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Seasonality in Economic Models¹

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Abstract

Seasonality has been a major research area in economics for several decades. The paper assesses the recent development in the literature on the treatment of seasonality in economics, and divides it into three interrelated groups. The first group, the Pure Noise Model, consists of methods based on the view that seasonality is noise contaminating the data or more correctly contaminating the information of interest for the economists. The second group, the Time Series Models, treats seasonality as a more integrated part of the modeling strategy, with the choice of model being data driven. The third group, Economic Models of Seasonality, introduces economic theory, i.e. optimizing behavior into the modeling of seasonality.

1 Introduction

1.1 Overview

Seasonality has been a major research area in economics for several decades. Starting with Sims (1974) and Wallis (1974), and continuing with two conferences organized by Arnold Zellner in 1976 and 1981, see Zellner (1978) and Zellner (1983), the foundation was created for an upsurge in the interest of economists and econometricians in the proper treatment of seasonality within economics. The literature on the treatment of seasonality in economics can be divided into three interrelated groups. The first group, the Pure Noise Model, consists of methods based on the view that seasonality is noise contaminating the data or more correctly contaminating the information of interest for the economists - a view dating back at least to Jevons (1862) see Hylleberg (1986). The second group, the Time Series Models, treats seasonality as a more integrated part of the modelling strategy, but in a time series fashion i.e. with the choice of model being data driven. The third group, Economic Models of Seasonality, introduces economic theory, i.e. optimizing behaviour into the modeling of seasonality. Obviously, the three groups are interrelated.

The most prominent method in the first group are the methods applied to create official seasonally adjusted data published by the statistical offices. The applications in economics of seasonally adjusted time series published by the official data gathering statistical offices are widespread. The most commonly applied official seasonal adjustment procedure has for many years been the X-11 method developed at the US Bureau of the Census, see Shiskin & Musgrave (1967). The X-11 procedure is described in Hylleberg (1986), reprinted in Hylleberg (1992). The procedure has now been replaced in some places by X-12, which no doubt is a major improvement over X-11. X-12¹ is described by Findley, Monsell, Bell, Otto, and Chen, see Findley, Monsell, Bell, Otto & Chen (1998) in the *Journal of Business and Economic Statistics* 1998, which also publishes discussions by Cleveland, Maravall, Morry and Chhab, Wallis, Ghysels, and Hylleberg. In some countries and in the EU statistical office, "Eurostat" TRAMO/SEATS developed by Victor Gomez and Augustin Maravall, see Gomez & Maravall. (1996) are applied as well². In the following we will not discuss the officially applied procedures, but refer to the excellent treatment in the recent book by Ghysels and Osborn, see Ghysels & Osborn (2001)³.

A much simpler filter often applied to clean up the data in empirical econometric work is seasonal dummy variables, which are added to the regression equations, see Lovell (1963), or the seasonal difference filter applied by Box & Jenkins (1970). The seasonal differencing of Box and Jenkins assumes that there are unit roots at the seasonal frequencies in the autoregressive representation, and they recommend that the seasonal difference filter is applied until the transformed series are stationary. Tests for seasonal unit roots/seasonal integration are suggested by Dickey, Hasza & Fuller (1984), Hylleberg, Engle, Granger & Yoo. (1990), Canova & Hansen (1995), Taylor

¹X-12 can be downloaded from the Bureau of Census website <http://www.census.gov/srd/www/x12a/>

²see also <http://www.modeleasy.com/tramosea.htm>

³The text book by Ghysels and Osborn treats several of the topics discussed in the below in detail and the book is recommended for those who want a thorough introduction into these areas

(1998), and Koop & VanDijk (2000), among others, and Arteche (1998), and Arteche & Robinson (2000), Gil-Alana & Robinson (1997) has extended the analysis to fractionally integrated or seasonally long memory models.

The filtering may also take place in the frequency domain, i.e. after a Fourier transformation of the data, as in Band Spectrum Regression, suggested by Engle (1974) and Engle (1980), and discussed in Hylleberg (1977), Hylleberg (1986), and Bunzel & Hylleberg (1982). Obviously, the general idea of band spectrum regression is almost identical to the now fashionable idea of the application of band pass filters, see Baxter & King (1999). The application of band pass filters, which is especially popular in the Real Business Cycle literature, is most often done to concentrate the attention on the business cycle components, and remove the higher and lower frequency components⁴.

The second group consists of five interrelated modeling approaches. The first approach was basically suggested by Box & Jenkins (1970) and the basic model is a multiplicative extension of the ARIMA model. The extension is based on adding stationary autoregressive- and invertible moving average lagpolynomials with coefficients specified at the seasonal lags to the original model in a multiplicative way and to seasonal differences as well as first differences to render the series stationary. The second approach, the unobserved components models, specify ARIMA models for the additive trend cycle component and seasonal ARIMA models for the additive seasonal component, and may be considered a restricted version of the general seasonal ARIMA model. These so-called UCARIMA models were advocated and applied by Nerlove (1967), Nerlove, Grether & Carvalho (1979), Engle (1978), Harvey & Todd (1983), Maravall & Pierce (1987), and Harvey & Scott (1994).

A third approach is based on time varying parameter models presented and discussed in Hylleberg (1986), based on earlier work of Pesando (1972), Gersovitz & MacKinnon (1978), and Trivedi & Lee (1981). Later on the periodic autoregressive model or PAR model was introduced into econometrics by Osborn (1988), Osborn & Smith (1989), Osborn (1990), Osborn (1991), Franses (1991), Franses (1993), Franses (1994), Franses (1995), Franses (1996), and Franses & Kloek (1995).

The fourth and closely related approach is based on the evolving seasonals models, originally suggested by E.J. Terrell & Tuckwell (1970), but reintroduced into econometrics by Hylleberg & Pagan (1997) as flexible models nesting several of the seasonal time series models such as the periodic model and the seasonal unit root model. The basic idea behind the evolving seasonals models is to decouple the existence of the seasonal pattern, established via additive cosine and sine terms from the nature of the seasonal pattern, established via the time varying coefficients to the additive cosine and sine terms. The flexibility of the evolving seasonal models has recently been exploited by Koop & VanDijk (2000) to construct a Bayesian test for seasonal integration.

Finally, the fifth approach is based on the idea that seasonality should be treated in a multivariate context. The idea, due to Granger in the early eighties and forcefully presented in Engle & Granger (1987), was that although a set of timeseries is integrated and each contains a unit root at the long run frequency, a linear combination of the series may not contain such a unit root. Hylleberg et al. (1990), Engle, Granger, Hylle-

⁴In most cases the band pass filtering is done in the time domain, but the analysis and the choice of the proper filters are done in the frequency domain.

berg & Lee (1993), and Engle, Granger & Hallman (1989) extended this concept to the seasonal frequency. Later on, the theory of seasonal cointegration model has been analysed by Osborn (1993), Lee (1992), Johansen & Schaumburg (1999) and Cubadda (2000).

Cointegration may also be considered season by season leading to periodic cointegration see Birchenhal, Bladen-Howell, Chui, Osborn & Smith (1989), Boswijk & Franses (1995), Franses (1993), Franses (1996), Franses & Kloek (1995), and recently Osborn (2000) suggest a framework in which to make the choice between seasonal and periodic cointegration.

Another alternative to modeling the common seasonal characteristics of economic time series by seasonal cointegration, requiring seasonal integration, is through so-called seasonal common features introduced by Engle & Hylleberg (1996) and further developed by Cubadda (1999).

1.2 The Definition of Seasonality

In Hylleberg (1992, p. 4) seasonality in economic time series is defined as *"the systematic, although not necessarily regular, intra-year movement caused by the changes of the weather, the calendar, and timing of decisions, directly or indirectly through the production and consumption decisions made by agents of the economy. These decisions are influenced by endowments, the expectations and preferences of the agents, and the production techniques available in the economy."*

The definition stresses both the characteristic features of the seasonal components, their causes, and the economic contents.

2 Applied Seasonal Models

2.1 The Noise Models

2.1.1 Seasonal Dummies.

The use of seasonal dummy variables to filter quarterly and monthly times series data is very popular in econometric applications. The dummy variable method was promoted by Lovell (1963) and it is designed to take care of a constant stable seasonal component. The popularity of the seasonal dummy variable method is partly due to its simplicity and the flexible way it can be used. By use of the famous Frisch & Waugh (1933) result, extended by Lovell, it can be shown that the OLS coefficient estimator is the same irrespective of whether the seasonal dummies have been introduced into the regression as in the quarterly model

$$y_t = \beta_0 + x_t' \beta + \delta_1 d_{1t} + \delta_2 d_{2t} + \delta_3 d_{3t} + \varepsilon_t, t = 1, 2, \dots, T \quad (1)$$

where x_t is a vector of explanatory variables observed in period t , and d_{jt} , $j = 1, 2, 3$ is a seasonal dummy variable with a value of one for $t = j, j+4, j+8, \dots$ and otherwise zero, or whether y_t and x_t or just x_t have been seasonally adjusted by regressing them on the seasonal dummies and the constant term before running a regression using the seasonally adjusted data and no seasonal dummies.

The application of seasonal dummies may be justified in some cases, but many economic time series exhibit a changing seasonal pattern implying that the seasonal dynamics at best show up in the general dynamic specification of the model, but often are buried in the errors, see Hylleberg, Jørgensen & Sørensen (1993).

2.1.2 Seasonal Integration and Seasonal Fractional Integration.

A simple filter often applied in empirical econometric work is the seasonal difference filter $(1 - L^s)$, where s is the number of observations per year, where typically $s = 2, 4, 12$ or 52 , see Box & Jenkins (1970). The seasonal differencing of Box and Jenkins assumes that there are unit roots at all the seasonal frequencies in the autoregressive representation, and they recommend that the seasonal difference filter is applied until the transformed series is stationary. The seasonal difference filter can be written as $(1 - L^s) = (1 - L)(1 + L + L^2 + \dots + L^{s-1})$, where the long run or zero frequency unit root is in the first factor and the seasonal unit roots are in the seasonal summation filter $S(L) = (1 + L + L^2 + \dots + L^{s-1})$. The seasonal summation filter has the real root -1 if $s = 2$, the real -1 , and the two complex conjugate roots $\pm i$ if $s = 4$, and one real and five pairs of complex conjugate roots if $s = 12$ etc.

Many empirical studies have applied the so-called HEGY test developed by Hylleberg et al. (1990) [HEGY] and Engle et al. (1993) for quarterly data and extended to monthly data by Franses (1990) and Beaulieu & Miron (1993). These tests are extensions of the well known Dickey- Fuller test for a unit root at the long-run frequency Dickey & Fuller (1979). Another test where the null is no unit root at the zero frequency is suggested by Kwiatkowski, Phillips, Schmidt & Shin (1992) [KPSS] and extended to the seasonal frequencies by Canova & Hansen (1995).

The existence of seasonal unit roots in the data generating process implies a varying seasonal pattern where "summer may become winter". In most cases such a situation is not feasible and the findings of seasonal unit roots should be interpreted with care and taken as an indication of a varying seasonal pattern where the unit root model is a parsimonious approximation and not the true DGP.

Recently, Arteche (1998) and Arteche & Robinson (2000) among others have extended the analysis to seasonal long memory or fractionally integrated models, for which a seasonal fractional difference filter would be appropriate in the Box-Jenkins spirit. Their estimation methods rely heavily on previous results from the analysis of standard (non-seasonal) long memory or fractionally integrated models, see Granger & Joyeux (1980), Hosking (1981), Geweke & Porter-Hudak (1983), Robinson (1995*b*), and Robinson (1995*a*).

One source of such fractionally integrated models is the aggregation of stationary dynamic models. Granger (1981) showed that aggregating many AR(1) models with random coefficients leads to a time series with long memory. Long memory models may also be the result of aggregation over time. Recently, this idea has been extended to the seasonal case by Lildholdt (2001) who considers aggregation of stationary seasonal AR models and also has an extensive simulation study. Common examples of such aggregated models are production, price indices, and many other macroeconomic and financial time series.

Seasonal Unit Roots In the standard unit root literature a time series is said to be integrated of order d if its d 'th difference has a stationary and invertible ARMA representation. Hylleberg et al. (1990) generalized this to seasonal integration and defined a real-valued stochastic process $\{y_t, t = 0, \pm 1, \dots\}$ to be integrated of order d at frequency ω if its spectral density satisfies

$$f(\omega + \lambda) \sim g |\lambda|^{-2d} \quad \text{as } \lambda \rightarrow 0, \quad (2)$$

where g is a positive constant, the symbol " \sim " means that the ratio of the left- and right-hand side tends to 1, ω is a seasonal frequency, and d is a non-negative integer. In case of quarterly data $\omega = \left\{0, \pi, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$ where the frequency is measured in radians. When convenient the seasonal frequency may also be presented as a fraction of a total circle i.e. as a fraction of 2π , or as $\theta = \left\{0, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right\}$, hence $\omega = 2\pi\theta$. Such a series is denoted $y_t \sim I_\theta(d)$, and an example is the process $y_t = (1 - L^4) \varepsilon_t, \varepsilon_t \sim iid(0, \sigma^2)$ which is integrated of order 1 at frequencies $\theta = \left\{0, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}\right\}$.

In general, consider the autoregressive representation

$$\phi(L)y_t = \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \quad (3)$$

where $\phi(L)$ is a finite lag polynomial. Suppose $\phi(L)$ has all its roots outside the unit circle except for possible unit roots at the long-run frequency $\omega = 0$ corresponding to $L = 1$, semiannual frequency $\omega = \pi$ corresponding to $L = -1$, and annual frequencies $\omega = \left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}$ corresponding to $L = \pm i$. The standard unit root literature considers the estimation and testing of hypotheses regarding the long-run unit root $L = 1$, and much of this work has now been generalized to include the seasonal cases $L = -1$ and/or $L = \pm i$.

Dickey et al. (1984) [DHF] suggested a simple test for seasonal unit roots in the spirit of the Dickey & Fuller (1979) test for long-run unit roots. They suggested estimating the auxiliary regression

$$(1 - L^s)y_t = \phi y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim iid(0, \sigma^2) \quad (4)$$

for seasons $s = 2, 4, 12$. The DHF test statistic is the 't-value' corresponding to π , which is non-standard distributed and thus tabulated in Dickey et al. (1984). This test, however, is a joint test for unit roots at the long-run and all the seasonal frequencies, for instance the polynomial $(1 - L^4)$ can be written as $(1 - L)(1 + L)(1 - iL)(1 + iL)$ and it has the roots $L = \{\pm 1, \pm i\}$.

In order to overcome the lack of flexibility in the DHF test, Hylleberg et al. (1990) refined this idea. By use of the result that any lag polynomial of order p , $\phi(L)$, with possible unit roots at each of the frequencies $\omega = 0, \pi, [\pi/2, 3\pi/2]$ can be written as

$$\begin{aligned} \phi(L) &= \sum_{k=1}^4 \frac{\zeta_k \Delta(L)(1 - \delta_k(L))}{\delta_k(L)} + \phi^*(L)\Delta(L) \quad (5) \\ \delta_k(L) &= 1 - \frac{1}{\varsigma_k} L, \varsigma_k = 1, -1, i, -i \\ \Delta(L) &= \prod_{k=1}^4 \delta_k(L) \end{aligned}$$

where ζ_k is a constant and $\phi^*(z) = 0$ has all its roots outside the unit circle, it can be shown that in the case of a quarterly time series, (3) can be written in the equivalent form

$$\phi^*(L)y_{4t} = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \varepsilon_t \quad (6)$$

This is a generalized version of (4) where

$$\begin{aligned} y_{1t} &= (1 + L + L^2 + L^3) y_t \\ y_{2t} &= -(1 - L + L^2 - L^3) y_t \\ y_{3t} &= -(1 - L^2) y_t \\ y_{4t} &= (1 - L^4) y_t. \end{aligned} \quad (7)$$

Notice that in this representation $\phi^*(L)$ is a stationary and finite polynomial if $\phi(L)$ from (3) only has roots outside the unit circle except for possible unit roots at the long-run, semiannual, and annual frequencies.

The HEGY tests of the null hypothesis of a unit root are now conducted by simple 't-value' tests on π_1 for the long-run unit root, π_2 for the semiannual unit root, and 'F-value' tests on π_3, π_4 for the annual unit roots. As in the Dickey-Fuller and DHF cases the statistics are not t or F distributed but have non-standard distributions, which for the "t" are tabulated in Fuller (1976) while critical values for the "F" test are tabulated in Hylleberg et al. (1990). The test for the complex unit roots may also be conducted by two tests based on the "t-value" on π_3 and the "t-value" on π_4 . The "t-value" on π_3 has a distribution as the DHF test with lag 2, provided $\pi_3 = 0$. Hence the test for complex unit roots using the "t-values" on π_3 and π_4 starts by testing $\pi_4 = 0$, by the "t-value" on π_4 , which under the null has a distribution tabulated by Hylleberg et al. (1990), and continue by testing $\pi_3 = 0$ as in the DHF case. The "F-value" can be shown to be the sum of the squared "t-values" on π_3 and π_4 . The "t-tests" are almost never used and that may be due to the fact that the "t-tests" cannot be saved by augmenting with lagged values of y_{4t} in case of autocorrelation as shown by Burridge & Taylor (2001). Tests for combinations of unit roots at the seasonal frequencies are suggested by Ghysels, Lee & Noh (1994). See also Ghysels & Osborn (2001), who correctly argues that if the null hypothesis is four unit roots i.e. the proper transformation is $(1 - L^4)$, the test applied should be an "F-test" of $\pi_i, i = 1, 2, 3, 4$ all equal to zero.

As in the Dickey-Fuller case the correct lag-augmentation in the auxiliary regression (6) is crucial. The errors need to be rendered white noise in order for the size to be close to the stipulated significance level, but the use of too many lag coefficients reduces the power of the tests.

Obviously, if the data generating process, the DGP, contains a moving average component, the augmentation of the autoregressive part may require long lags, see Hylleberg (1995). As is the case for the DF test, the HEGY test may be seriously affected by moving average terms with roots close to the unit circle, but also one time jumps in the series, often denoted structural breaks in the seasonal pattern and noisy data with outliers may cause problems as shown by a number of authors, such as Franses

& T. Vogelsang (1998), Ghysels & Lee (1996), Ghysels et al. (1994), Taylor Taylor & Smith (2001), Taylor & Smith (2001), Canova & Hansen (1995), Breitung & Franses (1998), Hassler & Rodrigues (2000), Lopes and Montanes (2000), Harvey & Scott (1994), Haldrup, Montanes & Sanso (2000), Kunst & Reuter (2000) and others.

The sensitivity of the HEGY test to so-called structural breaks has led to extensions, such as the one suggested by Hassler & Rodrigues (2000). They first examine the behaviour of several seasonal unit root tests in the context of structural breaks, and show that the HEGY test and an LM variant of the HEGY test Breitung & Franses (1998) or Rodrigues (2000) are asymptotically unaffected by a finite seasonal mean shift. However, in finite samples both these tests suffer from severe size and power distortions. To correct for this Hassler & Rodrigues (2000) propose a new break corrected LM type test, which has asymptotic distributions already tabulated in the literature, but is robust to seasonal mean shifts. Furthermore, it is shown by a Monte Carlo experiment that although the test assumes the break point is known a priori, it is robust to misspecification of the break time even in finite samples.

An alternate procedure was suggested by Canova & Hansen (1995) who extended the KPSS test to the seasonal case. The KPSS test for a unit root at the zero frequency is based on a state space representation of the process, often called the structural or unobserved components model, such as

$$\begin{aligned} y_t &= \tau_t + \epsilon_t \\ \tau_t &= \tau_{t-1} + e_t \end{aligned} \tag{8}$$

where e_t is white noise with variance σ_e^2 and the errors e_t and ϵ_s are independent for all t and s . The null hypothesis of no unit root is parameterized as $H_0 : \sigma_e^2 = 0$. Canova & Hansen (1995) extend the test to the seasonal case by constructing similar models as (8) for the zero frequency unit roots to each of the seasonal unit roots, see below in Section 2.2.3.

The CH test is an LM type test based on the residuals from the auxiliary regression

$$y_\omega = G\alpha + X\beta + e \tag{9}$$

where the regressand, y_ω , $\omega = 0, \pi$, or $\pi/2$ is a transformation of the observed variable leaving only the potential unit root in the series, and the regressors are deterministic seasonal terms (G) and other non-stochastic terms (X) present under the null hypothesis. In Canova & Hansen (1995) the first difference of the series is used as the regressand in order to remove a unit root at the zero frequency, but it is shown by Hylleberg (1995) and Hylleberg & Pagan (1997) that the regressand must also be free of seasonal unit roots at frequencies other than the one being under investigation in order for the assumption that the $T \times 1$ error term to be of the form $e = u + \tau C_\omega \zeta$ where u and ζ are independent white noise errors. The known $T \times T$ matrix C_ω projects ζ into a process with a unit root at frequency ω .

Then Canova & Hansen (1995) test the hypothesis $H_0 : \tau = 0$, i.e. the null is no unit root at the frequency ω , against the alternative of a unit root, and the test statistics suggested by Canova & Hansen (1995) is

$$L_\omega = \frac{1}{T\hat{\sigma}^2} \hat{e}' C_\omega C_\omega' \hat{e} \tag{10}$$

for frequencies $\omega = 0$, or π , or $\frac{\pi}{2}$, or $\left(\frac{3\pi}{2}\right)$. Here \hat{e} are the OLS residuals from the regression (9) and $\hat{\sigma}^2$ is a consistent estimate of the long-run variance of e_t . The distribution of the L_ω -test is non-standard but depends only on the number of unit roots being tested, and is tabulated in Canova & Hansen (1995).

Besides the problems caused by the test being conditional on assumptions about the integratedness at other frequencies, the introduction of lagged dependent variables into the auxiliary regression may cause problems for testing for seasonal unit roots, see Hylleberg (1995) and Canova & Hansen (1995) unless sufficient care is exercised when choosing the specific lags in the augmentation which do not conflict with the seasonal unit roots. Specifically, the use of lag 1 when testing for a semiannual unit root may ruin the test.

Recently, some authors have begun developing an optimality theory for seasonal unit root tests. Though no uniformly most powerful test has been proposed, several attempts have been made applying other optimality criteria. The CH test is extended by Caner (1998) who uses a parametric correction for autocorrelation instead of the non-parametric correction employed by CH. Thus he is able to prove that his test is Locally Best Invariant Unbiased (LBIU). In a Monte Carlo study this property is demonstrated to hold also in finite samples as his test shows considerable power gains over the CH test. A related approach is considered by Tam & Reinsel (1997) who develops a LBIU test and a Point Optimal Invariant (POI) test for a seasonal unit root in the MA representation corresponding to seasonal overdifferencing. Thus their null hypothesis is that of seasonal trend stationarity. By simulations it is shown that the LBIU test is approximately uniformly most powerful since its power curve is very close to the power envelope.

In Hylleberg (1995) it is argued that the CH test and the HEGY test complements each other but Kunst & Reuter (2000) considering the problem of choosing between the Caner test, the CH test, and the HEGY test, using a Bayesian decision setup and Monte Carlo experiments show that the gains of such combinations over just applying the HEGY test is small in most cases.

While unit root testing in the case of semiannual and quarterly data is relatively easy to perform in practice, and doable in case of monthly observations, it is not possible in practice to handle cases where the auxiliary regressions contain more than twenty regressors as would be the case for weekly or daily data.⁵

The results of a number of studies testing for seasonal unit roots in economic data series suggest the presence of one or more seasonal unit roots, but often not all required to the application of the seasonal difference filter, $(1-L^s)$, advocated by Box & Jenkins (1970) or the application of the seasonal summation filter, $S(L)$, should be modified by applying a filter which removes the unit roots at the frequencies where they were found, and not at the frequencies where no unit roots can be detected. Another and maybe more satisfactory possibility would be to continue the analysis applying the theory of seasonal cointegration which is the subject of section 2.2.5.

⁵Notice, that sometimes the techniques are used to detect weekly effects in daily data, but here at most 7 "seasons" are considered, and not 52 or 365.

Seasonal fractional integration Recently, Arteche (1998) and Arteche & Robinson (2000) have extended the analysis to include non-integer values of d in the definition (2) of an $I_\omega(d)$ process. In particular, let $\{y_t, t = 0, \pm 1, \dots\}$ be a real-valued stochastic process with spectral density satisfying (2) for any real number $d \in \left(-\frac{1}{2}, \frac{1}{2}\right)$. Then the fractionally integrated process is said to have strong dependence or long memory at frequency ω since the autocorrelations die out at a hyperbolic rate in contrast to the much faster exponential rate in the weak dependence case. The parameter d determines the memory of the process and its parameter space $d \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ is chosen to ensure that the process is stationary and invertible, i.e. has a one-sided linear representation. If $d = 0$, then the spectral density is bounded at ω , and the process has only weak dependence. For proofs of these properties and many more, see e.g. Granger & Joyeux (1980) or Hosking (1981).

When $\omega = 0$, the process has standard long memory and when ω is a seasonal frequency the process is said to have seasonal long memory. Many estimators of the memory parameter d and the scale parameter g have been developed in the standard long memory context, Robinson (1994b), Baillie (1996), and Beran (1994) provide overviews of both theoretical and empirical results in the area of (standard) long memory processes in econometrics and time series analysis up to about 1995. Basically there are two estimation methods. The semiparametric method (developed in Geweke & Porter-Hudak (1983), Robinson (1995b) or Robinson (1995a) and later improved by Phillips (1999), Andrews & Guggenberger (2000), Shimotsu & Phillips (2000b), and Shimotsu & Phillips (2000a)) assumes only the model (2) for the spectral density and then uses a degenerating part of the periodogram around ω to estimate the model. It therefore has the advantage of being invariant to any dynamics at other frequencies, e.g. when estimating standard long memory models the estimator is invariant to short-run dynamics. Some estimators based on fully specified parametric models have been developed in the probabilistic literature (e.g. Fox & Taqqu (1986), Dahlhaus (1989), Robinson (1994a), and Nielsen (2001)), which are much more efficient using the entire sample, but will be inconsistent if the parametric model is specified incorrectly.

One of the two commonly used semiparametric estimators is the Log Periodogram Estimator originally introduced by Geweke & Porter-Hudak (1983) and extended to seasonal long memory in Porter-Hudak (1990). Taking logs in (2) and inserting sample quantities we get the approximate regression relationship

$$\ln(I(\omega + \lambda_j)) = c + -2d \ln(\lambda_j) + error \quad (11)$$

where $\lambda_j = \frac{2\pi j}{n}$ are Fourier frequencies and $I(\lambda_j) = \frac{1}{2\pi n} \left| \sum_{t=1}^n (y_t - \bar{y}) e^{it\lambda_j} \right|^2$ is the periodogram of the observed process $\{y_t, t = 1, \dots, n\}$. The estimator \hat{d} is defined as the OLS estimator in the regression (11) using $j = \pm 1, \dots, \pm m$, where $m = m(n)$ is a bandwidth number which tends to infinity as $n \rightarrow \infty$. Under suitable regularity conditions including $\{y_t\}$ being Gaussian and a restriction on the bandwidth Arteche & Robinson (2000) showed consistency and asymptotic normality of the estimator.

The Gaussian Semiparametric Estimator (or local Whittle estimator) is attractive because of its nice asymptotic properties and very mild assumptions. The estimator is

defined as the pair (\hat{g}, \hat{d}) that minimizes the (local Whittle likelihood) function

$$Q(g, d) = \frac{1}{m} \sum_{j=1}^m \left\{ \log g \lambda_j^{-2d} + \frac{\lambda_j^{2d}}{g} I(\omega + \lambda_j) \right\}. \quad (12)$$

One drawback compared to log-periodogram estimation is that numerical optimization is needed. However, this estimator does not require the Gaussianity condition and Arteche & Robinson (2000) showed that $\sqrt{m}(\hat{d} - d) \xrightarrow{d} N\left(0, \frac{1}{4}\right)$, an extremely simple asymptotic distribution facilitating easy asymptotic inference.

The problem with the semiparametric approach is that only \sqrt{m} -consistency is achieved in comparison to \sqrt{n} -consistency in the parametric case. Thus, the semiparametric approach is much less efficient than the parametric one since $\frac{m}{n} \rightarrow 0$.

A more practical difficulty with the application of long memory seasonal models or seasonally fractional models is caused by estimating several d -parameters. Even in the standard long memory model with only one d parameter, difficulties may arise, but in case of quarterly data where there are three possible d parameters, the testing procedure becomes very elaborate, with a sequence of clustered tests as in Gil-Alana & Robinson (1997).

2.1.3 Band Spectrum Regression and Band Pass Filters

A natural way to analyse time series with a strong periodic component seems to be in the frequency domain, where the time series is represented as a weighted sum of cosine and sine waves. Hence, the time series are Fourier transformed and time domain tools as autocovariance functions and crosscovariance functions are replaced by frequency domain counterparts such as spectra and cross spectra, where the spectrum is the Fourier transformation of the autocovariance functions, and the autocovariance is the inverse Fourier transformation for the spectrum⁶. In the 1960s, frequency domain analyses applying spectra etc. were considered a very promising new tool of analysis, after the introduction of spectral analyses into economics by Granger & Hatanaka (1964). However, partly due to the often short economic times it proved less advantageous to do the actual empirical analysis in the frequency domain than expected and even if some of the concepts of frequency domain analysis have been used as an analytical effective tool, the actual modeling of economic time series is often not done in the frequency domain but in the time domain.

In some cases, though, the arguments in doing the analyses in the frequency domain are only based on the lack of convenient computer programs which may handle complex series. However, today such arguments are invalid, as the most popular computer programs do handle complex variables.

In the so-called Real Business Cycle literature it has become common practice to filter out components like the trend and also components with short periods like the seasonal component and concentrate on the so-called business cycle component. This

⁶In practice, estimation of the spectra often takes place as a histogram approximation or smoothing of the periodogram across adjacent frequencies. The periodogram is obtained as the norm of the Fourier transform time series.

is done by applying Band Pass Filters, which ideally should leave only the business cycle component in the series. For a recent discussion of Band Pass Filters, see Baxter & King (1999). However, application of such filters dates back a long time, see Hannan (1960). The application of Band Spectrum Regression was further developed and analysed by Engle (1974), Engle (1980), Hylleberg (1977), Hylleberg (1986), and Bunzel & Hylleberg (1982).

The Band Spectrum Regression. Band spectrum regression is based on a frequency domain representation of the time series. Let us assume that we have data series with T observations in a $T \times 1$ vector y and a $T \times k$ matrix X related by $y = X\beta + \varepsilon$, where ε is the disturbance term and β a $k \times 1$ coefficient vector. The finite Fourier transformations of the data series are obtained by premultiplying the data matrices by a $T \times T$ matrix Ψ , with the $k + 1$ row equal to

$$\Psi_k = \frac{1}{\sqrt{T}} \left\{ 1, e^{\frac{2\pi i k}{T}}, e^{\frac{2\pi i 2k}{T}}, \dots, e^{\frac{2\pi i (T-1)k}{T}} \right\}. \quad (13)$$

The Ψ matrix is complex and a Hermitian unitary matrix, i.e. $\Psi = \Psi^@$ and $\Psi^@\Psi = I$ where $\Psi^@$ is the transposed complex conjugate matrix of Ψ . Hence, the OLS estimate of the transformed series

$$\Psi y = \Psi X \beta + \Psi \varepsilon \quad (14)$$

is then the same as the original OLS estimate. Let us premultiply the transformed model by a diagonal $T \times T$ matrix A with zeros and ones on the diagonal to obtain

$$A \Psi y = A \Psi X \beta + A \Psi \varepsilon \quad (15)$$

The effect of having zeros in the diagonal of A is to plug out the corresponding frequency components in the Fourier transformed data series⁷. Hence, by an appropriate choice of zeros in the main diagonal of A the exact seasonal frequencies⁸ i.e. in the quarterly case the $(k + 1)$ th diagonal element of A where $\frac{2\pi k}{T} = \pi/2, \pi$, and $3\pi/2$ for $k = 0, 1, \dots, T - 1$, may be filtered from the series. In case of a varying seasonal pattern, frequencies in a band around the exact seasonal frequencies may be filtered from the series as well. In order for the estimates of the coefficients to be real the A matrix must be symmetric around the other diagonal⁹.

An obvious advantage for the band spectrum regression representation is that the model in (15) lends itself directly to a test for the appropriate filtering as argued in

⁷Notice, that A must be symmetric around the southwest-northeast diagonal in order for the coefficient estimates to be real.

⁸In case only the exact seasonal frequencies i.e. π and $\pi/2$ in the quarterly case are removed OLS on (15) will produce coefficient estimates which are identical to those obtained by adding quarterly seasonal dummies to the regression equation.

⁹The fact that most regression programs are unable to handle complex variables as in (15) implies that the filtered data should be transformed back to the time domain before applying the least squares algorithm. The inverse Fourier transformation of the transformed filtered variables in the model is obtained by premultiplying (15) with Ψ .

Engle (1974). In fact the test is just the well known so-called Chow test applied to the stacked model written under the alternative as

$$\begin{bmatrix} A\Psi y \\ (I-A)\Psi y \end{bmatrix} = \begin{bmatrix} A\Psi X & 0 \\ 0 & (I-A)\Psi X \end{bmatrix} \begin{bmatrix} \beta_A \\ \beta_{I-A} \end{bmatrix} + \begin{bmatrix} A\Psi v \\ (I-A)\Psi v \end{bmatrix} \quad (16)$$

with the null hypothesis as $H_0 : \beta_A = \beta_{I-A} = \beta$.

The Band Pass Filters Most of the literature on the use of band-pass filters is connected to the Real Business Cycle literature following Hodrick & Prescott (1980), Prescott (1986), and Kydland & Prescott (1990). The focus of that literature is the business cycle component, and the ideal band-pass filter is a filter which leaves out all the components not connected to the business cycle often defined as components with periods between 4 quarters and 32 quarters.

Obviously, similar procedures could be applied to leave out only the seasonal components. The ideal seasonal band-pass filter is a filter that passes through the non seasonal components of the economic series.

Following Baxter & King (1999) let us define a polynomial in the lag operator $a(L) = \sum_{k=-K}^K a_k L^k$ which is symmetric and where weights add up to zero i.e. $a(1) = \sum_{k=-K}^K a_k = 0$.

Applying this polynomial to a zero-mean time series, y_t , we get a new time series $y_t^* = a(L)y_t = \sum_{k=-K}^K a_k y_{t-k}$. In the frequency domain the time series y_t can be expressed as an integral of random periodic components $y_t = \int_{-\pi}^{\pi} \phi(\omega) d\omega$, where the $\phi(\omega)$ are mutually orthogonal for different ω . The filtered time series can be expressed as

$$y_t^* = \int_{-\pi}^{\pi} \sum_{h=-K}^K a_h e^{-i\omega h} \phi(\omega) d\omega = \int_{-\pi}^{\pi} \alpha(\omega) \phi(\omega) d\omega \quad (17)$$

where $\alpha(\omega)$ is the frequency-response function i.e. the function indicating to what extent y_t^* responds to y_t at the frequency ω . I.e. $\alpha(\omega)$ is the weight attached to the periodic component $\phi(\omega)$. The frequency response function may be applied to design filters that isolate the specific frequencies of the series in the frequency domain.

Consider an ideal band-pass filter, that passes through only frequencies $\underline{\omega} \leq \omega \leq \bar{\omega}$. This filter will have a frequency-response function given by

$$\beta(\omega) = \begin{cases} 1 & \text{for } \underline{\omega} \leq \omega \leq \bar{\omega} \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Denoting the representation of the ideal time domain filter $b(L) = \sum_{h=-\infty}^{\infty} b_h L^h$ we get the frequency response function by taking an inverse Fourier transformation of $\beta(\omega)$ in (18) or

$$b_h = \frac{1}{2\pi} \int_{-\pi}^{\pi} \beta(\omega) e^{i\omega h} d\omega \quad (19)$$

which evaluated yields the weights

$$b_h = \begin{cases} \frac{\bar{\omega} - \omega}{\pi}, & h = 0 \\ \frac{\sin(\bar{\omega}h) - \sin(\omega h)}{h\pi}, & h = 1, 2, \dots \end{cases} \quad (20)$$

However, the ideal filter $b(L) = \sum_{h=-\infty}^{\infty} b_h L^h$ is an infinite moving average, and in practice we are forced to apply an approximate finite moving average filter such as $a(L) = \sum_{k=-K}^K a_k L^k$ with frequency response function $a_K(\omega) = \sum_{k=-K}^K a_k e^{-i\omega k}$.

If the optimization criteria are based on the quadratic $\frac{1}{2\pi} \int_{-\pi}^{\pi} |\beta(\omega) - a_K(\omega)|^2 d\omega$ it can be shown that the optimal approximate Band Pass Filter is $a(L) = \sum_{k=-K}^K b_k L^k$ for a given truncation K . Hence, the optimal approximating Band Pass Filter $a(L)$ is constructed from the ideal Band Pass filter $b(L)$ by letting the weights be equal within the truncation lag i.e. $a_k = b_k$ for $k = 0, \pm 1, \dots, \pm K$.

The effects of the truncation with lag length K are to lose $2K$ observations at each end of the sample. However, a large K implies a better approximation. In fact a small K may result in admitting substantial components just above $\bar{\omega}$ and below $\underline{\omega}$, an effect called leakage, while the frequency response may be both below the unit frequency response (compression) and above (exacerbation). A similar effect will be present at the frequencies where the frequency response is zero, ideally. Hence, there exists a trade-off between large and small K 's for a given sample size T .

Obviously, the frequency components in the interval $\underline{\omega} \leq \omega \leq \bar{\omega}$ may be removed, and the frequency components outside be contained by applying the filter $1 - b(L)$, or rather $1 - a(L)$, instead of $a(L)$. In addition, a proper combination of Band Pass filters may remove frequency components around some or all of the seasonal frequencies.

Whether one applies filtering in the frequency domain as in Band Spectrum Regression or in the time domain as in Band Pass filtering the actual success of the filtering depends on the choice of bandwidth or K , and the spectral characteristics of the series at hand. Even a small leakage in the filter may give rise to severe disturbances if the spectrum of the filtered series has mass at particular frequencies.

2.2 The Time Series Models

2.2.1 The Box-Jenkins model.

In the traditional analysis of Box and Jenkins see Box & Jenkins (1970) the time series were made stationary by application of the filters $(1 - L)$ and/or $(1 - L^s) = (1 - L)(1 + L + L^2 + L^3 + \dots + L^{s-1})$ as many times as was deemed necessary from the form of the resulting autocorrelation function. After having obtained stationarity the filtered series were modelled as an Autoregressive Moving Average model or ARMA model. Both the AR and the MA part could be modelled as consisting of a non seasonal and seasonal lag polynomial. Hence, the so-called Seasonal ARIMA model has the form

$$\phi(L)\phi_s(L^s)(1 - L^s)^D(1 - L)^d y_t = \theta(L)\theta_s(L^s)\varepsilon_t \quad (21)$$

where $\phi(L)$ and $\theta(L)$ are invertible lag polynomials in L , while $\phi_s(L^s)$ and $\theta_s(L^s)$ are invertible lag polynomials in L^s .

In light of the results mentioned in the section on seasonal unit roots the modeling strategy of Box and Jenkins may easily be refined to allow for situations where the nonstationarity exists only at some of the seasonal frequencies.

The model in (21) lends itself to a straightforward extension to the multivariate case, but unless constraints are invoked the model will not be identified in the traditional econometric sense of the word. In Zellner & Palm (1974) the Box-Jenkins model and the traditional econometric modeling techniques are combined and Plosser (1978) extends the approach to the seasonal case. The simultaneous modeling of both the seasonal and the nonseasonal components applying time series as well as econometric techniques is further developed in Hylleberg (1986), which also contains a long list of references.

2.2.2 The "Structural" or Unobserved Components Model.

When modeling processes with seasonal characteristics, complicated and high ordered polynomials must be applied in the ARMA representation, see Hylleberg (1992). As an alternative to this the Unobserved Components (UC) model was proposed. The model can in its most general form be specified as

$$Y_t = \mu_t + \gamma_t + \varepsilon_t, \quad (22)$$

where μ_t is the trend-cycle component and γ_t the seasonal component, and ε_t is the irregular component. It is assumed that the μ_t and γ_t can be modelled as two distinct ARMA processes

$$\begin{aligned} A_C(L) \mu_t &= B_C(L) v_t \quad \text{and} \\ A_S(L) \gamma_t &= B_S(L) w_t \end{aligned} \quad (23)$$

where the processes v_t , w_t and ε_t are assumed to be independent, serially uncorrelated processes with zero means and variances σ_v^2 , σ_w^2 and σ_ε^2 . This class of model is also called Unobserved Components Autoregressive Integrated Moving-Average Models (UCARIMA) by Engle (1978).

Substituting (23) into (22) it is seen that the UC model equivalently can be specified as

$$\begin{aligned} A_C(L) A_S(L) X_t &= B_C(L) A_S(L) v_t \\ &+ B_S(L) A_C(L) w_t \\ &+ A_C(L) A_S(L) \varepsilon_t. \end{aligned} \quad (24)$$

From this, and the results of Granger & Morris (1976) it is seen that the UCARIMA model is a general ARIMA model with restrictions on the parameters. The restrictions may be derived from the estimated parameters of the unconstrained ARIMA model. Alternatively, the UC model may be specified as a so-called structural model following Harvey (1993).

The structural approach is based on a very simple and quite restrictive modeling of the components of interest such as trends, seasonals and cycles. The model is often

specified as (22) The trend is normally assumed only to be stationary in first or second differences whereas γ_t is stationary when multiplied by the seasonal summation operator. In the Basic Structural Model (BSM) the trend is specified as

$$\begin{aligned}\mu_t &= \mu_{t-1} + \beta_{t-1} + \eta_t \\ \beta_t &= \beta_{t-1} + \zeta_t\end{aligned}\tag{25}$$

where each of the error terms is independent and normally distributed.¹⁰ The seasonal component is specified as

$$S(L)\gamma_t = \sum_{j=0}^{n-1} \gamma_{t-j} = w_t\tag{26}$$

where s is the number of periods per year and where $w_t \sim N(0, \sigma_w^2)$.^{11,12} The BSM model can also be written as

$$y_t = \frac{\zeta_t}{\Delta^2} + \frac{w_t}{S(L)} + \varepsilon_t,\tag{27}$$

where $\zeta_t = \eta_t - \eta_{t-1} + \zeta_{t-1}$ is equivalent to an MA(1) process. Expressing the model in the form (27) makes the connection to the UCARIMA model in (24) clear.

Estimation of the general UC model is treated in Hylleberg (1986) and follows the same lines as the Box-Jenkins modeling procedure. Thus, the modeling procedure may be criticized on similar grounds as the general ARIMA models.

The statistical treatment in the structural approach is based on the state space formulation and the problems of specifying the ARMA models for the components is avoided by a priori restrictions. Harvey & Scott (1994) argue that the type of model above which has a seasonal component evolving relatively slowly over time can fit most economic time series. Nonetheless, the model presuppose a trend component with a unit root and a seasonal component with all possible seasonal unit roots present¹³.

2.2.3 The Time Varying Parameter Models and the Periodic Models

The periodic model extends the nonperiodic time series models by allowing the parameters to vary with the seasons. The periodic autoregressive (PAR) model assumes that the observations in each of the seasons can be described using different autoregressive models, and the same goes for the periodic extensions to the MA and ARMA models. Most of the research undertaken so far has focused on PAR models Franses (1996).

¹⁰If $\sigma_\zeta^2 = 0$ this collapses to a random walk plus drift. If $\sigma_\eta^2 = 0$ as well it corresponds to a model with a linear trend.

¹¹This specification is known as the dummy variable form, since it reduces to a standard deterministic seasonal component if $\sigma_w^2 = 0$.

¹²Specifying the seasonal component this way makes it slowly changing by a mechanism that ensures that the sum of the seasonal components over any s consecutive time periods has an expected value of zero and a variance that remains constant over time.

¹³The consumption function advocated by Harvey & Scott (1994) and found using the structural approach is identical to a consumption function obtained by the seasonal cointegration approach provided a common cointegration vector applies at the zero frequency and at all the seasonal frequencies.

Consider a quarterly times series y_t which is observed for N years. The PAR(h) model can be written as

$$y_t = \mu_s + \phi_{1s}y_{t-1} + \dots + \phi_{ps}y_{t-h} + \varepsilon_t \quad (28)$$

for $s = 1, 2, 3, 4$ and $t = 1, 2, \dots, T = 4N$, or as

$$y_t = \sum_{s=1}^4 \mu_s D_{s,t} + \sum_{s=1}^4 \phi_{1s} D_{s,t} y_{t-1} + \dots + \sum_{s=1}^4 \phi_{hs} D_{s,t} y_{t-h} + \varepsilon_t \quad (29)$$

where $D_{s,t}$ are seasonal dummies. The model may be estimated by Maximum Likelihood or OLS but under normality of the error process and with fixed starting values the parameter estimates are equal. Testing for periodicity in (29) amounts to testing the following hypothesis $H_0 : \phi_{is} = \phi_i$ for $s = 1, 2, 3, 4$ and $i = 1, 2, \dots, p$. This hypothesis can be tested with a likelihood ratio test which is asymptotically χ^2_{3p} under the null, irrespective of any unit roots in y_t , see Boswijk & Franses (1995). The order of the PAR(p) model can be found using an information criterion or using a general-to-specific approach on (29).

It has been shown that any PAR model can be described by a non-periodic ARMA model Osborn (1991). In general, however, the orders will be higher than in the PAR model. For example, a PAR(1) corresponds to a non-periodic ARMA(4,3) model. Furthermore, it has been shown that estimating a non-periodic model when the true DGP is a PAR can result in a lack of ability to reject the false non-periodic model Franses (1996). Fitting a PAR model does not prevent the finding of a non-periodic AR process, if the latter is in fact the DGP. In practice it is thus recommended that one starts by selecting a PAR(p) model and then tests whether the autoregressive parameters are periodically varying using the method described above.

A major weakness of the periodic model is that the available sample for estimation $N = n/s$ where s is the number of seasons, can be small. Furthermore, the identification of a periodic time series model is not as easy as it is for non periodic models since the periodic models have similarities with vector AR(MA) models. However, the models are easily estimated under the assumptions above, and standard type tests can be used to test the specification.

The periodic models can be considered special cases of what is referred to as the Time-Varying Parameter models, see Hylleberg et al. (1990).¹⁴ These are regression models of the form

$$Y_t = X_t' \beta_t + u_t$$

with seasonally varying coefficients, which in the most general form are specified as

$$B(L) (\beta_t - \bar{\beta}) = A\gamma_t + \xi_t \quad (30)$$

¹⁴In Hylleberg & Pagan (1997) it is also noted that the PAR models impose certain restrictions on the evolving seasonals model.

This model can be written in state-space form and estimated using the Kalman filter. A special case of (30) is the systematic nonrandom varying parameter model where

$$\beta_t = A\gamma_t$$

This model, in principle, constitutes no estimation problems if γ_t is known and the number of observations for each season is large.¹⁵

However, γ_t is seldom known and the number of observations for each season may often be small. The latter problem can be addressed by restricting the parameters. A sensible assumption is that the parameters varies smoothly over the seasons. This assumption was used by Gersovitz & MacKinnon (1978) applying Bayesian techniques. An alternative way to smooth the variation in the coefficients consists of restricting them to lie along low-ordered polynomials. Estimation of this model can be done using a method like the one suggested by Almon (1965) for distributed lag models.

2.2.4 The Evolving Seasonals Model

The evolving seasonals model was promulgated by Hannan in several articles in the 1960s, see E.J. et al. (1970). The model was revitalized by Hylleberg & Pagan (1997) and used to nest many of the most commonly applied seasonal models. Recently, the model has been used by Koop & VanDijk (2000) to analyze seasonal models from a Bayesian perspective.

The evolving seasonals model for a quarterly time series is based on a representation like

$$\begin{aligned} y_t &= \alpha_{1t} \cos(\lambda_1 t) + \alpha_{2t} \cos(\lambda_2 t) + 2\alpha_{3t} \cos(\lambda_3 t) + 2\alpha_{4t} \sin(\lambda_3 t) & (31) \\ y_t &= \alpha_{1t} + \alpha_{2t} \cos(\pi t) + 2\alpha_{3t} \cos(\pi t/2) + 2\alpha_{4t} \sin(\pi t/2) \\ y_t &= \alpha_{1t}(1)^t + \alpha_{2t}(-1)^t + \alpha_{3t}[i^t + (-i)^t] + \alpha_{4t}[i^{t-1} + (-i)^{t-1}] \end{aligned}$$

where $\lambda_1 = 0$, $\lambda_2 = \pi$, $\lambda_3 = \pi/2$, $\cos(\pi t) = (-1)^t$, $2 \cos(\pi t/2) = [i^t + (-i)^t]$, $2 \sin(\pi t/2) = [i^{t-1} + (-i)^{t-1}]$, $i^2 = -1$, while α_{jt} , $j = 1, 2, 3, 4$ is a linear function of its own past, and a stochastic term, e_{jt} , $j = 1, 2, 3, 4$. For instance

$$\begin{aligned} \alpha_{1t} &= \rho_1 \alpha_{1,t-1} + e_{1t}, & (32) \\ \alpha_{2t} &= \rho_1 \alpha_{2,t-1} + e_{2t}, \\ \alpha_{3t} &= \rho_3 \alpha_{3,t-2} + e_{3t}, \\ \alpha_{4t} &= \rho_4 \alpha_{4,t-3} + e_{4t} \end{aligned}$$

In such a model, $\alpha_{1t}(1)^t = \alpha_{1t}$ represents the trend component with the unit root at the zero frequency, $\alpha_{2t}(-1)^t$ represents the semiannual component with the root -1 , while $\alpha_{3t}[i^t + (-i)^t] + \alpha_{4t}[i^{t-1} + (-i)^{t-1}]$ represents the annual component with the complex conjugate roots $\pm i$. In Hylleberg & Pagan (1997) it is shown that the HEGY

¹⁵The same thing goes for the PAR model, for which γ_t is equal to seasonal dummies and where X_t consists of lagged values of the dependent variable.

auxiliary regression in (6) rewritten as $(1 - L^4)y_t = D(L)\varepsilon_t = \sum_{h=0}^{\infty} d_h \varepsilon_{t-h}$ has an evolving seasonals model representation with

$$e_{1t} = \frac{\varepsilon_t}{4} D(1) = \frac{\varepsilon_t}{4} \sum_h d_h \quad (33)$$

$$e_{2t} = \frac{\varepsilon_t}{4} D(-1)(-1)^t = \frac{(-1)^t \varepsilon_t}{4} \sum_h d_h (-1)^h \quad (34)$$

$$e_{3t} = e_{4t} = \frac{\varepsilon_t}{2(a_t + b_t)} \sum_h (-1)^h d_{2*h} \quad (35)$$

$$+ \frac{\varepsilon_{t-1}}{2} \sum_h (-1)^h d_{2*h+1}$$

where $a_t = i^t + (-i)^t$ and $b_t = i^{t-1} + (-i)^{t-1}$.

From (31) to (35) it is seen that the HEGY auxiliary regression is an evolving seasonals model with $\alpha_{3t} = \alpha_{4t}$ and the errors in the models for α_{jt} , $j = 1, 2, 3$ perfectly correlated. A unit root at a given frequency implies that the corresponding ρ_j , $j = 1, 2, 3$ is one. However, as α_{jt} , $j = 1, 2, 3$ are not observed they must be estimated. Hylleberg & Pagan (1997) shows that

$$\alpha_{1t} = \frac{y_{1t}}{4} + I(0) \quad (36)$$

$$\alpha_{2t} = \frac{-y_{2t}}{4(-1)^t} + I(0)$$

$$\alpha_{3t} = \frac{-y_{3t}}{2(a_t + b_t)} + I(0)$$

where y_{jt} , $j = 1, 2, 3$ is defined in Section 2.1.3 and $I(0)$ means a stationary error. Inserting these expressions in(32) produces

$$(1 - L)y_{1t} = y_{4t} = \pi_1 y_{1,t-1} + I(0) \quad (37)$$

$$(1 + L^2)y_{2t} = -y_{4t} = \pi_2 (-y_{2,t-1}) + I(0)$$

$$(1 + L^2)y_{3t} = -y_{4t} = \pi_3 (-y_{3,t-2}) + I(0)$$

where $\pi_j = \rho_j - 1$, $j = 1, 2, 3$.

As the terms in the HEGY auxiliary regression are uncorrelated one may perform the HEGY test by three regressions $y_{4t} = \pi_1 y_{1,t-1} + I(0)$, $y_{4t} = \pi_2 y_{2,t-1} + I(0)$, and $y_{4t} = \pi_3 y_{3,t-2} + I(0)$ and testing null of $\pi_j = \rho_j - 1 = 0$, $j = 1, 2, 3$ against the alternative that the π_j 's are less than zero. For $j=3$ the test assumes that $\pi_4 = 0$ i.e. $y_{3,t-1}$ is not in the HEGY regression¹⁶. But from (37) above it is seen that

¹⁶The HEGY "F" test is based on a regression of the form

$$y_{4t} = \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} + I(0)$$

where the null is that $(1 + L^2)y_{3t}$ is stationary and the alternative is that is that the model for y_{3t} has the form $(1 \pm \gamma_1 L + \gamma_2 L^2)y_{3t} = error$.

these regressions are exactly the regressions produced by the evolving seasonals model provided the errors (??) are perfectly correlated.

The Canova-Hansen test may also be presented in the framework of the evolving seasonals model as shown by Hylleberg & Pagan (1997). Rewriting (36) and adding (32) with $\rho_j = 1, j = 1, 2, 3$ produces three state space models

$$\begin{aligned}
 x_{1t} &= \frac{y_{1t}}{4} = \alpha_{1t} + I(0) & (38) \\
 \alpha_{1t} &= \alpha_{1,t-1} + e_{1t} \\
 x_{2t} &= \frac{-y_{2t}}{4(-1)^t} = \alpha_{2t} + I(0) \\
 \alpha_{2t} &= \alpha_{2,t-1} + e_{2t} \\
 x_{3t} &= \frac{-y_{3t}}{2(a_t + b_t)} = \alpha_{3t} + I(0) \\
 \alpha_{3t} &= \alpha_{3,t-1} + e_{3t}
 \end{aligned}$$

Applying a KPPS procedure as in Section 2.1.3. to each of the three state space models implies seasonal unit root tests which coincide with the Canova-Hansen tests under their assumption that no other root exists in the data than possibly at the frequency under consideration. However, applying the KPPS procedure directly to (38) has the advantage that unit root at the other frequencies does not matter, as shown by Hylleberg & Pagan (1997).

The PAR(p) may also be developed within the evolving seasonals model. Consider a simple version of (29) such as

$$y_t = \sum_{s=1}^4 \phi_s D_{s,t} y_{t-1} + \varepsilon_t \quad (39)$$

As $D_{s,t}$ is a seasonal dummy variable taking the value 1 in the s 'th quarter and zero elsewhere we can write the seasonal dummies as

$$\begin{aligned}
 D_{1,t} &= [1 + b_t - (-1)^t]/4 & (40) \\
 D_{2,t} &= [1 - a_t + (-1)^t]/4 \\
 D_{3,t} &= [1 - b_t - (-1)^t]/4 \\
 D_{4,t} &= [1 + a_t + (-1)^t]/4
 \end{aligned}$$

and it can be shown, see Hylleberg & Pagan (1997) that (39) implies an evolving sea-

sonals models such as (31) with $\alpha_{3t} = \alpha_{4t}$ and

$$\alpha_{1t} = \frac{\kappa_1 + \kappa_2 a_t + \kappa_3 b_t + \kappa_4 (-1)^t}{4} \alpha_{1,t-1} + \frac{\varepsilon_t}{4} \quad (41)$$

$$\alpha_{2t} = \frac{-\kappa_4 - \kappa_2 a_t + \kappa_3 b_t - \kappa_1 (-1)^t}{4} \alpha_{2,t-1} + \frac{(-1)^t \varepsilon_t}{4}$$

$$\alpha_{3t} = \frac{\kappa_4 - \kappa_2 a_t - \kappa_3 b_t - \kappa_1 (-1)^t}{4} \alpha_{3,t-1} + \frac{(-1)^t \varepsilon_t}{2(a_t + b_t)}$$

$$\kappa_1 = \phi_1 + \phi_2 + \phi_3 + \phi_4, \kappa_2 = -\phi_2 + \phi_4 \quad (42)$$

$$\kappa_3 = \phi_1 - \phi_3, \kappa_4 = -\phi_1 + \phi_2 - \phi_3 + \phi_4 \quad (43)$$

Hence, it is shown that several of the most popular seasonal models may be represented in the context of an evolving seasonals models¹⁷.

Koop & VanDijk (2000) applies the evolving seasonals model (31) to nest the HEGY and CH auxiliary regressions as

$$\phi^*(L) y_{4t} = \pi_1 y_{1,t-1} + \pi_2 y_{2,t-1} + \pi_3 y_{3,t-2} + \pi_4 y_{3,t-1} \quad (44)$$

$$+ \alpha_{1t} + \alpha_{2t} \cos(\pi t) + 2\alpha_{3t} \cos(\pi t/2) + 2\alpha_{4t} \sin(\pi t/2) + \varepsilon_t$$

$$\alpha_{jt} = \alpha_j + a_{j,t-1} + e_{jt}, j = 1, 2, 3, 4$$

$$\sigma_j^2 = \text{var}(e_{jt}), j = 1, 2, 3, 4 \quad (45)$$

where a drift term has been added to the state equations¹⁸

2.2.5 Seasonal Cointegration, Periodic Cointegration and Common Seasonal Features

The idea that the seasonal components of a set of economics time series are driven by a smaller set of common seasonal features seems a natural extension of the idea that the trend components of a set of economic time series are driven by common trends. In fact, the whole business of seasonal adjustment may be interpreted as an indirect approval of such a view.

If the seasonal components are integrated the idea immediately leads to the concept of seasonal cointegration, introduced in the papers by Engle et al. (1989), Hylleberg et al. (1990), and Engle et al. (1993). In case the seasonal components are stationary the idea leads to the concept of seasonal common features see Engle & Hylleberg (1996), while so-called periodic cointegration, see Birchenhal et al. (1989), Boswijk & Franses (1995), Franses (1993), Franses & Kloek (1995), Franses (1996), and Osborn (2000) consider cointegration season by season.

¹⁷From the evolving seasonals model representation of the PAR model it is seen that the PAR model assumes that the seasonals and also the stochastic trend evolve as a periodic model.

¹⁸In their empirical work the only drift term allowed is in the state equation containing α_{1t} .

Seasonal Cointegration Seasonal cointegration exists at a particular seasonal frequency if, at least one linear combination of series seasonally integrated at the particular frequency, is integrated of a lower order. For ease of exposition we will concentrate on quarterly time series integrated of order 1, but the theory is easily extended to daily, weekly or monthly data and to higher orders of integration. Quarterly time series may have unit roots at the annual frequency $\pi/2$ with period 4 quarters, or at the semiannual frequency π with period 2 quarters, and at the long run frequency 0. The cointegration theory at the semiannual frequency where the root on the unit circle is real is a straightforward extension of the cointegration theory at the long run frequency. However, the complex unit roots at the annual frequency leads to the concept of polynomial cointegration where cointegration exists if one can find at least one linear combination including a lag of the seasonally integrated series which is stationary.

In Hylleberg et al. (1990) and Engle et al. (1993) seasonally cointegration was analysed along the path set up in Engle & Granger (1987). Consider the quarterly VAR model

$$\Pi(L)X_t = \varepsilon_t, t = 1, 2, \dots, T \quad (46)$$

where $\Pi(L)$ is an pxp matrix of lag polynomials of finite dimension, X_t a $px1$ vector of observations on the demeaned variables, while, while the $px1$ disturbance vector is $\varepsilon_t \sim NID(0, \Omega)$. Under the assumptions that the p variables are integrated at the frequencies 0, $\pi/2$ ($3\pi/2$), and π , and that cointegration exists at these frequencies as well, the VAR model can be rewritten as seasonal error correcting model

$$\begin{aligned} \Phi(L)X_{4t} &= \Pi_1 X_{1,t-1} + \Pi_2 X_{2,t-1} + \Pi_3 X_{3,t-2} + \Pi_4 X_{3,t-1} + \varepsilon_t \\ \Pi_1 &= \alpha_1 \beta'_1, \Pi_2 = \alpha_2 \beta'_2, \Pi_3 = \alpha_4 \beta'_4 - \alpha_3 \beta'_3, \Pi_4 = \alpha_4 \beta'_3 + \alpha_3 \beta'_4 \end{aligned} \quad (47)$$

where the transformed $px1$ vectors $X_{j,t}$, $j = 1, 2, 3, 4$ are defined as in (7), and where $Z_{1t} = \beta'_1 X_{1t}$, and $Z_{2t} = \beta'_2 X_{2t}$ contain the cointegrating relations at the long run and semiannual frequencies, respectively, while $Z_{3t} = (\beta'_3 + \beta'_4 L)X_{3t}$ contains the polynomial cointegrating vectors at the annual frequency. In Engle et al. (1993) seasonal and non-seasonal cointegrating relations were analyzed between the Japanese consumption and income estimating the relations for Z_{jt} , $j = 1, 2, 3$ in the first step following the Granger-Engle two step procedure.

The well known drawbacks of this method, especially when the number of variables included exceeds 2, is partly overcome by Lee (1992) who extended the Maximum Likelihood based methods of Johansen (1988) for cointegration at the long run frequency to cointegration at the semiannual frequency π .

To adopt the ML based cointegration analyses at the annual frequency $\pi/2$ with the complex pair of unit roots $\pm i$, is somewhat more complicated, however.

To facilitate the analysis a slightly different formulation of the seasonal error correcting model is given in Johansen & Schaumburg (1999). In our notation the formu-

lation is

$$\begin{aligned}
\Phi(L)X_{4t} &= \alpha_1\beta'_1 X_{1,t-1} + \alpha_2\beta'_2 X_{2,t-1} + \alpha_*\beta'_* X_{*,t} + \alpha_{**}\beta'_{**} X_{**,t} + \varepsilon_t \quad (48) \\
2\alpha_* &= \alpha_3 + i\alpha_4, 2\alpha_{**} = \alpha_3 - i\alpha_4, \beta_* = \beta_3 + i\beta_4, \beta_{**} = \beta_3 - i\beta_4 \\
X_{*,t} &= (X_{t-2} - X_{t-4}) + i(X_{t-1} - X_{t-3}) = -X_{3,t-2} - iX_{3,t-1} \\
X_{**,t} &= (X_{t-2} - X_{t-4}) - i(X_{t-1} - X_{t-3}) = -X_{3,t-2} + iX_{3,t-1}
\end{aligned}$$

The formulation in (48), which is just a reformulation of (47)¹⁹, writes the error correcting model with two complex cointegrating relations $Z_{*,t} = \beta'_* X_{*,t}$ and $Z_{**,t} = \beta'_{**} X_{**,t}$ corresponding to the complex pair of roots $\pm i$.

Notice, (47) and (48) shows the isomorphi between polynomial lags and complex variables. The general results may be found in Johansen & Schaumburg (1999) and Cubadda (2000). The relation between the cointegration vector β_m and polynomial cointegration vector $\beta_m(L)$ is

$$\beta_m(L) = \begin{cases} \beta_m & \text{if } \omega_m = 0, \pi \\ [\text{Re}(\beta_m) - \text{Im}(\beta_m)] \frac{\cos(\omega_m)L - 1}{\sin(\omega_m)} & \text{for } \omega_m \in (0, \pi) \end{cases} \quad (49)$$

Brillinger (1981) extend the canonical correlation analysis to the case of complex variables and illustrates it similarities. Based on these results Cubadda (2000) then applies the usual Johansen approach based on canonical correlations to obtain tests for cointegration at all the frequencies of interest i.e. at the frequencies 0 and π with the real unit roots ± 1 and at the frequency $\pi/2$ with the complex roots $\pm i$.

Hence for each of the frequencies of interest the likelihood function is concentrated by a regression of X_{4t} and $X_{1,t-1}, X_{2,t-1}$ or the complex pair $(X_{*,t}, X_{**,t})$ on the other regressors, resulting in the complex residual matrices $U_{*,t}$ and $V_{*,t}$ with complex conjugates $U_{**,t}$ and $V_{**,t}$ respectively. After having purged X_{4t} and $X_{1,t-1}, X_{2,t-1}$ or the complex pair $(X_{*,t}, X_{**,t})$ for the effects of the other regressors the cointegration analysis is based on a canonical correlation analysis of the relations between $U_{*,t}$ and $V_{*,t}$. The product matrices are $S_{UU} = T^{-1} \sum_{t=1}^T U_{*,t} U'_{**,t}$, $S_{VV} = T^{-1} \sum_{t=1}^T V_{*,t} V'_{**,t}$, and $S_{UV} = T^{-1} \sum_{t=1}^T U_{*,t} V'_{**,t}$ and the Trace test of r or more cointegrating vectors is found as $TR = -2T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i)$ where $\hat{\lambda}_1 > \hat{\lambda}_2 > \dots > \hat{\lambda}_p$ are the ordered eigenvalues of the problem

$$\left| \lambda S_{VV} - S_{VU} S_{UU}^{-1} S_{UV} \right| = 0 \quad (50)$$

The corresponding (possibly) complex eigenvectors properly normalized are $v_j, j = 1, 2, \dots, p$ where the first r vectors forms the cointegrating matrix β .

Critical values of the trace tests for the complex roots are supplied by Johansen & Schaumburg (1999) and Cubadda (2000), while the critical values for cointegration at the real root cases are found in Lee (1992) and Osterwald-Lenum (1992).

Furthermore, tests of linear hypothesis on the polynomial cointegration vectors may also be executed as χ^2 test, similar to the test applied in the long run cointegration case.

¹⁹(47) is obtained from (48) by inserting the definitions of $\alpha_*, \beta_*, X_{*,t}$, and their complex conjugates $\alpha_{**}, \beta_{**}, X_{**,t}$, and order the terms.

Periodic cointegration In order to present some of the important concepts applied in the literature, see Franses (1996), Osborn (2000), and Ghysels & Osborn (2001) let us define the observations of the s quarter in year τ , $s = 1, 2, 3, 4$, $\tau = 1, 2, \dots, N$ as $y_{s\tau}$ while the 4×1 vector $Y_\tau = (y_{1\tau}, y_{2\tau}, y_{3\tau}, y_{4\tau},)$ contain the observations from year τ . Consider the VAR model

$$\Delta_4 Y_\tau = \Pi Y_{\tau-1} + \sum_{j=1}^{k-1} \Phi_j \Delta_4 Y_{\tau-j} + U_\tau, \quad (51)$$

$$U_\tau \sim NID(0, \Omega_U)$$

where Π and Φ_j are 4×4 coefficient matrices, Ω_U a 4×4 covariance matrix, and $\Delta_4 = (1 - L^4)$ where L operates on the seasonal index s and not on τ . Hence

$$L^m y_{s\tau} = \begin{cases} y_{s-m, \tau}, & m = 0, 1, 2, 3 \\ y_{s-m+4, \tau-1}, & m = 4, 5, 6, 7 \\ y_{s-m+8, \tau-2}, & m = 8, 9, 10, 11 \\ \text{etc} \end{cases}$$

The VAR model in (51) is written in error correction form and the number of cointegrating relations between the 4 series, one for each quarter, is determined by the rank of Π . Following Osborn (2000), we then have the following three definitions:

y_t , $t = 1, 2, \dots, T = 4N$ is *Integrated*, $y_t \sim I(1)$, if $\text{rank}(\Pi) = 3$ and the three cointegrating relations are $y_{2\tau} - y_{1\tau}$, $y_{3\tau} - y_{2\tau}$, and $y_{4\tau} - y_{3\tau}$ i.e. the quarterly changes are the cointegrating relations.

y_t , $t = 1, 2, \dots, T = 4N$ is *Periodically Integrated*, $y_t \sim PI(1)$, if $\text{rank}(\Pi) = 3$ and the three cointegrating relations are $y_{2\tau} - \beta_1 y_{1\tau}$, $y_{3\tau} - \beta_2 y_{2\tau}$, and $y_{4\tau} - \beta_3 y_{3\tau}$ with at least one $\beta_j \neq 1$, $j = 1, 2, 3$.

y_t , $t = 1, 2, \dots, T = 4N$ is *Seasonally Integrated*, $y_t \sim SI(1)$, if $\text{rank}(\Pi) = 0$, which imply $\Pi = 0$. Hence there is no cointegration between the series for the individual seasons $s = 1, 2, 3, 4$.²⁰

From Engle & Granger (1987) we have that two *Integrated* series $y_t \sim I(1)$ and $x_t \sim I(1)$ are Cointegrated if there exist a linear combination $y_t - \beta x_t$ which is stationary. The vector $(1, -\beta)$ is called the cointegration vector. In the notation above the series $y_t \sim I(1)$ and $x_t \sim I(1)$ are (*Nonperiodically*) *Cointegrated* if each pair of annual series $y_{s\tau}$, $x_{s\tau}$ are cointegrated with the same cointegration vector $(1, -\beta)$ for all $s = 1, 2, 3, 4$.

In the section on seasonal cointegration we defined *zero frequency cointegration* between two variables integrated at the zero frequency, $y_t \sim I_0(1)$ and $x_t \sim I_0(1)$, as existing if the transformed variables y_{1t} and x_{1t} , see (7) cointegrate, *semiannual cointegration* between two variables integrated at the semiannual frequency, $y_t \sim I_{\frac{1}{2}}(1)$ and $x_t \sim I_{\frac{1}{2}}(1)$, exist if the transformed variables y_{2t} and x_{2t} , see (7) cointegrate, while *annual*

²⁰In Hylleberg and al. (1990) this form of integration is denoted "Integration at frequency 0, 1/4, 1/2" $I_\theta(1)$, $\theta = 0, 1/4(3/4), 1/2$. Hence a *Seasonally Integrated* in the sense of Osborn series has unit roots at all the frequencies $0, \pi/2, (3\pi/2)$, and π .

cointegration between two variables integrated at the annual frequency, $y_t \sim I_{1/4}(1)$ and $x_t \sim I_{1/4}(1)$, exist if the transformed variables y_{3t} and x_{3t} and $x_{3,t-1}$, see (7) polynomially cointegrate. Annual cointegration may also be expressed in terms of the complex transformations, in fact *Annual cointegration* exist if the complex pairs $(y_{*,t}, x_{*,t})$ and $(y_{**,t}, x_{**,t})$ cointegrate.

Full Periodic Cointegration exist, see Osborn (2000) if each pair of annual processes $y_{s\tau}, x_{s\tau}$ cointegrate with cointegration vector $(1, -\beta_s^P)$, but not all $\beta_s^P = \beta^P$, for $s = 1, 2, 3, 4$. *Partial Periodic Cointegration* exist, see Osborn (2000) if some but not all annual processes $y_{s\tau}, x_{s\tau}$ for $s = 1, 2, 3, 4$.

Based on these definitions Osborn (2000) obtain a series of results, which are summarized in Table 1

Table1 Cointegration possibilities^c						
Two Processes	Long	Full	Annual	Semi	Full	Partial
x y	Run	Seasonal	Cointe-	Annual	Periodic	Periodic
	Cointe-	Cointe-	gration	Cointe-	Cointe-	Cointe-
	gration	gration		gration	gration	gration
	<i>CI</i>	<i>FSCI</i>	<i>ASCI</i>	<i>SASCI</i>	<i>FPCI</i>	<i>PPCI</i>
$I(1)$ $I(1)$	+	-	-	-	-	-
$PI(1)$ $PI(1)$	+	-	-	-	-	-
$I(1)$ $PI(1)$	-	-	-	-	-	-
$SI(1)$ $SI(1)$	+	+	+	+	+	-
$I_{\frac{1}{2}}(1)$ $I_{\frac{1}{2}}(1)$	-	-	-	+	-	-
$I_{\frac{1}{4}}(1)$ $I_{\frac{1}{4}}(1)$	-	-	+	-	-	-
$I_{\theta}(1)$ $I_{\theta^*}(1)$	-	-	-	-	-	-
$PI(1)$ $SI(1)$	-	-	-	-	+ ^a	+
$I(1)$ $SI(1)$	+ ^b	-	-	-	-	+

a) *FPCI* between $x_{s\tau}$ and $(1 + L + L^2 + L^3)y_{s\tau}$, $s = 1, 2, 3, 4$

b) *CI* between $x_{s\tau}$ and $(1 + L + L^2 + L^3)y_{s\tau}$, $s = 1, 2, 3, 4$

c) A + indicate a cointegration possibility, a - indicate that no cointegration is possible

Osborn (2000) also suggest a series of test for choosing between the different cointegration possibilities.

Common Seasonal Features Although economic time series often exhibits non-stationary behaviour, stationary economic variables exists as well, especially when conditioned on some deterministic pattern, such as linear trends and seasonal dummies, breaks etc. However, a set of stationary economic times series may also exhibits common behaviour, and for instance share a common seasonal pattern. The technique for finding such patterns, known as Common Seasonal Features are based on earlier contributions defining common features by Engle & Kozicki (1993) and Vahid & Engle (1993). The Common Seasonal Features were introduced by Engle & Hylleberg (1996), and further developed in Cubadda (1999).

Consider a multivariate autoregression written in error correcting form as

$$\Delta Y_t = \sum_{j=1}^p B_j \Delta Y_{t-j} + \Pi v_{t-1} + \Gamma z_t + \varepsilon_t, \quad (52)$$

$$t = 1, 2, \dots, T$$

where Y_t is $k \times 1$ vector of observations on the series of interest in period t and the error correcting term is Πv_{t-1} . The vector v_t contain the cointegrating relations in case of cointegration at the zero frequency, and the of cointegrating relations is equal to the rank of Π . If no cointegration exist Π has full rank equal to k and the series are stationary. In the quarterly case the vector z_t is a vector of trigonometric seasonal dummies, such as $\{\cos(2\pi ht/4 + 2\pi j/T), h = 1, 2; j \in (-\delta T \leq j \leq \delta T), \sin(2\pi h4 + 2\pi j/T), h = 1, 2; j \in (-\delta T \leq j \leq \delta T), j \neq 0, \text{ when } h = 2\}$. The use of trigonometric dummy variables facilitate the "modelling" of a varying seasonal pattern as proper choice of δ takes care of the neighbouring frequencies to the exact seasonal frequencies, see Hylleberg (1986). Notice, that if $\delta = 0$, the trigonometric dummies defined above are equivalent to the usual seasonal dummies as described in Section 2.1.2.

The implication of a full rank of the $k \times m$ matrix Γ equal to $\min[k, m]$ is that you need different linear combinations of the seasonal dummies in z_t to explain the seasonal behaviour of the variables in Y_t . However, there are common seasonal features in these variables we don't need all the different linear combinations, and the rank of Π is not full. Hence, a test of the number of common seasonal features can be based on the rank of Π , see Engle & Hylleberg (1996)

The tests is a reduced based on a reduced rank regression similar to the test for cointegration described earlier. Hence the hypothesis tested using a canonical correlation analysis between of z_t and ΔY_t , where both set of variables are purged of the effect from the other variables in (52)

This kind of analysis has proved useful in some situations, but is difficult to apply, if the number variables is large, and the results sensitive to the lag-augmentation, similarly to the case of cointegration. In addition, the somewhat arbitrary nature of the choice of z_t poses difficulties.

2.3 The Economics of Seasonality

Many economic time series have a strong seasonal component, and it is obvious that economic agents must react to that. Producers know that the demand for their products varies over the year, and the consumers know that certain products are only available at some periods or at least are cheaper in some periods than in others. Hence, the seasonal variation in economic time series must be an integrated part of the optimizing behaviour of economic agents, and the seasonal variation in economic time series must be a result of the optimizing behaviour of economic agents, reacting to exogenous factors such as the weather, the timing of holidays etc.

Hence, the mere fact that economic agents react and adjust to seasonal movements on the hand and influence them on the other, imply that the application of seasonal

data in economic analysis may widen the possibilities for testing theories about economic behaviour. The relative ease at which the agents may forecast at least some of the causes of the seasonality may be quite helpful in setting up testable models for production smoothing for instance.

However, apart from what is caused by the easiness of forecasting exogenous factors the type of optimizing behaviour and the agents reactions to a seasonal phenomena may be expected not to differ fundamentally from what is happening in a nonseasonal context. However, the recurrent characteristic of seasonality may be exploited. Such a recurrent characteristic may have important effects on adjustment cost etc. seen in relation to adjustments to other cyclical but less regular phenomena. The recurrent characteristic may also have effect on the short and long run reaction to seasonal phenomena. In the short run the farmers may adapt to the weather conditions in a passive way, but in the longer run investments in more effective species of grain, irrigation etc. may change or smooth the effect of the weather.

In the following we will discuss how agents react and interact when faced with seasonal fluctuations.

There is no real dominating approach found in the literature although there are two main branches - the Real Business Cycle approach (e.g. Christiano & Todd (2000), Chatterjee & Ravikumar (1992), or Braun & Evans (1995) These all work with an utility optimizing consumer faced with some feasibility constraint. Christiano & Todd (2000) also discuss whether this conventional RBC model is sensible to the approximations usually done in these model. A different approach is Ghysels (1988), Miron & Zeldes (1988) and Miron (1996) who examines whether firms actual smooth production.

Often the situation can be seen as though states changes in accordance with the seasons, past actions etc. whereafter the agents must decide on what to do. The problem can be made concrete by assuming that we can write the problem as an expected profit maximization

$$\max_{\{x_t\}_{t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} f(x_t, y_t)$$

subject to some constraints

$$y_{t+1} = g(x_t, y_t, \varepsilon_{t+1})$$

where x_t is vector containing e.g. sales or production controlled by the firm, and y_t is a vector of states such as interest rates, storage capacity, inventory as well as the direct seasonality i.e. average temperature. and the ε_{t+1} is a stochastic term.

If this problem obeys certain condition of regularity (see for instance Stokey, Lucas & Prescott (1989)) this can be rewritten as a recursive problem. For concreteness, let the discount factor be constant and as before let the objective function be additive separable. In addition let the seasonal shock have a Markov property.

We can then rewrite the problem as a Bellman problem

$$V(x) = \max_y [f(x, y) + \beta E[V(g(x, y, \varepsilon)) | x]]$$

This functional equation has almost always a solution, and we can derive a rule of how the firm should choose x_t given the state y_t . Unfortunately, the problem is in general too complicated to be given an analytical solution, and it is often necessary to find the solution by an iterative algorithm. See Rust (1996) and Judd (1998).

Todd (1990) provides a framework to obtain analytic solutions using the linear quadratic approach. The main idea is to have the objective function being quadratic and the transition equations being linear, and to model seasonality by a periodic representation. However, the linear quadratic approach is quite restrictive.

Nonetheless, this approach is widely used for mainly two reasons. Firstly, the procedure is after all general enough to capture a very broad set of problems. Secondly, when the decision is derived from the model, the Euler equations, the first order conditions, provide estimable econometric equations. If data is available for some of the states and the final decisions it is possible to test whether the model is compatible with actual observed behaviour, see Rust ().

In an intertemporal dynamic model a common result is that the presence of seasonality in an exogenous variable may induce power in the spectrum of the endogenous variables both at the seasonal and non- seasonal frequencies. The reason for this is that agents in general will react to changes in their environment as well as in their information set, when they know that actions today will effect the future. An examples which describes the intuition behind this result is the adjustment cost in production, e.g. if there is large cost associated with training workers, it may not be a good idea to fire workers in period with low sale, if the sales are expected to rise in the future.

There are several papers which illustrate this fact, Ghysels (1988) set up an intertemporal production model, with the above mentioned adjustment costs and an exogenous seasonal pattern introduced into the demand. It is then shown that seasonality is affecting the endogenous part of the model, not only at the seasonal frequencies but at all frequencies. This also illustrate the danger of applying prefiltered seasonal adjusted data. In some cases it is shown that the distortions may be small, however, see Christiano & Todd (2000).

Osborn (1988), extended Hall (1978) by including seasonal varying components into the utility function. The model is then tested with data from the U.K., and even though the model is rejected it still does better than the standard model.

A production smoothing model is analyzed in Ghysels (1988), Miron & Zeldes (1988) and Miron (1996), but the empirical evidence is negative, as the hypothesis of production smoothing is rejected. A finding which could be caused by the use of to aggregated data.

However, both these empirical findings may also be due to misspecified models. The functional forms may be too restrictive or perhaps the agents may not have all the information that model builder normally assume.

A rather new approach, called Robust Control, advocated by Hansen & Sargent (2000), attacks this problem. The idea is to allow for the fact that the agent thinks the model could be misspecified, and hence uses the information set available to him and allow for the uncertainty. Thus, the agent is assumed to think of the model as a possible misspecified model of the real model. It could be interesting to see this applied to models with seasonal characteristics.

3 Conclusions

Seasonality has been a major research area in economics for several decades. The paper assesses the recent development in the literature on the treatment of seasonality in economics, and divides it into three interrelated groups. The first group, the Pure Noise Model, consists of methods based on the view that seasonality is noise contaminating the data or more correctly contaminating the information of interest for the economists. The second group, the Time Series Models, treats seasonality as a more integrated part of the modeling strategy, with the choice of model being data driven. The third group, Economic Models of Seasonality, introduces economic theory, i.e. optimizing behaviour into the modeling of seasonality.

The recent development has been quite promising, where increasingly, the treatment of seasonal economic timeseries in economics is an integral part of the modelling process, and where there is an increased awareness that use of pre-seasonally adjusted data very easily leads to errors and in addition throw away valuable information. The next step in the development should be a more elaborate integration of economic theory into the modelling process. A development which is as needed in the area of modelling seasonal economic time series as it is in most other areas of econometrics, if not all.

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