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LABOUR, AN EQUIVOCAL CONCEPT
FOR ECONOMIC ANALYSES

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Labour, An Equivocal Concept for Economic Analyses

Abstract:
Ever since the adoption of the symbiosis of mathematical analytical methods and marginalism in economics, there has been a tendency of negligence in the sense that it is seldom to ask the dual question: what are the fundamental assumptions on which the above approach is resting and are they worth while as measured by realia? In a rather informal form we try to address some aspects of these basic questions. A main result is that it is very doubtful to rely too naively on main stream results, relying on a one-dimensional concept of homogenous labour. Most of the standard results on this foundation - including those of unemployment - are ambiguous, at best.

Keywords: heterogenous labour, measurement and mathematical representability, economic methodology
JEL: B4, C0, E1, J3

1. Introduction

One the one hand, the fundamental idea of this paper is not new. The article by Roy [1951] described a tribal population which could be divided into two groups of individuals, one of which was relatively more qualified in hunting and the other one relatively more qualified in fishing. Under straightforward assumptions[1] an optimal behaviour of the society meant that the 'hunters' went hunting and the 'fishermen' went fishing, cf the Ricardian theory on comparative advantages or in more general terms division of labour.

One the other hand the methodological implications of such a heterogeneity setup seem to have been left completely out of mind in main stream economic theory, where more or less tacitly it is assumed that heterogenous individual labour supplies are aggregable in homogenous units and hence are collectively representable by a one-dimensional quantity, denoted labour and assumed to consist of perfectly divisible and substitutable units.

In the sequel we shall try to refocus on the heterogeneity aspect of individual labour suppliers, investigate some of its implications and try to trace the compatibility of such a generalized approach with standard theoretical results.

Implicitly the concept L covers the aggregate of de-individualized labour suppliers or if individualized it is assumed to represent the product of the number of identical labourers times the number of their working hours, tacitly set to an arbitrary constant, say 1.

Our presentation takes the form of a bottom-up journey, running from N heterogenous individuals to a construction of an L, representing the labour supply of the whole population.

2. Heterogenous individuals mapped onto a measure space

Let us assume that the labour force is constituted by N heterogenous people and also explicitly that the number of working hours is equal to one time unit. The N individuals all offer their services at the labour market(s).

To operationalize the concept of labour supply we must introduce a series of assumptions, enabling us to create an economy-independent, de-personalized measure of labour services.

First we postulate that each individual is endowed with a (finite) set of relevant attributes that are measurable by means of some external 'yardstick'; we leave out redundant attributes, ie dimensions or characteristics that are irrelevant to

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[1] Eg excluding a lexicographic ordering of outcomes.

any environment in the context, eg hair colour. The attributes may be (partly)

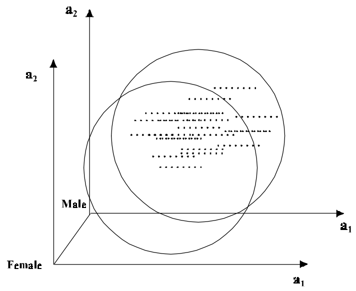

Figure 1. **Ability or attribute distribution**

interdependent; if exact relationships exist, some dimensions may be left out.

In mathematical terms the individual h is represented by a vector $a_h \in R^M$. By this assumption we have mapped the whole population onto an M-dimensional space, representing eg physical strength, weight, age, IQ, sex[2]etc.

For M=3 the idea could be sketched as a frequency distribution in R^M like figure 1.

3. Construction of capabilities

The second step is twofold. Firstly, we claim that the attributes are (partly) substitutable[3], enabling us to create a classification of individuals into equivalence classes and secondly that different kinds of jobs or sectors rank individuals differently.

For M=2 in figure 2 the line AA' collects those personal attribute combinations that identify the individuals that are equally productive or capable in sector A, say at some level X^1 ; likewise BB' collects equally productive individuals in sector B, say Y^1 . Hence, individual h is able to supply either a capability of X^1 in sector 1 or Y^1 in sector 2.

If for expository reasons we restrict ourselves to only two (S=2) job types - or sectors - we are able to illustrate the second main mapping, viz from the ability

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[2]It is no restriction to let some of the characteristics be represented by a finite set of discrete values, eg of the binary type.

[3]Of course it would be possible to apply a more general ordering relationship, eg of the lexicographic type; such an extension would not add any aspects at the level of principles to our setup, however.

space onto the capability space, R^S .

The mutually exclusive capabilities of h in the two jobs/sectors are indicated by a plot, highlighted in the general frequency distribution of individuals according to the S(=2) capabilities. Thus in figure 3 we have withheld the individuality, except

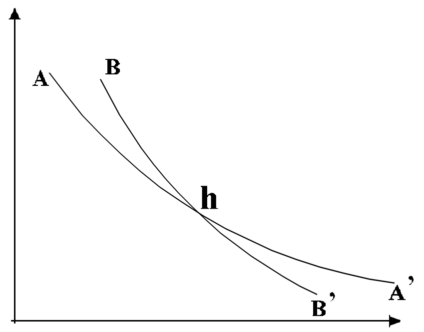

Figure 2. Ability space

for multiple plots; in principle we are able to identify individuals by an inverse tracing procedure.

Along the dotted line in figure 3 we find all the individuals that are equally

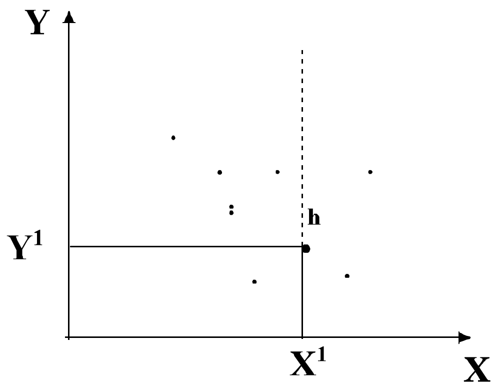

Figure 3. Capability space

capable in sector X; they are perfectly substitutable in that sector, but certainly not in sector Y. Referring to figure 2, the dotted line collects the AA'-people; obviously, under general assumptions they are not substitutable in sector Y - that would require AA' to coincide with BB'!

4. A one dimensional capability

Next it is natural to ask: Under what conditions will the scattered pattern in figure

3 degenerate into a straight line, starting in origo - or more generally into a continuous single line, tantamount to a unidimensional frequency distribution. In the

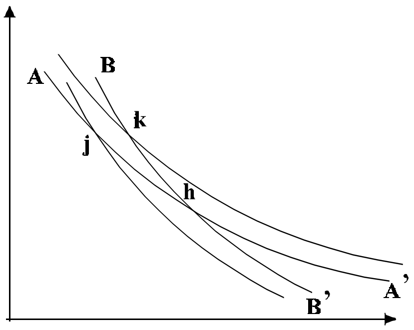

Figure 4

linear case the answer is simple and easily carries over to the general case.

Let us reproduce figure 2 in a slightly elaborated form. Here h is equivalent to j in sector X and equivalent to k in sector Y; hence, if those individuals that are equivalent to h in sector X should be equivalent to each other in sector Y as well, any crossing of the iso-curves is ruled out. All classification curves must be 'parallel', a result with a direct analogy to the Sono-Leontief aggregation requirement! Any homothetic ordering relation meets this requirement.

So, if we wish to operate with a universal concept of capability, we must accept that *differences in inherent attributes do not give rise to variations in the ranking of people according to job*; the intelligent person is able to compensate fully for lack of physical strength in any job etc. This is a very restrictive, let alone absurd, assumption which in fact is so restrictive that it ought to stop the aggregation approach, but in economic theory it does not.

5. Homogenous labour

By acceptance of the above absurd restrictive assumption and eo ipso having implied a one-dimensional frequency distribution in universal capability units, we now take the last step to arrive at the aggregate labour concept L.

The step implies that we transform the frequency distribution in figure 5 into some definite number L.

The measure sought should obey the rule of perfect substitutability which means that any combination of individual capabilities that *add up* to some specific, fixed sum is equivalent in all respects to a single individual who by himself is able to supply the same capability. If we denote individual capabilities by *l* and assume that some aggregate L is defined by

$$L = l_k + l_j \tag{1}$$

then the effect of using the two persons k and j is the same as that of using person h, if his $l_h = L$

In other words we rule out super- and sub additivity[4,5] between l's or in more plain words we disregard interdependencies–or neighbour effects–between

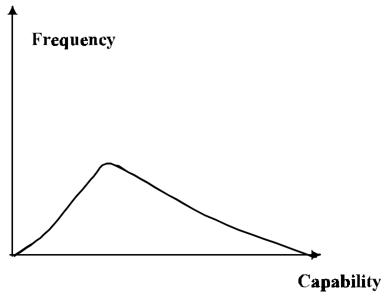

Figure 5. Universal capability distribution

individuals and furthermore: two stupid guys can substitute a clever one.

It should be noticed that we are not discussing the time dimension; if, in that respect we rule out super and sub additivity we would consider two equally potent persons on a half time basis as equivalent to one of them on a full time basis. But, that is another matter. In our situation all members of a team are working for the

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[4]By super additivity we mean that a convex combination of equally qualified individuals leads to a joint effect that exceeds that of one of the individuals alone, or

$$f(\varphi l_h + (1-\varphi) l_k) > \varphi f(l_h) + (1-\varphi) f(l_k)$$

$$where\ l_h = l_k\ and\ (0 \le \varphi \le 1)$$

Sub additivity corresponds to an inversion of the inequality sign.

[5]It should be noticed that super additivity between individuals means that the joint input of two individuals who represent the $[\varphi l_h\ and\ (1-\varphi) l_h]$ are more productive than individual h alone. The time dimension is fixed!

same fixed time period!

We can illustrate the situation for a three-person team by a simplex, representing the value L. Any team, consisting of (until) three persons, whose capabilities add up to L are equally productive in all respects.

Already having ruled out interdependencies, we have also ruled out the situation where a job cannot be solved by a single person, eg to lift a heavy stone. One person cannot, but two, working together can do the job. (This example is not

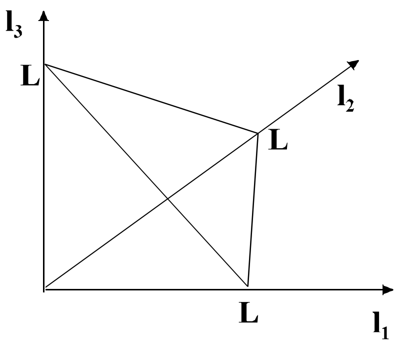

Figure 6

perfect!). cf Yndgaard [1997] on the interpretation of the marginal product of labour.

6. Refusal of substitutability

The restrictions needed to warrant a homogenous labour concept, are so unrealistic that we shall do without them; instead we shall build on the capability distribution directly, cf figure 3.

Assuming that the capabilities are mutually exclusive[6] and that individuals are searching the highest paid job, we can generate an allocative mechanism for the S-dimensional labour market.

Each sector alias job supplier pays a fixed amount per capability unit[7]: w_s where (s=1,...S). For S=2 we can illustrate the allocation mechanism by figure 7, where the division beam has a coefficient of inclination given by

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[6]The person is employed in one sector only - on a full time basis.

[7]This construct presumes that eg a double-qualified person is paid the double wage in comparison with a reference person; that is unlikely to be the case in practice, but that is another story, not to pe pursued here, cf deflection curves Yndgaard[1972].

$$\alpha = \frac{w_1}{w_2} \qquad \textbf{(2)}$$

since any wage earner looks for the maximum wage earning: $w_{max}=\{w_s\ l_{hs}\}$.

It might be defensible to assume that the conditions for setting up a homogenous labour concept within each sector are acceptable, but that is not very

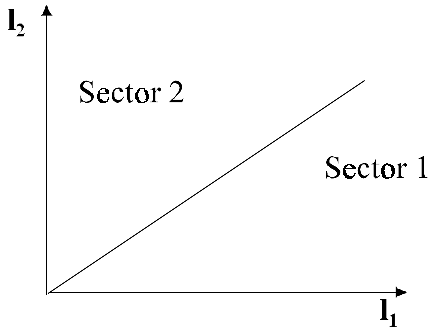

Figure 7

helpful - nor really well founded theoretically - since total labour supply is not well defined at all.

Of course, the initial distribution of capabilities is given and exogenous alias economy-independent; but actualized capabilities are endogenous, because the distribution on sectors is determined inter alia by the relative wage rates[8]! In other words, we end in the usual infinite regression: wage rates are determined by labour supply (and demand), but the potential labour supply is determined by wage rates. We are missing an equation, cf the Joan Robinson criticism on ia Solow.

To see this problem more clearly, we again restrict ourselves to the two-sector case, S=2 and let the abscissa axis represent the relative wage rates while we measure actual earnings-maximizing labour input in the two sectors on the ordinate axis. Here we have defined L_i as the simple sum of the capabilities of those individuals who prefer sector i; since a switch from one sector to another is not symmetric as measured by labour units; even under full employment total labour input is not constant. If we had defined labour by the number of individuals, obviously the two curves would have behaved symmetrically - that would correspond to identical individuals!

[8]It is worth stressing that we are not aiming at a segmentation of the labour market into disjoint subsets; such an approach would violate our basic idea of substitutability in basic abilities.

In short, even though total labour supply is fixed by the capability distribution, its actualization is endogenous!

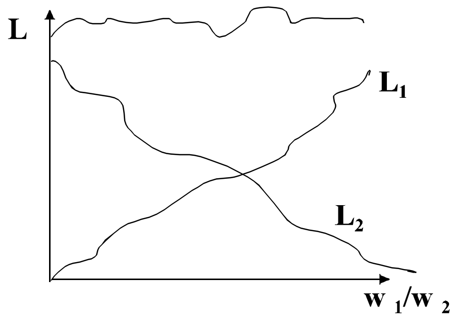

Figure 8

7. An operationalization of a labour market with heterogenous workers

Based on an allocation mechanism, cf below - and also Yndgaard [1978], let us begin with a situation where one sector/firm has hired a team of K workers:

$$l = \{l_1, l_2, \ldots l_K\} \qquad \textbf{(3)}$$

where the vector ***l*** is an enumeration of the individual capabilities in that sector.

If we rule out super- and sub additivity between the individuals and furthermore assume that any capability unit is equally productive, irrespective of its supplier, ie we assume perfect substitutability between all the units, supplied by the team, it is meaningful to define a gross labour concept for the sector looked at by adding the constituents or

$$L = \sum_K l_k \qquad \textbf{(4)}$$

Hence L can be interpreted as a point on a K-dimensional simplex.

Demand side for labour

By a very strong assumption the production function of the sector is defined in two factors, viz the L, as it was defined above and capital C, ie

$$z = f(C, L) \qquad \textbf{(5)}$$

This means that the marginal product w.r.t. labour is defined per capability unit, irrespective of origin.

To the firm, apparently this implies that it would not be concerned about the composition of the labour team, its number of participants and their capabilities, if only their total supply of capability units adds up to L. This is unrealistic. Hence, in our setup we produce a more satisfactory result than is usually provided by standard labour demand theory. In the latter case, firms do not care about the number of workers; they just hire the relevant number of homogenous units.

In reality, however, firms do care about *the number of employees*, because part of labour costs is proportional to that number, eg insurance costs, canteen space, toilet facilities, *capital equipment and composition* etc.

We shall proceed by setting up an optimization policy by means of a two-stage policy.

At the first stage, the firm finds out, what for some unit labour cost w the optimal L should be - this is standard[9] procedure and leads to an identification of some optimal $z_L^{'}$ and an implied optimal L.

At the second stage, firms strive to collect this optimal L by a minimum number of constituents, ie by solving the problem

$$\min_K \; s.t. \sum_K l_k = L \qquad \textbf{(6)}$$

a result that directly translates into the rule that firms 'engage in a top-down fashion'.

Supply side of labour

The other side of the labour market is the people looking for jobs; in accordance with common standard assumptions individuals are looking for the highest paid job.

The individual h having the potential capabilities ($l_1, l_2, l_3 ... l_S$) meets an S-dimensional wage rate vector: $w=(w_1, w_2, w_3, ... w_S)$, where the single w_s is the payment per capability unit in sector s; he prefers the sector, offering the highest earning or

[9]We disregard the (realistic) possibility that z_l' depends on the capital equipment and its composition!

$$\max_{s}\{w_s l_{sh}\} \quad \textbf{(7)}$$

which identifies the optimal sector to the worker, viz l_s where s follows from the above expression.

Since the wage rate is defined per L-unit, the individual is offered a personal earning that follows from

$$w_{hs} \leq p_s z'_{Ls} l_{hs} \quad \textbf{(8)}$$

in sector s.

To find out, if a person goes to sector s or sector q in the above notation he must calculate if

$$p_S z'_{L_s} l_s \geq p_Q z'_{L_Q} l_q \quad \textbf{(9)}$$

is met; if that is the case, he sticks to sector s else he prefers sector q.

In general–in the ‘two-dimensional’ case analysed here–this rule can be used to split the total labour force by means of a separating line with the equation

$$l_s \geq \frac{p_Q z'_{L_Q}}{p_S z'_{L_s}} l_q \quad \textbf{(10)}$$

cf figure 7 above.

8. Existence of an allocation

For a given vector of perfect competition prices: $\mathbf{p}=\{p_1,p_2,...p_S\}$ and a given labour-unit wage vector: $\mathbf{w}=\{w_1,w_2,...w_S\}$ the S sectors are able to define their marginal products of labour, measured in sectoral homogenous units and the corresponding optimal L-vector: $\mathbf{L}=\{L_1,L_2,...L_S\}$. We denote the marginal products by z and collect them in a diagonal matrix:

$$\mathbf{z} = \begin{Bmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_S \end{Bmatrix} \quad \textbf{(11)}$$

In this notation it follows that the optimal **L**s are found by solving the matrix equation,

$$\mathbf{pz} = \mathbf{w} \quad \textbf{(12)}$$

w.r.t. the implied Ls.

For now, let us assume that the prices and the wage rates represent a labour market with exactly full employment, then the single sector achieves exactly the L wanted and the single individuals are hired at the personal highest earnings possible. Let us elaborate at bit on the two-sector case.

For sector 1, as long as $p_1 z_1 \geq w_1$ the sector will hire from above the most capable workers, ie those with the highest l_{h1} who will be paid $p_1 z_1 l_{h1}$. Since their inclusion into the L_1 labour force reduces z_1 and under the assumption of full employment he can only come from sector 2, the z_2 goes up along with a reduction of L_2

$$pz = w \quad \textbf{(13)}$$

Let us illustrate the mechanism graphically. As drawn in figure 9, in both sectors the sectoral prices exceed those of the wage rates which means that both sectors drive the Ls level so high that the marginal product of one labour unit, ie z_1 becomes less than one, since in equilibrium this equation holds for all sectors. How does that result come about in the two sector case?

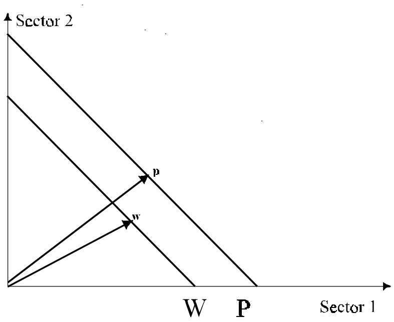

Figure 9. The simplices of w and p

In figure 10 let us walk along the abscissa axis from origo, ie where the number of employees in sector one $\#N_1$ is equal to zero. Here the marginal product is very high, but as the number grows, and $\#N_2$ falls accordingly, the z_1 will gradually fall. The rate of decrease will be irregular, since the employees contribute individually varying capabilities and not by one unit at a time. However, the two z's will cross somewhere[10].

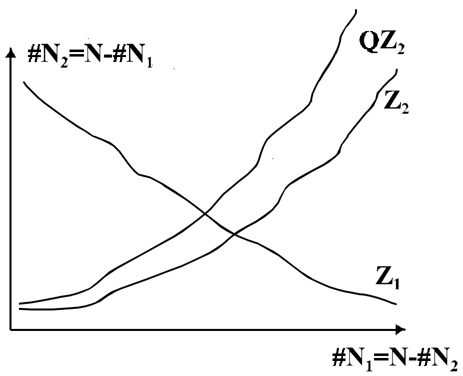

Figure 10. Two-sector allocation

From the marginal conditions for both sectors follow by division that

$$\frac{z_1}{z_2} = \left[\frac{\frac{w_1}{p_1}}{\frac{w_2}{p_2}} \right] \tag{14}$$

If we denote the relative real wage rates by Q we realize that there exists a solution (a fix point) where z_1=Qz_2, cf figure 10.

Obviously, this result carries over, mutatis mutandis to the S-dimensional case or stated differently, there exists a matrix **Z** so that the labourers are distributed onto all the S sectors. Of course, there may be persons who are indifferent between sectors; that situation corresponds to a representation of the z_s-curves in a hemi-continuous form in figure 10.

Summarizing, there exists a full employment allocation of N individuals on S sectors.

11. Classical and Keynesian unemployment

From formula (14) above it follows directly that for any set of relative wage rates–or a rather wide range–the relative prices can bring about a full-employment

[10]To carry through an existence proof, the use of Kakutani's fix-point theorem will be necessary.

allocation; likewise for a given price vector there can be found a wage vector that allocates all the individuals according to the behavioural foundation, set up above. Total specialization, ie all labourers employed in one sector, say one, follows from the w/p-ratio of that sector to be high enough in comparison with that of sector two and vice versa.

In figure 9, we have indicated the simplex levels of prices and wages by P and W respectively. This construct is helpful to illustrate how our model behaves under non full employment conditions.

If W is too high in comparison with P–in a standard model a too high real wage rate–the model produces a (neo-)classical unemployment situation. Two important differences are worth emphasizing though.

Firstly, the rate of unemployment or sectoral employment depends on the relative real wage rates; thus, it is not excluded that one sector has the same employment as it would have if the W had not been too high; conversely, if, ceteris paribus, a specific sector raises its wage claim too much–a sort of sectoral reservation wage–it may send labourers of all qualification levels to either other sectors or into unemployment. Its number of employees will fall relatively heavily, because it is rational to collect the reduced L by means of as few employees as possible.

Secondly, the unemployed individuals are identifiable as the least efficient or capable workers in general. This result seems to be in good accordance with stylized facts.

In a Keynesian setting, the marginal real wage rate is not prohibitive to an expansion of the labour force in that a marginal expansion of production would be profitable at the ruling prices and wage rates, if demand were not deficient. As an example of such a situation, let us imagine that the rate of interest is too high–perhaps for speculative reasons–to produce a demand volume that would match a full employment output. Obviously, here the firms would not use the above marginal condition as a foundation for hiring employees; instead they would replace the formula $pz_s = w_s$ by a formula like

$$p\lambda_s z_s = w_s \tag{15}$$

where λ_s is some systematic, sectoral under-estimation parameter of the marginal product of labour, ie $\lambda_s \leq 1$.

For an algorithm that solves such an allocation problem above in a general setting, cf Yndgaard [1978].

12. Concluding remarks

The concept of L - and C for that matter - as a unidimensional analytical tool for mathematical analysis of economic problems is widely used, but its foundation is seldom exposed to a close, more epistemological scrutiny. We have tried to demonstrate that many core results of economic analysis are ambiguous and that the precision apparently warranted by mathematical analysis is not generally substantiated.

In reality, the allocation of workers on activities is - at least partly - individualized, a result that is at odd with the whole idea of a homogenous labour concept. Other aspects of the labour market such as unemployment is founded on individuality, also; the distribution of unemployment on persons is not based on a random selection, corresponding to the implicit assumption of perfect substitutability between labour units, across individuals.

Even concepts like total labour supply and full employment are ambiguous, dependent as they are on the endogenous core phenomena of the economy, primarily prices and wage rates - and composition of capital equipment.

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