_{1t} - p_{2t}) + (p_{2t} - p_{3t}) = p_{1t} - p_{3t}$ is a stationary relation and hence p_{1t} and p_{3t} will co-move.

Observe, that much of the discussion in relation to the pairwise comparison of price data equally apply in the present context. The advantage of a multivariate analysis of the pairwise data, given that the data can be simplified in this fashion, is that the grouping of multiple price series can be much better given.

Simplifying the model to look at price differences is attractive because there is a straightforward interpretation of the (simplified) model in this case. When one-minus-one attractors cannot be identified in the data it still makes sense from a market delineation point of view to consider cointegrating relations of the form $p_{it} - a_{ij}p_{jt}$. In general terms, a matrix of cointegration relations corresponding to (22), for instance, can be described as

$$\beta = \begin{pmatrix} 1 & \beta_{12} & \beta_{13} \\ \beta_{21} & 1 & \beta_{23} \\ \beta_{31} & \beta_{32} & 1 \\ \beta_{41} & \beta_{42} & \beta_{43} \\ \beta_{51} & \beta_{52} & \beta_{53} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -a_{21} & 1 & 0 \\ 0 & -a_{32} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -a_{53} \end{pmatrix}$$

In this model the price series still co-move systematically constitute two separate markets, but movements within the single markets is not one-to-one.

Joint delineation of product and geographical markets. A major advantage of the modelling framework just presented is that the relevant product and geographical market can be delineated simultaneously. In most competition cases market delineation is undertaken sequentially: First the relevant product markets is determined, next the relevant geographical market is determined. The foregoing procedure based on multivariate cointegration analysis has the advantage that products and regions can be considered jointly. The testable implications of a joint delineation of the relevant product and geographical market amounts to testing of hypotheses and restrictions of parameters such as in (22) and with the separate prices reflecting particular products in particular regions. A limitation of such an analysis is, however, that only a limited number of products and regions can be modelled in order for the dimension of the system not to explode and hence making the testing infeasible in practice.

Dynamic considerations in market delineation. Separate tests that can be of interest in the delineation of the relevant geographical market concerns the adjustment (or loading) parameters α . For instance, consider the situation where the β matrix is given by (22). The adjustment matrix α will typically

take the unrestricted form

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ \alpha_{51} & \alpha_{52} & \alpha_{53} \end{pmatrix}$$

where the single entries determine how each of the price series change in response to the price differentials (21) being non-zero. For instance α_{11} tells (via the error correction model(17)) how much p_{1t} changes from the value in the previous period when $p_{1t} - p_{2t} \neq 0$. The size of the α coefficients measure the speed of the adjustment process. The imposition and statistical non-rejection of restrictions on $\boldsymbol{\alpha}$ will induce important structure concerning the dynamic adjustment in the system. For instance, one could ask whether it is such that particular country (or region) specific commodities are price leaders in the dynamic adjustment of prices. Consider the following example:

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ 0 & 0 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ 0 & 0 & 0 \end{pmatrix}$$

In this case there are 6 zero restrictions on the α coefficients. These indicate that when price differentials exist across the commodities of 5 different origins, then the adjustments are in p_{1t} , p_{2t} , and p_{4t} . If the restrictions on $\boldsymbol{\beta}$ are as given in (20), then this means that p_{3t} and p_{5t} (which belong to separate markets) *do not* reject in response to price differentials. However, the other prices *do* adjust. In this sense the commodities associated with p_{3t} and p_{5t} can be treated as exogenous and thus governing price adjustments of the other commodities.

In relation to the discussion of adjustment mechanisms it should also be noted that cointegration and the existence of error correction models in such cases is intimately related to the notion of Granger causality. In fact, through the error correction model, cointegration implies Granger non-causality in at least one direction thus meaning that there is at least one entry in the $\boldsymbol{\alpha}$ matrix which is non zero.

4.2.5 Obtaining consistent market delineation from different models

One problem with co-integration analysis of the type described above is that the conclusions to be drawn may differ depending upon which price variables are included in the VAR model. Such problems can occur for instance, due to poor data quality,. Also, nothing guarantees that working with VAR models of different orders, i.e. different values of q , will yield the same conclusion. Robust inference occurs when market delineation is insensitive to and consistent with different combinations of the variables included in the study. A procedure for

robustifying the analysis is to estimate VAR models and conduct co-integration analysis for models of an increasing complexity:

- First co-integrated analysis is undertaken for the pair-wise data, that is for $q = 2$.
- Next, all possible combinations of the prices for 3-dimensional systems, ($q = 3$), are considered.
- Next, 4-dimensional systems are addressed

and so forth. The analysis can be rather demanding in terms of the number of systems that need to be scrutinized. However, one can hope that a consistent pattern will emerge as to which countries segment into particular "groups". From a pragmatic point of view one will expect that some countries or regions are stronger associated than others, but at least the conclusions to be drawn should be consistent with and reasonably robust to the information set being used, i.e. the particular countries studied.

4.2.6 Practical problems and concerns

There is no doubt that when sufficiently many observation points are available and the price series appear to be non-stationary $I(1)$, then the Johansen procedure for the analysis of cointegration and common stochastic trends is a very powerful tool and an intuitive way of implementing the SSNIP test in the delineation of the relevant geographical market. However, as it has perhaps become clear from the foregoing description of the procedure, an advanced insight into the practical implementation of the technique is necessary. This is *not* an argument for *not* using the method.

The Engle-Granger method seems less attractive for multivariable analysis because the method presupposes that only a single cointegrating relation exists, and this can frequently not be assumed a priori. Instead, a pairwise analysis of the single series can be implemented by use of ADF-tests of the pairwise price differences amongst the different price processes. Often the limitation of data is itself a reason for using such a procedure.

There are a number of practical problems and concerns one should be aware of when implementing the Johansen procedure. These will be summarized below.

Lag truncation of the VAR. One of the major difficulties with the Johansen procedure is that a rather large number of parameters need to be estimated in the VAR model. This can be a practical problem if only observations for a limited number of years, quarters, or months, are available. Hence, limiting the number of lags is important. On the other hand, having too few lags in the model will induce autocorrelation in the errors of the single relations. This is equally serious because in this case the test of cointegration rank will become invalid.

Removal of autocorrelation by lag augmentation of the VAR is the most important problem to solve when implementing the Johansen procedure. Studies have shown that heteroscedasticity and deviation from the normality assumption are less important for practical hypothesis testing about the cointegration rank.

Seasonality. When analyzing data exhibiting seasonal variation it is frequently useful to include seasonal dummy variables in the VAR model (18). In fact, in many cases the number of lags needed in the VAR to render the errors free from autocorrelation can be reduced significantly by allowing for seasonal indicator variables.

Deterministic components and the distinction between absolute and relative price convergence. In addition to a constant and seasonal dummy variables a trend can sometimes be included in the VAR. A comprehensive discussion of the implication of doing so is rather advanced so here only an intuitive explanation will be given.

Consider first the situation with a constant in the model. It occurs that the constant will play different roles depending upon whether the model is looked at in the stationary (cointegrating) directions or the non-stationary (common stochastic trend) directions. The constant will have the dimension q . In the r stationary directions this constant plays the role of a *constant* in the sense that if this is non-zero, a relative rather than an absolute version of the law of one price can be interpreted (in terms of the examples given in the previous sections). However, in the non-stationary directions of which $q - r$ exist, the constant appears to become the drift of the processes. That is, a time trend in addition to the stochastic trend components. In fact, such a linear trend in price processes can be expected in most cases because it is an empirical regularity that most price series appear to be positively deterministically trending over long horizons. A test that is frequently of interest to conduct is that of testing whether there is *no* constant in the stationary directions. In other words, the testing of whether the price differences are stationary and have a zero mean. Most soft-ware can implement this test directly. The situation with no constant in the stationary directions will indicate absolute pairwise convergence of prices, c.f. the discussion in section 1.1.

Now turning to the case where a trend is included in the VAR a similar discussion can be made. In the r stationary directions the trend will produce a trend in the attractor relations and a quadratic trend in the $q - r$ non-stationary directions. In relation to the market delineation problem, the possibility of a trend in the stationary directions is *not* interesting and hopefully can be tested to zero. Otherwise it would mean that the prices systematically trend away from each other in the long run and hence naturally the underlying commodities cannot belong to the same geographical market. On the other hand, it is not unusual that price series tend to have a quadratic trend in their levels and hence it is often of use to allow for this possibility in estimation.

Testing the above kinds of restrictions on the deterministic components can

be undertaken rather easily in soft-ware implementing the Johansen procedure. Conditional on the cointegration rank, the tests can be conducted by comparing restricted and un-restricted model estimates and calculating the Likelihood Ratio test which will follow a χ^2 distribution with a number of restrictions corresponding to the number of overidentifying restrictions. Potentially, the test of deterministic components can be calculated jointly with any other test on the α and β parameters.

Outliers and aberrant observations. If outlying or aberrant observations appear to be present in the model this needs to be appropriately accounted for. Inclusion of impulse dummy variables does not cause any problems with the statistical tests. On the other hand, step dummy variables will generally cause the relevant distributions to change and hence this situation should be avoided. Specialized computer software will be needed in this case and will not be discussed in the present exposition.

4.3 A structural model multivariate stability test

A test of the joint stability of price differences can be made using a procedure for structural models suggested by Nyblom and Harvey (2000), and which is a multivariate generalization of the KPSS test of Kwiatkowski *et al* (1992) described in section 2.3. The test has similarities with the Johansen tests of cointegration in the sense that the stability of price differences is being (jointly) tested given that the single price series each exhibit stochastically trending behaviour. The difference is, however, that in the present case the null hypothesis of *stability* is being tested (rather than *instability*). This null hypothesis may seem more adequate in some cases because what is desirable to test is that prices comove and *have* converged, rather than they have *not* converged. The suggested test should be considered complementary to cointegration tests.

Initially, the model framework for the case of no serial correlation is given. This is unlikely to occur in practice, but it will be instructive first to present this simplified version of the test and then extend it to more appropriate situations. The model can be considered consisting of $q - 1$ price differences $d_t = p_{it} - p_{jt}$ which can be described as a multivariate random walk plus noise process:

$$\begin{aligned} \mathbf{d}_t &= \boldsymbol{\delta} + \boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_t, & \boldsymbol{\varepsilon}_t &\sim Nid(\mathbf{0}, \Sigma_\varepsilon) \\ \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} + \boldsymbol{\eta}_t, & \boldsymbol{\eta}_t &\sim Nid(\mathbf{0}, \Sigma_\eta) \end{aligned}$$

with

$$\mathbf{d}_t = (d_{1t}, d_{2t}, \dots, d_{q-1,t})', \quad \boldsymbol{\delta} = (\delta_1, \delta_2, \dots, \delta_{q-1})', \quad \text{and} \quad \boldsymbol{\mu}_t = (\mu_{1t}, \mu_{2t}, \dots, \mu_{q-1,t})'.$$

The vector $\boldsymbol{\delta}$ is a potentially nonzero intercept vector whose elements vary with the price pairs (measuring deviations from absolutely converged prices), and $\boldsymbol{\mu}_t$ is a random walk component for each of the price differences. Testing stationarity amounts to testing the null of no random walk components in the

price differences, i.e. the prices do not systematically deviate in the long run. One way of operationalizing this hypothesis is by setting $\Sigma_\eta = \mathbf{0}$. Under the alternative some of the price differences have unit roots. Rejection of the null hypothesis will thus indicate that no common geographical market can possibly characterize all of the products for which prices are available. This does not imply, however, that subgroups cannot be considered belonging to the same geographical market, but then a model involving a fewer number of price differences needs to be build and tested.

The test statistic takes the form

$$\xi_{q-1} = \text{trace} [S^{-1}C] \quad (23)$$

where S is an estimate of the so-called long-run error covariance matrix which can be computed as

$$\begin{aligned} S &= T^{-1} \sum_{t=1}^T (d_t - \bar{d})(d_t - \bar{d})' \quad \text{with } \bar{d} = T^{-1} \sum_{t=1}^T d_t \\ C &= T^{-2} \sum_{j=1}^T \left[\sum_{t=1}^j (d_t - \bar{d}) \right] \left[\sum_{t=1}^j (d_t - \bar{d}) \right]' \end{aligned}$$

The distribution of the Nyblom-Harvey test (23) is non-standard and has been tabulated in Nyblom and Harvey (2000).

Assuming that the test cannot be rejected this can be taken as evidence of prices having converged in relative terms. Absolute convergence would require $\delta = 0$. A likelihood ratio test of this hypothesis can be easily constructed by estimating the model by use of the *STAMP* programming language, see Koopman et al. (2000). A more pragmatic test involves testing each of the means of the price differences to equal zero. Logically, such a test should be conducted after the Nyblom-Harvey test.

4.3.1 Practical Problems and concerns.

Serial correlation The most important practical problem in conducting the test (23) is the fact that d_t generally will contain a lot of serial correlation. For the test to be useful in practice we thus need to allow for this feature. A correction can be made parametrically in the following way. For each pair of price differences, specify an autoregressive process on the form

$$\alpha_i(L)d_{it} = \delta_i + \mu_{it} + \varepsilon_{it}$$

where the index i signifies the i 'th price differences in the panel and $\alpha_i(L) = 1 - \alpha_{i1}L - \dots - \alpha_{ip_i}L^{p_i}$ is an autoregressive polynomial in the lag operator with p_i indicating the particular lags used in the i 'th pair of price differences. Note in particular, that the number of lags in each equation can be different. For the test to have power it is necessary to estimate the autoregressive parameters such

that these will be consistent under both the null and alternative hypothesis. In so doing the model

$$\alpha_i(L)\Delta d_{it} = (1 - \theta_i L)\zeta_{it} \quad (24)$$

can be estimated. $\alpha_i(L)$ is as defined before, and ζ_{it} is distributed as $iid(0, \sigma_{\xi_i}^2)$. Testing stationarity is now equivalent to testing whether $\theta_i = 1$. Subsequently, using standard soft-ware an ARMA($p_i, 1$) is fitted to Δd_{it} as in (24). Next, the residual series

$$d_{it}^* = d_{it} - \hat{\alpha}_i(L)d_{it}$$

is constructed and the Nyblom Harvey test is conducted for the corrected series $\mathbf{d}_t^* = (d_{1t}^*, d_{2t}^*, \dots, d_{q-1,t}^*)'$.

5 Price convergence - a time series perspective

5.1 Some motivation and conceptual considerations

In the presentation of quantitative techniques so far, it has been assumed as a benchmark that the various price series could be characterized as being *stable* (or being in a *steady state*) in the sense that either prices have converged or, alternatively, have *not* converged across separate geographical regions. However, when price data is available for longer periods it becomes relevant to ask whether price behaviour can be considered an on-going evolving process towards *increased* price convergence, when the initial state characterizes separate markets. The relevant question to ask is thus whether a stable state characterizes prices and price differences *or* whether there is convergence *towards* a stable state. In fact, a (non) stability test like a unit root test could reject convergence even though the transient dynamics of price adjustment will indicate movement towards a convergence state. In this section this possibility will be discussed in more detail by presenting a class of structural time series models which appear adequate to describe evolving price behaviour. The techniques to be discussed rely on recent research by Harvey and Carvalho (2002). As we shall see, it is possible to introduce a class of unobserved components models where latent factors describing the convergence components can be estimated. It turns out that the convergence components have error correction model features, as previously described, and hence will have an attractive interpretation. This provides insight into historic behaviour of the price series as well as a coherent procedure for the prediction of future observations and likely convergence properties of prices. Also, this class of models will allow the analyst to ask relevant questions concerning the speed of convergence and the size of the gap between converging prices.

5.2 Structural time series models of price differentials

5.2.1 A Simple Direct test of price convergence

The structural time series approach discussed in e.g. Harvey (1989) and Harvey and Carvalho (2002) will show useful for the modelling of price convergence as a process which evolves gradually over time. Assume we consider the price difference $d_t = p_{it} - p_{jt}$ between two price series for which observations are available for $t = 1, 2, \dots, T$. The price difference is assumed to consist of a stochastic trend component, μ_t , as well as a cyclical (ψ_t) and an irregular (ε_t) component. That is, the price difference can be written

$$\begin{aligned} d_t &= \mu_t + \psi_t + \varepsilon_t, & t = 1, \dots, T \\ \varepsilon_t &\sim Nid(0, \sigma_\varepsilon^2) \end{aligned} \quad (25)$$

The trend, μ_t , is considered to receive shocks to both the level and the slope and can be parameterized as

$$\begin{aligned} \mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t & \eta_t &\sim Nid(0, \sigma_\eta^2) \\ \beta_t &= \beta_{t-1} + \varsigma_t & \varsigma_t &\sim Nid(0, \sigma_\varsigma^2) \end{aligned}$$

The shocks ε_t , η_t , and ς_t are the irregular, level and slope disturbances, respectively. These are mutually independent - $Nid(0, \sigma^2)$ means normally and independently distributed with zero mean and variance σ^2 .

Observe, that if both variances σ_η^2 and σ_ς^2 are zero, then the trend is deterministic. When just σ_ς^2 is zero, then the trend will evolve as a random walk with the constant drift β .

The cyclical component ψ_t captures short run dynamics such as seasonal and other cyclical variation and can be modelled in a number of ways. For instance, an autoregressive specification can be used. Details will not be given here, but can be found in e.g. Harvey (1989), Harvey and Carvalho (2002) and Koopman *et al.* (2000). These references will also describe the details of how to estimate a structural model such as (25). In broad terms, unobserved components models can be estimated by the maximum likelihood method after setting up the model in so-called *state space form* and using the *Kalman filter*. It goes beyond the present presentation to describe these techniques. However, the estimation procedure has been implemented in computer software such as *STAMP*, see Koopman *et al.* (2000). In addition to delivering estimates of the unknown parameters, *STAMP* will deliver estimates of the latent components.

In the present context the model can be interpreted as follows. Assume we wish to look at the possible convergence of the price series without imposing a particular mechanism for the convergence process. Then the price difference d_t is made up of the stochastic trend (level) component μ_t together with the cyclical and irregular component. Simply estimating the smoothed trend component μ_t will describe the time path of the price difference. This is particularly relevant in situations where the data is flawed by noisy factors and hence the unobserved components model suggests a way of extracting the signal associated

with the convergence component of the price difference. A direct test of *absolute convergence* can be calculated by comparing the estimate of μ_T , the terminal value of the convergence component, with the estimated root mean square standard error (RMSE). This output is automatically delivered in *STAMP*. If a 95 % confidence interval, say, covers the value zero, this can be taken as evidence that absolute convergence has occurred. If it occurs that μ_t stabilizes at a non-zero value it can be taken as evidence that relative convergence has occurred. After all, the time path of μ_t will itself be very informative concerning the trend towards absolute or relative convergence.

The above model can be extended in *STAMP* to situations with multiple time series, N , such that joint tests of the absolute convergence hypothesis can be conducted. In this case the hypothesis can be formulated as $\mathbf{D}\boldsymbol{\mu}_T = 0$ where

$$\mathbf{D}\boldsymbol{\mu}_t = \begin{pmatrix} 1 & -1 & 0 & \dots & 0 \\ 0 & 1 & -1 & \dots & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & \dots & 1 & -1 \end{pmatrix} \begin{pmatrix} \mu_{1t} \\ \mu_{2t} \\ \vdots \\ \mu_{Nt} \end{pmatrix}$$

That is, for each of the convergence components their terminal condition will be zero. Harvey and Carvalho (2002) provide the details.

A limitation of the present set-up concerns the fact that convergence is measured entirely in terms of descriptive properties. Also, the use of non-stationary components to model convergence may seem counterintuitive because once convergence has taken place the series is stationary. The next section demonstrates how these limitations can be loosened.

5.2.2 Modelling the Transient Dynamics of price convergence

We have previously seen how an error correction model for cointegrated price series can be used as a device for identifying common geographical markets. Following a similar train of thought, the modelling of the transient dynamics characterizing price convergence can be made. The difference is that in the present context the error correction model is not constructed as a stable model but rather as a model allowing for the evolving dynamics in a situation where the initial value is somewhat away from zero. That is, if initially prices have not converged, the subsequent adjustment process is addressed. Harvey and Carvalho (2002) suggest the following model of (bivariate) price differences:

$$\begin{aligned} d_t &= \alpha + \mu_t, & t = 1, \dots, T \\ \mu_t &= \phi\mu_{t-1} + \eta_t \end{aligned}$$

with a fixed initial value μ_0 . When $0 < \phi < 1$, the price gap will narrow over time. The equivalent error correction model for μ_t reads

$$\Delta d_t = (\phi - 1) d_{t-1} + \delta + \eta_t$$

where $\delta = \alpha(1 - \phi)$. For data in logarithms, this shows that the growth rate in the current period is a negative fraction of the price gap after allowing for the permanent difference which will reflect relative convergence in case $\delta \neq 0$.

Of course, the above error correction model interpretation of convergence can be extended to allow for a richer dynamic structure, i.e. with further lags of the differenced series. It should be noted, however, that the above structure presupposes cointegration in the sense we have previously defined it, and hence assumes that a unit root is *not* present in d_t . This would contradict the convergence hypothesis. After all, the estimate of ϕ will indicate how fast price adjustments take place towards convergence which can be either relative or absolute depending upon the value of δ .

5.2.3 A Structural Unobserved Components ECM

The unobserved components method adds cycle and irregular components to the error correction model and in so doing avoids mixing up the transitional dynamics of the convergence process with that of short-term steady state dynamics, for instance seasonal variation. An especially attractive specification is the following

$$d_t = \alpha + \mu_t + \psi_t + \varepsilon_t, \quad t = 1, \dots, T$$

with the smooth transitional dynamics:

$$\begin{aligned} \mu_t &= \phi\mu_{t-1} + \beta_{t-1} \\ \beta_t &= \phi\beta_{t-1} + \varsigma_t. \end{aligned} \tag{26}$$

and an appropriate choice of the cyclical factor component ψ_t .

Alternatively, (26) can be written in error correction form:

$$\begin{aligned} \Delta\mu_t &= (\phi - 1)\mu_{t-1} + \beta_{t-1} \\ \Delta\beta_t &= (\phi - 1)\beta_{t-1} + \varsigma_t. \end{aligned}$$

whereby it can be seen that in log transformed data there is a convergence mechanism acting in both the level and the growth of price differences. Yet another way of writing this is

$$\Delta\mu_t = -(1 - \phi)^2\mu_{t-1} + \phi^2\Delta\mu_{t-1} + \varsigma_t$$

This shows in a more direct fashion, that changes in the convergence component not only depends on the gap itself, but also on the change in the previous period. Hence changes take place more slowly and the convergence component will thus evolve more smoothly. Again, the structural model can be estimated by maximum likelihood by use of the model state space form and the Kalman filter.

In theory it is possible to extend the above class of models to multiple price process. Harvey and Carvalho (2002) suggest how this can be done. It turns out, however, that estimation difficulties increase prohibitively with the dimension of the system. After all, pairwise comparisons of price differences

using the technology above is a useful device for clarifying any potential transient dynamics in a convergence process. The hope is, that by conducting the analysis pairwise for multiple price differences a common pattern will develop as to which groupings of regions exhibit convergence properties whereby a common geographical market can be delineated.

5.2.4 Practical problems and concerns

It is a clear advantage of the above procedures that a specification can be chosen where no assumptions are made concerning the stationarity properties of the single price series. What is of interest is to see whether a convergence component can be extracted and thus indicating a particular trend of the simultaneous price movements.

A general problem in relation to the estimation of unobserved components models concerns the fact that the model structure is determined a priori and a spectrum of different specifications exists. It is thus of importance to choose a specification where the signals to be extracted reflect what one has in mind. The models suggested in sections 5.2.1 and 5.2.3 have nice interpretations in terms the transitional dynamics when moving from a non-convergence to a convergence state. However, in order not to mix up this dynamics with short term adjustments in the form of cyclical and seasonal variation, it is of importance to appropriately choose a specification of ψ_t which captures these properties in the data. In Harvey (1989) and Koopman *et al.* (2000) a range of models to choose a specification of ψ_t are suggested. These are implemented in the *STAMP* programming package.

Model specification testing is equally important in the modelling of structural time series models. Once estimated, the fit of the model can be checked using standard time series diagnostics such as tests for residual correlation etc.

A common problem existing in the estimation of structural models of the above kind by use of maximum likelihood procedures, the state space form, and the Kalman filter, concerns the fact that estimation convergence to achieve ML estimates will sometimes fail. In these cases it is necessary to examine the initial conditions used to start up the appropriate algorithm. This will be clear from the programming software used for estimation. It turns out, that in certain cases estimation and (statistical) convergence of parameter estimates can be complicated by the fact that an identification problem exists. In these cases a respecification of the model will often be useful.

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