DEPARTMENT OF ECONOMICS

Working Paper

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Working Paper No. 2001-9
Centre for Dynamic Modelling in Economics

ISSN 1396-2426

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Stable Cartels Revisited

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October 24, 2001

Abstract

This paper analyzes cartel stability when firms are farsighted. It studies a price leadership model á la D' Aspremont et al. (1983), where the dominant cartel acts as a leader by determining the market price, while the fringe behaves competitively. According to D' Aspremont et al.'s (1983) approach a cartel is stable if no firm has an incentive to either enter or exit the cartel. In deciding whether to deviate or not, a firm compares its status quo with the outcome its unilateral deviation induces. However, the firm fails to examine whether the induced outcome will indeed become the new status quo that will determine its profits. Although the firm anticipates the price adjustment following its deviation, it ignores the possibility that more firms may exit (or enter) the cartel that may eventually stabilize in a very different situation from the one the firm originally induced. In other words, the firm does not consider the fact that the outcome immediately induced by its deviation may not be stable itself. We propose a notion of cartel stability that allows firms to *fully* foresee the result of their deviation. Our solution concept is built in the spirit of von Neumann and Morgenstern's (1944) stable set, while it modifies the dominance relation following Harsanyi's (1974) criticism.We show that there always exists a unique, non-empty set of stable cartels and provide an algorithm the determines it. *Journal of Economic Literature* Classification Numbers: C79, D43, D49, L13

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1 Introduction

The importance of cartel stability is manifested through the extend of the literature dedicated to the topic over the last decades. By using a conceptually different approach, we aspire to shed some light to the rather debatable aspects of cartel stability.

The classical doctrine about oligopolistic (finite number of firms) markets is that even though collusive behavior –all firms acting as one monopolist– is more profitable than competitive behavior –each firm maximizing independently its own profits while ignoring the strategic element inherent in the environment– collusion will not prevail. The reason being that *given* that every one else colludes and maintains a high price, each firm, unilaterally, has an incentive to deviate and free ride on the cartel's collusive efforts. The cartel would price and produce by maximizing aggregate profits, whereas the cheating firm would function as a price taker and set its marginal cost equal to the market price, as set by the cartel.

One of the most influential works studying cartel stability is the one by D' Aspremont et al. (1983). Their model, based on a general price-leadership framework, considered a finite economy where a dominant cartel sets the market price at a joint profit maximizing level, and a competitive fringe free-rides on the profit maximizing efforts of the cartel by overstepping the quota set by the cartel.

Although, price-leadership models were studied in earlier works and not necessarily in an attempt to study cartel behavior, the major contribution of D' Aspremont et al. (1983) was the observation that once a member of the cartel deviates and joins the fringe, the cartel is going to adjust its behavior by adjusting its quota (and thus the market price) in a manner that maximizes the new (shrunk by one member) aggregate cartel profits. Once such an adjustment is captured by the model, the result is that it is not always beneficial for a firm to exit the cartel and join the fringe. The profits the potential deviant may enjoy by increasing his output may be offset by the decrease in market price as brought about by the cartel adjustment. In particular, the authors formally define and show that there always exists a stable cartel, that is, a specific size of a cartel such that it is not profitable for a firm to violate the quota anymore and join the fringe (this aspect of cartel stability is defined as *internal stability*). Moreover, no more fringe members wish to join the cartel either (this aspect of cartel stability is defined as *external stability*). It is also shown that the result does not hold for the case where the economy is comprised of an infinite number of firms.

Along the same lines Donsimoni et al. (1986) study the same general model with the additional assumption of linear demand and marginal cost functions. Besides their different approach towards existence, the authors show that under some additional conditions on cost efficiency the stable cartel is unique.

Within the same institutional setting other works modified some of the assumptions of the model. Specifically, Donsimoni (1985) allowed for product heterogeneity, while Shaffer (1995) studied the case where the fringe does not behave "that competitively" anymore, instead the fringe members realize the strategic impact of their actions on the market price by behaving as in a Cournot competition.

Prokop (1999) uses extensive form games to describe the *process* of collusion. Each firm, consecutively and in an exogenously determined order, decides to enter the cartel or not. Depending on the order of moves some firms enjoy more profits than others. The interesting result is that applying subgame perfection to such a dynamic process yields the same results (stable cartels) with the D' Aspremont et al. (1983) approach.

The venue we follow in this paper is to assume that binding agreements are not possible, and therefore examine the immunity of cartels against potential deviations. We do not allow for coalitions to form, apart from the cartel itself and thus all deviations are unilateral. However, unlike previous works we allow each firm to compare the *ultimate outcome* of its deviation with its status quo instead of some intermediate situation that will not prevail.

1.1 A simple example

The following trivial example intends to clearly point out the myopia embedded in the analysis of stable cartels. The firms' behavior and the various institutional assumptions are identical to those in the D' Aspremont et al. model (1983).

Consider a market with $N = \{1, ..., 5\}$ identical firms and let $Q = \sum_{i \in N} q_i$ indicate the total quantity produced in the market, whereas q_i indicates the individual firm's quantity. Let P indicate the market price.

Consider the simple case of a linear demand where $Q = 100 - P$ and where the individual cost function is $TC(q_i) = \frac{1}{2}10q_i^2$. When a cartel of size $k \leq n$, denoted by C_k, forms it behaves like one firm by maximizing aggregate (with respect to its members) profits, and thus, it produces and prices at the point where the marginal revenue (derived from the residual demand) equals marginal cost. The fringe of size $n-k$, denoted by F_k, behaves competitively by producing at the point where market price is equal to marginal cost of each firm. The firms in the cartel are aware of the fringe's behavior, and this awareness is reflected on their consideration of the *residual demand* instead of the market demand.

The following table illustrates the profits per firm for the 5 different market structures:

Perfect competition $k = 0$		$\swarrow \pi^f(0) = 222$
$k = 1$	$\pi^c(1) = 223$	$\swarrow \pi^f(1) = 224$
$k = 2$	$\pi^c(2) = 226$	$\swarrow \pi^f(2) = 230$
$k = 3$	$\pi^c(3) = 231$	$\pi^f(3) = 241$
$k = 4$	$\pi^c(4) = 239 \nearrow$	$\pi^f(4) = 258$
Full cooperation $k = 5$	$\pi^c(5) = 250 \nearrow$	

Where $\pi^c(k)$ denotes the profits of a firm belonging to C_k, whereas $\pi^f(k)$ denotes the profits of a firm belonging to the fringe of C_k, F_k. Note that

$\pi^f(0)$ depicts the profits per firm of a perfectly competitive market, whereas $\pi^c(5)$ depicts the profits per firm of a perfectly collusive market.

According to D' Aspremont et al. 's (1983) approach only C_3 is stable. In particular, C_1 is not stable because members of its fringe wish to join in ($226 > 224$). Similarly, C_2 is not stable since members of its fringe wish to join in ($231 > 230$). Furthermore, C_5 is not stable since its members wish to exit and join F_4 ($258 > 250$). Lastly, C_4 is not stable since its members wish to join F_3 ($241 > 231$). C_3, however, is stable since no cartel member wants to leave ($231 > 230$) -*internally stable*,- while no fringe member wants to join ($241 > 239$) -*externally stable*.

Once C_5 is formed everyone is earning 250, and the reason of its instability is the incentive to unilaterally deviate (and join F_4) in order to earn 258. However, a farsighted firm would see ahead and realize that C_4 is not a stable state either, since once it becomes the status quo some other firm would deviate (and join F_3) in order to earn 241 vs 239. Once the cartel is down to size three no more firms would like to exit since being a member of C_3 is more beneficial than being a member of F_2 ($231 > 230$) or F_1 ($231 > 224$) or F_0 ($231 > 222$). Therefore, when the very first firm contemplates deviating from C_5, it should foresee that it will end up being a member of F_3 instead of F_4 and its final payoff would be 241 and not 258, which suggests against deviation since $241 < 250$.

It is apparent that the reason C_5 is characterized (according to the stability concept in the literature) unstable is because F_4 is preferred to (and can be induced from) C_5. Yet, F_4 is unstable as well since F_3 is preferred to (and can be induced from) C_4. In essence, the perfectly collusive situation is discredited by an outcome that is itself discredited. Since F_4 is going to be *replaced* by F_3, it is natural to compare C_5 with F_3. Such a comparison suggests that deviating from C_5 is not profitable and thus it should be considered internally stable. External stability is trivially satisfied since the fringe is empty, hence, C_5 is stable.

While analyzing the stability of C_5 and the instability of C_4 we implicitly assumed that C_3 is stable. Indeed, we have argued that it is internally stable since it is not beneficial for its members to exit. We still need to explain why it is externally stable, especially since we have repeatedly pointed out that members of F_3 prefer to be members of C_5. For a firm to proceed with a deviation it does not suffice that it prefers some outcome, but it is also required that this outcome can actually be *reached*. When a firm enters the cartel from F_3 it induces C_4. Once C_4 is the status quo *no more* fringe members will enter from F_4 to C_5 since $258 > 250$, *instead* members of C_4 (which is internally unstable) will induce F_3 again. The problem lies in the fact that a single firm *cannot induce*[1] C_5 from F_3 and what a firm can induce from F_3 *will not lead* to C_5. Thus, C_3 is stable since it is both internally and externally stable.

In the D' Aspremont et al. (1983) model and its variants, it is assumed that once a firm deviates from a cartel (fringe) of size k, the remaining (increased) cartel members will react to the deviation by adjusting the market price to its new *equilibrium* level (given the new residual demand that corresponds to a cartel of size $k-1$ $(k+1)$). In fact, the deviating agent's ability to foresee this price adjustment and compare its status quo with the equilibrium (in terms of price) resulting from its deviation is the very merit of such works. It is only natural thus, that we allow the deviating firm to *fully* foresee *all* reactions ignited by its deviation involving not only price but cartel size adjustments as well.

The myopia embedded in the D' Aspremont et al. (1983) approach is exactly the issue we attempt to resolve in this work. In particular, we propose a solution concept built in the spirit of von Neumann and Morgenstern's

[1] Such an inducement requires either a sequence of moves, which as we argued is not incentive compatible, or a group of firms (a coalition) moving. The latter is certainly a modification of our proposition one can consider with only one caveat: in the context of cartel formation binding agreements are not permitted and thus remaining within the non-cooperative spirit agents act one at time.

(1944) stable set that allows agents to count only on equilibrium outcomes when contemplating a deviation. However, we amend the dominance relation so that agents can examine which equilibrium outcomes will ensue their deviation in the event the outcome each agent directly induces is not an equilibrium itself. The result is a set of stable cartels that would survive *credible* deviations.

2 The model

Let the industry $N = \{1, 2, ..., n\}$ consist of a finite number of n identical firms producing a homogeneous output. The product's market demand is represented by a differentiable function $D(P) \geq 0$ such that $D'(P) < 0$, where P is the market price. Each firm has the same differentiable cost function $C(q_i)$ with marginal cost $C'(q_i) > 0$ and $C''(q_i) > 0$ where q_i is the quantity produced by firm i and $Q = \sum_{i \in N} q_i$ denotes the total quantity supplied in the market.

The classical price-leadership model that we analyze assumes that $k \in \{0, 1, ..., n\}$ firms form a cartel $C_k \subset N$ and the rest $n - k$ firms form a fringe $F_k = N \backslash C_k$. The cardinality of the sets will be denoted by $|C_k| = k$ and $|F_k| = n - k$. Note that the subscript of a cartel indicates its size, whereas the subscript of a fringe indicates the size of its corresponding cartel.

The fringe behaves competitively by setting its marginal cost equal to the price determined by the cartel, i.e., $C'(q_i) = P$. Since all firms are identical we will suppress the subscript of individual quantities. Instead we will denote each fringe member's output by $q^f(P)$, and the total fringe supply by $Q^f = (n - k)q^f$.

The cartel C_k is, in turn, faced with a residual demand, $RD(k, P) = D(P) - Q^f$. The cartel behaves like a monopolist by setting its marginal revenue with respect to the residual demand equal to its marginal cost. Such a joint profit maximization leads the cartel members to produce q^c each, and since they are identical $Q^c = kq^c$ as a group.

Let $P(k)$ indicate the market price a cartel C_k has chosen, and let $\pi^c(k)$ indicate the profits of each member of the cartel. Similarly, let $\pi^f(k)$ indicate the profits of each member of the fringe F_k. Observe that if $k = 0$ then $|F_0| = n$ and our industry is perfectly competitive, whereas if $k = n$ then $|C_n| = n$ and industry is perfectly collusive.

Furthermore, as has already been established by D' Aspremont et al. (1983) $\pi^c(k) < \pi^f(k)$ for $k > 0$. However, what is of interest to the agents is not the afore mentioned relationship, since once a firm exits the cartel and joins the fringe the size of the cartel is not k anymore but $k-1$. Such a size adjustment alters the residual demand function and therefore the market price $(P(k-1) \neq P(k))$ chosen by the shrunk cartel.

Previous works took this price adjustment into consideration by having a cartel member compare $\pi^c(k)$ to $\pi^f(k-1)$ when contemplating deviation, and a fringe member compare $\pi^f(k)$ to $\pi^c(k+1)$ when contemplating joining a cartel. As a result, if $\pi^c(k) \geq \pi^f(k-1)$ no firm would wish to exit the cartel and thus the cartel is characterized *internally stable*, whereas if $\pi^f(k) \geq \pi^c(k+1)$ no fringe member would wish to join the cartel, which is thus characterized *externally stable*. If a cartel C_k is both internally and externally stable then it is called *stable*.

2.1 Cartel Stability

The issue of credibility and foresight has risen on several occasions in economic models and more fundamentally in solution concepts within an economic or game theoretic context.

In the D' Aspremont et. al. (1983) approach when a firm contemplates exiting a cartel C_k it compares $\pi^c(k)$, that is, the profits it makes while a member of the cartel, with the profits it will make once it exits and joins the fringe F_{k-1}, that is, $\pi^f(k-1)$. The firm implicitly assumes that once it deviates, no one else will want to deviate and therefore it will enjoy profits $\pi^f(k-1)$ with certainty. But as we have already shown this is not always the case, in fact, it is possible that another firm may wish to exit cartel C_{k-1}, by now, and join the fringe F_{k-2}, and so on. Thus, the firm should compare its status quo $\pi^c(k)$ with the *final* outcome that will *result* once the firm ignites a sequence of events by exiting C_k. This *final* outcome can be characterized as such only if no more firms wish to exit and no more firms wish to join, if, in other words, it is stable itself. Put differently, we can determine whether a cartel is stable or not, only if we know what every other cartel is. Such a recursive approach is adopted by the classical notion of the abstract stable set.

The (abstract) stable set originally defined by von Neumann and Morgenstern (1944) is a solution concept that captures consistency. The stable set approach instead of characterizing each outcome independently, it characterizes a solution set, that is, a collection of outcomes that are stable, while those excluded from the solution set are unstable. In particular, no inner contradictions are allowed, that is, any outcome in the stable set cannot be dominated by another outcome *also* in the stable set. Note that, unlike other equilibrium notions, a stable outcome could be dominated by some other *non-stable* outcome since such a dominance would not be consistent (or credible)[2]. Similarly, every outcome excluded from the stable set is accounted for in a consistent manner by being dominated by some outcome *in* the stable set.[3] At this point we would like to clarify that by dominance we do not merely imply preference but we presume feasibility as well, in the sense that if some outcome a is dominated by some other outcome b via a

[2]This feature of the stable set is known as *Internal Stability*, yet we will avoid the terminology since it coincides with the one attributed to cartels and has been used in the cartel literature for some time. In this paper characterizing a cartel as internally stable implies that no members wish to exit the cartel, as is formalized in Definition 1.

[3]This feature of the stable set is known as *External Stability*. The same problem with terminology arises here as well. We will maintain the meaning of external stability as formalized in Definition 2.

group of agents, it must be the case that this group of agents both prefers b to a and can induce b from a.

Although the notion of the stable set is very appealing exactly due to the afore mentioned properties that attribute consistency[4] it has been criticized on two grounds: firstly, it does not always exist, and secondly, it suffers from myopia as well. The latter was originally criticized by Harsanyi (1974) and later on remedied in the works of Chwe (1994), Xue (1998) and others. The authors point out that if a group of agents prefers a stable outcome c to an outcome a but this same group cannot induce c from a, while it can induce some other outcome b from a which is (directly or indirectly) dominated by b, then a is indirectly dominated by c. Using the simple (direct) dominance relation outcome a would in fact be deemed stable due to the myopia of the group of agents that fail to see that they can get to c via b. In the notion we propose we replace the direct dominance by indirect to allow each agent to consider many steps ahead. Keep in mind that since, in our context, agents move unilaterally, each firm may not be able to induce upon deviating some stable outcome. Such an event should not deter the firm from deviating, instead the firm should examine the cause of instability and foresee its payoff once stability is reinstated. Only if that payoff is improving the firm should proceed with its deviation. This notion of foresight is captured by indirect dominance. The notion we propose differs from the recent works of Chwe (1994) and Xue (1998) that adopt the same blend by adapting the concept to the context of cartel formation[5].

[4]Its appeal is captured and improved upon by Greenberg (1994). In the *Theory of Social Situations (TOSS)*, a unifying approach towards cooperative and non-cooperative game theory, where any behavioral and institutional assumptions are explicitly defined, an equivalence is shown between the von Neumann & Morgenstern (vN-M) stable set and the *Optimistic Stable Standard of Behavior (OSSB)*, a solution concept built in the spirit of vN-M stability, yet with the precise assumption of optimistic behavior explicitly formalized. TOSS amplified the pertinence of stability by recasting the dominance relation into a broader concept beyond the boundaries of a binary relation. In doing so, behavioral assumptions can be imposed on the agents, and more complex institutional settings can be analyzed.

[5]Cartel formation (as is considered in this paper) involves simplifying assumptions,

We start by considering the set of all stable cartels, σ, i.e., $\sigma = \{C_k, C_h, ..., C_l\}$. A cartel C_k is stable, $C_k \in \sigma$ if and only if it is both internally and externally stable.

A cartel is internally stable if no firm wishes to exit. So far, a firm compared its current profits $\pi^c(k)$ with the profits of the fringe it would join $\pi^f(k-1)$. We claim that such a comparison is justified only if C_{k-1} is a stable cartel, i.e., $C_{k-1} \in \sigma$ as well, which would imply that if C_{k-1} becomes the status quo it would remain so. Otherwise, if $C_{k-1} \notin \sigma$ once at C_{k-1} some other agent wishes to either join it or exit it. Thus, the very first agent when contemplating whether to exit C_k or not he should compare it to the final stable outcome that will *arise*, i.e., $C_{k-m} \in \sigma$. More formally,

Definition 1 *A* ***cartel*** C_k *is* ***internally stable*** *(given σ) if and only if there does not exist a finite sequence of cartels $C_{k-1}, C_{k-2}, ...C_{k-j}, ..., C_{k-m}$, where $m \in \{1, 2, ..., k\}$ such that $C_{k-m} \in \sigma$ and $\pi^c(k-j) < \pi^f(k-m)$ for every $j = 0, 1, ..., m-1$.*

A parallel process describes external stability. A cartel C_k is externally stable if no firm wishes to join it. Again the firm makes such a decision by comparing its profits under the status quo $\pi^f(k)$ with the profits it will make once it joins, namely $\pi^c(k+1)$. Such a comparison is justified only if $C_{k+1} \in \sigma$ is a stable cartel itself and thus, no more agents wish to enter or exit. The firm should compare its status quo with the final outcome that will *arise*, i.e., $C_{k+m} \in \sigma$. Formally,

Definition 2 *A* ***cartel*** C_k *is* ***externally stable*** *(given σ) if and only if there does not exist a finite sequence of cartels $C_{k+1}, C_{k+2}, ...C_{k+j}, ..., C_{k+m}$, where $m \in \{1, 2, ..., n-k\}$, such that $C_{k+m} \in \sigma$ and $\pi^c(k+m) > \pi^f(k+j)$ for every $j = 0, 1, ..., m-1$.*

either to conceptually refine the notion, or to guarantee existence, that lead deviating agents to unique paths. Thus, no behavioral assumptions are necessary unlike the works of both Chwe (1994) and Xue (1997).

Finally, a cartel $C_k \notin \sigma$ is excluded from our solution set if it is not stable and therefore either its internal or external stability is violated. Conclusively,

Definition 3 *A* ***cartel*** C_k *is* ***stable*** *(given* σ*) if and only if it is both internally and externally stable (given* σ*).*

Note that the null cartel C_0 that contains no firms is trivially internally stable since there does not exist a firm to exit, and that a perfectly collusive market where the cartel C_n contains all the firms is trivially externally stable, since there do not exist more firms to join. When a cartel C_k is stable we characterize its fringe F_k as stable as well.

We mentioned earlier that one of the major drawbacks of the stable set is its difficulty to exist. Yet, we were able to establish a general existence result for our model. Another problem associated with the stable set is its multiplicity. More precisely the existence of more than one stable set, suggesting different groups of stable outcomes. Notice the difference between uniqueness of a stable cartel (there exists a σ that contains one element only, i.e., $\sigma = \{C^*\}$) and uniqueness of a set of stable cartels (there exists one σ only that may contain more than one stable cartels, i.e., $|\sigma| > 1$). The former, is obviously not generally true since the example we presented in the introduction admits a unique stable set σ that contains two stable cartels, i.e., $\sigma = \{C_3, C_5\}$. The latter is asserted in the following theorem were it is shown that our model admits a unique set of stable cartels.

Theorem 4 *With n-finite, there always exists a unique non-empty set of stable cartels,* σ*.*

Proof. In order to proceed with the proof we need to recall first the following results that were established by D' Aspremont et al.(1983):

(1) $\pi^c(k)$ is strictly increasing in k and

(2) For n-finite there exists $k \in \{1, ..., n\}$ such that $\pi^c(k) \geq \pi^f(k-1)$ and $\pi^f(k) \geq \pi^c(k+1)$.

The proof is by construction, that is, we built a non empty set σ of cartels and then we show it is stable. The construction of σ consists of two parts. First, we locate the smallest cartel that should be in the set by starting from the largest cartel possible (C_n) and continue the search along smaller cartels. Step 1 is capturing exactly this procedure as well as labeling some critical cartel sizes during the descent that are used in subsequent stages. In step 2 we use the finding of step 1 as a starting point and begin ascending, collecting cartels along the way and thus, forming σ. In step 3 we show that σ is indeed stable. And finally in step 4 we show that σ is unique. We urge the reader to consult the graph depicted in the Appendix, which is meant to facilitate the understanding of the proof.

STEP 1(Descent)

Let $C_{\hat{k}}$ denote a cartel that is 'stable' a la D' Aspremont et al. (1983), that is, $\pi^c(\hat{k}) \geq \pi^f(\hat{k}-1)$ and $\pi^f(\hat{k}) \geq \pi^c(\hat{k}+1)$. Let the largest $C_{\hat{k}}$ be $C_{\hat{k}_1}$. If $\pi^c(\hat{k}_1) > \pi^f(k)$ for every $k \leq \hat{k}_1 - 1$ then $C_{\hat{k}_1} \equiv C_{k^1} \in \sigma$ and we jump to step 2. If not, we start decreasing $\hat{k}_1$ until we locate (the largest) $C_{\bar{k}_2}$ below $C_{\hat{k}_1}$ such that $\pi^f(\bar{k}_2) \geq \pi^c(\hat{k}_1)$.

We continue decreasing until we locate the first $C_{\hat{k}}$ below $C_{\bar{k}_2}$, let that be $C_{\hat{k}_2}$. Note that since $\pi^f(\bar{k}_2) \geq \pi^c(\hat{k}_1)$, $\pi^c(k)$ is strictly increasing in k, and $\hat{k}_1 \geq \bar{k}_2+1$ it is the case that $\pi^f(\bar{k}_2) \geq \pi^c(\bar{k}_2+1)$ and since $\pi^c(1) > \pi^f(0)$ there exists such $C_{\hat{k}_2}$. If $\pi^c(\hat{k}_2) > \pi^f(k)$ for every $k \leq \hat{k}_2 - 1$ then $C_{\hat{k}_2} \equiv C_{k^1} \in \sigma$ and we jump to step 2.

If not, we continue the search in this manner by decreasing k until we find $C_{\hat{k}_m}$ such that $\pi^c(\hat{k}_m) > \pi^f(k)$ for every $k \leq \hat{k}_m - 1$, where $C_{\hat{k}_m}$ is the first $C_{\hat{k}}$ below $C_{\bar{k}_m}$, and $\pi^f(\bar{k}_m) \geq \pi^c(\hat{k}_{m-1})$, then $C_{\hat{k}_m} \equiv C_{k^1} \in \sigma$ and we jump to step 2. We now have a subset of 'stable' a la D' Aspremont et al. (1983) cartels $C_{\hat{k}_m}, ..., C_{\hat{k}_i}, ..., C_{\hat{k}_1}$ where $\hat{k}_m < \hat{k}_i < \hat{k}_1$[6].

[6]If any two *adjacent in the collection* cartels are really consequtive, i.e., $\hat{k}_i = \hat{k}_{i-1} - 1$ then it must be the case that $\pi^c(\hat{k}_{i-1}) = \pi^f(\hat{k}_{i-1} - 1) = \pi^f(\bar{k}_i) = \pi^f(\hat{k}_i)$. Since from the construction of our collection $\hat{k}_i \leq \bar{k}_i \leq \hat{k}_{i-1}$ it must be that $\hat{k}_i = \bar{k}_i = \hat{k}_{i-1} - 1$,

Note that there may exist a unique $C_{\hat{k}}$ in which case it is immediately $C_{\hat{k}_1} \equiv C_{k^1} \in \sigma$ and we jump to step 2. (The uniqueness of $C_{\hat{k}}$ implies that $\pi^c(\hat{k}) > \pi^f(k)$ for every $k \leq \hat{k} - 1$. Otherwise, there exists $k' \leq \hat{k} - 2$ such that $\pi^f(k') \geq \pi^c(\hat{k}) > \pi^c(k' + 1)$, let k'' denote the smallest k below $\hat{k}$ such that $\pi^f(k'') > \pi^c(k'' + 1)$, but since k'' is the smallest and $\pi^c(1) > \pi^f(0)$, we have $\pi^c(k'') \geq \pi^f(k'' - 1)$ making k'' a $\hat{k}$.) Furthermore, it may be the case that this unique $C_{\hat{k}} = C_n$ in which case $\sigma = \{C_n\}$ and we jump to step 3.

Step 2(Ascent)

$C_{\hat{k}_m} \equiv C_{k^1}$ is the smallest size cartel belonging to σ, with C_{k^1} as a starting point we begin our ascent, collecting cartels along the way, by increasing the size of cartels by one at a time until we reach C_k where $k > k^1 = \hat{k}_m$ and $\pi^c(k) \geq \pi^f(k^1)$. If such a k exists and if $k \in (\hat{k}_f, \bar{k}_f] \cup (\hat{k}_1, n]$ for $f = 1, ..., m$ then $C_k \equiv C_{k^2}$ and it is the second smallest cartel in σ[7].

If $k \in (\bar{k}_f, \hat{k}_{f-1}]$ for $f = 2, ..., m$ then let $C_{k^2} \equiv C_{\hat{k}_{f-1}} \in \sigma$. If such a k does not exist then $\sigma = \{C_{k^1}\}$ and we jump to step 3.

We continue in this manner by increasing the size of k. That is, given that $C_{k^i} \in \sigma$, where $\hat{k}_f \leq k^i \leq \bar{k}_f$ for some $1 < f \leq m$ or $\hat{k}_1 \leq k^i$, locate C_k such that $k > k^i$ and $\pi^c(k) \geq \pi^f(k^i)$. If such a k exists, and $k \in (k_i, \bar{k}_f] \cup (\hat{k}_{f-j}, \bar{k}_{f-j}] \cup (\hat{k}_1, n]$ for $j = 1, ..., f - 2$ then $C_k \equiv C_{k^{i+1}}$ and it is the next smallest cartel in σ.

If $k \in (\bar{k}_{f-j}, \hat{k}_{f-j-1}]$ for $j = 0, ..., f - 2$ then let $C_{k^2} \equiv C_{\hat{k}_{f-j-1}} \in \sigma$. If such a k does not exist then $\sigma = \{C_{k^1}, ..., C_{k^i}\}$ and we jump to step 3.

We continue the ascent until we find $C_{k^l} \in \sigma$ in the above prescribed manner and no $k > k^l$ such that $\pi^c(k) \geq \pi^f(k^l)$. We then have a set $\sigma = \{C_{k^1}, C_{k^2}, ..., C_{k^i}, ..., C_{k^l}\}$ where $l \leq n$, and $k_l \leq n$.

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moreover $\pi^f(\bar{k}_i) \geq \pi^c(\hat{k}_{i-1})$, while from $\hat{k}_{i-1}$'s D' Aspremont et al. (1983) stability we have $\pi^c(\hat{k}_{i-1}) \geq \pi^f(\hat{k}_{i-1} - 1) = \pi^f(\hat{k}_i) = \pi^f(\bar{k}_i)$.

[7] Note that if $k \in (\hat{k}_m, \bar{k}_m]$ then C_k will not immediately succeed C_{k^1}, that is $k > k^1 + 1$, otherwise $\pi^c(k) \geq \pi^f(k^1) \Leftrightarrow \pi^c(k) \geq \pi^f(k-1)$ which would contradict the original finding of $C_{k^1} \equiv C_{\hat{k}_m}$—during the original descent from $C_{\bar{k}_m}$ we should have stopped at C_k instead, which would have been the first $C_{\hat{k}}$ below $C_{\bar{k}_m}$.

To summarize the construction of σ, we know that $\pi^c(k^{i+1}) \geq \pi^f(k^i)$ for $i = 1, ..., l-1$ while $\pi^c(k^1) > \pi^f(k)$ for every $k \leq k^1 - 1$. Note that $\sigma \neq \emptyset$ since, as argued earlier, we can always find at least C_{k^1}. Also note that for $l = 1$ and $k^l = n$ we have the special case of $\sigma = \{C_n\}$.

Step 3 (Stability)

No inner contradictions.

We will show that all the elements inside σ are both internally and externally stable. If $|\sigma| = 1$, that is, it is a singleton then the one cartel in it is trivially both internally and externally stable since firms have essentially no other alternative, no other stable cartel (or stable fringe) to want to deviate to.

If $|\sigma| > 1$, that is, σ contains more than one cartel, we start by considering C_{k^1}. It is internally stable since no smaller stable cartel belongs to σ, therefore firms have nowhere (stable) to go to if they exit the cartel. Similarly, C_{k^l} is externally stable since there is no other larger stable cartel for the fringe members to want to join in. If $k^l = n$ then the cartel is even more trivially externally stable since everyone has joined in already.

Let us consider some $C_{k^i} \in \sigma$ where $k^1 < k^i \leq k^l$. It is internally stable because $\pi^c(k^i) \geq \pi^f(k^{i-1})$ by construction of σ. Therefore, no cartel member wishes to exit and end up at $F_{k^{i-1}}$.

Now let us consider $C_{k^i} \in \sigma$ where $k^1 \leq k^i < k^l$ then it is the case that $\hat{k}_f \leq k^i \leq \bar{k}_f$ for some f such that $1 < f \leq m$, or $\hat{k}_1 \leq k^i$. C_{k^i} is externally stable. By the construction of σ we have $\pi^f(k-1) \geq \pi^c(k)$ for all $k \in (\hat{k}_f, \bar{k}_f] \cup (\hat{k}_1, n]$ for all $f = 2, ..., m$. Thus, if $k^{i+1} \in (k^i, \bar{k}_f] \cup (\hat{k}_{f-j}, \bar{k}_{f-j}] \cup (\hat{k}_1, n]$ where $j = 1, ..., f-2$, then even though $\pi^c(k^{i+1}) \geq \pi^f(k^i)$ by the construction of σ, at the last step of the sequence $(F_{k^{i+1}-1})$ that would lead us from F_{k^i} to $C_{k^{i+1}}$ no agent wishes to join in $C_{k^{i+1}}$ since $\pi^f(k^{i+1} - 1) \geq \pi^c(k^{i+1})$.

If $C_{k^{i+1}} \equiv C_{\hat{k}_{f-j}}$ where $j = 1, ..., f-1$, even though $\pi^c(k^{i+1}) \geq \pi^f(k^i)$, by the construction of σ, there exists $\bar{k}_{f-j+1}$ such that $k^i \leq \bar{k}_{f-j+1} < \hat{k}_{f-j}$ and

$\pi^f(\bar{k}_{f-j+1}) \geq \pi^c(\hat{k}_{f-j}) = \pi^c(k^{i+1})$. Thus, at $F_{\bar{k}_{f-j+1}}$ along the sequence that would lead us from F_{k^i} to $C_{k^{i+1}} \equiv C_{\hat{k}_{f-1}}$ no agent wishes to join in.

Accounting for every exclusion.

Now we will argue that every $C_h \notin \sigma$ is accounted for, that is either its internal or external stability is violated.

If $k^1 \geq 2$, we consider $C_h \notin \sigma$, where $0 < h \leq k^1 - 1$. Since by construction of σ, $C_{\hat{k}_m} \equiv C_{k^1}$ and $C_{\hat{k}_m}$ is such that $\pi^c(\hat{k}_m) > \pi^f(k)$ for every $k \leq \hat{k}_m - 1$, cartels $C_1, ..., C_h, ..., C_{k^1-1}$ are externally unstable since members of their fringes want to join in and reach C_{k^1}.

If $k^l \leq n-1$, we consider $C_h \notin \sigma$ where $k^l+1 \leq h \leq n$. By construction of σ we know that $\pi^f(k^l) > \pi^c(n), ..., \pi^f(k^l) > \pi^c(h), ..., \pi^f(k^l) > \pi^c(k^l+1)$, thus cartels $C_n, C_{n-1}, ..., C_h, ..., C_{k^l+1}$ are internally unstable since their members wish to join the fringe F_{k^l}.

If $|\sigma| > 1$, that is, it contains more than one cartel, we consider $C_h \notin \sigma$ where $k^i < h < k^{i+1}$ and $k^i \in [\hat{k}_f, \bar{k}_f] \cup [\hat{k}_1, n]$ for some $f = 2, ..., m$.

If $k^{i+1} \in (\hat{k}_{f-j}, \bar{k}_{f-j}] \cup (\hat{k}_1, n]$ for some $j = 0, ..., f-2$ then $\pi^c(k^{i+1}) \geq \pi^f(k^i)$ and $\pi^f(k^i) > \pi^c(h)$ for every h such that $k^i \leq h < k^{i+1}$. Thus, C_h is internally unstable since its members wish to exit and join F_{k^i}.

If $k^{i+1} = \hat{k}_{f-j}$ for some $j = 1, ..., f-1$ and $h \in (\bar{k}_{f-j+1}, \hat{k}_{f-j})$ for $j = 1, ..., f-1$ then $\pi^c(k^{i+1}) = \pi^c(\hat{k}_{f-j}) > \pi^f(h)$ for every h such that $\bar{k}_{f-j+1} < h < \hat{k}_{f-j}$. Therefore, C_h is externally unstable since everyone from F_h wants to join in and reach $C_{k^{i+1}} \equiv C_{\hat{k}_{f-j}}$.

If $k^{i+1} = \hat{k}_{f-j}$ for some $j = 1, ..., f-1$ and $h \in (k^i, \bar{k}_{f-j+1}]$ for $j = 1, ..., f-1$ then $\pi^f(k^i) > \pi^c(h)$ for every h such that $k^i \leq h \leq \bar{k}_{f-j+1}$. Therefore, C_h is internally unstable since its members wish to exit and join F_{k^i}.

Step 4 (Uniqueness)

Uniqueness stems from the fact that C_{k^1} is always internally and externally stable, regardless of σ. In particular, it is internally stable because

$\pi^f(k) < \pi^c(k^1)$ for every $k \leq k^1 - 1$, which means that no firm will ever wish to exit since every fringe thereafter is worse than the cartel. It is externally stable since even if there exists some $k^* > k^1$ such that $\pi^c(k^*) > \pi^f(k^1)$ then it cannot be the case that $\pi^c(k^*) > \pi^f(k)$ for every k such that $k^1 \leq k < k^*$. For if it was the case, since $\pi^c(k^*) > \pi^f(k^1) > \pi^c(k^1) > \pi^f(k)$ for every $k \leq k^1 - 1$ we would have $\pi^c(k^*) > \pi^f(k)$ for every $k \leq k^* - 1$ and since $\pi^f(k^*) \geq \pi^c(k^* + 1)$ then k^* should be k^1. Note that if $\pi^f(k^*) < \pi^c(k^* + 1)$ then we keep on searching upwards until we locate $k^{**} > k^*$ such that $\pi^c(k^{**}) > \pi^f(k)$ for every $k \leq k^{**} - 1$ and $\pi^f(k^{**}) \geq \pi^c(k^* + 1)$ then $C_{k^{**}} \equiv C_{k^1}$. If we cannot find it and reach n then $C_n \equiv C_{k^1}$.

Since C_{k_1} is both internally and externally stable regardless σ, it will be included in every σ. An immediate consequence is that every other cartel of smaller size is externally unstable since $\pi^c(k^1) > \pi^f(k)$ for every $k \leq k^1 - 1$. Similarly, all subsequent cartels such that $\pi^f(k^1) > \pi^c(k^1)$ are also excluded from σ since they are internally unstable (their members wish to join F_{k^1}).

Now, we will show that $C_{k^2} \in \sigma$ as well, regardless of σ -of course, we use the fact that $C_{k^1} \in \sigma$, since this true for every σ.

If $k^2 \in (\hat{k}_f, \bar{k}_f] \cup (\ \hat{k}_1, n]$ for some $f = 2, ..., m$ then $\pi^c(k^2) \geq \pi^f(k^1)$ and $\pi^f(k^1) > \pi^c(h)$ for every h such that $k^1 \leq h < k^2$. Then C_{k^2} is obviously internally stable (all the other cartels below it, down to k^1, are excluded from σ).

It is externally stable as well since there does not exists some $k^* > k^2$ such that $\pi^c(k^*) > \pi^f(k^2)$ and $\pi^c(k^*) > \pi^f(h)$ for every h such that $k^2 \leq h < k^*$. Recall that such a $k^* \notin (\hat{k}_{f-j}, \bar{k}_{f-j}] \cup (\hat{k}_1, n]$ for all $j = 0, ..., f-2$, since for every $h \in (\hat{k}_{f-j}, \bar{k}_{f-j}] \cup (\hat{k}_1, n]$ we have that $\pi^f(h-1) \geq \pi^c(h) \Longleftrightarrow \pi^f(k^* - 1) \geq \pi^c(k^*)$. In addition, $k^* \notin (\bar{k}_{f-j}, \hat{k}_{f-j-1}]$ for all $j = 1, ..., f-2$, since for every $h \in (\bar{k}_{f-j}, \hat{k}_{f-j-1}]$ we have $\pi^f(\bar{k}_{f-j}) > \pi^c(h) \Longleftrightarrow \pi^f(\bar{k}_{f-j}) > \pi^c(k^*)$.

Now if $k^2 = \hat{k}_f$ for some $f = 1, ..., m-1$, again $\pi^c(k^2) \geq \pi^f(k^1)$ but it may not be the case that $\pi^f(k^1) > \pi^c(h)$ for every h such that $k^1 \leq h < k^2$. Nevertheless, although not all C_h's below C_{k^2} are excluded C_{k^2} still remains both internally and externally stable.

In particular, $\pi^c(k^2) \geq \pi^f(k^1) > \pi^c(\bar{k}_{f-1}) \Rightarrow \pi^f(k^1) > \pi^c(h)$ for every h such that $k^1 \leq h \leq \bar{k}_{f-1}$, rendering all such cartels internally unstable due to $C_{k^1} \in \sigma$. While for all h such that $\bar{k}_{f-1} < h < k^2$ it is the case, by construction of σ that $\pi^f(h) < \pi^c(k^2)$, therefore, no member of C_{k^2} would want to exit and end up there (in case they are stable since we haven't characterized them yet). The argument for external stability remains the same as in the case where $k^2 \in (\hat{k}_f, \bar{k}_f] \cup (\hat{k}_1, n]$.

Once we conclude that $C_{k^2} \in \sigma$ as well, then it is immediate that all those C_h where $\bar{k}_{f-1} < h < k^2$ are externally unstable since $\pi^f(h) < \pi^c(k^2)$ and members of their fringe wish to join in.

So far we have shown that always $C_{k^1} \in \sigma$ and as a consequence of this fact $C_{k^2} \in \sigma$ as well, while $C_h \notin \sigma$ for all $h \in [1, k^1) \cup (k^1, k^2)$. The same procedure, if iterated, shows that if $C_{k^i} \in \sigma$ so should $C_{k^{i+1}} \in \sigma$ and $C_h \notin \sigma$ for all $h \in (k^i, k^{i+1})$. It is obvious that, in this manner we can construct only one stable set, the one we described above. ■

It is important to point out that steps 1 (descent) and 2(ascent) of the above proof can serve as an algorithm to find the set of stable cartels in any specific situation. Moreover, the following corollary which stems directly from the same steps formalizes the relationship between our characterization of stable cartels and that of D' Aspremont et al. (1983).

Corollary 5 *Let σ' denote the collection of all stable a la D' Aspremont et al. (1983) cartels. The intersection of σ' and σ is always non empty and it contains at least the smallest cartel belonging to σ, that is, C_{k^1}.*

Indeed, in the case of the example presented in the introduction $\sigma' = \{C_3\}$ where $C_3 = C_{k^1}$ while $\sigma = \{C_3, C_5\}$.

3 Conclusion

What we attempted to accomplish in this paper is to encompass foresight and consistency in the study of cartel stability. In doing so, we employed a modified, in the spirit of Harsanyi's (1974) criticism, version of the notion of von Neumann & Morgenstern (1944) stability to analyze oligopolistic markets. The stable set in conjunction with indirect dominance (in Harsanyi's spirit) provided us with a solution concept that embodies the desired features. Namely, agents foresee finitely many steps ahead and make their decisions by comparing the final stable outcomes resulting their actions. We showed that there always exists a unique set of stable cartels.

In our model the cartel behaves as a price leader and the fringe behaves competitively. Our solution concept could very well be applied to different models where the fringe behaves a la Cournot, or where the product is not homogeneous, or where the firms are not identical, etc. Moreover, it is explicit that each firm enters and exits independently, and does not consider colluding with others while doing so. A natural extension of the model would be the permission of coalitional moves allowing groups of firms to enter or exit simultaneously.

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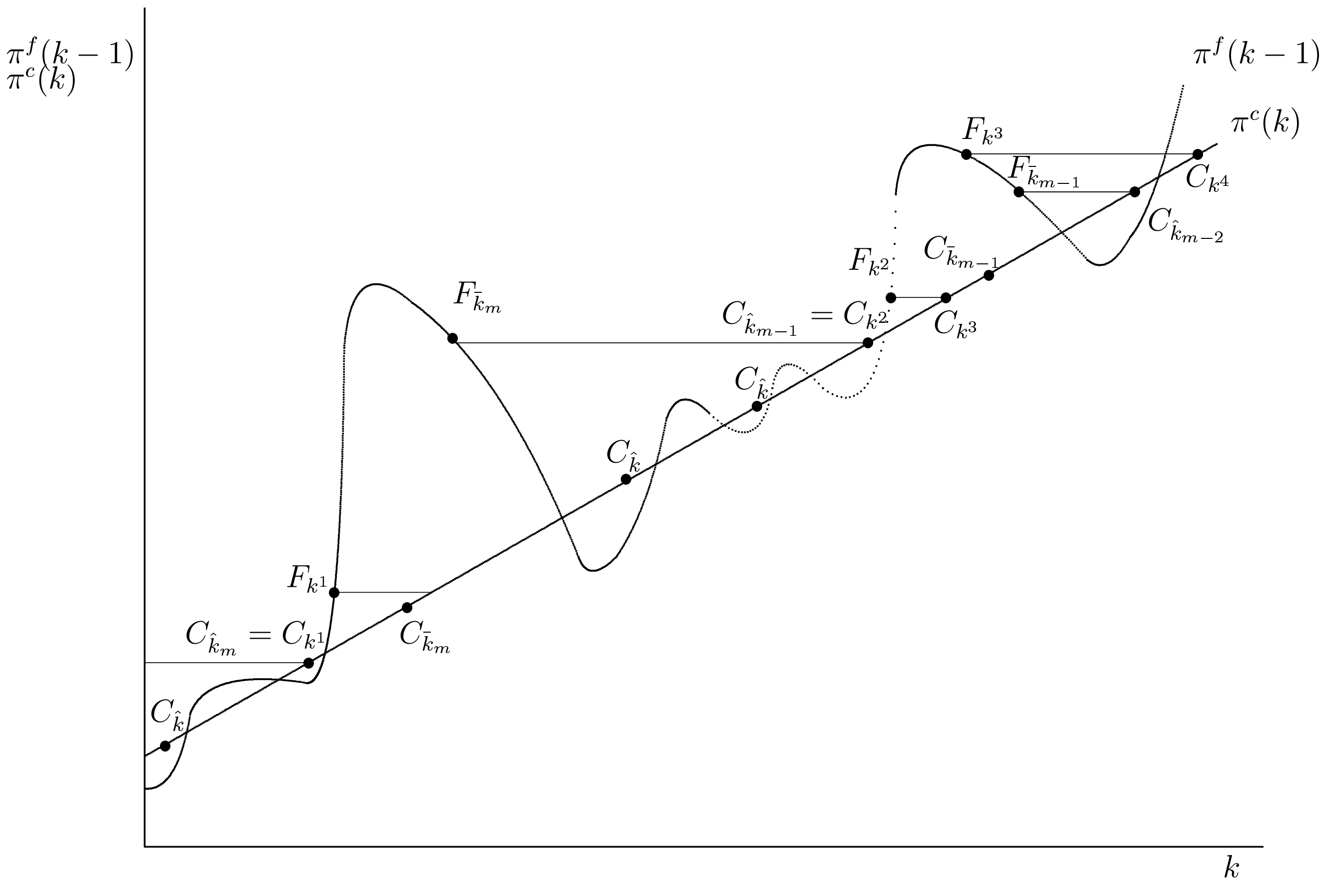
5 Appendix

Graphical Aid to the Proof of Theorem 4



The above graph does not depict the entire proof. It only shows the last part of step 1, that is, the location of the smallest cartel in σ and the beginning of step 2 illustrating how we ascent.

The straight upward slopping line depicts the cartel profits for various sizes k. The non monotonic line depicts the fringe profits for various sizes as well *but* it is shifted one size ahead since we always compare $\pi^c(k)$ with $\pi^f(k-1)$ and $\pi^f(k)$ with $\pi^c(k+1)$. This is why for a given cartel that appears on the graph, its corresponding fringe is always a bit to the right. As a result, all $C_{\hat{k}}$, that is, all stable a la D' Aspremont et al. (1983) cartels are always located just before (or on) the point where the fringe profit curve jumps above the cartel profit curve. In this manner, $\pi^c(\hat{k})$ is above (higher) than $\pi^f(\hat{k}-1)$, whereas $\pi^f(\hat{k})$ is above (higher) than $\pi^c(\hat{k}+1)$.

Therefore, $C_{\hat{k}_m}$ is located before an intersection and its above the entire fringe profit curve to its left indicating that $\pi^c(\hat{k}_m) > \pi^f(k)$ for every $k \leq \hat{k}_m - 1$. The rest of the graph is described in the proof.

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