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# TAXES, QUOTA AND EXTERNALITIES IN MONOPOLISTIC COMPTITION

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Taxes, Quotas and Externalities in Monopolistic Competition

by

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Abstract: In this paper we investigate environmental regulation by taxes and quotas in the context of

a monopolistically competitive industry. Firstly, we find the combination of a quota and a tax

supporting the first best solution. Secondly, we explain why the allocative equivalence between the

two instruments vanish in monopolistically competitive industries. Thirdly, we show that if the

regulator is to choose between the two instruments then quotas are unambiguously preferable to

taxes if income effects can be ignored.

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### Taxes, Quotas and Externalities in Monopolistic Competition

Abstract: In this paper we investigate environmental regulation by taxes and quotas in the context of a monopolistically competitive industry. Firstly, we find the combination of a quota and a tax supporting the first best solution. Secondly, we explain why the allocative equivalence between the two instruments vanish in monopolistically competitive industries. Thirdly, we show that if the regulator is to choose between the two instruments then quotas are unambiguously preferable to taxes if income effects can be ignored.

In Pigou's (1947) study on public finance there is a remarkably simple advice on how to deal with environmental externalities: when a pollution tax equals the marginal environmental damage the market guides the economy to its social optimum. It is straightforward that a tradeable quota can replace the tax. These results presuppose an otherwise perfectly competitive economy where product variety problems, and other sources of imperfect competition, are. To the contrary, in the Dixit-Stiglitz-Spence model of monopolistic competition there is a relation between product variety and profit which covers set-up costs. In such an economy a tax changes output per firm as well as the firms ability to cover the set-up costs. From the welfare point of view, the latter of these effects suggests a quota facilitating entry into the industry whereas a tax lowers the number of firms. It is this topic we address here.<sup>1</sup>

We employ the monopolistically competitive setting found in Spence (1976) and Dixit and Stiglitz (1977) to address the regulation problem including the question of equivalence between taxes and quotas.<sup>ii</sup> Models like these focus on the market's actual product choice relative to the welfare maximizing product choice. The idea is that firms must pay some set-up costs before trade. In this situation the individual firm's revenue plays an important role regarding the socially optimal kinds and quantities of commodities. Equality between the price and marginal cost, a condition for a socially optimal allocation, leaves the firm with negative profit. Allowing some degree of monopoly each firm can recover the set-up costs but at a price above marginal costs. Spence (1976) and Dixit and Stiglitz (1977) show that product variety can be below the first best level in such circumstances. Here we introduce environmental externality problems into the monopolistic competition model. There are now three sources to misallocation: One is the distortion due to the externality, one is the individual firm's incentive to produce too little and the final distortion is that the market supports too few firms. For the symmetric case we find the combination of a quota and a tax implementing the first best solution. When this ideal solution is infeasible, for political reason for example, and the regulator is to choose between the two instruments then he should opt for a quota, at least for some specifications of the welfare function. iii

Nowadays there is a lot of interest in environmental regulation under imperfect competition. The issue of negative externalities in monopolistically competitive markets is not, however, addressed in existing literature. In the short run (when the income distribution is left aside) taxes and quotas are essentially equivalent policy measures. Or as Cropper and Oates (1992) put it: the regulator can, in short, set either "price" or "quantity." This point of view has been challenged on several occasions. Concentrating upon the monopoly case Buchanan (1969) argues that a tax equal to the

marginal external damage can be detrimental to welfare, see also Barnett (1980). Considering the long run Carlton and Loury (1980) argues that the Pigouvian output tax must be supplemented by a lump sum subsidy in order to support the optimal number of firms in the market. Spulper (1985) demonstrates that a tax or a quota on emissions, rather than output, implements the first best. That is, it is the choice of the instruments and not their number which is important. Our analysis shows that this argument does not extend to the case of monopolistically competitive markets. More recently the focus has been upon environmental policy in oligopolistic markets, like for example Carraro et al. (1996). These studies concentrate on firms' market shares and they ignore set-up costs as well as consumers' preferences for product variety. The issue of oligopoly is also addressed in Santoni (2000), Denicolo and Matteuzzi (2000) and Haworth (1998). Product variety and search is discussed in Anderson and Renault (1999). Neither of these recent studies include externalities. Goering and Boyce (1999) discusses emissions taxation and imperfect competition but in an oligopoly market for durable goods.

#### I. The model

The basic model we use has been used by Spence (1976) and Dixit and Stiglitz (1977) to study optimum product diversity. In addition to the original specification we add external costs. The economy is divided into a monopolistically competitive industry and a numeraire good summarizing the remainder of the economy. Within the industry firms are characterized by scale economies and they are linked together by significant cross-elasticities. The number of active monopolists, called n, is also the number of the different goods in the monopolistically competitive industry and this number is determined by a zero profit condition. Following Dixit and Stiglitz (1977) scale economies are modelled by supposing that production involves a fixed set-up cost and

constant marginal cost according to  $(y_i+c_i)x_i+F_i$  where  $F_i$  is the firm's set-up cost and  $x_i$  its output. The firm experiences constant marginal costs divdided into private marginal costs,  $c_i$ , and external marginal costs,  $y_i$ . Consumers are assumed to be large in number and pricetaking. We know from Spence (1976) and Dixit and Stiglitz (1977) that the market generates too little product variety relative to the socially optimum kinds of commodities when we ignore externalities. The basic reason for this is that profit and product variety are related and marginal cost pricing leaves firms with a negative profit.

Our concern is the choice of environmentally motivated taxes or quotas or a combination of both. If a regulator is to choose between one or the other of the two instruments profit's welfare role in such an environment clearly points to a quota as a superior instrument relative to a tax since a quota facilitates entry. On the other hand, since output and profits (and through profits the number of active firms) should be regulated it is no surprise that the first best solution can be supported only when the regulator uses a combination of quotas and a tax. In order to focus on this matter as simply as possible all commodities in the group have identical fixed and marginal costs (from now on denoted c and c, respectively). This symmetry also goes for external costs (from now on denoted by c) assumed to be a linear function of firm output. Due to the symmetry assumption we can write the utility of consumption as:

(1a) 
$$u = G(m) + x_0, G'(m) > 0, G''(m) < 0$$

(1b) 
$$m = nax^B, 0 < B < 1$$

where a is a constant. The industry's commodities are less than perfect substitutes with the assumption on B's value. In the case of B=1 goods are perfect substitutes corresponding to the case of perfect competition. Conventionally, G says that consumers' utility is increasing but at a decreasing rate in m, to be thought of as an index of congestion in the industry. Finally  $x_0$  is the numeraire good entering additively and we are, thus, ignoring income effects. This approach is also used in Spence (1976) and makes the welfare analysis of the industry amenable to a partial equilibrium approach.

Turn now to the question of the optimum kinds and quantities of commodities and the optimal tax. There are two sources of market failure present in the economic environment. Firstly, due to fixed costs there are scale economies effects. In relation to a comparison between the social optimum and the market equilibrium fixed cost has at least two implications. They restrict commodity variety and possibly also the volume of each produced commodity. And they are a source of non-competitive pricing. In the current model this contributes to suboptimality in the form of too little product diversity, cf. Spence (1976) and Dixit and Stiglitz (1977). The second source to market failure is production's external effect. Other things equal this implies that firm output is inefficiently large. Firm output, however, contributes to the congestion index which is increasing in firm output. That is, the two sources of suboptimality in the market point in opposite directions.

To be more precise about the nature of the problem involved consider the welfare function. Setting the price of the numeraire good to one, consumers spend npx+x0 on the total consumption bundle and net-utility is therefore G(m)-npx. Total industry profits are npx-n(cx+F). Including external costs and rewriting n in terms of m and x the welfare function becomes:

(2) 
$$W = G(m) - \frac{m}{ax^{B}}((y+c)x+F)$$

Maximizing (2) with respect to x and m the social optimum is:

(3a) 
$$x^* = \frac{BF}{(1-B)(v+c)}$$

(3b) 
$$G'(m^*) = \frac{(y+c)x^* + F}{ax^{*B}}$$

It follows that the individual firm's output is decreasing in marginal external costs. Clearly, m is a decreasing function of marginal external cost. Consider the relationship between external costs and the optimal number of firms, n. A necessary condition for an optimal resource allocation is that a given market congestion is achieved at the least possible costs. Iso-congestion curves are defined by  $m_f = nax^B$  (where  $m_f$  denotes a fixed value for m) and they are clearly convex to the origin. The isocost function for the industry is  $TC_f = n((y+c)+F)$  and its slope is negative and we have  $\partial(\partial n/\partial x)/\partial y < 0$ . That is, the trade off between the number of firms (commodities) and the output per firm changes in the favour of increased product variety as marginal external cost increases. This is the reason why a regulator is in need of both quotas and a tax. Using the quota he can regulate output per firm and using the tax, which may well be expected to be negative, he can adjust the number of active firms.

#### II. Taxes and Quotas

When we ignore complex relationships between firms' output and the external effect, uncertainty and so on, taxes and quotas are equivalent and the regulator can set either price or quantity to support efficiency. Considering the merits of the two instruments in the context of a polluting monopolistically competitive industry the starting point is that imperfect competition, at least in the models discussed here, gives too little product variety. This, naturally, gives quotas an advantage in direct comparison to a tax since an output quota facilitates additional entry into the industry. To demonstrate the potential of quotas let us (as a reference situation) consider the case without externalities. Let the regulator fix a quota given by  $x_q = ((1-B)c)^{-1} BF$  (setting y=0 in (3a)). The demand function for the representative firm in the industry follows immediately from (1a) and (1b):  $p = G'(m)aBx^{B-1}$  and firm profit is  $G'(m)aBx^B-cx-F$ . Entry into the industry is regulated by the zero profit condition and we have  $G'(m)aBx^B = cx + F$ . But this is (3b) for the case without externalities. Thus, in the case without externalities the proper quota supports the first best allocation. An output tax (or subsidy) can not achieve this. This is quite general. When, in the unregulated economy, the situation is characterised by too few firms each producing too much a tax can cut output per firm but the tax drives the number of firms in the wrong direction. A subsidy can attract firms to the industry but the subsidy pushes firm output in the wrong direction.

Externalities in a monopolistically competitive industry can be dealt with by a combination of a tax and a quota. This is easely seen. Fix the output quota at  $x_q = x^*$ , where  $x^*$  is given by (3a):

(4a) 
$$x_q = x^* = \frac{BF}{(1-B)(y+c)}$$

The firm's profit under a tax, t, is  $G'(m)aBx^B - ((t+c)x+F)$  and with respect to entry we thus have:

(4b) 
$$G'(m)aBx^{B} = (t+c)x + F$$

The optimal allocation can be supported by a quota and a tax by adjusting that tax so as to satisfy (4b) for  $x = x_q = x^*$  and G'(m) as defined by (3b). This comes down to a tax rate satisfying (t+c)/(y+c) = (2B-1)/B, or  $t = y-B^{-1}(1-B)(c+y)$ . Denoting the elasticity  $e = (dp/p)(dx/x)^{-1} = B-1$  the optimal tax is expressed as  $t = y - e^{-1}(1-e)(y+c)$ . We state this as proposition 1. vii

Proposition 1. A quota fixed at  $x^*$  in combination with a tax given by  $t = y - e^{-1} (1-e)(y+c)$  implements the first best.

There are two things to notice with respect to the tax. Firstly, the tax is less than the value of the marginal external damage when B < 1. This in turn implies that the firm's preferred output under a tax, called x(t), is equal to or greater than  $x_q$  making the quota a binding restriction. Of course, the quota would not be binding in cases where the tax for some reason exceeds the marginal damage's value. Secondly, the tax is not necessarily positive. It can turn out to be a subsidy. We have  $t \ge 0$  for  $y/c \ge (1-B)/(2B-1)$  and t < 0 for y/c < (1-B)/(2B-1). It is straightforward to understand this. Consider the congestion index in the market economy for  $x = x^*$ :

(5a) 
$$G'(m_q(x^*)) = \frac{cx^* + F}{aBx^{*B}}$$

and compare this to the congestion index's first best value:

(5b) 
$$G'(m(x^*)) = \frac{(y+c)x^* + F}{ax^{*B}}$$

We have  $m(x^*) > m_q(x^*)$  for y/c > (1-B)/(2B-1). That is, the market delivers to many firms and proposition 1 tells us that the quota must be supplemented by a tax. Contrary, for y/c < (1-B)/(2B-1) the market undersupplies with respect to the number of firms and the quota should be combined with subsidy. The parameter B together with the external damage's (relative) value determines whether the situation calls for a tax or a subsidy. A tax is less likely for a combination of strong monopoly effects and neglible marginal external (relative to private) costs. As noted, the role of 0 < B < 1 has to do with the describtion of competition in the goods market. The case B=1 corresponds to perfect competition (with the goods being perfect substitutes) and as B decreases substitution possibilities are decreasing and each firm's monopoly power increases.

Despite of similarities our results differ from those set forth in Schulze and D'Arge (1974) and Carlton and Loury (1980). They show how a combination of the traditional Pigouvian output tax and a lumpsum transfer supports the first best allocation, a result qualitatively like our proposition 1. The tax constrains firms' output to the socially efficient level and the transfer implements long run efficiency in the number of firms. In our setting a tax will never be optimal. The optimality of a tax requires that it is set at the marginal external cost but such a tax gives an insufficient number of firms. ix

However, Spulper (1985) shows that a source to inefficiencies can be whether the tax is levied on firm output or firm emission. If emissions are taxed the proper emission tax supports the first best (Spulper, 1985, proposition 5) and the problem is, thus, the choice of instruments rather than the number of instruments. Our result on the necessity of two instruments does not depend upon the choice of the base of the tax, contrary to the analysis in Spulper (1985). To see this assume that firms have constant returns to scale so that x is both input and output and assume further that emission per firm is linear in its input. In this case it is immaterial whether it is emssions or output which is taxed showing that two instruments are needed in the presence of monopolistic firm behaviour.

#### III. Taxes versus Quotas

In this section we discuss the regulator's problems when he is to choose between a tax and a quota (cf. Kelman (1999)). The ideal solution is to restrict output by a quota and regulate the number of firms by a tax or a subsidy as needed. In this section we assume that the regulator is constrained to use either a tax or a quota.

Based on the result of the previous section it can one can ask why not go for both of the instruments. This presupposes that the regulator can introduce the

Consider section I's remarks on the trade off between the number of goods and their quantities in the presence of negative externalities: the trade off between the number of commodities and the output of each commodity changes in the favour of increased product variety as marginal external cost increases showing that a tax can be problematic. A tax accomplishes lower output at the firm

level but it also extracts revenue from each firm and this, other things equal, leaves room for fewer firms. The problem here derives from the zero profit condition in combination with fixed costs. It is straightforward to understand the problem. Use the expression for optimal output under a tax  $x(t) = ((1-B)(t+c))^{-1}$  and notice that the firm's total cost equals F/(1-B) irrespective of the tax. A typical firm's revenue is  $G'(m)aBx^B$  and the firm's adjustment towards lower production as a result of the tax will lower its revenue. Thus, each firm's demand curve should be shifted up for the zero-profit condition to be met. A decreasing value of the congestion index is the only way that this can happen. And the upper limit on the congestion index sets the limit for the number of commodities in the industry. The, thus, drive out firms from the industry.

Ignoring externalities a quota outperforms a tax. In our version of the Dixit-Stiglitz-Spence model this is accounted for by the fact that firm output in the laizzes-faire economy corresponds to the first best but with too few firms. The quota facilitates entry into the industry. Introducing externalities puts a qualifier on this result and it is of interest to establish whether a quota can outperform a tax if the regulator is to choose between the two. Depending upon the type of quota system quotas may have a different effect upon firm profit relative to a tax and, thus, on product variety. Firms must pay for the right to pollute as they would have to under a tax if the quotas, or permits, are auctioned off. But rather than introducing quotas by auction they can be initiated with a one time distribution free of charge. The distribution can follow some form of grandfathering to allocate the quotas between firms. This is assumed here. Once a firm has been granted a quota it is marketable. Since firms are identical they are all given the same quota and they will not find it favorable to engage in selling or buying quotas and we ignore the issue of tradeable versus non-tradeable quotas in the current setting.

To address the issue of characterizing situations in which a tax and those in which a quota is preferable we inquire into the optimal tax. Let us denote -mG''/G' by g(m) where g > 0 follows from the conventional assumptions on G's curvature. In the appendix (available upon request) the welfare maximizing tax is found to:

(6) 
$$t = \frac{B^2 + (1-B)Bg(m)}{B^2 + (1-B)Bg(m) + (1-B)} (y+c) - c$$

This equation together with the firm's first order condition and the zero profit condition determines the tax rate, output per firm and the number of firms. The tax, of course, differs from the one discussed in the previous section since the tax is now the only instrument and not considered as a supplement to a quota. We shall compare the use of the optimal tax to the use of a quota. In the appendix (under the heading of lemma 1 and lemma 2) we show that it is possible to find a quota which is better than the best tax if the optimal tax is positive or zero (in the cases of  $y/c \ge (1-B)/(B^2+B(1-B)g(m))$ ).

Proposition 2. When income effects can be ignored a quota is better than a (positive) tax.

Proposition 2 suggests that a regulator should never go for a tax when the goods market is characterised by monopolistic competition. To inquire into the generality of this result it is relevant to consider whether the result on quotas' superiority can be extended to more general environments. Our conjecture is that this will not be the case. To see why strong results seem unlikely in other settings let us consider the general equilibrium analysis in Dixit and Stiglitz (1977). They consider more general specifications like  $u = U(x_0, (\sum x_i^z)^{1/z})$ , 0 < z < 1 and  $u = x_0^{1-z} (\sum v(x_i))^z$ , 0 < z < 1,

where v is some concave function. The first of these has constant elasticity whereas the latter is of the variable elasticity type. Let us consider the number of firms and output per firm for these specifications.

Consider the first of the two utility functions. For the case without externalities it can be shown that each firm's output equals the first best output but with insufficient product variety. Thus the starting point is qualitatively like the starting point for less general seperable utility function employed in this paper. Consequently, a policy restricting firm output and pushing up the number of firms must be expected to be better than a policy having restrictive effects on both output and the number of firms.

Turning attention to the last of the two utility functions the number of firms need not be too low.

Referring to Dixit and Stiglitz (1977), section 2 for details the market equilibrium and the first best equilibrium are characterised by:

(7a) 
$$((c+y)x^*+F)^{-1} (c+y)x^* = v(x^*)^{-1} x^*v'(x^*)$$

(7b) 
$$n^* = (F + (c + y)x^*)^{-1}z$$

and

(8a) 
$$(cx+F)^{-1} cx = v'(x)^{-1} (v'(x)+xv''(x))$$

(8b) 
$$n = (cx+F)^{-1} w(x)$$

where  $w(x) = (z\rho(x)+1-z)^{-1} z\rho(x)$ . It can be shown that  $\rho(x) < 1$ . The starting point of proposition 2 is that the market generates insufficient product variety relative to the first best in the case without externalities but firm output is efficient. Let us consider equations (7) and (8), ignoring externalities, and let us see whether the number of firms in the market is too low for the more general specification. Setting y = 0 in (7a) we have that  $x > x^*$  when v(x)/x + v(x) v''(x)/v'(x) > v'(x) which may well be the case. Consider next the number of firms. From (7b) and (8b) we have  $n^*/n = ((cx^*+F)z)/((cx+F)w(x))$ . Comparing z and w(x) we have z/w(x) > 1. On the other hand, assuming we have  $x > x^*$  we see that  $(cx^*+F)/(cx+F) < 1$ . Thus  $n^*/n$  may well be less than one or  $n^* < n$ . In this situation the market supports inefficiently high output per firm and too many firms even abstracting from external effects. In a situation like this it is of course much less clear that a qouta is the best of the two instruments since we want lower output per firm and fewer firms.

#### V. Conclusions

In this paper we have considered taxes and quotas in monopolistically competitive economies. The first rather obvious result is that in the presence of externalities and monopolistic competition neither of the instruments can support the first best on their own. We have derived the combination of a quota and a tax supporting the first best equilibrium. Spulper (1985) considers the issue of the number of instruments versus the tax base and argue that an emissions tax can realise the first best. Our results suggest that this does not extend to monopolistic competition. Relative to Carlton and Luory (1980) a Pigou tax supporting the first best in their setting will not do so here.

If the regulator is to choose between a tax and a quota we have shown that it is best to go for a quota. This can perhaps be seen as an answer to Cropper and Oates' (1992) observation: the regulator can choose price or quantity but he seems to prefer the latter. If the two types of instruments are allocatively alike, as they are under perfect competition, the explanation for this can appeal to the nature of the policy making process, for example that direct control is liked over indirect measures like a tax. Focusing on a partial equilibrium analysis this paper's comparison of taxes and quotas offers the simple explanation that quotas are better from a welfare point of view. If they are then this can be a much more straightforward explanation for regulators' liking for quotas.

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Appendix.

## The optimal tax

With a tax, t, we have (dropping t to save notation):

(1) 
$$x = \frac{BF}{(1-B)(t+c)}$$

(2) 
$$G'(m) = \frac{(t+c)x+F}{aBx^B}$$

Now, from (1):

(3) 
$$\frac{dx}{dt} = -\frac{BF}{(1-B)(t+c)^2}$$

Using (3) in (2):

(4) 
$$G'(m) = \frac{F}{(1-B)aB} (\frac{(1-B)(t+c)}{BF})^{B}$$

From (4):

(5) 
$$\frac{dm}{dt} = \frac{BG'(m)}{(t+c)G''(m)}$$

and:

(6) 
$$\frac{d}{dx}(\frac{(y+c)x+F}{ax^{B}}) = \frac{(1-B)(y+c)-BF/x}{ax^{B}}$$

Consider now welfare:

(7.1) 
$$W = G(m) - \frac{m}{\alpha x^{B}} ((y+c)x + F)$$

We have the optimal tax defined by dW/dt = 0:

(7.2) 
$$\frac{dW}{dt} = (G'(m) - \frac{(y+c)x+F}{ax^B})\frac{dm}{dt} - m\frac{d}{dx}(\frac{(y+c)x+F}{ax^B})\frac{dx}{dt}$$

Now, using equations (3), (4), (5) and (6) in (7.2), and cancelling, the optimal tax is defined by:

(7.3) 
$$\frac{G'(m)}{mG''(m)}((t+c)x+F-B((y+c)x+F)) + \frac{(1-B)(y+c)-BF/x}{ax^B}BF = 0$$

Using the definition of x (equation (1)):

(7.4) 
$$\frac{G'(m)}{mG''(m)} \left(\frac{F}{1-B} + F - B\left(\frac{B}{1-B} + \frac{y+c}{t+c} + F\right)\right) + \frac{(y+c) - (t+c)}{(t+c)} BF = 0$$

or:

(7.5) 
$$\frac{G'(m)}{mG''(m)} \left(\frac{1}{1-B} - B \frac{B}{1-B} \frac{y+c}{t+c} - B\right) + B \frac{(y+c)}{(t+c)} - B = 0$$

Solving we have:

(8.1) 
$$\frac{(y+c)}{(t+c)} = 1 + \frac{(1-B)}{B^2 + (1-B)g(m)}$$

where g(m) = -mG''(m)/G'(m), or:

(8.2) 
$$t = \frac{B(B + (1 - B)g(m))}{B^2 + (1 - B)Bg(m) + (1 - B)}(y+c)-c$$

### **Proof of proposition 2**

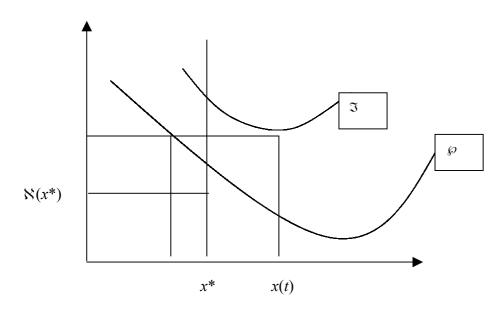
We shall prove that there exists a quota outperforming the best positive tax. Notice first that since the tax is less than the marginal external damage's value we have  $x(t) > x^*$ .

Consider the relations:  $\aleph = ((y+c)x+F)/(ax^B)$ ,  $\Im = ((y+t)+F)/(aBx^B)$  and  $\wp = (cx+F)/(aBx^B)$ . The first of these gives the optimal market congestion value for a given output per firm. When  $x = x^*$  this curve defines the first best. The  $\wp$ -curve is placed below the  $\Im$ -curve for all values of x and the minimum of the  $\wp$ -curve is somewhere to the right of the minimum of the  $\Im$ -curve (when the tax is positive).

Case 1. Look at x(t) and define  $x_m$  by  $(c x_m + F)/x_m^B = ((c+t)x(t) + F)/x(t)^B$ , where x(t) is firm output under the best tax. That is, the market congestion index under the tax and under the (thus defined) quota are identical. Notice that we have  $x_m < x(t)$  because of the relation between the  $\wp$ - and 3-curves. Now, when  $x(t) > x_m > x^*$  a quota at  $x_m$  is better than the tax since the instruments support the same congestion index but the quota brings output per firm closer to its optimal value. This proves proposition 2 when  $x(t) > x_m > x^*$ .

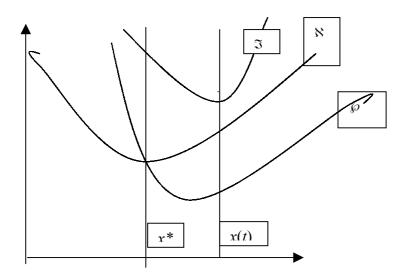
Case 2a. Let us consider the case of  $x_m < x^* < x(t)$ . Since, this defines the case,  $x_m < x^* < x(t)$  we know that the intersection between the  $\wp$ -curve and the vertical line at  $x^*$  is placed below the horizontal line passing htrough the minimim of the 3-curve. When  $\aleph(x^*) < \wp(x^*)$  we can use  $x^*$  as a quota: this supports  $x=x^*$  while the congestion value is more favourable under the quota compared to its value under the tax, see figure A1. Consider  $\aleph(x^*) < \wp(x^*)$ . We have  $x^* = BF/((1-B)(y+c))$  and the inequality reduces to 2B-1 < (c/(y+c))B. This is satisfied when  $1/2 \ge B$  and when  $1/2 \ge B$  and when  $1/2 \ge B$  and  $1/2 \ge B$  and 1

Figure A.1



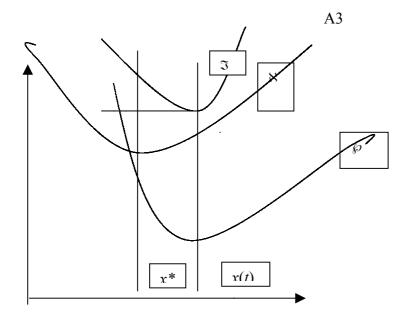
We are left with case of  $B > \frac{1}{2}$  and y > (1-B)/(2B-1)c. We know that a quota given by  $x^*$  is optimal when y = (1-B)/(2B-1)c, cf. proposition 1. In this case the  $\wp$ -curve passes through the  $\aleph$ -curve in  $x^*$ . The  $\Im$ -curve is placed above the  $\aleph$ -curve at x = x(t). This is seen noticing that  $\Im(x) > \aleph(x)$  for x < ((1-B)F)/(B(c+y)-(c+t)). Comparing this expression with x(t) we find that the two curves intersect to the right of x(t). This applies for all parameter configurations. The situation for y = (1-B)/(2B-1)c is depicted in figure A2.

Figure A2



Now, when y increases (from (1-B)/(2B-1)c) the  $\wp$ -curve stay put while the  $\Im$ -curve and the  $\aleph$ -curve moves upwards and to the left and  $\Im(x(t)) > \aleph(x(t))$ . Using figure A3 we can construct a quota as follows. Take the horizontal line through the  $\Im$ -curve at x=x(t) an pick out the intersection with the (fixed)  $\wp$ -curve and use this as the quota. This supports the same congestion index by unit costs will fall.





### Endnotes:

<sup>&</sup>lt;sup>1</sup> Pigouvian taxes versus quotas is discussed in Weitzman (1974) and Roberts and Spence (1976) with focus on uncertain pollution control costs. Recently this line of thinking has been extended by Hoel and Karp (1999) to include asymmetric information. In this paper such kind of uncertainties are not the issue.

ii Hart (1979, 1985a, 1985b) also discusses this problem. The approach taken by Spence and Dixit and Stiglitz is discussed in Heidjra and Yang (1993), D'Aspremont et al. (1996) and Benassy (1996). Different approaches to monopolistic competition are Caplin and Nalebuff (1991) and Deneckere and Rothschild (1992).

iii Kelman (1999), in relation to the U.S., discusses the problems involved in the choice between regulation and taxation including limitations in choice imposed by the Constitution.

<sup>iv</sup> Carlton and Loury consider a situation where external effects are introduced into an otherwise perfectly competitive economy. They show that a Pigouvian tax supports short and long-run efficiency when external damage is a function of total output. This is actually the case analysed here and a Pigouvian tax is not efficient in our setting. In the more general case where the external damage function is defined over output and the number of firms a tax and a subsidy must be used together but the Pigouvian tax is efficient when the number of firms is fixed.

<sup>v</sup> Spence does not assume symmetry and his discussion is thus more general. In a subsection he uses the specification applied here. Dixit and Stiglitz consider the case of a separable utility function used here. They also consider less restrictive formulations but still assuming symmetry. Later we will comment on the implications for our analysis of these more general utility specifications.

 $\label{eq:weak_problem} $$^{\text{vi}}$ We have $G'(m)=(ax^B)^{-1}((y+c)x+F)$ . From (3a:) $d((ax^B)^{-1}((y+c)x+F))/dx=0$ so that $$dm/dy=(G''(.)ax^{1-B})^{-1}<0$ .$ 

vii Even though we have many firms we have only two market failures since the firm is representative. When this is the case it is unsurprising that two (linearly independent) instruments support the desired objective. This argument, of course, will not apply when firms differ through their cost functions, say.

viii Notice that this tax shall not be compared to Barnett (1980) since the tax rate he derives is a second best tax rate.

<sup>ix</sup> Consider an output tax. Firm output with a tax is BF/((1-B)(t+c)). From the zero profit condition we have:

$$G'(m(t)) = \frac{F}{(1-B)aB} (\frac{(1-B)(t+c)}{BF})^{B}$$

Now, if the regulator sets t = y in order to achieve  $x(t) = x^*$  then  $m(t) < m^*$  (by comparison of (3b) in the text).

Notice that a quota can support the first best but only for a specific parameter configuration. If a quota is to implement the first best solution when, of course, it is defined by  $x_q = x^*$ . From the zero profit condition we have:

$$G'(m_q)aBx^{*B} = cx*+F$$

Comparing this to (3b) we have  $m_q \neq m^*$  unless  $B = (c + 2y)^{-1}(c + y)$ .

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