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- BETWEEN WEAK AND PEARCE'S EXTENSIVE
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A Notion of Consistent Rationalizability

– between weak and Pearce's extensive form rationalizability

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Abstract

Ben-Porath (1997) characterizes the strategies consistent with common certainty of rationality (CCR) at the origin of a generic game of perfect information. More generally, the notion of "weak extensive form rationalizability" (weak EFR) captures the implications initial CCR in an extensive form game. We go one step further by ascertaining at which additional information sets initial CCR can be maintained "consistently". Our consistency notion has two aspects: we examine whether there is "internal consistency" in assuming CCR at a given collection of information sets by using Battigalli and Siniscalchi's (1999) recent result while we introduce "external consistency" to account for all reachable information sets. For a class of games, including all belief-consistent games [cf. Reny (1993)], we identify a unique collection of information sets and hence a unique set of strategy profiles; moreover, we show that in this case our notion is outcome-equivalent to Pearce's (1984) EFR. But in general our notion is between weak and Pearce's EFR.

Journal of Economic Literature Classification Number: C72

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1 Introduction

The notion of rationalizability introduced by Bernheim (1984) and Pearce (1984) captures the implications of common belief (CBR henceforth) or certainty of rationality

*This paper succeeds my earlier work entitled "Consistent solutions for extensive games – How far can rationality go?" (April 1999). I thank Geir B. Asheim and two anonymous referees for comments and suggestions. All errors are mine.

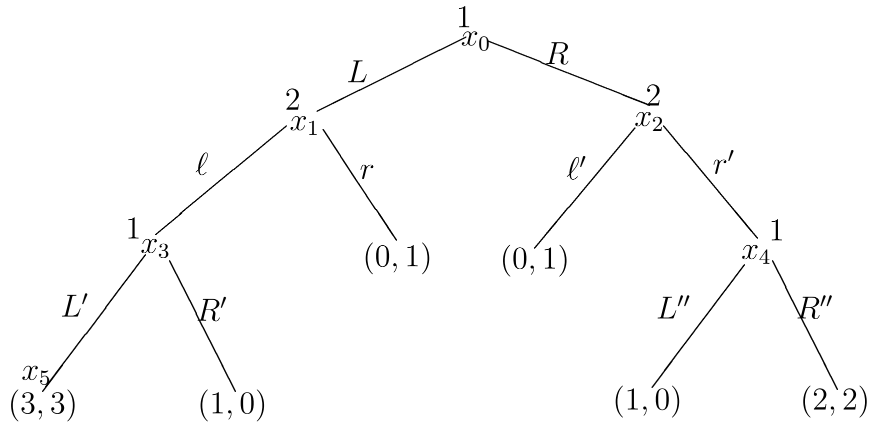
(CCR[1] henceforth) in a strategic form game. In extensive form games, however, it is in general impossible to maintain common certainty of rationality at every information set [see, e.g., Reny (1993)] and extensive form rationalizability [Pearce (1984), EFR henceforth] is subject to criticism for failing to take this fact into account. As Börgers (1991, p.4) wrote, "at early stages of the procedure [that defines EFR] a strategy of a player may be regarded as 'rational' and hence used to impose restrictions for admissible beliefs (and hence strategies) of the other players even though at later stages this strategy is eliminated".

To escape from the above difficulties, Ben-Porath (1997) considers the following questions: What is the consequence of CCR at the beginning of an extensive form game? Such a question is not vacuous because rationality in extensive form games (i.e., sequential rationality) is stronger than that in strategic games. Using a finite epistemic model, he showed that in a generic game of perfect information, the strategy profiles consistent with initial CCR are precisely those surviving the Dekel-Fudenberg (1990, DF henceforth) procedure which comprises one round deletion of weakly dominated strategies followed by iterated deletion of strictly dominated strategies. More generally, "weak extensive form rationalizability" (weak EFR) captures the implications of initial CCR [see, e.g., Bonanno and Battigalli (1999)] and amounts to iterated deletion of strategies that are not sequentially best responses, but at each stage of the iteration, the strategies at the previous stage are used to impose restrictions on admissible beliefs of the players *only at the beginning of the game*.[2] This is in contrast with Pearce's EFR procedure which, at each stage, uses the strategies surviving the previous stage of the procedure to impose restrictions on admissible beliefs of the players at all information sets reachable by these strategies.

However, the DF procedure and more generally weak EFR are too permissive. This is not surprising given that CCR is assumed *only* at the beginning of a game. It is therefore interesting to investigate at which additional information sets CCR can be maintained "consistently". To illustrate the problem at hand, consider an example due to Reny (1993).

[1]CBR or CCR [see, e.g., Ben-Porath(1997)] means that every player is rational, everyone assigns probability 1 to the event that everyone else is rational, everyone assigns probability 1 to the event that everyone else assigns probability 1 to the event that everyone is rational, and so forth. In strategic games, CCR is equivalent to common knowledge of rationality.

[2]This is equivalent to deletion of strictly dominated strategies at each information against beliefs reaching this information set or "conditionally dominated" strategies at each information set (Shimoji and Watson, 1998). See also Battigalli and Bonanno (1999, Lemma 4.9).



	$\ell\ell'$	$\ell r'$	$r\ell'$	rr'
LL'	3,3	3,3	0,1	0,1
LR'	1,0	1,0	0,1	0,1
RL''	0,1	1,0	0,1	1,0
RR''	0,1	2,2	0,1	2,2

Figure 1[3]

This game is used in Ben-Porath (1997) to exemplify the difficulty in maintaining initial CCR at additional vertices. Strategies consistent with initial CCR are those surviving the DF procedure: Both LR' and RL'' are weakly dominated strategies for player 1 and are removed in the first round; in the second round, $r\ell'$ being a strictly dominated strategy for player 2, is removed. The resulting strategies are as follows.

	$\ell\ell'$	$\ell r'$	rr'
LL'	3,3	3,3	0,1
RR''	0,1	2,2	2,2

Can we maintain CCR at some additional vertices?

> "It is interesting to examine the possibility of CCR (or CKR) at some subset of the vertices. $\cdots$ It turns out that is not clear what is the set of vertices where CCR would be a 'good' assumption. $\cdots$ insisting on CCR at both x_1 and x_2 is problematic." [Ben-Porath (1997, p. 44)]
>
> "... This is not because one of $[x_1]$ or $[x_2]$ is inconsistent with CBR, but because *together* they are inconsistent with CBR".[Reny (1993, p.268)]

[3]For conciseness, reduced strategic form is used in every example. This is innocuous since sequential rationality here is defined for "plan of actions" (see Section 2).

Indeed, if initial CCR were imposed at both x_1 and x_2, there would be a contradiction because x_2 could no longer be reached; using Reny's (1993) terminology, there cannot be a "jointly rational belief system" for x_0, x_1, and x_2. Recently, Battigalli and Siniscalchi (1999) provided an epistemic characterization of the strategies consistent with CCR at an exogenously given collection of information sets H^*; these strategies are precisely those surviving a procedure of iterated deletion of non-sequential best responses, but at each step of the iteration, only beliefs at information sets in H^* are restricted. In the case of Figure 1, the set of strategies consistent with CCR given $\{x_0, x_1, x_2\}$ is empty.

Our objective is to identify those "interesting" information sets for an extensive form game where initial CCR (i.e. CCR assumed at the beginning of the game) can be maintained "consistently". Using Battigalli and Siniscalchi's (1999) result, we can identify Σ_{H^*}, the set of strategy profiles consistent with CCR at a given collection of information sets H^*. If Σ_{H^*} is nonempty, then H^* is "internally consistent" in that there is no internal contradiction or inconsistency in assuming CCR at the information sets in H^*. However, H^* may lack "external consistency" because H^* may be a strict subset[4] of $H(\Sigma_{H^*})$, the collection of information sets reachable by the strategy profiles in Σ_{H^*}, in which case, it is not justified why there is no CCR at the information sets in $H(\Sigma_{H^*}) \setminus H^*$. Consider again Figure 1. $\Sigma_{\{x_0,x_2\}} = \{LL', RR''\} \times \{\ell r', rr'\}$; hence there is no internal inconsistency in maintaining initial CCR at x_2. But , $\Sigma_{\{x_0,x_2\}}$ reaches x_1 as well and yet CCR is not maintained at x_1. Thus, $\{x_0, x_2\}$ lacks external consistency. At the same time, there cannot be CCR at x_0, x_1, and x_2. However, $\Sigma_{\{x_0,x_1\}} = \{LL'\} \times \{\ell r', rr'\}$, which reaches x_1 but does not reach x_2. This points to the fact that $\{x_0, x_1\}$ has the external consistency property that $\{x_0, x_2\}$ lacks and also is exactly the reason why $\Sigma_{\{x_0,x_1,x_2\}}$ is empty.

Thus, external consistency of H^* requires us to justify why there is no CCR at the information sets in $H(\Sigma_{H^*}) \setminus H^*$. One natural (and also the strongest) external consistency condition is that H^* contains all information sets reachable by strategy profiles in Σ_{H^*}, i.e., $H^* \supset H(\Sigma_{H^*})$. Obviously, if H^* is both internally and externally consistent, then $H^* = H(\Sigma_{H^*})$, i.e., H^* has the *fixed point* property. Therefore, CCR is abandoned if and only if players are "surprised" in that an information set precluded by Σ_{H^*} is reached.[5] Such an external consistency condition is weaker than the "best

[4]Note that if there is CCR at information sets in H^*, then $H^* \subset H(\Sigma_{H^*})$; that is, Σ_{H^*} necessarily reaches information sets in H^* [see Battigalli and Siniscalchi (1999) and Section 2 in this paper].

[5]This is also similar to one of Basu's (1990) restrictions on solution concepts in extensive form

rationalization principle" in Battigalli and Siniscalchi's (1997) characterization of EFR, which maintains highest possible order of belief in rationality at every information set. Thus, the "best rationalization principle" entails a particular restriction on players' beliefs when they are surprised.

If $H^* = H(\Sigma_{H^*})$, we shall call the pair (H^*, Σ_{H^*}) a *consistent solution.* We show that a game admits at most one consistent solution. Indeed, if a game admits a fixed point H^* such that $H(\Sigma_{H^*}) = H^*$, then it is the unique fixed point. We label a game *consistent* if it admits a consistent solution. The game in Figure 1 admits a fixed point $H^* = \{x_0, x_1, x_3, x_5\}$ which comprises precisely all the vertices on the unique backward induction path and $\Sigma_{H^*} = \{LL'\} \times \{\ell\ell', \ell r'\}$. Thus, consistent application of CCR yields the backward induction outcome in this game because CCR on the backward induction path "solves" the game. Note that this game is not "belief-consistent" in the sense of Reny (1993) because there cannot be CCR at all "relevant" vertices[6] which include both x_1 and x_2. Thus, Reny's definition is too demanding as he essentially requires external consistency to account for all "relevant" vertices. In fact, if a game is belief-consistent *a la* Reny, then it is also consistent according to our definition. It is straight forward to define a notion of consistent rationalizability for a consistent game since its unique consistent solution has the property that there is CCR at all reachable information sets.

However, CCR at all reachable information sets is not always possible. In this case, the following weaker external consistency condition provides a natural alternative: A collection of information sets H^* is deemed weakly externally consistent, if any $h \notin H^*$ would be a "cause" of the internal consistency of $H^* \cup \{h\}$ in that when identifying the strategies consistent with CCR given $H^* \cup \{h\}$, h fails to be reached after some step of the iterative procedure. If a game is consistent, this weaker external consistency condition is equivalent to its stronger counterpart that stipulates "CCR at all reachable information sets". If H^* has both the internal consistency and the weak external consistency properties, then we shall call the pair (H^*, Σ_{H^*}) a *weak consistent solution.* While a game admits at most one consistent solution, it can have multiple weakly consistent solutions. A notion of (weakly) consistent rationalizability needs to accommodate such a possibility.

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games: Agents start off with certainty of rationality of their opponents and remain certain until actions that are inconsistent with any rational strategy are observed. [see also Dekel and Gul (1997)].

[6]See Section 2 for the definition of "relevant vertices".

The rest of the paper is organized as follows. Section 2 formalizes the notions discussed thus far; properties of our notions are also studied (e.g., connection to "common knowledge of iterated weak dominance"). Section 3 illustrates these notions through more examples. The last section discusses some related literature and epistemic foundations of our notions.

2 Consistent Rationalizability

We consider a class of *finite* extensive form games with perfect recall and no chance moves and use the following notations as in Battigalli and Siniscalchi (1997).

- N is the set of players.
- H_i is the set of information sets of player $i \in N$ and $H = \cup_{i \in N} H_i$. $h^0 \in H$ denotes the beginning of the game. H is partially ordered by $\leq$: for $h, h' \in H$, $h \leq h'$ if and only if *some* vertex in h appears (weakly) before *some* vertex in h'.
- S_i is the set of pure strategies of player $i \in N$. As usual, $S = \prod_{i \in N} S_i$. $S_{-i} = \prod_{j \neq i} S_j$.
- $u_i : S \to \Re^1$ is the normal form payoff function of player $i \in N$.
- $S(h)$ is the set of strategy profiles reaching $h \in H$ and $S_i(h)$ is the set of strategies of player $i \in N$ that reach h; that is,
$$S_i(h) = \{s_i \in S_i \mid (s_i, s_{-i}) \in S(h) \text{ for some } s_{-i} \in S_{-i}\}.$$
- $H_i(s_i) = \{h \in H_i \mid s_i \in S_i(h)\}$ is the set of information sets of player $i \in N$ that are reachable by his strategy $s_i \in S_i$. $H(s)$ is the set of information sets reached by $s \in S$ and for $T \subset S$, $H(T) = \cup_{s \in T} H(s)$.
- $\mathcal{B}_i(H_i) = \{B \mid B = S(h) \text{ for some } h \in H_i\}$.
- For $i \in N$, a conditional probability system (or CPS) on (S, H_i) is a mapping
$$\mu(\cdot \mid \cdot) : 2^S \times \mathcal{B}_i(H_i) \to [0,1]$$
such that

– for all $B \in \mathcal{B}_i(H_i)$, $\mu(\cdot \mid B)$ is a probability measure on S such that $\mu(B \mid B) = 1$;

– (Bayes' rule) for all $A \subset S, B, C \in \mathcal{B}_i(H_i)$, if $A \subset B \subset C$, then $\mu(A \mid C) = \mu(A \mid B)\mu(B \mid C)$

and $\Delta^{H_i}(S)$ is the set of possible CPS's.

As most of the literature, we use the following definition of "weak sequential rationality", which is a notion of sequential rationality for "plans of action" [see, e.g., Battigalli and Bonanno (1999)].

Definition 1 *Weak sequential rationality: Let $T \subset S$ and $\mu_i \in \Delta^{H_i}(S)$. $s_i \in T_i$ is a weak sequential best response in T to μ, written $s_i \in r_i(\mu_{-i}, T)$, if for all $s_i' \in S_i(h) \cap T_i$,*

- $\mu(\{s_i\} \times S_{-i}(h) \mid S(h)) = 1.$
- $\sum_{s_{-i} \in S_{-i}} [u_i(s_i, s_{-i}) - u_i(s_i', s_{-i})] \, \mu_i(\{(s_i, s_{-i})\} \mid S(h)) \geq 0.$

Pearce's EFR can be defined by the following procedure of iterated deletion of non-sequential best responses [see, e.g., Battigalli and Siniscalchi (1997)].

Definition 2 *Let $\Sigma_P^0 = S$. Assume that $\Sigma_P^0, \cdots, \Sigma_P^k$ have been defined. Then $s \in \Sigma_P^{k+1}$ if and only if $s \in \Sigma_P^k$ and for all $i \in N$, there exists a CPS $\mu_i \in \Delta^{H_i}(S)$ such that*

- *For every $h \in H_i(s_i)$, if $S(h) \cap \Sigma_P^k \neq \emptyset$, then $\mu_i(\Sigma_P^k \mid S(h)) = 1$.*
- *$s_i \in r_i(\mu_i, \Sigma_P^k)$.*

The set of EFR strategy profiles is given by $\Sigma_P = \bigcap_{k>0} \Sigma_P^k$.

Battigalli (1997) shows that replacing $r_i(\mu_i, \Sigma_P^k)$ by $r_i(\mu_i, S)$ yields an equivalent definition. Note that at every step of the procedure, the strategies surviving the previous step are used to restrict beliefs at all information sets reachable by these strategies and given these beliefs, strategies that are not sequential best responses are removed. Note that it is well possible that for some $h \in H, S(h) \cap \Sigma_P^k \neq \emptyset$ for some k but $S(h) \cap \Sigma_P = \emptyset$. The following more general form of iterative procedure allows us to vary the set of information sets at which beliefs are restricted.

Definition 3 *Let $H^* \subset H$ and $\Sigma^0 = S$. Assume that $\Sigma^0_{H^*}, \cdots, \Sigma^k_{H^*}$ have been defined. Then $s \in \Sigma^{k+1}_{H^*}$ if and only if $s \in \Sigma^k_{H^*}$ and for all $i \in N$, there exists a CPS $\mu_i \in \Delta^{H_i}(S)$ such that*

- *For every $h \in H(s_i) \cap H^*$, if $S(h) \cap \Sigma^k_{H^*} \neq \emptyset$, then $\mu_i(\Sigma^k_{H^*} \mid S(h)) = 1$.*
- *$s_i \in r_i(\mu_i, \Sigma^k_{H^*})$.*

$$\Sigma_{H^*} = \bigcap_{k>0} \Sigma^k_{H^*}.$$

Again, $r_i(\mu_i, \Sigma^k_{H^*})$ can be replaced by $r_i(\mu_i, S)$. Using Shimoji and Watson's (1998) result, Definition 3 is equivalent to[7]

Definition 3$'$ *Let $H^* \subset H$ and $\Sigma^0_{H^*} = S$. Then $s \in \Sigma^1_{H^*}$ if and only if $s \in S$ and for all $i \in N$, there does not exist $h \in H$ such that s_i is strictly dominated in $S(h)$. Assume that $\Sigma^1_{H^*}, \cdots, \Sigma^k_{H^*}$ have been defined. Then $s \in \Sigma^{k+1}_{H^*}$ if and only if $s \in \Sigma^k_{H^*}$ and for all $i \in N$, there does not exist $h \in H^*$ such that s_i is strictly dominated in $\Sigma^k_{H^*} \cap S(h)$.*

$$\Sigma_{H^*} = \bigcap_{k>0} \Sigma^k_{H^*}.$$

In Definition 3, we obtain Pearce's EFR by setting $H^* = H$ and weak EFR [see Bonanno and Battigalli (1999)] by setting $H^* = \{h^0\}$. For generic games of perfect information, $\Sigma_{\{h^0\}}$ coincides with strategies surviving the DF procedure. The procedure in Definition 3 takes after that of Battigalli and Siniscalchi (1999) which identifies the set of strategy profiles consistent with CCR given H^*; the only difference between our procedure and theirs lies in the additional constraint $S(h) \cap \Sigma^k_{H^*} \neq \emptyset$ in part (1) of Definition 3. The set of strategy profiles consistent with CCR given H^* (identified by Battigalli and Siniscalchi's procedure) is nonempty and coincides with Σ_{H^*} if and only if $\Sigma_{H^*} \cap S(h) \neq \emptyset$ for all $h \in H^*$, i.e., $H^* \subset H(\Sigma_{H^*})$. We shall say that H^* is *internally consistent* if $H^* \subset H(\Sigma_{H^*})$, i.e., there is no internal contradiction in assuming CCR at information sets in H^*.

[7] Let $T \subset S$. For $i \in N$, s_i is strictly dominated in T if there exists a mixed strategy $\lambda_i \in \Delta(T_i)$ such that $u_i(\lambda_i, s_{-i}) > u_i(s_i, s_{-i})$ for all $s_{-i} \in T_{-i}$. s_i is "conditionally dominated at h" in the sense of Shimoji and Watson (1998) if it is strictly dominated in $S(h)$.

The reason we introduce the additional constraint in Definition 3 is twofold: First, we can compare Σ_{H^*} directly with Pearce's EFR strategies. Secondly and more importantly, when the set of strategy profiles consistent with CCR given H^* is empty, we can pinpoint the source of internal inconsistency of H^* by seeking out the information sets in $H^* \setminus H(\Sigma_{H^*})$. Indeed, if $h \in H^* \setminus H(\Sigma_{H^*})$, then there exists an integer $\bar{k}$ such that $h \in H(\Sigma_{H^*}^{\bar{k}}) \setminus H(\Sigma_{H^*}^{\bar{k}+1})$. Thus, internal inconsistency of H^* results from imposition of CCR at information sets in $H(\Sigma_{H^*}) \setminus H^*$ rather than in $H^* \cap H(\Sigma_{H^*})$. This allows us to formalize the following external consistency condition.

Weak external consistency Let H^* be internally consistent, i.e. $H^* \subset H(\Sigma_{H^*})$. Then H^* has the *weak external consistency* property if $h \in H(\Sigma_{H^*}) \setminus H^*$ implies that $h \notin H(\Sigma_{H^* \cup \{h\}})$.

That is, H^* has the weak external consistency property if whenever $h \in H(\Sigma_{H^*}) \setminus H^*$, $H^* \cup \{h\}$ is internally inconsistent and the internal inconsistency stems from imposing CCR at h. Note that such an external consistency condition is stronger than CCR at a maximal collection of information sets. To see this, let $H^* = \{x_0, x_2, x_4, x_6\}$ in Figure 1. It is easy to show that H^* is a maximal collection of information sets where there can be CCR. Σ_{H^*}, however, reaches x_1. In fact, x_1 is also reachable by $\Sigma_{H^* \cup \{x_1\}}$. Thus, H^* does not have the weak external consistency property.

Weak external consistency formalizes the intuitive idea of "CCR as much as possible" and is implied by the following stronger external consistency condition of "CCR at all reachable information sets".

External consistency $H^* \subset H$ is externally consistent if $H(\Sigma_{H^*}) \setminus H^* = \emptyset$ or $H^* = H(\Sigma_{H^*})$.

Note that internal consistency of H^* is also embedded in $H^* = H(\Sigma_{H^*})$. We shall call the pair (H^*, Σ_{H^*}) a *consistent solution* if $H^* = H(\Sigma_{H^*})$. The following proposition states that a game admits at most one consistent solution and it is outcome-equivalent to Pearce's EFR.

Proposition 1 *There exist at most one $H^* \subset H$ such that $H^* = H(\Sigma_{H^*})$. Moreover, if $H^* = H(\Sigma_{H^*})$, then $H^* = H(\Sigma_P)$.*

Proof. We first show that if $H^* \subset H'$, then $\Sigma_{H^*} \supset \Sigma_{H'}$. It is sufficient to show that if $\Sigma_{H^*}^k \supset \Sigma_{H'}^k$, then $\Sigma_{H^*}^{k+1} \supset \Sigma_{H'}^{k+1}$. Indeed, $s \in \Sigma_{H'}^{k+1} \subset \Sigma_{H'}^k \subset \Sigma_{H^*}^k$ implies that for all $i \in N$, there exists a CPS $\mu_i \in \Delta^{H_i}(S)$ such that

(a) For every $h \in H(s_i) \cap H'$, if $S(h) \cap \Sigma^k_{H'} \neq \emptyset$, then $\mu_i(\Sigma^k_{H'} \mid S(h)) = 1$.

(b) $s_i \in r_i(\mu_i, \Sigma^k_{H'})$.

Since $H^* \subset H'$ and $\Sigma^k_{H^*} \supset \Sigma^k_{H'}$, (a) implies

(a′) For every $h \in H(s_i) \cap H^*$, if $S(h) \cap \Sigma^k_{H^*} \neq \emptyset$, then $\mu_i(\Sigma^k_{H^*} \mid S(h)) = 1$.

Since $r_i(\mu_i, \Sigma^k_{H'}) = r_i(\mu_i, S)$, we have $s \in \Sigma^{k+1}_{H^*}$ and hence $\Sigma^{k+1}_{H'} \subset \Sigma^{k+1}_{H^*}$.

We proceed to show that if H^* is a fixed point and $H' \supset H^*$, then $H(\Sigma_{H'}) = H^*$. In view of the previous arguments, we have $\Sigma_{H^*} \supset \Sigma_{H'}$ and hence $H^* = H(\Sigma_{H^*}) \supset H(\Sigma_{H'})$. We need to show that $H^* \subset H(\Sigma_{H'})$. To this end, we shall show by induction that if $s_i \in \Sigma_{H^*,i}$, and $t_i \in \Sigma_{H',i}$, then $s'_i \in \Sigma_{H',i}$ where $s'_i(h) = s_i(h)$ for all $h \in H^* \cap H_i$ and $s'_i(h) = t_i(h)$ otherwise. Assume that if $s_i \in \Sigma_{H^*,i}$, and $t_i \in \Sigma^k_{H',i}$, then $s'_i \in \Sigma^k_{H',i}$ where $s'_i(h) = s_i(h)$ for all $h \in H^* \cap H_i$ and $s'_i(h) = t_i(h)$ otherwise. Using Definition $3'$, it is easy to see that for every $h \in H^*$, s_i is not strictly dominated in $\Sigma^k_{H'} \cap S(h)$. Thus, if $t'_i \in \Sigma^{k+1}_{H',i}$, then $s''_i \in \Sigma^{k+1}_{H',i}$ where $s''_i(h) = s_i(h)$ for all $h \in H^* \cap H_i$ and $s''_i(h) = t'_i(h)$ otherwise.

To complete the proof of the first statement, let $H^\star$ be another fixed point. Then $H(\Sigma_{H^* \cup H^\star}) = H^*$ and $H(\Sigma_{H^* \cup H^\star}) = H^\star$; hence $H^* = H^\star$.

The second statement follows from $\Sigma_H = \Sigma_P$. ■

Call a game *consistent* if it admits a (unique) consistent solution. The following corollary is immediate and it states that a generic perfect information game is consistent if and only if CCR along the backward induction path "solves" the game.

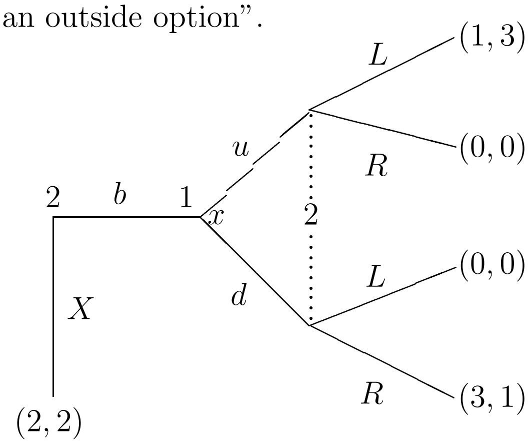
Corollary 1 *A finite generic extensive form game of perfect information is consistent if and only if $H(\Sigma_{H(s^*)}) = H(s^*)$, where s^* is the unique subgame perfect equilibrium.*

Proof. The "if" part is true by definition. To show the "only if" part, recall that $H^* = H(\Sigma_{H^*})$, then $H(\Sigma_{H^*}) = H(\Sigma_P)$. For a generic game of perfect information, $H(\Sigma_P) = H(s^*)$ [see, e.g., Theorem 4 in Battigalli (1997)]. ■

It is straightforward to define a notion of "consistent rationalizability" for consistent games, since a consistent game has a unique consistent solution and there is CCR at all reachable information sets.

Definition 4 *For a consistent game whose consistent solution is (H^*, Σ_{H^*}), the set of consistently rationalizable (CR) strategy profiles at $h \in H^*$ is given by $\Sigma_{H^*} \cap S(h)$.*

No prediction is made once some $h \in H \setminus H^*$ is reached. The game in Figure 1 is a consistent game and $\{LL'\} \times \{\ell\ell', \ell r'\}$ is the set of CR strategy profiles. Indeed, let $H^* = H(\{LL'\} \times \{\ell\ell', \ell r'\})$; then $\Sigma_{H^*} = \{LL'\} \times \{\ell\ell', \ell r'\}$ and $H(\Sigma_{H^*}) = H^*$. To further demonstrate consistent rationalizability, consider the following game of "battle of the sexes with an outside option".



	X	L	R
u	2,2	1,3	0,0
d	2,2	0,0	3,1

Figure 2

Let $H^* = H(\{(u, L)\})$. At the first step of the reduction procedure, R, not being sequentially rational, is deleted. Consequently, at the second step player 1's belief at his information set is restricted to reflect the removal of R. Then player 1 will remove d that is not a sequentially best response. At the third step, player 2's belief at the origin is restricted and X is subsequently deleted. (u, L) is thus the unique strategy profile that survives the procedure; that is, $\Sigma_{H^*} = \{(u, L)\}$. Obviously, $H(\Sigma_{H^*}) = H^*$. Like EFR, consistent rationalizability captures *forward induction*: 1 believes 2 is rational and hence will not play R; then player 1 will not play d since at his information set he believes that R is removed and knows X has not been played.

Within the class of generic games of perfect information, Reny (1993) defined "belief-consistent games". A vertex $h \in H$ is *relevant* if (1) h is consistent with rationality: $h \in H(\Sigma^1_{\{h^0\}})$ and (2) if $h \in H_i$, then player i does not have a dominant continuation strategy at h.[8] A game is *belief-consistent* if there is CCR at all

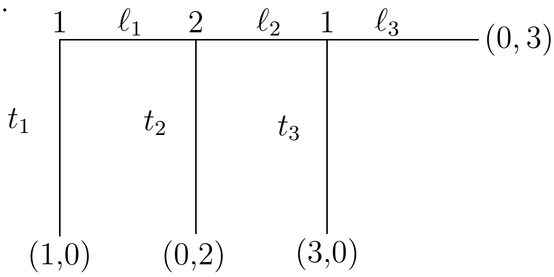
[8] Player i has a dominant continuation strategy at h if he has a strategy $s_i \in S_i(h)$ such that, for all $s'_i \in S_i(h)$ such that $s'_i(h) \neq s_i(h)$, $U_i(s_i, s_{-i}) > U_i(s'_i, s_{-i})$ for all $s_{-i} \in S_{-i}(h)$.

relevant vertices. Belief consistency is a strong requirement because a game typically has "too many" relevant vertices. Consequently, the class of belief-consistent games is very small. In fact, Reny shows that a game is belief-consistent if and only if the set of relevant nodes comprises precisely those on the unique BI path. Figure 1 is not a belief-consistent game according to Reny (1993): x_0, x_1 and x_2 are all relevant vertices, yet there cannot be CCR at all these vertices. This game is, however, a consistent game according to our notion: CCR along the BI path is sufficient to identify the BI path and thus solve the game; hence, among the set of relevant vertices of in Figure 1, only those on the BI path are "interesting". Thus, Reny's definition of belief consistency is more demanding than needed. The following lemma is immediate.

Lemma 1 *If a generic game of perfect information is belief-consistent (a la Reny), then it is a consistent game.*

Our definition of consistent games also applies to games without perfect information. Indeed, "battle of sexes with an outside option" in Figure 2 is consistent.

CCR, however, at all reachable information sets is not always possible and hence existence of consistent rationalizability is not guaranteed. Consider the following "take it or leave it" (TOL) game.



	t_2	ℓ_2
t_1	1,0	1,0
$\ell_1 t_3$	0,2	3,0
$\ell_1 \ell_3$	0,2	0,3

Figure 3

Initial CCR implies the DF procedure which eliminates only $\ell_1\ell_3$. If CCR is maintained after history (ℓ_1), then player 2, upon, reaching his vertex, believes that $\ell_1\ell_3$ has been deleted and hence will delete ℓ_2. The initial CCR implies that 1 will delete $\ell_1 t_3$, leaving (t_1, t_2) to be the only surviving strategy profile. Obviously, player 2's vertex can no longer be reached. Thus, maintaining CCR after history (ℓ_1) is inconsistent with initial CCR. Consequently, CCR should be abandoned once history (ℓ_1) transpires. Given that $\{\emptyset, (\ell_1)\} \subsetneq \Sigma_{\{h^0\}}$ and $(\ell_1) \notin \Sigma_{\{h^0,(\ell_1)\}}$, consistent rationalizability fails to exist.

Thus for inconsistent games like the one depicted by Figure 3, CCR at all reachable information sets is *not* possible and we need resort to the weaker external consistency defined earlier in this section. In this case, we obtain the following notion of "weakly consistent solution".

Definition 5 *Let $H^* \subset H$. (H^*, Σ_{H^*}) is called a weakly consistent solution if it satisfies the following conditions:*

(1) if $h \in H^*$ and $h' < h$, then $h' \in H^*$;

(2) internal consistency: $H^* \subset H(\Sigma_{H^*})$; and

(3) weak external consistency: $h \in H(\Sigma_{H^*}) \setminus H^*$ implies that $h \notin H(\Sigma_{H^* \cup \{h\}})$.

Obviously, (1) implies that H^* contains the beginning of the game; moreover, by imposing (1), we take the view that once CCR collapses, rationality and CCR offer no guidance when analyzing the rest of the game. This is a natural requirement because we are ascertaining the set of information sets where initial CCR can be maintained consistently.[9] The following result is easy to verify.

Lemma 2 *If (H^*, Σ_{H^*}) is a weakly consistent solution, then $\Sigma_{H^*} \supset \Sigma_P$.*

The inclusion may well be strict as is the case for Figure 3. Given a weakly consistent solution, the "prediction" is $\Sigma_{H^*} \cap S(h)$ for each $h \in H^*$ and $S(h)$ (thus, everything is possible) for each $h \in H \setminus H^*$.

Reny (1993) shows that in Figure 3 once player 1 leaves the first dollar, it is impossible to have CCR and this result can be generalized to any finite TOL game (or

[9]Gul (1996) and Ben-Porath (1997), among others, express a similar view. This is in contrast to Pearce's EFR which entails that players maintain the highest possible order of belief in rationality [see Battigalli and Siniscalchi's (1977)].

centipede game).[10] Thus, it is only possible to maintain initial CCR if player 1 takes the first dollar. Consequently, a finite TOL or centipede game has a unique weakly consistent solution (H^*, Σ_{H^*}) where $H^* = H(s^*)$ and s^* is the subgame perfect equilibrium. In fact, H^* is the unique maximal collection of information set that satisfies (1) and (2) in Definition 5; that is, H^* is the unique maximal set that is internally consistent and contains the beginning of the game.

Remark 1 *In a TOL or centipede game, although initial CCR does not single out BI outcome (recall that strategy profiles consistent with initial CCR are those surviving the DF procedure), initial CCR together with "*ex post *CCR" (i.e., initial CCR is maintained till the end of the play) do imply BI outcome.*[11] *But a consistent solution does not always identify the BI path as the unique outcome even if we insist upon ex post CCR. Figure 1 illustrates this point.*

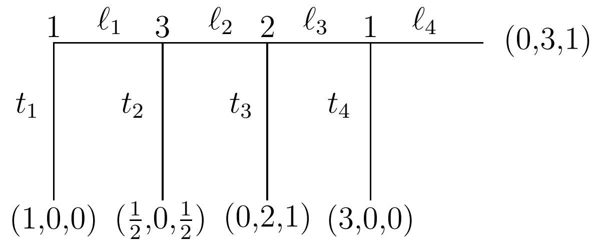


Figure 4

The game in Figure 4 has a unique consistent solution (H^*, Σ_{H^*}) where H^* consists of all the vertices on the paths (t_1) and (ℓ_1, t_2). But the latter path is not a backward induction path.

Each of the examples we have examined so far admits a unique weak consistent solution. It is possible, however, for a game to have multiple consistent solutions [see the appendix for an example). How can we define a notion of rationalizability to accommodate such a multiplicity? Consider two (different) weak consistent solutions (H^*, Σ_{H^*}) and $(H^\star, \Sigma_{H^\star})$. By Definition 5, if $h \in (H^* \cup H^\star) \setminus (H^* \cap H^\star)$, then $h \notin H(\Sigma_{H^* \cup H^\star})$. Thus, players cannot assume CCR at the information sets in $(H^* \cup H^\star) \setminus (H^* \cap H^\star)$ when they are uncertain about weak consistent solutions. The following definition of weakly consistent rationalizability (WCR henceforth) captures this point.

[10] Using Definition 3, we can show that if H^* contains the beginning of the game and player 2's first vertex, then Σ_{H^*} does not reach player 2's vertex.

[11] See Aumann (1998) for a related result.

Definition 6 *Let $\mathcal{W}$ be the set of weak consistent solutions for an extensive form game and let* $\bar{H} = \bigcap_{(H^*, \Sigma_{H^*}) \in \mathcal{W}} H^*$. *The the set of WCR strategy profiles at $h \in \bar{H}$ is given by* $\Sigma_{\bar{H}} \cap S(h)$.

For consistent games, WCR is equivalent to consistent rationalizability (CR henceforth). In some sense, WCR is the strongest notion relying on CCR. It incorporates the intuitive idea of "maintaining CCR as far as possible" but put no restriction on players behavior once CCR fails. Consequently, WCR refines weak EFR without getting into difficulty when dealing with "counter-factuals".

It is interesting to compare the strategies obtained by WCR, EFR, and iterated weak dominance[12]. EFR entails "partial"[13] reduction by some order of elimination of weakly dominated strategies in the strategic form. The EFR strategies, like those surviving iterated weak dominance, may lack "external consistency" with respect to weak dominance in that some eliminated strategies may no longer be weakly dominated given the set of EFR strategies. In contrast, WCR has external consistency property with respect to weak dominance.

Proposition 2 *Let $\bar{H}$ be defined by Definition 6 and $\Sigma_{\bar{H}}$ be the set of WCR strategy profiles. Then for each $i \in N$, if $s_i \in S_i \setminus \Sigma_{\bar{H},i}$, then s_i is weakly dominated in $\Sigma_{\bar{H}}$.*

Proof. Consider Definition $3'$. If $s_i \in \Sigma^1_{\bar{H},i} \setminus \Sigma_{\bar{H},i}$, then s_i is strictly dominated in $\Sigma^k_{\bar{H}} \cap S(h)$ for some $k \geq 1$ and $h \in \bar{H}$. Since $\bar{H} \subset H(\Sigma_{\bar{H}})$, s_i is strictly dominated in $\Sigma_{\bar{H}} \cap S(h)$ and hence weakly dominated in $\Sigma_{\bar{H}}$. If $s_i \in \Sigma^1_{\bar{H},i} \setminus \Sigma^2_{\bar{H},i}$, then s_i is strictly dominated in $S(h)$ at some $h \in H$. If $h \in H(\Sigma_{\bar{H}})$, then s_i is strictly dominated in $\Sigma_{\bar{H}} \cap S(h)$ and hence weakly dominated in $\Sigma_{\bar{H}}$. If $h \notin H(\Sigma_{\bar{H}})$, then there exists some k such that $h \in H(\Sigma^k_{\bar{H}}) \setminus H(\Sigma^{k+1}_{\bar{H}})$. Let $t_i \in \Sigma^k_{\bar{H},i}$; then s_i is strictly dominated by t_i in $\Sigma^k_{\bar{H}} \cap S(h)$. If $t_i \in \Sigma_{\bar{H},i}$, we are done. Otherwise, t_i is strictly dominated in $\Sigma^\ell_{\bar{H}} \cap S(h')$ for some $h' < h$ and $\ell > k$ and hence s_i is strictly dominated in $\Sigma^\ell_{\bar{H}} \cap S(h')$. Continuing in this fashion, we can show that s_i is strictly dominated in $\Sigma_{\bar{H}} \cap S(h^\star)$ for some $h^\star \in \bar{H}$.
■

Note that WCR may lack internal consistency with respect to weak dominance in that it can contain weakly dominated strategies (see, e.g., the TOL game in Figure 3).

[12]Let $T \subset S$. For $i \in N$, s_i is weakly dominated in T if there exists a mixed strategy $\lambda_i \in \Delta(T_i)$ such that $u_i(\lambda_i, s_{-i}) \geq u_i(s_i, s_{-i})$ for all $s_{-i} \in T_{-i}$ with strict inequality for some $s_{-i} \in T_{-i}$.

[13]as opposed to "full" reduction entailed by iterated weak dominance [see Marx and Swinkels (1997)].

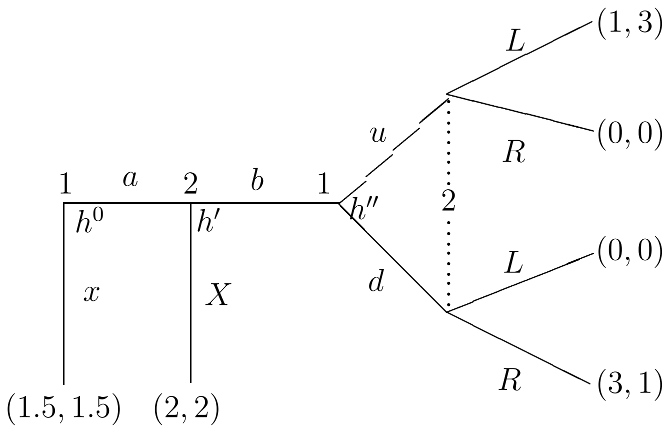
If a generic game of perfect information is consistent, then WCR, which is equivalent to CR, also has internal consistency with respect to weak dominance. Thus, "common knowledge of weak dominance" can be achieved in the sense of Samuelson (1992).

Corollary 2 *Consider a consistent generic game of perfect information and let* (H^*, Σ_{H^*}) *be the unique consistent solution. Then* Σ_{H^*} *is both internally and externally consistent with respect to weak dominance. That is, for each* $i \in N$, $s_i \in \Sigma_{H^*}$ *if and only if* s_i *is not weakly dominated in* Σ_{H^*}.

Proof. External consistency follows directly from Proposition 2. To show internal consistency, note that if $s_i \in \Sigma_{H^*}$, then $s_i \in r_i(\mu_i, \Sigma_{H^*})$ for some $\mu_i \in \Delta^{H_i}(S)$ such that $\mu_i(\Sigma_{H^*} \mid S(h)) = 1$ for every $h \in H_i \cap H^*$ and that $H^* = H(\Sigma_{H^*})$. Using Ben-Porath's (1997, Lemma 1.2), s_i is not weakly dominated in Σ_{H^*}. ■

3 Further Examples

Consider "the battle of sexes with outside options for both players". Such a game has been used to illustrate the tension between forward and backward induction [see, e.g., van Damme (1989) and Dekel and Fudenberg (1990)].

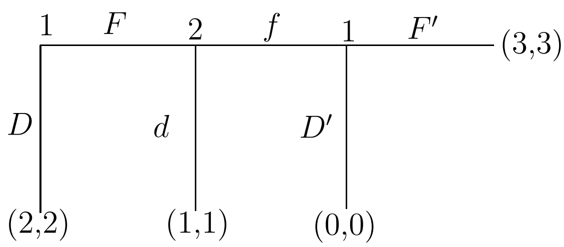


	X	L	R
x	1.5,1.5	1.5,1.5	1.5,1.5
u	2,2	1,3	0,0
d	2,2	0,0	3,1

Figure 5

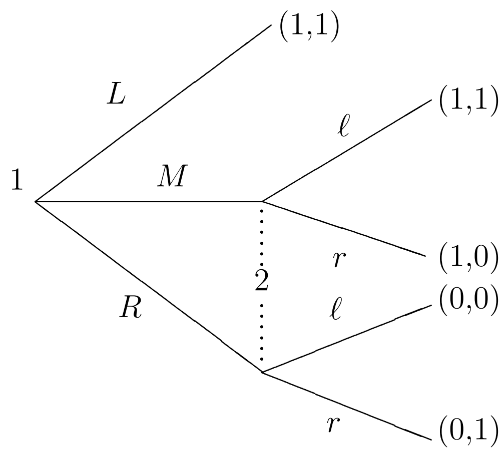
Initial CCR eliminates only R. Thus, $\Sigma_{\{h^0\}} = \{x, u, d\} \times \{X, L\}$. Can we maintain CCR at the first information set of player 2? The answer is affirmative since $\Sigma_{\{h^0, h'\}} = \{x, u, d\} \times \{X, L\}$. We cannot, however, maintain CCR at h''. Indeed, $\Sigma_{\{h^0, h'\}} = \{(x, L)\}$. Evidently, h'' can no longer be reached. The set of WCR strategy profiles is $\{x, u, d\} \times \{X, L\}$. No conflict arises between forward and backward induction because CCR cannot be maintained at h''.

The following example is used by Asheim and Dufwenberg (1998) to illustrate that Ben-Porath's approach or the DF procedure is too permissive. Indeed, the DF procedure eliminates only FD'. However, it is easy to see that $\Sigma_{H^*}(H(\{(F', f)\}) = \{(F', f)\}$. Consistent rationalizability yields a unique strategy profile (F', f) and CCR can be maintained after all histories induced by (F', f).



	d	f
D	2,2	2,2
FD'	1,1	0,0
FF'	1,1	3,3

Figure 6



	ℓ	r
L	1,1	1,1
M	1,1	1,0
R	0,0	0,1

Figure 7

The last example is similar to the one in Pearce (1984, p. 1044) and is used also by Asheim and Dufwenberg (1998). Initial CCR eliminates only R. It is easy to show that initial CCR can be maintained at 2's information set. In fact, this game is consistent. Indeed, let $H^* = H(\{(L, M)\} \times \{\ell\}$; then $\Sigma_{H^*} = \{(L, M)\} \times \{\ell\}$. Therefore, if player 2's information set is reached, he will play ℓ, given his belief that 1 must have chosen M. No strategies can be further eliminated.

4 Concluding Remarks

Pearce's extensive form rationalizability can be criticized on the grounds that it maintains rationality postulate even at information sets that are not reached by rationalizable strategies [see, e.g., Reny (1993), Basu (1988), and Battigalli (1997)]. Weak extensive form rationalizability captures the implications of CCR at *only the beginning* of an extensive form game. Our analysis aims to go one step further than Ben-Porath's (1997) result and weak extensive form rationalizability by identifying the information sets where CCR can be maintained *consistently.* The external consistency conditions introduced in this paper capture the intuitive idea of "maintaining CCR as much as possible". In this aspect, our notion resembles that of Pearce. However, as Battigalli and Siniscalchi's (1997) characterization indicates, Pearce's notion does not only maintain CCR as much as possible but also attributes the highest possible order of belief in rationality when CCR fails; put differently, "players bestow the highest possible degree of strategic sophistication upon their opponents" (Battigalli and Siniscalchi,1997). This implies that players' strategic sophistication differs across information sets and they know it. Our notion takes the view that once CCR fails, it is difficult to make the case that one theory is more plausible than the other [see, e.g., Ben-Porath (1997)]. Nevertheless, our notion also capture some aspects of forward induction like EFR.

By endogenously determining the "interesting information sets", our notion also constitutes an important application of Battigalli and Siniscalchi's (1999) recent contribution, which characterizes the implications of CCR at an exogenously given collection of information sets. Thus, an epistemic characterization of a (weakly) consistent solution can be borrowed directly from them (note that their characterization were obtained for multistage games). Alternatively, to characterize (weak) consistent rationalizability, a weaker notion than Battigalli and Siniscalchi's (1997) "strong belief in rationality" is called for.

The notion proposed here bears certain similarities with that of Asheim and Dufwenberg (1998), who characterize common knowledge of "full admissible rationality". Our notions predict the same outcomes in many of the examples in this paper (e.g., Figures 1 2, 6, and 7). At the heart of their definition of rationality are *caution* (each player takes into account all vectors of opponents' strategies) and *opponents optimization* (a player deems any vector where all opponents choose maximal elements infinitely more likely than any vector not having this property). Thus, their notion does not preclude any information set from being reached, and at an information set that is not consistent with any maximal strategy, nothing is imposed on the likelihood of non-maximal strategies. However, their rationality is stronger than ours because they imposed caution. This enables them to delete d (a weakly dominated strategy) in Figure 5.

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5 Appendix: A Game with Multiple Weakly Consistent Solutions

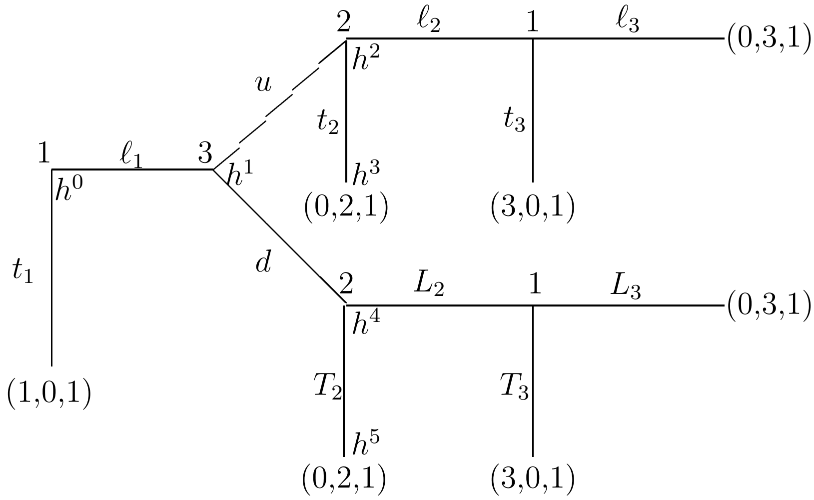

Figure 8

This game has two weakly consistent solutions $(H^{*}, \Sigma_{H^{*}})$ and $(H^{\star}, \Sigma_{H^{\star}})$ such that $H^{*} = \{h^0, h^1, h^2, h^3\}$ and $H^{\star} = \{h^0, h^1, h^4, h^5\}$.

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