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Multicointegration in US consumption data.*

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Abstract

The present paper tests for the existence of multicointegration between real per capita private consumption expenditure and real per capita disposable personal income in the USA. In doing so, we exploit the fact that the flows of disposable income and consumption expenditure on the one hand, and the stock of consumers' wealth, which can be considered as cumulative past discrepancies between the flows of income and expenditure, on the other hand, can be thought of as a stock-flow model, in which multicointegration is likely to occur. We apply recently developed I(2) techniques for testing for multicointegrating relations and find supporting evidence for the existence of multicointegration in this simple bivariate model.

Keywords: Cointegration, multicointegration, I(2) processes, consumption.

JEL code: C12, C13, C22, C32, C51, E21.

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1 Introduction.

Explaining and modeling of consumer's expenditure has been a long-standing occupation of several generations of economists as well as econometricians. From the economists' side a number of prominent theories has been put forward that shaped the economic views on consumption for years to come. These include, amongst others, the formulation of the consumption function suggested in Keynes (1936), the permanent income hypothesis (PIH) of Friedman (1957), and the life-cycle hypothesis of Ando & Modigliani (1963).

From the side of the applied economists and econometricians the doctrine of error-correction models, that dominates modern time series econometrics, was initially suggested in Davidson, Hendry, Srba & Yeo (1978). The notion of error-correction mechanisms were introduced into economics in Phillips (1954) and Phillips (1957) who borrowed the ideas from the control engineering literature. The error-correction models, which were somewhat later statistically justified by the theory of cointegration (see Engle & Granger (1987), *inter alia*), came about as a response to the fact that the theoretical economic models often only stipulate the long-run or equilibrium relations between the economic variables. In doing so, they often fail or are unable to describe the dynamic adjustment towards these equilibrium relations as well as to take the characteristic features of the actual data into consideration. Thus, the approach started in Davidson et al. (1978), while taking its inspiration in the formal theoretical economic models, specifically focuses on designing empirical models that explicitly take the salient features of the data into account. The relevance of the empirical models are judged on the basis of several design criteria developed for this purpose. This data-driven approach, which subsequently evolved to what is known as the London School of Economics (LSE) approach, offers a practical way of modeling economic relations in general and modeling e.g. consumption function in specific.

In this paper we model the consumers' expenditure in the US following the LSE traditions. The motivation of the paper is as follows. Following Davidson et al. (1978), we assume the existence of the long-run equilibrium relation between consumption expenditure and disposable income, which are assumed to be well approximated by I(1) processes in the sequel. In reality, however, this relation need not to be held exactly in every time period. In other words, we assume that consumption expenditure and disposable income are cointegrated. This means that the resulting savings variable, or cointegration error, defined as the difference between consumption expenditure and disposable income, is stationary. Intuitively, this is appealing since in general one cannot spend income without earning it and saving income without spending it also makes little sense.

Furthermore, suppose that in a given period saving is the increment to household wealth such that the savings accumulated over time represent a measure of private wealth. Hence, the three variables: income and expenditure flows, and the stock of cumulated savings (or wealth), taken together form a so-called stock-flow model. Granger & Lee (1989) and Granger & Lee (1991) were the first to suggest the possible existence of a second cointegrating relation in stock-flow type models, i.e. when the flow variables cointegrate with the stock variable (itself being created from the historic flows). In our case, this corresponds to cointegration between the stock of wealth and the flows of expenditure and income. Then we say that the income and expenditure variables are multicointegrated in the sense of Granger & Lee (1989). Thus, in a bivariate system, multicointegration means that there exist two cointegrating relations formed by the two original time series and their transformations. This is opposite to the usual cointegration case where only a single cointegrating relation is allowed in a model with only two variables. As we have seen, the first cointegrating relation arises between the levels of the flow variables, whereas the second cointegrating relation involves the cumulated equilibrium errors, obtained in the first step, as well as the original variables in levels.

The idea of approximating the stock of wealth by summation of the past discrepancies between disposable income and consumption expenditure is not new in the econometrics literature. In fact, Stone (1966) and Stone (1973) can be credited to be the first who approximated the stock of wealth held by consumers by cumulating past savings in his study of the UK consumers' expenditure. Clearly, the introduction of wealth effects into the study of consumption behaviour of the economic agents seems not to be unwarranted as some (unobservable to econometrician) wealth stock must undergo some changes when income and expenditure flows fail to match each other. Elaborating on the above mentioned study of Davidson et al. (1978), Hendry & von Ungern-Sternberg (1981) were the first who, next to the assumed long-run relation between the disposable income and the consumption expenditure, incorporated the measure of wealth into the celebrated error-correction framework by stipulating existence of the long-run relation between the stock of wealth on the one hand and the disposable income on the other hand. In other words, using the modern terminology, the consumption function, developed in Hendry & von Ungern-Sternberg (1981), can be considered as a multicointegrating system, which can be statistically tested using already available techniques.

So far, the literature on multicointegration has been rather limited. Granger & Lee (1989) and Granger & Lee (1991) found support for the presence of multicointegrating relations existing between sectorial production and sales figures across a range of US industries and industrial aggregates. Hence, in this case two cointegrating relations were found amongst the production and sales variables and the stock of inventory defined as the cumulative historic discrepancies between production and sales.

In succession, Lee (1992), Lee (1996), and Engsted & Haldrup (1999) detected multicointegration in data for US housing. They found a stationary linear relation amongst the flows of housing units started and completed as well as the stock of housing units under construction; the latter being defined as the history of cumulated quasi-difference between the number of housing units started and completed in a given period.

While Granger & Lee (1989), Granger & Lee (1991), Lee (1992), and Lee (1996) estimated the multicointegrating relations using only the original I(1) variables, Engsted & Haldrup (1999) showed that the statistical inference and estimation of the multicointegrating relations could be carried out in the framework of the Johansen FIML procedure for I(2) variables, see Johansen (1995). In order to apply this procedure, we first need to transform the original I(1) flow variables into their cumulated stock variants which then become I(2) series by construction. As advocated by Engsted & Johansen (1999), this transformation of variables is necessary because the I(1) analysis turns out to be invalid in the presence of multicointegration.

In the present paper we address the detection and estimation of a possible multicointegrating relation in the US consumption data set used in Campbell (1987) by employing the recent technique developed in Engsted & Haldrup (1999).

The plan of the paper is as follows. In Sections 2 and 3, we provide the formal definition of multicointegration in the sense of Granger and Lee together with a brief description of the Johansen FIML I(2) estimation technique which we use to make statistical inference as well as for estimation of the multicointegrating relation as it was originally done in Engsted & Haldrup (1999). Next, we present the stock-flow vector error correction models - henceforth VECM - for the multicointegrating variables in Section 4. The data set and the empirical results are described in Section 5. We draw conclusions and discuss possible extensions and limitations of this study in Section 6.

2 The Statistical Model.

We use the consumption-income example presented above for the formal definition of multicointegration. Suppose that income, y_t, and consumption variables, c_t, are integrated of order one. Moreover, assume that the variables in question are cointegrated, i.e. such that there exists some stationary linear combination of these variables:

$$s_t = y_t - \frac{1}{\gamma}c_t \;\sim\; I(0). \tag{1}$$

The I(0) variable on the left hand side of (1), s_t, represents the cointegration error. Multicointegration occurs when the cumulated cointegration error, which is an I(1) stock variable by construction, forms a cointegrating relation CI(1,1) with either one of the original flow variables or both [1]:

$$\sum_{j=1}^{t}\left(y_j - \frac{1}{\gamma}c_j\right) + \phi_1 y_t + \phi_2 c_t \;\sim\; I(0). \tag{2}$$

Notice that (2) represents the stationary linear combination between the flows of consumption and income and the wealth variable which is the cumulated stock of the past discrepancy between income and consumption.

Furthermore, if we adopt the convention that the generated I(2) variables are denoted in capital letters,

$$Y_t = \sum_{j=1}^{t} y_j,\; C_t = \sum_{j=1}^{t} c_j,\; \Delta Y_t = y_t,\; \Delta C_t = c_t,$$

then we can write our multicointegrating relation (2) in the form of a polynomial cointegrating relation

$$Y_t - \frac{1}{\gamma}C_t + \phi_1 \Delta Y_t + \phi_2 \Delta C_t \;\sim\; I(0), \tag{3}$$

which occurs when I(2) variables cointegrate with their first differences. Note that both layers of cointegration are encompassed in (3).

Observe that in the first cointegrating relation (1) we estimate the parameter γ. In Campbell (1987) the γ parameter is defined as the marginal propensity to consume out of the hypothetical permanent income y_t^p. Campbell (1987) estimates this parameter in equation (1) using both a method of the grid-search and the two-step Engle-Granger procedure, see Engle & Granger (1987). The method applied in this paper can be considered as an

[1] We assume zero initial values for y_t and c_t. Such scaling has no implications for the further analysis, except that proper allowance for deterministic components in the model will be needed.

alternative estimation method of the parameter of interest. Note that the parameter γ is expected to be either equal to one or be a positive fraction, i.e. $\gamma \leq 1$.

As mentioned by Engsted & Haldrup (1999), the existence of the stationary relation between the stock and flow variables (3) would imply that we can estimate the parameter γ in the first step cointegrating relation (1) at the fast rate of consistency, $O_p(T^{-2})$.

Having defined the statistical and economic models, we consider the estimation and inference procedures as well as the VECM representations for the multicointegrating variables.

3 Estimation and Inference Procedures.

Initially[2], consider the following unrestricted VAR model of order k for the $p \times 1$ vector of variables X_t integrated of order two:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_k X_{t-k} + \varepsilon_t, \quad t = 1, ..., T, \tag{4}$$

where we assume fixed initial values. The error term is identically, independently distributed N(0,Ω). We also assume here that the roots of the characteristic polynomial of (4) either take value of unity or lie outside the unit circle.

Following Johansen (1995), as an intermediate step we can reparametrize (4) as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \tag{5}$$

where

$$\Pi = \sum_{i=1}^{k} \Pi_i - I, \ \Gamma_i = - \sum_{j=i+1}^{k} \Pi_j, \ i = 1, ..., k-1.$$

Finally, after one more rearrangement we arrive at

$$\Delta^2 X_t = \Pi X_{t-1} - \Gamma \Delta X_t + \sum_{i=1}^{k-2} \Phi_i \Delta^2 X_{t-i} + \varepsilon_t, \tag{6}$$

where

$$\Gamma = I - \sum_{i=1}^{k-1} \Gamma_i, \ \Phi_i = - \sum_{j=i+1}^{k-1} \Gamma_i, \ i = 1, ..., k-2.$$

[2]The I(2) analysis in VAR models is technically involved. Therefore, in the further discourse we mainly present the skeleton of the inference and estimation procedures we use. For a recent review of the I(2) analysis as well as for the further technical details, see e.g. Haldrup (1999) and the references therein.

The last reformulation (6) is convenient for the subsequent analysis because it displays rather explicitly the reduced rank conditions that characterize the model with I(2) variables. Hence, according to Johansen (1995) the I(2) model nested in the unrestricted VAR involves the following two reduced rank conditions:

$$\underset{p\times p}{\Pi} = \alpha\beta' \qquad \underset{(p-r)\times(p-r)}{\alpha'_{\perp}\Gamma\beta_{\perp}} = \xi\eta',$$

where α and β are $p\times r$ matrices, and $\alpha_{\perp}$ and $\beta_{\perp}$ are the respective orthogonal complements of dimension $p\times(p-r)$ with $r<p$, such that by definition we have that $\alpha'_{\perp}\alpha = 0$ and $\beta'_{\perp}\beta = 0$. The matrices ξ and η have the dimensions $(p-r)\times s$ with $(p-r)>s$. Further description of an I(2) model requires more notation. Denote $\overline{\alpha} = \alpha\left(\alpha'\alpha\right)^{-1}$ such that $P_{\alpha} = \overline{\alpha}\alpha'$ is the orthogonal projection matrix onto the vector space spanned by the columns α and correspondingly $\alpha'\overline{\alpha} = \mathbf{I}$ is the identity matrix. Then in addition to already introduced matrix $p\times r$ α we can define the following matrices $\alpha_1 = \overline{\alpha}_{\perp}\xi$ and $\alpha_2 = \alpha_{\perp}\xi_{\perp}$ of the corresponding dimensions of $p\times s$ and $p\times(p-r-s)$ in such a way that α, α_1, and α_2 provide an orthogonal basis for the $p-$dimensional vector space. The same holds for the following matrices β, $\beta_1 = \overline{\beta}_{\perp}\eta$, and $\beta_2 = \beta_{\perp}\eta_{\perp}$ which have dimensions of $p\times r$, $p\times s$, and $p\times(p-r-s)$, respectively.

Using this notation, we can give the condition that rules out the presence of variables which are integrated of order higher than two, i. e. the following matrix needs to be of full rank:

$$\alpha'_2\Theta\beta_2 = \alpha'_2\left\{\Gamma\overline{\beta}\overline{\alpha}'\Gamma + \sum_{i=1}^{k-1} i\Gamma_i\right\}\beta_2.$$

In the following we will refer to the numbers r, s, and $p-r-s$ as the *integration indices.* Given the fact that we have p variables in the system (6), these integration indices, respectively, indicate the number of I(0), I(1), and I(2) relations present in the model. Thus, the I(2) model is characterized by the following. There are $p-r-s$ linear combinations that do not cointegrate and represent the common stochastic I(2) trends:

$$p-r-s: \qquad \beta'_2X_t \sim I(2).$$

There are s linear combinations of the X_t variables that cointegrate to the I(1) level referred to as the common stochastic I(1) trends:

$$s: \qquad \beta'_1X_t \sim I(1).$$

The remaining r linear combinations of the X_t variables and often its first differences, ΔX_t, cointegrate to the I(0) level:

$$r: \qquad \beta' X_t - \delta\beta_2' \Delta X_t \sim I(0), \tag{7}$$

where $\delta = \overline{\alpha}'\Gamma\overline{\beta}_2$ is the $r \times (p-r-s)$ matrix with $r \geq (p-r-s)$.

It is important to note that for a bivariate I(2) system with $X_t = (Y_t, C_t)'$ this linear combination 7 constitutes the only possible polynomially cointegrating relation defined in (3) with $\beta = (1, -1/\gamma)'$ and $\delta\beta_2' = (\phi_1, \phi_2)'$. Therefore this relation is of our primary interest. Hence, we would expect in the multicointegrating system to have one stationary relation[3], $r = 1$, no common I(1) trends, $s = 0$, and one common I(2) trend, $p - r - s = 1$.

In order to address the question of how the models with different integration indices are related we need the following notation. First, consider the restricted I(1) model without any I(2) trends. This corresponds to the case when $p - r = s$, i.e. the matrix $\alpha_\perp'\Gamma\beta_\perp$ has full rank. Thus we have only one reduced rank condition left. Therefore, we denote H_r as a model that has $rank(\Pi) \leq r < p$, whereas H_r^0 denotes the model with the $rank(\Pi) = r$. Therefore H_r^0 is a submodel of H_r or $H_r = \cup_{i=0}^r H_i^0$.

Similarly, we define the more general hierarchical ordering of the models by allowing for the I(2) relations as well. The model with H_{rs} involves two reduced rank conditions: $rank(\Pi) = r < p$ and $rank(\alpha_\perp'\Gamma\beta_\perp) \leq s < (p-r)$. It nests the sub-models H_{rs}^0 with $rank(\alpha_\perp'\Gamma\beta_\perp) = s$ such that the various models are related as follows: $H_{rs} = \cup_{i=0}^s H_{ri}^0$ and $H_{r0} \subset H_{rs} \subset ... \subset H_{rp-r} = H_r^0 \subset H_r \subset H_p$.

The relations amongst the various bivariate models with the different integration indices are viewed best when presented in Table 1, adapted from Johansen (1995).

Recapitulating, the upper-left corner of Table 1 houses the most restricted model H_{00} with $\Pi = \Gamma = 0$ such that we have only the noncointegrating I(2) variables present. This corresponds to the VAR in second differences, see (6). The unrestricted model placed in the lower-right corner is H_p with $p = 2$, where we have only I(0) variables. The remaining models comprise one or another form of cointegration as discussed above. The exception is the model H_0 in the upper-right corner which contains only the noncointegrating I(1) variables such that it corresponds to the VAR model in first differences.

This order of how the various models are nested determines the sequence of the testing

[3] Observe that by using the I(2) formulation of the problem the single multicointegrating relation involves the two layers of cointegration that follow from the usual I(1) analysis.

Table 1: Hierarchy of the various models for $p = 2$.

r	*I(2) model*				*I(1) model*				*I(0)model*	
0	H_{00}	$\subset$	H_{01}	$\subset$	$H_{02} = H_0^0$	$\subset$	H_0			
							$\cap$			
1			H_{10}	$\subset$	$H_{11} = H_1^0$	$\subset$	H_1	$\subset$	H_2	
$p-r-s$	2		1		0				0	

Adapted from Johansen (1995).

procedure for the integration indices in our model. We start testing with the most restrictive model against the unrestricted alternative. In case we reject the hypothesis in question, we proceed to the less restrictive model and so on until the first hypothesis that we cannot reject. This determines the integration indices.

Notice that here we have ignored the deterministic terms for expositional simplicity. There are two readily developed parametrizations of the deterministic terms in the I(2) models such as Rahbek, Kongsted & Jørgensen (1999) and Paruolo (1994). As we will argue below, each of these parametrizations has its own appealing properties when used for the data at hand. We review each parametrization in turn starting with the suggestion of Rahbek et al. (1999).

First notice that from equation (1) we have that if the estimated savings variable has a nonzero mean then this results in a trend-stationary multicointegrating relation after we obtain the measure of the wealth stock as an integral of the savings. In order to allow for a possible trend-stationary multicointegrating relation, we adopt the restrictions on the deterministic terms suggested in Rahbek et al. (1999). These restrictions allow us to rule out the nonlinear deterministic trend terms and at the same time to retain at most a linear trend in each of the I(0), I(1), and I(2) directions.

In the absence of the deterministic terms the process X_t in equation (4) can be given the following *common stochastic trends representation* [4]:

$$X_t = C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + C_1 \sum_{i=1}^{t} \varepsilon_i + C^* (L) \varepsilon_t, \tag{8}$$

[4]Abstracting from the nuisance parameters that depend on the initial conditions.

where

$$C_2 = \beta_2 \left(\alpha_2' \theta \beta_2\right)^{-1} \alpha_2'$$
$$\beta' C_1 = \overline{\alpha}' \Gamma C_2$$
$$\beta_1' C_1 = \overline{\alpha}_1' \left(I - \theta C_2\right),$$

and all the characteristic roots of a matrix lag polynomial $C^* (L)$ lie strictly outside the unit circle.

Based on equation (8), Rahbek et al. (1999) introduce the following model:

$$X_t = C_2 \sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + C_1 \sum_{i=1}^{t} \varepsilon_i + C^* (L) \varepsilon_t + \tau_0 + \tau_1 t, \tag{9}$$

such that linear deterministic trends are allowed in every I(0), I(1), and I(2) directions. Thus, the I(2) process X_t has a linear trend $\tau_1 t$. As shown in Rahbek et al. (1999), the following $r + s$ I(1) linear combinations $\beta' X_t$ and $\beta_1' X_t$ possess the linear deterministic trends given by $\beta' \tau_1 t$ and $\beta_1' \tau_1 t$, respectively. Next, the $p - r - s$ common stochastic I(2) trends $\beta_2' X_t$ are with linear trends $\beta_2' \tau_1 t$ as well. Finally, the r multicointegrating relations $\beta' X_t - \delta \beta_2' \Delta X_t$ are trend-stationary around a linear trend $\beta' \tau_1 t$.

Next we consider the following argument in favor of the model suggested by Paruolo (1994). Because the data in levels are trending, as seen at Figure 1, their cumulative counterparts are likely to have quadratic trends [5]. Hence, the restrictions of Rahbek et al. (1999) are valid only if the nonlinear quadratic trends cancel out in the resulting system. Unfortunately, the limitations of the available software does not allow us to test this assumption. As an alternative route, we have considered the VAR model (4) with an unrestricted constant, i.e.:

$$X_t = \Pi_1 X_{t-1} + ... + \Pi_k X_{t-k} + \mu + \varepsilon_t, \quad t = 1, ..., T, \tag{10}$$

According to Paruolo (1994), this results in the presence of linear and, most importantly, quadratic trends in the data. This is best seen in the corresponding common stochastic trends representation of the model given in (10):

$$\begin{aligned} X_t &= C_2 \textstyle\sum_{s=1}^{t} \sum_{i=1}^{s} (\varepsilon_i + \mu) + C_1 \sum_{i=1}^{t} (\varepsilon_i + \mu) + C^* (L) \varepsilon_t, \\ X_t &= \tfrac{1}{2} C_2 t^2 + \left(\tfrac{1}{2} C_2 + C_1 + B\right) t + C_2 \textstyle\sum_{s=1}^{t} \sum_{i=1}^{s} \varepsilon_i + C_1 \sum_{i=1}^{t} \varepsilon_i + C^* (L) \varepsilon_t. \end{aligned} \tag{11}$$

The model (11) allows for the I(2) process X_t with quadratic trends given by $\frac{1}{2} C_2 t^2$. Next, by defining the I(1) relations as $\beta_1' X_t$ we annihilate both the stochastic I(2)- and the quadratic

[5]This point has been made by Hans Cristian Kongsted.

deterministic trends such that the resulting I(1) linear combinations have only at most a linear deterministic trend given by $\beta_1' C_1 t$. Finally, due to the equality restriction $\beta' C_1 = \overline{\alpha}' \Gamma C_2$ there are no linear deterministic trends in the multicointegrating relations $\beta' X_t - \delta \beta_2' \Delta X_t$.

Notice that the model (11) does not allow for different stochastic and deterministic orders in either of I(2), I(1), or I(0) directions. In particular, as opposed to the parametrization of Rahbek et al. (1999) it does not allow for the trend-stationary multicointegrating relations. Again, within a given model of Paruolo (1994) this remains an untestable assumption.

To summarize, it seems that the available parametrizations of the system with I(2) variables are somewhat restrictive and, therefore, are not general enough to allow us to test the assumptions on the deterministic terms that we are involuntarily imposing on the system by resorting to either of the existing specifications in our empirical application. Hence, in order to assess the legitimacy of the imposed restrictions we need a system for I(2) variables that would encompass both parametrizations suggested in Rahbek et al. (1999) and Paruolo (1994) by allowing for the trend-stationary multicointegrating relations and for the quadratic trends in the data.

Rahbek et al. (1999) and Paruolo (1994) show that in the presence of the imposed restrictions on the deterministic terms inference on the integration indices is performed in the likelihood-based two-step procedure similar to Johansen (1995). Essentially, at the first step we address the reduced rank of $\Pi = \alpha\beta'$ by testing the restricted model H_r against the unrestricted alternative H_p. For later use in the empirical section, we denote the corresponding test statistics as $Q(r)$. Then, by fixing the rank of the matrix Π at each of the following values $r = 0, ..., p-1$ we address the reduced rank of the other matrix $\alpha_\perp' \Gamma \beta_\perp = \xi\eta'$ by testing the restricted model H_{rs} against the alternative $H_{r,p-r}$ model. Finally, because of the fact that in practice the reduced rank of the matrix Π is unknown and since the models are nested as discussed earlier, inference on the integration indices is based on the joint hypothesis of H_{rs} against the unrestricted H_p model. The relevant test statistics is referred to as $S(r, s)$.

We have argued above that the bivariate multicointegrating model contains two cointegrating vectors that essentially appear in the form of a single polynomially cointegrating vector. In the next section we demonstrate how these equilibrium relations can be incorporated into VECM representations for multicointegrated variables.

4 The VECM for Multicointegrating Variables.

Engsted & Haldrup (1999) suggest two types of vector error correction models (VECM) for the multicointegrating variables. The first type shows how the flow variables react to deviations from an equilibrium. The second type shows disequilibrium responses in the stock variables.

Engsted & Haldrup (1999) present the VECM for the general case, potentially embracing more than two variables. Since we operate in the bivariate system, we know that multicointegration in such a system implies the following integration indices: $r = 1$, $s = 0$, and $p - r - s = 1$. This knowledge allows us to simplify significantly the presentation of the VECM for the multicointegrating variables which is given below.

Definition 1 *The bivariate flow VECM representation for the multicointegrating variables:*

$$\Delta x_t = \alpha[Q_{t-1} - \delta\beta_2 x_{t-1}] - \zeta_1 \Delta Q_{t-1} + \Phi(L)\Delta x_t + \varepsilon_t, \tag{12}$$

where $x_t = (y_t, c_t)'$, $Q_t = \sum_{j=1}^{t} \beta' x_j$ represents the stock of cumulative equilibrium errors and $\Phi(L) = \sum_{i=1}^{k-2} \Phi_i L^i$ contains the coefficients of the short-run dynamics. The adjustment coefficients α and $\zeta_1 = \Gamma\overline{\beta}$ have the equal dimensions of $p \times r$, with $\overline{\beta} = \beta(\beta'\beta)^{-1}$. The matrix $\delta = \overline{\alpha}\Gamma\overline{\beta}_2$ is $r \times (p - r - s)$, where $\overline{\alpha}$ and $\overline{\beta}_2$ are defined similarly to $\overline{\beta}$.

Notice that this VECM incorporates several control mechanisms, as discussed in Hendry & von Ungern-Sternberg (1981), for example. For instance, the integral control mechanism, $[Q_{t-1} - \delta\beta_2 x_{t-1}]$, represents the multicointegrating relation. The proportional control mechanism, $\Delta Q_{t-1} = \beta' x_t$, represents the first step cointegrating relation between the variables in levels, and lastly, the derivative control mechanism is given by the lagged Δx_t's.

Definition 2 *The bivariate stock VECM representation for the multicointegrating variables*

$$\begin{aligned} \Delta\widetilde{x}_t &= M\alpha[Q_{t-1} - \delta\beta_2 x_{t-1}] - \widetilde{\zeta}_1 \Delta Q_{t-1} + \\ &\quad + \widetilde{\Phi}(L)\Delta\widetilde{x}_t + M\Phi(L)\overline{\beta}_2 \Delta\beta_2' x_{t-1} + M\varepsilon_t, \end{aligned} \tag{13}$$

where in addition to the variables and the model parameters defined above we have $\Delta\widetilde{x}_t = (\Delta Q_t', \Delta x_t'\beta_2)'$, $M = (\beta, \beta_2)'$, $\widetilde{\zeta}_1 = (\iota_r - M\zeta_1)$ with $\iota_r = (1, 0)'$, and $\widetilde{\Phi}(L) = M\Phi(L)M^{-1}D_\perp(1)$

such that $M^{-1} = (\overline{\beta}', \overline{\beta}'_2)$ *and*

$$D(L) = \begin{pmatrix} \Delta & 0 \\ 0 & 1 \end{pmatrix} \quad D_\perp(L) = \begin{pmatrix} 1 & 0 \\ 0 & \Delta \end{pmatrix} \quad D(L)D_\perp(L) = \begin{pmatrix} \Delta & 0 \\ 0 & \Delta \end{pmatrix}.$$

The latter VECM representation is worth commenting further on. First, note that the equilibrium relations are the same as in the former representation. Secondly, the variables that adjust to the previous period disequilibrium state are the stock variable, $Q_t = \sum_{j=1}^{t} \beta' x_j$, as well as the flow variables that appear in the VECM as the first difference of the I(2) trends, $\beta_2' x_t$. Finally, note that the "$\sim$" parameters in (13) retain the same dimension as the parameters without the "$\sim$" sign in (12).

Additionally, keep in mind that for both VECM representations we impose the restrictions on the deterministic terms (not shown) in accordance with Rahbek et al. (1999).

5 The Empirical Application.

In this study we use the same data set as in Campbell (1987) [6]. This data set contains quarterly data for the period of 1953:2 to 1984:4 with 127 observations. The data are the seasonally adjusted time series of real disposable income and real total private consumption expenditure taken from the National Income and Product Accounts (NIPA) with some adjustments made by Blinder & Deaton (1985). The data are in per capita values in units of thousands US$. We want to address this period in order to be able to compare our results with those of Campbell.

The upper panel of Figure 1 displays the actual values of the real total disposable income, y_t, and the real total private consumption expenditure, c_t. As seen, both the time series develop very synchronously. Given the results of the ADF test reported in Campbell (1987) and Table 2 that these variables are I(1), this is the first sign that they might be cointegrated. However, we are interested in testing whether the variables in question are multicointegrated.

In order to sort this out, we first transform the variables into their cumulative counterparts, Y_t and C_t, which are shown in the lower panel of Figure 1. Next we use these newly generated I(2) variables to form a parsimonious bivariate VAR(7) model. Table 3 summarizes the results of both the univariate and multivariate diagnostic tests of the estimated residuals. The univariate diagnostic tests comprise: F_{AR8} - test for autocorrelation of most

[6] This dataset has been used extensively in the literature, for example, in Blinder & Deaton (1985), Campbell & Deaton (1989), Flavin (1993), and Vahid & Engle (1993).

Table 2: Results of the ADF test.

Variable	Deterministic terms	Augmentation	t-ratio	5% critical value
y_t	Constant, Trend	1,5	-2.027	-3.45[a]
c_t	Constant, Trend	2	-2.224	-3.45

The critical values are reported after Fuller (1976).

Table 3: VAR (7). Residual diagnostic tests.

Univariate analysis			Multivariate analysis	
Y_t :	$F_{AR8}(8, 96)$	= 1.2282 [0.29]	$F^v_{AR8}(32,174)$	= 0.87379 [0.66]
C_t :	$F_{AR8}(8, 96)$	= 1.4550 [0.18]		
Y_t :	$Normality\ \chi^2(2)$	= 2.8825 [0.23]	$Normality^v\ \chi^2(4)$	= 12.803 [0.01]
C_t :	$Normality\ \chi^2(2)$	= 16.952 [0.00]		
Y_t :	$F_{HET}(30, 73)$	= 1.0256 [0.45]	$F^v_{HET}(90,213)$	= 0.8521 [0.80]
C_t :	$F_{HET}(30, 73)$	= 1.2864 [0.19]		
Y_t :	$F_{ARCH4}(4, 96)$	= 0.2913 [0.88]		
C_t :	$F_{ARCH4}(4, 96)$	= 0.2305 [0.92]		

The corresponding p-values are reported in the square brackets.

Table 4: VAR(7): Dynamic analysis.

real	complex	modulus	real	complex	modulus
-0.7265	0	0.7265	0.6528	0.4812	0.811
-0.5342	0.2513	0.5904	0.6528	-0.4812	0.811
-0.5342	-0.2513	0.5904	0.8663	0.287	0.9126
-0.197	0.7467	0.7722	0.8663	-0.287	0.9126
-0.197	-0.7467	0.7722	0.9697	0.05923	0.9715
0.06783	0.6227	0.6264	0.9697	-0.05923	0.9715
0.06783	-0.6227	0.6264	1.005	0	1.005

Eigenvalues of the companion matrix of equation (4).

8$^{\text{th}}$ order (see Godfrey (1978)); *Normality* - test for the normally distributed residuals (see Doornik & Hansen (1994); F_{HET} - White (1980) test for heteroscedasticity based on the original and squared regressors; F_{ARCH4} - Engle (1982) test for the *4*$^{\text{th}}$ order AutoRegressive Conditional Heteroscedasticity. The multivariate test statistics denoted with the superscript v were derived in Doornik & Hansen (1994) for vector normality, and in Doornik (1995) for vector autocorrelation and vector heteroscedasticity. The graphics and residual diagnostic tests were calculated using GiveWin and Pc-Fiml 9.3 , see Hendry & Doornik (1999).

Taken as a whole, it seems that the model residuals do not display autocorrelation, ARCH effects, and heteroscedasticity when judged on the basis of both from the results of the univariate and the multivariate specification tests. However, their is some deviation from the normality assumption in the equation for C_t. An additional information can be obtained from Figure 2, which provides a graphical analysis of the estimated residuals. It contains the estimated residuals, their correlogram, spectral density, and histogram. As seen, the deviation from normality in the equation for C_t occurs due to a amall number of large negative residuals. The most important assumption we require to be fulfilled is the absence of autocorrelation in the VAR residuals, since it introduces nuisance parameters in the limiting distribution of the test statistics. This invalidates the asymptotic critical values that we use in our statistical inference procedure. On the other hand, Gonzalo (1994) showed that the FIML Johansen procedure is rather robust to minor departures from the model assumptions due to non-normality.

Table 5: Test for integration indices.

$p-r$	r	$S(r,s)$		$S(r)$
2	0	49.92**	32.22*	25.39*
		47.5	*34.4*	*25.4*
1	1		**11.67**	8.76
			19.9	*12.5*
$p-r-s$		2	1	0

**,* indicate rejection at the 5% and 10% significance levels, respectively.
The asymptotic 95% quantiles are reported in italics, see Table 1 in Rahbek et al. (1999).

The system dynamics is summarized by the eigenvalues of the companion form of (4), see Table 4. *A priori*, in the bivariate multicointegrating model we would expect two unit roots corresponding to the one common I(2) trend. As seen for the given realization of the stochastic variables in our model we have one explosive eigenvalue, but it needs not be significantly different from unity. Hence, we assume it to be a unit root in the sequel. Furthermore, we have two pairs of comparatively large complex conjugate eigenvalues of moduli 0.97 and 0.91, respectively. The remaining eigenvalues of rather smaller magnitude lie at some other different from the zero frequencies. Thus, the unrestricted VAR model seems to contain at least two unit roots or, possibly, more.

The statistical inference[7] of testing sequentially the hypotheses of the restricted submodel H_{rs} against the unrestricted alternative H_p yields the results displayed in Table 5.

The empirical results suggest that we have one polynomially cointegrating relation, $r = 1$, and one common I(2) trend, $p-r-s = 1$, and no common I(1) trends, $s = 0$, in our bivariate system. These are the values of the integration indices we would have expected to find if the series were multicointegrated. The estimated trend stationary multicointegrating relation reads:

$$\sum_{j=1}^{t} (y_j - 1.000c_j) - 5.253y_t - 5.253c_t + constant + trend. \tag{14}$$

[7]All I(2) analysis has been performed using the I(2) procedure written by Clara M. Jørgensen for the CATS in RATS package, see http://www.estima.com/procs/i2index.htm

Table 6: VECM representations, $r=1$, $s=0$, $p-r-s=1$..

Multicointegrating relation	$\widehat{\beta}' = (1, -1.000)\,, \widehat{\delta} = 5.253$
	$\widehat{\delta}\widehat{\beta}_2' = (5.253, 5.253)$
Adjustment coefficients	$\widehat{\alpha}' = (-0.002, 0.010)$
I(1) trend	$\widehat{\beta}_1 = 0$
I(2) trend	$\widehat{\beta}_2' = (1, 1.000)$
M-matrix	$\widehat{M} = \left(\widehat{\beta}, \widehat{\beta}_2\right)' = \begin{pmatrix} 1 & -1.000 \\ 1 & 1.000 \end{pmatrix}$
Γ-matrix	$\widehat{\Gamma} = \begin{pmatrix} -0.017 & -0.005 \\ -0.261 & 0.369 \end{pmatrix}$
Flow VECM	**Stock VECM**
Derived adjustment parameters.	
$\widehat{\alpha} = \begin{pmatrix} -0.002 \\ 0.010 \end{pmatrix}$	$\widehat{M}\widehat{\alpha} = \begin{pmatrix} -0.012 \\ 0.008 \end{pmatrix}$
$\widehat{\zeta}_1 = \widehat{\Gamma}\widehat{\overline{\beta}} = \begin{pmatrix} -0.006 \\ -0.315 \end{pmatrix}$	$\widehat{\widehat{\zeta}}_1 = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \widehat{M}\widehat{\zeta}_1\right] = \begin{pmatrix} 0.619 \\ 0.321 \end{pmatrix}$

Rahbek et al. (1999) restrictions on deterministic terms.

Incidentally, we estimated $\widehat{\gamma}$ to be exactly 1.000. This means that the flows of income and expenditure variables are cointegrated with the vector $\widehat{\beta} = (1, -1.000)'$. This contrasts the findings of Campbell (1987) who specifically reports that the linear combination $y_t - 1.000c_t$ is non-stationary, see Campbell (1987), Table I p. 1260[8]. The facts that we allow for a linear trend in the stationary directions and consider a VAR approach with I(2) variables could account for some of the difference.

Furthermore, notice the equal coefficients on the flow variables that enter the multicointegrating relation (14) in form of the differenced I(2) trend, $\beta_2'\Delta X_t$. This is due to the fact that the matrices $\widehat{\beta}$ and $\widehat{\beta}_2$ that provide subspaces for the I(0) and I(2) directions, respectively,

[8] In addition, we note here that Campbell (1987), using the two–step Engle–Granger method, obtains the estimate $\widehat{\gamma} = 0.941$, which corresponds to the cointegrating vector of $(1, -1.062)$. Vahid & Engle (1993) in Table I report that testing for unit root in the logarithmic transformation of the income-consumption ratio, $(\log y_t - \log c_t)$, with an intercept and three lags in augmentation results in t-statistics of -3.77, which is significant at the 5% level.

are mutually orthogonal as noted above. Hence, as summarized in Table 6, additionally to $\widehat{\beta} = (1, -1.000)'$ our estimates of the remaining coefficients in the multicointegrating relation are $\widehat{\delta} = 5.253$ and $\widehat{\beta}_2 = (1, 1)'$ such that the orthogonality condition holds, i.e. $\widehat{\beta}'\widehat{\beta}_2 = 0$.

Figure 3 displays the estimated multicointegrating vector (14) in deviations from the linear trend with subtracted mean. It looks reasonably stationary. It is interesting to compare graphically the original flow income and expenditure time series with the estimated measure of wealth. This is done in Figure 4. Notice that to ease comparisons, in the figure we have adjusted the displayed time series to have the same mean and range although not in the rest of the paper. As noted above, the income and expenditure time series move rather closely with one another, while at the same time their joint development could be seen as a cyclical movement around the estimated stock of accumulated savings which is much smoother than either of the flow variables. As the final exercise we place the estimated equilibrium relations in the VECM discussed in Section 4.

Using Table 6 the flow VECM looks as follows[9]:

$$\begin{aligned} \begin{pmatrix} \Delta y_t \\ \Delta c_t \end{pmatrix} &= \begin{pmatrix} -0.002 \\ 0.010 \end{pmatrix} (Q_{t-1} - 5.253 y_{t-1} - 5.253 c_{t-1}) - \\ &\quad - \begin{pmatrix} -0.006 \\ -0.315 \end{pmatrix} (\Delta Q_{t-1}) + lags\{\Delta y_t, \Delta c_t\} + \\ &\quad + constant + trend + error\ term. \end{aligned} \tag{15}$$

The stock VECM is

$$\begin{aligned} \begin{pmatrix} \Delta Q_t \\ \Delta \beta_2' (y_t, c_t)' \end{pmatrix} &= \begin{pmatrix} -0.012 \\ 0.008 \end{pmatrix} (Q_{t-1} - 5.253 y_{t-1} - 5.253 c_{t-1}) + \\ &\quad + \begin{pmatrix} 0.619 \\ 0.321 \end{pmatrix} (\Delta Q_{t-1}) + lags\{\Delta y_t, \Delta c_t\} + \\ &\quad + constant + trend + error\ term, \end{aligned} \tag{16}$$

where the stock variable is $Q_t = \sum_{j=1}^{t} (y_j - 1.000 c_j)$ in both the VECM's.

The magnitude of the adjustment coefficients for the multicointegrating control mecha-

[9]In Table 6 due to the limitations of the software we have neither the standard errors of the estimates nor the estimated coefficients on a intercept and a linear trend.

nism is much smaller than for the adjustment coefficients on the proportional control mechanism in either of the two VECM models. This indicates the rather slow adjustment to the deviations from the multicointegrating equilibrium that takes place. On the other hand, there is the quite rapid adjustment to the first step equilibrium error. Hence the "stock effect" only appears to play a minor role.

In general, it is difficult to sign *a priori* the adjustment coefficients due to the complex simultaneous interaction between the two equilibrium errors, but still it is possible to draw some conclusions. First consider the flow model (15). The positive (though numerically small) adjustment coefficient on the integral control mechanism for the consumption variable implies that consumption increases in response to the situation when the wealth variable is above its equilibrium level and *vice versa*. Additionally, notice that the magnitude of the income adjustment coefficient is rather small as well. The likelihood ratio test for restricting the latter coefficient to zero yields the value of 0.15, which corresponds to p-value 0.70 when compared with $\chi^2(1)$ distribution.

Next, we notice that the consumption adjustment coefficient for the proportional control mechanism has the expected sign. This means that consumption rightly adjusts to the past period equilibrium error between the flow variables. As regarding the income adjustment coefficient for the proportional control mechanism it seems to have a counterintuitive sign. In addition it is rather small compared to the consumption adjustment coefficient. Although we lack the formal testing procedure here, it seems safe to conclude that the income variable is rather less responsive to the past equilibrium errors represented by the multicointegrating relation as well as the simple cointegrating relation. Thus it seems that the adjustment in the flow error correction model mainly occurs through the consumption channel.

Considering the stock VECM (16) we notice that the adjustment coefficients to the integral control mechanism have the intuitively expected signs. This means that both the stock and flow variables adjust to correct for past deviations from the estimated equilibrium relation between the stocks and flows in our model. Finally, it should be noticed that the adjustment coefficients for the other cointegrating relation are both positive.

Next we present the estimation results of the model of Paruolo (1994). The obtained results of inference on the integration indices and estimation of the system with imposed one multicointegrating relation are displayed in Tables 7 and 8. As seen from Table 7 the rank determination is problematic as practically every hypothesis is rejected either at the 10% or 5% significance level. As discussed above, this might be due to the restrictive assumptions on the deterministic terms that we impose. Nevertheless, we proceeded further by imposing

Table 7: Test for integration indices.

$p-r$	r	$S(r,s)$		$S(r)$
2	0	30.14*	21.49**	20.75**
		30.25	*19.79*	*15.4*
1	1		7.02**	5.33**
			5.99	*3.8*
$p-r-s$		2	1	0

**,* indicate rejection at the 5% and 10% significance levels, respectively.

The asymptotic 95% quantiles are reported in italics, see Table A1 in Paruolo (1994).

Table 8: VECM representations, $r=1$, $s=0$, $p-r-s=1$.

Multicointegrating relation	$\widehat{\beta}' = (1.000, -1.014)\,,\ \widehat{\delta} = 5.253$
	$\widehat{\delta}\widehat{\beta}_2' = (5.741, 5.664)$
Adjustment coefficients	$\widehat{\alpha}' = (0.001, 0.011)$
I(1) trend	$\widehat{\beta}_1 = 0$
I(2) trend	$\widehat{\beta}_2' = (1.014, 1.000)$
M-matrix	$\widehat{M} = \left(\widehat{\beta}, \widehat{\beta}_2\right)' = \begin{pmatrix} 1.000 & -1.014 \\ 1.014 & 1.000 \end{pmatrix}$
Γ-matrix	$\widehat{\Gamma} = \begin{pmatrix} 0.000 & 0.014 \\ -0.193 & 0.323 \end{pmatrix}$
Flow VECM	**Stock VECM**
Derived adjustment parameters.	
$\widehat{\alpha} = \begin{pmatrix} 0.001 \\ 0.010 \end{pmatrix}$	$\widehat{M}\widehat{\alpha} = \begin{pmatrix} -0.010 \\ 0.012 \end{pmatrix}$
$\widehat{\zeta}_1 = \widehat{\Gamma}\widehat{\overline{\beta}} = \begin{pmatrix} -0.007 \\ -0.256 \end{pmatrix}$	$\widehat{\widetilde{\zeta}}_1 = \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \widehat{M}\widehat{\zeta}_1\right] = \begin{pmatrix} 0.746 \\ 0.263 \end{pmatrix}$

Paruolo (1994) restrictions on deterministic terms.

the restrictions that correspond to the presence of one multicointegrating relation. As seen from Table 8, both estimates of the parameters of the multicointegration relation and the estimates of the adjustment coefficients in the VECM formulations are very much alike with those obtained when parametrization of Rahbek et al. (1999) was used, see Table 6. We display the estimated multicointegrating relation in Figure 5. Given the estimation results, it looks quite similar to that obtained from the model of Rahbek et al. (1999) as well, see Figure 3.

6 Conclusions.

Using the same data set as in Campbell (1987), this study has detected the presence of multicointegration between the consumption expenditure and disposable income flows that was anticipated by Granger & Lee (1989) and Granger & Lee (1991). As it was initailly suggested by Engsted & Johansen (1999) and implemented in Engsted & Haldrup (1999), we perform statistical inference and estimation of the multicointegrating relation using the I(2) technique based on the Johansen (1995) FIML procedure.

The existence of a multicointegrating relation implies that there are two layers of cointegrating relations in the bivariate model. We incorporated these two estimated equilibrium relations in the error correction models for the multicointegrating variables that were initially proposed by Engsted & Haldrup (1999). It was found that the disposable income responds to the past disequlibria specified by estimated (multi-) cointegrating relations to the much lesser degree than does the consumption expenditure variable.

In our study we employed two available parametrizations on the deterministic terms developed for the I(2) systems in Rahbek et al. (1999) and Paruolo (1994). While both models have the appealing properties that match certain features of the data at hand, each of them is based on certain restrictive assumptions. A limitation of the present study is that we could not apply any formal statistical procedure in order to verify legitimacy of the imposed restrictions in each of the models. Therefore the tasks for further research are to develop the theoretical model for I(2) variables that would bridge both models of Rahbek et al. (1999) and Paruolo (1994) and in doing so would allow comparison of these models on the basis of formal statistical criteria as well as to include more variables relevant for modeling of the consumption function that incorporates the multicointegrating relations.

References

Ando, A. & Modigliani, F. (1963), 'The "life-cycle" hypothesis of saving: Aggregate implications and tests', *American Economic Review* **53**(1), 55–84.

Blinder, A. S. & Deaton, A. S. (1985), 'The time-series consumption revisited', *Brooking Papers on Economic Activity* pp. 465–521.

Campbell, J. Y. (1987), 'Does saving anticipate declining labour income? an alternative test of the permanent income hypothesis', *Econometrica* **55**, 1249–1273.

Campbell, J. Y. & Deaton, A. S. (1989), 'Why consumption so smooth?', *Review of Economic Studies* **56**, 357–374.

Davidson, J. E. H., Hendry, D. F., Srba, S. & Yeo, S. (1978), 'Econometric modelling of the aggregate time-series relationship between consumers' expenditure and income in the United Kingdom', *Economic Journal* **80**, 661–692.

Doornik, J. (1995), 'Testing vector autocorrelation and heteroscedasticity in dynamic models', mimeo, Nuffield College, Oxford.

Doornik, J. A. & Hansen, H. (1994), 'A practical test for univariate and multivariate normality', Discussion Paper, Nuffield College, Oxford.

Engle, R. F. (1982), 'Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation', *Econometrica* **50**, 987–1007.

Engle, R. F. & Granger, C. W. J. (1987), 'Cointegration and error correction: Representation, estimation and testing', *Econometrica* **55**, 251–276.

Engsted, T. & Haldrup, N. (1999), 'Multicointegration in stock-flow models', *Oxford Bulletin of Economics and Statistics* **61**, 237–254.

Engsted, T. & Johansen, S. (1999), Granger's representation theorem and multicointegration, *in* R. Engle & H. White, eds, 'Cointegration, Causality and Forecasting, Festschrift in Honour of Clive Granger', Oxford University Press., Oxford.

Flavin, M. (1993), 'The excess smoothness of consumption: Identification and interpretation', *The Review of Economic Studies* **60**(3), 651–666.

Friedman, M. (1957), *A Theory of the Consumption Function*, Princeton University Press, Princeton, NJ.

Fuller, W. A. (1976), *Introduction to Statistical Time Series*, New York, Wiley.

Godfrey, L. G. (1978), 'Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables', *Econometrica* **46**, 1303–1313.

Gonzalo, J. (1994), 'Five alternative methods of estimating long-run equilibrium relationships', *Journal of Econometrics* **60**, 203–233.

Granger, C. W. J. & Lee, T. (1989), 'Investigaion of production, sales and inventory relations using multicointegration and non-symmetric error correction models', *Journal of Applied Econometrics* **4**(Supplement), S145–S159.

Granger, C. W. J. & Lee, T. H. (1991), Multicointegration, *in* R. F. Engle & C. W. J. Granger, eds, 'Long-Run Economic Relationships. Reading in Cointegration', Advanced Texts in Econometrics, Oxford University Press, Oxford.

Haldrup, N. (1999), 'A review of the econometric analysis of I(2) variables', *Journal of Economic Surveys* **12**(5), 595–650.

Hendry, D. F. & Doornik, J. A. (1999), *Empirical Econometric Modelling Using PcGive 9.0 for Windows*, 2nd edn, London: Timberlake Consultants Press.

Hendry, D. F. & von Ungern-Sternberg, T. (1981), Liquidity and inflation effects on consumers' expenditure, *in* A. S. Deaton, ed., 'Essay in the Theory and Measurement of Consumers' Behaviour', Cambridge University Press, Cambridge.

Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Advanced Texts in Econometrics, Oxford University Press, Oxford.

Keynes, J. M. (1936), *The General Theory of Employment, Interest and Money*, Macmillan, London.

Lee, T. H. (1992), 'Stock-flow relationships in US housing construction', *Oxford Bulletin of Economics and Statistics* **54**, 419–430.

Lee, T. H. (1996), 'Stock adjustment for multicointegrated series', *Empirical Economics* **21**, 63–639.

Paruolo, P. (1994), 'The role of the drift in I(2) systems', *Journal of the Italian Statistical Society* **3**(1), 93–123.

Phillips, A. W. (1954), 'Stabilization policy in the closed economy', *The Economic Journal* **64**, 290–323.

Phillips, A. W. (1957), 'Stabilization policy and the time form of lagged responses', *The Economic Journal* **67**, 265–77.

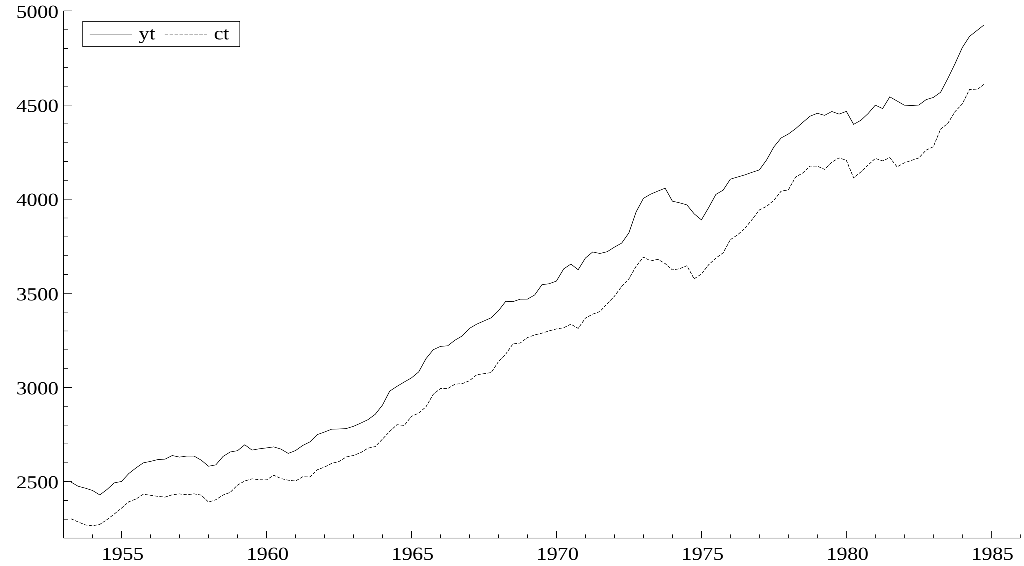
Rahbek, A., Kongsted, H. . C. & Jørgensen, C. (1999), 'Trend-stationarity in the I(2) cointegration model', *Journal of Econometrics* **90**, 265–289.

Stone, R. (1966), 'Spending and saving in relation to income and wealth', *L'industria* **4**, 471–499.

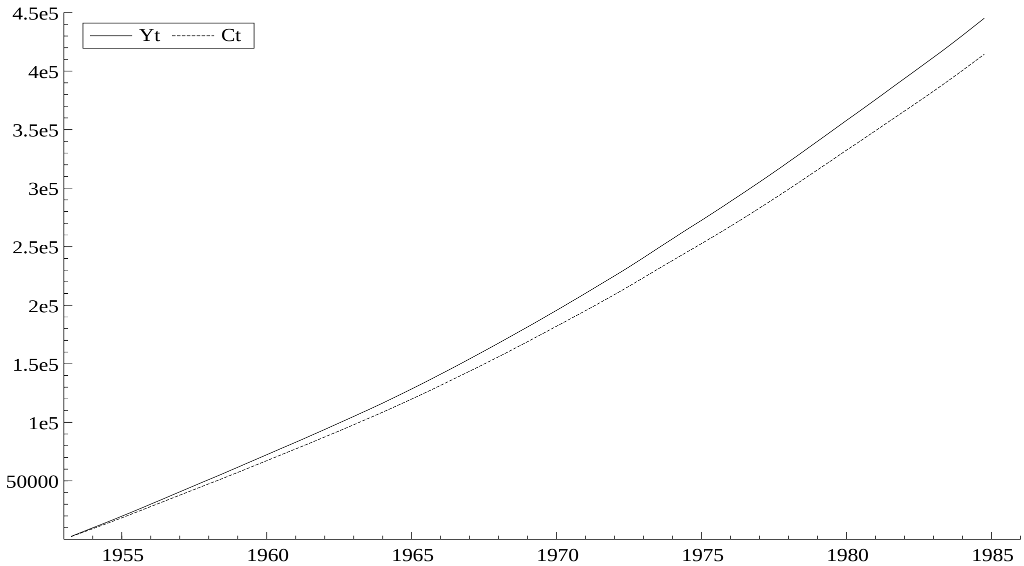
Stone, R. (1973), Personal spending and saving in postwar Britain, *in* H. C. Bos, H. Linneman & P. de Wolff, eds, 'Economic Structure and Development (Essays in Honour of Jan Tinbergen)', North-Holland Publishing Co, Amsterdam.

Vahid, F. & Engle, R. (1993), 'Common trends and common cycles', *Journal of Applied Econometrics* **8**, 341–360.

White, H. (1980), 'A heteroscedastic-consistent covariance matrix estimator and a direct test for heteroscedasticity', *Econometrica* **48**, 817–838.



(a) Total real consumption expenditure, c_t, total real disposable income, y_t, per capita values in thousands US$, seasonally adjusted.



(b) The cumulative series of c_t and y_t, denoted C_t and Y_t, respectively.

Figure 1: Data.

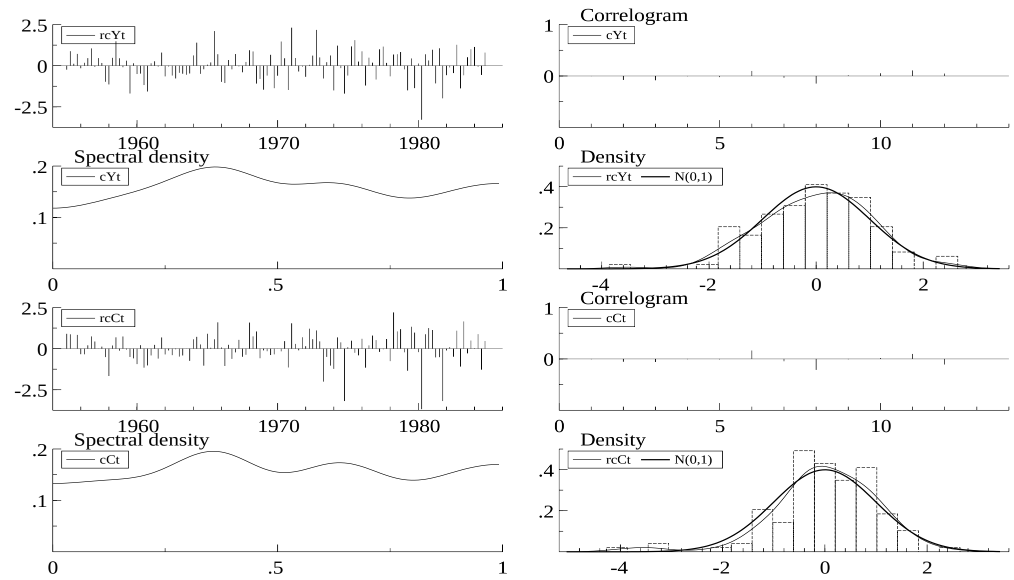


Figure 2: Estimated residuals, their correlogram, spectral density, and histogram.

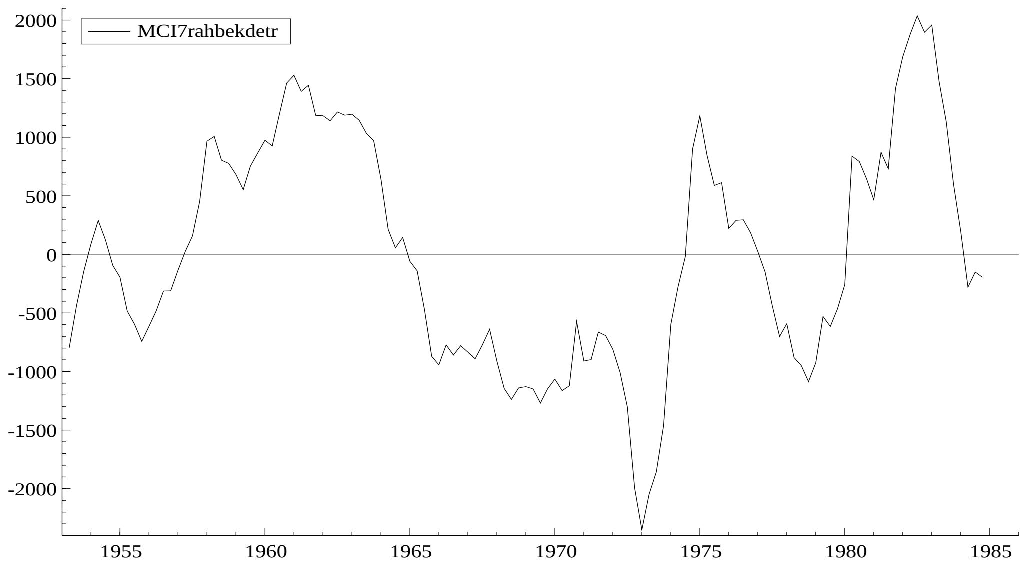


Figure 3: Rahbek et al. (1999) specification. Estimated multicointegrating relation, $\sum_{j=1}^{t}(y_j - 1.000c_j) - 5.253y_t - 5.253c_t$. Detrended with subtracted mean.

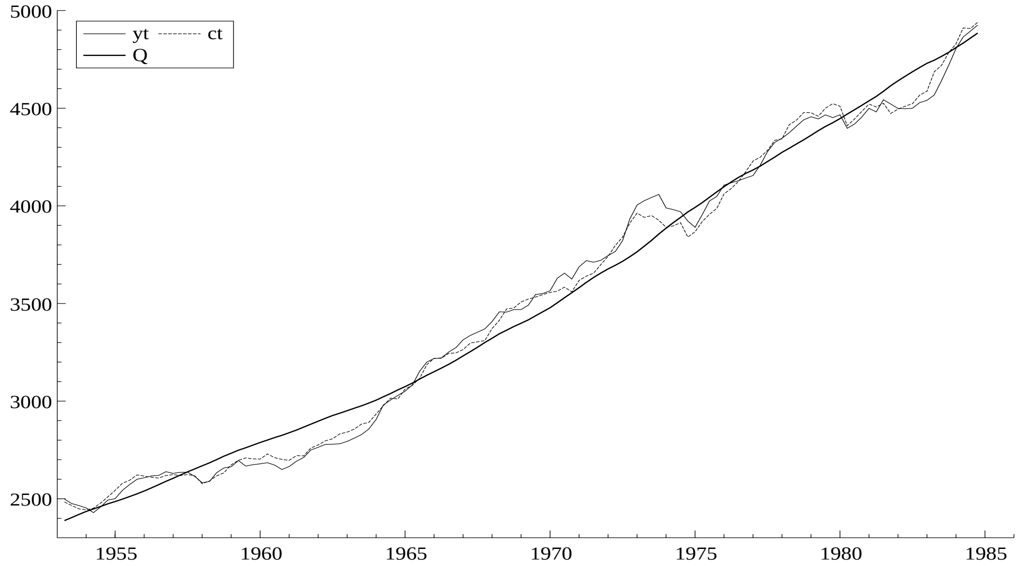


Figure 4: Total real consumption expenditure, c_t, total real disposable income, y_t, estimated stock of wealth, $Q_t = \sum_{j=1}^{t}(y_j - 1.000c_j)$. All series are adjusted to have the same mean and range.

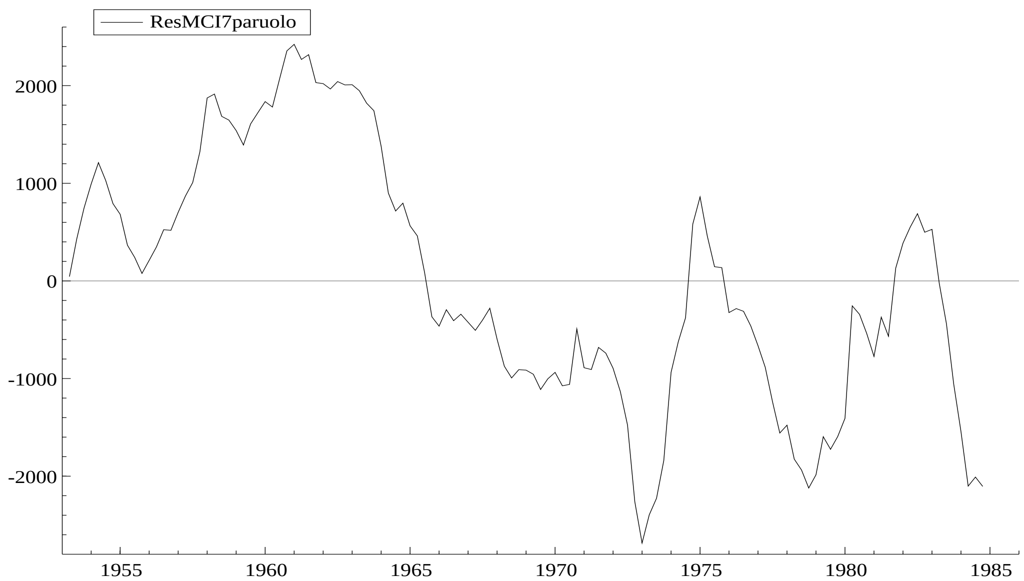


Figure 5: Paruolo (1994) specification. Estimated multicointegrating relation, $\sum_{j=1}^{t}(y_j - 1.014c_j) - 5.741y_t - 5.664c_t$. in deviations from the mean.

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