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ENVIRONMENTAL TAXES IN
MONOPOLISTIC COMPETITION

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Environmental taxes in monopolistic competition

by

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*Abstract:* Taxes can change product selection bias in markets where set-up or fixed costs play an important role. We demonstrate that the Pigouvian correction for externalities will intorduce products in the socially optimal order but too few firms survive. Allowing for firm specific taxes we find the optimal tax which is strictly less than the Pigouvian tax rate. The optimal tax leaves the monopolistically-competitive industry with too few firms.

## 1.Introduction

A tax equal to emissions' marginal damage is efficient in the otherwise perfectly-competitive economy (see e.g. Spulber, 1985). The assumption made with respect to goods in a setting of perfect competition renders superfluous concerns about product differentiation and consumers' taste for product variety. Chamberlin's (1933) view of the monopolistically-competitive market, in contrast, supposes that there are firms producing goods sufficiently close to form a reference set and yet they are not identical. Introducing "taste for variety" the papers of Spence (1976) and Dixit and Stiglitz (1977) suggest a formal model of monopolistic competition. The symmetric version of that model has since been applied in diverse areas as international trade, patents, macroeconomics and growth.

In this paper we discuss environmental taxation of a monopolistically-competitive industry. Spence (1976) argues that the market's choice of the kind and quantities of goods is inefficient in monopolistically-competitive markets with some increasing returns. In particular, when firms have different fixed costs there is a bias against products with high fixed costs. Assuming away income effects it can be shown that the market provides too little product variety. The issue of environmental taxation is the subject of a huge number of analyses but despite of the special welfare problems environmental externalities and taxation is not discussed in a setting of monopolistic competition in existing literature.[1]

The normative theory of environmental regulation has attracted attention since the Pigouvian tax was introduced; for a survey see Cropper and Oates (1992). Closely related to this paper is the seminal work by Buchanan (1969) who argues that a tax equal to the marginal external cost could actually reduce welfare in the case of monopoly. Later the optimal tax has been derived by Lee (1975) and Barnett (1980). In our setting a tax can not support the first best allocation. A recent paper by Lee (1999) uses an n-firm oligopoly model with free entry to derive the second best tax. This approach and other similar approaches (e.g. Carraro and Soybeyran, 1996) ignore fixed cost and the question of product selection bias.

Carlton and Loury (1980 and 1986), in a competitive model with free entry and u-shaped average cost curves, show that a Pigouvian tax will not in general result in an efficient allocation. The point is that, in a competitive economy, firms' output is the same with and without the tax since it is determined by the minimum of the average cost curve. But the socially optimal output per firm will in general differ from the output minimizing the firm's cost. In a later paper Carlton and Loury

[1] Hart (1979, 1985a, 1985b) also discusses this problem. The approach taken by Spence and Dixit and Stiglitz is discussed in Heidjra and Yang (1993), D'Aspremont et al. (1996) and Benassy (1996). Different approaches to monopolistic competition are Caplin and Nalebuff (1991) and Deneckere and Rothschild (1992).

(1986) extend the reasoning to include taxes on emissions. It is shown that taxes and subsidies can support the first best allocation. These papers do not focus on the product diversity problem and one of the conclusions, positive net taxes, does not survive in our setting. Finally, Spulber (1985) argues that taxes on emissions support the first best but this is not the case here.

In section 2 of the paper we extend some results on production selection bias and taxes. In section 3 we discuss optimal and second best taxes while the discussion is extended to more general settings in section 4. The conclusion is in section 5 and proofs in the appendix.

## 2. Product selection bias and environmental taxes

Product selection bias, i.e., the market's inefficient choice of the kinds and quantities of different products, arises in monopolistically-competitive markets when firms have fixed cost. With some degree of monopoly power the firm can recover the fixed costs but pricing above marginal cost is incompatible with efficient allocations. On the other hand, with fixed costs firms will loose by marginal cost pricing since the fixed costs can not be covered by firms' revenue.

The few results on efficiency and product selection bias (in the case where firms have different costs functions) are due to Spence (1976): Following Spence the economic environment is made up of a group of price-taking consumers and a monopolistically-competitive industry. Each commodity is produced by only one firm. Product variety, the number of commodities or firms, is determined by a zero-profit condition relating to the industry's marginal firm. Within the industry, firms are characterized by scale economies, and they are linked together by significant cross-elasticities. Scale effects in production are modelled by supposing that production efforts involve a fixed set-up cost. The notion of product selection bias is that a firm's contribution to the social surplus will, in general, not be matched by the firm's profit. That is, a firm's contribution to the social surplus need not be reflected in the firm's running profits and it is a problem to cover the fixed cost. Spence demonstrates that there is, roughly speaking, a bias against firms with high fixed costs.

With respect to externalities we assume that the individual firm uses capital ($F(i)$) and some ressource ($R(i)$). We will let $F(i)$ be the firm's fixed cost. Externalities in production are modelled by supposing that the firm's private costs are $R(i) + F(i)$ whereas social costs are $sR(i) + R(i) + F(i)$ or $(1+s)R(i) + F(i)$. For now, external costs are thus assumed to be proportional to the total resource use.[2] We shall relax this assumption later. When we consider the situation with externalities it is most reasonable to let resource use be taxed since this is the externality source.[3] That is, the firm's profit under a tax on ressource use is given by $(1+t(i))R(i) + F(i)$. Our first results, following immediately from Spence. The first result shows the circumstances where a Pigou tax will support the first best. These are ideal circumstances and the second result shows the kind of inefficiency

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[2] More generally a firm's pollution would be described absctractly as some function of its input, and the specifics of the functional form then describes how each firm handles its inputs. Since there is a one-to-one relation between the firm's input and its costs, we can rewrite to obtain $EC(i) = S(c(i)x(i)+F(i),i)$. Our assumption regarding external costs can be justified, firstly, if fixed costs are immaterial for the difference between firms' external effects and, secondly, when the $S$– function is the same for different firms. When this is the case a firm with high marginal costs will also have high external cost. If we think of marginal costs being cost of energy this is not unreasonable: firms with high marginal cost needs (relatively speaking) more energy and they pollute more.

[3] Spulber (1985) discusses the relation between emissions and taxes and shows that a properly defined emissions tax implements the first best.

arising usinf a Pigou tax when the requirements of proposition 1 fails to be met. We have propositions 1 and 2 as follows.[4]

Proposition 1. When firms can price discriminate the Nash equilibrium with a Pigouvian tax coincides with the first best allocation.

Proposition 2. In the absence of price discrimination the optimal output per firm is higher than that generated by the market regulated by a Pigouvian tax. Furthermore, the market may possibly deliver the wrong products relative to the set of socially optimal products.

The propositions follow directly from Spence and we state them without a formal proof. The intuition is as follows. Consider a tax on firms' resource use. When the tax rate per unit of resource use is set at the firm's marginal external damage, marginal damage becomes just another private cost. Additionally, it is well known that perfect price discrimination removes the monopoly distortion. By the proper tas and assuming perfecvt price discrimination both of the market failures are removed and Proposition1 follows unsurprisingly. Proposition 2 is also straightforward. There are two market failures and the tax corrects for only one of these. Since, in this kind of model, the laissez faire economy tends to give a bias against high fixed costs (this is what Spence shows) the second result follows.n 2 then reflects that the Pigouvian tax can deal with one disturbance only.

With respect to welfare evaluation the basic requirement behind propositions 1 and 2 is that consumers' utility function is additively seperable in two groups:  the goods in the

[4] Edlin et al. (1998) discuss this further and show that Pareto optima can be incompatible with surplus maximization even when firms can price discriminate.

monopolistically-competitive sector and all other goods. This assumption defines away income effects and permits a partial equilibrium analysis. In this case the net surplus to society of a bundle, $\boldsymbol{x} = (\ldots x(i) \ldots)$ is given by $T(\boldsymbol{x}) = u(\boldsymbol{x}) - \sum_i sc(i, x(i)) + F(i))$, where the requirements to the $u$-function are fairly weak (see Spence, 1976, page 222 for details). In this expression the term $sc(i, x(i))$ refers to the total social cost of a firm's production. Further results require that we are able to say which products should ideally be in the market and which ones should be left out. It is clear from Spence and from Dixit and Stiglitz (1985) that even within the kind of model considered here this requires some further assumptions.

## 3. Pigou tax and optimal taxes

Propositions 1 and 2 it must be noted, implicitly assume that the firm's variable cost is independent of fixed cost. When fixed cost is marketing cost and the like, as it is in Spence, this is reasonable. The interpretation used in this section and throughout the paper is based on the presumption that fixed and variable costs are negatively related.[5] For this reason Proposition 1 and 2 are only suggestive with respect to the general problems of environmental taxates and product selection bias. Instructive results on taxation requires that we are able to say which products should ideally be in the market and compare these to the ones that are actually supported by the market. This is possible by focussing to the following economic environment introduced by Spence.

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[5] Dixon (1986) analyses a two-stage duopoly model with a somewhat similar cost structure. In the first stage firms choose capital and in the second stage they choose output. Firms' production is determined by capital and some (short-run) variable input according to $x(i) = \sqrt{R(i)K(I)}$. Once capital is fixed the firm's cost function becomes: $(w/K(i))x(i)^2 + \mathrm{r}K(i)$ where $rK(i)$ is capital cost and $(w/K(i))x(i)$ is private marginal cost falling in the firm's holdings of capital.

Firstly, let us assume that there is a continuum firms within the monopolistically-competitive sector and let them be distributed on $[i_l , i_u ]$, and assume that firms are ranked so that $dF(i)/di > 0$, i.e.,

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the higher is the index of the firm the higher is the firm's fixed cost. In addition, let there be one numeraire good with a price equal to one. In this case consumers' expenditure on the total consumption bundle ($\boldsymbol{x}$) is $\int_i p(i)x(i) + x_m x_m$ , $i \in \tau$, where $\tau$ are the commodities actually produced (out of the overall potential in the monopolistically-competitive sector) and $x_m$ is the numeraire. Consumers' utility function and the production function have the following forms:

(1) $\quad u(\boldsymbol{x}) = G(m) + x_m , \qquad m = \int_i x(i)^b \quad i \in \tau$

(2) $\quad x(i) = R(i)^{\mathrm{a}} F(i)^{1-\mathrm{a}} , \qquad 0 < \mathrm{a} < 1$

Here $G' > 0$, $G'' < 0$. We can think of (1) as the limiting case of $u(\boldsymbol{x}) = G(\sum_i x(i)^b ) + x_m$ . In this variant of the model we have many products each of which constitutes a small fraction of sales in the industry's market. The production function have continuous marginal cost functions, the restriction used by Spence, and in addition marginal costs are increasing. We take capital to be fixed prior the decisions on how much to produce. In our context fixed costs are thus capital costs. From these equations we can solve for the firm's resource use given its fixed costs. In the Cobb-Douglas case private and social costs per firm (in section 4 we discuss general specifications of external effects) are given by:

(3a) $TC(x(i)) = x(i)^{1/a} F(i)^{-(1-a)/a} + F(i)$

and

(3b) $STC(x(i)) = (1+s)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i)$

We shall use the assumption that $dm/dx(i) = 0$, together with the specification of consumers' utility, to describe a situation where the individual firm is negligible relative to the market while it faces a downward-sloping demand curve. This follows Spence and a similar argument is used in Dixit and Stiglitz (1977) assuming that firms are symmetric. It is argued in Hart (1979, 1985a and 1985b) that fixed costs is incompatible with small firms in a finite economy. This does not apply to Dixit and Stiglitz's approach since they explicitly consider the case where the number of firms goes to infinity. On the other hand, Dixit and Stiglitz requires a discontinuity in the marginal rate of substitution at the equilibrium.[6] In Hart (1979) it is shown that the monopolistically competitive equailibrium is Pareto optimal when firms are small (in the replicated economy) and when a firm's

[6] Consumers are identical and they use all goods. For this reason the equilibrium price of a monopolistically competitive good is determined by the marginal rate of substitution between the good and a numeraire good. When the marginal rate of substitution is continuous function of per capita consumption the price is insensitive to a firm's supply since the individual firm's supply is only a small fraction of total supply. To obtain monopoly power one must assume that the marginal rate of substitution is discontinuous.

output is distributed accross all of the consumers in the economy. On the other hand, when each consumer uses only  subset of all of the potential goods it can be shown that each firm can be small ($dm/dx(i) = 0$) while it has monopoly power (Hart 1985a). In general, there can be too many or too few products in the (symmetric) equilibrium (Hart 1985b). Here we follow Spence but in the discussion we shall come back to Hart's analysis of the problem.

The major analytical problem in the monopolistic-competition model with differentiated goods is to describe which products we ideally would like to be in the market and compare this set of products to the products that are able to survive. With respect to this problem the utility and cost functions specified here allow us to concentrate on (calculable and monotone) survival coeffcients. The social survival coefficient is defined as the ratio between the firm's cost and its contribution to social welfare. With the utility function in equation (1) the latter is given by $G'(m)x(i)^b$ . In this case the problem of minimizing $STC(x(i))/ G'(m)x(i)^b$ has a well-defined solution. This solution, the social survival coeffecient is a measure of the (most favourable) social cost of the firm's activities relative to the benefit they generate, and the social survival coefficient thus tells us how products compete from the socially optimal point of view. In a similar way we can define a market survival coefficient. This is a measure of the ratio between the firm's cost and its revenue and it defines the firm's ability to survive in the market.

3.1 The Social Optimum

Consumers' surplus is:

(4) $$CS\,(x(i)) = G(m) - \int_i p(i)x(i)di \qquad i \in \tau$$

Taking into account firms' profit, which account for private costs, and subtracting the external costs, the welfare function is:

(5) $$W(\boldsymbol{x}) = G(m) - \int_i ((1+s)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i))di \quad i \in \tau$$

Formally, the problem is to maximize welfare with respect to $\tau$ and $x(i)$. Owing to Spence the easy way to recover the social optimum is to consider how a given value of *m* is attained. That is, we address directly the question of which commodities to introduce and which to leave out. Clearly, for the allocation to be optimal for a given *m*, say *m**, *m** should be provided at minimal social costs. That is, firms are to be introduced into the market according to their social survival coeffcients and each with a production volume satisfying:

(6) $$\rho(i) = \min_{x(i)} \; x(i)^{-b} ((1+s)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i))$$

Firms are introduced according to $\rho(i)$, from smallest to largest, and introduced in that order. The solution to (6) is:

(7a) $$x^*(i) = \psi(1+s)^{-a} F(i), \; \psi = ((1/a-b)^{-1} b)^a$$

Consider the ranking of the different commodities and their fixed cost. We have $\rho(i) = \rho' F(i)^{1-b}$ with $\rho' = (1-ab)^{-1} \psi^{-b} (1+s)^{ab}$ with $d\rho(i)/dF(i) > 0$ using $x^*(i)$ in the expression for the social survival coeffiecient. Since $d\rho(i)/dF(i) > 0$ commodities are to be introduced from the firm having lowest set-up cost and towards firms with higher cost, that is from $i_l$ and upwards. The full optimization problem is solved for:

(7b) $G'(m) = \rho(n)$

where $n$ is the marginal product. With respect to the social survival coefficient, this is increasing as set-up costs are increased by introducing more firms. On the other hand, the marginal utility of yet another firm (an increase in $m$) is decreasing and the social optimum is, thus, uniquely defined. Notice that (7b) implies that the marginal firm's profit is exactly zero, while it is positive for inframarginal firms.

Notice that $d\rho(i)/dF(i) > 0$, that is, firms are ranked like in Spence and this implies that we ideally want to see firms with low costs in the market. In our paper high fixed cost is a drawback since it tends to increase the value of the social survival coefficient through the last component in the expression for a firm's costs: $(1+s)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i)$. On the other hand, the social survival coeffcient tends to fall with increases in the fixed cost since high fixed cost substitutes ressource. In the case discussed here the former effect dominates. In section 4 we consider a situation where $d\rho(i)/dF(i) < 0$.

3.2 Resource Taxes

Using the utility function in equation (1) the firm's inverse demand function is given by $p(i) = G'(m)bx(i)^{b-1}$ . The regulator controls firm output through the tax. But the tax also changes firms' profit, and, through the marginal firm's zero-profit restriction, the tax is also a determinant of the number of firms operating in the monopolistically-competitive industry. With firm specific taxes firm $i$'s profit is $\pi(i) = G'(m)bx(i)^b - ((1+t(i))x(i)^{1/a} F(i)^{-(1-a)/a} + F(i))$. The quantity $m$ is important. When it increases (and it will all other things equal when more firms are introduced), $G'(m)$ falls, because of the concavity of $G(m)$ and the firm's ability to survive in the private market (as opposed to the social optimum) is determined by how small the number $(bx(i)^b)^{-1}((1+t(i)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i))$ can be made. In the market economy, firms are entering according to the following market survival coefficients:

(8) $$\eta(i) = \min_{x(i)} (bx(i)^b)^{-1}((1+t(i)x(i)^{1/a} F(i)^{-(1-a)/a} + F(i))$$

The choice of $x$(i) minimizing (8) is:

(9) $$x'(i) = \psi(1+t(i))^{-a} F(i), \psi = ((1/a-b)^{-1} b)^a$$

and using this in the expression for the market survival coefficient we have: $\eta(i) = \rho' b^{-1} F(i)^{1-b}$ with $\rho' = (1-ab)^{-1} \psi^{-b} (1+t(i))^{ab}$. The first thing to notice with respect to the market survival coefficient is that the ranking under the tax follows the first best ranking for $t(i) = s$. The values of the market and the social survival coefficients differ since it is clear that $\eta(i) > \rho(i)$ from $b^{-1} > 1$. Notice also (compare (7a) and (9)) that $x'(i) = x^*(i)$. Proposition 3 is proved in the appendix.

Proposition 3. The Pigouvian tax, $t(i) = s$ introduces firms in the socially optimal order, but each active firm over-supplies relative to the first best optimum and the number of firms is too low relative to the optimum.

Consider now the optimal tax. The optimal tax structure maximizes the market's performance from the social welfare point of view. The optimal tax is found straightforwardly paying attention to the assumption that changes in firm output leaves market congestion unaffected. That is, when the regulator considers firm *i*'s tax, he needs only to take into consideration the *i*'th commodity. To optimize welfare, the regulator must set the tax on this good so that its marginal contribution to consumers' utility equals its marginal social cost. We have the (socially relevant) marginal contribution given by $G'(m)bx(i)^{b-1}$ and we have the social marginal costs given by $1/a(1+s)x(i)^{1/a} F(i)^{-(1-a)/a} x(i)^{-1}$. Combining this with $G'(m)b^2 x(i)^{b-1} = 1/a(1+t(i)x(i)^{1/a} F(i)^{-(1-a)/a} x(i)^{-1}$ we arrive at $b(1+s) = 1+t(i)$, or $t(i) = b(1+s) - 1$. In the appendix we prove:

Proposition 4. The optimal resource tax is given by $t = b(1+s) - 1$ per unit of ressource use. Compared to the first best allocation each firm over-supplies and the number of firms is too low.

When $b = 1$, the case of perfect competition, the tax reduces to the marginal external damage but otherwise it is strictly less. It is straightforward to understand the problem. Consider the marginal firm, and notice that the firm's total cost equals $F/(1-ab)$ irrespective of the tax. The firm's revenue is $G'(m)bx(n)^b$, and the adjustment towards lower production as a result of the tax will lower its revenue. Thus, for a fixed congestion value this firm cannot break even. The marginal firm's demand curve should be shifted up for the zero-profit condition to be met. A decreasing value of the congestion index is the only way that this can happen. And the upper limit on the congestion index sets the limit for the already-too-small number of commodities in the industry.

4. Discussion

So far we have analyzed the case where the socially-optimal allocation calls for firms with low fixed costs. An example where high fixed cost firms are advantageous is the case of $c(i) = K/F(i)$ and $b > ½$. Here high fixed cost is an advantage since the external effect is sufficiently strong relative to the effect of costs upon consumers surplus: $d\rho(i)/d\,F(i) < 0$.[7] Commodities are, consequently, to be introduced from the firm having highest set-up cost and towards firms with lower set-up cost, that is from $F(i_u)$ and downwards. The full optimization problem is solved for $G'(m) = \rho(n)$, where $n$ is the marginal product. Like before, the social optimum is uniquely defined: This follows since the social planning survival coefficient is decreasing as set-up costs are decreased by introducing more firms while the marginal utility of yet another firm (an increase in $m$) is decreasing.

Consider now optimal firm-specific taxes. Profit maximization calls for $G'(m)b^2 x(i)^{b-1} = c(i) + t(i)$. Once again, when changes in firm output leaves market congestion unaffected the regulator considers firm $i$'s tax without paying attention to other goods. He should therefore set the tax on this good so that its marginal contribution to consumers' utility equals its marginal social cost. That is, the tax on this commodity satisfies $G'(m)bx(i)^{b-1} = (1+s)c(i)$. combing we have $t(i) =$

[7] The relevant survival coefficient is $\rho(i) = \min_{x(i)} x(i)^{-b}((1+s)c(i)x(i)+F(i))$. The solution is $x^*(i) = ((1-b)(1+s)c(i))^{-1} bF(i)$, $c(i) = K/F(i)$. Using this we have $\rho(i) = (1-b)^{-1}(b^{-1}(1-b)(1+s))^b F(i)^{1-2b}$ and $d\rho(i)/d\,F(i) < 0$, for $b > ½$.

$b(1+s)c(i)$. The firm $i$'s profit is $\pi(i) = G'(m)bx(i)^b - ((c(i)+t(i))x(i) + F(i))$ and the tax introduces firms in the socially optimal order.[8]

So far we have derived the expression for the optimal tax using specific functional forms. The purpose of the choice of the functional forms is that we can solve for the precise ranking of the firms. To investigate the question of the optimal tax more generally assume that the tax picks out firms in the socially-optimal order. Assuming this our results generalize. Take the optimal tax given by $1 + t = b(1 + s)$. Denoting by $D'(.)$ the marginal damage of firm $i$'s resource use we can express the optimal tax in terms of elasticities and marginal damage. Since $m$ is fixed from the individual firm's point of view we have $\mathrm{d}P/\mathrm{d}x(i) = G'(m)(b-1)bx(i)^{b-2}$ and it follows that $E^{-1} = b - 1$ where $E$ is the price elasticity relevant to the $i$'th firm. We can rewrite and obtain $1 + t = (1 + E^{-1})(1+D'(.))$. Let us write $P_i(x(i), m)$ for the price of good $i$. In monopolistic competition we have $\mathrm{d}m/\mathrm{d}x(i) = 0$. The firm's private costs are $(1+ t(i))R_i(x(i), F(i))$ where $R_i(x(i), F(i))$ is resource use and the firm's external effect is given by $D(R_i, R_j)$, $j \neq i$. Proposition 5 is proved in the appendix:

Proposition 5. The optimal tax is defined by $(\mathrm{d}R_i(x(i), F(i))/\mathrm{d}x(i) + \mathrm{d}D(R_i, R_j)/\mathrm{d}x(i))(1 + E(x(i))^{-1}) = (1+ t(i))\mathrm{d}R_i(x(i), F(i))/\mathrm{d}x(i)$, where $x(i) = x(i, t(i), F(i))$, when the tax so defined picks out firms in the socially optimal order.

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[8] The relevant survival coefficient is $\eta(i) = \min_{x(i)} (bx(i)^b)^{-1}((t(i) + c(i))x(i)+F(i))$. The solution is $x'(i) = ((1-b)(t(i) + c(i))^{-1} bF(i), = K/F(i)$. Using this we have $\eta(i) = (b(1-b))^{-1}((1-b)(b(1+s)+1)b^{-1})^b K^b F(i)^{1-2b}$, for $b > ½$.

Proposition 5 reflects that the tax levied at the individual firm serves two purposes: it should correct for the externality and it should correct for the divergence between price and marginal revenue due to monopolistic competition. When the regulator sets a tax he manipulates the firm's choice of output. This in turn affects the difference between the price and the marginal revenue and also the total marginal costs. The tax is choosen so as to bring equality between the marginal benefit and the marginal cost. The tax is strictly less than the marginal external cost. Some papers discuss output externalities and output taxes; Buchanan (1969), Lee (1975) and Barnett (1980) discuss the monopoly case, Lee (1999) analyses homogenous oligopoly with free (non-strategic entry) while Carlton and Loury (1980) discusses pollution taxes in perfect competition economies with free entry and U-shaped average cost curves. In the case of output externalities we have that the damage function is given by $D(x_i, x_j)$ and the optimal output tax in the monopolistically competitive environment is described in proposition 6 (proved in the appendix).

Proposition 6. The optimal output tax is defined by $(\mathrm{d}R_i(x(i), F(i))/\mathrm{d}x(i) + \mathrm{d}D(x_i, x_j)/\mathrm{d}x(i))(1 + E(x(i))^{-1}) = \mathrm{d}R_i(x(i), F(i))/\mathrm{d}x(i) + t(i)$, where $x(i) = x(i, t(i), F(i))$, when the tax so defined picks out firms in the socially optimal order.

Our result on the optimal tax structure is qualitatively like that obtained in monopolistic competition situations, the formula for the optimal tax is the same. The tax is strictly less than the marginal external damage and the explanation borrows clearly from the discussion of externalities in monopoly. Ignoring external effects, a monopolist produces at a point where the price is strictly above marginal cost. Thus, the starting point is that production should be regulated in the upwards direction. Adding external effects to this picture may, once the externality is sufficiently important, call for lower production relative to the monopoly output but the tax will (obviously) never exceed the marginal external cost. On the other hand, the right tax (which can be negative) would implement the first best in the monopoly situation. This does not extend to the case of monopolistic competition. We have given two examples (one where low cost firms are desireable from the social point of view and one where high cost firms are preferable) where the best tax leaves the monopolistically competitive sector with too little product variety. The problem here derives from the zero profit condition in combination with fixed costs and the interaction between firms' decisions.

Propositions 5 and 6 assume that the tax picks out firms in the socially optimal order and that a change in one firm's output leaves the overall market unaffected. When the latter assumption applies it does not matter whether the market supports too few or too many firms. In our examples there are too few firms in the market but Hart (1985b) shows that we can not establish clear cut results on the number of firms in the market relative to the socially optimal number of firms. In the symmetric case, $F(i) = F(j)$, we only have to deal with the number of firms or goods since it is irrelevant form a welfare point of view if one firm is replaced with another. In this case we can use the results in Hart (1985b) to generalize our findings. A symmetric monopolistically competitive equilibrium $(p^*, y)$ satisfies: $p = p^*$ maximises $pd(p,p^*,y) – C[d(p,p^*,y)] - td(p,p^*,y) – F$ and $pd(p,p^*,y) – C[d(p,p^*,y)] - t\, d(p,p^*,y) – F = 0$ where $p$ is the price set by one firm when other firms set $p^*$, $d(p,p^*,y)$ is the demand for the firm's product, $C[]$ is variable cost and $t$ is the tax on the firm in question. The variable $y$ is the fraction of operating firms out of all of the potential firms. In the limiting (replicated) economy the entry of one more firm leaves $y$ unaffected. Profit maximization by the individual firm calls for: $p^*[1+ E(y)^{-1}] = C'[d(p,p^*,y)]$.

When the regulator sets a tax, price shall equal marginal costs (here including external costs). If this is not the case the tax can be adjusted marginally leaving *y* unchanged but bringing each active firm's decisions more in line with the socially-optimal allocation. That is: $p^* = C'[d(p,p^*,y)] + EC'[d(p,p^*,y)]$. Combing with $p^*[1+ E(y)^{-1}] = C'[d(p,p^*,y)]$ we have $(C'[d(p,p^*,y)] + EC'[d(p,p^*,y)])[1+ E(y)^{-1}] = C'[d(p,p^*,y)]$. Despite of the similarity with propositions 5 and 6 this result is of a different nature. The most important differences are found in the utility functions applied by Hart and in the explicit treatment of the economy's size. In this case the utility function does not rely on discontinuity in consumers' marginal rate of substitution (in equilibrium). What is required here is that each consumer consumes only some of the potential product available. This rule out what is known as neighbouring goods. Since the individual firm does not have neighbours it can safely ignore (strategic) reactions from other firms. When this is the case a firm can be small and yet enjoy monopoly power. With respect to the size of the economy the number of firms goes to infinity. In the finite economy equilibrium profits are zero for all firms when they are symmetric and nearly zero when they differ. In the limiting economy profits are zero in both cases. Since the number of firms are explicitly considered one can not argue in this case that fixed costs make the economy small.

## 5. Conclusion

Taxes are much favoured in environmental regulation because of their efficiency properties. In this paper we consider environmental regulation in a market distorted also by product selection bias. Imperfect competition and environmental taxation is studied in a number of contributions, for example studies on monopoly and environmental taxes (Buchanan, 1969; Lee, 1975 and Barnett, 1980), and environmental taxes in oligopolistic markets (Lee, 1999, Katsoulacos and Xepapadeas, 1996; Carraro and Soybeyran, 1996).

but the problem analysed here. Our starting point is Spence's (1976) original analysis of product selection bias, and we have added external effects.

Three aspects have been emphasized. Firstly, and following directly from Spence's analysis, a tax can change product selection bias in markets where set-up or fixed costs play an important role. When firms' marginal costs and their variable costs are unrelated, as is the case in Spence, there is bias against products involving high fixed costs. Secondly, in a model where marginal costs are negatively related to fixed costs we show that the Pigouvian tax introduces products in the socially optimal order (for the generalized constant elasticity of substitution utility function employed here). This result is suggestive with respect to policy implications. One can imagine that the Pigou tax picks out the wrong kinds of products, i.e., it directs the economy towards a situation where the monopolistically-competitive industry is dominated by the worst firms (from the social point of view). This is not the case and one can appeal to Oates and Strassmann (1984) to argue in favour of a simple Pigouvian tax. The point made in Oates and Strassman is that when the regulator, in the context of a monopoly market, neither has the information nor the authority to impose a tax reflecting monopoly power, then the loss of using the Pigouvian tax (relative to the optimal) is very limited. In our setting the divergence between marginal revenue and the price is less than under pure monopoly and this makes Oates and Strassmannn's point stronger. Thirdly, even if it is somehow politically feasible to set firm-specific taxes, the regulator is unable to support the first best, showing that the situation of externalities in monopoly markets does not generalize to monopolistic competition.

Spulber (1985) discusses another type of problem related to the Pigouvian tax. Typically, this tax is analysed in the context of simple models where output or input is related in a simlple way to the final value of emissions. However, in a multi-input case the relationship between inputs and emission is complex and a tax on one or more inputs or a tax on the firm's output fail to support the first-best allocation. Spulber (1985) shows that a tax based on the value of the firm's emission (rather than on inputs or output) will actually support the first best. That is, the problem here is the base of the tax. This is not the case in our setting. Thinking in terms of a constant returns to scale production function and assuming making our simplifying assumption on the marginal external damage we can not find a set of (firm specific) taxes supporting the first best. We need more instruments (one set to control for emissions and one to correct for product heterogenity). Our problem seems to be in the number of taxes rather than in the base of the tax.

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Appendix

Proof of proposition 3. Consider the two relations characterizing the market. Profit maximization calls for equality between marginal revenue and marginal cost at the firm level. As noted, the market selects commodities in the socially optimal order, and therefore we have the second relation: Firms enter from the top until the profit of the marginal firm or product (called $n$) is zero. That is:

(A.1) $G'(m)b^2 x(i)^{b-1} = 1/\mathrm{a}(1+t(i)x(i)^{1/\mathrm{a}} F(i)^{-(1-\mathrm{a})/\mathrm{a}} x(i)^{-1}$

(A.2) $G'(m)bx(n)^b = (1+t(n))x(n)^{1/\mathrm{a}} F(n)^{-(1-\mathrm{a})/\mathrm{a}} + F(n)$

Equation (A.1) defines firm output as a function of the model's parameters and $m$, the market congestion index, and the firm specific tax. Consider the marginal firm. Eliminating $G'(m)$ and rewriting we have $x(n) = \psi(1+t(i))^{-\mathrm{a}} F(i)$, that is, $G'(m) = \eta(n)$. Using the relationship between $\eta(.)$ and $\rho(.)$ we have $G'(m) = b^{-1} \rho(n)$ . Consider an intermediate firm with profits $G'(m)\, bx(i)^b$ - $(1+s)x(i)^{1/\mathrm{a}} F(i)^{-(1-\mathrm{a})/\mathrm{a}}$ - $F(i)$, where $t(i)$ is replaced by $s$. Let us define $\eta'(i) = (bx(i)^b)^{-1}$ [ $(1+s)^{1/\mathrm{a}} F(i)^{-(1-\mathrm{a})/\mathrm{a}}$ - $F(i)$] and rewrite profit as $\pi(i) = (G'(m)- \eta'(i))bx(i)^b$ . The first order condition for a profit maximum is $b^2 x(i)^{b-1} ((G'(m)- \eta'(i)) - bx(i)^b \mathrm{d}\eta'(i)/\mathrm{d}x(i) = 0$. With respect to the marginal firm the zero-profit condition implies $G'(m)- \eta'(n) = 0$ and we have $\mathrm{d}\eta'(n)/\mathrm{d}x(n) = 0$. Inframarginal firms have positive profits, $G'(m) > \eta'(i)$, and we have $\mathrm{d}\eta'(i)/\mathrm{d}x(i) > 0$, that is $x(i) > x'(i)$. However, with $t(i) = s$ we have $x'(i) > x^*(i)$ and we see that each inframarginal firm over-supplies relative to the social optimum. Consider the number of firms. When a firm is introduced into the market it over-supplies and because of this $G'(m)$ tends to fall more quickly when compared to the social optimum. At the same time we have that the market survival coefficient is above the social survival coefficient and this gives the conclusion.

Proof of proposition 4. Let us consider the survival coefficients. We have $\eta_s(i) = b^{-1}\rho(i)$ with $t(i) = s$. Now, for $t(i) = b(1+s)-1$ we find $\eta_t(i) = b^{ab} b^{-1}\rho(i)$. That is: $\eta_s(i) > \eta_t(i) > \rho(i)$. Consider firm output. From the proof of proposition 3 we have compared firms' output under the tax $t(i) = s$ and in the socially optimal allocation. When the tax is lowered to $b(1+s) - 1$ output increases even further. For this reason $G'(m)$ will decrease more rapidly with the introduction of firms.

Proof of Proposition 5. In the monopolistically competitive market firm i's product will sell at $P_i(x(i), m)$ where monopolistic competition is characterised by $dm/dx(i) = 0$. The firm's private marginal cost is defined by its resource use and we will write it as $R_i(x(i), F(i))$. Since we tax resource use the cost becomes $(1+t(i))R_i(x(i), F(i))$ when the tax is introduced. Profit maximization calls for:

$$\text{(A.3)} \qquad P_i(x(i), m) + dP_i(x(i), m)/dx(i) = (1+t(i))dR_i(x(i), F(i))/dx(i)$$

Equation (A.3) defines $x(i)$ as a function of $t(i)$ and $F(i)$. Let us write this as $x(i) = x(i, t(i), F(i))$ and rewrite to:

$$\text{(A.4)} \qquad P_i(x(i), m)(1 + E(x(i),t(i))^{-1}, m) = (1+t(i))dR_i(x(i), F(i))/dx(i)$$

Once again, when the market congestion index is unaffected by the action of the individual firm, firm $i$'s production must satisfy

(A.5) $P_i\,(x(i), m) = \mathrm{d}R_i\,(x(i), F(i))/\mathrm{d}x(i) + \mathrm{d}D(R_i\,, R_\mathrm{j}\,)/\mathrm{d}x(i), \quad \mathrm{j} \neq \mathrm{i}$

Combining we have

(A.6) $$(\mathrm{d}R_i\,(x(i), F(i))/\mathrm{d}x(i) + \mathrm{d}D(R_i\,, R_j\,)/\mathrm{d}x(i))\ (1 + E(x(i),t(i))^{-1}\ , m)$$

$$= (1+ t(i))\mathrm{d}R_i\,(x(i), F(i))/\mathrm{d}x(i)$$

where $x(i) = x(i, t(i), F(i))$.

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