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Can Nominal Wage and Price Rigidities Be Equivalent Propagation Mechanisms? The Case of Open Economies*

Bo William Hansen[†] Lars Mayland Nielsen[‡]

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Abstract

Does it matter for the propagation mechanism following nominal shocks whether nominal rigidities are specified as sticky wages instead of sticky prices? We analyze the question in a standard dynamic general equilibrium "new open macro-economy" model, which is solved analytically. By comparing the adjustment patterns of the terms of trade, in an otherwise unchanged model under, respectively, nominal wage and price rigidities, we find that the two type of rigidities give rise to the same persistence pattern. Specifically, nominal wage and price rigidities are equivalent "impact adjusted" propagation mechanisms. Results are presented for one-period nominal rigidities and two-period nominal staggering.

Keywords: nominal shocks; nominal rigidities; propagation; persistence; staggering

JEL classification: E32; F41

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1 Introduction

The interest of this paper concerns nominal rigidites. By building on a standard "new open macro-economy" model that uses nominal wage instead of price rigidities we ask; *does it matter whether nominal rigidities are specified as sticky prices or as sticky wages?* The question is relevant since the two cases might have different implications for the behavior of the labor and goods markets.

As Lane (2001) notes the literature has largely focused on price rigidities as the locus of nominal stickiness. Why some authors choose sticky prices rather than sticky wages (or vice versa) is often a matter of preference, e.g. Kimball (1995) argues in favor of price stickiness while Bergin (1995) views wage stickiness as preferable. To date only a few papers have studied the issue theoretically and, indeed, it appears to be an implicit assumption in most of the literature that there is no qualitative difference between the two cases. Cho (1993) calibrates a general equilibrium model of the real business-cycle type under both a one-period nominal wage contract and a one-period nominal price contract. It is found that the nominal wage contract improves the fit of the model in virtually all aspects. Moreover, the price contract has some very counter-intuitive effects. Andersen (1998) demonstrates that wage staggering is more likely to generate persistence than price staggering, since wage stickiness implies that labor demand rather than labor supply will determine quantities in the labor market. Therefore, the elasticity of labor supply with respect to the real wage - shown to be critical for persistence by Chari, Kehoe and McGratten (2000) - becomes irrelevant in determining short-run marginal costs.

In this paper we compare the adjustment patterns of the terms of trade under respectively nominal wage and price rigidities in an otherwise unchanged model. The setup and solution method follow Andersen and Beier (1999) who analyze the propagation of nominal shocks under sticky wages. Thus, we essentially substitute the assumption of sticky wages in the Andersen and Beier model with that of sticky prices. We find that under certain conditions *nominal wage and price rigidities are equivalent propagation mechanisms.* In particular, price and wage rigidities are equivalent "impact adjusted" propagation mechanisms; once the terms of trade impulse response functions (following a nominal monetary shock) are divided by their respective impact effects, the adjustment dynamics become completely identical for nominal wage and price rigidities. We also find that the two model characteristics (1) a random walk in relative consumption and (2) satisfaction of purchasing power parity, nicely separate the "real" part of the propagation mechanism from the nominal shock. This might be taken as the explanation of the observed equivalence. As such, we feel that the paper provides a good understanding of how nominal rigidities influence this kind of model.

To solve the model we log-linearize it around a symmetric steady state. This way, we can disregard all constant level effects such as risk terms and monopoly mark-ups as these do not affect the dynamic processes.[1] A crucial element of the analysis is that we use the method of undetermined coefficients to solve the model analytically thus avoiding the black box that often dominates calibrations of complicated models. Even more important, we are able to explicitly compare the dynamic adjustment patterns following nominal shocks under the two rigidity regimes. Precisely how the underlying source of stickiness turn up will not be modelled here, instead we simply introduce rigidities as an exogenous feature of the environment. We present analytical results for one-period nominal rigidities and two-period nominal staggering. However, as we briefly note, the main equivalence result easily generalizes to X-period nominal staggering.

The paper is organized as follows. In section 2 we introduce the basic model framework. Section 3 introduces nominal wage and price rigidities and derives the implied goods and labor market equilibrium conditions. In section 4 we compare the general equilibrium terms of trade solutions. Finally, section 5 concludes.

[1] We assume risk terms (variances and covariances of the endogenous variables) to be constant over time in our model.

2 The Model

The basic framework is an off-the-shelf "new open macro-economy" two-country general equilibrium model with a flexible exchange rate [essentially it is the classic *Redux* model by Obstfeld and Rogoff (1995)]. Both countries, labeled Home and Foreign, produce a single separate good demanded by consumers in both countries. All goods are tradable and there is a riskless real bond which is traded in an internationally integrated market. The model is symmetric and we denote all Foreign variables by an asterisk.

2.1 Preferences

We consider an infinitely lived representative consumer. Utility is of the CES form and additively separable in consumption (C), real money balances $\left(\frac{M}{P}\right)$, and disutility of effort (N). The consumption index is normalized and money balances and disutility of effort are weighted with λ and κ, respectively,

$$U_t = E_t \sum_{j=0}^{\infty} \delta^j \left[\frac{\sigma}{\sigma - 1} C_{t+j}^{\frac{\sigma-1}{\sigma}} + \frac{\lambda}{1-\varepsilon} \left(\frac{M_{t+j}}{P_{t+j}} \right)^{1-\varepsilon} - \frac{\kappa}{1+\mu} N_{t+j}^{1+\mu} \right],$$

where $\sigma > 0$, $\lambda > 0$, $\varepsilon > 0$, $\kappa > 0$, $\mu > 0$, $0 < \delta \leq 1$. The dynamic budget constraint for the representative Home agent in nominal terms is

$$P_t B_t + M_t + P_t C_t = (1 + r_{t-1}) P_t B_{t-1} + M_{t-1} + W_t N_t + \Pi_t + P_t \tau_t,$$

where the right hand side is available resources in period $t-1$, defined as the sum of gross return on bondholdings $(1 + r_{t-1}) P_t B_{t-1}$, initial money holdings M_{t-1}, nominal labor income $W_t N_t$, nominal profits Π_t, and transfers from the government $P_t \tau_t$. Resources can be allocated in period t to bonds, money holdings, and consumption. The government runs a balanced budget each period and there is no government spending. This way the government's role is reduced to collecting taxes and redistributing money in a lump sum fashion. The government budget constraint becomes $(M_t - M_{t-1}) = \tau_t P_t$. The real Home consumption index is defined over the consumption of the Home good and the Foreign good as $C_t = \left[\left(\frac{1}{2}\right)^{\frac{1}{\rho}} (C_t^h)^{\frac{\rho-1}{\rho}} + \left(\frac{1}{2}\right)^{\frac{1}{\rho}} (C_t^f)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}$, $\rho \geq 1$, with the price index $P_t = \left[\frac{1}{2}(P_t^h)^{1-\rho} + \frac{1}{2}(P_t^f)^{1-\rho}\right]^{\frac{1}{1-\rho}}$.[2] We assume that there is a home bias to money. Let P_t^h (P_t^{*h}) be the price of the Home good and P_t^f $\left(P_t^{*f}\right)$ the price of the Foreign good, where an asterisk refers to Foreign currency denomination. Assume that the law of one price (LOP) holds for both goods, $P_t^h = S_t P_t^{*h}$ and $P_t^f = S_t P_t^{*f}$, where S_t is the nominal exchange rate defined as the Home price of Foreign currency. With no barriers to trade the purchasing power parity (PPP) holds, $P_t = S_t P_t^*$. Note, however, that relative product prices between the Home and the Foreign good - the terms of trade - need not remain constant. Home consumers' demand for the Home and the Foreign good is

$$D_t^h = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t, \quad D_t^f = \frac{1}{2}\left(\frac{P_t^f}{P_t}\right)^{-\rho} C_t.$$

By symmetry we can obtain the Foreign consumers' demand. Aggregating demands the world demand facing the Home producers becomes

$$D_t = D_t^h + D_t^{*h} = \frac{1}{2}\left(\frac{P_t^h}{P_t}\right)^{-\rho} C_t^W,$$

where $C_t^W = C_t + C_t^*$ denotes world consumption.

[2]Note, ρ is the elasticity of substitution between Home and Foreign goods. For $\rho = 1$, the trade balance is always unchanged as no wealth reallocations take place. For $\rho > 1$, the Marshall-Learner condition is fulfilled. We rule out the implausible case of $\rho < 1$ where a monetary expansion induces an appreciation of the nominal exchange rate. As Andersen and Beier (1999) note, this might lead to a perverse adjustment pattern of the terms of trade.

2.1.1 Relative Consumer First-order Conditions and The Nominal Exchange Rate

In order to obtain an analytical solution for the model we log-linearize it around a country symmetric steady state where initial net Foreign assets are zero. Under the assumed stochastic process for the exogenous variable (money) the variables of the model are log-normally distributed (c.f. below). Since we solve the model relative to the Foreign country, we are interested in *relative* versions of the (standard) consumer first-order conditions. We find

$$E_t \Delta c_{t+1} = \Delta c_t \Leftrightarrow \Delta c_t = \Delta c_{t-1} + \eta_{cu} u_t, \tag{1}$$

$$\Delta m_t^d = (1-\delta)\eta_{mc}\Delta c_t + \left(1+\eta_{mp}\right) s_t - \eta_{mp} E_t s_{t+1}, \tag{2}$$

$$\Delta n_t^s = \eta_{nw}\psi_t + \eta_{nc}\Delta c_t, \tag{3}$$

$$\Delta d_t = -\rho q_t. \tag{4}$$

To fully characterize the equilibrium and rule out no-Ponzi schemes, we have assumed that the transversality condition holds.[3] Let $\psi_t = \Delta w_t - s_t$ denote relative wages measured in Home currency and let $q_t = p_t^h - p_t^f$ represent the terms of trade. All constants (including variance terms which are assumed fixed) have been neglected since we are only interested in the dynamic processes. Equation (1) is the Euler equation determining the optimal path of relative consumption and eq. (2) is the familiar relative real money demand equation from money-in-the-utility-function models. The labor-leisure trade-off condition in eq. (3) determines relative individual labor supply. Finally, eq. (4) is relative output demand. Together the four equations determine optimal behavior on the consumer side of the economy.

Notice that relative consumption follows a random walk (martingale) pro-

[3] Lower-case letters denote log-deviations from steady state of the corresponding upper-case variables. Moreover, let η_{xv} denote the elasticity of X with respect to V and $\Delta x_t = x_t - x_t^*$ denote that a Home log-variable is evaluated relative to its Foreign counterpart.

cess. The u_t in eq. (1) denotes a shock to relative money supply (c.f. below). Hence, the elasticity η_{cu} captures the impact effect of a monetary shock on relative consumption. This way η_{cu} is a composite parameter reflecting the general equilibrium effect of a nominal shock on relative consumption. In general the size of η_{cu} depends on whether we choose nominal wages or prices to be fixed, since the size of the expenditure switch following a nominal shock depends on the rigidity specification (see Appendix A). As it turns out, the terms of trade impact effect can be shown to be numerically larger with nominal price rather than nominal wage rigidities. Accordingly the parameter η_{cu} is larger under nominal price rigidities.[4]

In order to solve the model analytically we need to specify a process for the money supply. Assume that the exogenous relative money supply follows a random walk, $\Delta m_t^s = \Delta m_{t-1}^s + u_t$, where $u_t \sim nid\,(0, \sigma_u^2)$ (the random walk assumption is merely to ease exposition and does not affect any conclusions). The money market equilibrium ($\Delta m_t^s = \Delta m_t^d$) is independent of the type of nominal rigidity. Using the method of undetermined coefficients we obtain the following solution for the nominal exchange rate

$$s_t = -\frac{1}{\sigma\varepsilon}\Delta c_t + \Delta m_t. \tag{5}$$

Inserting the expressions for Δc_t and Δm_t we obtain

$$s_t = s_{t-1} + \left(1 - \frac{1}{\sigma\varepsilon}\eta_{cu}\right) u_t. \tag{6}$$

Hence, since relative consumption follows a random walk so does the nominal

[4]This result is very intuitive. With fixed prices the terms of trade obviously must fall 1:1 (following a positive domestic money shock) with the depreciation in the nominal exchange rate. In contrast, with wage rigidities the terms of trade only reacts partially to the depreciation in nominal exchange rate. The reason is that with rigid wages the exchange rate depreciation is partly offset by a second-order rise in perfect competition prices since aggregate demand for the Home good increases.

exchange rate. Additionally, the subsequent terms of trade effect on the nominal exchange rate is eliminated because PPP holds. In the following we use the convenient relationships, $E_t s_{t+j} = s_t$ for all $j > 0$, and, $s_t - E_{t-1} s_t = s_t - s_{t-1} = \left(1 - \frac{1}{\sigma\varepsilon}\eta_{cu}\right) u_t$.

2.2 Technology

The representative firm's production function exhibits decreasing returns to scale. Labor input N is the only factor needed for production

$$Y_t^h = N_t^\gamma, \quad 0 < \gamma < 1.$$

Each firm only produces in its own country using the domestic labor force. This concludes the description of the model framework.

3 Nominal Rigidities

We now, in turn, introduce nominal wage and price rigidities and describe the resulting labor and product market equilibrium conditions implied by the model. Analytical results are presented for one-period nominal rigidities and two-period nominal staggering.

3.1 Nominal Wage Rigidities

We start by analyzing the case of nominal wage rigidities. As a benchmark it will be useful to first consider the case where wages are flexible. Assume workers are organized in identical monopoly unions, and that each utilitarian union represents a small subset of workers supplying labor to a given group of firms. Moreover, assume a "right-to-manage" structure such that employment is determined by firms given the wage. Since all the unions are identical we can write the *flexible* *wage* decision problem of the representative union as

$$\max_{W_t} \ \Sigma_t = \varsigma_t \frac{W_t}{P_t} N_t - \frac{\kappa}{1+\mu} N_t^{1+\mu},$$

where $\varsigma_t = C_t^{-\frac{1}{\sigma}}$ is the shadow value of wage income to the household. With *one-period nominal wage contracts* all unions maximize $E_{t-1}\Sigma_t$, whereas with *two-period staggered nominal wage contracts* half the unions maximize $E_{t-2}(\Sigma_{t-1} + \delta\Sigma_t)$ and the other half maximizes $E_{t-1}(\Sigma_t + \delta\Sigma_{t+1})$. By solving these three maximization problems we obtain

$$w_t^{0,w} = \eta_{wp}[(p_t^h)^{0,w} - p_t^{0,w}] + p_t^{0,w} + \eta_{wc} c_t^{0,w}, \tag{7}$$

$$w_t^{1,w} = E_{t-1} w_t^{0,w}, \tag{8}$$

$$w_t^{2,w} = \frac{1}{2(1+\delta)} \left(E_{t-2} w_{t-1}^{0,w} + \delta E_{t-2} w_t^{0,w} + E_{t-1} w_t^{0,w} + \delta E_{t-1} w_{t+1}^{0,w} \right), \tag{9}$$

where $w_t^{0,w}$, $w_t^{1,w}$, and $w_t^{2,w}$ denote log-wages given, respectively, no contracts, one-period fixed contracts, and two-period staggered contracts.[5] As before we have neglected all constants since we are only interested in dynamic adjustments. This includes the various risk terms (variances and covariances) of the endogenous variables in eqs. (8) and (9) due to presetting of nominal wages under uncertainty. The constant monopoly union mark-up $(1/\gamma)$ has also been eliminated since it is not relevant for wage adjustment. Accordingly, optimal pre-set wages equal rational expectations of the flexible wage causing equilibria with preset wages to differ from ones with flexible wages only because of the effects of unanticipated shocks.

Given optimal wages we derive the three labor market equilibria corresponding

[5] From now on, in a pair of superscripts (z, x) we let z refer to the *rigidity-structure* and x to the *rigidity-type*. Superscript z takes the values $\{0, 1, 2\}$ denoting, respectively, no rigidities, one-period rigidities and two-period staggered contracts. Superscript x can be $\{w, p\}$ representing either nominal wage or price rigidities. When the *rigidity-type* superscript is neglected the variable or parameter being analyzed is identical across the given *rigidity-structure*.

to eqs. (7), (8) and (9). Equilibrium relative wages become

$$\psi_t^{0,w} = \eta_{\psi q} q_t^{0,w} + \eta_{\psi c} \Delta c_t^{0,w}, \tag{10}$$

$$\psi_t^{1,w} = \eta_{\psi q} E_{t-1} q_t^{1,w} + \eta_{\psi c} \Delta c_{t-1}^{1,w} - (s_t^{1,w} - E_{t-1} s_t^{1,w}), \tag{11}$$

$$\begin{aligned} \psi_t^{2,w} = &\frac{\eta_{\psi q}}{2(1+\delta)} \left(E_{t-2} q_{t-1}^{2,w} + \delta E_{t-2} q_t^{2,w} + E_{t-1} q_t^{2,w} + \delta E_{t-1} q_{t+1}^{2,w} \right) \\ &+ \eta_{\psi c} \Delta c_{t-1}^{2,w} - \frac{1}{2} \eta_{\psi c} (\Delta c_{t-1}^{2,w} - \Delta c_{t-2}^{2,w}) \\ &- (s_t^{2,w} - E_{t-1} s_t^{2,w}) - \frac{1}{2} (E_{t-1} s_t^{2,w} - E_{t-2} s_{t-1}^{2,w}). \end{aligned} \tag{12}$$

Given the random walk processes for relative consumption and the nominal exchange rate we already begin to see how equilibrium wages behave.

3.2 Nominal Price Rigidities

To justify nominal price rigidities we assume the product market is characterized by imperfect competition. Hence, the representative firm maximizes profits subject to world demand and afterwards distributes all profits back to households. The representative Home firm's problem under *flexible prices* is thus

$$\max_{P_t^h} \; \Omega_t = P_t^h \left(\frac{P_t^h}{P_t} \right)^{-\rho} - W_t \left(\frac{P_t^h}{P_t} \right)^{-\frac{\rho}{\gamma}}.$$

With *one-period fixed prices* firms maximize $E_{t-1}\Omega_t$, whereas under *two-period staggered price setting* half of the firms maximize $E_{t-2}(\Omega_{t-1} + \delta\Omega_t)$ and the other half $E_{t-1}(\Omega_t + \delta\Omega_{t+1})$. Optimal monopoly Home prices for the three cases can be derived as

$$(p_t^h)^{0,p} = \eta_{pw}(w_t^{0,p} - p_t^{0,p}) + p_t^{0,p}, \tag{13}$$

$$(p_t^h)^{1,p} = E_{t-1}(p_t^h)^{0,p}, \tag{14}$$

$$(p_t^h)^{2,p} = \frac{1}{2(1+\delta)}\left(E_{t-2}(p_{t-1}^h)^{0,p} + \delta E_{t-2}(p_t^h)^{0,p} + E_{t-1}(p_t^h)^{0,p} + \delta E_{t-1}(p_{t+1}^h)^{0,p}\right), \tag{15}$$

where constant risk terms and the monopoly firm mark-up $[\rho/(\rho-1)]$ have been deleted. The structure behind these firm price-setting equations clearly resembles that of the union wage-setting decisions in eqs. (7), (8) and (9).[6]

Given optimal prices the corresponding product market equilibria become

$$q_t^{0,p} = \eta_{pw}\psi_t^{0,p}, \tag{16}$$

$$q_t^{1,p} = \eta_{pw}E_{t-1}\psi_t^{1,p} - (s_t^{1,p} - E_{t-1}s_t^{1,p}), \tag{17}$$

$$q_t^{2,p} = \frac{\eta_{pw}}{2(1+\delta)}\left(E_{t-2}\psi_{t-1}^{2,p} + \delta E_{t-2}\psi_t^{2,p} + E_{t-1}\psi_t^{2,p} + \delta E_{t-1}\psi_{t+1}^{2,p}\right) - (s_t^{2,p} - E_{t-1}s_t^{2,p}) - \frac{1}{2}(E_{t-1}s_t^{2,p} - E_{t-2}s_{t-1}^{2,p}). \tag{18}$$

The equations are obviously analogous to those derived for the labor market with nominal wage rigidities. The main difference is that the terms of trade have switched side with relative wages hence implying a different interpretation of the coefficients. In addition, relative consumption does not enter the product market

[6] One apparent difference is that, unlike the flexible union wage in eq. (7), the flexible firm price in eq. (13) does not depend *directly* on consumption. This is not to say that the wealth effect is non-existent under price setting. Instead it has merely become a second order effect on price dynamics. For example, an increase in wealth makes household enjoy both more consumption and more leisure. This reduction in labor supply puts pressure on wages which eventually feeds into prices. Thus wealth does affect prices, however, only *indirectly* through its effect on wages. Such an observation might lead one to draw the (wrong) conclusion that the wage-price spiral is stronger with nominal price rigidities compared to wage rigidities. Fortunately, the analytical approach taken here enables us to show that in fact the two models encompass equivalent spirals.

equilibrium equations (see also footnote 6). As relative consumption follows a random walk this indicates that the initial expenditure switch differs across the two rigidity types.

4 General Equilibrium

Having derived equilibrium conditions under different rigidity regimes we are now able to make explicit analytical comparisons of the implied terms of trade processes.

4.1 Flexible Equilibrium

The flexible equilibrium is independent of market imperfections since market power is absorbed by constant terms when log-linearizing around a steady state (monopoly mark-ups become additive when log-linearizing and disappear under the "relative" approach taken here). The terms of trade solution can thus be found straightforwardly from either the flexible monopoly union or monopoly firm problem. It takes the simple form

$$q_t^0 = \eta_{qc}^0 \Delta c_t^0, \qquad \eta_{qc}^0 > 0, \tag{19}$$

(where we leave out the *rigidity-type* superscript to indicate cross-rigidity uniformity, see footnote 5). Only relative consumption determines long-run dynamics for the terms of trade, and given the random walk of relative consumption we also obtain a unit root in the terms of trade, $E_t q_{t+1}^0 = q_t^0$.

4.2 One-period Nominal Rigidities

Comparing the general equilibrium terms of trade processes becomes more complicated when nominal rigidities are introduced. Using eq. (6) for the nominal exchange rate we start by rewriting eqs. (11) and (17). This yields two very similar expressions

$$q_t^{1,w} = \eta_{qq}^1 E_{t-1} q_t^{1,w} + \eta_{qc}^1 \Delta c_{t-1}^{1,w} - \eta_{qu}^{1,w} u_t, \tag{20}$$

$$q_t^{1,p} = \eta_{qq}^1 E_{t-1} q_t^{1,p} + \eta_{qc}^1 \Delta c_{t-1}^{1,p} - \eta_{qu}^{1,p} u_t, \tag{21}$$

where the following elasticity restrictions hold

$$\eta_{qq}^1 < 0, \qquad \eta_{qc}^1 > 0, \qquad 0 < \eta_{qu}^{1,w} < \eta_{qu}^{1,p}.$$

Notice the cross-rigidity uniformity of η_{qq}^1 and η_{qc}^1. As expected the terms of trade impact effect is larger under price rigidities.

In order to say more we need analytical solutions for both terms of trade processes. These can be obtained using the method of undetermined coefficients along with similar guesses of the form $q_t^{1,x} = \pi_{qc}^1 \Delta c_{t-1}^{1,x} + \pi_{qu}^{1,x} u_t$, where $x \in \{w, p\}$. The solution procedure is not especially illuminating and we refer the interested reader to Andersen and Beier (1999) for further details. Since one can show that all the π parameters are different from zero we conclude that nominal money shocks have real effects. The resulting terms of trade process is an ARIMA (0,1,1) where the random walk is due to the random walk in relative consumption. A central feature of the solution procedure is that the elasticity of the lagged terms of trade (η_{qq}^1) can be found separately without taking relative consumption or the nominal exchange rate into account. It turns out that this is only possible as long as relative consumption follows a random walk and PPP holds. A violation of PPP breaks the random walk property of the nominal exchange, because then the expected terms of trade are included in eq. (5) for s_t. This affects the elasticity of the lagged terms of trade (η_{qq}^1) in such a way that it will no longer be equal across the two rigidity types (due to the different impact effects). For the same reason, the elasticity of relative consumption (η_{qc}^1) will only be equal across the two rigidity types as long as relative consumption follows a random walk. Given a solution for the lagged terms of trade elasticity the lagged relative consumption elasticity (η_{qc}^{1}) can similarly be solved autonomously. Hence, the solutions for "real" elasticities are derived independently of nominal rigidities since the latter enter the general equilibrium through changes in the nominal exchange rate. In view of that it is not surprising that the terms of trade processes only have different impact effects and otherwise equivalent adjustment dynamics. To see this analytically we divide the terms of trade processes by their respective impact effects.[7] We find

$$\frac{q_t^{1,w}}{\eta_{qu}^{1,w}} = \frac{q_t^{1,p}}{\eta_{qu}^{1,p}} \Leftrightarrow \frac{\Delta c_t^{1,w}}{\eta_{qu}^{1,w}} = \frac{\Delta c_t^{1,p}}{\eta_{qu}^{1,p}}. \tag{22}$$

Consequently, we can conclude that once the terms of trade processes are "impact adjusted" they become identical. The same is true for relative consumption as it is proportional to the terms of trade impact effect. This observation is interesting since it implies that, irrespective of the nominal rigidity type, the model always produces the same adjustment pattern towards long-run equilibrium following a shock. Of course, in this setting where the adjustment takes place over one period the result is not very attractive. However, as the next section confirms, our finding carries over to more sophisticated rigidity specifications.

4.3 Two-period Staggering

In this section we compare the terms of trade processes entailed by two-period nominal wage and price staggering. The modus operandi is the same as for one-period nominal rigidities. Thus, we begin by rewriting eqs. (12) and (18) using

[7] Specifically, we divide $q_t^{1,w}$ by $\pi_{qu}^{1,w}$ and $q_t^{1,p}$ by $\pi_{qu}^{1,p}$. By comparing eqs. (20) and (21) with the form of the method of moments guesses we obviously end up with the parameter restrictions $\pi_{qu}^{1,w} = \eta_{qu}^{1,w}$ and $\pi_{qu}^{1,p} = \eta_{qu}^{1,p}$.

eq. (6) for the nominal exchange rate. We find expressions of the following form

$$\begin{aligned} q_t^{2,w} &= \frac{1}{2}\frac{1}{1+\delta}\eta_{qq}^2\left(E_{t-2}q_{t-1}^{2,w} + \delta E_{t-2}q_t^{2,w} + E_{t-1}q_t^{2,w} + \delta E_{t-1}q_{t+1}^{2,w}\right) \\ &\quad + \eta_{qc}^2\Delta c_{t-1}^{2,w} - \frac{1}{2}\eta_{qc}^2\left(\Delta c_{t-1}^{2,w} - \Delta c_{t-2}^{2,w}\right) - \eta_{qu}^{2,w}u_t - \frac{1}{2}\eta_{qu}^{2,w}u_{t-1}, \end{aligned}$$

$$\begin{aligned} q_t^{2,p} &= \frac{1}{2}\frac{1}{1+\delta}\eta_{qq}^2\left(E_{t-2}q_{t-1}^{2,p} + \delta E_{t-2}q_t^{2,p} + E_{t-1}q_t^{2,p} + \delta E_{t-1}q_{t+1}^{2,p}\right) \\ &\quad + \eta_{qc}^2\Delta c_{t-1}^{2,p} - \frac{1}{2}\eta_{qc}^2\left(\Delta c_{t-1}^{2,p} - \Delta c_{t-2}^{2,p}\right) - \eta_{qu}^{2,p}u_t - \frac{1}{2}\eta_{qu}^{2,p}u_{t-1}, \end{aligned}$$

together with elasticity restrictions

$$\eta_{qq}^2 > 0, \qquad \eta_{qc}^2 > 0, \qquad 0 < \eta_{qu}^{2,w} < \eta_{qu}^{2,p}.$$

The two processes look very similar. Notice the cross-rigidity uniformity of η_{qq}^2 and η_{qc}^2 and the higher impact effect under nominal price rigidities. Thus, as under one-period stickiness we have that all the elasticities concerning "real" variables are independent of the rigidity type. Only the impact elasticities on u_t (and u_{t-1}) differ.

To find analytical solutions for the terms of trade processes under staggering we use the method of undetermined coefficients along with guesses of the form $q_t^{2,x} = \pi_{qq}^2 q_{t-1} + \pi_{qc}^2 \Delta c_{t-1} + \pi_{qu}^{2,x} u_t + \frac{1}{2}\pi_{qu}^{2,x} u_{t-1}$, where $x \in \{w, p\}$. In this case the resulting terms of trade follow an ARIMA(1,1,2) process and so staggering has added an extra AR and MA part to the terms of trade dynamics. The dynamics are tangled by staggering but the principal underlying propagation mechanism is still the powerful consumption smoothing from the random walk. Again, dividing both processes by their respective impact effects we obtain

$$\frac{q_t^{2,w}}{\eta_{qu}^{2,w}} = \frac{q_t^{2,p}}{\eta_{qu}^{2,p}} \Leftrightarrow \frac{\Delta c_t^{2,w}}{\eta_{qu}^{2,w}} = \frac{\Delta c_t^{2,p}}{\eta_{qu}^{2,p}}.$$

Accordingly, we once again have that nominal wage and price rigidities are equivalent "impact adjusted" propagation mechanisms. The intuition is the following. The AR terms across the two processes coincide since the "real" elasticities are determined independently of the MA terms and therefore the rigidity structure. Moreover, since the new MA term (u_{t-1}) clearly is proportional to the initial shock (u_t), it does not change the basic feature of relative consumption being proportional to the terms of trade impact effect (the proportionality factor is, of course, changed).

To illustrate the result graphically we have calibrated the impulse response functions for the two terms of trade processes under staggering. They are displayed below in Figures 1a and 1b.[8]

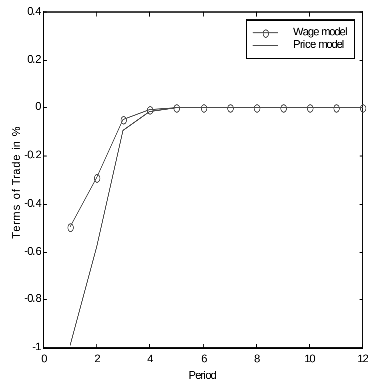

Figure 1a. $\gamma = 2/3$.

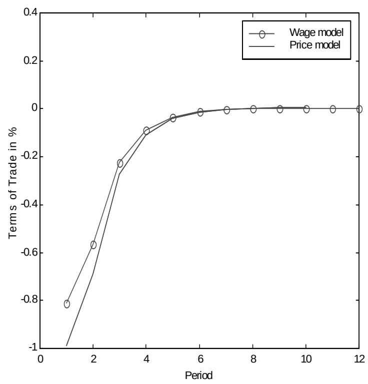

Figure 1b. $\gamma = 0.90$.

In both figures the terms of trade adjustment patterns look somewhat similar.[9] The remarkable finding is that, taking any of the two figures, if we divide

[8] The impulse response functions have been calculated given a one percent increase in the relative domestic money supply in period 1. The baseline parameter values used are $\rho = 2$, $\gamma = 2/3$, $\mu = 10$, $\sigma = 3/4$, $\varepsilon = 9$, $\delta = 1/1.05$. These are used in Figure 1a. For descriptive reasons we also show the case where $\gamma = 0.90$ in Figure 1b. This creates more persistence and makes it easier to see that the two processes feature the same dynamics following an impact. We refer the reader to Andersen and Beier (1999) for a thorough discussion of persistence in this model.

[9] In Figures 1a and 1b the long-run effects are actually positive, but almost zero, for both nominal wage and price rigidities. The reason that long-run effects are quantitatively insignificant compared to impact effects is that wealth effects are smoothed in an infinite horizon setting with consequent small effects within a given period. In the figure it also looks as if the impulse

the impulse response functions by their respective impact effects they become identical.

5 Conclusion

We have shown that nominal wage and price rigidities can be equivalent "impact adjusted" propagation mechanisms. Results were shown for nominal one-period rigidities and two-period staggering. However, they easily generalize to X-period staggering.[10] Our finding supports the implicit assumption in the previous literature that there is no qualitative difference between the two types of rigidities.

The results, of course, rely on log-linearization to remove market imperfections. Yet, what other features are causing the equivalence can only be speculated. We found that the two model characteristics (1) a random walk in relative consumption and (2) a binding PPP, nicely separate the "real" part of the propagation mechanism from the nominal shock. However, whether a violation of these conditions also implies a qualitatively different propagation mechanism between nominal wage and price rigidities is unknown. It might be, that once the nominal shock has found its way through the wage-price spiral these differences in "real" elasticities cancel out. Only an analytical solution of the model under these circumstances could tell.

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response functions overlap (have identical long-run responses). It can be shown, however, that the nominal price model in fact always has a strictly higher long-run wealth effect.

[10] Including another period of staggering simply adds an extra AR and MA part to the terms of trade process. The model can still be solved using the method of moments with identical guesses and the resulting terms of trade process have an ARIMA$(X-1,1,X)$ form. All AR part elasticies are cross-rigidity identical as they are solved independently of the MA parts, which remain the only difference between the two processes. All MA parts turn up proportional to the respective impact effects of the terms of trade processes. Hence, relative consumption will still be proportional to the terms of trade impact effect.

A Relative Consumption

To see that the size of η_{cu} depends on the size of the expenditure switch, observe that the Home national budget constraint is defined as

$$B_t = (1 + r_{t-1}) B_{t-1} + Y_t - C_t,$$

which takes the following steady state log-linearized form

$$\Delta b_t = \delta^{-1} \Delta b_{t-1} + (1 - \rho) q_t - \Delta c_t \Leftrightarrow$$
$$\Delta c_t = \delta^{-1} \Delta b_{t-1} - \Delta b_t + (1 - \rho) q_t.$$

Assume for a moment that there is a one-period nominal rigidity in either wages or prices. The intuition is then well-known from the classic *Redux* model by Obstfeld and Rogoff (1995). Following e.g. a one-time positive relative money shock the initial nominal exchange rate depreciation, working through q_t, induces an expenditure switch. This increases individual wealth and hence relative consumption rises by the same amount in all future periods due to the random walk property. The impact effect on relative consumption (η_{cu}) must therefore be *proportional* to the impact effect on the terms of trade.

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