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The Macroeconomic Policy Mix in a Monetary Union with Flexible Inflation Targeting*

Torben M. Andersen

January 3rd, 2003

Abstract

Policy mix problems may arise in a monetary union with centralized monetary policy and decentralized fiscal policy. A consequence of this may be an inappropriate stabilization of shocks. This paper addresses how policy coordination problems are affected by the objectives of the monetary authority. It is shown that non-coordinated fiscal policies tend to be too counter-cyclical in the case of aggregate shocks, and that this bias can be reduced by lowering the weight to output stability in monetary policy. Oppositely, for country-specific shocks non-coordinated fiscal policies tend to be too pro-cyclical and this bias can be reduced by increasing the weight to output stability in monetary policy. Considering the assignment of policy tasks – within the set of binding policy rules for fiscal- and monetary policy – it is found that flexible inflation targeting dominates strict inflation targeting.

JEL Classification: E52, E58, E61, E62 and E63

Keywords: EMU, policy-mix, shocks and policy cooperation.

1 Introduction

The European Monetary Union has a centralized monetary policy and a decentralized fiscal policy. A recurrent issue is whether this decision structure leads to systematic problems with the macroeconomic policy mix, that is, would there be a tendency that monetary and fiscal policy are in conflict seen relative to the business cycle situation?

Although the sample period is short the available evidence for the EMU seems to indicate a tendency towards policy mix problems in the sense that the macroeconomic policy stance is characterized by an expansionary fiscal policy and a contractionary monetary policy (1998, 2000) or the reverse (1999), see, e.g. EU (2002). Interestingly, similar problems have not arisen for the US. It is an open question whether this difference is robust and whether the difference

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between the EMU and the USA can be attributed to a difference between decentralized and centralized fiscal policy or different monetary policy objectives.

Recent work has shown that policy mix problems arise in a currency union even with monetary commitment to price stability (strict inflation targeting) and therefore a clear assignment of macroeconomic policy instruments and objectives to monetary and fiscal authorities. The reason is that certain policy interdependencies are not fully taken into account in decentralized fiscal decisions (see, e.g. Sveen (2001), Andersen (2002b), Uhlig (2002)).

The presence of systematic policy mix problems under strict inflation targeting raises two important issues. First, strict inflation targeting may be considered as a too rigid interpretation of the monetary policy being pursued in the monetary union, and if so it is relevant to consider what kind of policy mix problems may arise under flexible inflation targeting. In particular since it has been shown that policy mix problems in particular relate to aggregate shocks, it may be questioned whether flexible inflation targeting would ensure a centralized stabilization response via monetary policy which is to be preferred to the decentralized response running via fiscal policy. Second, a consideration of flexible inflation targeting is also interesting from a normative perspective, since if flexible inflation targeting reduces the policy mix problems this may be an argument for reconsidering the monetary policy objectives in the monetary union.

A premise for the following analysis is that there is commitment to a particular monetary strategy and that this is known and credible. Thereby attention is restricted to a situation where commitment and credibility problems do not arise. This surpasses the issue of the leader-follower relations between monetary and fiscal authorities which has been extensively analysed in the literature[1]. In the following it is thus assumed that the monetary policy rule is credible and well known to fiscal decision makers, who therefore can take into account the possible monetary policy feedbacks to fiscal policy decisions. Specifically, monetary policy is designed so as to minimize a loss function defined over inflation and aggregate output fluctuations within the monetary union (implying a Taylor type policy reaction function). Fiscal authorities control domestic fiscal policy instruments aiming at stabilizing domestic output and avoiding variations in the policy instruments. The question addressed is both the positive implications of monetary policy targets for policy coordination problems and the normative of optimal assignments of tasks to fiscal and monetary policy authorities. These issues are considered in the case of both aggregate and country-specific shocks within the currency union.

The aim of the following is thus to bring forth some basic mechanisms in the interaction between monetary and fiscal policy. Accordingly, the model is as simple and stylized as possible to focus on the basic mechanisms and to avoid technical complexities. At the same time it is rich enough to capture basic international interdependencies in policies, as well as various types of shocks.

[1] See Dixit and Lambertini (2000 a,b), Beetsma and Bovenberg (1998, 1999)and Beetsma, Debrun and Klaassen (2001). For a survey and further references see Andersen (2002a).

The paper is organized as follows: The model is set up in section 2. Fiscal policy decision making in the non-cooperative and the cooperative case are considered in section 3, and section 4 analyses the coordination problems in fiscal policy in the case of both aggregate and country-specific shocks. The optimal assignment of the role of stabilizing output between fiscal and monetary authorities is addressed in section 5. A numerical illustration of key results is provided in section 6 and section 7 summarizes and concludes.

2 A model for a monetary union

Following the tradition in the recent literature on macroeconomic policy rules[2], the model structure builds on an aggregate supply curve (expectations augmented Phillips-curve relation), and an aggregate demand relation. In the present context it is important to be specific about the interactions between member countries of the monetary union, both in respect to inflation and output. To simplify, the currency union is considered to be a closed area to focus on the direct interdependencies between member countries of the currency area[3]. The currency area has n member countries indexed by i, which are all symmetric up to the realization of shocks (see below). The policy sequence is such that fiscal authorities choose fiscal policy under full knowledge of the monetary policy objectives, and thus the monetary policy responses to fiscal policy. All variables – except the interest rate – are defined by the log value of the variable in question measured relative to its long-run value.

Wage and Price Setting[4]
Assume that inflation in country i in period t (π_i) is determined as follows

$$\pi_{it} = \pi_t^e + \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \qquad ;\omega_y > 0, \omega_g > 0 \tag{1}$$

Country-specific inflation depends on expected union wide inflation π_t^e via e.g. wage setting (commodities from all member countries enter equally into the consumption basket). Moreover, price setting is affected by country-specific activity (y_{it}), fiscal policies[5] [6] (g_{it}), and shocks (u_{it}) (for interpretation see below).

[2] See, e.g. Svensson (1997), Dixit (2000), Dixit and Lambertini (2000a,b), Leitemo (2000), Sveen (2000) and Beetsma, Debrun and Klaassen (2001).

[3] Since the consolidated trade share for EU is about 10% of GDP this assumption is a reasonable approximation.

[4] The model disregards forward-looking elements to simplify the analysis. However, under strict inflation targeting this has no major qualitative implications.

[5] It is well known that higher income taxes may lead to an increased labour supply and thus a downward pressure on the wage rate provided income effects are sufficiently strong, see e.g. Dixon (1987) and Baxter and King (1992). It can also arise via a production subsidy as analysed in e.g. Dixit and Lambertini (2000a).

[6] Note that a change in fiscal policy may in general increase or decrease the inflationary pressure in the economy, depending on the particular instrument used. Attention is here restricted to the case where a fiscal expansion has a direct inflationary impact. See Andersen (2002) for an analysis allowing for both cases. Finally, including fiscal policy seperately captures that the inflationary consequences of private and public demand in general are different since a larger fraction of the latter turns to domestic production.

Aggregating over country-specific producer inflation we get

$$\pi_t = \pi_t^e + \omega_y y_t + \omega_g g_t + \omega_u u_t \tag{2}$$

It follows that there is no long-run trade-off between activity and inflation. Note that the Phillips-curve implies that long-run or structural output ($\pi_t = \pi_t^e$) is given by

$$y_t = \frac{-1}{\omega_y}\left(\omega_g g_t + \omega_u u_t\right)$$

Aggregate Demand
The microfoundations for the demand relation are not given explicitly, but can be derived in the context of a specialized production structure as in Obstfeld and Rogoff (1995) or through an allocation of production across countries depending on differences in comparative advantages and trade frictions (see, e.g. Andersen (2001)). Likewise intertemporal substitution (via the real rate of interest) is introduced in a straightforward and traditional way.

The output relation for country i in period t is given as

$$\begin{aligned} y_{it} = \quad & -\delta_r(i_t - \pi_t^e) - \delta_\tau(\pi_{it} - \pi_t) + \delta_y y_t + \delta_g g_{it} + \delta_u u_{it} \\ & ; \;\; \delta_r > 0, \delta_\tau > 0, \delta_y > 0, \delta_g > 0 \end{aligned} \tag{3}$$

where the first term captures the negative effect of an increase in the real rate of interest on demand[7], the second term the role of competitiveness in the sense that a higher increase in domestic producer prices than in foreign producer prices shifts demand away from domestic producers (competitiveness effect), the third term captures how aggregate income[8] in the union ($y \equiv \frac{1}{n}\sum_j y_j$) affects demand for the products of country i, the fourth term captures the demand effect of fiscal policy, and finally we have the effect of the contemporaneous innovation to the shock[9].

Note that different interpretations of the shocks are possible, if $\delta_u > 0$ and $\omega_u > 0$, the shock has an interpretation as a demand shock, and if $\delta_u > 0$ and $\omega_u < 0$, the shock has an interpretation as a supply shock. It is assumed that shocks have expected value zero, i.e.

$$E_t u_{it+j} = 0 \qquad \forall i,\ \forall j > 0$$

[7] Note that since consumer price inflation is the same in all countries, the real rate of interest is also the same. However, allowing for different consumption bundles such that $\pi_{it}^c = \lambda\pi_{it} + (1-\lambda)\pi_t$ would imply that $-\delta_r(i_t - \pi_{it}^c) - \delta_\tau(\pi_{it} - \pi_t) = -\delta_r(i_t - \pi_t) - (\delta_\tau + \lambda\delta_\tau)(\pi_{it} - \pi_t)$, hence under a straightforward reinterpretation of the coefficients of the demand relation, the analysis applies for the case of different consumption bundles across countries.

[8] Notice, that aggregate income is specified so as to be invariant to change in the number of fiscal decision makers (n).

[9] Note that the specification implies that only unanticipated shocks to the inflationary process has real effects.

and the variance is denoted σ_u^2. The aggregate value of shocks across the monetary union is given as[10]

$$u_t \equiv \frac{1}{n}\sum_j u_{jt}$$

The effect of domestic fiscal policy on domestic inflation is given by

$$\frac{\partial \pi_i}{\partial g_{it}} = \omega_g + \omega_y \delta_g$$

that is, there is both a direct effect (ω_g) and an indirect effect running via activity ($\omega_y \delta_g$). In the following attention is restricted to cases where $\omega_g + \omega_y \delta_g > 0$. In the same vein it is assumed that the impact effect of a fiscal change on activity ($\delta_g - \delta_\tau \omega_g$) is positive.

It follows straightforwardly from (3) that aggregate output is given as

$$y_t = \frac{1}{1-\delta_y}\left[-\delta_r(i_t - \pi_t^e) + \delta_g g_t + \delta_u u_t\right] \tag{4}$$

and that

$$y_{it} - y_t = -\delta_\tau(\pi_{it} - \pi_t) + \delta_g(g_{it} - g_t) + \delta_u(u_{it} - u_t) \tag{5}$$

that is, output in a member country differs from the mean output in the monetary union if it either i) has a different producer price inflation (wage increases), ii) pursues a different fiscal policy, or iii) is affected by country-specific shocks.
Monetary policy
The monetary authority is controlling the nominal interest rate (i) which for given inflation expectations implies that the expected real rate of return ($r \equiv i - \pi^e$) is effectively determined. Monetary policy is determined at a centralized level and is rule-determined (rules out time-inconsistency problems) so as to minimize a loss function penalizing fluctuations in output and inflation, i.e.

$$L_t^M = E_t \sum_{k=0}^{\infty}(1+\rho)^{-k}\left(\frac{1}{2}\alpha_M \left(y_{t+k}\right)^2 + \frac{1}{2}(\pi_{t+k})^2\right)$$

where ρ is the subjective discount rate, and α_M the weight attached to stabilize output relative to stabilizing inflation. Note that the loss function is specified only over aggregate variables in the union. In its decision the monetary authority recognizes that output is determined by (2) and inflation by (4).

The first order condition to the monetary policy problem reads

$$E_t \sum_{k=0}^{\infty}(1+\rho)^{-k}\left(\alpha_M y_{t+k}\frac{\partial y_{t+k}}{\partial r_t} + \pi_{t+k}\frac{\partial \pi_{t+k}}{\partial r_t}\right) = 0$$

[10] One may think of $u_{it} = \epsilon_t + \varepsilon_{it}$, where ϵ is the aggregate part of the shock, and ε_i the country-specific or idiosyncratic part (where $\sum \varepsilon_j = 0$).

In the appendix it is shown that the implicit reaction function can be written

$$y_t = -\phi_\pi \pi_t \quad , \qquad \phi_\pi = \frac{\omega_y}{\alpha_M} > 0 \tag{6}$$

implying that monetary policy places the union on a trade-off between inflation and output (this is the implicit monetary reaction function). The less concerned the monetary authority is about output stabilization (the lower α_M) the more steep is the trade-off between output and inflation faced by fiscal policy makers. Notice that in the limiting case where $\alpha_M = 0$ we have strict inflation targeting i.e. $\pi_t = 0$. Since monetary policy decisions can be made after fiscal decisions it follows that the trade-off given above becomes a constraint on fiscal policy (see, e.g. Dixit (2000)).

Inserting in the aggregate output relation we have that the monetary reaction function can be written

$$r_t = \frac{1-\delta_y}{\delta_r} \left[\phi_\pi \pi_t^e + \delta_g g_t + \delta_u u_t \right]$$

Monetary policy will be tightened when expected inflation is increasing, fiscal policy is expanded and shocks expanding output occur.

Inserting the inflation process (2) in (6) we get that aggregate output can be written

$$y_t = \frac{-\phi_\pi}{1+\omega_y \phi_\pi} \left[\pi_t^e + \omega_g g_t + \omega_u u_t \right]$$

The response of output to fiscal policy is interesting, and brings out important lessons on the interaction between fiscal and monetary policy under rule-based monetary policy. The fiscal policy instrument affects output and inflation in the same directions. However, with monetary commitment the monetary reaction function (6) becomes a constraint on fiscal policy. Therefore, at the aggregate level the supply effects of fiscal policy become of importance[11]. At the country level, it is, however, still the case that the usual aggregate demand effects of fiscal policy play a role, and therefore determine the relative output position of a country, cf (5). This difference is the source of the coordination problem in fiscal policy.

As a consequence the shock variable has a contractionary effect if it is a demand shock ($\omega_u > 0$), and expansionary if it is a supply shock ($\omega_u < 0$). The reason is also here to be found in the monetary response to the shock which in turn depends on how the shock affects inflation. Since a demand shock boosts inflation a monetary contraction is necessary, and oppositely for a supply shock.

Finally, note that it is an implication of the monetary policy response that

$$\pi_t^e = 0 \quad \forall t$$

Equilibrium output

[11] Observe that actual output become proportional to structural output.

In the appendix it is shown that equilibrium output in any country i in period t can be written

$$y_{it} = \sum_j b_{ij} u_{jt} + \sum_j c_{ij} g_{jt}$$

where

$$\begin{aligned} b_{ii} &= \frac{-\phi_\pi \omega_u}{1+\omega_y \phi_\pi} \frac{1}{n} + \frac{\delta_g - \delta_\tau \omega_u}{1+\delta_\tau \omega_y}(1-\frac{1}{n}) \\ b_{ij} &= -\left[\frac{\phi_\pi \omega_u}{1+\omega_y \phi_\pi} + \frac{\delta_g - \delta_\tau \omega_u}{1+\delta_\tau \omega_y}\right]\frac{1}{n} \qquad for \quad i \neq j \\ c_{ii} &= \frac{-\phi_\pi \omega_g}{1+\omega_y \phi_\pi} \frac{1}{n} + \frac{\delta_g - \delta_\tau \omega_g}{1+\delta_\tau \omega_y}(1-\frac{1}{n}) \\ c_{ij} &= -\left[\frac{\phi_\pi \omega_g}{1+\omega_y \phi_\pi} + \frac{\delta_g - \delta_\tau \omega_g}{1+\delta_\tau \omega_y}\right]\frac{1}{n} \qquad for \quad i \neq j \end{aligned}$$

The c-parameters are crucial for the following. c_{ii} measures the effect domestic fiscal policy has on domestic activity, and c_{ij} measures the fiscal spill-over effect on activity. The first term in the expression for c_{ii} captures that a fiscal expansion increases inflation which in turn induces a monetary contraction, and the more steep the trade-off between output and inflation, the more strongly the monetary authority reacts to an increase in inflation. This effect is perceived by fiscal policy makers because the monetary policy response is known and credible. Consequently, the expansionary effect of fiscal policy is curtailed through this monetary policy spill-over. Notice that the less steep the output-inflation trade-off (ϕ_π) and the larger n the smaller this effect. The second term in the expression for c_{ii} captures the direct activity effect of fiscal policy which is made up of the direct demand effect less the demand loss from a deterioration of competitiveness. This is multiplied by $(1-\frac{1}{n})$ which measures the extent to which domestic fiscal policy is perceived to differ from the aggregate fiscal stance in the currency union, cf (5). It is a straightforward implication that

$$\frac{\partial c_{ii}}{\partial n} = \frac{1}{n^2}\left(\frac{\phi_\pi \omega_g}{1+\omega_y \phi_\pi} + \frac{\delta_g - \delta_\tau \omega_g}{1+\delta_\tau \omega_y}\right) > 0$$

that is, the more decentralized fiscal policy decisions are, the more policy makers perceive that a fiscal expansion will boost activity. The reason is that the policy maker in this case perceives to have less influence on the aggregate situation within the currency union. Hence, there is a value of n, $\underline{n}$, such that $c_{ii} > 0$ for $n > \underline{n}$. Finally, notice that the fiscal externality $(c_{ij}, i \neq j)$ is unambiguously negative.

3 Fiscal Policy

We can now turn to fiscal policy decisions under the assumption that the national fiscal authorities choose fiscal policy so as to minimize a loss function given as

$$L_{it}^{G} = E_t \sum_{k=0}^{\infty} (1+\rho)^{-k} (\frac{1}{2}\alpha_G \left(y_{it+k}\right)^2 + \frac{1}{2}(g_{it+k})^2)$$

that is, the loss function penalizes variations in output (around its steady state level) and variations in the fiscal instrument (public sector activity). Notice, that fiscal policy makers are assumed not to be directly concerned about inflation, since the task of controlling inflation is delegated to a monetary authority targeting inflation. However, the policy maker is indirectly concerned about country specific inflation since it affects competitiveness and thus in turn activity. The weight of output stabilization relative to instrument stability is $\alpha > 0$, and $\rho \geq 0$ is the subjective discount rate of the policy maker.

Non-cooperative policy making
In the non-cooperative case each national fiscal authority decides on fiscal policy taking the fiscal decisions of other member states as given (Cournot-Nash game), but taking into account the monetary policy reaction function. The fiscal policy reaction function can be written (see appendix) as depending on country-specific activity , i.e.

$$g_{it} = \kappa_y^{nc} y_{it}$$

where

$$\kappa_y^{nc} = -\alpha_G c_{ii} \gtrless 0$$

It is seen that fiscal policy responds counter-cyclically to domestic activity provided that fiscal policy is expansionary ($c_{ii} > 0$), while it moves pro-cyclically if fiscal policy is contractionary ($c_{ii} < 0$). Note that the policy reaction function given above is not a closed form solution since y_{it} is endogenous.

Aggregating we find that the aggregate fiscal stance is determined as

$$g_t = \kappa_y^{nc} y_t \tag{7}$$

Using (7) we get that the aggregate fiscal stance can be written

$$g_t = \frac{-\phi_\pi^{nc}\kappa_y^{nc}}{1+\omega_y\phi_\pi^{nc}+\omega_g\phi_\pi^{nc}\kappa_y^{nc}}\omega_u u_t$$

Hence, a supply shock tends to imply a fiscal expansion and a demand shock a fiscal contraction.[12]

Similarly, aggregate activity can be written

$$y_t = \frac{-\phi_\pi^{nc}}{1+\omega_y\phi_\pi^{nc}+\omega_g\phi_\pi^{nc}\kappa_y^{nc}}\omega_u u_t$$

if the denominator is positive, it follows that aggregate output moves counter-cyclically to demand shocks, and pro-cyclically to supply shocks.

[12] Interestingly in Andersen and Holden (2001) this is found to be the optimal policy response in an explicit intertemporal model in which policy is designed so as to minimize risks.

Cooperative policy making

The cooperative solution is found under the assumption of a utilitarian criterion implying that fiscal policy for a country is determined as

$$L_t^G = \frac{1}{n}\sum_i L_{it}^G = E_t \frac{1}{n}\sum_i \sum_{k=0}^{\infty}(1+\rho)^{-k}(\frac{1}{2}\alpha_G\left(y_{it+k}\right)^2 + \frac{1}{2}(g_{it+k})^2)$$

We have that the policy reaction function for country i can be written

$$g_{it}^c = -\alpha_G\left((c_{ii}-c_{ji})y_{it} + nc_{ji}y_t\right)$$

which is seen to be unambiguously counter-cyclical to local output variations, but pro-cyclical to aggregate output variations.

In the aggregate we have

$$g_t^c = \kappa_y^c y_t$$

where

$$\kappa_y^c = \alpha_G \frac{\phi_\pi^c \omega_g}{1+\phi_\pi^c \omega_y} > 0$$

as would be expected κ_y^c is independent of n.

Accordingly, we get in the cooperative case

$$\begin{aligned} g_t &= \frac{-\phi_\pi \kappa_y^c}{1+\omega_y\phi_\pi^c + \omega_g\phi_\pi^c\kappa_y^c}\omega_u u_t \\ y_t &= \frac{-\phi_\pi^c}{1+\omega_y\phi_\pi^c + \omega_g\phi_\pi^c\kappa_y^c}\omega_u u_t \end{aligned}$$

Hence, the cooperative policy has aggregate fiscal policy to be expansionary to supply shocks and contractionary to demand shocks, and output expands to supply shocks and contracts to demand shocks.

Considering country differences we have

$$g_{it}^c - g_t^c = -\alpha\delta_G(y_{it} - y_t)$$

It follows that countries experiencing output levels about the average should contract fiscal policy relative to the average, and vice versa. Note this holds irrespective of the sign of ω_g.

4 Fiscal policy coordination problems

The fiscal response in the non-cooperative and cooperative case can now be compared. Since the outcome depends critically on the nature of shocks, it is useful to take the two extreme cases, namely, the one where shocks are common for all member countries (aggregate shocks), i.e. $u_i = u_j \forall i, j$, and the other where the shocks are country-specific with no aggregate consequences, i.e. $u = 0$.

Aggregate shocks

Considering first the aggregate response of fiscal policy to the aggregate level of activity within the monetary union, we have that

$$\kappa_y^c - \kappa_y^{nc} > 0 \quad for\ n > 1$$

Hence, for the monetary union as a whole the non-cooperative fiscal policy stance is unambiguously too counter-cyclical relative to the cooperative policy. The intuition is that each separate fiscal authority does not sufficiently take into account the policy responses of the other fiscal authorities, and they all perceive that they can change output to their advantage.

This inefficiency in fiscal policy is increasing in the number of fiscal decision makers (n), i.e.

$$\frac{\partial\left(\kappa_y^c - \kappa_y^{nc}\right)}{\partial n} > 0$$

since the larger the monetary union the less effect does each separate fiscal authority perceive that its actions have on the aggregate policy stance within the monetary union. To put it differently, more fiscal decentralization reinforces the coordination problem in fiscal policy. Note this comparison only makes sense if there are aggregate shocks in the monetary union i.e. $u_t \neq 0$ (see below on country-specific shocks).

Since the weight given to output in the monetary objective function affects both the non-cooperative and the cooperative fiscal policy parameter it follows that the fiscal policy coordination problem may differ across regimes distinguished by the weight given to output stability in monetary policy. It can be shown that (see appendix)

$$\frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial \alpha_M} < 0$$

that is, the larger the output weight in monetary policy the less the fiscal coordination problem. This is tantamount to saying that the fiscal authorities are better off if the monetary authority assigns a larger weight to output stabilization.

A different question is whether the weight in the monetary objective function could be set so as to eliminate the fiscal coordination problem. The answer is in general affirmative, that is, there exists a $\widetilde{\alpha}_M$ which ensures the cooperative fiscal response for a given α_M, i.e. should the objective of the monetary authority be biased relative to the "social preferences" to minimize the fiscal coordination problem (cf Rogoff (1985)). The interpretation is that by letting the weight to output stabilization in the monetary policy objective function differ from the social value ($\alpha_M \neq \widetilde{\alpha}_M$) inefficiencies in fiscal policy can be eliminated. Since the problem is that non-coordinated fiscal policies are too counter-cyclical it follows that

$$\widetilde{\alpha}_M < \alpha_M \quad ; \quad \frac{\partial \widetilde{\alpha}_M}{\partial n} < 0$$

i.e. the output weight should be set below the true value to eliminate the fiscal coordination problem. With more decentralization of fiscal policy the weight to output stability has to be lower to ensure a given aggregate fiscal response.

Finally, given the inefficiency in fiscal policies due to coordination problems, it is worth asking whether fiscal activism is always desirable, or whether non-coordinated fiscal policy can end up being destabilizing. To see this compare output fluctuations under the optimal non-cooperative policy and in the case of passive fiscal policy ($\kappa_y^{nc} = 0$). We have that non-cooperative policies imply more output variability if

$$\kappa_y^{nc} < 0$$

Note that $\kappa_y^{nc} < 0$ is possible if n is sufficiently large. Hence, in a sufficiently large monetary union it is possible that fiscal policy activism may be destabilizing. The intuition is that in a large union, the decentralized fiscal authorities perceive that a fiscal expansion will boost activity, but they end up causing an inappropriate policy mix which is detrimental to output stability. Note that fiscal policy activism (by definition) is always beneficial in the cooperative case ($\kappa_y^c > 0$).

Since $\kappa_y^c > \kappa_y^{nc}$ it follows that aggregate output is unambiguously more stable with coordinated fiscal policies than with non-cooperative policy. Moreover, since $\frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial n} > 0$ it follows that this effect is stronger, the larger the union.

Country-specific shocks

In the case where the shocks are only country-specific or idiosyncratic ($u = 0$) we have $y_t = \pi_t = g_t = 0$ (for technical details see appendix). In this case the non-cooperative policy function reads

$$g_{it} = \kappa_y^{nc} y_{it}$$

and the cooperative

$$g_{it}^c = \kappa_{yi}^c y_{it}, \qquad \kappa_{yi}^c = -\alpha_G \frac{\delta_g - \delta_\tau \omega_g}{1 + \delta_\tau \omega_y} < 0$$

Note that the cooperative policy in this case is unambiguously countercyclical. The reason being that there is no monetary spill-over effect to take into account in this case. We have that[13]

$$\kappa_{yi}^c < \kappa_y^{nc} \qquad for \quad n \geq 2$$

It follows that idiosyncratic output fluctuations are stabilized too little as compared to the cooperative case. The demand spill-over effect is causing this difference between cooperative and non cooperative policy making, and this reflects that country-specific fiscal authorities do not take into account the spill-over effects of fiscal policy to its trading partners[14]. However, the inefficiency is decreasing in the number of fiscal decision makers, i.e.

[13] Observe that idiosyncratic shocks are only well-defined for $n \geq 2$.

[14] Observe that the fiscal spill-over is always negative ($c_{ji} < 0$). However, with idiosyncratic shocks, it follows that if the domestic country is experiencing a recessionary shock, the trading partners face an expansionary shock, and therefore a domestic fiscal stabilization contributes to stabilize foreign output.

$$\frac{\partial(\kappa_y^{nc} - \kappa_{yi}^c)}{\partial n} < 0$$

Since the weight given to output in the monetary objective function affects both the non-cooperative and the cooperative fiscal policy parameter it follows that the fiscal policy coordination problem may differ across regimes distinguished by the weight given to output stabilization policy. It can be shown that (see appendix)

$$\frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial \alpha_M} > 0$$

that is, the larger the output weight in monetary policy the stronger the fiscal coordination problem. Fiscal authorities are worse of the larger weight the monetary authority assigns to output stabilization.

Turning to the question whether the weight in the monetary objective function could be set so as to eliminate the fiscal coordination problem we also have in the case of idiosyncratic shocks that the answer in general is affirmative. That is, there exists an $\widetilde{\alpha}_M$ which ensures the cooperative fiscal response for a given α_M . The interpretation is that by letting the weight to output stabilization in monetary policy differ from the social value ($\alpha_M \neq \widetilde{\alpha}_M$) inefficiencies in fiscal policy can be eliminated. Since the problem is that non-coordinated fiscal policies are too counter-cyclical it follows that

$$\widetilde{\alpha}_M > \alpha_M \qquad ; \qquad \frac{\partial \widetilde{\alpha}_M}{\partial n} < 0$$

i.e. the output weight should be set below the true value to eliminate the fiscal coordination problem, and with more decentralization of fiscal policy the weight to output stability has to be lower to ensure a given aggregate fiscal response.

5 Optimal Policy Assignment

The coordination problem has so far been discussed as a coordination problem between fiscal decision makers given the decentralized decision structure for fiscal policy. In general there is also a coordination issue between fiscal and monetary policy. In the EMU this problem has been addressed by granting monetary authority to an independent institution with price stability as its main objective. Given the coordination problem in fiscal policy and between fiscal and monetary policy it may be asked how the objectives of monetary policy should be defined to minimize overall coordination problems.

To consider this issue the super-cooperative outcome is used as reference point, that is, the policy setting which would arise under coordinated determination of both fiscal and monetary policy. This policy minimizes the loss

$$L_t^S = L_t^M + L_t^G$$

The policy outcome in this case depends on the aggregate weight to output stabilization ($\alpha_M + \alpha_G = \alpha$), see appendix. The weight α is interpreted as the social weight to output stability which is then divided between the weight the monetary and the fiscal authorities should assign to output stabilization. Strict inflation targeting for monetary policy corresponds in this case to $\alpha_M = 0$, implying that the task of stabilizing output is left with the fiscal authorities ($\alpha_G = \alpha$).

The optimal assignment problem can now be interpreted as how to divide the task of stabilizing output between the fiscal and monetary authority so as to minimize the difference between the overall loss in the non-cooperative case relative to the case of cooperation between fiscal and monetary authorities. This is here considered for the case of aggregate shocks, and it is shown in the appendix that flexible inflation targeting dominates strict inflation targeting, i.e.

$$\alpha_M^* > 0$$

In interpreting this result it should be noted that it addresses the ex-ante assignment of objectives presuming that policy coordination is not possible in the specific policy response to shocks. It is therefore an implication that if flexible inflation targeting is to be implemented it requires that fiscal authorities reduce the weight given to output stabilization in the policy design ($\alpha_G^* < \alpha$). To put it differently, flexible inflation targeting is optimal only if accompanied by some form of fiscal restraint.

6 Numerical illustrations

A key aspects of the preceding analysis is the inefficiencies arising due to decentralized fiscal policy decision making. It is therefore of interest to clarify how sensitive the inefficiencies are to the number of fiscal authorities and thus the degree of decentralization in fiscal policy. Figure 1 and 2 provide a numerical illustration[15] of the relative loss to fiscal authorities in the non-cooperative to the cooperative case for aggregate and country-specific shocks, respectively. The figures confirms the analytical results that inefficiencies in respect to stabilization of aggregate shocks are increasing in the number of fiscal decision makers, while they are decreasing in the case of country-specific shocks. Interestingly the figure shows that in both cases the inefficiencies are relatively invariant to the number of decision makers when it is 10 or more. This suggests that enlargement of the EMU would not to a significant extent reinforce coordination problems already present. Moreover, in a large monetary union the problem of handling aggregate shocks is larger than that of addressing country-specific shocks.

Turning to the assignment figure 3 shows how the optimal assignment[16] depends on the number of fiscal decision makers. It is seen that more fiscal decentralization implies an increase in the output weight in monetary policy and

[15] The illustrations are made for the following parameter values: $\omega_y = \omega_g = 0.5, \omega_u = 0.2, \delta_r = -0.2, \delta_\tau = 0.2, \delta_y = \delta_g = 0.5, \delta_u = 0.1$, and $\alpha_G = 2, \alpha_M = 0.25$.

[16] When the social weight equals 2.

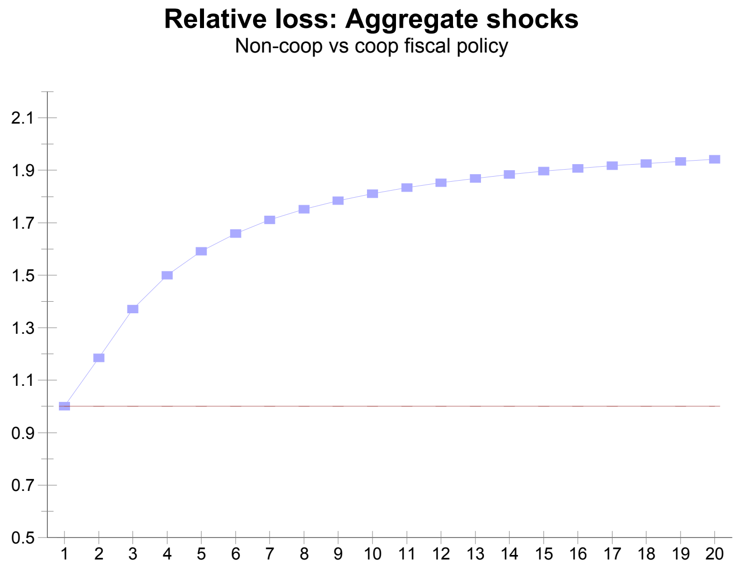

Figure 1:

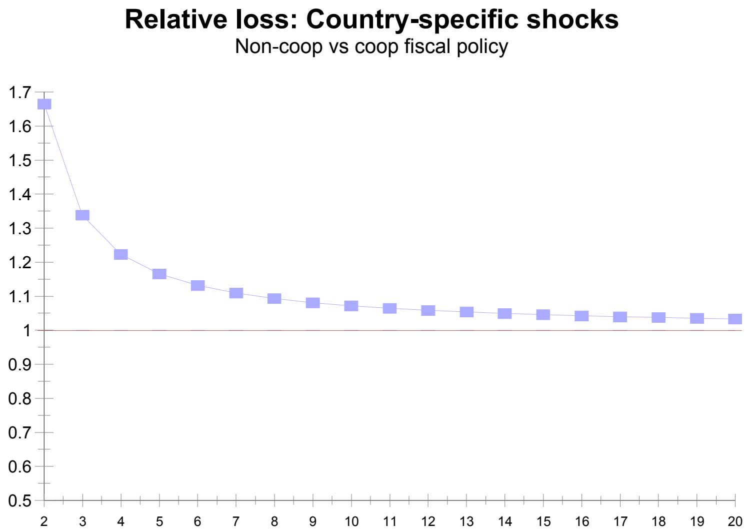

Figure 2:

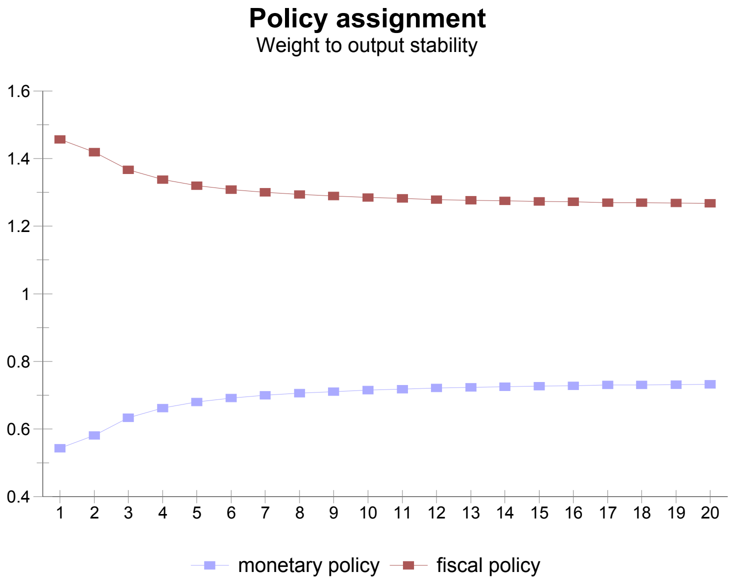

Figure 3:

a reduction in fiscal policy. Again the optimal weights are relatively insensitive to the number of decision makers once it is 10 or larger. Finally, it is seen that the output weight in fiscal policy is larger than in monetary policy, i.e. the optimal assignment implies that flexible inflation targeting is optimal, but fiscal authorities should still take a larger responsibility for output stabilization.

7 Concluding remarks

This paper has considered basic interactions between centralized monetary policy and decentralized fiscal policy in a currency union. Fiscal policy coordination problems are present, and the policy mix tends to be inefficient. However, the bias is shock contingent since it depends critically on whether shocks are aggregate or country specific. Accordingly, simple and general policy restraints will be targeting the inefficiencies imprecisely. It follows that more flexible inflation targeting is not necessarily overcoming the stabilization problems arising from non-coordinated policy making.

The model used in the present paper has purposely been kept simple to avoid technical difficulties, and to focus on the basic questions. The analysis is therefore only a first step in identifying the key issues. In the respect it is reassuring that Beetsma and Jensen (2002) find that the gains from (coordinated) fiscal stabilization policies in a monetary union can be large. In Andersen and Spange

(2002) it is shown in an explicit general equilibrium model that a stabilization deficit in respect to country-specific shocks arises if there is some wage rigidity.

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APPENDIX A

Monetary policy

From (4) and (2) aggregate output and inflation are given as

$$y_t = \frac{1}{1-\delta_y}\left[-\delta_r r_t + \delta_g g_t + \delta_u u_t\right]$$

$$\pi_t = \pi_t^e + \omega_y y_t + \omega_g g_t + \omega_u u_t$$

The first order condition to the decision problem of the monetary authority reads

$$E_t \sum_{k=0}^{\infty} (1+\rho)^{-k} (\alpha_M y_{t+k} \frac{\partial y_{t+k}}{\partial r_t} + \pi_{t+k} \frac{\partial \pi_{t+k}}{\partial r_t}) = 0$$

which can be written

$$\alpha_M y_t \frac{-\delta_\tau}{1-\delta_y} + \pi_t \frac{-\delta_\tau \omega_y}{1-\delta_y} = 0$$

or

$$\begin{aligned} y_t &= -\phi_\pi \pi_t \quad , \\ \phi_\pi &\equiv \frac{\omega_y}{\alpha_M} > 0 \end{aligned}$$

Equilibrium output

Initially note that we have

$$\begin{aligned} y_{it} - y_t &= -\delta_\tau (\pi_{it} - \pi_t) + \delta_g (g_{it} - g_t) + \delta_u (u_{it} - u_t) \\ \pi_{it} - \pi_t &= \omega_y (y_{it} - y_t) + \omega_g (g_{it} - g_t) + \omega_u (u_{it} - u_t) \end{aligned}$$

Hence

$$\begin{aligned} y_{it} - y_t &= -\delta_\tau \left[\omega_y (y_{it} - y_t) + \omega_g (g_{it} - g_t) + \omega_u (u_{it} - u_t) \right] + \delta_g (g_{it} - g_t) + \delta_u (u_{it} - u_t) \\ &= \frac{1}{1+\delta_\tau \omega_y} \left[(\delta_g - \delta_\tau \omega_g)(g_{it} - g_t) + (\delta_u - \delta_\tau \omega_u)(u_{it} - u_t) \right] \end{aligned}$$

Given the monetary reaction function and $\pi^e = 0$ we have that aggregate output is given as

$$y_t = \frac{-\phi_\pi}{1+\omega_y \phi_\pi} \left[\omega_g g_t + \omega_u u_t \right]$$

and country-specific output as

$$y_{it} = y_t + \frac{1}{1+\delta_\tau \omega_y} \left[(\delta_g - \delta_\tau \omega_g)(g_{it} - g_t) + (\delta_u - \delta_\tau \omega_u)(u_{it} - u_t) \right]$$

Using the definitions of π, g and u we have that equilibrium output in country i in period t can be written (using $\pi_t^e = 0$)

$$y_{it} = \sum_j b_{ij} u_{jt} + \sum_j c_{ij} g_{jt}$$

where

$$
\begin{aligned}
b_{ii} &= \frac{-\phi_\pi \omega_u}{1+\omega_y \phi_\pi}\frac{1}{n} + \frac{\delta_u - \delta_\tau \omega_u}{1+\delta_\tau \omega_y}(1-\frac{1}{n}) \\
b_{ij} &= \left[\frac{-\phi_\pi \omega_u}{1+\omega_y \phi_\pi} - \frac{\delta_u - \delta_\tau \omega_u}{1+\delta_\tau \omega_y}\right]\frac{1}{n} \qquad for \quad i \neq j \\
c_{ii} &= \frac{-\phi_\pi \omega_g}{1+\omega_y \phi_\pi}\frac{1}{n} + \frac{\delta_g - \delta_\tau \omega_g}{1+\delta_\tau \omega_y}(1-\frac{1}{n}) \\
c_{ij} &= \left[\frac{-\phi_\pi \omega_g}{1+\omega_y \phi_\pi} - \frac{\delta_g - \delta_\tau \omega_g}{1+\delta_\tau \omega_y}\right]\frac{1}{n} \qquad for \quad i \neq j
\end{aligned}
$$

For later use note that

$$
\begin{aligned}
\pi_{it} &= \pi_t + \omega_y(y_{it} - y_t) + \omega_g(g_{it} - g_t) + \omega_u(u_{it} - u_t) \\
&= (1+\omega_y \phi_\pi)\,\pi_t + \omega_y y_{it} + \omega_g(g_{it} - g_t) + \omega_u(u_{it} - u_t)
\end{aligned}
$$

and aggregate inflation can therefore be written as a function of past inflation, fiscal policy and aggregate shock.

$$
\pi_t = \frac{1}{1+\omega_y \phi_\pi}\left[\omega_g g_t + \omega_u u_t\right]
$$

Hence

$$
\pi_{it} = \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it}
$$

We thus have

$$
\begin{aligned}
\frac{\partial \pi_{it}}{\partial g_{it}} &= \omega_y c_{ii} + \omega_g \\
\frac{\partial \pi_{jt}}{\partial g_{it}} &= \omega_y c_{ji} \\
\frac{\partial \pi_{t+1}}{\partial g_{it}} &= 0
\end{aligned}
$$

Fiscal Policy: Non-cooperative

The loss function of the fiscal authority is given as

$$
L_G = E_t \sum_{k=0}^{\infty}(1+\rho)^{-k}(\frac{1}{2}\alpha_G \left(y_{it+k}\right)^2 + \frac{1}{2}(g_{it+k})^2)
$$

The first order condition reads (non-cooperative)

$$
\alpha_G c_{ii} y_{it} + g_{it} = 0
$$

or

$$
g_{it} = -\alpha_G c_{ii} y_{it}
$$

Inserting it follows that the fiscal policy reaction function can be written:

$$g_{it} = \kappa_y^{nc} y_{it}$$

where

$$\kappa_y^{nc} = -\alpha_G c_{ii} \gtreqless 0$$

Fiscal Policy: Cooperative

The loss function of the fiscal authority is given as

$$L_G = E_t \sum_i \sum_{k=0}^{\infty} (1+\rho)^{-k} (\frac{1}{2}\alpha_G (y_{it+k})^2 + \frac{1}{2}(g_{it+k})^2)$$

Conjecture that aggregate fiscal policy in the cooperative case can be written

$$g_t = \kappa_y^c y_t$$

Using the same procedure as above for the solution to the monetary policy problem we find

$$y_t = -\phi_\pi \pi_t$$

where

$$\phi_\pi = \frac{\omega_y}{\alpha_M}$$

and where the η^c and χ^c are defined as above for the cooperative values of the fiscal policy parameters.

The first order condition to the fiscal policy problems reads in the cooperative case (Utilitarian criterion)

$$\alpha_G \sum_j y_{jt} c_{ji} + g_{it} = 0$$

$$g_{it} = \sum \kappa_{yj}^c y_{jt}$$

where

$$\kappa_{yj}^c = -\alpha_G c_{ji}$$

Comparison cooperative and non-cooperative outcome - aggregate shocks

(I) Aggregate shocks ($u_{it} = u_t$ for all i)

Aggregating we have

$$g_{it} = (\sum \kappa_{yj}^c) y_t$$

where

$$\sum \kappa_{yj}^c = \kappa_y^c = \alpha_G \frac{\phi_\pi \omega_g}{1 + \omega_y \phi_\pi} = \alpha_G \frac{\omega_y \omega_g}{\alpha_M + \omega_y^2}$$

We have

$$\begin{aligned}\kappa_y^{nc} &= -\alpha_G c_{ii} = \alpha_G \left[\frac{1}{n}\frac{\phi_\pi \omega_g}{1+\omega_y\phi_\pi} - \frac{\delta_g - \delta_\tau\omega_g}{1+\delta_\tau\omega_y}(1-\frac{1}{n})\right] \\ &= \alpha_G \left[\frac{1}{n}\frac{\omega_y\omega_g}{\alpha_M+\omega_y^2} - \frac{\delta_g - \delta_\tau\omega_g}{1+\delta_\tau\omega_y}(1-\frac{1}{n})\right]\end{aligned}$$

Hence

$$\kappa_y^c - \kappa_y^{nc} = \alpha_G(1-\frac{1}{n})\left[\frac{\omega_y\omega_g}{\alpha_M+\omega_y^2} + \frac{\delta_g - \delta_\tau\omega_g}{1+\delta_\tau\omega_y}\right]$$

Observing that

$$\kappa_y^{nc} = \kappa_y^c \quad for \quad n = 1$$

and using that $\frac{\partial\kappa_y^{nc}}{\partial n} < 0$ it follows that

$$\begin{aligned}\kappa_y^{nc} &\leq \kappa_y^c \\ \frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial n} &> 0\end{aligned}$$

Note that the higher α_M the lower κ_y^c and κ_y^{nc}, that is, higher weight to output stabilization for the monetary authority, the less pro-cyclical is fiscal policy. However,

$$\frac{\partial(\kappa_y^c - \kappa_y^{nc})}{\partial\alpha_M} < 0$$

Next consider whether there exists a choice of $\widetilde{\alpha}_M$ which delivers a given fiscal policy response κ^* under non-cooperative policy making, i.e. $\kappa_y^{nc} = \kappa^*$. This requires

$$\kappa^* = \alpha_G \left[\frac{1}{n}\frac{\omega_y\omega_g}{\widetilde{\alpha}_M+\omega_y^2} - \frac{\delta_g - \delta_\tau\omega_g}{1+\delta_\tau\omega_y}(1-\frac{1}{n})\right]$$

Hence, such a choice exists in general, and if $\kappa^* > \kappa_y^{nc}$ then $\widetilde{\alpha}_M < \alpha_M$. Note that $\widetilde{\alpha}_M$ is decreasing in n.

Aggregate output can be written

$$y_t = \frac{-\phi_\pi}{1+\omega_y\phi_\pi}\left[\omega_g g_t + \omega_u u_t\right]$$

In the non-cooperative case we have

$$g_t = \kappa_y^{nc} y_t$$

and hence

$$y_t = \frac{-\phi_\pi}{1+\omega_y\phi_\pi+\omega_g\phi_\pi\kappa_y}\omega_u u_t$$

The variance of output is given as

$$VAR^{nc}(y) = \left[\frac{\phi_\pi\omega_u}{1+\omega_y\phi_\pi+\omega_g\phi_\pi\kappa_y^{nc}}\right]^2\sigma_u^2$$

and

$$VAR^{nc}(g) = \left[\kappa_y^{nc}\right]^2 VAR^{nc}(y)$$

Similar relations hold in the cooperative case, i.e.

$$VAR^{c}(y) = \left[\frac{\phi_\pi \omega_u}{1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^c}\right]^2 \sigma_u^2$$

and

$$VAR^{c}(g) = \left[\kappa_y^{c}\right]^2 VAR^{c}(y)$$

Note that since $\kappa_y^{nc} \leq \kappa_y^c$

$$VAR^c(y) \geq VAR^{nc}(y)$$

The relative loss

$$\begin{aligned}
\frac{L^{nc}}{L^c} &= \frac{(\alpha_G + \left[\kappa_y^{nc}\right]^2)}{(\alpha_G + \left[\kappa_y^{c}\right]^2)} \frac{VAR^{nc}(y)}{VAR^{c}(y)} \\
&= \frac{(\alpha_G + \left[\kappa_y^{nc}\right]^2)}{(\alpha_G + \left[\kappa_y^{c}\right]^2)} \frac{\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^c\right)^2}{\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)^2}
\end{aligned}$$

Observe that the unconditional mean of output and hence the fiscal instrument is zero.

Note that

$$\frac{\partial}{\partial n}\left(\frac{L^{nc}}{L^c}\right) = \frac{1}{L^c}\frac{\partial}{\partial n}\left(L^{nc}\right)$$

where

$$\begin{aligned}
\frac{\partial}{\partial n}\left(L^{nc}\right) &= \frac{2\kappa_y^{nc}\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)^2 - 2(\alpha_G + \left[\kappa_y^{nc}\right]^2)\omega_g \phi_\pi \left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)}{\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)^4} \frac{\partial \kappa_y^{nc}}{\partial n} \\
&= \frac{2\left[\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)\right]\left[\kappa_y^{nc}\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right) - (\alpha_G + \left[\kappa_y^{nc}\right]^2)\omega_g \phi_\pi\right]}{\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)^4} \frac{\partial \kappa_y^{nc}}{\partial n} \\
&= \frac{2\left[\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)\right]\left[\kappa_y^{nc}\left(1 + \omega_y \phi_\pi\right) - \alpha_G \omega_g \phi_\pi\right]}{\left(1 + \omega_y \phi_\pi + \omega_g \phi_\pi \kappa_y^{nc}\right)^4} \frac{\partial \kappa_y^{nc}}{\partial n}
\end{aligned}$$

Using that $\kappa_y^{nc}\left(1 + \omega_y \phi_\pi\right) - \alpha_G \omega_g \phi_\pi < 0$ we have

$$\frac{\partial}{\partial n}\left(\frac{L^{nc}}{L^c}\right) > 0$$

Cooperation Fiscal and Monetary Policy

The objective function for society is given as the sum of the loss function for the Monetary Authority and the Fiscal Authorities, i.e.,

$$L_t^S = L_t^M + L_t^G = \frac{1}{2}(\alpha_M + \alpha_G) y_t^2 + \frac{1}{2} g_t^2 + \frac{1}{2}\pi_t^2$$

And we have

$$\begin{aligned} y_t &= \frac{1}{1-\delta_y}\left[-\delta_r r_t + \delta_g g_t + \delta_u u_t\right] \\ \pi_t &= \pi_t^e + \omega_y y_t + \omega_g g_t + \omega_u u_t \end{aligned}$$

The first-order condition to the problem of choosing fiscal and monetary policy to minimize the loss function for society is

$$\begin{aligned} (\alpha_M + \alpha_G) y_t \left(\frac{\delta_g}{1-\delta_y}\right) + g_t + \pi_t \left(\frac{\omega_y \delta_g}{1-\delta_y} + \omega_g\right) &= 0 \\ (\alpha_M + \alpha_G) y_t \left(\frac{-\delta_\tau}{1-\delta_y}\right) + \pi_t \left(\frac{-\delta_\tau \omega_y}{1-\delta_y}\right) &= 0 \end{aligned}$$

The latter condition implies

$$y_t = \frac{-\omega_y}{\alpha_M + \alpha_G}\pi_t$$

implying

$$\begin{aligned} g_t &= \left[\left(\frac{\omega_y \delta_g}{1-\delta_y} + \omega_g\right)\frac{\alpha_M + \alpha_G}{\omega_y} - (\alpha_M + \alpha_G)\left(\frac{\delta_g}{1-\delta_y}\right)\right] y_t \\ &= (\alpha_M + \alpha_G)\frac{\omega_g}{\omega_y} y_t \\ &= \kappa^{sc} y \end{aligned}$$

It furthermore follows that

$$\begin{aligned} -\frac{\alpha_M + \alpha_G}{\omega_y} y_t &= \omega_y y_t + \omega_g g_t + \omega_u u_t \\ y_t &= \frac{-\frac{\omega_y}{\alpha_M + \alpha_G}}{1 + \omega_y \frac{\omega_y}{\alpha_M + \alpha_G}}\left[\omega_g g_t + \omega_u u_t\right] \end{aligned}$$

Hence

$$\begin{aligned} y_t &= \frac{-\frac{\omega_y}{\alpha_M + \alpha_G}}{1 + \omega_y \frac{\omega_y}{\alpha_M + \alpha_G} + \omega_g \kappa^{sc} \frac{\omega_y}{\alpha_M + \alpha_G}} \omega_u u_t \\ \pi_t &= -\frac{\alpha_M + \alpha_G}{\omega_y} y_t \end{aligned}$$

It follows that the loss is

$$L^{S,C} = \left[\frac{1}{2}(\alpha_M + \alpha_G) + \frac{1}{2}(\kappa^s)^2 + \frac{1}{2}\left(-\frac{\alpha_M + \alpha_G}{\omega_y}\right)^2\right] VAR^s(y_t)$$

where

$$\begin{aligned} VAR^s(y_t) &= \left[\frac{-\frac{\omega_y}{\alpha_M+\alpha_G}}{1+\omega_y\frac{\omega_y}{\alpha_M+\alpha_G}+\omega_g\kappa^{sc}\frac{\omega_y}{\alpha_M+\alpha_G}}\right]^s \omega_u^2\sigma_u^2 \\ &= \left[\frac{-\omega_y}{\alpha_M+\alpha_G+\omega_y\omega_y+\omega_g\kappa^{sc}\omega_y}\right]^2 \omega_u^2\sigma_u^2 \end{aligned}$$

In the non-cooperative case we have

$$L^{S,NC} = \left[\frac{1}{2}(\alpha_M+\alpha_G)+\frac{1}{2}\left(\kappa^{nc}\right)^2+\frac{1}{2}\left(\frac{\alpha_M}{\omega_y}\right)^2\right] VAR^{nc}(y_t)$$

Optimal assignment: consider the choice of α_M and α_G for a given value of $\alpha_M + \alpha_G$.

$$\begin{aligned} \min_{\alpha_M}\ L^{S,NC} &= \left[\frac{1}{2}(\alpha_M+\alpha_G)+\frac{1}{2}\left(\kappa^{nc}\right)^2+\frac{1}{2}\left(\frac{\alpha_M}{\omega_y}\right)^2\right]\left[\frac{\omega_y\omega_u}{\alpha_M+\omega_y\omega_y+\omega_g\omega_y\kappa_y^{nc}}\right]^2\sigma_u^2 \\ st\ \alpha_M+\alpha_G &= \alpha \\ \kappa_y^{nc} &= \alpha_G\left[\frac{1}{n}\frac{\omega_y\omega_g}{\alpha_M+\omega_y^2}-\frac{\delta_g-\delta_\tau\omega_g}{1+\delta_\tau\omega_y}(1-\frac{1}{n})\right] \end{aligned}$$

Hence

$$\begin{aligned} \frac{\partial L^{S,NC}}{\partial\alpha_M} &= \left[-\left(\kappa^{nc}\right)^2\alpha_G^{-1}+\kappa_y^{nc}\frac{\partial\kappa_y^{nc}}{\partial\alpha_M}+\frac{\alpha_M}{\omega_y^2}\right]\left[\frac{\omega_y\omega_u}{\alpha_M+\omega_y\omega_y+\omega_g\omega_y\kappa_y^{nc}}\right]^2\sigma_u^2 \\ &\quad -\frac{1}{2}\left[(\alpha_M+\alpha_G)+\left(\kappa^{nc}\right)^2+\left(\frac{\alpha_M}{\omega_y}\right)^2\right]\left[\frac{(\omega_y\omega_u)^2}{(\alpha_M+\omega_y\omega_y+\omega_g\omega_y\kappa_y^{nc})^3}\right]\sigma_u^2\left[1-\omega_g\omega_y\frac{\partial\kappa_y^{nc}}{\partial\alpha_M}\right] \end{aligned}$$

It follow straightforward that

$$\frac{\partial L^{S,NC}}{\partial\alpha_M} < 0 \quad \text{for} \quad \alpha_M = 0$$

Considering the optimal value of α_M we have the necessary condition

$$-\left(\kappa^{nc}\right)^2\alpha_G^{-1}+\kappa_y^{nc}\frac{\partial\kappa_y^{nc}}{\partial\alpha_M}+\frac{\alpha_M}{\omega_y^2} > 0$$

or

$$\frac{\alpha_M}{\omega_y^2} > -\left(\kappa^{nc}\right)^2\alpha_G^{-1}+\kappa_y^{nc}(\alpha-\alpha_M)\frac{1}{n}\frac{\omega_y\omega_g}{(\alpha_M+\omega_y^2)}$$

Idiosyncratic shocks $(u_t = 0)$
In this case $y_t = \pi_t = 0$.

We have

$$\begin{aligned}
g_{it}^c &= \sum \kappa_{yj}^c y_{jt} \\
&= -\alpha_G c_{ii} y_{it} - \alpha_G c_{ji} \sum_{j \neq i} y_{jt} \\
&= -\alpha_G (c_{ii} - c_{ji}) y_{it} - \alpha_G c_{ji} n y_t \\
&= -\alpha_G \frac{\delta_g - \delta_\tau \omega_g}{1 + \delta_\tau \omega_y} y_{it}
\end{aligned}$$

Hence

$$\kappa_{yi}^c = -\alpha_G \frac{\delta_g - \delta_\tau \omega_g}{1 + \delta_\tau \omega_y} < \alpha_G \left[\frac{1}{n} \frac{\omega_y \omega_g}{1 + \omega_y^2} - \frac{\delta_g - \delta_\tau \omega_g}{1 + \delta_\tau \omega_y} (1 - \frac{1}{n}) \right] = \kappa_y^{nc} \quad \text{for} \quad n \geq 2$$

and

$$\kappa_y^{nc} - \kappa_{yi}^c = \alpha_G \left[\frac{\omega_y \omega_g}{\alpha_M + \omega_y^2} + \frac{\delta_g - \delta_\tau \omega_g}{1 + \delta_\tau \omega_y} \right] \frac{1}{n}$$

From which it follows that

$$\begin{aligned}
\frac{\partial(\kappa_{yi}^{nc} - \kappa_{yi}^c)}{\partial n} &< 0 \\
\frac{\partial(\kappa_{yi}^{nc} - \kappa_{yi}^c)}{\partial \alpha_M} &> 0
\end{aligned}$$

Using

$$\begin{aligned}
y_{it} &= -\delta_\tau \pi_{it} + \delta_g g_{it} + \delta_u u_{it} \\
\pi_{it} &= \omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \\
g_{it} &= \kappa y_{it}
\end{aligned}$$

We have

$$\begin{aligned}
y_{it} &= -\delta_\tau \left[\omega_y y_{it} + \omega_g g_{it} + \omega_u u_{it} \right] + \delta_g g_{it} + \delta_u u_{it} \\
&= -\delta_\tau \omega_y y_{it} + (\delta_g - \delta_\tau \omega_g) g_{it} + (\delta_u - \delta_\tau \omega_u) u_{it} \\
&= \left[\kappa(\delta_g - \delta_\tau \omega_g) - \delta_\tau \omega_y \right] y_{it} + (\delta_u - \delta_\tau \omega_u) u_{it} \\
&= \frac{\delta_u - \delta_\tau \omega_u}{1 - \kappa(\delta_g - \delta_\tau \omega_g) + \delta_\tau \omega_y} u_{it}
\end{aligned}$$

Hence

$$\begin{aligned}
L^{nc} &= \left[\alpha_G + (\kappa_y^{nc})^2 \right] \left[\frac{\delta_u - \delta_\tau \omega_u}{1 - \kappa^{nc}(\delta_g - \delta_\tau \omega_g) + \delta_\tau \omega_y} \right]^2 \sigma_u^2 \\
L^c &= \left[\alpha_G + (\kappa_y^c)^2 \right] \left[\frac{\delta_u - \delta_\tau \omega_u}{1 - \kappa^c(\delta_g - \delta_\tau \omega_g) + \delta_\tau \omega_y} \right]^2 \sigma_u^2
\end{aligned}$$

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